Enforcing weak patents: the role of litigation costs, and damage rules in patent disputes*

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Abstract

This paper compares the effects of two rules used by courts of law in assessing awards for damages to patentees (Lost Profit vs Unjust Enrichment). Our model captures situations in which two innovative firms compete on a market for a homogenous good, but only one (the Patentee) is successful in innovating first and filing a patent. The other firm (the Infringer) strategically infringes the patent in order to secure a licensing agreement. In this set up, we show that the Unjust Enrichment rule yields fewer trials conditional on patent enforcement than the Lost Profit rule. However, regarding three main objectives of intellectual property laws (protection of innovator’s profits, incentives to invest in R&D, and social welfare), we find that neither rule is better than the other, generally speaking. Our model also illustrates how the combination of litigation costs, the size of negotiation gains, and the strength of IPRs shapes the incentives to enforce and infringe IPRs, but in a way which is damages specific. Interestingly enough, we find that the Lost Profit rule preserves the incentives to enforce weak patents, which may be deterred under the Unjust Enrichment rule.

Keywords: intellectual property, patent litigation, pretrial negotiations.

JEL classification: 03, L1, L4, D8, K2, K4.

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1 Introduction

In this paper we discuss two inter-related issues in the area of intellectual property rights (IPRs) which have been overlooked in the literature up to now. The first relates to the discretionary power given to courts in most industrialized countries regarding the assessment of damages to be awarded to patent holders. This has long been a matter of debate in the empirical literature (see e.g., Galini, 2002; Lanjow and Lerner, 1998; Lanjow and Schankerman, 2001; Reitzig, Henkel and Heath, 2007; Somaya, 2003) and a motivation to promote reforms of intellectual property laws in order to introduce guidelines for the assessment of damages (see for USA: Opderbeck, 2009). A neglected aspect of this debate is that any rule designed to provide practical solutions to the ex post problems of assessing the compensation to be awarded to the patentee will modify ex ante the incentives of both the infringer and the patentee, and will shape the competition at the market stage. In this paper, we illustrate this point, comparing two well known rules: Lost Profit and Unjust Enrichment.

The second issue concerns the development of licensing agreements and patent settlement practices in businesses sustained by an intensive activity of innovation and high levels of R&D. Such behavior prompted some early interest in the literature because of its anti-competitive effects (see e.g., Balto, 2000; Langenfeld and Li, 2003; Shapiro, 2003) while more recently interest has shifted to the welfare effects of alternative design of licensing contracts (see e.g., Farrell and Shapiro, 2008) under specific market structures. However, the literature has largely ignored the point that licensing agreements and patent settlements may be part of a litigation strategy in patent infringement cases. In this paper, we develop this argument, showing how court decisions about the choice of a damages rule affect parties’ litigation and market strategies alike. More specifically, we consider the case where the goods supplied by competing firms are perfect substitutes and are produced by patentable technologies; all firms competing on the market are innovators, but one of them succeeds in innovating first and patents its discovery. A typical example is the medicine and drugs industry with branded versus generic drug manufacturers.

Note that in this paper, we ignore the issue of what in the courts’ enforcement of IP laws explains their decision whether to uphold an IPR, neither do we discuss whether exclusive IPRs should be preferred to other weaker forms of protection allowing some degree of imitation. Our purpose here is rather to focus on the various consequences of courts’ decisions in terms of awards, working on the assumption that IPRs have ex ante an uncertain value. Since alternative damages rules define different allocations of industry profits between the patentee and the potential infringers, we analyze their impact on the patentee’s incentives to enforce its rights, on the incentives for
the infringer to strategically infringe the patent and then secure a licensing agreement; finally, we discuss the welfare effects of the different rules. In this way, we introduce a useful distinction into the definition of the value of a patent. A first determinant of a patent’s value is given by the likelihood that it will be upheld (or not) in the event of a trial; from this point of view, we assume here that a court invalidates a patent with an exogenous probability. Another important determinant of a patent’s value is related to the maintaining of the patentee’s profits thanks to the litigation system; this issue forms the crux of the paper.

In section 2, we explain our work in the perspective of recent studies and highlight our main results. Section 3 introduces the basic framework with non drastic innovations and private information about the patentee’s legal costs, discusses the properties of the equilibria obtained when courts award Lost Profits or enforce the Unjust Enrichment rule. Section 4 highlights the performances of the Lost Profit and Unjust Enrichment rules regarding the impact on courts’ activity, the safeguarding of firms’ profits, and social welfare. Section 5 concludes.

2 Motivations and highlights

Earlier literature on innovation\(^1\) took it for granted that the entitlement of rights enshrined in existing IP laws affords perfect protection to innovators and as a consequence IPRs provide efficient incentives to innovators and promote R&D investments. However, plentiful empirical evidence shows that the existing IP rules are not "self-sufficient" and that the courts play an active role in enforcing IPRs. Lanjouw and Lerner (1998) and Bessen and Meurer (2005) first reported the dramatic increase in the number of patent litigation cases over the two last decades. Based on all litigated patent cases, Allison and Lemley (1998) also reported a figure close to 50% for the plaintiffs’ rate of success, associated with high rates of reversal of judgments on appeal. Moreover, evidence suggests that many patentees forgo suing infringers because of the duration and cost of litigation in patent cases (Gallini, 2002; Lanjouw and Schankerman, 2001; Pagano and Rossi, 2009). In a sense, the evidence suggests that the effective value of existing IPRs is probabilistic, and in practice there is a risk that existing IPRs will be found invalid in the event of trial.

The more recent literature recognizes that the effective value of IPRs cannot be fully determined ex ante. On the contrary, the validity of IPRs is the outcome of the litigation process and is thus contingent on the intervention of enforcing institutions, whether patent and trademark offices or the courts of law.\(^2\) In this perspective, two of the main issues are whether strategic infringement can

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\(^1\) For a classical survey, see Reinganum (1989).

\(^2\) Numerous papers have pointed out the role of Patent and Trademark Offices in most industrial countries (see
be deterred and what the consequences are for courts’ workloads. Several papers have dealt with an explicit analysis of the patent litigation process and have found that the threat of a trial fails to deter the infringement of a probabilistic patent. This holds under alternative assumptions about the strategic interactions between innovative firms (i.e. quantity vs price competition, or vertical relationships) or the legal environment (i.e. use of preliminary injunctions, role of alternative damage rules, limits of antitrust and competition law), but also for drastic innovations (the cost advantage to the innovative firm enables it to potentially monopolize a market to the extent that no infringement occurs; Choi, 2009) and non drastic ones (the innovators’ market power increases but not in such a way that competing firms will exit; Anton and Yao, 2006).

The fact that IP laws and court intervention grant only imperfect protection to patentees potentially has major consequences for social welfare and the incentives to innovate. Existing works on this point find that the effects of strategic infringement are channeled through the impacts on production costs and/or product differentiation at the market stage only (Anton and Yao, 2003, 2006; Choi, 2009). This means that, surprisingly, the usual determinants of behavior at the litigation stage (Spier, 2007) seem to have no effect on social welfare and incentives to innovate. This contrasts with what is suggested in the empirical literature (Gallini, 2002; Lanjouw and Schankerman, 2001; Pagano and Rossi, 2009) and the ongoing debate about the increasing social costs of IP laws due to the explosion in patent litigation (Lanjouw and Lerner (1998) and Bessen and Meurer (2005)).

Keeping this in mind, a first innovative feature of this paper is that it addresses two restrictive assumptions usually adopted in this literature and which partly explain the zero deterrence result. First, the literature neglects the existence and strategic role of the legal costs borne by both the patentee and the infringer at trial. They are usually seen as unavoidable fixed costs incurred by litigants and are assumed to be equal to zero for simplicity. Second, this literature assumes that whenever an infringement is detected, a trial occurs for sure, although the verdict is uncertain. Thus it ignores the point that both litigants may find a settlement preferable to a trial. On the one hand, when the court finds in favor of the plaintiff with a probability of less than one

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4 Two noticeable exceptions are Meurer (2000) and Ottoz and Cugno (2012), who analyze the influence of fee shifting on patent litigations.

5 The exception is Crampes and Langinier (2002) but they consider Nash negotiations; thus, the probability of trial is trivially either 1 or 0.
(IPRs are probabilistic), going to trial appears to the patentee to be a risky strategy; on the other hand, settling out of court allows both parties to save some costs and avoid potential additional losses due to the time required to obtain a judgment. In our paper, then, we combine a model of settlement litigation in the Law & Economics tradition (see the surveys by Spier, 2007; Daughety and Reinganum, 2012) with a model of oligopolistic competition. In this setup, we analyze how the choice between two damages rules, Lost Profit versus Unjust Enrichment, affects both social welfare and incentives to innovate.6

The second original feature of our paper is that we consider a set of strategies for firms that is richer than is usually assumed in the more recent literature. Here several decisions made by the patentee or the infringer are endogenous: the decisions to infringe and to enforce a patent, the decision to settle or litigate the case, as well as the decision to produce the good in the first place. This sheds some new light on the incentives of potential infringers and nuances the zero deterrence result; but it also raises the issue of patentees’ incentives to enforce their rights. More specifically, we will show that considering a complete litigation game after infringement with an explicit pretrial negotiation stage is not anecdotal but is of major importance for comparing damages rules performances.

Given that the medicine and drugs industry is a typical laboratory for such analysis, we focus on the case of a non drastic innovation and assume Cournot competition between the potential infringer and the patent holder. For ease of exposition, we also assume linear demand and constant marginal production costs for both firms. Returning to the two main issues developed at the beginning of the introduction, we show that a court’s decision to use one rule rather than the other for assessing the damages to be awarded to the patentee entails different selection effects at the pretrial stage, thus inducing a different probability of trial/settlement. Although the argument is routine in the Law & Economics literature on litigation, its main implication has been ignored in existing analysis of patent disputes: basically, the court’s decision (in terms of damages awarded at trial) will serve as an anchor for both parties during the bargaining process. Thus, the value of the patent in terms of the licensing price negotiated in the shadow of the court reflects its value at the threat point where a trial occurs and the judge intervenes. This in turn may provide different incentives to infringe and to enforce a patent.

As we will see, as a major consequence, comparing profits or welfare associated with each rule may prove difficult or even meaningless. The reason is that one parameter of the model, the strength of the patent, is central to our analysis: depending on the damages rule, we find that

6These two alternative rules are currently used by courts both in the U.S. and in Europe; see Choi (2009) and Elkin-Koren and Salzberger (2000) for a discussion of the doctrines.
different equilibria may arise and are supported by different values of "patent strength". More specifically, we find that only weak patents are potentially enforceable under the Lost Profit rule, in the sense that only patents weak enough will be associated with positive harm in the event of infringement (see also Anton and Yao, 2006). In contrast, under the Unjust Enrichment rule, the patentee may waive enforcing its patent when it is weak enough, despite the existence of actual harm. We also show that the Lost Profit rule yields a (conditional) probability of trial which is always at least as high as with the Unjust Enrichment rule. Regarding the objectives of patentee’s protection, R&D incentives, and social welfare maximization, we find that generally speaking, neither rule is better than the other. When the patent is weak enough, the patentee’s profit is higher under the Lost Profit rule. When the patent is strong enough, social welfare is higher (lower) under the Lost Profit (Unjust Enrichment) rule when the patentee is more (respectively less) efficient than the infringer.

We now turn to the formal analysis and explicit resolution of our model.

3 A model for non drastic innovation

Our simple game-based framework combines a basic model of imperfect competition (Cournot duopoly) with a model of strategic pretrial negotiations (under asymmetric information). In the rest of the paper, we use the terms "patentee" and "infringer", "innovator" and "imitator" only for ease of exposition. We capture competitive contexts where all firms/competitors are innovators and invest in R&D. But (for reasons that need not be modeled here) there is a single first innovator, i.e. the efforts of one firm enable it to succeed first and then to patent its discovery. The other firm enters the market and may strategically infringe the patent.

In order to simplify the analysis of the pretrial negotiation process, we consider the case of unilateral asymmetric information, assuming that the patentee’s litigation cost is private information. Such an assumption may be seen as an analytical short-cut capturing the fact that parties opposed in litigation may initially be unequally endowed in terms of skill or ability to predict the outcome of the trial, which is not easy to verify for the other party. But our interpretation of these litigation costs is general enough to capture, aside from legal expenses associated with the proceedings (taxes, lawyer’s fees, and so on), several other opportunity costs due to the length of

\[\text{See Chopard et al. (2010). For example, parties will experience } ex \ post \text{ differences in legal expenditure as long as, regarding the technology of information production, there are significant differences in the marginal products of the individual effort. As long as the characteristics of the technology of information production are private information, these investments in information acquisition are not observable by the other party.}\]
the proceedings and the delays in obtaining a judgment (losses of potential business, additional R&D expenditure required to provide new evidence and so on) which are non verifiable by the other party. The point of interest that results from this simplification is that our model is a simple screening game.\textsuperscript{8}

### 3.1 Model and assumptions

The game between the Infringer and the Patentee has four main stages:

- At stage 1, Nature chooses the type of the Patent Holder \( c_h \). The Infringer knows only that the Patent Holder’s cost (Patent Holder’s type in the rest of the paper) is a random variable \( c_h \in [c_h, \bar{c}_h] \) distributed according to a cumulative function \( F(c_h) \) and a density \( f(c_h) \). In contrast, the Infringer’s litigation costs, denoted \( c_e \), are common knowledge.\textsuperscript{9}

- At stage 2, the Infringer competes with the Patent Holder (the potential market is a duopoly) and has to decide whether to enter without infringement (choose \textit{Non Infringe}) such that the Patent Holder produces the output \( y_h^N \) and earns \( \pi_h^N \), while the Infringer produces the output \( y_e^N \) and earns \( \pi_e^N \); or whether to enter with infringement (choose \textit{Infringe}), in which case, the game moves on to stage 3.

- At stage 3, the Patent Holder chooses either \textit{Accommodate} to adapt the entry of the Infringer (thus, they produce their duopoly outputs and earn their duopoly profits, respectively \( y_e^A, \pi_e^A \) for the Infringer and \( y_h^A, \pi_h^A \) for the Patent Holder); or it chooses \textit{Litigate} such that the game reaches stage 4, i.e. the case may be defended at trial or may be settled (at the market stage, the Patent Holder produces the output \( y_h^L \) and earns the market profit \( \pi_h^L \), while the Infringer produces the output \( y_e^L \) and earns \( \pi_e^L \)).

- At stage 4, the pretrial bargaining process takes place: the Infringer makes a (take-it-or-leave-it) \textit{Settlement offer} \( L \) to the Patent Holder, corresponding to a price for the patent agreement (or fees for the normal use of the patent). On the one hand, if the Patent Holder chooses \textit{Accept}, they settle their dispute amicably - and they earn their duopoly profits up to the

\textsuperscript{8}But this is wlog, the reason being that when legal costs are private information, their signalling value typically does not exist if basic rules of legal costs allocation are chosen by Courts: this is the case of the American rule considered here (see Chopard et al., 2010). On the other hand, a complete analysis of fee shifting is an issue beyond the scope of our paper. We focus rather on the effects of two specific damage rules.

\textsuperscript{9}In words, we consider the limit case where individual legal expenditures have no effect on the plaintiff’s probability of prevailing at trial. This is a simplification in order to focus on the role of informational asymmetries resulting from the parties’ skill in predicting a given outcome at trial, requiring, all else equal, different amounts of investments in information production.
cost and price of licensing, respectively \( u_e(L) = \pi_e^L - L \) for the Infringer and \( u_h(L) = \pi_h^L + L \) for the Patent Holder. On the other hand, if the Patent Holder chooses \textit{Reject}, then a trial occurs. The Court finds for the Patent Holder with probability \( \theta \in (0, 1) \), i.e. claims that the patent is valid; the verdict consists in damages that the Infringer must pay to the Patent Holder, denoted as \( D \); then, each party earns its duopoly profits minus its legal costs incurred at trial, respectively \( \pi_e^L - D - c_e \) for the Infringer and \( \pi_h^L + D - c_h \) for the Patent Holder. In contrast, the Court finds in favor of the Infringer with probability \( 1 - \theta \); in this event, each party earns its duopoly profits minus its legal costs incurred at trial, respectively \( \pi_e^L - c_e \) for the Infringer and \( \pi_h^L - c_h \) for the Patent Holder.

For the sake of simplicity, we will also assume that:

**Assumption 1:** the ratio \( \frac{1-E}{f} \) is decreasing on \([c_h, \bar{c}_h]\), and satisfies: \( \left( \frac{1-E}{f} \right)_{c_h} > c_e + \bar{c}_h \) and \( \left( \frac{1-E}{f} \right)_{\bar{c}_h} < c_e + c_h \).

which avoids technical developments regarding the issue of the existence and uniqueness of the solution at the pretrial stage, which is not central here.

**Assumption 2:** the market demand is linear with a demand price given by: \( P = a - b(y_h + y_e) \), where \( a, b > 0 \) and \( y_h, y_e \) denote the quantity produced by the Patent Holder and the Infringer, respectively.

**Assumption 3:** the technology of production entails constant marginal costs of production, respectively: \( \tilde{k} < a \) (before innovation occurs, for both firms), \( k_h < \tilde{k} \) (for the Patent Holder, after innovation) and \( k_e < \tilde{k} \) (for the Infringer).

We do not impose \textit{a priori} either \( k_h < k_e \) or \( k_h > k_e \). This allows us to capture two opposing situations: on the one hand the (maybe imperfect) imitation of the patent (\( k_h \leq k_e \)) and on the other, the case of incremental innovation (\( k_h > k_e \)) by the Infringer. The levels of R&D expenditures required in each case are likely to be very different (higher in the latter case than in the former). However, we omit those (fixed) costs in the present model.

We assume that at equilibrium, we have:

**Assumption 4:** \( \theta D - \tilde{c}_h > 0 \) for \( D \in \{ D_{LP} = \pi_h^N - \pi_h^L, D_{UE} = \pi_e^L - \pi_e^N \} \).

which holds both under the Lost Profit rule \( D_{LP} \) and under the Unjust Enrichment rule \( D_{UE} \). Thus, the credibility issue of the Patent Holder’s threat associated with the decision to sue at the pretrial bargaining stage is beyond the scope of our paper (see Meurer (1989)).
Finally, we introduce an upper bound on the Infringer’s legal costs:

**Assumption 5:** $c_e < \pi^A_e - \pi^N_e$.

This assumption is useful in order to rule out cases of deterrence which are not essential here. For example, assumption 5 prevents the size of the legal costs from deterring patent infringement despite a very low probability of losing at trial (i.e. in the neighborhood of $\theta = 0$; see the formal justification in the appendix).

### 3.2 Equilibria under the Lost Profit rule

Under the **Lost Profit rule**, we have $D_{LP} = [\pi^N_h - \pi^L_h]$, which implies that at the trial stage, the expected outcome for any Patent Holder $c_h \in [c_h, \bar{c}_h]$ is:

$$u_h(c_h) = \pi^L_h + \theta [\pi^N_h - \pi^L_h] - c_h = \theta \pi^N_h + (1 - \theta) \pi^L_h - c_h$$

while for the Infringer, it is defined as:

$$u_e(c_e) = \pi^L_e - \theta [\pi^N_h - \pi^L_h] - c_e$$

The central results in this case are provided in the next proposition:

**Proposition 1**

1. If $D_{LP} \geq 0 \Leftrightarrow \pi^N_h \geq (\pi^L_h)_{LP}$, if $\theta \leq \theta_{LP}$, then $\pi^N_h$ is the value of the patent strength satisfying $D_{LP} = \pi^N_h - (\pi^L_h)_{LP} = 0$. Thus, the restriction $\theta < \theta_{LP}$ is required in order to have $D_{LP} = \pi^N_h - (\pi^L_h)_{LP} = 0$.

2. If $\theta \in (0, \bar{\theta})$, with $\bar{\theta} \leq \theta_{LP}$, then the equilibrium under the Lost Profit rule is such that:

   - the Infringer chooses Infringe and produces $y^L_L = \frac{a-2k_e+k_h-k(3-\theta)}{k(3-\theta)}$, and at the pretrial stage it makes a licensing demand whose value is $L^* = \theta [\pi^N_h - (\pi^L_h)_{LP}] - c_h^* > 0$, where $c_h^*$ is defined according to $(\frac{1-\theta}{\theta})|c_h^* = c_h^* + c_e$.

   - the Patent Holder chooses Litigate and produces $y^L_L = \frac{a-2k_e+k_h}{\theta(3-\theta)}$, and then at the pretrial stage it chooses Reject if its type is $c_h \in [c_h, c_h^*)$, or Accept if its type is $c_h \in [c_h^*, \bar{c}_h]$.

See the proof in the appendix. We highlight here the basic principles driving the incentives to enforce or infringe the patent. To begin with, note that $\theta_{LP}$ is the value of the patent strength satisfying $D_{LP} = \pi^N_h - (\pi^L_h)_{LP} = 0$. Thus, the restriction $\theta < \theta_{LP}$ is required in order to have...
\( D_{LP} > 0 \), i.e. positive lost profit at equilibrium.\(^{10}\) In this case, proposition 1 says that the Patent Holder always enforces its right under the Lost Profit rule whatever its type, and either there is a trial, or a licensing agreement is reached. The incentives to defend the patent run as follows. Given that any type \( c_h \) may settle for a licensing price \( L^* > 0 \), and given that at the market stage we have (under \( \theta < \theta_{LP} \)) \( \pi^N_h > (\pi^L_h)_{LP} > \pi^A_h \), then the Patentee reaches (at least) the minimum expected benefit \( u_h(L^*) \) when it chooses \textit{Litigate} such that: \( u_h(L^*) = \pi^h_L + \theta (\pi^N_h - (\pi^L_h)_{LP}) - c^*_h > \pi^A_h \); thus, whatever its type, the Patent Holder is always better off enforcing its right rather than choosing \textit{Accommodate}.

Let us come now to the incentives to infringe the patent. The Infringer’s expected total benefit associated with the decision \textit{Infringe} (and given the strategies of the Patent Holder according to its type) is:

\[
U_e (L^*; c^*_h) = (1 - F(c^*_h)) ((\pi^L_e)_{LP} - L^*) + F(c^*_h) ((\pi^L_e)_{LP} - \theta (\pi^N_e - (\pi^L_e)_{LP}) - c_e)
\]

\[
= (\pi^L_e)_{LP} - \theta (\pi^N_e - (\pi^L_e)_{LP}) - c^*
\]

where \( c^* = F(c^*_h) c_e - (1 - F(c^*_h)) c_e - (1 - F(c^*_h)) (c_e + c^*_h) \) is the difference between the Infringer’s legal costs \( c_e \) and the expected gains from the negotiation \( (1 - F(c^*_h)) (c_e + c^*_h) \) (the probability of settlement, times the negotiation gains). Note that \( c^* \) may be either positive or negative; moreover, it is increasing in \( c_e \). Thus, the Infringer chooses \textit{Infringe} as long as \( U_e (L^*; c^*_h) > \pi^N_e \) or equivalently:

\[
((\pi^L_e)_{LP} - \pi^N_e) - \theta (\pi^N_e - (\pi^L_e)_{LP}) > c^*
\]

Strictly speaking\(^{11}\), when \( c^* \leq 0 \), it is sufficient that \( \theta < \theta_{LP} \), to have \( U_e (L^*; c^*_h) > \pi^N_e \); in contrast when \( c^* > 0 \), the condition must be strengthened: it must be that \( \theta < \tilde{\theta} \), where \( \tilde{\theta} < \theta_{LP} \) satisfies \( U_e (L^*; c^*_h) = \pi^N_e \).

Note the specific role of legal costs under the Lost Profit rule. On the one hand, we find that they induce a screening effect between high Patentee’s types (settling for \( L^* \)) and low types (going to trial) at the pretrial stage; but under assumption 4 (credibility of claims), the amount paid by the Patentee in the event of trial is not high enough to be better off by accommodating patent infringement. As a result, patent infringement is always litigated under the Lost Profit rule. On the other hand, the influence of legal costs on the Infringer’s side depends on the strength of the patent. When the Infringer’s legal costs are lower than the expected gains from the negotiation

\(^{10}\)In other words, \( \theta \geq \theta_{LP} \) implies that infringement is not harmful to the Patentee.

\(^{11}\)See appendix: at \( \theta = \theta_{LP} \), we have \( \pi^N_h = (\pi^L_h)_{LP} \) and \( (\pi^L_e)_{LP} = \pi^N_e \).
(c* ≤ 0) and the patent is weak enough, i.e. θ ≤ θ_{LP}, infringement is never deterred; in contrast, when the Infringer’s legal costs are higher than the expected gains from negotiation (c* > 0) and the the patent is strong enough, i.e. θ > θ, the Infringer is deterred. To sum up, enforcement is never deterred under Lost Profit, whereas infringement may be deterred in the event of strong patents associated with high legal costs (as compared to the expected gains from the negotiation) on the Infringer’s side.

Finally, the output decisions are also worth mentioning, since the feedback effect of the trial stage on the competition stage may be easily understood. As also noticed in the literature (Anton and Yao, 2006; Choi, 2009), under the Lost Profits rule the feed-back effect of the trial stage on the market stage results from an externality effect that this damages rule imposes on the Infringer which modifies its best response function specifying the quantity produced. This point is made clearer, coming to the first order conditions $\frac{\partial}{\partial y_h} u_h(c_h) = 0$ and $\frac{\partial}{\partial y_e} u_e(c_e) = 0$, since they can be written as:

\begin{align*}
(1 - \theta) \frac{\partial \pi^L_h}{\partial y_h} &= 0 \\
\frac{\partial \pi^L_e}{\partial y_e} + \theta \frac{\partial \pi^L_h}{\partial y_e} &= 0
\end{align*}

Along the equilibrium path where the Patent Holder chooses Litigate (either the case is settled, or there is a trial), the Patentee’s best response function is not affected (same FOC) by the occurrence of a trial, while the Infringer’s best response function shows that the Infringer’s incentives to produce will be reduced ($\frac{\partial \pi^L_e}{\partial y_e} + \theta \frac{\partial \pi^L_h}{\partial y_e} < \frac{\partial \pi^L_e}{\partial y_e}$). This externality also operates when the case is settled, since the "take-it-or-leave-it" offer corresponds to the expected outcome at trial of the marginal Patentee $c_h^*$.

### 3.3 Equilibria under the Unjust Enrichment rule

Under the Unjust Enrichment rule, we have $D_{UE} = [\pi_e^L - \pi_e^N]$, which implies that at the trial stage for any Patent Holder $c_h \in [c_h, \bar{c}_h]$ we get:

\begin{align*}
u_h(c_h) &= \pi^L_h + \theta D - c_h \\
&= \pi^L_h + \theta \left[\pi^L_e - \pi_e^N\right] - c_h
\end{align*}

and for the Infringer:
\[
  u_e(c_e) = \pi_e^L - \theta [\pi_e^L - \pi_e^N] - c_e \\
  = \theta \pi_e^N + (1 - \theta) \pi_e^L - c_e
\]

The central results in this case are provided in the two next propositions. A first difference with the previous case is worth noticing: \(D_{UE}\) is always positive. The second main difference is that an equilibrium may exist associated with the same qualitative features as under the Lost Profit rule, but it may be that other kinds of equilibria also arise where at least some Patent Holder types have an incentive to choose \textit{Accommodate} rather than \textit{Litigate} the case.

**Proposition 2** If \(\theta \in (\theta_{UE}^l, \theta_{UE}^r)\), where \(\theta_{UE}^r \leq 1\), the equilibrium obtained under the Unjust Enrichment rule is such that:

- the Infringer chooses \textit{Infringe} and produces \(y_e^L = \frac{a-2k_e+k_h}{b(3-\theta)}\), and at the pretrial stage it makes a licensing demand whose value is \(L^{**} = \theta \left[ (\pi_e^L)_{UE} - \pi_e^N \right] - c_h^* > 0\).
- the Patent Holder chooses \textit{Litigate} and produces \(y_h^L = \frac{a-2k_h+k_e-\theta(a-k_e)}{b(3-\theta)}\), and then at the pretrial stage it chooses \textit{Reject} if its type is \(c_h \in (c_h^*, c_h^+])\), or \textit{Accept} if its type is \(c_h \in [c_h^-, c_h^+]\).

See the proof in the appendix. The incentives to enforce the patent or to infringe it now run as follows. On the one hand, the equilibrium of proposition 2 requires that\(^{12}\) \(u_h(L^{**}) = (\pi_h^L)_{UE} + \theta \left[ (\pi_e^L)_{UE} - \pi_e^N \right] - c_h^* > \pi_h^A\) (where \(c_h^*\) is still defined as under the LP rule), where we now have \((\pi_h^L)_{UE} < \pi_h^A\) and \((\pi_e^L)_{UE} > \pi_e^N\). Let us assume that there exists a \(\theta_{UE}^r\) for which \(u_h(L^{**}) = \pi_h^A\). Then in order for the Patentee to choose \textit{Litigate} at equilibrium whatever its type, it must be that \(\theta > \theta_{UE}^r\), i.e. the Patent Holder’s case must be strong enough.

On the other hand, the Infringer’s best decision at the initial node is \textit{Infringe} if: \(U_e(L^{**}; c_h^*) > \pi_e^N\), which can now be written:

\[
  (1 - \theta) \left[ (\pi_e^L)_{UE} - \pi_e^N \right] > c^*
\]

where \(c^* \geq 0\). Once more (see in appendix), we may distinguish between the case where \(c^* \leq 0\) and the case where \(c^* > 0\). When \(c^* \leq 0\), then it can be shown that \(U_e(L^{**}; c_h^*) > \pi_e^N \forall \theta \in [0, 1]\); in contrast when \(c^* > 0\), the condition must be strengthened: \(U_e(L^{**}; c_h^*) > \pi_e^N\) only if \(\theta < \theta_{UE}^r\), where \(\theta_{UE}^r < 1\) satisfies \(U_e(L^{**}; c_h^*) = \pi_e^N\).

\(^{12}\text{Under assumption 4, we still have } \theta \left[ (\pi_e^L)_{UE} - \pi_e^N \right] - c_h^* > 0. \text{ However under the Unjust Enrichment rule, the inequality } (\pi_h^L)_{UE} < \pi_h^A \text{ now holds (see in the appendix).}\)
Thus under the Unjust Enrichment rule, the Patentee’s legal costs also entail a screening effect at the pretrial stage between high (now settling for $L^{**} > L^*$) and low (going to trial) Patentee types; but now, the amount paid by the marginal Patentee (indifferent between a settlement and a trial) in the event of trial may be high enough to make it better off accommodating patent infringement, despite the credibility of claims (assumption 4): patent infringement is now litigated only when the patent is strong enough ($\theta > \theta'_{UE}$). On the other hand, the effect on the Infringer is qualitatively similar to the effect observed for the Lost Profit case: when the Infringer’s legal costs are lower than the expected gains from negotiation ($c^* \leq 0$), infringement is never deterred regardless of patent strength; in contrast when the Infringer’s legal costs are higher than the expected gains from negotiation ($c^* > 0$), the Infringer is deterred if the patent is strong enough, i.e. $\theta > \theta'_{UE}$.

The consequences for the outputs at equilibrium may also be easily understood. With the Unjust Enrichment rule (see also Anton and Yao, 2006; Choi, 2009), the externality effect is now shifted to the Patent Holder and modifies its best response function. The first order conditions $\frac{\partial}{\partial y_h} u_h(c_h) = 0$ and $\frac{\partial}{\partial y_e} u_e(c_e) = 0$ can be written by symmetry as:

$$\begin{align*}
\frac{\partial \pi_h^L}{\partial y_h} + \theta \frac{\partial \pi_e^L}{\partial y_e} &= 0 \\
(1 - \theta) \frac{\partial \pi_h^L}{\partial y_e} &= 0
\end{align*}$$

Along a path where the Patent Holder chooses Litigate (either the case is settled, or there is a trial), the Infringer’s best response function is now not affected (same FOC) by the occurrence of a trial, while the Patentee’s best response function shows that its incentives to produce will be reduced ($\frac{\partial \pi_h^L}{\partial y_h} + \theta \frac{\partial \pi_e^L}{\partial y_e} < \frac{\partial \pi_h^L}{\partial y_h}$).

However, alternative equilibria may be obtained for weaker patents, that we describe in the following proposition:

**Proposition 3** Assume that$^{13}$ $\theta < \theta'_{UE}$; if there exists a $c_h^* \in [\bar{c}_h, \bar{c}_h^*]$ defined by$^{14}$ $u_h(c_h^*) = \pi_h^A$, two alternative equilibria may be obtained under the Unjust Enrichment rule:

i) The first kind of equilibrium holds when $(\pi_h^L + \pi_e^L)_{UE} > \pi_h^A + \pi_e^A$, and is such that:

- the Infringer chooses Infringe and produces $y_e^L = \frac{a - k_h - k_e}{b(3 - a)}$, and at the pretrial stage it makes a licensing demand whose value is $\bar{L} = \pi_h^A - (\pi_h^A)_{UE} > 0$.

---

$^{13}$Alternatively, it may be the case that $\theta'_{UE}$ does not exist, i.e. there exists no value of $\theta$ for which $u_h(L^{**}) = \pi_h^A$.

$^{14}$If $\tilde{c}_h \leq \bar{c}_h$, then any equilibrium may be built such that whatever the settlement offer made by the Infringer at the pretrial stage, every Patent Holder types choose Accommodate, while the Infringer chooses Infringe. The outputs $(y_h^A, y_e^A)$ are obtained at the market stage.
- the Patent Holder chooses Litigate and produces \( y_h^L = \frac{a-2k_b+k_e-\theta(a-k_e)}{b(3-\theta)} \), and then at the pretrial stage it chooses Reject if its type is \( c_h \in [\hat{c}_h, \tilde{c}_h) \) or Accept if its type is \( c_h \in [\tilde{c}_h, \check{c}_h] \).

ii) The second kind of equilibrium holds when \( (\pi_h^L + \pi_e^L)_{UE} < \pi^A_h + \pi^A_e \), and is such that:

- the Infringer chooses Infringe and produces \( y_e^L = \frac{a-2k_b+k_e-\theta(a-k_e)}{b(3-\theta)} \) conditional on \( c_e \leq \check{c}_h \), or \( y_e^A = \frac{a-2k_b+k_e}{3b} \) conditional on \( c_e > \check{c}_h \), and then at the pretrial stage it may make any licensing demand\(^{15} \)

\( \hat{L} \in [0, \check{L}] \).

- the Patent Holder chooses Litigate and produces \( y_h^L = \frac{a-2k_b+k_e-\theta(a-k_e)}{b(3-\theta)} \), and at the pretrial stage it chooses Reject, if its type is \( c_h \in [\hat{c}_h, \tilde{c}_h) \).

- but it chooses Accommodate and produces \( y_h^A = \frac{a-2k_b+k_e}{3b} \) if its type satisfies \( c_h \in [\check{c}_h, \tilde{c}_h] \).

See the proof in the appendix. Proposition 3 says that under the Unjust Enrichment rule, different kinds of equilibria may arise when the patent is weak \((\theta < \theta'_{UE})\). Assume there exists a \( \tilde{c}_h \in [c_h, c_h'] \) satisfying \( u_h(\tilde{c}_h) = \pi^A_h \Leftrightarrow \tilde{c}_h = ((\pi_h^L)_{UE} - \pi^A_h) + \theta ( (\pi_e^L)_{UE} - \pi^A_e) \) and consider the licensing price \( \hat{L} = \pi^A_h - (\pi_h^L)_{UE} \). On the one hand, if the Infringer proposes \( \hat{L} \), there is a separation of the population of Patent Holder’s types between those who choose \((\text{Litigate}, \text{Reject})\) (types \( c_h \in [\hat{c}_h, \tilde{c}_h) \)) and those who choose \((\text{Litigate}, \text{Accept})\) (types \( c_h \in [\tilde{c}_h, \check{c}_h] \)). On the other hand, if the Infringer proposes any \( \check{L} \in [0, \hat{L}] \), there is a separation of Patentee’s types between those who choose \((\text{Litigate}, \text{Reject})\) (\( c_h \in [\hat{c}_h, \tilde{c}_h) \)) and those who choose \( \text{Accommodate} \) (\( c_h \in [\check{c}_h, \tilde{c}_h] \)). The offer \( \tilde{L} \) is part of an equilibrium if \( U_e(\tilde{L}; \check{c}_h) > (\text{or} <) U_e(\check{L}; \check{c}_h) \), which requires the condition \( (\pi_h^L + \pi_e^L)_{UE} > (\text{or} <) \pi^A_h + \pi^A_e \) to hold (see the appendix).

In a sense, what proposition 3 says is that holding a weak patent is not a sufficient condition for the Patent Holder to forgo enforcing its right under the Unjust Enrichment rule – its is only a necessary condition\(^{16} \). In an equilibrium where \( (\pi_h^L + \pi_e^L)_{UE} > \pi^A_h + \pi^A_e \Leftrightarrow (\pi_e^L)_{UE} - \pi_e^A > \pi^A_h - (\pi_h^L)_{UE} \), the Patent Holder will always enforce its patent despite its weakness. The condition is not always satisfied given that under the Unjust Enrichment rule we have at the same time \( (\pi_e^L)_{UE} - \pi_e^A > 0 \) and \( \pi^A_h - (\pi_h^L)_{UE} > 0 \). Then the case is litigated only as long as \( (\pi_e^L)_{UE} - \pi_e^A > \pi^A_h - (\pi_h^L)_{UE} = \hat{L} \), which means that the Infringer must win more than the Patent Holder loses, as compared to a situation where the infringement is accommodated. The driving force is that

\(^{15}\)Indeed, there exists a continuum of equilibria of the third type, each being associated with a specific value of \( \hat{L} \in [0, \hat{L}] \), but yielding the same outcome since they induce the same separation between the Patent Holder types.

\(^{16}\)Choi (proposition 5 p 153, 2007) for the case of a drastic innovation and zero litigation costs, finds that any probabilistic patent is enforced and infringed, whatever its strength. Schankerman and Scotchmer (2011) obtain the deterrence of infringement for an ironclad patent, under a condition of dilution of industry profits (difference between industry profit with and without infringement) – notice that our condition is different (difference between industry profit under infringement and litigation, and industry profit under infringement and accomodation).
the Infringer has the opportunity to offer a licensing price \( \tilde{L} \) which is large enough (\( \tilde{L} > \hat{L} \)) to be accepted by the Patent Holder in case it has the highest legal costs – the counterpart is that whatever its type, the Patent Holder always enforces its right. In contrast, to induce the Patent Holder not to (always) enforce its right, sufficiency requires that \( (\pi_h^L + \pi_e^L)_{UE} < \pi_h^A + \pi_e^A \iff (\pi_e^L)_{UE} - \pi_e^A < \pi_h^A - (\pi_h^L)_{UE} \). This means that the highest licensing price the Infringer has the opportunity to offer is lower than the minimum accepted by the Patent Holder (\( \tilde{L} < \hat{L} \)). As a result, the patent is not enforced except by the smallest Patent Holder types. Otherwise, the infringement is accommodated.

Thus, the characteristic features of these equilibria rely on different screening effects\(^{17}\), and thus whether the Infringer is better off in reaching a settlement at the pretrial stage with a subset of Patentees, or whether it prefers to induce some of them to accommodate infringement, while the others litigate. We now emphasize the role of the Infringer’s legal costs (see the appendix for the formal analysis). Considering the equilibrium of Part i), the influence of the Infringer’s legal costs are similar to those observed before: when the Infringer’s legal costs are lower than the expected gains from negotiation (\( \tilde{c} = c_e - (1 - F(\tilde{c}_h)) (c_e + \tilde{c}_h) \leq 0 \), with \( \tilde{c}_h \) the new marginal Plaintiff being indifferent between accommodating and litigating the case), infringement is never deterred however strong the patent; in contrast when the Infringer’s legal costs are higher than the expected gains from negotiation (\( \tilde{c} > 0 \)), the Infringer is deterred if the patent in a sense is weak (\( \theta < \theta_{UE}^0 \)) but not too weak (see appendix). On the other hand considering now the equilibrium of Part ii), assumption 5 is sufficient to ensure that infringement is never deterred.

4 Damages rules and patent protection: impacts on settlements, profits, and social welfare

Propositions 1 to 3 show that the two damages rules may have very different selection effects on patent cases, both regarding the Infringer’s incentives and the Patentee’s incentives. In this section, we investigate some of the consequences of these differences. We will focus on the Patent Holder’s protection afforded by each rule, and the issue of social welfare.

\(^{17}\)Either there is a separation between high (now settling for \( \tilde{L} \)) and low (going to trial) Patentee’s types; or there is a separation between high (now accommodating) and low (still going to trial) Patentee’s types.
4.1 Frequency of settlement and trials

Regarding the effect of the choice of a damages rule on pre-trial negotiations and on the frequency of trial, the main consequence of the previous section is:

**Proposition 4** The Lost Profit rule yields a conditional probability of trials at least as large as the Unjust Enrichment rule.

*Proof.* Consider propositions 1 and 2; then conditional to enforcement, both rules are associated with the same conditional probability of trials $F(c_h)$; however, comparing propositions 1 and 3, the Lost Profit rule engenders more trials since $F(c_h) > F(\tilde{c}_h)$.

However, unconditional probabilities of trial cannot be readily compared without additional assumptions. There are two main reasons for this. On the one hand, it may be impossible to enforce the patent under the Unjust Enrichment rule at least for some Patent Holder types (the largest values for $c_h$); these will be better off accommodating the infringement of the patent rather than litigating it, given the impossibility of reaching a settlement. On the other hand, under Lost Profit, only weak patents are potentially enforceable, in the sense that the Patentee is allowed to argue that infringement has been harmful to him.

Note also that the effect on the equilibrium licensing price is ambiguous. As explained in the next paragraph, this is because the impact on the equilibrium value of damages is ambiguous.

4.2 The allocation of industry profits

Regarding the issue of the Patent Holder profits protection, the comparison between $u_h(c_h)_{LP}$ and $u_h(c_h)_{UE}$ makes sense only for $\theta \in (0, \theta_{LP}]$: this is because there is no damage under the Lost Profit rule if $\theta > \theta_{LP}$, i.e. $D_{LP} = 0(< D_{UE})$. On the other hand, it can be seen that for any $\theta \in (0, \theta_{LP})$, the difference:

$$u_h(c_h)_{LP} - u_h(c_h)_{UE} = \left(\pi^L_h + \theta \left[\pi^N_h - \pi^L_h\right]\right)_{LP} - \left(\pi^L_h + \theta \left[\pi^L_e - \pi^N_e\right]\right)_{UE}$$

can be rearranged as the sum of two effects. On the one hand, the difference between the value of expected damages $\theta \left[\pi^N_h - \pi^L_h\right]_{LP} - \theta \left[\pi^L_e - \pi^N_e\right]_{UE}$; this is a direct effect, in terms of compensation *per se* allowed to the Patent Holder in the event of trial, and its sign is ambiguous. On the other hand, there is an indirect (feedback) effect since the choice of the damages rule affects the Patentee’s market profits, and thus the difference $\left(\pi^L_h\right)_{LP} - \left(\pi^L_h\right)_{UE}$; this reflects the market discipline imposed by each rule on competitors.
In the specific case where \( k_h = k_e \), it can be verified that \((\pi_h^L)_{LP} = (\pi_e^L)_{UE}\) and \((\pi_h^L)_{UE} = (\pi_e^L)_{LP}\), which implies that \(u_h(c_h)_{LP} > u_h(c_h)_{UE}\), and conversely \(u_e(c_e)_{LP} < u_e(c_e)_{UE}\). In the general case where \( k_h \neq k_e \), we sum up our results in the next proposition.

**Proposition 5** A) If \( \theta \leq \hat{\theta} \) where \( \hat{\theta} < \theta_{LP} \), then the Lost Profit rule affords the Patentee better protection than the Unjust Enrichment rule. If \( \theta > \hat{\theta} \), the comparison is ambiguous.

B) If \( \theta \geq \hat{\theta} \), then the Infringer earns higher profits under the Unjust Enrichment rule than under the Lost Profit rule. If \( \theta < \hat{\theta} \), the comparison is ambiguous.

**Proof.** A) Using the equilibrium values of profits (see in appendix) and rearranging, we find that \( D_{LP} > D_{UE} \) if \((3 - \theta)^2 > 9 \left( \frac{a - 2k_h + k_e}{a - 2k_h + k_e} \right)^2 + \frac{(a - 2k_h + k_e)^2}{(a - 2k_h + k_e)^2} \) or equivalently if \( \theta < \hat{\theta} \equiv 3 \left( 1 - \frac{\sqrt{((a - 2k_h + k_e)^2 + (a - 2k_h + k_e)^2)}}{(a - 2k_h + k_e)^2 + (a - 2k_h + k_e)^2} \right) \). Simple but tedious manipulations\(^{18}\) show that \( \hat{\theta} < \theta_{LP} \). Conversely, \( D_{LP} < D_{UE} \) if \( \theta > \hat{\theta} \). Hence, considering the case for a strong (weak) patent, i.e. for \( \theta \) large (respectively, small) enough, the Unjust Enrichment rule grants higher (respectively smaller) damages than the Lost Profit rule.

The second effect \((\pi_h^L)_{LP} - (\pi_h^L)_{UE}\) is unambiguously positive since we have shown that: \(\pi_h^N > (\pi_h^L)_{LP} > (\pi_h^L)_{UE} > (\pi_e^L)_{UE}\) (given \( \theta < \theta_{LP} \)). Hence, regarding this indirect effect on market profits, the Lost Profit rule gives a higher profit to the Patent Holder than the Unjust Enrichment rule.

Collecting both effects, we obtain that \( \theta \leq \hat{\theta} \) implies:

\[
(\pi_h^L + \theta [\pi_h^N - \pi_h^L])_{LP} \geq (\pi_h^L + \theta [\pi_e^L - \pi_e^N])_{UE}
\]

otherwise, the result is ambiguous.

B) Note that the consequences for the Infringer’s gross profits are also easy to describe; given that \(u_e(c_e)_{LP} - u_e(c_e)_{UE} = (\pi_e^L)_{LP} - (\pi_e^L)_{UE} - \theta(D_{LP} - D_{UE})\), with \(\pi_e^N < (\pi_e^L)_{LP} < \pi_e^A < (\pi_e^L)_{UE}\), the result is direct by symmetry. ■

A main consequence of proposition 4 is worth noting, since it is related to the analysis of R&D incentives. As discussed by Choi (2009)\(^{19}\) the incentives to invest in R&D in non tournament models of innovation (a single investor; cf Reinganum (1989)) are driven by the Patentee’s expected profits; in contrast, in tournament models (multiple investors), they depend on the difference between the Patentee’s expected profit and the potential competitor’s profit. As a result, our

\(^{18}\)Available on request.

\(^{19}\)Recall that Choi (2009) discusses the case for a drastic innovation.
analysis implies that in tournament models, the Lost Profit rule generates more R&D investments than the Unjust Enrichment rule only for weak patents, i.e. only if $\theta < \hat{\theta}(< \theta_{LP})$; in a sense, this can be understood as a pervasive effect of the Lost Profit rule, since it preserves the incentives to innovate in the range of patents having a high risk of infringement. Otherwise, the effect is ambiguous. Finally, in tournament models, no rule is clearly better than the other, whatever the strength of the patent.

4.3 Social welfare

Social welfare will be defined here as the standard Marshallian surplus (sum of the consumers’ surplus and the firms’ gross profits, including firms’ legal costs). When the Unjust Enrichment rule is used, several kinds of equilibria may occur; in order to distinguish them, let us denote as:

- $SW_{1,LP}$ the social welfare associated with proposition 1;
- $SW_{2,UE}$ the social welfare associated with proposition 2;
- $SW_{3i,UE}$ the social welfare associated with proposition 3i);
- $SW_{3ii,UE}$ the social welfare associated with proposition 3ii).

Now defining as:

$$SW^L_i = \int_0^{(y^L_i + y^L_i)_h} P(x) dx - k_h(y^L_h)_i - k_e(y^L_e)_i \text{ for } i = LP, UE$$

$$SW^A = \int_0^{(y^A_h + y^A_e)} P(x) dx - k_h y^A_h - k_e y^A_e$$

it can be verified that:

$$SW_{1,LP} - SW_{2,UE} = SW^L_{LP} - SW^L_{UE}$$

$$SW_{1,LP} - SW_{3i,UE} = SW^L_{LP} - SW^L_{UE} - \int_{\tilde{c}_h}^{c_h} (c_h + c_e) dF(c_h)$$

$$SW_{1,LP} - SW_{3ii,UE} = SW^L_{LP} - SW^L_{UE} - \int_{\tilde{c}_h}^{c_h} (c_h + c_e) dF(c_h) - (1 - F(\tilde{c}_h)) (SW^A - SW^L_{UE})$$

Notice that the comparisons of welfare make sense only for properly chosen values of $\theta$; this means that proposition 1 and proposition P2 (or P3) can only be compared when the value for $\theta$ supports both equilibria (see the appendix). Notwithstanding these qualifications, we can show that the next results hold:
Proposition 6  

A) (proposition 1 vs proposition 2) Social welfare is higher under the Lost Profit (Unjust Enrichment) rule if \( k_h < k_e \) (resp. \( k_h > k_e \)).

B) (proposition 1 vs proposition 3) i) When \( (\pi_h^L + \pi_e^L)_{UE} > \pi_h^A + \pi_e^A \), social welfare is higher under the Unjust Enrichment rule if \( k_h > k_e \). The result is ambiguous if \( k_h < k_e \). ii) When \( (\pi_h^L + \pi_e^L)_{UE} < \pi_h^A + \pi_e^A \), the result is ambiguous whatever the sign of \( k_h - k_e \).

Proof. A) We have to sign \( SW_{1,LP} - SW_{2,UE} \). Given that both damages rules are associated with the same value for the marginal Plaintiff (Patentee), and thus with the same expected legal costs, this implies that \( SW_{1,LP} - SW_{2,UE} = SW_{L,LP}^L - SW_{UE}^L \). Thus, the proof is the same as in Choi (proposition 3, 2009).\(^{20}\)

B) The relevant welfare comparison is based either on \( SW_{1,LP} - SW_{3i,UE} \) or \( SW_{1,LP} - SW_{3ii,UE} \), meaning that we have to take into account that the value for the marginal Patentee is no longer the same between the equilibria compared here, and/or that the Patentee may accommodate at equilibrium. Then additional effects must be considered due to legal costs and existing differences in the probability of a trial. For this purpose, let us distinguish two different cases.

i) First, consider the case where \( (\pi_h^L + \pi_e^L)_{UE} > \pi_h^A + \pi_e^A \). Then \( SW_{1,LP} - SW_{3i,UE} \) depends, beyond \( SW_{L,LP}^L - SW_{UE}^L \) (which behaves as before), on the difference in legal costs \( \int_{c_h}^{c_h^*} (c_h + c_e) dF(c_h) \). As there are more trials under the Lost Profit rule than those under the Unjust Enrichment rule (since \( c_h^* > c_h \)), the expected legal costs under the Lost Profit rule are higher than the ones under the Unjust Enrichment rule: \( \int_{c_h}^{c_h^*} (c_h + c_e) dF(c_h) > 0 \). As a consequence, if \( k_e < k_h \), the social welfare is larger under the Unjust Enrichment rule; but if the condition \( k_e > k_h \) holds, the comparison is ambiguous.

ii) Second, assume now that \( (\pi_h^L + \pi_e^L)_{UE} < \pi_h^A + \pi_e^A \). In this case, the sign of \( SW_{1,LP} - SW_{3ii,UE} \) is also governed by a third effect, \( (1 - F(c_h)) (SW_A - SW_{L,UE}^L) \), reflecting that under the Unjust Enrichment rule there is a positive probability that the Patentee opts for accommodating the entry of the Infringer, rather than enforcing its right. Thus, we have to establish\(^{21}\) the conditions under which it is socially desirable that under the Unjust Enrichment rule at least some Patentee’s types accommodate rather than sue. Solving explicitly, we find that:

\[
SW_{UE}^L - SW_A = \frac{\theta(a - 2k_e + k_h)}{18b(3 - \theta)^2} \left[ - (6 - \theta) (a - 2k_e + k_h) + 12 (3 - \theta) (k_h - k_e) \right]
\]

The bracketed term is negative when \( k_h < k_e \), which implies that \( SW_A - SW_{UE}^L > 0 \) (and thus the third effect runs counter to the first two). But the sign is indeterminate when \( k_h > k_e \), and likewise for the sign of \( SW_A - SW_{L,UE}^L \).

\(^{20}\)The result does not depend on whether the innovation is drastic or not.

\(^{21}\)The sign of \( SW_{L,LP} - SW_{UE}^L \) is obtained under the same conditions as in case i).
In part A), \( SW_{1,LP} - SW_{2,UE} = SW_{LP}^L - SW_{UE}^L \) depends on two effects working in opposite directions, respectively the integral term and the production costs. On the one hand, the Patent Holder – whatever the sign of \( k_h - k_e \) – produces a larger share of the total output under the Lost Profit rule \( (y_h^L)_{LP} > (y_h^L)_{UE} \), whereas the reverse is true for the potential infringer \( (y_e^L)_{LP} < (y_e^L)_{UE} \); this effect in turn affects production costs accordingly. Thus, when \( k_h > k_e \), an inefficient allocation of market shares is obtained under the Lost Profit rule since the Patentee produces more than the Infringer despite higher production costs; but, when \( k_h < k_e \), there is no inefficiency associated with the Lost Profit rule. On the other hand, it can be verified that the market output under the Lost Profit rule \( (y_h^L + y_e^L)_{LP} \) is larger (smaller) than that under the Unjust Enrichment rule \( (y_h^L + y_e^L)_{UE} \) if \( k_h > k_e \) (respectively if \( k_h < k_e \)), which yields that the integral term under Lost Profit is larger (respectively smaller). It can be shown\(^{22}\) that when \( k_h > k_e \) the inefficiency effect associated with Lost Profit dominates (is dominated by) the first (respectively, second) one such that \( SW_{LP}^L - SW_{UE}^L < 0 \); but when \( k_h > k_e \), it is direct that \( SW_{LP}^L - SW_{UE}^L > 0 \).

To sum up, our analysis shows that the comparison of social welfare is sensitive to the patent strength \( \theta \), to the accommodation of infringement, and to the sign of the difference in marginal costs \( k_h - k_e \). Comparing equilibria where any Patentee enforces his right and never accommodates (specifically when \( \theta_{UE} < \theta_{LP} < \theta_{UE}' \)), it appears that the Lost Profit rule is better from a social point of view as long as the Patent Holder is at least as efficient as the Infringer \( (k_h \leq k_e) \). Otherwise, the Unjust Enrichment rule is better from a social point of view. But considering a weak patent \( (\theta < \theta_{UE}' \)\), the advantages of one rule over the other do not necessarily hold for a given condition on marginal costs. As long as infringement is not accommodated and the Infringer becomes more efficient than the Patent Holder \( (k_h > k_e) \), the Unjust Enrichment rule should be preferred from a social point of view however strong the patent. In other cases, including the possibility of accommodation of infringement, we find that neither rule is better than the other.

5 Conclusion

Our paper introduces a comprehensive model of patent litigation with pretrial negotiations, which allows us to analyze the decisions to enforce and to infringe a patent and captures the feedback influences of amicable settlements on the market stage. We show that when the value of a patent is probabilistic (the verdict at trial is uncertain) and the occurrence of a trial is not certain (negotiations may produce an agreement before trial), the choice made by courts between two general damages rules has a complex effect on the Infringer’s decision whether to infringe a patent, on

\(^{22}\)The proof is similar to Choi (2009), who considers a drastic innovation.
the Patentee’s decision whether to sue rather than accommodate infringement, and then on the decision whether to settle rather than take the case to court. As a result, while the Unjust Enrichment rule never yields more trials than the Lost Profit rule, we find that generally speaking, neither rule is better than the other regarding the preservation of Patentee’s profits and incentives to invest in R&D, or regarding social welfare. We also find that such an ambiguity arises whatever the difference between marginal costs of production between the Patentee and the Infringer.

Our paper also sheds some new light on the controversy about the existence of infringement at equilibrium under the Lost Profit rule (Shankerman and Scotchmer, 2001; Anton and Yao, 2006; Choi, 2009), and broadens the view in several ways – specifically regarding some weaknesses of the Unjust Enrichment rule which have been overlooked up to now. Shankerman and Scotchmer (2001) consider ironclad patents and find that infringement occurs under the Lost Profit rule only when it dissipates the industry profits (i.e. when the sum of firms’ profits is lower under infringement than without infringement). In Anton and Yao (2006), infringement of a probabilistic patent corresponding to a non drastic innovation always exists at equilibrium under the Lost Profit rule since the imitator has the opportunity to engage in passive infringement where it produces the non infringement output at a lower cost, but such that it is never liable (since there is no harm at equilibrium). Finally, Choi (2009) shows for drastic innovations and probabilistic IPRs that infringement always occurs under both rules.

Thus our paper is useful since it provides a fuller picture of the incentives of both the Patentee (to sue) and the Infringer (to infringe) and how they are shaped by the choice of the damages rule at the trial stage. The interest and novelty of our analysis is also that the paper emphasizes the role of the timing of decisions as well as the importance of the set of strategies held by competing firms. On the one hand, central to our analysis is the assumption that the Infringer and the Patentee move sequentially, implying that the opt-out options are firm-specific at the main stages of the game (for example, non infringement vs accommodation). On the other hand, also important to our results is the introduction of a pretrial stage and its consequences for firms’ decisions such as enforcing and infringing a patent.

It is important to underline here that infringement may not always exist at equilibrium in our model because of patent strength and the Infringer’s legal costs; nevertheless, to hammer home the point, the argument is not trivial. We have assumed that the Infringer’s legal costs are bounded above in order to rule out cases of deterrence which are not essential here – such as when patent infringement is deterred although it might be expected that infringement would be accommodated.

23 As a result, the dissipation of industry’ profits does not mean the same thing for both firms, nor does it mean the same thing as compared to the literature.
given the very low probability of losing in court. Thus our findings show more specifically that the incentives to infringe the patent are generally driven by the difference between the Infringer’s legal costs and the expected value of the negotiation gains (the size of which depends on the marginal Plaintiff’s legal costs, up to the Infringer’s costs), under both damages rules. To sum up, when the Infringer’s legal costs are low compared to the expected value of the negotiation gains, then the Infringer always infringes a patent (weak enough); otherwise, the Infringer may be deterred from doing so. But regarding the incentives to enforce a patent, we have shown that they may depend on the strength of a patent and the size of the Patentee’s legal costs, although in a way specific to each damages rule. More specifically, we have found that any patent is enforced under the Lost Profit rule (provided it is weak enough to be associated with a harm when infringed). In contradistinction, under the Unjust Enrichment rule high legal costs may induce accommodation by some Patentee types, rather than litigating the case; this is more likely to occur when the patent is weak enough and when infringement is associated with the dissipation of the industry’s profits (but in a different sense than in Schankerman and Scotchmer: the sum of firms’ profits is lower under infringement than under accommodation).

To conclude, let us underline some limits of our work, but nevertheless its general nature. On the one hand, several results may be very specific to the timing of the model introduced here. The specific influence of the order of moves will be assessed in future research, since the incentives to enforce as well infringe the patent are closely related to the specific opt-out option of firms. We have also highlighted the importance of two parameters of litigation technology: the intrinsic strength of the patent and the distribution of legal costs, which are given exogenously here. Given that the strength of the patent may summarize the probability of detecting infringement and the probability of securing a court ruling, it would be worth relating them to the Patentee’s behavior as well as the behavior of the courts and thus the distribution of both public and private legal costs. This is also left for future research. On the other hand, our work is also a contribution to the theoretical literature on litigation more broadly speaking. Recent research displays an increasing interest for the effects of judges’ behavior on private decisions outside the litigation process, specifically at the market stage. For example the feedback influences of damages setting on care activity levels are considered in Landeo and Nikitin (2012) or Polinsky and Shavell (2012). Their consequences in terms of output decisions or R&D investments are described by Baumann and Heine (2012) and Endres, Friehe and Rundshagen (2014). The parallel with the literature on tort law and liability rules is not accidental; several legal commentators have observed the shift in the application of IP laws from the strict observance of a doctrine in property law to an interpretation in terms of liability rules (Elkin-Koren and Salzberger, 2000; Opderbeck, 2009). Hence, our paper is also a
contribution to the analysis of liability rules in contexts where strategic interactions exist between parties, outside of the litigation process. In this sense, our results regarding the implications of alternative damages rules are not limited to IPRs but have a broader scope.

References


6 Appendix

6.1 Proof of Proposition 1

The complete proof requires some intermediate results to be established.

- Stage 4

The pretrial bargaining game in stage 4 is solved as follows. The reason why the Infringer has to litigate (reaches stage 4) is that at least some (types of) Patent Holders preferred to choose Litigate rather than Accommodate at stage 3. Let us assume that there exists a cut-off value $\hat{c}_h$ with $c_h < \hat{c}_h \leq \check{c}_h$ such that Patent Holders with a $c_h < \hat{c}_h$ chooses Litigate, in contrast to Patent Holders with a $c_h \geq \check{c}_h$ who choose Accommodate. The intuition is that a Patent holder having large litigation costs should be prone to accommodate.

Conditional on the fact that at stage 4 the Infringer expects to face Patent Holders with a $c_h < \hat{c}_h$, the settlement offer $L$ may separate the Patent Holders between those who accept $L$ - any Patent Holder who litigates and for whom $u_h(L) \geq u_h(c_h)$ will accept - and those who reject $L$ - any Patent Holder who litigates and for whom $u_h(L) < u_h(c_h)$ will reject the offer. Let us denote
as $c_L$ the marginal holder’s type who is indifferent between litigate and settle; its type is given by:

$$u_h(c_L) = u_h(L) \iff \theta \pi_h^N + (1 - \theta) \pi_h^L - c_L = \pi_h^L + L \text{ or rearranging:}$$

$$c_L = \theta [\pi_h^N - \pi_h^L] - L \quad (1)$$

Thus, the Infringer faces with probability $\left(1 - \frac{F(c_L)}{F(c_h)}\right)$ a Patent Holder who will be prone to chose Accept (the settlement demand $L$); and with probability $\frac{F(c_L)}{F(c_h)}$, it faces a Patent Holder who will prefer to chose Reject (go to trial).

As a result, the best licensing price $L^*$ for the patent infringer can now be defined as the solution of the maximization of its ex ante total benefit for the case:

$$U_e(L; c_L) = \left(1 - \frac{F(c_L)}{F(c_h)}\right) u_e(L) + \frac{F(c_L)}{F(c_h)} u_e(c_e)$$

$$= \pi_e^L - \left(1 - \frac{F(c_L)}{F(c_h)}\right) L - \frac{F(c_L)}{F(c_h)} (\theta [\pi_h^N - \pi_h^L] + c_e) \quad (2)$$

**Lemma 7** Under assumption 1, the solution to the maximization of $(3)$ under $(2)$, is unique and corresponds to the licensing offer $L^*$ and the cut-off value for the Patent Holder’s type $c_L^* = c_h^*$ which are implicitly obtained by solving the system:

$$L^* = \theta [\pi_h^N - \pi_h^L] - c_h^* \quad (3)$$

$$\left(\frac{F(c_h) - F}{f}\right)_{c_h^*} = c_e + c_h^* \quad (4)$$

**Proof.** The derivative of $U_e(L; c_L)$ in $L$ is:

$$\frac{\partial}{\partial L} U_e(L; c_L) = - \left(1 - \frac{F(c_L)}{F(c_h)}\right) + \frac{f(c_L)}{F(c_h)} \left[u_e(L) - u_e(c_e)\right] \quad (5)$$

The first term is the marginal cost of the offer $L$. Indeed, the Infringer will get an increase in its cost of making an offer with a probability of $1 - \frac{F(c_L)}{F(c_h)}$. The second term is the marginal benefit of the licensing offer, which is the result of the impact of the offer:

- on the probability of an amicable settlement: $\frac{d}{dL} \left(1 - \frac{F(c_L)}{F(c_h)}\right) = \frac{f(c_L)}{F(c_h)}$,

- on the gains of the negotiation evaluated for the marginal Patent Holder: $u_e(L) - u_e(c_e) = c_e + c_L > 0$.

Substituting for $u_e(L)$, $u_e(c_e)$ and $L = u_h(c_L) - \pi_h^L$, and rearranging the terms gives the first order condition for an interior solution $(L^*, c_h^*)$:

$$0 = - \left(1 - \frac{F(c_h^*)}{F(c_h)}\right) + \frac{f(c_h^*)}{F(c_h)} [c_e + c_h^*] \quad (6)$$
which leads to condition (3). Under assumption 1, it is easy to verify that the second order condition holds (implying more generally that $U_e(L; c_L)$ is concave). Given that the RHS in (3) is increasing in $c_h$, it is obvious that both the existence and uniqueness result from assumption 1. ■

Since under assumption 4, we have $\theta \left[\pi_h^N - \pi_h^L\right] > c_h^*$, the equilibrium of the entry game is associated with a frequency of trial $F(c_h^*)$ and a frequency of settlement $1 - F(c_h^*)$, with $L^* > 0$, where $c_h^*$ is the solution to (5).

- **Stage 3**

Let us show first that by sequential rationality, we have: $\hat{c}_h = \check{c}_h$. As previously shown, Patent Holders with a $c_h < \check{c}_h$ choose Litigate (and, either reach a settlement, or end up in court), implying that they obtain at least $u_h(L^* = u_h(c_h^*)$. Litigating is the best strategy for any $c_h < \check{c}_h$ if:

$$u_h(L^*) = \pi_h^L + \theta \left(\pi_h^N - \pi_h^L\right) - c_h^* > \pi_h^A$$

(7)

But if this inequality holds, it precludes that accommodating for Patent Holders with a $c_h \geq \check{c}_h$ be sequentially rational: given that in the bargaining process, any type $c_h \in [c_h^*, \check{c}_h]$ is pooled together with $c_h^*$, observe that any $c_h \geq \check{c}_h$ would be better off litigating and settling its case rather than accommodating. Thus, by sequential rationality, we must have: $\hat{c}_h = \check{c}_h$. We show now that the inequality (7) always holds, considering the market conditions.

Under the **Lost Profits rule**, with a linear demand $P = a - b(y_h + y_e)$, the equilibrium is achieved, first solving for the first best response function of both firms. In the post-entry game where the Infringer chooses not to infringe, $(y_h^N, y_e^N)$ is the solution to the system:

$$y_h^N = y_h(y_e^N) = \arg\max_{y_h} \left\{\pi_h^N = (P - k_h)y_h \text{ s.t. } P = a - b(y_h + y_e^N)\right\}$$

$$y_e^N = y_e(y_h^N) = \arg\max_{y_e} \left\{\pi_e^N = (P - k) y_e \text{ s.t. } P = a - b(y_e + y_h^N)\right\}$$

In the post-entry game where the Infringer chooses to infringe but the Patent Holder accommodates, $(y_h^A, y_e^A)$ is the solution to the system:

$$y_h^A = y_h(y_e^A) = \arg\max_{y_h} \left\{\pi_h^A = (P - k_h)y_h \text{ s.t. } P = a - b(y_h + y_e^A)\right\}$$

$$y_e^A = y_e(y_h^A) = \arg\max_{y_e} \left\{\pi_e^A = (P - k_e) y_e \text{ s.t. } P = a - b(y_h^A + y_e)\right\}$$

27
When the Patent Holder chooses to litigate, the Infringer’s expected total benefit associated with the decision to infringe is:

\[
U_e (L^*; c_h^e) = (1 - F (c_h^e)) (\pi_e^L - L^*) + F (c_h^e) (\pi_h^L - \theta [\pi_h^N - \pi_h^L] - c_e)
\]

\[
= \pi_e^L - (1 - F (c_h^e)) L^* - F (c_h^e) (\theta [\pi_h^N - \pi_h^L] + c_e)
\]

Substituting with \(L^*\) gives:

\[
U_e (L^*; c_h^e) = \pi_e^L - \theta [\pi_h^N - \pi_h^L] + (1 - F (c_h^e)) c_h^e - F (c_h^e) c_e
\]

Thus, the CN equilibrium \((y_h^L, y_e^L)\) is the solution to the system:

\[
y_h^L = y_h (y_e^L) = \arg \max_{y_h} \{u_h (c_h) = \theta \pi_h^N + (1 - \theta) (P - k_h) y_h - c_h \text{ s.t. } P = a - b(y_h + y_e^L)\}
\]

\[
y_e^L = y_e (y_h^L) = \arg \max_{y_e} \left\{ U_e (L^*; c_h^e) = (P - k_e) y_e - \theta [\pi_h^N - (P - k_h) y_h] + (1 - F (c_h^e)) c_h^e - F (c_h^e) c_e \right\}
\text{ s.t. } P = a - b(y_h^L + y_e)
\]

Under assumptions 2 and 3, the characteristic features of the three market equilibria are described in the next schedule:

<table>
<thead>
<tr>
<th></th>
<th>(y_h^N)</th>
<th>(y_e^N)</th>
<th>(P^N)</th>
<th>(\pi_h^N)</th>
<th>(\pi_e^N)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No Infringement</strong></td>
<td>(\frac{a-2k_h+k}{3b})</td>
<td>(\frac{a-2k_h+k}{3b})</td>
<td>(\frac{a+k_h+k}{3})</td>
<td>(b (y_h^N)^2)</td>
<td>(b (y_e^N)^2)</td>
</tr>
<tr>
<td><strong>Accommodation</strong></td>
<td>(\frac{a-2k_h+k}{3b})</td>
<td>(\frac{a-2k_h+k}{3b})</td>
<td>(\frac{a+k_h+k}{3})</td>
<td>(b (y_h^A)^2)</td>
<td>(b (y_e^A)^2)</td>
</tr>
<tr>
<td><strong>Litigation</strong></td>
<td>(\frac{a-2k_h+k}{b(3-\theta)})</td>
<td>(\frac{a-2k_h+k-\theta(a-k_h)}{b(3-\theta)})</td>
<td>(\frac{a+k_h+k-\theta k_h}{3-\theta})</td>
<td>(b (y_h^L)^2)</td>
<td>(b (y_e^L)^2)</td>
</tr>
</tbody>
</table>

According to the conditions imposed on marginal costs, we have:

\[
y_h^N > y_h^A \text{ and } y_h^L > y_h^A
\]

\[
y_e^N < y_e^A \text{ and } y_e^L < y_e^A
\]

but we need that \(\tilde{k} - k_e > \frac{\theta}{3} (a - 2k_h + \tilde{k}) \Leftrightarrow \theta < \theta_{LP} \equiv 3 \left(\frac{\tilde{k} - k_e}{a - 2k_h + \tilde{k}}\right)\) to have \(y_h^N > y_h^L\) and \(y_e^N < y_e^L\) – i.e. for Lost Profit to be positive, since, given the ranking of output levels, we have the ranking of equilibrium profits:

28
\[ \pi^N_h > \pi^L_h > \pi^A_h \]
\[ \pi^N_e < \pi^L_e < \pi^A_e \]

we also have under this inequality: \( P^L < P^N \).

Thus, the inequality (7) always holds, and we have: \( \hat{\epsilon}_h = \hat{\epsilon}_h \). Then, the belief consistency requirement implies that \( \frac{F(c^*_h)}{F(c^*_h)} = F(c^*_h) \). Finally, the equilibrium in the subgame corresponding to the one described in lemma 5 satisfies the conditions:

\[
\begin{align*}
L^* &= \theta \left[ \pi^N_h - \pi^L_h \right] - c^*_h \\
\left( \frac{1 - F}{f} \right) |_{c^*_h} &= c_e + c^*_h
\end{align*}
\]

**Stage 2**

The Infringer chooses the entry with infringement if \( U_e (L^*; c^*_h) \geq \pi^N_e \) or:

\[
(\pi^L_e - \pi^N_e) - \theta \left[ \pi^N_e - \pi^L_e \right] \geq F(c^*_h) c_e - (1 - F(c^*_h)) c^*_h
\]

Under Lost Profit, the condition \( \theta < \theta_{LP} \) implies \( \pi^N_h - \pi^L_h > 0 \) and \( \pi^L_e - \pi^N_e > 0 \). Thus, both the RHS and the LHS in (8) have an ambiguous sign. It can be verified that:

- \( c^* = F(c^*_h) c_e - (1 - F(c^*_h)) c^*_h \) is an increasing function in \( c_e \) since:

\[
\frac{\partial c^*}{\partial c_e} = F(c^*_h) - ((1 - F(c^*_h)) - f(c^*_h) [c_e + c^*_h]) \frac{\partial c^*_h}{\partial c_e} = F(c^*_h) > 0
\]

- for \( \theta = 0 \), we have \( \pi^L_e = \pi^A_e \) and thus \( U_e (L^*; c^*_h) - \pi^N_e = (\pi^A_e - \pi^N_e) - c^* \). Thus assumption 5 is sufficient to have \( U_e (L^*; c^*_h) - \pi^N_e > 0 \) at \( \theta = 0 \): assumption 5 prevents the size of the legal costs from deterring the Infringer from infringing the patent in the neighborhood of \( \theta = 0 \).

- for \( \theta = \theta_{LP} \), we have \( \pi^N_h = \pi^L_h \) and \( \pi^N_e = \pi^N_e \), which implies that \( U_e (L^*; c^*_h) - \pi^N_e = -c^* \leq 0 \). On the other hand, note also that we have (making use of the envelope theorem):

\[
\frac{\partial U_e}{\partial \theta} (L^*; c^*_h) = \frac{\partial \pi^L_e}{\partial y^L_h} \frac{\partial y^L_h}{\partial \theta} - \left[ \pi^N_h - \pi^L_h \right]
\]

where it can be verified that under the Lost Profit rule: \( \frac{\partial \pi^L_e}{\partial y^L_h} < 0 \) and \( \frac{\partial y^L_h}{\partial \theta} > 0 \) (given our assumptions 2 and 3; see also Choi (2009)). Hence:

- for \( \theta < \theta_{LP} \), we obtain that \( \frac{\partial U_e}{\partial \theta} (L^*; c^*_h) = \frac{\partial \pi^L_e}{\partial y^L_h} \frac{\partial y^L_h}{\partial \theta} - \left[ \pi^N_h - \pi^L_h \right] < 0 \).
- for \( \theta = \theta_{LP} \), we obtain that \( \frac{\partial U_c}{\partial \theta} (L^*; c_h^*) = \frac{\partial U_c^L}{\partial \theta} \frac{\partial y_k^L}{\partial \theta} < 0. \)

To sum up, under the condition \( (\pi^A_e - \pi^N_e) - c_e > 0: \)

- if \( c^* = 0 \) then \( \theta < \theta_{LP} \) implies \( U_e(L^*; c_h^*) > \pi^N_e. \)
- if \( c^* < 0 \) then \( \theta \leq \theta_{LP} \) implies \( U_e(L^*; c_h^*) > \pi^N_e. \)
- if \( c^* > 0 \) then \( \theta < \bar{\theta} < \theta_{LP} \) (where \( \bar{\theta} \) satisfies \( U_e(L^*; c_h^*) = \pi^N_e \)) implies \( U_e(L^*; c_h^*) > \pi^N_e. \)

### 6.2 Proof of propositions 2 and 3

By the same argument as was developed for proposition 1, it is easy to verify that at stage 4 (pretrial bargaining stage) lemma 5 may be substituted by lemma 6, where \( D_{UE} = \pi^L_e - \pi^N_e \) and:

\[
c_L = \theta [\pi^L_e - \pi^N_e] - L
\] (2')

**Lemma 8** Under assumption 1, the solution to the maximization of (3) under (2'), is unique and corresponds to the licensing offer \( L^* \) and the cut-off value for the Patent Holder’s type \( c_L^* = c_h^* \) which are implicitly obtained by solving the system:

\[
L^{**} = \theta [\pi^L_e - \pi^N_e] - c_h^*
\] (9)

\[
\left( \frac{F(c_h) - F}{f} \right)_{|c_h^*} = c_e + c_h^*
\] (10)

We omit the proof since it is similar to the proof of proposition 1. Conditions (4) and (10) are identical. The main differences comparing the Unjust Enrichment rule and the Lost Profit rule arise at stages 2 and 3:

- given that the best settlement offer is now \( L^{**} = \theta [\pi^L_e - \pi^N_e] - c_h^* \), Litigate is the best action for any \( c_h \in [c_h, \bar{c}_h] \) only if:

\[
u_h(L^{**}) = \pi^L_h + \theta [\pi^L_e - \pi^N_e] - c_h^* > \pi^A_h
\] (11)

We will show that this inequality may not always hold, at least for the highest values of \( c_h \).

- let us consider the market stage. When the case is litigated, the Infringer’s expected total benefit associated with the decision to infringe is:
\[ U_e(L^{**}; c_h) = \left(1 - \frac{F(c_h^*)}{F(\hat{c}_h)}\right) (\pi_e^L - L^{**}) + \frac{F(c_h^*)}{F(\hat{c}_h)} (\pi_e^L - \theta [\pi_e^L - \pi_e^N] - c_e) \]

\[ = \pi_e^L - \theta [\pi_e^L - \pi_e^N] + \left(1 - \frac{F(c_h^*)}{F(\hat{c}_h)}\right) c_h^* - \frac{F(c_h^*)}{F(\hat{c}_h)} c_e \]

\[ = \theta \pi_e^N + (1 - \theta) \pi_e^L + \left(1 - \frac{F(c_h^*)}{F(\hat{c}_h)}\right) c_h^* - \frac{F(c_h^*)}{F(\hat{c}_h)} c_e \]

Thus, the Cournot-Nash equilibrium \((y^L_e, y^L_e)\) is the solution to:

\[
y^L_h = y_h(y^L_e) = \arg \max_{y_h} \left\{ u_h(c_h) = (P - k_h) y_h + \theta [(P - k_v) y_e - \pi_e^N] - c_h \text{ s.t. } P = a - b(y_h + y_e) \right\}
\]

\[
y^L_e = y_e(y^L_h) = \arg \max_{y_e} \left\{ U_e(L^{**}; c_h^*) = \theta \pi_e^N + (1 - \theta) (P - k_v) y_e + \left(1 - \frac{F(c_h^*)}{F(\hat{c}_h)}\right) c_h^* - \frac{F(c_h^*)}{F(\hat{c}_h)} c_e \text{ s.t. } P = a - b(y_h + y_e) \right\}
\]

with:

<table>
<thead>
<tr>
<th>Litigation</th>
<th>[ y^L_h = \frac{a - 2k_h + k_v - (a - k_v)}{b(3 - \theta)} ]</th>
<th>[ \pi^L_h = b (y^L_h)^2 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[ y^L_e = \frac{a - 2k_v + k_h}{b(3 - \theta)} ]</td>
<td>[ \pi^L_e = b (y^L_e)^2 ]</td>
</tr>
</tbody>
</table>

| \[ P^L = \frac{a + k_h - k_v}{3 - \theta} \] |

and according to the conditions imposed on marginal costs, it follows directly that24:

\[
y^N_h > y^A_h > y^L_h \Rightarrow \pi^N_h > \pi^A_h > \pi^L_h
\]

\[
y^N_e < y^A_e < y^L_e \Rightarrow \pi^N_e < \pi^A_e < \pi^L_e
\]

This implies that (11) may not hold since under assumption 4: \( \theta [\pi^L_e - \pi^N_e] - c_h^* > 0 \), but now with \( \pi^A_h > \pi^L_h \). Thus, different kinds of equilibria may be obtained in the pretrial negotiation subgame:

- assume \( \pi^L_h + \theta [\pi^L_e - \pi^N_e] - c_h^* > \pi^A_h \): in this case, once more it is easy to see that by sequential rationality \( \hat{c}_h = c_h \). Thus any Patent Holder \( c_h \in [c_h^*, \hat{c}_h] \), where \( c_h^* \) is the same as under the Lost Profit rule, chooses (Litigate, Accept), while any Patent Holder \( c_h \in [\hat{c}_h, c_h^*] \) chooses (Litigate, Reject). In this case, the belief consistency requirement implies that at stage 2, the Infringer obtains \( U_e(L^{**}; c_h^*) \) and Infringe is the best decision if \( U_e(L^{**}; c_h^*) \geq \pi^N_e \) or:

\[ k_e < \hat{k} + \frac{\theta}{6} (a - 2\hat{k} + k_h) \]

which is always satisfied.

\[ ^{24} \text{The condition required for Unjust Enrichment, i.e. to have } \pi^L_e - \pi^N_e > 0, \text{ is:} \]

\[ k_e < \hat{k} + \frac{\theta}{6} (a - 2\hat{k} + k_h) \]
(1 - \theta) (\pi_e^L - \pi_e^N) \geq F (c_h^*) c_e - (1 - F (c_h^*)) c_h^*

Let us investigate the values of \theta that are consistent with such an equilibrium. First, it is easy to verify that for \theta = 0, we have \pi_e^L = \pi_e^A and \pi_h^L = \pi_h^A and thus \mu_h (L^{**}) - \pi_e^A = -c_h^*. Second, as \theta increases, there are two opposites forces driving the sign of \mu_h (L^{**}) - \pi_e^A: on the one hand \pi_h^L - \pi_h^A - c_h^* has a negative impact on \mu_h (L^{**}) - \pi_e^A; on the other hand, \theta [\pi_e^L - \pi_e^N] has a positive impact. Formally, making use of the envelop theorem, we have:

\[ \frac{\partial \mu_h}{\partial \theta} (L^{**}) = \frac{\partial \pi_e^L}{\partial \theta} \frac{\partial h_e^L}{\partial \theta} + \left[ \pi_e^L - \pi_e^N \right] \]

where it can be verified that under the Unjust Enrichment rule: \frac{\partial \pi_e^L}{\partial \theta} < 0 and \frac{\partial h_e^L}{\partial \theta} > 0 (given our assumptions 2 and 3; see also Choi (2009)). Hence, we obtain that:

\[ \frac{\partial \mu_h}{\partial \theta} (L^{**}) = \frac{\partial \pi_e^L}{\partial \theta} \frac{\partial h_e^L}{\partial \theta} + \left[ \pi_e^A - \pi_e^N \right] \]

It can be conjectured that in the neighborhood of \theta = 0, the first term dominates the second such that \frac{\partial \mu_h}{\partial \theta} (L^{**}) < 0; in contrast as \theta becomes large enough, the second term is the dominant one, such that \frac{\partial \mu_h}{\partial \theta} (L^{**}) > 0. In this perspective, let us assume that there exists a \theta'_{UE} < 1 for which \mu_h (L^{**}) = \pi_h^A \Leftrightarrow \theta'_{UE} \equiv \frac{\pi_h^A - (\pi_e^L)_{UE} + c_h^*}{(\pi_e^L)_{UE} - \pi_e^A}. Then: if \theta > \theta'_{UE}, we have \mu_h (L^{**}) > \pi_h^A; i.e. the Patent Holder’s case must be strong enough for it to choose Litigate at equilibrium, whatever its type.

Turning to the Infringer, the best decision of the Infringer at the initial node is Infringe if:

\[ U_e (L^{**}; c_h^*) > \pi_e^N, \text{ which can now be written:} \]

\[ (1 - \theta) \left[ (\pi_e^L)_{UE} - \pi_e^N \right] - c^* > 0 \]

Thus, notice that:

- for \theta = 0, \mu_e (L^{**}; c_h^*) = \pi_e^A - \pi_e^N - c^*; thus, assumption 5 is still sufficient to obtain \mu_e (L^{**}; c_h^*) - \pi_e^A > 0 at \theta = 0.

- for \theta \in (0, 1), (1 - \theta) \left[ (\pi_e^L)_{UE} - \pi_e^N \right] > 0.

- for \theta = 1, \mu_e (L^{**}; c_h^*) = -c^*.

Hence, when c^* \leq 0, then \mu_e (L^{**}; c_h^*) > \pi_e^N \forall \theta \in [0, 1]; but when c^* > 0, let us define as \theta''_{UE} < 1, the patent strength for which \mu_e (L^{**}; c_h^*) = \pi_e^N \Leftrightarrow \theta''_{UE} \equiv 1 - \frac{c^*}{(\pi_e^L)_{UE} - \pi_e^N}; it must be that \theta < \theta''_{UE}, i.e. the Patent Holder’s case must be weak enough for Infringe to be the best
decision for the Infringer: \( U_e(L^*; c_h^*) > \pi_e^N \). Thus, if \( \theta'_{UE} < \theta'_{UE} \), the equilibrium is that described in proposition 2.

- Assume \( \pi_h^L + \theta [\pi_e^L - \pi_e^N] - c_h^* < \pi_h^A \iff \theta < \theta'_{UE} \): in this case, if \( \pi_h^L + \theta [\pi_e^L - \pi_e^N] - c_h^* < \pi_h^A \), then any equilibrium may be built such that whatever the proposal \( L \geq 0 \) made by the Infringer at the pretrial stage, every Patent Holder type chooses \textit{Accommodate}, and the Infringer chooses \textit{Infringe}. In terms of outcomes, all these equilibria are associated with the licensing price \( y_h^A \).

- \textbf{case 1:} Consider that the Infringer proposes \( \hat{L} \) at stage 4; this implies that any Patent Holder \( c_h \in [\hat{c}_h, c_h^*] \) chooses \textit{Litigate} and accepts \( \hat{L} \), while any Patent Holder \( c_h \in [\hat{c}_h, c_h^*] \) chooses \textit{Litigate} and rejects \( \hat{L} \). At stage 2: the Infringer chooses the entry with infringement if \( U_e(\hat{L}; \hat{c}_h) > \pi_e^N \) or:

\[
F(\hat{c}_h) \left( \pi_e^L - \theta (\pi_e^L - \pi_e^N) - c_e \right) + (1 - F(\hat{c}_h)) (\pi_e^L - \hat{L}) \geq \pi_e^N \\
\Downarrow \\
(1 - \theta) (\pi_e^L - \pi_e^N) - F(\hat{c}_h)c_e + (1 - F(\hat{c}_h)) \hat{c}_h \geq 0
\]

Given assumption 5, and \( \pi_e^L - \pi_e^N > 0 \), then once more either \( \hat{c} = F(\hat{c}_h)c_e - (1 - F(\hat{c}_h)) \hat{c}_h = c_e - (1 - F(\hat{c}_h)) (c_e + \hat{c}_h) \leq 0 \), and then \( U_e(\hat{L}; \hat{c}_h) > \pi_e^N \forall \theta \in (0, \theta'_{UE}] \); or \( \hat{c} > 0 \) and \( U_e(\hat{L}; \hat{c}_h) > \pi_e^N \) only for \( \theta \in (0, \theta_{UE}] \) where \( \theta_{UE} < \theta'_{UE} \) is such that \( U_e(\hat{L}; \hat{c}_h) = \pi_e^N \).

Note that the Infringer has no incentive to increase \( L \) over \( \hat{L} \) (in order to induce the separation of Patent Holder’s types between \textit{Accept} and \textit{Reject} with a cut-off value less than \( \hat{c}_h \)). By definition, \( U_e(L; c_L) \) reaches its maximum for \( (L^*; c_h^*) \): by the second order condition (concavity), \( U_e(L; c_L) \) is thus decreasing (respectively increasing) for any \( c_h < (>) c_h^* \), or equivalently, \( U_e(L; c_L) \) is decreasing (respectively increasing) for any \( L > (>) L^* \). Thus for any \( L > \hat{L} \) (associated with \( c_h < \hat{c}_h \)) we have: \( U_e(L; c_L) < U_e(\hat{L}; \hat{c}_h) \).

This is the case described in proposition 3i); however, to complete the proof that this is an equilibrium we must show that it also resists a reduction of \( L \) under \( \hat{L} \). Let us analyze this point:

- \textbf{case 2:} Consider now that the Infringer proposes any \( \hat{L} \in [0, \hat{L}) \) at stage 4; since \( \hat{L} < \hat{L} \) then \( u_h(\hat{L}) < \pi_h^A \), and thus (since a \( \tilde{c}_h \in [c_h, c_h^*] \) exists) any Patent Holder \( c_h \in [\tilde{c}_h, \hat{c}_h] \) now chooses...
Accommodate, while any Patent Holder \( c_h \in [c_h, \bar{c}_h] \) chooses (Litigate, Reject) and obtains a judgment. At stage 2, the Infringer chooses entry with infringement if \( U_e(\hat{L}; \bar{c}_h) > \pi_e^N \) or:

\[
F(\hat{c}_h) \left( \pi_e^L - \theta \left( \pi_e^L - \pi_e^N \right) - c_e \right) + (1 - F(\hat{c}_h)) \pi_e^A > \pi_e^N \\
\downarrow
\]

\[
F(\hat{c}_h) \left( 1 - \theta \right) \left( \pi_e^L - \pi_e^N \right) + (1 - F(\hat{c}_h)) \left( \pi_e^A - \pi_e^N \right) > F(\hat{c}_h)c_e
\]

Under assumption 5 we have: \( \pi_e^A - \pi_e^N - F(\hat{c}_h)c_e > 0 \). For \( \theta = 0 \), we have \( \pi_e^L = \pi_e^A \), and thus \( U_e(\hat{L}; \bar{c}_h) - \pi_e^N = \pi_e^A - \pi_e^N - F(\hat{c}_h)c_e > 0 \); for \( \theta = 1 \) then \( U_e(\hat{L}; \bar{c}_h) - \pi_e^N = (1 - F(\hat{c}_h)) \left( \pi_e^A - \pi_e^N \right) - F(\hat{c}_h)c_e > 0 \). Moreover for any \( \theta \in (0, 1) \):

\[
\frac{\partial U_e}{\partial \theta}(\hat{L}; \bar{c}_h) = (1 - \theta) \frac{\partial \pi_e^L}{\partial y_h} \frac{\partial y_h}{\partial \theta} > 0
\]

since \( \frac{\partial \pi_e^L}{\partial y_h} < 0 \) and \( \frac{\partial y_h}{\partial \theta} < 0 \). As a result, the inequality \( U_e(\hat{L}; \bar{c}_h) > \pi_e^N \) holds once more for any \( \theta \in (0, \theta_{UE}) \). This is the case described now in proposition 3ii).

Thus, in the case \( \theta < \theta_{UE} \) and if there is a \( c_h \in [c_h, \bar{c}_h] \) for which \( \pi_h^A + \theta [\pi_e^L - \pi_e^N] - c_h = \pi_h^A \), then:

- when \( U_e(\hat{L}; \bar{c}_h) > U_e(\hat{L}; \bar{c}_h) \), which requires that: \( \pi_e^A > \pi_e^L - \hat{L} \Leftrightarrow \pi_h^A + \pi_e^A > \pi_e^L + \pi_h^L \), the equilibrium has the features described in proposition 3ii).

- in contrast, when the opposite inequality holds \( \pi_h^A + \pi_e^A < \pi_e^L + \pi_h^L \), the equilibrium is as described in proposition 3i).

### 6.3 Consistency of welfare comparisons

The next table illustrates the difficulties associated with the comparisons of welfare, through a simple numerical simulation.

**Calibration:** \( a = 70; b = 1; k = 20; c_h^* = 0 = c^* \)

<table>
<thead>
<tr>
<th>( k_h = 15; k_e = 18 )</th>
<th>( k_h = 15 = k_e )</th>
<th>( k_h = 4 = k_e )</th>
<th>( k_h = 15; k_e = 8 )</th>
<th>( k_h = 8; k_e = 18 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{LP} )</td>
<td>0.1</td>
<td>0.25</td>
<td>0.58</td>
<td>0.6</td>
</tr>
<tr>
<td>( \theta'_{UE} )</td>
<td>0.99</td>
<td>0.79</td>
<td>0.57</td>
<td>0.35</td>
</tr>
<tr>
<td>( \theta''_{UE} )</td>
<td>0.9</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
This is only illustrative, since we do not provide a comprehensive analysis of sensitivity, and moreover we ignore the influence of legal costs. However, it shows that the result may be very sensitive to the difference between marginal production costs. In this set of computations, we find that the comparison of welfare is consistent (given that $\theta'_{UE} < \theta_{LP} < \theta''_{UE}$) only for $k_h = 15, k_e = 8$ (fourth column).