Social Wealth and Optimal Care*

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Abstract

When accidents result in noncompensable losses, a monetary payment is not enough to compensate the victim. We study the characteristics of optimal levels of care and distribution of risk under these circumstances and show that care depends on the aggregate wealth of society but does not depend on wealth distribution. We then examine whether ordinary liability rules, regulation, insurance, taxes and subsidies can be used to implement the first-best outcome (in terms of both care and risk). Finally, our results are discussed in the light of fairness considerations (second best) and in the special case of accidents between individuals and a firm.

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1 Introduction

Should rich countries spend more on accident prevention than poor countries? Should compensation be paid for noncompensable losses and how much? These questions are intertwined seen as most accidents result in bodily injury or death, in addition to pecuniary losses. Medical malpractice cases, workplace injuries, industrial and traffic accidents (the greatest contributor to tort litigation) routinely involve such losses, which are difficult or impossible to compensate. For instance, in 1997 the estimated number of deaths in medical malpractice cases in the US alone is between 44,000 and 98,000 (Kessler and Rubinfeld, 2007, p. 352). This is not to say that these losses will not be evaluated in monetary terms by courts. Rather, no amount of money can bring, say, a paraplegic victim or the relatives of a deceased back to their previous levels of welfare (Viscusi, 2000, p. 660). Another example of noncompensable losses may be found with environmental pollution, when the harm to the environment is irreversible or results in permanent damage for large numbers of individuals.

We study the optimal design of liability rules for accidents causing irreparable harm, considering both the allocation of risk and the level of care. We focus on the question whether the level of care taken by injurers should depend on individuals’ risk aversion and wealth. With respect to wealth, we look at both the aggregate wealth of society (that is, we compare rich and poor countries) and its distribution among individuals (that is, we compare countries with a relatively equal distribution of wealth with countries with unequal wealth distribution).

Our approach departs from previous literature in three ways. First, we consider noncompensable harm. The literature on tort liability has mainly analyzed accidents where the harm is pecuniary and, thus, can be perfectly compensated, at least in principle (for a review see Arlen, 2000). This may be the case when the accident results in minor property damage only. There is consensus on the fact that, if harm is pecuniary, the first-best solution consists of separating the incentive problem from the risk-sharing and the wealth-distribution problems. Consequently, liability rules should provide incentives for injurers to take the level of care that minimizes the total accident costs, insurance should provide for an optimal allocation of risk, and the income tax system should provide for redistribution (Shavell, 1981; Kaplow and Shavell, 1994 and 2000).

In other cases, harm is compensable but insurance is not available (Arlen, 1992a; Miceli and Segerson, 1995) or insurers cannot cope with the moral hazard problem effectively (Shavell, 1982 and 1987). Examples of the former scenario are situations in which the risk is large and systemic or when the law or commercial practices forbid insurance (for instance, flood insurance is unavailable in the Netherlands: Botzen and van den Bergh, 2008). Examples of the former scenario are those situations in which the insured party can take important but

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1Calabresi (1970) indicated both incentives and risk-sharing as two important goals for the liability system (the third being the minimization of the administrative costs of the system). Subsequent economic literature first tackled the incentive problem, starting from Brown (1973), and later focused also on the risk-sharing problem, beginning with Shavell (1982). See also Landes and Posner (1987) and Shavell (1987).
unverifiable precautionary measures to reduce the entity of the harm. In those cases, perfectly separating risk-sharing from incentives is not possible and the socially optimal level of care depends on the parties’ risk aversion. Likewise, when redistribution by independent means is not possible, the socially optimal level of care depends on the distribution of wealth among individuals (Arlen, 1992a; Greenwood and Ingene, 1978; Graff Zivin and Small 2003; Segerson, 1986). However, the role played by society’s aggregate wealth has not been investigated yet.

In order to bring noncompensable harm into the analysis, we employ a state-dependent expected utility (SDEU) representation for the individuals’ utilities (Hirschleifer, 1970; Keeney and Raiffa, 1976; Karni, 1985) and compare our results with those above. This is in order to account for the fact that, after an accident, victims may find themselves in an inferior state (suffering from posttraumatic stress disorder, being on a wheelchair or having lost a child), in which their utility is less than it was before the accident. Thus, damages are not enough to bring victims back to their initial utility, irrespective of the magnitude of the compensation (Cook and Graham, 1977). Injurers, in turn, might also be in an inferior state after the accident. They might suffer from bad reputation, loss of job opportunities, or remorse as a result of being involved in an accident in addition to having to pay damages (Nielson and Winter, 1997). Moreover, there is evidence that not only do such injuries change absolute levels of utility but they also affect the marginal utility of money by increasing or reducing it. For instance, Viscusi and Evans (1990) report a reduction in the marginal utility of wealth following a job injury and, more generally, existing literature considers a reduction of marginal utility more likely and hence advises for leaving noncompensable losses uncompensated and supplementing liability with fines (Spence, 1977; Calfee and Rubin, 1992; Shavell, 2004, pp. 269-275). We do not take sides concerning this empirical question and examine both increased and reduced marginal utility.

Second, with the exception of Cooter and Schäfer (2009), existing literature does not account for the fact that accidents have a negative effect on the overall wealth of society. This effect is particularly evident in large-scale disasters, such as the Bhopal gas leak in 1984, the Chernobyl nuclear plant explosion in 1986 and the New Orleans levee system failure due to hurricane Katrina in 2005, which caused enormous and lasting financial and human costs. After an accident, society as a whole is poorer than it was before and, hence, social wealth becomes an important variable in the determination of the optimal response to accidents. We take this effect into account and examine how social wealth impacts the optimal level of care.

The third departure from existing literature consists of bringing into consideration the distinction between precautions that affect the magnitude of the harm (self-insurance) and precautions that affect the probability of accidents (self-protection). Safety measures such as evacuation plans, timely alarms, or helmets reduce the magnitude of the harm but not the probability of the accident; thus, accidents requiring these safety measures are of the self-insurance type. Conversely, routine safety checks and procedures in a nuclear power plant
or on an airplane, radars and sonars on a ship, or the containment of pollutants (Kornhauser and Revesz, 1990) reduce the probability, but not (or only marginally) the magnitude of the loss; in this case we speak of self-protection. The distinction between self-insurance and self-protection has been developed in the insurance literature (Ehrlich and Becker, 1967; Sweeney and Beard, 1992), which deals with a party who takes precautions in order to prevent harm to himself. In contrast, our model involves an injurer creating risks for passive victims. Tort literature has exploited this distinction solely with reference to the incentive problem (Boyd and Ingberman, 1994; Dari-Mattiacci and De Geest, 2005), while papers dealing with incentives and risk focus exclusively on self-protection. The present analysis bridges this gap in the literature by also considering accidents of the self-insurance type, where the probability of occurrence is exogenous, but the injurer can reduce the magnitude of the harm by taking care. Our model first describes those situations and focuses both on risk and on incentives. Then, we extend some of our results to other scenarios, including the case of self-protection.

We first describe a care-free world—in which the only problem is to allocate risk optimally—and a risk-free world—in which the only problem is to induce an optimal level of care. In relation to these two benchmark cases, we study the properties of the first-best allocation of risk and the first-best level of care in a world in which both aspects are equally important. Starting from self-insurance, we focus on four characteristics of the first best and demonstrate the following results:

1. Mutuality: The costs of accidents should be borne in some proportion by both parties. Consequently, an accident results in a reduction of wealth and utility for both parties, which runs against principles of full compensation of innocent victims and the traditional results obtained in case of compensable losses.

2. Care and risk: Compared to accidents involving only pecuniary losses, accidents involving noncompensable losses do not necessarily require higher levels of care. If the marginal utility of money is greater after an accident, a higher level of care is socially optimal; however, the opposite might be true if the marginal utility of money is less after an accident.

3. Care and society’s wealth: The socially optimal level of care depends on society’s initial wealth. In contrast, the distribution of the social wealth among individuals is irrelevant for the level of care, if wealth can be redistributed by other means. Whether care increases or decreases in society’s wealth depends in turn on the individuals’ tolerance to risk. This result crucially depends on the assumption that both parties have state-dependent preferences. If this is not the case—if, for instance, the injurer is a firm—then care does not depend on society’s wealth.

4. Care and probability of accidents: The socially optimal level of care increases when the probability of accidents increases.

\footnote{See also Jullien, Salanié and Salanié (1999) focusing on the relationship between risk-aversion and care.}
It is shown that these characteristics apply to the first best—where the allocation of risk is optimal—but are also (weakly) persistent in the second best—where fairness concerns suggest that one party (usually the victim) should be fully insured against accidents. The analysis is then extended to a setting with varying numbers of victims holding total harm fixed. The presence of multiple victims allows for a natural spreading of risk and is shown to have the same consequences as an increase in social wealth.

Although it is plausible that both the victim and the injurer in traffic accidents or medical malpractice cases have state-dependent preferences, it might be argued that in some industrial accidents the injurer (for instance, a firm with diffuse shareholding) is risk-neutral and his preferences are not state-dependent. In this case, we show that the optimal level of care is the same as in the risk-free world and does not depend on social wealth, while all other results still apply. This finding suggests that industrial accidents in the developing world should require the same level of care as expected in richer countries.

After describing the characteristics of first- and second-best policies concerning care and risk-sharing, we investigate what instruments can be used to implement these policies. A mix of regulation of safety, taxes and subsidies is enough to achieve the first best. This should be no surprise. In fact, the policy maker has three instruments to reach three goals: enforce the desired level of care and redistribute wealth in the good and the bad state of the world. We further examine whether ordinary liability rules such as strict liability and simple negligence, possibly paired with some forms of redistributive taxation or private insurance can reach the first or the second best. In general, contrary to the literature dealing with compensable losses (Shavell, 1982; Miceli and Segerson, 1995) we find that the second best is attainable while the first best requires additional instruments. Our results are different because we consider instances in which risk is not perfectly diversifiable. At the end of each subsection we explain how the results change if the injurer is a firm: in general, in this case it is easier to attain the first best.

Further, we analyze two extensions of our model: self-protection and the case in which the noncompensable loss depends on care. We find that the mutuality principle extends to these cases and qualify other results. In particular, if the noncompensable loss depends on care, we find that more risk increases the level of first-best level of care. Moreover, the result about care and society’s wealth partially applies to self-protection. The optimal level of care depends on the aggregate wealth of society but does so in an ambiguous way: care might increase or decrease with social wealth. The same ambiguity surrounds the analysis of care and risk and care and magnitude of accidental losses, the self-protection equivalent of the problem care and probability of accidents that we analyze under self-insurance. Our analysis suggests that the level of care that should be enforced by means of regulation or liability rules depends on the aggregate wealth of society but not on the distribution of such a wealth among individuals. The reason is intuitive: any distributional issue can be corrected by redistributing wealth, while the total wealth of society is an insurmountable constraint. However, if the injurer is a firm, care should not depend on social
This paper is structured as follows: in Section 2, we present the properties of the first- and second-best levels of care and distribution of risk. In Section 3, we discuss the implementation of the first and second best by means of ordinary liability rules, regulation, insurance, taxes and subsidies. In Section 4, we present a discussion of the results and two extensions—self-protection and the case in which the magnitude of the noncompensable loss depends on care—discuss the case of bilateral accidents and briefly comment on other possible extensions including levels of activity and insolvency. In Section 5, we conclude.

2 Analysis

2.1 The basic framework

We consider a simple society with two different groups of identical individuals, injurers and victims, who are initially endowed with wealth \( w = w_0 \) and \( y = y_0 \), respectively, and are strangers to each other. Note that \( W_0 = w_0 + y_0 \) represents society’s initial aggregate wealth—henceforth simply society’s wealth.

An injurer’s activity may result in an accident with an exogenous probability \( p > 0 \); if an accident occurs, a victim suffers a pecuniary loss \( h(x) \), which depends on the injurer’s pecuniary investment in care \( x \), with \( h'(x) < 0 \), \( h''(x) > 0 \), \( h(0) = H > 0 \), and \( h'(\infty) \to 0 \). We assume that it is always profitable both for the injurer and for society that the injurer undertakes such an activity. Thus, our analysis does not address questions concerning the optimal level of activity.

The loss also implies the destruction of an irreplaceable asset for both parties. To capture this aspect, we employ a SDEU representation of individuals’ utilities: \( u_i(w) \) and \( v_i(y) \) denote the injurer’s and the victim’s utility, which are functions of their wealth, with \( u''_i(w), v''_i(y) < 0 \); \( i = b \) in the accident state (the “bad” state) and \( i = g \) in the no-accident state (the “good” state). We also require the following assumption to hold:

**A1:** The good state is better than the bad state. An individual never prefers the bad state over the good state, whatever his wealth; \( \forall w : u_g(w) \geq u_b(w) \) and \( \forall y : v_g(y) \geq v_b(y) \).

We first consider the case in which a benevolent social planner can command a certain level of care \( x \) and a certain allocation of risk \( (w_b, w_g; y_b, y_g) \). The planner’s objective is to maximize social welfare, defined as follows:

\[
SW = p [u_b(w_b) + v_b(y_b)] + (1 - p) [u_g(w_g) + v_g(y_g)]
\]

subject to the resource constraints \( w_b + y_b = w_0 + y_0 - h(x) - x \), in the bad state, and \( w_g + y_g = w_0 + y_0 - x \), in the good state.
2.2 Benchmark cases

We will analyze the social planner’s problem against two standard benchmark cases: one in which the planner only deals with risk-spreading and another in which the planner only deals with incentives.

B1: The care-free world. The first scenario concerns a world without safety technology, in which the injurer is not able to reduce the pecuniary loss $H$ to the victim in case of an accident (that is, all accidents are due to natural events) and hence the planner only optimizes the spreading of risk. We refer to this scenario as the care-free world. Under standard assumptions, a Pareto-efficient allocation of risk implies that both the injurer’s and the victim’s wealth are larger in the good state than in the bad one: $y_b < y_g$ and $w_b < w_g$. This allocation of risk is said to be comonotonic in the sense that, the richer the society as a whole in a state, the richer its members individually.$^3$

B2: The risk-free world. In the second scenario, there is a safety technology but individuals are risk-neutral and have state-independent utility functions—that is, the harm is entirely pecuniary: a risk-free world. In this case, any feasible allocation of risk is Pareto efficient, and the first-best level of care satisfies:

$$ph’(\hat{x}) + 1 = 0$$

The socially optimal, risk-free level of care $\hat{x}$ increases in $p$ but depends neither on society’s aggregate wealth, nor on the distribution of wealth among individuals.

2.3 First-best allocation of risk and level of care

Our model encompasses situations in which both risk and care are important; thus, the first best is characterized by a certain level of care $x$ and a certain sharing of the aggregate risk in each state $(w_b, w_g; y_b, y_g)$ which maximize (1). In this section, we examine the characteristics of the first best and relate them to the benchmark cases discussed above.

2.3.1 Mutuality

Accidents reduce the aggregate wealth of society. However, this loss can in principle be allocated in many different ways between the parties involved. The following proposition puts some restrictions on such feasible allocations of risk.

$^3$The literature on optimal risk sharing does not consider care choices by the parties but consistently assumes that the aggregate risk is a nondiversifiable risk and parties are (state independent) risk-averse; that is, no one is risk-neutral and no institution exists to deal with the aggregate risk. This is what we call “care-free world”. Note that in this world, by construction, the property of comonotonic allocation of risk also holds when one of the agents is risk neutral, although in a weak sense: $y_g \geq y_b$ (every agent obtains a constant allocation $y_g = y_b$, except for the risk neutral agent for whom $y_g > y_b$). See Arrow (1964), Borch (1962) and Raviv (1979) for Expected Utility models; extensions to Non-expected Utility models are provided by Landsberger and Meilijson (1994) and Chateauneuf, Dana and Tallon (2000).
Proposition 1  The first-best allocation of risk is comonotonic, \( w_b \leq w_g \) and \( y_b \leq y_g \) (mutuality principle), and satisfies the Borch conditions, \( u'_b(w_b) = v'_b(y_b) \) and \( u'_g(w_g) = v'_g(y_g) \).

Proof.  See appendix.

According to the mutuality principle already established in the care-free world, it is efficient to give both individuals a level of wealth in the good state that is at least as large as in the bad state, since the aggregate social wealth in the good state is always larger than in the bad state. Proposition 1 extends such a principle to accident cases in which the cost of care is also considered. Note, however, that not all comonotonic allocations are efficient but only those which satisfy the Borch conditions. Those conditions ensure that an efficient allocation of risk is reached when, in each state, the aggregate social wealth is shared so that the injurer’s marginal utility of wealth equals the victim’s marginal utility of wealth.

It is important to note that the optimal allocation of risk implies that, in relation to the parties’ wealth, neither the injurer nor the victim obtains full insurance, that is, neither of them obtains the same wealth in the bad state as in the good state. This implies that in relation to the parties’ utility, neither the injurer nor the victim is protected against adverse changes in his utility, that is, their utility is necessarily less in the bad state than in the good state. This result matches what already observed about the allocation of risk in a care-free world.

2.3.2  Care and risk

In order to characterize the first-best level of care and compare this general scenario with the benchmark case of a risk-free world, we need to impose some structure regarding the way in which the accident affects the marginal utility of wealth for the parties. We consider two alternative situations:

\( A2a: \) Money is more useful in the good state. The parties’ marginal utility is never greater in the bad state than in the good state, at any level of wealth; \( \forall w : u'_g(w) \geq u'_b(w) \) and \( \forall y : v'_g(y) \geq v'_b(y) \).

\( A2b: \) Money is more useful in the bad state. Under this alternative assumption, the marginal utility of money is never less in the bad state than in

\(^4\) Note that we arrive at a unique solution in that we have implicitly assigned the same weight to the victim and the injurer in the calculation of social welfare. More generally, given different weights one can arrive at different Pareto optimal allocations.

\(^5\) Unless some restrictive conditions are satisfied, the relations in Proposition 1 are strict inequalities.

\(^6\) The situations we are about to describe have been extensively discussed in the literature about irreplaceable commodities (Cook and Graham, 1977) or about self-protection expenditures and the willingness to pay for safety, health and life (Dehez and Drèze, 1987; Jones-Lee, 1974).
the good state, at each level of the parties’ wealth; \( \forall w : u’_g(w) \leq u’_b(w) \) and \( \forall y : v’_g(y) \leq v’_b(y) \).

The next proposition addresses the question how the general case compares with the risk-free benchmark. In particular, is the level of care greater or less than in a risk-free world?

**Proposition 2** For any first-best allocation of risk:

i) Under assumption A2a, the first-best level of care may be greater or less than the risk-free level of care;

ii) Under assumption A2b, the first-best level of care is greater than the risk-free level of care.

**Proof.** In the appendix, it is shown that given a first-best allocation of risk, the first-best level of care \( x^* \) satisfies the following condition:  
\[
ph'(x^*) + 1 = (1 - p) \left( 1 - \frac{v'_g(y_g)}{v'_b(y_b)} \right) \tag{3}
\]
Moreover, according to the Borch conditions, the allocation of risk satisfies:
\[
\frac{v'_g(y_g)}{v'_b(y_b)} = \frac{u'_g(w_g)}{u'_b(w_b)} \tag{4}
\]
The condition in (3), implies that the optimal level of care \( x^* \) does not necessarily minimize the expected cost of accidents as it is the case in an risk-free world. In contrast, the optimal level of care is adjusted to respond to risk. Consequently, the first-best level of care \( x^* \) is greater than the first-best level of care \( \hat{x} \) in a risk-free world if the right-hand side of (3) is positive, and is less otherwise.

Recall that, according to Proposition 1, we must have \( y_g \geq y_b \) and, by concavity, \( v'_g(y_g) \leq v'_b(y_b) \). Therefore: i) Under assumption A2a we have \( v'_g(y_g) \geq v'_b(y_b) \), such that it may be the case that \( \frac{v'_g(y_g)}{v'_b(y_b)} \geq 1 \) and, thus, also \( x^* \geq \hat{x} \). ii) Under assumption A2b we have \( v'_g(y_b) \leq v'_b(y_b) \), which implies \( \frac{v'_g(y_b)}{v'_b(y_b)} \leq 1 \) and \( x^* \geq \hat{x} \).  

These results imply that the first-best level of care is generally different from the level of care that minimizes the sum of accident loss and cost of care, and it depends on the difference in marginal utilities between states. Our findings can be explained as follows: Care expenditures are a cost both in the good state and in the bad state, while reducing the magnitude of the loss in the bad state only. More precisely, socially optimal investment in care reduces social wealth in the good state—a self-evident observation—while increasing social wealth in the bad state.  

It is as if care realized an implicit transfer of wealth from the

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\footnote{Note that the condition in the text does not guarantee the uniqueness of the result, since the right-hand side in (3) is not monotonic in \( x \).}

\footnote{In the appendix, it is shown that (3) may be written as \( h'(x^*) + 1 = -\frac{1 - p}{p} \frac{v'_g(y_g)}{v'_b(y_b)} < 0 \), which implies that the total accident cost in the bad state (victim’s loss plus cost of care) is decreasing at the socially optimal level of care.}
good state to the bad state, with the effect of reducing the difference in wealth between states.

When the marginal utility of money is larger in the bad state (A2b), it is clear that increasing care expenditures above the risk-free level is advantageous. When, instead, the marginal utility of money is larger in the good state, two contrasting forces govern the result. On the one hand, transferring money from the good state to the bad state is less desirable and might result in first-best care being less than in the risk-free world. On the other hand, risk-aversion might impose higher care expenditures, reversing the result. The following corollary clarifies the role of risk-aversion in determining the first-best level of care.

**Corollary 3** For any first-best allocation of risk, if one party is risk-neutral:

i) Under assumption A2a, the first-best level of care may be greater or less than the risk-free level of care;

ii) Under assumption A2b, the first-best level of care is greater than the risk-free level of care.

**Proof.** If the injurer is risk-neutral (but has state-dependent preferences), his marginal utility is constant in wealth although not equal across states. According to the Borch conditions, this implies: $\frac{v_0(y_g)}{v_0(y_b)} = k$, where $k$ is a constant satisfying $k > 0$ and $k \neq 1$. Thus, from Proposition 2 it is easy to see that the relation between $x^*$ and $\hat{x}$ depends on whether $k$ is greater or less than 1, which in turn is determined by whether A2a or A2b applies.

The logic of this result is that the risk-neutral party (for instance, the injurer) can indemnify the risk-averse party (for instance, the victim), partially countering the effect of risk aversion. The risk-averse victim will not exhibit the same marginal utility across states but her marginal utilities will be proportional: $v_0(y_g) = kv_0(y_b)$. Thus, the marginal utility of money is still a relevant factor determining departures from the risk-free level of care.

### 2.3.3 Care and society’s wealth

An often debated question is whether the first-best level of care increases or decreases when society’s wealth increases. That is, are rich societies better off by exerting greater or less care than poor ones? The next proposition shows that the relationship between care and society’s wealth depends on society’s *tolerance to risk* in the good state relative to that in the bad state.\(^9\)

Society’s *tolerance to risk* in a given state is defined as the sum of the individual indexes of tolerance to risk in that state. In turn, an individual’s index of risk tolerance is simply the inverse of his index of absolute risk aversion. That is, an individual who is only slightly risk averse is said to have a high tolerance to risk. Formally,

\[^9\]Karni (1983 and 1985) compares risk aversion across individuals. He shows that the measure of the risk premium for state-dependent preferences is the mathematical expectation of the Arrow-Pratt premium across states. Thus, an individual is more risk averse than another if and only if his Arrow-Pratt premium is larger in each state of the world. Our results require the comparison of risk aversion across states rather than across individuals and, hence, we allow for risk aversion to vary in different states.
let \( t_v^g = -\frac{u'_v(w_0)}{v'_g(y_0)} \), \( t_b^g = -\frac{u'_v(w_0)}{v'_g(y_0)} \), and \( t_v^b = -\frac{u'_v(w_0)}{v'_b(w_0)} \), be the indexes of absolute tolerance to risk, respectively for the victim and the injurer in the good and bad states—where the arguments of the indexes (individuals’ wealth) are omitted in order to keep notation simple.

**Proposition 4** For any first-best allocation of risk:

i) If society’s risk tolerance is larger in the good state, then the first-best level of care decreases in society’s wealth.

ii) If society’s risk tolerance is larger in the bad state, then the first-best level of care increases in society’s wealth.

**Proof.** Society’s initial wealth is \( W_0 = w_0 + y_0 \). In the appendix, it is shown that:

\[
\frac{\partial x^*}{\partial W_0} = \frac{\frac{t_v^u + t_v^g}{t_v^b + t_v^g} - 1}{\frac{h''(x^*)}{t_v^u + t_v^g} (t_v^u + t_v^g) + \frac{t_v^u + t_v^g}{t_v^b + t_v^g} (h'(x^*) + 1) - 1}
\]  

(5)

Given that according to the second order condition for the first best,\(^{10}\) the denominator has a negative sign, we have:

\[
\text{sign} \frac{\partial x^*}{\partial W_0} = \text{sign} \left( 1 - \frac{t_v^u + t_v^g}{t_v^b + t_v^g} \right)
\]

yielding the results. □

Four important implications follow from this proposition. First, the first-best level of care depends on society’s wealth. However, the direction of this relation in turn depends on society’s tolerance to risk. Thus, for richer societies it might be optimal to take either more or less care than for poorer societies, depending on a further condition.

Second, this condition states that society should take more care when richer than when poorer if its risk tolerance is higher in the bad state and, vice versa, society should take less care when richer than when poorer if the opposite applies. As we have previously observed, the fact that the harm can be reduced by taking care implements an implicit transfer of wealth from the good state to the bad state, thereby reducing the spread between the states. Thus, as society becomes richer, the issue is to decide in which state to benefit from this increase in wealth. According to Proposition 4, more wealth should be available in the state where society is less sensitive to the dispersion in the distribution of wealth between the states.

Third, the first-best level of care only depends on society’s initial wealth and not on its distribution among individuals in the following sense: purely redistributive changes in individuals’ initial wealth that keep the initial aggregate wealth constant do not affect care. This means for example that two equally rich countries differing only with regard to the distribution of wealth\(^{11}\) should invest

\(^{10}\)See the proof of Proposition 2 in the appendix.

\(^{11}\)Consider, for instance, the countries \((w_0, y_0)\) and \((w'_0, y'_0)\) with \(w_0 + y_0 = W_0 = w'_0 + y'_0\).
the same amount of resources in care. This result follows from the assumption that wealth be redistributed freely among individuals.

Finally, due to the mutuality principle, as a society becomes richer both parties become richer in both states. However, which party most benefits from this improvement depends on risk tolerance. From the Borch conditions we can derive the following relationships (see appendix).

\[
\frac{\partial y_b}{\partial W_0} = \frac{t_v^b}{t_u^b} \frac{\partial w_b}{\partial W_0}
\]

\[
\frac{\partial y_g}{\partial W_0} = \frac{t_v^g}{t_u^g} \frac{\partial w_g}{\partial W_0}
\]

These conditions imply that the party exhibiting larger tolerance to risk in a state will capture a larger portion of the increase in society’s wealth in that state. This in turn implies that if the injurer’s risk tolerance is larger in both states, the injurer should pay a decreasing portion of the total cost of accidents as society becomes richer. This result can lead to a decrease of the injurer’s liability towards the victim. Vice versa, if the victim’s risk tolerance is larger in both states, the injurer should pay an increasing portion of the total cost of accidents as society becomes richer, yielding the opposite result.

2.3.4 Care and probability of accidents

**Proposition 5** An increase in the probability of accidents leads to an increase in the first-best level of care, an increase in both individuals’ wealth in the bad state, and a decrease in both individuals’ wealth in the good state.

**Proof.** See appendix. ■

An increase in $p$ implies that the bad state becomes relatively more probable than the good state. Hence, all else equal, society moves to a more risky (in the sense of the first order stochastic dominance) distribution of wealth between states, which deteriorates its welfare. An increase in care implicitly transfers some wealth from the good state to the bad state, as previously explained. This transfer is advantageous because it realizes a less spread distribution of wealth between states (in the sense of the second order stochastic dominance), reducing society’s exposure to risk. As care increases and wealth is implicitly transferred from the good state to the bad state, the mutuality principle guarantees that both individuals become richer in the bad state and, consequently, poorer in the good state.

2.4 Fairness and second best: One party is fully insured

In this section, we focus on situations in which one party obtains full insurance against accidents. Full insurance implies that a party receives the same wealth, irrespective of the state of the world that materializes. Fully insuring one party realizes a second-best (Pareto-constrained) outcome, since we have seen that in
the first best both parties should bear some risk, even when one of them is risk neutral.

Accident prevention policies might be constraint by several considerations. Fairness, for instance, might require that innocent victims do not suffer any reduction in their wealth as a consequence of accidents that they were not in a position to avoid. We refer to this outcome as second best. Note, however, that the same reasoning could in theory apply to injurers involved in activities that are perceived as socially valuable, such as doctors or rescue teams in relation to harm arising from unsuccessful rescue operations. In this case we speak of dual second best; although the results below are derived with respect to the second best, they also apply to the dual second best.

Consider the case in which the victim is fully insured so that his wealth is constant across states: \( y_b = y_g = \bar{y} \). A specific case of full insurance obtains when \( \bar{y} = y_0 \), that is, the victim is guaranteed its initial level of wealth in all states. In this section, we treat this problem as a special case of the more general framework in which \( \bar{y} > y_0 \), that is, the victim’s wealth is constant across states but might be either greater or less than his initial wealth.

More specifically, we obtain the following results regarding the properties of the second best:

**Proposition 6** When fairness is considered:

i) The second-best allocation of risk is comonotonic;

ii) Under assumption A2a, the second-best level of care may be greater or less than the risk-free level of care; under assumption A2b, the second-best level of care is greater than the risk-free level of care;

iii) If the injurer’s risk tolerance is larger in the good state, then the second-best level of care decreases in society’s wealth; if the injurer’s risk tolerance is larger in the bad state, then the second-best level of care increases in society’s wealth;

iv) The second-best level of care increases with the probability of accidents.

**Proof.** Claim i) is self-evident. Concerning claims ii) to iv), given that the victim’s wealth is constant across states, the social problem in (1) simplifies to:

\[
SW_{\bar{y}} = pu_b(w_0 + y_0 - \bar{y} - h(x) - x) + (1 - p)u_g(w_0 + y_0 - \bar{y} - x)
\]

where the only independent variable is the level of care \( x \). The solution, denoted as \( x_{\bar{y}} \), satisfies the following first order condition:

\[
ph'(x_{\bar{y}}) + 1 = (1 - p) \left( 1 - \frac{u'_g(w_0 + y_0 - \bar{y} - x_{\bar{y}})}{u'_b(w_0 + y_0 - \bar{y} - h(x_{\bar{y}}) - x_{\bar{y}})} \right)
\]

(6)

The rest of the proof is analogous to the proofs of propositions 1 to 5. ■

Accordingly to this Pareto-constrained solution, the injurer obtains \( w_b = w_0 + y_0 - \bar{y} - x - h(x) \), in the bad state, and \( w_g = w_0 + y_0 - \bar{y} - x \), in the good state. It is easy to see that this allocation of risk does not generally correspond to the
first best, since there is no guarantee that the Borch conditions in Proposition 1 are satisfied. Nevertheless, the resulting second-best allocation of risk and level of care have most of the characteristics found for the first best.

2.5 Harm is spread among \( N \) victims

Where an accident involves \( N \) victims, we can compare situations in which the harm resulting from an accident is spread among a varying number of victims. The first best is found by maximizing:

\[
SW = pu_b(w_b) + (1 - p)u_g(w_g) + \sum_{i=1}^{N} (pu_b^i(y_b^i) + (1 - p)u_g^i(y_g^i))
\]

subject to:

\[
w_b + \sum_{i=1}^{N} y_b^i = W_0 - h(x) - x
\]

\[
w_g + \sum_{i=1}^{N} y_g^i = W_0 - x
\]

\[
w_0 + \sum_{i=1}^{N} y_0^i = W_0
\]

with \( h(x) \) denoting the aggregate harm to all victims. Conditions (3) and (4) still hold and the Borch conditions now are:

\[
u_0^i(b) = u_b^i(y_b^i) \quad \forall i = 1, ..., N
\]

\[
u_0^i(g) = u_g^i(y_g^i) \quad \forall i = 1, ..., N
\]

such that (5) becomes:

\[
\frac{\partial x^*}{\partial W_0} = \frac{T^*_b - 1}{T^*_b (h''(x)) + 1 - 1}
\]

where: \( T_g = t^u_g + \sum_{i=1}^{N} t^v_i g^i \) and \( T_b = t^u_b + \sum_{i=1}^{N} t^v_i b^i \). It is easy to see that propositions 1 to 5 apply.

In the specific case where all victims have the same initial wealth \( y_0 \) and the same preferences, we have \( W_0 = w_0 + Ny_0 \), \( T_g = t^u_g + Nt^v_g \) and \( T_b = t^u_b + Nt^v_b \); hence, an increase in \( N \) yields an increase in \( W_0 \) such that we have \( \frac{\partial W_0}{\partial N} = y_0 \), which in turn implies that an increase in the number of victims affects care in the same way as an increase in society’s wealth:

\[
\frac{\partial x^*}{\partial N} = y_0 \frac{\partial x^*}{\partial W_0}
\]

Consequently, Proposition 4 can be extended to describe the effects on a varying number of victims on care. These results can be summarized by the following proposition:
Proposition 7 For any first-best allocation of risk:
i) As the number of victims increases, if the injurer’s wealth in one state increases (decreases) then the victims’ wealth must also increase (decrease). More precisely:
   ii) If society’s risk tolerance is larger in the good state and if $y_0 \in ]y_b, y_g[$, then both individuals’ wealth increases in the bad state but decreases in the good state as $N$ increases;
   iii) If society’s risk tolerance is larger in the bad state and if $y_0 < y_b$, then both individuals’ wealth decreases in the bad state as $N$ increases, while the effect is ambiguous in the good state;
   iv) If society’s risk tolerance is larger in the bad state and if $y_0 > y_g$, then both individuals’ wealth increases in the good state as $N$ increases, while the effect is ambiguous in the bad state.

Proof. See the appendix.

If the harm is spread among many victims—each of them bearing a smaller share in the harm—and risk tolerance is larger in the good state, the first- and second-best level of care decreases with the number of victims. Vice versa, if risk tolerance is larger in the bad state, care increases with the number of victims. Increasing the number of victims has an effect that is qualitatively the same as the one associated to an increase in the victims’s initial wealth. As shown in the proposition, the way a change in the number of victims affects the results depends on the distribution of wealth between injurer and victims prior to the change. Moreover, given that $\frac{\partial y_b}{\partial N} = \frac{v_b}{w_b} \frac{\partial w_b}{\partial N}$ and $\frac{\partial y_g}{\partial N} = \frac{v_g}{w_g} \frac{\partial w_g}{\partial N}$, the way in which a change in the number of victims affects the final allocation of risk depends on the relative risk tolerance of injurers and victims. Thus, if the injurer’s risk tolerance is less than the victims’ risk tolerance in both states ($\frac{v_b}{w_b}, \frac{v_g}{w_g} > 1$), an increase in the number of victims must affect the victims’ allocation in the good and the bad state more than the injurer’s, and vice versa.\textsuperscript{12}

2.6 The injurer is a firm

Although it is plausible that both the victim and the injurer in traffic accidents or medical malpractice cases have state-dependent preferences, it might be argued that in some industrial accidents the injurer (for instance, a firm with diffuse shareholding) is risk neutral and its preferences are not state-dependent (thus excluding reputational effects). Thus, the case in which only one party has state-dependent preferences may be a realistic description of accidents between a firm and an individual. Yet, also these types of accidents may cause major or even fatal physical injuries, which are neither fully reparable nor fully compensable.

\textsuperscript{12}Nell and Richter (2003) also present a model with $N$ victims but focus on the effect that risk spreading has on the desirability of different liability rules, in the case of a pure pecuniary loss. They show that an increase in the number of victims makes it desirable to allocate more risk to the victims due to better risk-spreading.
The case where the injurer is risk neutral but has state-dependent preferences has been reviewed in corollary 3. We consider now the case of injurers with state-independent preferences. In this case, the marginal utility of money for the injurer is constant across states (because his preferences are independent of the state) and independent of wealth (because he is risk neutral), such that:

\[
\frac{u'(g)(w_g)}{u'(b)(w_b)} = 1
\]

Thus, the Borch conditions imply that also the victim’s marginal utilities across states must be equal:

\[
v'(g)(y_g) = v'(b)(y_b) \tag{7}
\]

Hence, concerning the optimal level of care, it is easy to see that (3) becomes equal to (2) implying that the socially optimal level of care is the level of care that would be optimal in a risk-free world. In other words, the optimal level of care and the optimal allocation of risk become independent issues. The intuition is straightforward: since the marginal utility of money is the same in both states for both parties, the minimization of the total accident loss is not affected by the need to use care to transfer some wealth from a state to the other, as it is the case when both parties have state-dependent preferences. It is easy to verify that all other results apply. In particular, since the victim is risk averse, the equality of his marginal utilities in (7) implies \( y_g > y_b \); that is, the victim is not perfectly insured against his losses, which is in accordance with the mutuality principle.

3 Implications for the design of liability rules

So far, we have studied care and risk-sharing policies by a benevolent planner, who can directly implement both of them. In the following, we extend the analysis to consider whether these two objectives can be reached by means of ordinary policy instruments, such as regulation, taxes and subsidies, insurance, and tort liability.

3.1 Regulation with taxes and subsidies

As shown in propositions 1, 4 and 5, the efficient allocation of wealth is not linear, in the sense that an increase in society’s wealth does not result in a proportional change in the individuals’ wealth. The specific shape of this relationship depends on the curvature of the individuals’ utility functions and on the safety technology. However, for practical purposes, effective insurance policies, regulation or liability ought to be simple enough to be implementable.

Consider a regulatory standard \( X \) paired with a simple sharing rule for both the harm and the cost of care, such that the injurer bears an amount \( \beta h(x) \) of the harm and an amount \( ax \) of the cost of care. It is very easy to verify that the first-best level of care can be obtained by setting \( X = x^* \) and the first-best
allocation of risk can be reached by choosing $\alpha$ to allocate wealth optimally between the victim and the injurer in the good state and $\beta$ in the bad state. Alternatively, the solution just described may also be implemented through a mix of regulation and lump-sum transfers. Regulation sets and enforces the required level of care, while taxes and subsidies realize the desired transfers of wealth between the parties in each state. These results can easily be understood considering the fact that there are as many mechanisms used as there are goals to be reached.

### 3.2 Strict liability

We now turn to liability rules, which are alternative tools to reallocate wealth and give incentives to take care. An important result of the previous analysis is that the first best requires enough instruments to reallocate wealth across states. However, liability rules allow transfers between the injurer and the victim in the bad state—in the form of damages payments—while ruling out any payment in the good state. Thus, it may be expected that liability falls short of controlling all of the three variables pertaining to risk-sharing and care and hence will not be enough to implement the first best. In the following, we also compare the performance of liability rules to the second best.

Consider first strict liability: the injurer pays damages equal to $\lambda h(x)$ whenever an accident occurs, where $\lambda > 0$. With $\lambda = 1$, the injurer pays perfectly compensatory damages to the victim—the victims obtains full compensation for his pecuniary losses and, thus, has a constant wealth across states $y_b = y_g = y_0$. However, strict liability can also be designed as to allow for supracompensatory damages ($\lambda > 1$, such as punitive damages) or infracompensatory damages ($\lambda < 1$), in which cases the victim receives a state-dependent wealth which is $y_0 + (\lambda - 1)h(x)$ in the bad state, and $y_0$ in the good state.

**Proposition 8** Under strict liability with perfectly compensatory damages $\lambda = 1$, the injurer chooses a second-best level of care. The associated allocation of risk is neither a first best nor a second best.

**Proof.** Assume that the liability rule is strict liability $\lambda$. Under this liability rule, the injurer will take care as to maximize:

$$pu_b(w_0 - \lambda h(x) - x) + (1 - p)u_g(w_0 - x)$$

Let $x_\lambda$ denote the injurer’s level of care, which satisfies the following first order condition:

$$p\lambda h'(x_\lambda) + 1 = (1 - p) \left( 1 - \frac{u'_g(w_0 - x_\lambda)}{u'_g(w_0 - \lambda h(x_\lambda) - x_\lambda)} \right)$$

Thus, when $\lambda = 1$ the injurer’s choice of care is the same as in the second best in (6), where the wealth of the victim is the constant allocation $\bar{y} = y_0$. 


which provides him with full insurance. Hence, $\lambda = 1$ allows to implement the second best both in terms of care and of risk, when fairness requires the victim to be fully compensated.

When $\lambda \neq 1$, strict liability generally does reach neither a first best nor a second-best allocation of risk. With supracompensatory damages, the victim receives a greater wealth in the bad state than in the good state: $y_0 + (\lambda - 1)h(x) > y_0 = y_0$, implying that the associated allocation of risk is not comonotonic, hence it cannot be first-best efficient. In contrast, with infracompensatory damages we have $y_0 + (\lambda - 1)h(x) < y_0$; then, the allocation is comonotonic but it will be only by chance that the Borch conditions are met; moreover care is not set at the first-best level. ■

From this proposition it emerges that increasing or decreasing the amount of damages affects both the level of care and the sharing of the risk, bringing the outcome away from the second best (but possibly improving over it), without being able to reach the first best.

One of the sources of inefficiency for the strict liability rule is that it only allows for a transfer in the bad state. This shortcoming could be corrected by adding a tax $\tau$ in the good state, while $\lambda$ takes care of the transfer in the bad state. The planner’s problem is to maximize:

$$SW = p u_b(w_0 - \lambda h(x) - x) + (1 - p) u_g(w_0 - x + \tau) + p u_b(y_0 + (\lambda - 1)h(x)) + (1 - p) v_g(y_0 - \tau)$$

which gives a result similar to the Borch conditions:

$$u'_b(w_0 - \lambda h(x) - x) = v'_b(y_0 + \lambda h(x))$$
$$u'_g(w_0 - x + \tau) = v'_g(y_0 - \tau)$$

However, the injurer chooses $x$ as to maximize

$$p u_b(w_0 - \lambda h(x) - x) + (1 - p) u_g(w_0 - x + \tau)$$

and hence we have:

$$p \lambda h'(x) + 1 = (1 - p) \left(1 - \frac{u'_g(w_0 - x + \tau)}{u'_b(w_0 - \lambda h(x) - x)}\right)$$

It is easy to see that in general $\lambda = 1$ does not satisfy the Borch conditions; however, the only way to obtain first best care is to set $\lambda = 1$, which in turn would be in contrast with the first-best allocation of risk. Thus, a simple tax in addition to strict liability cannot reach the first best as the same instrument $\lambda$ is used to allocate wealth in the bad state and to regulate caretaking. In the special case of accident between a firm and an individual (Section 2.6), it is easy to see that $\lambda = 1$ combined with a simple transfer is first-best optimal.

### 3.3 Negligence

Under the negligence rule, the injurer pays damages only if negligent, that is if his level of care is below $X$. Here the only policy instrument is the due care
level $X$. In fact, if the injurer abides by the standard of care, he does not pay damages to the victim, thus $\lambda$ becomes irrelevant as concerns the allocation of risk.

However, the parameter $\lambda$ is important in respect of the question of incentive compatibility. When the standard of care is set at the level $X$, the utility level of the injurer is defined as:

$$U(w_0, x) = \begin{cases} pu_b(w_0 - x) + (1 - p)u_g(w_0 - x), & \text{if } x \geq X \\ pu_b(w_0 - \lambda h(x) - x) + (1 - p)u_g(w_0 - x), & \text{otherwise} \end{cases}$$

As a result, under the negligence rule, the injurer obtains a sure outcome $(w_0 - X)$ if he adheres to the due care standard, and a risky outcome $(p, w_0 - \lambda h(x) - x; 1 - p, w_0 - x)$ if he does not. Note that according to the first line of (8), the injurer has no incentives to choose $x > X$. According to the second line of (8), when he does not comply with $X$, the injurer chooses the same level of care as under strict liability; $x_\lambda$ denotes this level of care.

Thus, negligence raises two issues: Will the injurer comply with the due care? Assuming he does, how does the outcome compare with the first and second best?

**Proposition 9** Under the negligence rule with a due care standard $X$:

i) If $X \leq x_\lambda$, then the injurer complies with the due care standard;

ii) If $X > x_\lambda$, then the injurer complies with the due care standard only if the following condition is satisfied:

$$pu_b(w_0 - \lambda h(x_\lambda) - x_\lambda) + (1 - p)u_g(w_0 - x_\lambda) \leq pu_b(w_0 - X) + (1 - p)u_g(w_0 - X)$$

iii) The allocation of risk is generally not first best. If the injurer complies, the allocation of risk is dual second best.

**Proof.** i) If $X \leq x_\lambda$, then:

$$u_b(w_0 - \lambda h(x_\lambda) - x_\lambda) \leq u_b(w_0 - x_\lambda) \leq u_b(w_0 - X)$$

which implies in turn:

$$pu_b(w_0 - \lambda h(x_\lambda) - x_\lambda) + (1 - p)u_g(w_0 - x_\lambda)$$

$$\leq pu_b(w_0 - x_\lambda) + (1 - p)u_g(w_0 - x_\lambda)$$

$$\leq pu_b(w_0 - X) + (1 - p)u_g(w_0 - X)$$

Thus, the injurer complies with due care.

ii) if $X > x_\lambda$, then:

$$pu_b(w_0 - x_\lambda) + (1 - p)u_g(w_0 - x_\lambda)$$

$$\geq pu_b(w_0 - X) + (1 - p)u_g(w_0 - X)$$

but condition (9) is not always satisfied. In several cases, the injurer may prefer to be found liable and bear the loss rather than comply with the due care standard.

iii) When the injurer complies, the victim is not compensated for his loss. The injurer only bears the cost of care and does not face any risk. This outcome
is the dual of the second best described above, where the victim did not face any risk.

Given the costs allocation associated to the negligence rule, the outcome in terms of risk sharing can never be first best. However, it is easy to see that whatever the standard $X$, the injurer complies as far as it entails a risk reduction as compared to not complying. This requirement is obviously met once we have $X \leq x_\lambda$; hence, the first best in term of prevention may be obtained if $X = x^* \leq x_\lambda$. Moreover, by setting $X = x_\lambda$ and $\lambda = 1$ the planner can reach for sure the second-best level of care. Concerning the allocation of risk, note that the negligence rule implements a second-best allocation of risk where the injurer, rather than the victim, is fully insured. Finally, the level of care that is second best when the injurer is fully insured can be reached provided that the incentive-compatibility conditions set in the proposition above are satisfied. In the special case of accident between a firm and an individual (Section 2.6), it is easy to see that $X = \hat{x}$ and $\lambda = 1$ implements the first-best level of care, while a transfer is needed to optimize risk.

3.4 Liability and insurance

We now assume that insurance is available and consider the issue of liability and insurance combined. Assume that the injurer has the opportunity to buy third-party liability insurance in an insurance market and the insurer can perfectly verify the injurer’s level of care; that is, there is no moral hazard. Under strict liability, the amount of insurance coverage $q$ and the level of care chosen by the injurer maximize

$$pu_b(w_0 - h(x) - x + q - m) + (1 - p)u_g(w_0 - x - m)$$

The insurer charges a price over the pure premium $m = \theta pq$, where $\theta - 1 \geq 0$ is the loading factor. The first order condition that characterizes the equilibrium insurance purchase $\tilde{q}$ is thus:

$$pu'_b(w_0 - h(x) - x + \tilde{q} - \tilde{m}) = \theta p^2 u'_b(w_0 - h(x) - x + \tilde{q} - \tilde{m})$$

with $\tilde{m} = \theta pq$. Condition (10) is well-known in insurance literature: the injurer buys an insurance coverage such that its marginal benefit adjusted for risk on the left-hand side equals its marginal cost adjusted for risk on the right-hand side. It is equally well-known that, in such a case, the injurer may buy

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13It is well known that the insurability for large, catastrophic losses implies only partial insurance at the individual level and requires insurance companies to have the opportunity to transfer the aggregate risk to financial markets. We do not consider these issues here.

14The analysis can also be easily extended to negligence and first-party insurance for the victims.

15An alternative model would be to define $q = th(x)$ where $t \in [0, 1]$ is the proportion of insurance coverage. This model and the one presented in the text are equivalent, since $x$ is perfectly observable.
partial insurance, full insurance or overinsurance (Cook and Graham, 1977). Furthermore, the individually efficient level of care \( \hat{x} \) satisfies:

\[
p h'(\hat{x}) + 1 = (1 - p) \left( 1 - \frac{u_g'(w_0 - \hat{x} - \hat{m})}{u_g'(w_0 - h(x) - \hat{x} + \hat{q} - \hat{m})} \right)
\]

which is a special case of (6), that is \( \hat{x} \) is second best. If there is no loading on the actuarially fair insurance premium, \( \theta = 1 \), the condition in (10) becomes:

\[
u_g'(w_0 - h(x) - \hat{x} + \hat{q} - p\hat{q}) = u_g'(w_0 - \hat{x} - p\hat{q})
\]

implying that condition (11) reduces to (2) and, hence, \( \hat{x} = \hat{x} \). Thus, the level of care that the injurer chooses is the same as the level of care that would be optimal in a risk-free world. The reason is that when \( \theta = 1 \), insurance markets allow the injurer to reallocate his wealth among different states of nature, in such a way that his marginal utility of wealth be equal between states.\(^{16}\) Nevertheless, this does not represent a Pareto efficient allocation for this economy, since according to the Borch conditions, the first best allocation of risk is only obtained through transfers between individuals, state by state, in order that individual marginal utilities are equal in each separate state.

Finally, note that if accidents occur between an individual and a firm (which is risk neutral and has state-independent preferences, Section 2.6), the level of care and the allocation of risk are first-best optimal. The reason is that, since the firm’s utility is constant across states, the setting of the level of care is independent of how risk is allocated between the parties. However, we have noted in the introduction how reputation effects might also make the firm’s utility functions dependent on the state. In this case, the results presented in this section would be applicable.

4 Discussion

The analysis focuses on accidents having two characteristics: the harm has a noncompensable component and care reduces the magnitude of the harm but not the probability of accidents. Our framework departs from previous literature, which considers either perfectly compensable losses and \( f \) or contexts in which care reduces the probability of accidents (Nell and Richter, 2003; Graff Zivin, Just and Zilberman, 2006).

Our findings shed new light on the question whether and to what extent safety standards should depend on wealth (Arlen, 1992a; Miceli and Segerson, 1995; Shavell, 1982). We find that both in the first best and in the second best—when either the victim or the injurer is fully insured—care does depend on society’s wealth, but it does not depend on the distribution of such wealth.

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\(^{16}\)Although the injurer’s choice of insurance coverage is designed to equalize his marginal utility of money between states he does not buy full insurance: \( \hat{q} \neq h(x) \). He will overinsure, \( \hat{q} > h(x) \), if money is more useful in the good state (under A2a) and underinsure, \( \hat{q} < h(x) \), if money is more useful in the bad state (under A2b); see Cook and Graham (1977).
among individuals. Moreover, the effect of an increase in wealth on the socially desirable level of precaution is not straightforward, as it is not always desirable to increase precaution when society becomes richer.

Since taking care creates a cost in the good state (when there is no accident) and a benefit in the bad state (when an accident occurs), we have described the problem of deciding whether to increase care when society becomes richer as a problem of allocating such additional wealth between the good state and the bad state. As we have shown, more wealth should be allocated to the state where society has more tolerance to risk. This implies that care increases with wealth if society is more tolerant to risk in the bad state and, vice versa, care decreases with wealth if society is more tolerant to risk in the good state.

Extending the one-agent framework introduced in the insurance literature (Ehrlich and Becker, 1967; Jones-Lee, 1974) to a two-agent framework, we have tackled a second basic question, whether a risk-averse society should take more or less care than a risk-neutral society. Also in this case, the answer is not unambiguous and depends on whether the marginal utility of wealth is larger in the good or in the bad state. If accidents cause temporary impairment, such as a reversible medical condition, which trigger some monetary expenditures, it might be worthwhile for a risk-averse society to spend more on precaution than a risk-neutral society. In contrast, if the accident causes a permanent disability which reduces the individual’s capacity to derive utility from money, risk aversion reduces the socially desirable level of care instead of increasing it.

In relation to the literature on the optimal allocation of risk (Borch, 1962; Arrow, 1964), we have shown that the mutuality principle—yielding that both parties should bear some risk in equilibrium—extends to situations in which the magnitude of the accident is endogenous and depends on care choices by one of the parties. Moreover, we have shown that the optimal allocation of risk depends on the (exogenous) probability of accidents, a result that is obtained even though the parties share the same information and beliefs concerning the probability of the different states of the world.

After having tackled the problem of optimally allocating risk and choosing care at an abstract level, we examine different policy measures to implement the desired levels of risk and care. Besides first-best solutions we also consider the implementability and characteristics of second-best outcomes where one party is fully insured against the accident loss. Although the optimal risk-sharing rules are complex and non-linear, we show that they can be mimicked by implementing simple linear rules according to which the parties have to share both the cost of care and the accident loss.

The issue whether strict liability or negligence is the superior liability rule has been discussed in the literature. Nell and Richter (2003) show that the negligence rule has the advantage of spreading risk among several parties rather than concentrating risk on the injurer; hence negligence might induce a better allocation of risk. Moreover, if harm is nonmonetary, the common wisdom (Shavell, 2004, ch. 11; Arlen, 1992a, note 3) is that the negligence rule is preferable because it allows a separation between the incentive problem and the risk-allocation problem. We show that the superiority of either rule is not
established: the negligence rule might produce better incentives to take care but strict liability induces an allocation of risk that is preferable when fairness consideration impose the compensation of innocent victims.

In this section, we extend some of the main results to accident contexts having specific features—self-protection activities and the case in which the noncompensable loss depends on care—discuss the case of bilateral accidents and briefly comment on other possible extensions including levels of activity and insolvency.

4.1 Self-protection

In case of self-protection, the magnitude of the loss $h$ is exogenous, while the probability of occurrence $p(x)$ is a function of the injurer’s care, with $p'(x) < 0$, $p''(x) > 0$, $p(0) > 0$, and $p'(\infty) \to 0$. The social planner’s objective is now to maximize:

$$SW = p(x)[u_b(w_b) + v_b(y_b)] + [1 - p(x)]\left[u_g(w_g) + v_g(y_g)\right]$$

subject to a new resource constraint for the bad state, $w_b + y_b = w_0 + y_0 - h - x$, while the resource constraint for the good state remains the same, $w_g + y_g = w_0 + y_0 - x$.

Note that with self-protection, care does not transfer wealth between states but reduces wealth in both states in the same way—this is because $h$ does not depend on $x$ but, in contrast, $x$ affects the probability that a particular state of the world materializes $p(x)$. Therefore, this model presents some challenges that make the analysis more complex than with self-insurance. Nevertheless, it is easy to verify that the two benchmark cases remain valid, with optimal care in a risk-free world satisfying $p'(\hat{x})h + 1 = 0$.

It is also easy to verify that Proposition 1 (mutuality) still applies here: the intuition is once more that for any $x > 0$, the social planner must distribute wealth among individuals in order to make the individuals’ marginal utilities equal state by state. Therefore, the costs of accidents should be borne in some proportion by both parties. Moreover, for any first-best allocation of risk, it is easy to show that the associated optimal level of care, $x^{**}$, satisfies:

$$-\frac{1}{p'(x^{**})} = \frac{v_g(y_g) - v_b(y_b)}{(1 - p(x^{**}))v'_g(y_g) + p(x^{**})v'_b(y_b)} + \frac{u_g(w_g) - u_b(w_b)}{(1 - p(x^{**}))u'_g(w_g) + p(x^{**})u'_b(w_b)}$$

(13)

The condition in (13) is a generalization of a similar condition obtained in purely individual-decision models (Chiu, 2000; Dehez and Drèze, 1987; Lee, 1998 and 2005; Sweeney and Beard, 1992). Note that (13) equalizes the marginal social cost of safety on the left-hand side with the marginal social willingness to pay for safety (Jones-Lee, 1974)—the sum of the victim’s and the injurer’s marginal willingness to pay for safety at $x^{**}$—on the right-hand side. In the
literature on public goods, the condition in (13) is known as the *Samuelson-Bowen-Lindhal condition*. Hence, the choice of a socially efficient level of care is akin to the problem of providing a pure public good: At the efficient level of care each individual pays a portion of the cost of care equal to his willingness to pay for safety. Note also that the extension of this result from 2 to \(N\) victims is straightforward.

Proposition 4 (*care and society’s wealth*) also carries over to the case of self-protection. Taking into account (13) together with the resources constraints, it is evident that the socially optimal level of care depends again on society’s wealth \(w_0 + y_0\) but not on its distribution among individuals. In public goods literature, this result is known as *neutrality of public good production to distribution* (Bergstrom, Blume and Varian, 1986; Cornes and Sandler, 1985). However, the usual property according to which the amount of a public good increases with the aggregate wealth of the pool of contributors (Bergstrom, Blume and Varian, 1986), does not necessarily apply. In fact, as Proposition 4 shows for self-insurance, the relationship between care and social wealth is ambiguous. In the case of self-protection, this ambiguity persists and the analysis becomes so complex that it exceeds the scope of this paper. The same is true for the remaining two points, *care and risk* and *care and probability of accidents*. Briys and Schlesinger (1990) explain that the difficulty with the analysis of individual self-protection decisions is that they simultaneously entail a mean-preserving spread and a mean-preserving contraction of risk.\(^{17}\)

Finally, the failure of a single liability rule to reach the first best (whether it is negligence or strict liability, and with or without insurance) follows directly from the fact that liability rules only implement a transfer in the bad state but leave the distribution of wealth in the good state unaffected, which is clearly not enough.

### 4.2 The noncompensable loss depends on care

In the analysis of self-insurance, we have implicitly assumed that while the magnitude of the monetary loss depends on care, the noncompensable loss is an exogenous parameter. This assumption is reasonable in a broad set of cases, in which the noncompensable loss is due to the mere fact of having been in an accident, irrespective of how big the monetary component of the loss was. Consider for instance the case in which the victim of an accident experiences a near-death situation with long-lasting psychological consequences or when an injurer suffers from a feeling of guilt due to the experience of an accident (again independently of the magnitude of the loss). However, there are many other real-life situations in which care affects the seriousness of the injury suffered by the victim—for instance, a high level of care results in posttraumatic stress disorder (PSD) for the victim while a low level of care also results in paraplegia or tetraplegia in addition to PSD. In these cases, both the monetary and the noncompensable loss vary with the level of care.

\(^{17}\)For example, this requires to define an appropriate probability threshold to obtain unambiguous results (Chiu, 2000).
In order to take into account the effect of care on the noncompensable loss, assume that the victim holds a nonmonetary asset the value of which is \( \ell_0 \) if there is no accident; if the asset is damaged due to an accident it takes the value \( \ell(x) \) with \( \ell'(x) > 0, \ell''(x) > 0 \) such that and for any \( x > 0 : \ell(x) < \ell_0 \). The victim’s preferences are represented by a two-argument utility function \( v(y, \ell) \), increasing and concave. The social welfare function is:

\[
SW = p [u_b(w_b) + v(y_b, \ell_0)] + (1 - p) [u_g(w_g) + v(y_g, \ell(x))]
\]

subject to the same resource constraints as before: \( w_b + y_b = w_0 + y_0 - h(x) - x \) and \( w_g + y_g = w_0 + y_0 - x \). It is easy to verify that Proposition 1 (mutuality) continues to hold. The same applies to Proposition 2 (care and risk). It can be shown that for any allocation of risk, the optimal level of care, \( \bar{x} \), satisfies:

\[
1 + p \left( h'(\bar{x}) - \frac{v''_{y_b}(y_b, \ell(\bar{x}))}{v'_1(y_b, \ell(\bar{x}))} \ell'(\bar{x}) \right) = (1 - p) \left( 1 - \frac{v'_1(y_g, \ell_0)}{v'_1(y_b, \ell(\bar{x}))} \right) \quad (14)
\]

Note that the condition in (14) is a generalization of a similar condition obtained in an individual-decision model by Lee (2005) and it is an intuitive modification of condition (4). If the noncompensable loss depends on care, safety choices should respond not only to risk (the right-hand side in (14)) but also to the monetary equivalent of the nonmonetary asset. More specifically, the term \( -\frac{v''_{y_b}(y_b, \ell(\bar{x}))}{v'_1(y_b, \ell(\bar{x}))} \ell'(\bar{x}) \) in the the left-hand side of (14) is the (monetary equivalent of the) marginal cost of accident due to the damage to the nonmonetary asset—that is, the marginal rate of substitution between wealth and nonmonetary asset \( \frac{v'_2(y_b, \ell(\bar{x}))}{v'_1(y_b, \ell(\bar{x}))} \) times the marginal reduction in the the nonmonetary loss \( \ell'(\bar{x}) \).

Concerning care and risk, it is easy to verify that \( \bar{x} \) may be smaller or larger than the risk neutral case \( \bar{x} \), depending on the sign of the cross derivative \( v''_{12}(y, \ell) \).

Note that on the one hand, the left-hand side satisfies

\[
1 + p \left( h'(\bar{x}) - \frac{v''_{y_b}(y_b, \ell(\bar{x}))}{v'_1(y_b, \ell(\bar{x}))} \ell'(\bar{x}) \right) < 1 + ph'(\bar{x}),
\]

indicating that the reduction in the nonmonetary loss obtained through safety reduces (at equilibrium) the expected marginal cost of accidents; this effect unambiguously leads to an increase in the level of care. On the other hand, if \( v''_{12}(y, \ell) < 0 \Rightarrow v'_1(y, \ell_0) > v'_1(y, \ell(x)) \), then by concavity we have (since \( y_g \geq y_b \)) \( \frac{v'_2(y_b, \ell_0)}{v'_1(y_b, \ell(\bar{x}))} < 1 \) implying that the right-hand side (marginal benefit of safety in utility terms) is positive: this means that risk aversion per se (both for wealth dispersion and nonmonetary loss) requires that the level of care should be further increased as compared to the risk-neutral case \( \bar{x} \). But if \( v''_{12}(y, \ell) > 0 \Rightarrow v'_1(y, \ell_0) > v'_1(y, \ell(x)) \), then the sign of \( 1 - \frac{v'_1(y_b, \ell_0)}{v'_1(y_b, \ell(\bar{x}))} \) is ambiguous (so is the right-hand side): specifically when the sharing of risk leads to a situation where \( 1 - \frac{v'_1(y_b, \ell_0)}{v'_1(y_b, \ell(\bar{x}))} < 0 \), the level of care may be reduced as compared to the risk-neutral case \( \bar{x} \).

---

18 We denote as \( v'_1(y, \ell) = \frac{\partial v(y, \ell)}{\partial \ell} \) and \( v''_{12}(y, \ell) = \frac{\partial^2 v(y, \ell)}{\partial \ell \partial y} \).

19 Where \( v''_{12}(y, \ell) = \frac{\partial^2 v(y, \ell)}{\partial \ell \partial y} \).
Finally, the same general comments provided for self-protection apply here, regarding the relationship between care and society’s wealth (Proposition 4 carries over to this case) and the comparative statics on the relationship between care and probability of accidents (which is too complex to be analyzed here). Finally, also in this case simple liability rules fail. The reason is twofold: the transfer in the bad state of the victim’s monetary loss is not enough and, due to the effect of care on the noncompensable component, a need for a transfer concerning the noncompensable component of the loss arises.

4.3 Bilateral accidents, activity levels and insolvency

This paper focuses on situations known as unilateral accident cases, where only the victim suffers harm and only the injurer can take care. The bilateral accident case (bilateral care and/or damage) is also an important one, which is extensively discussed in the literature. Arlen (1992b) has considered the state-dependent representation of preferences to assess the functioning of tort law when both the injurer and the victim take care; however, a main difference with our paper is that Arlen (1992b) assumes small risks for which perfect insurance exist.

Moreover, it is well known that the predictions obtained in the bilateral accident context are very sensitive to the timing of individual decisions (simultaneous choice of care v. sequential choices of care; see Endres, 1992; Frieh, 2009; Leong, 1989; Shavell, 1983; Winter, 1994). More specifically, an often identified failure of liability rules regarding the bilateral harm case is that it is doubtful that judges have consideration for injurers’ losses (Cooter and Porat, 2000). It could be easily expected that introducing the issue of risk reallocation (for aggregate, noninsurable risks) in the bilateral accident model aggravates rather than mitigates this problem.

Dharmapala and Hoffmann (2005) and Kim and Feldman (2006) consider the case where one party’s choice of precaution typically corresponds to self-insurance activities (i.e., activities designed to reduce one’s own loss in case of accident) but increases the magnitude of harm to another party. For example, heavy cars incur less damage than smaller cars (or pedestrians and cyclists) in case of a collision, and they may often inflict larger damages upon others simply because they are heavier. In these contexts, an additional externality arises since the parties’ costs of precaution become interdependent. As a result, simple liability rules fail to reach efficient care levels because they require loss transfers only in the accident state (Dharmapala and Hoffmann, 2005), which is not enough to reallocate the full costs of care implied by interdependent precautions. Yet, this is what aggregate risks would require since, as we have shown, both injurers and victims should bear at least a portion of both the total costs of accident and care.\footnote{An interpretation of our result is that, considering efficiency in both care and risk sharing, there is no \textit{a priori} reason to focus on the parties’ role-type, \textit{i.e.} the fact that we can identify ex ante injurers and victims, in order to assign liability for the residual loss. The issue of role-}
Finally, many other relevant variables have been left out of the analysis. Of importance for the implementation of our recommendations could be issues concerning activity levels and potential insolvency, which are extensively analyzed in previous literature (Shavell, 1980; Gauza and Gomez, 2008). The presence of a noncompensable component may make the analysis of activity level incentives more complex (victims are likely to internalize losses that will not be compensated by injurers) and change the way in which insolvency affects the results, due to the fact that insolvency concerns the monetary loss only.

5 Conclusions

The analysis of accidents where harm has a noncompensable component and care reduces the magnitude of the harm but not the probability of accidents sheds new light on some fundamental questions about care, risk and wealth. We find that care does depend on society’s wealth but it does not depend on the distribution of such wealth among individuals. Moreover, the effect of an increase in wealth on the socially desirable level of precaution is not straightforward, as it is not always desirable to increase precaution when society becomes richer. These results do not support the idea that poorer countries should always invest less in safety than richer countries, given that the effect of wealth on care depends on society’s risk-tolerance. Different risk-scenarios may yield different recommendations. In particular, if the injurer is a firm, the same level of care may be required in poor and rich countries.

We have also tackled a second important question, whether a risk-averse society should take more or less care than a risk-neutral society. Also in this case, the answer is not unambiguous and depends on whether the marginal utility of wealth is larger in the good or in the bad state. Essentially, wealth and risk affect care choices in different ways depending on the characteristics of the good and the bad state: the effect of wealth on care depends on risk-tolerance while the effect of risk on care depends on the marginal utility of money.

References


Type uncertainty is discussed by Kim and Feldman (2006) and Friese (2007) in the risk-neutral case with sequential choices of care.


APPENDIX

Proof of Proposition 1: We assume that there exists some values of $x > 0$ such that $h(x) + x < H$. Assume now that the feasible allocation $[(w_b, w_g); (y_b, y_g)]$ with $w_b \leq w_g$ and simultaneously $y_b > y_g$, associated to a care level $x$, is Pareto optimal. Now for the same level of care, define an alternative feasible allocation $[(\hat{w}_b, \hat{w}_g); (\hat{y}_b, \hat{y}_g)]$ where $\hat{w}_b \leq \hat{w}_g$ and simultaneously $\hat{y}_b = \hat{y}_g$, such that:

\[
\begin{align*}
\hat{w}_b &= w_b + (1 - p)(y_b - y_g) \\
\hat{w}_g &= w_g - p(y_b - y_g) \\
\hat{y}_b &= py_b + (1 - p)y_g = \hat{y}_g \\
\hat{w}_b + \hat{y}_b &= w_b + y_b \\
\hat{w}_g + \hat{y}_g &= w_g + y_g
\end{align*}
\]

By definition, both individuals obtain the same expected individual wealth irrespective of the allocation we choose, since: $py_b + (1 - p)y_g = py_b + (1 - p)y_g$ for the victim and $p\hat{w}_b + (1 - p)\hat{w}_g = pw_b + (1 - p)w_g$ for the injurer. However, $(\hat{w}_b, \hat{w}_g)$ is less spread than $(w_b, w_g)$ in the sense of the second-order stochastic dominance, given that for the same probabilities $(p, 1 - p)$ we have the following ordering of the injurer’s wealth in the different states: $w_b < \hat{w}_b \leq \hat{w}_g < w_g$. The allocation $(\hat{y}_b, \hat{y}_g)$ is also less spread than $(y_b, y_g)$ since we have: $y_b > \hat{y}_b = \hat{y}_g > y_g$. Recall that, by assumption, both individuals are risk averse to second-order dominance shifts in risk. Thus $[(\hat{w}_b, \hat{w}_g); (\hat{y}_b, \hat{y}_g)]$ Pareto dominates $[(w_b, w_g); (y_b, y_g)]$; hence a contradiction.

Define now two real numbers $\mu_b$ and $\mu_g$ as the shadow prices of the aggregate resource constraints of society. If an interior solution exists for the problem of the social planner, then it corresponds to a vector $(x, w_b, w_g, y_b, y_g)$ which satisfies the following conditions (which are the derivatives of the social problem with respect to $x, w_b, w_g, y_b$ and $y_g$ respectively):

\[
\begin{align*}
-\mu_b h'(x) - (\mu_g + \mu_b) &= 0 \quad \text{(A)} \\
p u'_b(w_b) - \mu_b &= 0 \quad \text{(B)} \\
(1 - p)u'_g(w_g) - \mu_g &= 0 \quad \text{(C)} \\
p u'_b(y_b) - \mu_b &= 0 \quad \text{(D)} \\
(1 - p)v'_g(y_g) - \mu_g &= 0 \quad \text{(E)}
\end{align*}
\]

Condition (A) characterizes efficient care. Conditions (B) to (E) define the rule that should be used by the planner to implement a first-best allocation of risk. Using (B) and (D) together, and (C) and (E) together, we obtain:

\[
\begin{align*}
u'_b(w_b) &= v'_b(y_b), \text{ with } w_b + y_b = w_0 + y_0 - x - h(x) \\
u'_g(w_g) &= v'_g(y_g), \text{ with } w_g + y_g = w_0 + y_0 - x
\end{align*}
\]
which are known as the Borch conditions. ■
Proof of Proposition 2: Summing up conditions (D) and (E) yields:

\[ \mu_b + \mu_g = pv'_b(y_b) + (1 - p)v'_g(y_g) \]

Substituting in (A) and rearranging with (D), we have that the first-best level of care satisfies the condition:

\[ -h'(x) = 1 + \frac{1 - p \; v'_g(y_g)}{v'_b(y_b)} \]  

which we can also write as: \( h'(x) + 1 = -\frac{1 - p \; v'_g(y_g)}{v'_b(y_b)} \), implying that at the optimum \( h'(x) + 1 \leq 0 \), that is, the total cost of the accident decreases in \( x \) in the bad state. Second order conditions require the following inequality to hold:

\[ \frac{h''(x^*)}{1 + h'(x^*)}(t'^u_b + t'^u_g) + \frac{t'^v_b + t'^v_g}{t'^g_b} (1 + h'(x^*)) - 1 < 0 \]

where: \( t'^u_b, t'^u_g, t'^v_b, t'^v_g \) are defined in the text; this last inequality is satisfied, since we have \( h'(x) + 1 < 0 \) and, by assumption, \( h''(x^*) > 0 \).

Finally, after some straightforward manipulations of (F), it is easy to obtain condition (3) in the text. ■

Proof of Proposition 4: Recall that the aggregate wealth is \( W_0 = y_0 + w_0 \). Thus, the impact of an increase in \( W_0 \) on the risk sharing rules is obtained by first totally differentiating the Borch conditions to obtain:

\[ u''_b(w_b) \frac{\partial w_b}{\partial W_0} = v''_b(y_b) \frac{\partial y_b}{\partial W_0} \]
\[ u''_g(w_g) \frac{\partial w_g}{\partial W_0} = v''_g(y_g) \frac{\partial y_g}{\partial W_0} \]

taking again into account the Borch conditions gives us:

\[ \frac{u''_b(w_b) \; \partial w_b}{u'_b(w_b) \; \partial W_0} = \frac{v''_b(y_b) \; \partial y_b}{v'_b(y_b) \; \partial W_0} \]
\[ \frac{u''_g(w_g) \; \partial w_g}{u'_g(w_g) \; \partial W_0} = \frac{v''_g(y_g) \; \partial y_g}{v'_g(y_g) \; \partial W_0} \]

or equivalently:

\[ \frac{\partial y_b}{\partial W_0} = \frac{t^v_b \; \partial w_b}{t^u_b \; \partial W_0} \]  

\[ \frac{\partial y_g}{\partial W_0} = \frac{t^v_g \; \partial w_g}{t^u_g \; \partial W_0} \]  

Now, totally differentiating the resource constraints with respect to \( W_0 \) gives \( \frac{\partial w_b}{\partial W_0} + \frac{\partial w_g}{\partial W_0} = 1 - (1 + h'(x)) \frac{\partial x}{\partial W_0} \) and \( \frac{\partial y_b}{\partial W_0} + \frac{\partial y_g}{\partial W_0} = 1 - \frac{\partial x}{\partial W_0} \); thus, after
substituting \( \frac{\partial w_b}{\partial W_0} = - \frac{\partial u_b}{\partial W_0} + 1 - (1 + h'(x)) \frac{\partial x}{\partial W_0} \) in (G), and \( \frac{\partial w_g}{\partial W_0} = - \frac{\partial u_g}{\partial W_0} + 1 - \frac{\partial x}{\partial W_0} \) in (H), we obtain:

\[
\frac{\partial y_b}{\partial W_0} = \frac{t_b}{t_b + t_b} \left( 1 - (1 + h'(x)) \frac{\partial x^*}{\partial W_0} \right) \quad (I)
\]

\[
\frac{\partial y_g}{\partial W_0} = \frac{t_g}{t_g + t_g} \left( 1 - \frac{\partial x^*}{\partial W_0} \right) \quad (J)
\]

Then, differentiating condition (F) with respect to \( W_0 \) and rearranging, we obtain:

\[
\frac{h''(x^*)}{1 + h'(x^*)} \frac{\partial x^*}{\partial W_0} = \frac{1}{t_b} \frac{\partial y_b}{\partial W_0} - \frac{1}{t_g} \frac{\partial y_g}{\partial W_0} \quad (K)
\]

After substituting (I) and (J) into (K), we have Exp. (5). Note that, as one might expect from the mutuality principle, it is easy to verify that:

\[
1 > (1 + h'(x^*)) \frac{\partial x^*}{\partial W_0} \Rightarrow \frac{\partial u_b}{\partial W_0} > 0 \text{ and } 1 > \frac{\partial u_g}{\partial W_0} \Rightarrow \frac{\partial u_g}{\partial W_0} > 0. \]

**Proof of Proposition 5:** We proceed as for the proof of proposition 4. The impact on risk sharing of an increase in \( p \) can be obtained once more by first totally differentiating the Borch conditions:

\[
u''(w_b) \frac{\partial w_b}{\partial p} = \nu''(y_b) \frac{\partial y_b}{\partial p} \Rightarrow \frac{\partial y_b}{\partial p} = \frac{t_b}{t_b} \frac{\partial w_b}{\partial p}
\]

\[
u''(w_g) \frac{\partial w_g}{\partial p} = \nu''(y_g) \frac{\partial y_g}{\partial p} \Rightarrow \frac{\partial y_g}{\partial p} = \frac{t_g}{t_g} \frac{\partial w_g}{\partial p}
\]

According to the resource constraints, we also have \( \frac{\partial w_b}{\partial p} + \frac{\partial w_g}{\partial p} = - (1 + h'(x^*)) \frac{\partial x^*}{\partial p} \) and \( \frac{\partial y_b}{\partial p} + \frac{\partial y_g}{\partial p} = - \frac{\partial x^*}{\partial p} \). Substituting in the conditions above, we obtain:

\[
\frac{\partial y_b}{\partial p} = - \frac{t_b}{t_b + t_b} (1 + h'(x^*)) \frac{\partial x^*}{\partial p} \quad (L)
\]

\[
\frac{\partial y_g}{\partial p} = - \frac{t_g}{t_g + t_g} \frac{\partial x^*}{\partial p} \quad (M)
\]

Then, differentiating condition (3) in \( p \) and rearranging, we obtain:

\[
\frac{h''(x^*)}{1 + h'(x^*)} \frac{\partial x^*}{\partial p} - \frac{1}{t_b} \frac{\partial y_b}{\partial p} + \frac{1}{t_g} \frac{\partial y_g}{\partial p} = \frac{-1}{p(1-p)} \quad (N)
\]

Substituting (L) and (M) in (N) gives:

\[
\frac{\partial x^*}{\partial p} = \frac{\frac{t_g + t_b}{p(1-p)}}{\frac{t_b}{1 + h'(x^*)} \left( t_g + t_g \right) + \frac{t_b}{1 + h'(x^*)} \left( 1 + h'(x^*) \right)} > 0
\]

Considering again (L) and (M), and recalling that at the optimum we must have \(- (1 + h'(x^*)) > 0\), it follows that \( \frac{\partial u_b}{\partial p} > 0 \) and \( \frac{\partial u_g}{\partial p} < 0. \)
Proof of Proposition 7: Starting with the Borch conditions, we have (as in the proof of Proposition 4):

\[
\begin{align*}
\frac{\partial y_b}{\partial N} &= \frac{t^*_b}{v_b} \frac{\partial w_b}{\partial N} \Rightarrow \text{sign} \frac{\partial y_b}{\partial N} = \text{sign} \frac{\partial w_b}{\partial N} \\
\frac{\partial y_g}{\partial N} &= \frac{t^*_g}{v_g} \frac{\partial w_g}{\partial N} \Rightarrow \text{sign} \frac{\partial y_g}{\partial N} = \text{sign} \frac{\partial w_g}{\partial N}
\end{align*}
\]

which proves i). Now, using \( \frac{\partial x^*}{\partial N} = y_0 \frac{\partial x^*}{\partial W_0} \) and the resource constraints, we have:

\[
\begin{align*}
\frac{\partial w_b}{\partial N} + N \frac{\partial y_b}{\partial N} &= (y_0 - y_b) - \left( h'(x^*) + 1 \right) \frac{\partial x^*}{\partial N} \\
&= y_0 \left( 1 - \left( h'(x^*) + 1 \right) \frac{\partial x^*}{\partial W_0} \right) - y_b \\
\frac{\partial w_g}{\partial N} + N \frac{\partial y_g}{\partial N} &= (y_0 - y_g) - \frac{\partial x^*}{\partial N} \\
&= y_0 \left( 1 - \frac{\partial x^*}{\partial W_0} \right) - y_g
\end{align*}
\]

such that the sign of the right-hand side in each expression gives the sign of the left-hand side. Using \( 1 > (1 + h'(x^*)) \frac{\partial x^*}{\partial W_0} \) and \( 1 > \frac{\partial x^*}{\partial W_0} \), the results ii) to iv) of the proposition are proven.