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Optimal Growth in Overlapping Generations with a Directly Polluting Sector and an Indirect One.

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Abstract

We study the optimal growth path and its decentralization in an overlapping generations model with two consumption goods and pollution effect. We consider two production sector *i.e.* one with a direct effect of pollution and the other with an indirect pollution effect by using energy. In the presence of externalities, decentralization of an optimal path needs some specific taxes in addition to lump-sum transfers. The introduction of a market for pollution permits, concerning only the polluting sector, neutralizes the external environmental effects. We show that there is a unique management of permits such that the equilibrium coincides with the optimal path: all permits should be auctioned *i.e.* no free permits to firms. This conclusion is in contradiction with the usual practice of grandfathering.

JEL Classification: D61,D9,Q28

Key words: Optimal growth, environment, market of permits.

1 Introduction

The aim of this paper is to show that in an economy with two sectors and an environmental externality (with a direct and an indirect source of pollution), it is possible to use, for only one sector, a market of permits with lump-sum transfers to decentralize the optimal growth path only if no free permit are given to this sector.

Since Montgomery (1972), it is well known that the way permits are issued is irrelevant for welfare. But such a result applies only to a static world in partial equilibrium. Standard text books show that a common resource generally leads to the Tragedy of Commons and the Coase theorem (Coase (1960)) may be irrelevant for property right of a common property, like environmental quality in an intertemporal general equilibrium framework. In a one sector framework, Jouvet, Michel, Rotillon (2005) show that, in an overlapping generation framework (OLG), the decentralization of the optimal path is obtained with lump-sum transfers only if no free permits are given to firms. This result contrast with the standard OLG model (Allais (1947), Diamond (1965)) without environmental constraint, where the optimal policy is decentralized with lump-sum transfers without any other conditions (De La Croix, Michel, 2003). With an environmental externality, free permits act as a subsidy which raises the return to the owners of the firm's capital and therefore subsidizes investments, causing a distortion in the economy. This result also contrast with effective markets of permits. Despite that recent research using general equilibrium models (Parry *et al.*, 1999) suggests that auction or taxes generally dominate a permits system based upon a criterion allocation in the presence of distorted factor markets, free permits via grandfathering allocation, is the chosen form of market-based approach in the United States. And the Kyoto Protocol has popularized the idea of setting up pollution rights freely allocated as an instrument of environmental policy for the reduction of greenhouse gas. Stavins (1998) states the reasons that many actors have to favor freely allocated tradeable permits. Among them, "existing firms favor freely allocated tradeable permits because they convey rents to them". These permits also act to entry barriers since new firms must purchase permits from installed firms (Koutstaal (1997)). In fact, grandfathering is made primarily on grounds of political acceptability because free allocation is strongly favored by most of the sectoral interests involved. An additional argument against auctioned permits in a sectoral economy comes from a lack of competition for the sectors which must buy the permits.

In this paper, we show that the decentralization of the optimal path is obtained with lump-sum transfers only if no free permits are given to firms even in an economy with two sectors. In fact, profits per unit of capital are the return of investment attributed to shareholder, the owners of the capital stock. With a free allocation of permits, the return of investment in the sector which receives pollution permits, its equal to the marginal productivity of capital plus the value of permits. Therefore, this return does not satisfy the neo-classical property of equality of factor income with marginal productivity. The Coase theorem does not apply and the optimal growth path can not be decentralized with free pollution permits. The intuition is that grandfathering permits acts as a subsidy to firms.

The paper is organized as follows. Section 2 presents the model. The optimal growth problem is stated in section 3 where we characterize the marginal optimality conditions. Section 4 defines the equilibrium with pollution permits and our main result is proved in section 5. We summarize our results in section 6.

2 The model

We consider a two sectors model, one with a direct effect of pollution and the other with an indirect pollution effect by using energy. The Energy sector produces electricity by using capital, labor and emissions of polluant.The other sector, call the Main sector, produces one good by using capital, labor and energy. Consumers also have an indirect effect on environment by consuming the energy good.

2.1 Technologies

The output Y_t of the Main sector occurs in each period according to a standard production function $F(.)$ homogeneous of degree one of capital, K_t, labor, L_t and energy Z_t^E. The Energy sector produce an output Y_t^E with a constant returns to scale production function $F^E(.)$, using capital, K_t^E, labor, L_t^E and emissions E_t. Y_t^E can be used as well as an intermediary good for the Main sector as a final good for consumers. The production of consumption good is defined by

$$Y_t = F(K_t, L_t, Z_t^E) \tag{1}$$

and the energetic production is defined by

$$Y_t^E = F^E(K_t^E, L_t^E, E_t) \tag{2}$$

The index ”^{E}“ represents input or output used or produced by the Energy sector.

2.2 Pollution and abatement

The dynamic of the stock of pollution at time t, P_t, is defined by

$$P_t = (1 - h)P_{t-1} + g(E_t, X_t) \tag{3}$$

where h is the natural level of pollution absorption, $0 \leq h \leq 1$ and $g(.)$ is the net pollution flow with X_t the spending of pollution abatement.

2.3 Consumers

We consider an overlapping generation model with two consumption goods and pollution effect. Individuals live two periods. The number of agent born at date t, N_t, is exogenous. Each agent young at period t, supplies inelastically one unit of labor in period t. She derives utility from the consumption of the two goods during the two periods - *i.e.* c_t and c_t^E in period t and d_{t+1} and d_{t+1}^E when agent is old. The negative effect of pollution occurs in the two periods - *i.e.* P_t and P_{t+1}. The agent's preferences are represented by a general utility function

$$U_t = U(c_t, c_t^E, P_t, d_{t+1}, d_{t+1}^E, P_{t+1}) \tag{4}$$

The function $U(.)$ is strictly concave, increasing with respect the two consumption goods, decreasing with respect pollution, twice continuously differentiable and satisfies the Inada conditions.

3 Optimal growth

3.1 The resource constraints

For the Main sector using capital K_t, labor L_t and energy Z_t^E we have the following resource constraints

$$Y_t = F(K_t, L_t, Z_t^E) = N_t c_t + N_{t-1} d_t + \overline{K}_{t+1} + X_t \tag{5}$$

where $\overline{K}_{t+1}$ is the total investment in the next period capital stock and X_t is the spending of pollution abatement.

Similarly with capital K_t^E, labor L_t^E and emissions E_t the resource constraint for Energy sector is

$$Y_t^E = F^E(K_t^E, L_t^E, E_t) = N_t c_t^E + N_{t-1} d_t^E + Z_t^E \tag{6}$$

We assume total depreciation of capital during one period. Thus the capital resource constraint implies

$$\overline{K}_t = K_t + K_t^E \tag{7}$$

and the labor resource constraint is

$$N_t = L_t + L_t^E \tag{8}$$

The objective of the central planner is to maximize the welfare of all agents of all generations, with a discount factor, γ, $0 < \gamma < 1$,

$$\sum_{t=-1}^{+\infty} \gamma^t N_t U_t$$

given the initial stocks $\overline{K}_0$ and P_{-1} and the past values for the first old.

3.2 Optimality conditions

The central planner chooses the level of consumptions, c_t, c_t^E, d_{t+1} and d_{t+1}^E, capital K_t, K_t^E, emission E_t, depollution X_t and the intermediary consumption Z_t^E. We substitute $K_t^E = \overline{K}_t - K_t$ and $L_t^E = N_t - L_t$. Denoting by λ_t and λ_t^E respectively the Lagrangian multiplier of the resources constraints (5) and (6) and by μ_t the Lagrangian multipliers of the dynamic of the stock of pollution (3), the Lagrangian is defined by

$$\gamma^{-1}U_{-1} + \sum_{t=0}^{+\infty} \gamma^t \left\{ \begin{array}{c} N_t U_t + \lambda_t \left[F(K_t, L_t, Z_t^E) - N_t c_t - N_{t-1} d_t - \overline{K}_{t+1} - X_t \right] \\ +\lambda_t^E \left[F^E(\overline{K}_t - K_t, N_t - L_t, E_t) - N_t c_t^E - N_{t-1} d_t^E - Z_t^E \right] \\ +\mu_t \left[P_t - (1-h) P_{t-1} - g(E_t, X_t) \right] \end{array} \right\}$$

One obtains thereby the first order conditions,

- for the first period consumptions

$$\frac{\partial U_t}{\partial c_t} = \lambda_t \text{ and } \frac{\partial U_t}{\partial c_t^E} = \lambda_t^E$$

- for the second period consumption

$$\frac{1}{\gamma}\frac{\partial U_{t-1}}{\partial d_t} = \lambda_t \text{ and } \frac{1}{\gamma}\frac{\partial U_{t-1}}{\partial d_t^E} = \lambda_t^E$$

- Use of energy Z_t^E, optimal emissions E_t and depollution X_t satisfy

$$\lambda_t F_{Z_t^E} = \lambda_t^E \ ; \ \lambda_t^E F_{E_t}^E = \mu_t g_{E_t} \ ; \ \lambda_t = -\mu_t g_{X_t}$$

We denote by $F_{Z_t^E}$ the derivative of $F(K_t, L_t, Z_t^E)$ with respect to the third argument and similarly for $F_{E_t}^E$, g_{E_t} and g_{X_t}.

- The arbitrage conditions for capital and labor between the two sectors

$$\lambda_t F_{K_t} = \lambda_t^E F_{K_t^E}^E \ ; \ \lambda_t F_{L_t} = \lambda_t^E F_{L_t^E}^E$$

The dynamics of the shadow prices are obtained by differentiating the Lagrangian with respect to $\overline{K}_{t+1}$ and P_t, $\forall t \geq 0$

$$\lambda_t = \gamma \lambda_{t+1}^E F_K^E(\overline{K}_{t+1} - K_{t+1}, N_{t+1} - L_{t+1}, E_{t+1})$$

and

$$\mu_t = \gamma \mu_{t+1}(1-h) + N_t \frac{\partial U_t}{\partial P_t} + \frac{N_{t-1}}{\gamma}\frac{\partial U_{t-1}}{\partial P_t}$$

The tranversality condition is (Michel (1990))

$$\lim_{t \to +\infty} \gamma^t \left(\lambda_t \overline{K}_{t+1} + \mu_t P_t\right) = 0$$

3.3 Arbitrage optimality conditions

After eliminating the shadow prices for physical capitals and for pollution and rearranging the terms, we explicit the different trade-offs faced by the central planner.

The ratio λ_t/λ_t^E is equal to

$$\frac{1}{F_{Z_t^E}} = \frac{F^E_{K_t^E}}{F_{K_t}} = \frac{F^E_{L_t^E}}{F_{L_t}} \tag{9}$$

- Trade-off between consumptions on life cycle (using the dynamic equation of λ_t)

$$\frac{\partial U_t}{\partial c_t} = \frac{\partial U_t}{\partial d_{t+1}} F_{K_{t+1}} \tag{10}$$

- Trade-off between the consumptions of the two goods

$$\frac{\partial U_t}{\partial c_t^E} = F_{Z_t^E} \frac{\partial U_t}{\partial c_t} \text{ and } \frac{\partial U_{t-1}}{\partial d_t^E} = F_{Z_t^E} \frac{\partial U_{t-1}}{\partial d_t} \tag{11}$$

4 Equilibrium with pollution permits

In the economy with a market of tradeable permits of pollution, the government policy consists of issuing a quantity of permits, $\overline{E}_t$, allocating permits $\overline{E}_t^E$ to firms of energy sector, and the difference, $\overline{E}_t - \overline{E}_t^E$, is auctioned. It also makes a transfers, τ_t, to the young agent and θ_t to each old agent. Its budget is balanced at each period t. The consumption good of the Main sector is taken as numeraire and we denote by p^E the price of the energy. The price on the pollution permits market is denoted q_t.

4.1 Consumers

Consumers take the environment as given. At the first period of life, the young agent earns the wage w_t and receives a transfer τ_t which may be positive or negative. She consumes, c_t, c_t^E and saves s_t. Then, the first period budget constraint is

$$w_t + \tau_t = c_t + p_t^E c_t^E + s_t \tag{12}$$

When she is old, in the second period of life, she is retired and receives a transfer θ_{t+1} in addition of the return to her savings , $R_{t+1}s_t$ with R_{t+1} the gross interest rate. The old agent consumes d_{t+1} and d_{t+1}^E with all her income. Then, the second period budget constraint is

$$d_{t+1} + p_{t+1}^E d_{t+1}^E = R_{t+1}s_t + \theta_{t+1} \tag{13}$$

The agent maximizes utility (4) by choosing consumptions subject to the budget constraints (12) and (13). Given prices and pollution, P_t and P_{t+1}, the first order conditions of arbitrage between the two goods are:

$$\frac{\partial U_t}{\partial c_t} = p_t^E \frac{\partial U_t}{\partial c_t^E} \tag{14}$$

$$\frac{\partial U_t}{\partial d_{t+1}} = p^E_{t+1} \frac{\partial U_t}{\partial d^E_{t+1}} \tag{15}$$

and the intertemporal arbitrage

$$\frac{\partial U_t}{\partial c_t} = R_{t+1} \frac{\partial U_t}{\partial d_{t+1}} \tag{16}$$

The relation (14) and (15) corresponds to the trade-off between consumptions and relation (16) corresponds to the trade-off between consumptions on life cycle.

4.2 Firms

Firms are competitive. We consider two representatives firms. At the first step we consider the capital stock K_t of the Main sector and the capital K^E_t of the Energy sector as given. The firms take prices, w_t, p^E_t and q_t as given and maximize there net revenue. The Main sector's firm maximizes with respect L_t and Z^E_t,

$$F(K_t, L_t, Z^E_t) - w_t L_t - p^E_t Z^E_t \tag{17}$$

The first order conditions are

$$F_L(K_t, L_t, Z^E_t) = w_t \tag{18}$$

and

$$F_{Z^E}(K_t, L_t, Z_t^E) = p_t^E \tag{19}$$

The corresponding profit per unit of capital π_t/K_t is equal to the marginal productivity of capital (from the Euler equation) *i.e.* $\pi_t/K_t = F_{K_t}(K_t, L_t, Z_t^E)$.

Given K_t^E and $\overline{E}_t^E$, the Energy representative firm maximizes,

$$p_t^E F^E(K_t^E, L_t^E, E_t) - w_t L_t^E - q_t(E_t - \overline{E}_t^E) \tag{20}$$

The first order conditions are

$$F^E_{L_t^E}(K_t^E, L_t^E, E_t) = \frac{w_t}{p_t^E} \tag{21}$$

and

$$F^E_{E_t}(K_t^E, L_t^E, E_t) = \frac{q_t}{p_t^E} \tag{22}$$

The corresponding profit per unit of capital π_t^E/K_t^E is equal to,

$$\frac{\pi_t^E}{K_t^E} = p_t^E F^E_{K_t^E} + q_t \frac{\overline{E}_t^E}{\overline{K}_t^E} \tag{23}$$

Profits per unit of capital are the return of investment attributed to shareholder, the owners of the capital stock. The Energy sector in addition to the marginal productivity of capital there is the value of permits given to firms. Therefore, this return does not satisfy the neo-classical property of equality of factor income with marginal productivity.

4.3 Equilibrium

At each period, the government budget is balanced, *i.e.* satisfied,

$$X_t + N_t\tau_t + N_{t-1}\theta_t = q_t(\overline{E}_t - \overline{E}_t^E) \tag{24}$$

The intertemporal equilibrium is defined, for a given sequence of government decisions (τ_t, θ_t, X_t, $\overline{E}_t$, $\overline{E}_t^E$) which satisfy (24) with q_t the equilibrium price of permits market. It is a sequence of prices (p_t^E, w_t, q_t), individual variables (c_t, c_t^E, s_t, d_{t+1}, d_{t+1}^E) and aggregate variables (K_t, L_t, Z_t, Y_t), (K_t^E, L_t^E, E_t, Y_t^E), P_t and $\overline{K}_{t+1}$ satisfying all the equilibrium conditions. The government decisions satisfies its budget constraint. Consumers decisions maximize their utility. Each firm maximizes its profit. The return to savings is equal to the average productivities of capital,

$$\frac{\pi_t}{K_t} = \frac{\pi_t^E}{K_t^E} = R_t \tag{25}$$

The total capital stock is equal to savings i.e. $K_t + K_t^E = \overline{K}_t = N_{t-1}s_{t-1}$. The markets of labor, permits and goods clear.
In addition, the dynamic equation of pollution holds.

The first old consumption satisfies her constraint and the optimal trade-off conditions,

$$p_0^E d_0^E + d_0 = R_0 s_{-1} + \theta_0 \text{ and } \frac{\partial U_{-1}}{\partial d_0} = p_0^E \frac{\partial U_{-1}}{\partial d_0^E} \tag{26}$$

and the initial capital stock $\overline{K}_0 = N s_{-1}$ is given.

We explicitly define the equilibrium of the economy as follows:

Definition 1 *For a given policy $(\overline{E}_t, \overline{E}_t^E, X_t, \tau_t, \theta_t)_{t\geq 0}$, an equilibrium is defined by*

- sequence of prices $(q_t, p_t^E, w_t, R_t)_{t\geq 0}$,

- sequence of individuals variables $(c_t, c_t^E, s_t, d_{t+1}, d_{t+1}^E)$ satisfying relations (12) to (16) and d_0 and d_0^E satisfies (26),

- sequence of aggregate variables (K_t, L_t, Z_t, Y_t), (K_t^E, L_t^E, E_t, Y_t^E), P_t and $\overline{K}_{t+1}$ satisfying (18) to (23).

such that the following equilibrium conditions hold:

- the government budget is balanced, (24),

- the capital stock $\overline{K}_t = K_t + K_t^E$ is equal to savings $N_{t-1}s_{t-1}$. With K_t and K_t^E satisfying $\pi_t/K_t = \pi_t^E/K_t^E$,

- the market of labor, permits and good clear:

$L_t + L_t^E = N_t$, $F_L = p_t^E F_{L_t}^E = w_t$

$E_t = \overline{E}_t$

- the resources constraints (5)and (6) hold

- the dynamic of pollution is defined by relation (3) .

Proposition 2 *The equilibrium satisfies the optimality condition (9) if and* *only if zero permits are attributed to firms,* $\overline{E}_t^E = 0$.

Proof. The equilibrium condition of equality of average returns of capital is

$$\frac{\pi_t}{K_t} = \frac{\pi_t^E}{K_t^E} \Leftrightarrow F_K(K_t, L_t, Z_t^E) = p_t^E F_K^E + q_t \frac{\overline{E}_t^E}{K_t^E}$$

and with equation (19) implies

$$\frac{\pi_t}{K_t} = \frac{\pi_t^E}{K_t^E} \Leftrightarrow F_K(K_t, L_t, Z_t^E) = F_{Z_t^E} F_K^E + q_t \frac{\overline{E}_t^E}{K_t^E}$$

The optimality condition (9) is equivalent to $\overline{E}_t^E = 0$ ■

Proposition 3 *The optimal path* $(c_t, c_t^E, d_t, d_t^E, K_t, L_t, Z_t^E, X_t, E_t, P_t, \overline{K}_{t+1})_{t \geq 0}$ *is an equilibrium with public decisions* X_t, $\overline{E}_t = E_t$, $\tau_t = c_t + p_t^E c_t^E + s_t - w_t$ *and* $\theta_t = d_t + p_t^E d_t^E - R_t s_t$
where $p_t^E = 1/F_{Z_t^E}$, $w_t = F_{L_t}$, $R_t = F_{K_t}$, $q_t = F_{E_t}^E / F_{Z_t^E}$ *and* $s_t = \overline{K}_{t+1}/N_t$

Proof. Straightforward : it is sufficient to verify that all equilibrium conditions are satisfied. ■

In fact, any path satisfying the resources constraints and the optimality condition (9) is an equilibrium.

5 Conclusion

We have shown that it is possible to decentralize the optimal growth path with only lump-sum transfers and a market for permits concerning only the

Energy sector. But a necessary condition to realize such a decentralization is to allocate no permits to firms, which rules out practices such as grandfathering. Such practices subsidize firms by raising the return on investment. As a consequence the interest rate is not equal to the marginal productivity of capital at the decentralized equilibrium.

References

Allais M., 1947, *Economie et Intérêt*, Imprimerie Nationale.

Coase M.A. [1960], "The Problem of Social Cost", *Journal of Law and Economics*, **3**, 1-44.

De la Croix, D. and Ph., Michel, 2002, *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*, Cambridge University Press.

Diamond P.A. [1965], "National Debt in a Neoclassical Growth Model", *American Economic Review*, **30**, 657-740.

Jouvet P.A., Ph. Michel and G. Rotillon, 2005, "Optimal Growth with Pollution : How to Use Pollution Permits", Journal of Economic Dynimics and Control, 29, 1597-1609.

Koutstaal P., 1997, *Economic Policy and Climate Change: Tradeable Permits for Reducing Carbon Emissions*, Edward Elgar Publishing Limited, UK.

Michel Ph., 1990,"Some Clarifications on Traversality Conditions", *Econometrica*, 58, 705-723.

Montgomery D.W., 1972, "Markets in Licenses and Efficient Pollution Control Programs", *Journal of Economic Theory*, 5, 395-418. Parry Ian W.H., Roberton C. Williams, and Lawrence H. Goulder, 1999,"When Can Carbon Abatement Policies Increase Welfare ? The Fundamental Role of Distorted Factor Markets" *Journal of Environmental Economics and Management*, 37, 52-84

Stavins Robert N., 1998,"What Can We Learn from the Grand Policy Experiment ? Lessons from SO2 Allowance Tradin", *Journal of Economic Perspectives*, 12 (3), 69-88.