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Moneychangers and Commodity Money*

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Abstract

We study the role played by coin experts, called moneychangers, in the metallic money system. To do that, we introduce intermediaries that can expertise and certify coins into the Velde Weber and Wright's (1999) model of commodity money with imperfectly recognizable coins. We show under which conditions buyers have their coins certified, how *circulation by weight* and *circulation by tale* equilibria are affected by moneychangers, and whether moneychangers increase welfare.

Keywords: Commodity money, Search, Information, Moneychangers.
JEL Classification: D82, E42, N23.

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1 Introduction

In the metallic money system, the intrinsic content of coins offered in payment by buyers was not readily assessable by sellers. This created an asymmetric information problem between sellers and buyers who supposedly had more information on their own coins. Velde Weber and Wright (1999), hereafter VWW, show that this asymmetric information problem generates two inefficient outcomes. When the discount rate is high, heavy coins that are not recognized by sellers trade below their full information value. And when the discount rate is low, unrecognized heavy coins are simply hoarded. The authors refer to the first situation as *circulation by tale* and to the second as *circulation by weight*.

In this paper we amend VWW by introducing a special class of intermediaries, called moneychangers, that can expertise and certify the intrinsic content of coins for a fee. Our goal is to evaluate their impact on coins circulation and welfare. Precisely, we ask: 1) Under which conditions do buyers ask moneychangers to certify their coin? 2) How do moneychangers impact the circulation by weight and circulation by tale equilibria displayed in VWW? 3) Do moneychangers increase welfare?

We show that the certification of heavy coins is always an equilibrium as long as the discount rate is neither too low nor too high. We also show that moneychangers limit circulation by weight and circulation by tale to situations in which the fraction of informed sellers is high, and that circulation by tale always survives the introduction of moneychangers for high values of the discount rate. In terms of welfare, the introduction of moneychangers that triggers a transition from a by-weight equilibrium to an active moneychangers equilibrium is always welfare improving, but welfare is higher in circulation by weight when the two equilibria coexist. By contrast, the introduction of moneychangers that triggers a transition from a by-tale equilibrium to an active moneychangers equilibrium is always welfare worsening, and welfare is also higher in circulation by tale when the two equilibria coexist.

2 The Model

The environment is VWW to which we add coin assayers. There is a $[0,1]$ continuum of infinitely lived agents and there are $I \geqslant 3$ types of goods. A type $k \in I$ agent consumes good k and produces good $k+1$. Agents meet bilaterally according to an anonymous random matching process with arrival rate α. Money is in the form of gold coins coming in light (L) and heavy (H) weight. Let M_i be the measure of agents endowed with coins of type $i = \{L, H\}$ so $M = M_H + M_L$ represents the fraction of agents that are buyers and $1 - M$ represents the fraction of agents that are sellers, also called producers. Each coin yields a positive periodic flow of utility γ_i proportional to its weight (or intrinsic content) so that $\gamma_H > \gamma_L$. A buyer always knows the type of his coin. By contrast, when matched with a buyer, the seller receives a common knowledge signal that reveals the weight of the coin held by the buyer with probability θ. The signal is uninformative with probability $1 - \theta$.

To circumvent the information problem, buyers can pay δ to a moneychanger in order to have a certificate of quality attached to their coin. We assume that moneychangers accept all goods in payment and value them identically so that each buyer can resort to a moneychanger. Also, if a seller is to accept a coin with its certificate, he must pay δ to the assayer to use the certificate the next period.[1]

Let V_0, V_L and $W = \max(V_H, V_{HC} - \delta)$ be the steady state value functions for, respectively, a producer, a holder of a light coin and a holder of a heavy coin before he decides whether to visit an assayer, where V_H is the steady-state value function of not expertising the heavy coin, and V_{HC} that of expertising the heavy coin exclusive of the fee δ. When trade takes place, a type i coin is exchanged against a quantity q_i of goods given by a take-it-or-leave-it offer made by the buyer. Producing q_i costs $c(q_i) = q_i$ and consuming q_i yields $u(q_i) = (q_i)^\alpha$ with $\alpha \in (0, 1)$. As in VWW, we note $\bar{q}$ the amount a buyer gets from an uninformed seller and $\hat{q} = 1$ the quantity such that $u(\hat{q}) = \hat{q}$. We define $q = (q_L, q_H)$ as the quantity vector.

Also, as in VWW, we note λ_{ij} the probability that a buyer with a coin of type $i \in \{L, H\}$ wants to trade with a seller of type $j \in \{K, U\}$ where K means that the weight of the coin is known and U the weight of the coin is unknown to the seller. Similarly, let λ_C be the probability that a buyer with a certified heavy coin wants to trade with a seller. Finally, let Σ be the probability that a random buyer with a heavy coin have it certified and let σ be the

[1] Coin assaying by moneychangers (also called coin-assayers or money-assayers) involved a series of costly operations. Weight was determined using precise scales, and fineness was estimated using a set of touchstones. The touchstone test consisted in rubbing a coin on a special stone and comparing the color of the trace left with that of needles of known fineness. A more precise assay consisted in melting down the coin in order to weight the remaining precious metal (Gandal and Sussman, 1997, p. 443-444). See also De Roover (1948) for the study of moneychangers in medieval Bruges.

corresponding individual best response. At the symmetric Nash equilibria, $\sigma = \Sigma$. We define $\Psi = (\lambda_{LK}, \lambda_{LU}, \lambda_{HK}, \lambda_{HU}, \lambda_C, \sigma)$ as the strategy vector. Note that holders of light coins never visit assayers.

Noting $\beta = \frac{\alpha}{I}(1 - M)$, the Bellman equation for a buyer with a light coin is[2]

$$rV_L = \gamma_L + \beta\theta \max_{\lambda_{LK}} \lambda_{LK} \left[u(q_L) - q_L\right] + \beta (1 - \theta) \max_{\lambda_{LU}} \lambda_{LU} \left[u(\bar{q}) - q_L\right] = rq_L. \tag{1}$$

Eq. (1) sets the flow return to a buyer holding a light coin, rV_L, equal to three parts. The first part corresponds to the return on holding the light coin, γ_L. The second part corresponds to the probability that he meets an informed producer, $\beta\theta$, times the net gain from trading the light coin against q_L, $u(q_L) - q_L$, times the probability that he decides to trade with him, λ_{LK}. The last part corresponds to the probability that he meets an uninformed seller, $\beta (1 - \theta)$, times the net gain from trading the light coin against the quantity traded in uninformed meetings $\bar{q}$, $u(\bar{q}) - q_L$, times the probability that he decides to trade with her, λ_{LU}.

The Bellman equation for a buyer with a heavy coin, before he decides whether to certify the coin, is

$$W = \max_{\sigma} \{\sigma (V_{HC} - \delta) + (1 - \sigma) V_H\} = q_H. \tag{2}$$

If a buyer does not certify his heavy coin, his payoff is

$$rV_H = \gamma_H + \beta\theta \max_{\lambda_{HK}} \lambda_{HK} \left[u(q_H) - q_H\right] + \beta (1 - \theta) \max_{\lambda_{HU}} \lambda_{HU} \left[u(\bar{q}) - W\right] + W - V_H. \tag{3}$$

[2] The equations (1) to (4) below are derived from the developed model displayed in the appendix.

If he does, his payoff is

$$rV_{HC} = \gamma_H + \beta \max_{\lambda_C} \lambda_C \left[u(q_H) - W\right] + W - V_{HC}. \tag{4}$$

When $\sigma = \Sigma = 0$, inserting the bargaining solution $W = V_H = q_H$ in equation (3) gives

$$rq_H = \gamma_H + \beta\theta \max_{\lambda_{HK}} \lambda_{HK} \left[u\left(q_H\right) - q_H\right] + \beta\left(1-\theta\right) \max_{\lambda_{HU}} \lambda_{HU} \left[u\left(\bar{q}\right) - q_H\right]. \tag{5}$$

When $\sigma = \Sigma = 1$, inserting the bargaining solution $W = V_{HC} - \delta = q_H$ in equation (4) gives

$$rq_H = \gamma_H + \beta \max_{\lambda_C} \lambda_C \left[u\left(q_H\right) - q_H\right] - \left(1+r\right)\delta. \tag{6}$$

Equations (3) and (4) below have a similar interpretation to (1).

The λ_{ij} must satisfy the following incentive conditions: for $i \in \{L, H\}$

$$\lambda_{iK} = \begin{cases} 1 & \text{if } u(q_i) - q_i \geq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

$$\lambda_{iU} = \begin{cases} 1 & \text{if } u(\bar{q}) - q_i \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

Finally σ is given by (2) and from (3) and (6) we conclude that $\lambda_C = \lambda_{HK}$ since $W = q_H$.

3 Equilibria

Definition 1 *A symmetric steady state equilibrium is a vector of quantities q and strategies Ψ such that: (i) : q satisfy (1) and (5) or (6); (ii) : Ψ satisfy (2), (7) and (8).*

To see how moneychangers affect the pure strategy equilibria displayed in VWW, we distinguish between equilibria where moneychangers are active, $\sigma = \Sigma = 1$, and equilibria where they are not, $\sigma = \Sigma = 0$. The generic condition for moneychangers to be inactive is given by

$$V_H > V_{HC} - \delta. \tag{9}$$

Focussing on equilibria where recognized coins circulate ($\lambda_{LK} = \lambda_{HK} = \lambda_C = 1$), using (3) and (4), replacing W by $V_H = q_H$ on the left side of (9) and W by $V_{HC} - \delta = q_H$ on the right side, this inequality transforms into

$$\beta\theta\left[u(q_H) - q_H\right] + \beta\left(1-\theta\right)\lambda_{HU}\left[u(\bar{q}) - q_H\right] > \beta\left[u(q_H) - q_H\right] - (1+r)\,\delta. \tag{10}$$

3.1 Equilibria with inactive moneychangers

Moneychangers are inactive when buyers holding heavy coins have no incentive to pay for their service ($\sigma = \Sigma = 0$). Then, circulation by weight corresponds to a situation where heavy coins are not traded in uninformed meetings but still agents do not certify them. The strategy whether to trade a certified heavy coin is then irrelevant so that $\Psi = (1, 1, 1, 0, \_, 0)$.

As in VWW, the two coins circulate in informed meetings if $u(q_L) \geq q_L$ and $u(q_H) \geq q_H$, which simplifies into $r > \gamma_H$,[3] and heavy coins do not circulate when not recognized if $u(q_L) \leq q_H$ so that the *by-weight frontier* $\theta = f_w(r)$ is defined by

$$\begin{aligned} rq_L &= \gamma_L + \beta\left[u(q_L) - q_L\right] && (11) \\ rq_H &= \gamma_H + \beta\theta\left[u(q_H) - q_H\right] && (12) \\ q_H &= u(q_L). && (13) \end{aligned}$$

[3] To see this note from (7) that $\lambda_{iK} = 1 \Longleftrightarrow u(q_i) - q_i > 0 \Longleftrightarrow q_i < \hat{q}$ or $rq_i < r\hat{q}$. Inserting $\hat{q}$ into (11) and (12) to obtain $rq_L = \gamma_L$ and $rq_H = \gamma_H$ and using $\hat{q} = 1$, this yields $r > \gamma_H > \gamma_L$.

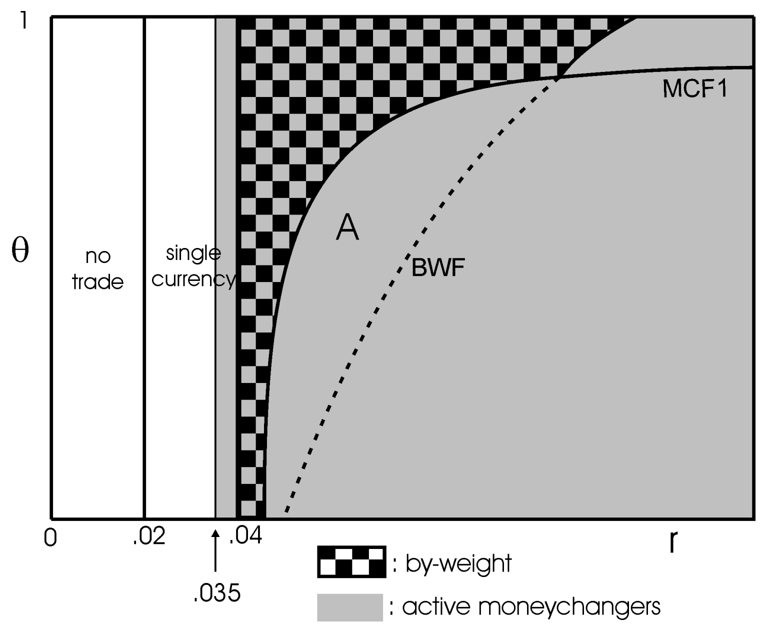

Figure 1: By-weight and active moneychangers equilibria

In addition to VWW, inserting $\lambda_{HU} = 0$ in (10) yields the following condition for moneychangers to be inactive

$$(1 + r)\,\delta > \beta\,(1 - \theta)\,[u\,(q_H) - q_H]\,. \tag{14}$$

This inequality says that for moneychangers to be inactive, we need the cost of expertise to exceed the benefit which is the gain from trade on unrecognized heavy coins. This condition generates a first moneychanger frontier, $MCF1$, as the solutions to (12) that satisfy (14) with equality for a given δ. It is represented in Fig. (1). All points in the (r, θ) space above this frontier are consistent with inactive moneychangers in the by-weight equilibrium.[4]

[4] This first moneychanger frontier, as for the next one, cannot be characterized analytically so that it is simulated using Mathematica algorithms available on request. We use the same parameter values as in VWW, that is $\alpha = 0.7$, $\gamma_L = 0.02$, $\gamma_H = 0.04$, $M_H = 0.2$, $M_L = 0.3$ and set additionally $\beta = 0.67$ and $\delta = 0.005$.

In circulation by tale, agents do not certify heavy coins so that they trade at the same price as light coins in uninformed meetings, that is $\Psi = (1, 1, 1, 1, \_, 0)$. As in VWW, the two coins circulate in informed meetings if $u(q_L) \geq q_L$ and $u(q_H) \geq q_H$, which again simplifies into $r > \gamma_H$, and heavy coins circulate when not recognized if $u(\bar{q}) \geq q_H$ so that the *by-tale frontier* $\theta = f_t(r)$ is defined by

$$
\begin{aligned}
rq_L &= \gamma_L + \beta\theta \left[u(q_L) - q_L\right] + \beta\left(1-\theta\right)\left[u\left(\bar{q}\right) - q_L\right] && (15)\\
rq_H &= \gamma_H + \beta\theta \left[u(q_H) - q_H\right] + \beta\left(1-\theta\right)\left[u\left(\bar{q}\right) - q_H\right] && (16)\\
q_H &= u(\bar{q}). && (17)
\end{aligned}
$$

Inserting $\lambda_{HU} = 1$ in (10) yields the following condition for moneychangers to be inactive

$$
\left(1+r\right)\delta > \beta\left(1-\theta\right)\left[u\left(q_H\right) - u(\bar{q})\right] \tag{18}
$$

which says that the cost of expertise needs exceed the benefit, which is the difference between what a buyer gets with a recognized heavy coin and an unrecognized heavy coin[5]. This condition generates a second moneychanger frontier, *MCF2*, as the equilibrium solutions of q_H to (15) and (16) that satisfy (18) with equality for a given δ. It is represented in Fig. (2). We note $\bar{r}_1$ the threshold value for r such that *MCF2* reaches the horizontal axis when $\theta = 0$. All points in the (r, θ) space above this frontier are consistent with inactive moneychangers in the by-tale equilibrium. Note that this frontier as for *MCF1* shift up as δ decreases.

[5] Note that this condition and (14) have a lot in common with the condition under which agents put their money into banks in He, Huang and Wright (2005). In He, Huang and Wright, agents put their money into banks for safety reasons, while here agents buy a certificate for recognizability reasons.

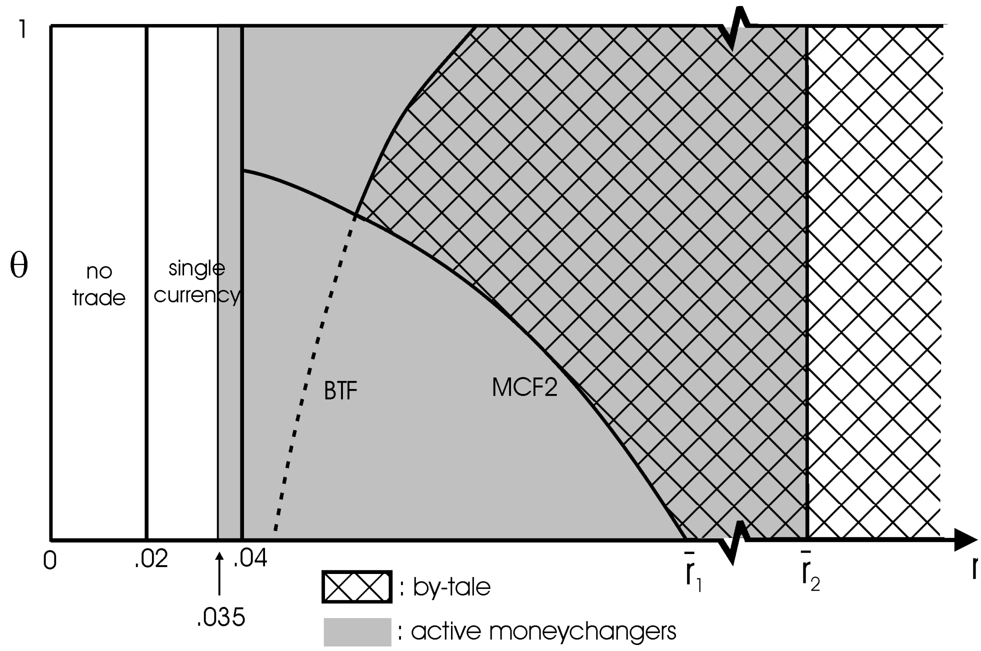

Figure 2: By-tale and active moneychangers equilibria

3.2 Equilibria with active moneychangers

If moneychangers are active, then $\sigma = \Sigma = 1$ and the strategies whether to trade uncertified heavy coins (λ_{HK} and λ_{HU}) are irrelevant so that $\Psi = (1, 1, -, -, 1, 1)$. Also, the information problem has disappeared so that light coins always trade at their full information value. The model becomes

$$rq_L = \gamma_L + \beta\,[u(q_L) - q_L] \tag{19}$$

$$rV_H = \gamma_H + q_H - V_H \tag{20}$$

$$rq_H = \gamma_H + \beta\,[u(q_H) - q_H] - (1 + r)\,\delta. \tag{21}$$

Using the same procedure as in footnote 3, recognized light and certified heavy coins circulate if $r > \frac{\gamma_H - \delta}{\hat{q}+\delta} \approx 0.035$ with our parameters so that $\gamma_H > \frac{\gamma_H - \delta}{\hat{q}+\delta} > \gamma_L$. In addition, note from (21) that the "real" periodic return on certified heavy coins is now $\gamma_H - (1+r)\,\delta$. This imposes that r must below some threshold, noted $\hat{r}$, otherwise (21) would not have a solution (more is said on this below).

For moneychangers to be active, the payoff must satisfy $V_{HC} - \delta > V_H$ which transforms into

$$\beta\,[u\,(q_H) - q_H] > (1+r)\,\delta. \tag{22}$$

This inequality says that the benefit derived from trade needs exceed the cost of expertise. This defines a third and last moneychanger frontier, *MCF 3*, as the solutions to (21) that satisfy (22) with equality for a given δ. Contrary to *MCF1* and *MCF2*, because the recognizability problem has vanished thanks to moneychangers, this third frontier simplifies into $r < \bar{r}_2$ with $\bar{r}_2$ independent of θ. It is easy to see that $\hat{r} > \bar{r}_2 > \bar{r}_1$ so that in the end heavy coins are certified by moneychangers if $r \in \left[\frac{\gamma_H - \delta}{\hat{q}+\delta}, \bar{r}_2\right]$ as represented on Fig. (2).[6]

4 Results and Welfare

First, moneychangers certifying heavy coins is an equilibrium if the discount rate has reasonable values. If it is too small, heavy coins are hoarded; if it is too big, gains from trade are too small

[6] The proof is by contradiction. Assume $\bar{r}_2 > r > \hat{r}$. According to $\bar{r}_2 > r$, heavy coins are certified. According $r > \hat{r}$, certified heavy coins do not circulate, in which case they are not certified, which contradicts $\bar{r}_2 > r$. Therefore $\bar{r}_2 < \hat{r}$. Assume now that $\bar{r}_1 > r > \bar{r}_2$. From $\bar{r}_1 > r$ we derive that moneychangers cannot be inactive in by-tale equilibrium, and from $r > \bar{r}_2$ we derive that moneychangers cannot be active, which contradicts $\bar{r}_1 > r$. Therefore $\bar{r}_1 < \bar{r}_2$

to compensate for the cost of certification. Second, moneychangers reduce both circulation by weight and circulation by tale to situations in which information on coins is abundant. Intuitively, should a seller not recognize the buyer's heavy coin, the buyer can still wait for another seller who will recognize his coin with a high probability. Third, circulation by tale always survives as an equilibrium for high values of the discount rate whatever the cost of expertise. Intuitively, when r is high q_L and q_H are low, so that in unrecognized meetings certification (which is costly) does not improve the situation compared to trading heavy coins at $\bar{q}$.

What about moneychangers' contribution to the economy's welfare? The welfare function in each equilibrium is defined as the weighted average of lifetime utilities across agents' types (appendix 2 displays these functions). The following propositions resumes our results:

Proposition 1 *The introduction of moneychangers that triggers a transition from a by-weight equilibrium to an active moneychangers equilibrium is always welfare improving. When the two equilibria coexist, welfare is higher in circulation by weight.*

Proof. See Appendix. ■

Proposition 2 *The introduction of moneychangers that triggers a transition from a by-tale equilibrium to an active moneychangers equilibrium is always welfare worsening. When the two equilibria coexist, welfare is also higher in circulation by tale.*

Proof. See Appendix ■

Certificates are purchased by buyers holding heavy coins only if the net benefit is positive. Therefore, in the by-weight equilibrium, since buyers holding light coins are indifferent to moneychangers, whether welfare is higher with active moneychangers entirely relies on buyers holding heavy coins. If they choose to pay for a certificate, their welfare must be higher (zone A in Fig. 1) and the economy's welfare is higher. In the by-tale equilibrium, however, the decision by heavy buyers to visit moneychangers also impacts on the welfare of buyers holding light coins in a way that makes the economy overall worse-off with active moneychangers. To see this, note first that in circulation by tale, agents are better off when coins are imperfectly recognizable than when there are not. As a matter of fact, because of risk aversion, agents prefer to get $\bar{q}$ in $(1-\theta)$ meetings than q_L in $(1-\theta)(1-\pi)$ meetings and q_H in the remaining $(1-\theta)\pi$ meetings. Second, when coins are fully recognizable, agents are better off in circulation by tale than with active yet costly moneychangers. These two observations imply that agents are always better off in a by-tale equilibrium than in an active moneychangers equilibrium.

To summarize, moneychangers help mitigate the asymmetric information problem in the metallic money system. Depending on the price they charge, circulation by weight and circulation by tale more or less survive as equilibrium circulation. Because of the fee, however, circulation by tale remains the only option when a high discount rate drives the gains from trade to negligible quantities. In terms of welfare, moneychangers only improve the situation of heavy coins holders so that welfare increases from circulation by weight, but falls from circulation by tale.

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Appendix

A.1. The developed model

Noting $x = \alpha/I$ and q_C the quantity traded for a certificate, we have

$$V_0 = \frac{1}{1+r}\left\{\begin{array}{c} xM_H\left\{(1-\Sigma)\,\theta\left[\lambda_{HK}\left(W-q_H\right)+\left(1-\lambda_{HK}\right)V_0\right]+\Sigma\left[\lambda_C\left(W-q_C\right)+\left(1-\lambda_C\right)V_0\right]\right\} \\ +xM_L\theta\left[\lambda_{LK}\left(V_L-q_L\right)+\left(1-\lambda_{LK}\right)V_0\right]+xM_L\left(1-\theta\right)\left[\lambda_{LU}\left(V_L-\bar{q}\right)+\left(1-\lambda_{LU}\right)V_0\right] \\ +xM_H\left(1-\Sigma\right)\left(1-\theta\right)\left[\lambda_{HU}\left(W-\bar{q}\right)+\left(1-\lambda_{HU}\right)V_0\right]+\left(1-xM\right)V_0 \end{array}\right\}$$

$$V_L = \frac{1}{1+r}\left\{\begin{array}{c} \gamma_L + x\left(1-M\right)\theta\max_{\lambda_{LK}}\left[\lambda_{LK}u(q_L)+\left(1-\lambda_{LK}\right)V_L\right] \\ +x\left(1-M\right)\left(1-\theta\right)\max_{\lambda_{LU}}\left[\lambda_{LU}u(\bar{q})+\left(1-\lambda_{LU}\right)V_L\right] \\ +\left[1-x\left(1-M\right)\right]V_L \end{array}\right\}$$

$$V_H = \frac{1}{1+r}\left\{\begin{array}{c} \gamma_H + x\left(1-M\right)\theta\max_{\lambda_{HK}}\left[\lambda_{HK}u(q_H)+\left(1-\lambda_{HK}\right)W\right] \\ +x\left(1-M\right)\left(1-\theta\right)\max_{\lambda_{HU}}\left[\lambda_{HU}u(\bar{q})+\left(1-\lambda_{HU}\right)W\right] \\ +\left[1-x\left(1-M\right)\right]W \end{array}\right\}$$

$$V_{HC} = \frac{1}{1+r}\left\{\begin{array}{c} \gamma_H + x\left(1-M\right)\max_{\lambda_C}\left[\lambda_C u(q_C)+\left(1-\lambda_C\right)W\right]+ \\ +\left[1-x\left(1-M\right)\right]W \end{array}\right\}$$

Take-it-or-leave-it offers by buyers are such that

$$\begin{aligned} W-q_H &= V_0 \\ W-q_C &= V_0 \\ V_L-q_L &= V_0 \\ \pi W+\left(1-\pi\right)V_L-\bar{q} &= V_0 \end{aligned}$$

where

$$\pi = \frac{M_H\lambda_{HU}\left(1-\Sigma\right)}{M_L\lambda_{LU}+M_H\lambda_{HU}\left(1-\Sigma\right)}$$

such that $V_0 = 0$. Then $q_L = V_L$ and $q_H = q_C = W$ and $\bar{q} = \pi q_H + (1-\pi) q_L$.

A.2. Proof of Proposition 1

In general, equilibrium value q_L and q_H are not the same across equilibria. We note q_i^{bw}, q_i^{bt} and q_i^{am} the equilibrium quantity traded respectively for a coin of type i in a by-weight, by-tale and active-moneychangers equilibrium. They are given by (11)-(12), (15)-(16), and (19) and (21).

In each equilibrium, welfare noted WE is given by

$$rWE_{bw} = M_L \left\{ \beta \left[u(q_L^{bw}) - q_L^{bw} \right] + \gamma_L \right\} + M_H \left\{ \beta\theta \left[u(q_H^{bw}) - q_H^{bw} \right] + \gamma_H \right\}, \quad (23)$$

$$\begin{aligned} rWE_{bt} &= M_L \left\{ \beta\theta \left[u(q_L^{bt}) - q_L^{bt} \right] + \beta(1-\theta) \left[u(\bar{q}^{bt}) - q_L^{bt} \right] + \gamma_L \right\} \\ &\quad + M_H \left\{ \beta\theta \left[u(q_H^{bt}) - q_H^{bt} \right] + \beta(1-\theta) \left[u(\bar{q}^{bt}) - q_H^{bt} \right] + \gamma_H \right\} \end{aligned} \quad (24)$$

and

$$rWE_{am} = M_L \{ \beta [u(q_L^{am}) - q_L^{am}] + \gamma_L \} + M_H \{ \beta [u(q_H^{am}) - q_H^{am}] + \gamma_H - (1+r)\delta \} \quad (25)$$

From (14), on MCF1 we have

$$(1+r)\delta = \beta(1-\theta) \left[u\left(q_H^{bw}\right) - q_H^{bw} \right]. \quad (26)$$

Inserting this into (12) gives $rq_H^{bw} = \gamma_H + \beta \left[u(q_H^{bw}) - q_H^{bw} \right] - (1+r)\delta$ identical to (21) so that $q_H^{bw} = q_H^{am}$ on MCF1. From (11) and (19) it is clear that $q_L^{bw} = q_L^{am}$ as well whatever

$(r, \theta) \in [\gamma_H, \infty] \times [0, 1]$. Therefore, using (26), (23) can be rewritten

$$rWE_{bw} = M_L \left\{ \beta \left[u(q_L^{am}) - q_L^{am} \right] + \gamma_L \right\} + M_H \left\{ \beta \left[u(q_H^{am}) - q_H^{am} \right] + \gamma_H - (1 + r) \delta \right\}.$$

so that $rWE_{bw} = rWE_{am}$ on MCF1.

Now note that $d(rWE_{am})/d\theta = 0$. However

$$\frac{d(rWE_{bw})}{d\theta} = \beta \left\{ u \left[q_H^{bw}(\theta) - q_H^{bw}(\theta) \right] + \theta . \frac{dq_H^{bw}}{d\theta} . \left(u' \left[q_H^{bw}(\theta) \right] - 1 \right) \right\}.$$

Totally differentiating (12) yields $r.\frac{dq_H^{bw}}{d\theta} = \beta \left\{ u \left[q_H^{bw}(\theta) - q_H^{bw}(\theta) \right] + \theta . \frac{dq_H^{bw}}{d\theta} . \left(u' \left[q_H^{bw}(\theta) \right] - 1 \right) \right\}$ so that $d(rWE_{bw})/d\theta = r.\frac{dq_H^{bw}}{d\theta} > 0$ since equilibrium q_H^{bw} increases with θ. Therefore, because WE_{am} is constant whatever θ, $WE_{bw} \geq WE_{am}$ for any value of θ equal or above that of the MCF1, and $WE_{bw} < WE_{am}$ for any value of θ below that of the MCF1.

A.3. Proof of Proposition 2

Grouping all the terms in θ and $(1 - \theta)$ in the by-tale equilibrium, we obtain

$$\begin{aligned} rWE_{bt} &= \theta \left\{ M_L \beta \left[u(q_L^{bt}) - q_L^{bt} \right] + M_H \beta \left[u(q_H^{bt}) - q_H^{bt} \right] \right\} \\ &\quad + (1 - \theta) \beta \left\{ (M_L + M_H) u(\bar{q}^{bt}) - \left(M_L q_L^{bt} + M_H q_H^{bt} \right) \right\} + M_L \gamma_L + M_H \gamma_H \end{aligned}$$

From the concavity of u, we have $u(\bar{q}^{bt}) = u \left[\pi q_H^{bt} + (1 - \pi) q_L^{bt} \right] > \pi u(q_H^{bt}) + (1 - \pi) u \left(q_L^{bt} \right)$. Using the definition of π we get $(M_L + M_H) u(\bar{q}^{bt}) > M_L u \left(q_L^{bt} \right) + M_H u \left(q_H^{bt} \right)$ so that $rWE_{bt} >$

$rWE\left(\theta\right)$ whatever θ with

$$\begin{aligned} rWE\left(\theta\right) &= \theta\left\{M_L\beta\left[u(q_L^{bt})-q_L^{bt}\right]+M_H\beta\left[u(q_H^{bt})-q_H^{bt}\right]\right\} \\ &\quad +\left(1-\theta\right)\beta\left\{M_L u\left(q_L^{bt}\right)+M_H u\left(q_H^{bt}\right)-\left(M_L q_L^{bt}+M_H q_H^{bt}\right)\right\}+M_L\gamma_L+M_H\gamma_H \end{aligned}$$

which simplifies into

$$rWE\left(\theta\right)=M_L\left\{\beta\left[u(q_L^{bt})-q_L^{bt}\right]+\gamma_L\right\}+M_H\left\{\beta\left[u(q_H^{bt})-q_H^{bt}\right]+\gamma_H\right\}.$$

Note now from (15)-(16) and (19) that when $\theta=1$, $q_L^{bt}=q_L^{am}$, and from (15)-(16) and (21) that $q_H^{bt}>q_H^{am}$ so that $rWE\left(\theta=1\right)>rWE_{am}$. Because rWE_{am} is invariant with θ but $rWE_{bt}>rWE\ \left(\theta=1\right)$, we conclude $rWE_{bt}>rWE_{am}$.