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Abstract

For contemporary legal theory, law is essentially an interpretative and hermeneutics practice (Ackerman (1991), Horwitz (1992)). A straightforward consequence is that legal disputes between parties are motivated by their divergent interpretations regarding what the law says on their case. This point of view fits well with the growing evidence showing that litigants' cognitive performances display optimistic bias or self-serving bias (Babcock and Lowenstein (1997)). This paper provides a theoretical analysis of the influence of such a cognitive bias on pretrial negotiations. However, we also consider that this effect is mitigated because of the litigants' confidence in their own ability to predict the verdict; we model this issue assuming that litigants are risk averse in the sense of Yaari (1987), *i.e.* they display a kind of (rational) probability distortion which is also well documented in experimental economics. In a model \dot{a} *la* Bebcuck (1984), we show that the consequences of self-serving bias are partially consistent with the "optimistic model", but that parties' risk aversion has more ambiguous/unpredictable effects. These results contribute to explaining that the beliefs in the result of the trial are not sufficient in themselves to understand the behaviors of litigants. As suggested by legal theory, the confidence the parties have in their beliefs is probably more important.

JEL classification: D81, K42.

Keywords: litigation, self-serving bias, risk aversion.

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1 Introduction

The resolution of legal conflicts is an important topic in Law and Economics. Models relating to the outcome of trials attempt to describe and explain the choice of the parties between litigation and settlement. The ambition is to evaluate the consequences of strategic interactions between parties, rendered even more complex as one of the litigants may hold private information or present characteristics the other party cannot observe. The approach supposes that the court's role consists in identifying *ex post* an optimal allocation (i.e. after the apparition of the dispute). This is the position taken by Posner (1992) when he considers that the court's role is to "mimic" the transaction at zero cost. However, before the judge's decision intervenes there are several stages to be covered, each requiring a decision on the part of the litigants. The first representations of litigation (Landes (1972), Gould (1973)) did not really aim to describe the negotiations which may have occurred before the judge's decision. The aim was rather to identify the incentives of the parties in their capacity of the rational agents to solve a dispute, and to explain why negotiations fail in certain cases, thus requiring the intervention of a judge. The answer to this question comes from excessive optimism by the parties concerning their chances of winning the trial, i.e. a misperception of the surplus emanating from negotiation. Even though these models do give a reasonably convincing explanation of how disputes are settled out of court, they remain vague as to the way in which litigants are over-optimistic. From this point of view, strategic models propose a richer description of the legal process whether or not it achieves an agreement between the parties. The strategic nature of negotiation between litigants has generally been captured with games involving imperfect information. Informational asymmetry produces differences in beliefs as to the outcome of a trial and final decisions may be the result of optimal strategies by the litigants (P'ng (1984); Bebchuck (1984); Nalebuff (1987)). In this way, pretrial negotiation may also break down because the parties fail to agree on how to split the surplus accruing from settlement (Cooter, Marks and Mnookin (1982)); such failure can also be derived from asymmetric information on each other's bargaining power (Farmer and Pecorino (1994, 2002)). A different reason why parties' expectations may diverge emerges from experimental evidence, pointing to the fact that disputants make self-serving valuations of their probability to winning the case. There is a systematic tendency for one to believe he or she has better chances than his or her counterpart (Loewenstein, Issacharo, Camerer, and Babcock (1993)). Bar-Gill (2006) justifies the persistence of biased beliefs as an evolutionary stable equilibrium.

However, some problems remain. From an empirical point of view, the diversity of situations is quite puzzling. It is difficult to explain why so few settlements take place in certain field of law (e.g. labor litigation) compared with other civil litigations while there is no significant difference in informational asymmetries. The topic of the paper is to investigate the reasons why settlement may fail in pretrial negotiation in such a context. Obviously, settlements suppose that the parties reach an agreement before the trial. As such, the respective risk attitudes of the parties play a important role in the willingness of the plaintiff to accept or reject the offer proposed by the defendant. This is a key element which the Law and Economics literature emphasizes.

The originality of the paper is that it adds another dimension, namely self-serving bias. The decision by the plaintiff to accept or reject settlement crucially depends on the representation he has about his probability of winning or losing his case. More specifically, the crux of the litigation process is certainly the way the judge will decide the case. In other words, it is crucial to focus on the manner in which the judge will interpret the existing legal text and law to settle the case. More precisely, we consider that the litigation may be understood as a conflict between different individual judgments about the interpretation of the law by the courts. Using the terminology of decision theory, litigation comes mainly from different expectations about the case.

From the legal theorists' perspective, the true nature of a legal dispute is explainable by the fact that the parties make divergent interpretations about what law says. By emphasizing this feature, we adopt the point of view of contemporary legal theory which considers law as an interpretative and hermeneutics practice. There are a lot of reasons why divergent interpretations may arise. This is the case notably because legal rules are never clear in themselves. And contemporary legal theorists acknowledge that fundamentally law is an interpretative process (Ackerman (1991); Horwitz (1992)): the methods of interpretation, the nature of interpretation and the consequences of the judge's ability to assign - or to determine - a specific meaning to a law are now the heart of the legal theory (Rosenfeld (1998)). Economists interested in legal problems need to take account of this feature when analyzing litigations: it is precisely because the meaning of a norm needs to be rebuilt by the judiciary that conflict may arise. What we call interpretation in law is the other face of the cognitive problem for the parties to assess the probability that they will win the case. As litigation results from divergent legal interpretations of the case, litigation in economics results from divergent expectations about the chance of winning. We consider that interpretational issues in law are the twin aspect of cognitive aspects in economics. That is why we introduce a selfserving bias into our model in order to capture the fact that interpretation is never clear and that parties may fail to perfectly interpret the law. The problem is different from those considered in "strategic models" because the problem lies not with informational asymmetries between litigants but with the interpretative mechanism of the courts. In a way, we are going back to the "optimistic approach" developed by Posner. Here, biases are considered as describing a twofold process: the

one by which people interpret the law and the one by which people form their expectations about the legal decision. The aim of the paper is to put the stress on the different effects of self-serving bias - that is to say, a bias in the interpretation of the meaning of the law.

This approach is original because it puts the emphasis not only on the parties' beliefs about the result of the trial, but on the faith they have in those beliefs. This is the great interest of the debates introduced by legal theory to consider the interpretative process. The problem cannot simply be defined in terms of optimism or pessimism. Of course the question of judicial interpretation refers clearly to the activity of the judge¹. However, a common interpretation of a legal rule generally emerges from the interaction between the judges and the parties. Consequently, it is crucial to appreciate whether the problems of confidence or errors aversion may modify the conclusions of the economic models built on the simple hypothesis of litigant's optimism.

To investigate this problem, we use the dual theory of Yaari (1987) which provides us with sound, axiomatically-founded but simple arguments to rationalize another channel for the disagreement between individuals' beliefs: these are not pure probabilistic decision weights, but reflect the preferences of the individuals and specifically their attitude towards risk which is termed *proba*bilistic risk aversion. In other words, the parties are characterized by some objective information (probabilistic) with regards to the interpretation of the law by the judge that they use in assessing their own individual beliefs, but this information is distorted according to their risk aversion, since litigants also take into account the fact that they may make mistakes. Decidue and Wakker (2001) and Weber and Kirsner (1997) rationalize such a probability transformation process as reflecting that individuals take into account and minimize the loss, disappointment or pain they will suffer when making an error in the assessment of the outcome associated with their decisions. In a sense, probabilistic risk aversion reflects the degree of confidence of an individual regarding his priors and his ability to make the best decision; while the self-serving bias is supported by a theory of self-manipulation of beliefs reflecting that, even if they have true and objective information about the situation, individuals are prone (more or less deliberately) to reinterpret the facts in a way that is favorable to themselves.

The paper is organized as follow. In the second part, we propose a model of litigation which is mainly a model à la Bebchuk. But we assume that the plaintiff is both risk-adverse and bear a cognitive bias concerning his own case, a self-serving bias. In the third part, we analyze the comparative statics of the model in order to discriminate the effects due to the self-serving bias from those due to risk-aversion. Our findings are quite different from those of Pecorino and Farmer

¹Moreover, Ichino, Polo and Rettore (2003), Marinescu (2007) and Viscusi (2001) give evidence on Courts and jurys biases of several kinds.

(2002), and in a sense confirm that the order of play between parties is important for the influence of their respective cognitive biases. We also find that changes in the possible plaintiff' types have effects that Bebchuck (1984)'s seminal work cannot capture, thus showing the importance of both cognitive bias and risk aversion in order to have a more comprehensive view of the litigation process.

2 The model

Bebchuk's model (1984) is modified in the following way. We consider a plaintiff who is hurt by an accident that may be the result of negligence or wrongdoings by another party, the defendant. The loss borne by the plaintiff in the accident is D > 0. It corresponds to the damages awarded by the court if the plaintiff wins the case. Compensation D is public information. Nevertheless, p the probability that the judgment at trial be in favour of the victim is private information (p is the type of the plaintiff). We assume that the defendant only knows that² $p \in [a, b]$ and is distributed according to a probability function characterized by the cumulative function G(p) and the density g(p), which are public information. In order to rule out secondary difficulties, we introduce the following assumption:

Assumption 1: the rate of hazard $\frac{G}{g}$ is increasing.

Such an assumption allows both second order condition to be verified and that the equilibrium to be unique.

2.1 assumptions about the preferences of the parties

On the one hand, we assume that the plaintiff displays a self-serving bias ($\sigma_p > 1$) and thus has an assessment of the prior corresponding to his chances of winning at trial denoted $\sigma_p p$ which is larger than his true probability. In other words, the plaintiff interprets the facts of the case as more favorable to himself than they really are from an objective point of view. On the other hand, the plaintiff has preferences which satisfy the axiomatic of Yaari's model (Yaari (1987)); thus, there exists a probability transformation function $\varphi(p)$ which is endowed with the basic properties that $\varphi : [0, 1] \rightarrow [0, 1]$ is unique, continuous and increasing in p, with $\varphi(0) = 0$ and $\varphi(1) = 1$. We will assume that $\varphi(p)$ is (at least twice) differentiable, with:

Assumption 2: $\forall p \in [0,1] : \varphi''(p) > 0.$

²We assume that a > 0 in order to rule out the case of frivolous suits, and b < 1.

which simply says that the plaintiff is a probabilistic risk-averse decision maker (which is equivalent to risk averse d.m. in the strong sense of Rothschild-Stiglitz: see Yaari (1987)). Given the presence of the plaintiff's self-serving bias, the anticipated utility of the plaintiff of type p in case of trial corresponding to a prospect $Xp = (x_1, 1 - p; x_2, p)$, with $x_1 < x_2$, is written:

$$E_{\varphi op}(Xp;\sigma_p) \equiv (1 - \varphi(\sigma_p p)) x_1 + \varphi(\sigma_p p) x_2$$

In words, $E_{\varphi op}(Xp; \sigma_p)$ is the subjectively transformed expected outcome at trial, since the probability of each outcome at trial is replaced by a subjective weight of likelihood, namely the transformation of its prior. Specifically, according to the convexity of $\varphi(p)$, the plaintiff assesses a weight of likelihood to the worst (best) outcome at trial which is larger (smaller) than its prior : *i.e.* $1 - \varphi(\sigma_p p) > 1 - \sigma_p p$ (respectively $\varphi(\sigma_p p) < \sigma_p p$). Interestingly enough, $E_{\varphi op}(Xp; \sigma_p)$ also provides us with the certainty-equivalent of the risky prospect at trial X (see Roëll (1987) and Yaari (1987)), or equivalently his willingness to pay for the risk at trial, but expressed in the terms of the prior.

Finally, we consider here that the defendant is a risk neutral individual, but also displays a self-serving bias denoted by $\sigma_d < 1$; this implies that facing the risky prospect of trial $Xp = (x_1, 1-p; x_2, p)$, the defendant's expected outcome is $E(Xp; \sigma_d) \equiv (1 - \sigma_d p)x_1 + \sigma_d px_2$.

We will focus on the case where (σ_p, σ_d) are public information.

2.2 the pretrial negotiation game

The negotiation game has two main stages, after Nature has chosen the type of the plaintiff p in [a, b], and after the plaintiff has filed his case:

- In the first stage, the defendant makes a "take-it-or-leave-it" offer to the plaintiff, denoted s, in order to reach an amicable settlement of the case.

- In the second stage, depending on his type, the plaintiff accepts the offer (thus, the case is settled) or rejects it, in which case parties go to trial.

The American rule is introduced to describe the allocation of the costs borne by each party at trial. We denote: $C_p > 0$ the plaintiff's costs and $C_d > 0$ the defendant's costs.

Formally, the p plaintiff's anticipated utility in case of trial corresponding to the prospect $Xp = (D - C_p, p; -C_p, 1 - p)$, is written:

$$E_{\varphi op}(Xp;\sigma_p) \equiv \varphi(\sigma_p p)D - C_p$$

We suppose that $\sigma_p a D - C_p > 0$ meaning that the weakest type for the plaintiff, *i.e.* when he knows he will face the defendant with the best chances of being seen as not liable by the Court, always has an incentive to go to trial. Nevertheless, given that the plaintiff is a risk-averse individual, he will not always go to trial since for all p: $E_{\varphi op}(Xp;\sigma_p) < \sigma_p pD - C_p$.

On the defendant's side, the risky trial is a prospect denoted $Xd = (-(D + C_d), p; -C_d, 1 - p)$, and the anticipated loss borne by the defendant when he faces a type p for the plaintiff at trial is:

$$E(-Xd;\sigma_d) \equiv \sigma_d p D + C_d$$

Finally, we consider situations where $D > C_p + C_d$ implying that the case to be solved is socially valuable (*i.e.* we exclude the case for frivolous suits).

2.3 the separating equilibrium

The equilibrium is described in terms of the amount for which the parties settle s (the equilibrium offer of the defendant to the plaintiff) and of the probability of a trial corresponding to the marginal plaintiff p(s), the one who is indifferent between accepting the offer or rejecting it and going to trial.

In the second stage, the plaintiff p chooses between a sure prospect: accepting the offer s, and a risky prospect: going to trial Xp, the certainty-equivalent of which is $\varphi(\sigma_p p)D - C_p$. As a result, plaintiff p accepts the offer s as soon as: $s \ge \varphi(\sigma_p p)D - C_p$. Otherwise, he rejects it. Let us denote as p(s) the marginal plaintiff that is, the one who is indifferent between both prospects:

$$\varphi(\sigma_p p(s)) D - C_p = s \tag{1}$$

Given the existence of the self-serving bias σ_p , any plaintiff having a case weaker than the marginal plaintiff (any p < p(s)) also accepts the offer, while any plaintiff having a stronger case (p > p(s)) goes to trial.

Coming back to the first stage, we are allowed to write the loss function used by the defendant to set his best offer. With probability G(p(s)), the defendant knows he will face a plaintiff prone to accept his offer, and thus bear the cost s to settle the case. But with probability 1 - G(p(s)), the defendant knows he will face a case stronger than the marginal one, and thus will have to bear the cost $\sigma_d \left(\int_{p(s)}^b p \frac{g(p)}{1 - G(p(s))} dp \right) D + C_d \equiv E(Xd; \sigma_d | p > p(s))$ to solve the case. The term $\int_{p(s)}^b p \frac{g(p)}{1 - G(p(s))} dp \equiv \mu(s)$ denotes the mean type of the plaintiff conditional to the population having rejected the amicable offer and going to trial. The defendant will announce the best offer $\hat{s} \geq 0$, which minimizes the loss function:

$$L(s) = G(p(s)) \times s + (1 - G(p(s))) \times E(Xd; \sigma_d | p > p(s))$$

= $s + (1 - G(p(s))) \times (E(Xd; \sigma_d | p > p(s)) - s)$ (2)

under the condition (1). The second line in (2) may be understood as follows: the defendant starts with the expenditure he would bear in case of settlement, and then he assesses the additional costs he would bear in case of trial, weighted by the probability of a trial.

We obtain the first proposition:

Proposition 1:

In an interior equilibrium, the offer \hat{s} and the marginal plaintiff \hat{p} are set according to the following conditions:

$$\hat{s} = \varphi(\sigma_p \hat{p}) D - C_p \tag{3}$$

$$\left(\frac{G}{g}\right)_{|\hat{p}} = \frac{1}{\varphi'(\sigma_p \hat{p})} \left(\frac{\sigma_d}{\sigma_p} \hat{p} \left(1 - \frac{\varphi(\sigma_p \hat{p})}{\sigma_d \hat{p}}\right) + \frac{C}{\sigma_p D}\right)$$
(4)

where $C \equiv C_p + C_d$, such that the probability of a trial is $\hat{\pi} = 1 - G(\hat{p})$.

Proof: If $\hat{s} > 0$ and $\hat{p} \in]\tilde{p}, b[$ are an admissible interior solution for the minimization of (2), then the First Order Condition writes as:

$$G(\hat{p}) - g(\hat{p}) \frac{1}{\sigma_p \varphi'(\sigma_p \hat{p}) D} \left((\sigma_d \hat{p} - \varphi(\sigma_p \hat{p})) D + C_p + C_d \right) = 0$$
(5)

The first LHS term in (5) is the marginal cost of the defendant's offer: raising the offer leads to an increase in the loss incurred by the defendant equal to the probability of settlement. The second LHS term in (5) is the marginal benefit of the offer which may be split into two components:

- on the one hand, the effect of raising the offer on the probability of trial, $\frac{d}{ds}(1 - G(p(s)) = -g(\hat{p}) \frac{1}{\sigma_p \varphi'(\sigma_p \hat{p}) D}$; this term reflects the efficiency of the screening of the various plaintiff's types due to an increase in the settlement offer;

- on the other hand, the gains of the negotiation³ expressed in terms of the marginal plaintiff $(\sigma_d \hat{p} - \varphi(\sigma_p \hat{p}))D + (C_p + C_d)$; this one obviously reflects the gain associated with the screening of the plaintiffs according to their type.

³Were the parties both risk neutral and having no self-serving bias, these gains would be reduced to the aggregate transaction costs of a trial : $C_p + C_d$. But, as the parties do not have the same perception of the risk of a trial (on the one hand, they have different priors; on the other they do not have the same sensibility to risk) the negotiation gains are different from the transaction costs of a trial: $E(Xd; \sigma_d | p > p(s)) - s \neq C_p + C_d \Leftrightarrow \sigma_d \hat{p} - \varphi(\sigma_p \hat{p}) \neq 0$.

Rearranging the various terms leads to (4).

Let us give a brief sketch of the conditions under which (3)-(4) have a unique solution.

Note that under assumption 1, the LHS in (4) is an increasing function of p. However under assumption 2, the RHS is not (necessarily) a monotonically decreasing function of p: thus, there may exist several extrema (several values of p satisfying (4) with their associated offer satisfying (2)). When this is the case, inspection of the second order condition which requires that the marginal cost of the offer increases more than its marginal benefit (or: $L''(s) \ge 0$):

$$\left(\frac{G}{g}\right)'_{|\hat{p}} + \left(\frac{G}{g}\right)_{|\hat{p}} \times \left(\frac{\varphi''}{\varphi'}\right)_{|(\sigma_p \hat{p})} \times \sigma_p + 1 - \frac{1}{\varphi'(\sigma_p \hat{p})} \frac{\sigma_d}{\sigma_p} \ge 0$$

enables us to identify which of those extrema are local minima: equivalently set, the SOC requires that or any extremum candidate to an equilibrium, the RHS in (4) must increase less than the LHS.

FIGURE 1



In FIGURE 1 (with: $H(p) = \frac{1}{\varphi'(\sigma_p \hat{p})} \left(\frac{\sigma_d}{\sigma_p} p \left(1 - \frac{\varphi(\sigma_p p)}{\sigma_d p} \right) + \frac{C}{\sigma_p D} \right)$), there exist three extrema, but the smallest and the largest values only are two local minima while the intermediate one is a local maximum Finally, substituting each value of the admissible minimum in L(s) provides us with the global minimum, which is the way we implicitly proceed in proposition 1.

In order to understand the driving forces of the model, three benchmark models may be used.

Benchmark 1 (Bebchuk, 1984):

The first one corresponds to Bebchuk's seminal model, for which the equivalent to condition (4) is (in (4), let us set $\varphi(p) = p$ and $\sigma_d = 1 = \sigma_p$):

$$\left(\frac{G}{g}\right)_{|\hat{p}} = \frac{C}{D}$$

Inspection of (4) shows that in equilibrium it must be that $\frac{\sigma_d}{\sigma_p}\hat{p}\left(1-\frac{\varphi(\sigma_p\hat{p})}{\sigma_d\hat{p}}\right)+\frac{C}{\sigma_p D}>0$. Risk aversion implies $\sigma_p\hat{p}>\varphi(\sigma_p\hat{p})$, but as $\sigma_d<\sigma_p$ the sign of $\sigma_d\hat{p}-\varphi(\sigma_p\hat{p})$ is ambiguous. Thus: $1-\frac{\varphi(\sigma_p\hat{p})}{\sigma_d\hat{p}} \geq 0$, and it is surprising to find here that the parties may prefer to settle their case despite the gains of the negotiation being *smaller* than the transaction costs at trial. In other words, risk aversion may both (and thus ambiguously) explain that the rate of trials is larger or smaller than in Bebchuk's seminal paper.

Benchmark 2 (Farmer and Pecorino, 2002):

The second one is the Plaintiff-informed version of the model proposed by Farmer and Pecorino (2002), when the parties are both risk neutral⁴ but display a self-serving bias; the condition equivalent to (4) writes (in (4), let us set $\varphi(p) = p)^5$:

$$\left(\frac{G}{g}\right)_{|\hat{p}} = \left(\frac{\sigma_d}{\sigma_p} - 1\right)\hat{p} + \frac{C}{\sigma_p D}$$

Knowing that $\sigma_d < 1 < \sigma_p$ and thus by assumption: $\frac{\sigma_d}{\sigma_p} - 1 < 0$ and $\frac{C}{\sigma_p D} < \frac{C}{D}$, it is obvious that, as it may be expected, the existence of litigants' self-serving bias *per se* reduces the rate of settlement (increases the frequency of trials) as compared to Bebchuk's model. As the proponents of the "optimistic approach" of litigations explained (Shavell, 1982)), the optimism of the parties may even prevent them from reaching an amicable settlement of their case, in the sense that a major disagreement between litigants in the sense that $\sigma_d << 1 << \sigma_p$ may prevent a separating equilibrium from existing⁶ - for if there is a (sufficiently) large disagreement between the litigants' priors, then all cases go to trial and none are settled.

Benchmark 3:

⁴Farmer and Pecorino (2002) focus on the defendant as the informed party, and assume that the plaintiff has a distorted perception of the defendant's self-serving bias.

⁵Note that the ambiguity concerning the role of σ_d obtained by Farmer and Pecorino (2002) disappears here when the plaintiff is the informed party.

⁶Once more, for an interior separating equilibrium to exist, with some plaintiff's types going to trial and the others accepting the settlement offer, it must be that $\left(\frac{\sigma_d}{\sigma_p}-1\right)\hat{p}+\frac{C}{D}>0.$

The third and last benchmark model corresponds to a situation where parties have no selfserving bias but where the plaintiff is risk averse; in such a case, we have the following FOC:

$$\left(\frac{G}{g}\right)_{|\hat{p}} = \frac{1}{\varphi'(\hat{p})} \times \left(\hat{p} - \varphi(\hat{p}) + \frac{C}{D}\right)$$

where by the convexity assumption: $\hat{p} - \varphi(\hat{p}) > 0$. This means that risk aversion *per se* is not an obstacle to the settlement of the case, and in fact increases the gains of the negotiation $(\hat{p} - \varphi(\hat{p}))D + C$ for the parties: now interestingly enough, the larger the divergence in their perception of the risk at trial (the higher the difference $\hat{p} - \varphi(\hat{p})$), the larger the gains of the negotiation - and thus, the more likely the settlement of the case. However, given that $\varphi'(\sigma_p \hat{p}) \ge 1$, risk aversion may either improve the impact of the gains of the negotiation (if $\varphi'(\hat{p}) - 1 < 0$) or dampen them (when $\varphi'(\hat{p}) - 1 > 0$).

We investigate more precisely these effects and their meaning in the comparative statics part of the paper.

3 Comparative statics

First, we focus on the role of fee-shifting. The consequences of each specific rule for the allocation of the legal expenditures between parties may be easily obtained through a general specification. Let us define α as the proportion of the defendant's costs borne by the plaintiff when the plaintiff looses at trial with probability $1-\pi$; and similarly: β is the proportion of the plaintiff's costs borne by the defendant when the plaintiff wins at trial with probability π . Several well known rules⁷ may be characterized as special cases of this general parametrization. The American rule where each party simply bears its own costs is obtained when $\alpha = \beta = 0$. The British rule, for which the party loosing at trial has to bear the aggregate costs of the trial, is the case where $\alpha = \beta = 1$. Finally, the French or Continental rule, where the judge as the discretionary power to transfer to the loosing party to bear some kinds of the winner's costs (which is called "depens" such as taxes, experts expenditures, but excluding attorney's fees) corresponds to the situation where $\alpha \in [0, 1]$ and $\beta \in [0, 1]$. The costs allocation rule is also a public information.

⁷See also Shavell (1982). The "proplaintiff" rule whereby the plaintiff bears no litigation costs when he wins but only his own costs when he looses is obtained for $\alpha = 0$ and $\beta = 1$. The symmetrical case of the "prodefendant" rule holds for $\alpha = 1$ and $\beta = 0$. Our parameterization is general enough to also encompass a greater variety of rules, including the Marshall, Quayle and Matthew rules discussed in Baye, Kovenock and de Vries (2005) - which simply requires that $\alpha \in [0, \infty)$ and $\beta \in [0, \infty)$.

With the general parameterization of the fee-shifting rule, it can be shown that condition (4) is now written as:

$$\left(\frac{G}{g}\right)_{|\hat{p}} = \frac{1}{\varphi'(\sigma_p \hat{p})} \left(\frac{\sigma_d}{\sigma_p} \hat{p} \left(1 - \frac{\varphi(\sigma_p \hat{p})}{\sigma_d \hat{p}}\right) + \frac{1}{\sigma_p} \left(\frac{C}{D + \beta C_p + \alpha C_d}\right)\right) \tag{6}$$

Hence, we obtain the following result:

Proposition 2:

The rate of trial is the smallest under the American rule, the largest under the English rule, and intermediate under the Continental rule.

Proof. Straightforward since all else being equal, we obtain:

$$\frac{D}{C} < \frac{D + \beta C_p + \alpha C_d}{C} < \frac{D + C_p + C_d}{C}$$

and according to (6), the larger $\frac{C}{D+\beta C_p+\alpha C_d}$, the higher \hat{p} and the smaller $1-G(\hat{p})$.

The result is the same in the three benchmark models.

Then, we study the impact of the individual optimistic biases and risk aversion.

Proposition 3:

I) The marginal plaintiff decreases (hence the probability of a trial increases) and the equilibrium offer increases, with σ_p .

II) The marginal plaintiff increases (hence the probability of a trial decreases) but the equilibrium demand may decrease or increase, with σ_d .

Proof. Given the SOC which requires the RHS to increase less than the LHS, it is easy to verify that the RHS in (4) increases with σ_d but decreases with σ_p ; hence the result, given that the LHS in (4) increases with p. Differentiating (2), we obtain respectively:

$$\begin{array}{lcl} \displaystyle \frac{d\hat{s}}{d\sigma_d} & = & \sigma_d \varphi'(\sigma_p \hat{p}) D \frac{d\hat{p}}{d\sigma_d} > 0 \\ \displaystyle \frac{d\hat{s}}{d\sigma_p} & = & \left(\hat{p} + \sigma_p \frac{d\hat{p}}{d\sigma_p} \right) \varphi'(\sigma_p \hat{p}) D \lessgtr 0 \end{array}$$

The ambiguity with the variation of σ_p is explained by the effect on the marginal type, given that: $\frac{d\hat{p}}{d\sigma_p} < 0.$

The defendant's bias has a direct and positive effect on the gains of the negotiation. The plaintiff's bias in contrast has two negative effects: it decreases the efficiency of the separation through $-g(\hat{p}) \frac{1}{\sigma_p \varphi'(\sigma_p \hat{p})D}$ and it decreases the gains of the negotiation. It may be worth considering the impact of risk aversion in this case on the first term; note that:

$$\frac{d}{d\sigma_p}\left(\frac{1}{\sigma_p\varphi'(\sigma_p\hat{p})D}\right) = -\frac{1}{\sigma_p\varphi'(\sigma_p\hat{p})D}\left(\frac{1}{\sigma_p} + \frac{\varphi''(\sigma_p\hat{p})}{\varphi'(\sigma_p\hat{p})}\hat{p}\right) < 0$$

showing that risk aversion in this case enhances the impact of the plaintiff's self-serving bias: the larger the probabilistic aversion index $\frac{\varphi''(\sigma_p \hat{p})}{\varphi'(\sigma_p \hat{p})}\hat{p}$, the larger the loss of efficiency in the separation of types.

Proposition 3 I is consistent with, but Part II is in contradiction to Farmer and Pecorino (2002), who found that the defendant's self-serving bias has an ambiguous effect both on the probability of trial and on the equilibrium settlement amount. This suggests at least that the specific way the optimistic bias of the defendant affects the frequency of trials at equilibrium depends on the order of play between the parties.

More generally, given that $\sigma_p > 1$ and $\sigma_d < 1$, proposition 3 means word by word that a rise in σ_d corresponds to the case where the defendant becomes less optimistic: as σ_d increases, the bias regarding his perception of the chances that the plaintiff will prevail is reduced, and his own assessment of the likelihood of winning becomes closer to the true probability. The symmetric occurs for a rise in σ_p . Hence, this result in proposition 3 is exactly the one more usually obtained in the "optimistic model" where litigants may fail to reach a settlement agreement when both parties overestimate their chances at trial, the more optimistic they are the higher the likelihood of a trial (Priest and Klein (1984), Waldfogel (1995,1998)).

We now evaluate the impact of shifts in the distribution of the plaintiff's characteristics, either (roughly speaking) a monotonic increase of its possible types, or a proportional spread of them. The following property is useful in the next proposition:

Property I: since φ is increasing and convex, with $\varphi(0) = 0$ and $\varphi(1) = 1$, there always exists a $q_0 \in]0, 1[$ such that $p \ge q_0 \Rightarrow \varphi'(p) \ge 1$, but $p \le q_0 \Rightarrow \varphi'(p) \le 1$. In the next proposition, we assume that $q_0 \in]a, b[$.

In other words, $\varphi'(p) \leq 1$ means that risk aversion increases with the plaintiff's prior in the area $]0, q_0[$ (the distortion of the probability $p - \varphi(p)$ is enhanced); and in contrast, $\varphi'(p) \geq 1$ means that risk aversion increases with the plaintiff's prior in the domain $]q_0, 1[$ (respectively, the transformation of the probability is dampened).

Proposition 4: An additive shift to the right in the range of plaintiff's types:

I) increases the marginal type. Moreover, it implies a less than proportional increase in the marginal type if $\sigma_p \hat{p} > q_0$; otherwise, there may exist a more than proportional increase in the marginal type.

II) increases the probability of trial if $\sigma_p \hat{p} > q_0$; otherwise, the probability of trial may decrease. III) increases the equilibrium offer.

Proof. We define (see also Bebchuk (1984)) an additive shift to the right of the range of plaintiff's types as a t-translation of plaintiff's types, such that p is now distributed in the interval [a + t, b + t] (with $t \ge 0$) with the cumulative $\Gamma(p)$ and the density $\gamma(p)$ satisfying the following correspondences with the primitives G(p) and g(p):

$$\Gamma(p) = G(p-t)$$

$$\gamma(p) = g(p-t)$$

In fact, these two conditions characterize a family of distribution functions which is parameterized by $t \ge 0$, where t = 0 gives us the primitives, and t > 0 leads to a distribution with a higher mean type but having identical higher order moments. In this case, the condition (4) may be substituted with the general formulation:

$$\left(\frac{G}{g}\right)_{|(\hat{p}-t)} = \frac{1}{\varphi'(\sigma_p \hat{p})} \left(\frac{\sigma_d}{\sigma_p} \hat{p} \left(1 - \frac{\varphi(\sigma_p \hat{p})}{\sigma_d \hat{p}}\right) + \frac{C}{\sigma_p D}\right)$$
(7)

with $\hat{\pi} = 1 - G(\hat{p} - t)$ and \hat{s} given by (3). I) Differentiating (7) gives:

$$\frac{d\hat{p}}{dt} = \frac{1}{\Omega} \left(\frac{G}{g}\right)'_{\mid (\hat{p}-t)}$$

with: $\Omega \equiv \left(\frac{G}{g}\right)'_{|(\hat{p}-t)} + \left(\frac{G}{g}\right)_{|(\hat{p}-t)} \times \left(\frac{\varphi''}{\varphi'}\right)_{|(\sigma_p\hat{p})} \times \sigma_p + 1 - \frac{1}{\varphi'(\sigma_p\hat{p})} \times \frac{\sigma_d}{\sigma_p} > 0$ according to the second order condition. Thus, it is obvious that $\frac{d\hat{p}}{dt} > 0$ since under assumption 2 the numerator is also positive: $\left(\frac{G}{g}\right)'_{|(\hat{p}-t)} > 0$. Given that under assumption 1: $\left(\frac{\varphi''}{\varphi'}\right)_{|(\sigma_p\hat{p})} > 0$, then $\sigma_p\hat{p} > q_0 \Rightarrow \frac{d\hat{p}}{dt} < 1$; in contrast, if $\sigma_p\hat{p} < q_0$ then $\frac{d\hat{p}}{dt} \ge 1$. II) As a result $\hat{\pi} = 1 - G(\hat{p} - t)$ increases with t if $\sigma_p\hat{p} > q_0$. Otherwise, the effect is ambiguous. III) Given that the marginal type increases with t, it is also straightforward to see that the equilibrium offer $\hat{s} = \varphi(\sigma_p\hat{p})D - C_p$ also increases with t.

We may also consider an alternative definition of the expansion of plaintiff's type. In the next proposition, we denoted: $\mu = E(p)$ the mean type of the plaintiff.

Proposition 5. A mean-preserving proportional shift in the range of plaintiff's types:

I) decreases the marginal type if $\hat{p} < \mu$; otherwise, the effect is ambiguous;

II) has an ambiguous effect on the probability of trial;

III) decreases (increases) the equilibrium offer if the marginal type decreases (respectively, increases).

Proof. We define (see also Bebchuk (1984)) a mean-preserving proportional shift in the range of plaintiff's types as the composition of an additive shift to the left (a $\mu(1-t)$ -translation, with $t \ge 1$ and $\mu = E(p)$) plus a multiplicative shift of plaintiff's types, such that p is now distributed in the interval $[ta + \mu(1-t), tb + \mu(1-t)]$ with a cumulative probability function $\Gamma(p)$ and a density $\gamma(p)$ satisfying the following correspondences with the primitives G(p) and g(p):

$$\begin{split} \Gamma(p) &= G\left(\frac{p-\mu}{t}+\mu\right) \\ \gamma(p) &= \frac{1}{t}g\left(\frac{p-\mu}{t}+\mu\right) \end{split}$$

Once more, these two conditions characterize a family of distribution functions which is parameterized by $t \ge 1$, where t = 1 gives us the primitives, and t > 1 gives us a new distribution with the same mean $\mu = E(p)$ but which is more spread than the primitive distribution; thus it has moments of higher orders which are larger than those of the primitive. In this case, the condition (4) may be now substituted with the general formulation:

$$t \times \left(\frac{G}{g}\right)_{\left|\left(\frac{p-\mu}{t}+\mu\right)\right|} = \frac{1}{\varphi'(\sigma_p \hat{p})} \left(\frac{\sigma_d}{\sigma_p} \hat{p} \left(1 - \frac{\varphi(\sigma_p \hat{p})}{\sigma_d \hat{p}}\right) + \frac{C}{\sigma_p D}\right)$$
(8)

with $\hat{\pi} = 1 - G\left(\frac{\hat{p}-\mu}{t} + \mu\right)$. I) Differentiating (8) gives:

$$\frac{d\hat{p}}{dt} = \frac{1}{\Omega} \times \left[\left(\frac{G}{g} \right)'_{|\left(\frac{\hat{p}-\mu}{t}+\mu\right)} \times \left(\frac{\hat{p}-\mu}{t} \right) - \left(\frac{G}{g} \right)_{|\left(\frac{\hat{p}-\mu}{t}+\mu\right)} \right]$$

It is obvious that $\hat{p} < \mu \Rightarrow \frac{d\hat{p}}{dt} < 0$ although if $\hat{p} > \mu$ then $\frac{d\hat{p}}{dt}$ has an ambiguous sign. II) Similarly, $\hat{\pi}$ may decrease or increase with t since:

$$\frac{d\hat{\pi}}{dt} = -\frac{1}{t} \times g\left(\frac{\hat{p}-\mu}{t}+\mu\right) \times \left(\frac{d\hat{p}}{dt}-\frac{\hat{p}-\mu}{t}\right)$$

Hence the result. III) Given the ambiguity on the marginal type, it is also straightforward to see that the equilibrium offer $\hat{s} = \varphi(\sigma \hat{p})D - C_p$ may increase (if the marginal type increases) as well as decrease (respectively if the marginal type decreases) with t.

Both propositions 4 and 5 give predictions concerning the effects of such shifts in the range of possible types for the plaintiff⁸ which are different as compared to those of Bebchuk (1984). For Bebchuk, the additive shift has no effect on the frequency of trial and a positive effect on the settlement offer, while the mean-preserving proportional shift increases the likelihood of a trial and has an ambiguous effect on the amount for which the parties settle. To be brief, our results suggest that any definition of expansion in the range of unobservable types has effects on the equilibrium when the parties are optimistically biased and/or risk averse; however, these consequences may be quite imprecise.

At the same time, it is worth noting that the effects we obtain here are also different from those predicted by the "optimistic model"; to see this, we can interpret the mean-preserving proportional shift in the range of plaintiff's types as representing, from the defendant's point of view, more variability in the prediction of plaintiff's type (less precision in the assessments of the true value for the plaintiff's chances of prevailing) since it corresponds to more dispersion in possible plaintiff's types. We should also remember that Priest and Klein (1984) and Waldfogel (1998) showed that when the errors made by the litigants in predicting the outcome at trial increase, then the likelihood of a trial increases: this is because the chances are raised that the plaintiff's optimistic estimate of prevailing at trial is larger than defendant's.

Finally, we investigate the impact of more risk aversion for the plaintiff. According to Yaari (1987), a plaintiff having a probability transformation function ψ is more risk averse than a plaintiff characterized by φ iff ψ is a positive and convex transformation of φ . This implies that for all p: $\psi(\sigma p) < \varphi(\sigma p)$ (Roëll (1987)). This also introduces a useful property used in the next proposition:

Property II: since ψ and φ are both (strictly) increasing and (strictly) convex, such that for all p: $\psi(\sigma p) < \varphi(\sigma p)$ and with $\psi(0) = \varphi(0) = 0$ and $\psi(1) = \varphi(1) = 1$, then: $1 < \psi'(0) < \varphi'(0)$ but $\psi'(1) > \varphi'(1) > 1$. By continuity, there always exists a (unique) $q_1 \in]0, 1[$ such that $p \ge q_1 \Rightarrow$ $\psi'(p) \ge \varphi'(p)$, but $p \le q_1 \Rightarrow \psi'(p) < \varphi'(p)$. In the following, we assume that $q_1 \in]a, b[$.

Proposition 6: If the plaintiff becomes more risk averse, then:

I) the marginal plaintiff increases (hence the probability of trial decreases) if $\sigma_p \hat{p} < q_1$; otherwise the effect is ambiguous.

II) the equilibrium offer may increase or decrease. Specifically, it decreases if the marginal plaintiff decreases; otherwise, the effect is ambiguous.

⁸In the model of Bebchuk (1984) where the private information is p, it does not matter whether the informed party is the defendant or the plaintiff in the analysis of such shifts in the range of unobservable types.

Proof. Assume that plaintiff ψ is more risk averse than plaintiff φ . I) As a result, since $\frac{\psi(\sigma_p p)}{\sigma_d p} < \frac{\varphi(\sigma_p p)}{\sigma_d p}$ meaning that when the plaintiff becomes more risk averse, the RHS in (4) increases due to the gains of the negotiation which are raised all else being equal. Moreover, if $\psi'(\sigma_p p) < \varphi'(\sigma_p p)$, then the efficiency of the separation between plaintiffs' types is also improved. Thus, if $\sigma_p \hat{p} < q_1$, the RHS globally increases meaning an increase in the marginal plaintiff. In contrast, note that if $\sigma_p \hat{p} > q_1$ which may imply (necessary but not sufficient) $\psi'(\sigma_p p) > \varphi'(\sigma_p p)$, then this second effect dampens the influence of the first one, such that the net effect on the RHS of (4) is ambiguous.

II) When the plaintiff becomes more risk averse, then there are two effects on the equilibrium offer: all else being equal, the anticipated utility of the marginal plaintiff decreases, which allows the defendant to reduce his offer; on the second, the type of the marginal plaintiff may increase or decrease. Hence the result. \blacksquare

The observation that an increase in risk aversion may have an ambiguous effect on the predicted behavior of an individual is not a new result and has led to a lot of literature in insurance economics, portfolio choices and so on (see Ross (1981)). What is really new here is that the comparative statics of risk aversion depends on $\sigma \hat{p}$ the initial prior of the marginal plaintiff and the way it affects the distortion of probability. Consider the situation where the marginal plaintiff is weakly biased and assesses a small prior for his chances of prevailing in the sense that $\sigma \hat{p} \leq q_1$; then according to proposition 6, as he becomes more sensitive to risk (more risk averse) such a plaintiff will be prone to settle the case as will some of the near-marginal plaintiff individuals who previously prefer to go before the court. In contrast, when the marginal plaintiff is highly biased in the sense that $\sigma \hat{p} > q_1$ then an increase in risk aversion on the one hand suggests that more plaintiffs agree to settle their case, and on the other that the less sensitive they are to the probability of prevailing, the more plaintiffs may go to trial.

4 Discussion and conclusion

Law and Economics scholars have to take into consideration the interpretation of legal norms by the courts. It is precisely because the meaning of a norm always needs to be reconstructed by the judiciary that conflict may arises. What we call interpretation in law is the other face of the cognitive problem for the parties to assess the probability that they will win the case. As litigation results from divergent interpretations of the legal aspects of the case, parties diverge in their expectations about the chance of winning. We consider that interpretational issues in law are the twin aspects of cognitive issues in economics. In this paper, we have considered a self-serving bias to capture the fact that interpretation is never clear and that parties may fail to perfectly interpret the law. The problem is different from those considered in "strategic models" because the problem lies not with asymmetries of information *per se* but with the interpretative mechanism of the courts. It also contributes to clarifying the foundations of the traditional "optimistic models". The reason is quite clear. In the optimistic approach, the reason why parties could present optimistic beliefs is exogenous. The origins of optimism are not developed by authors who prefer to refer in general terms to informational advantages. Our main result is that the introduction of cognitive biases into the model contribute to clarifying the debate.

In our paper, we observe that the defendant's self-serving bias has an ambiguous effect both on the probability of trial and on the equilibrium settlement amount. The paper also suggests that the specific way the optimistic bias of the defendant affects the frequency of trials at equilibrium depends on the organization of the trial, and particularly on the parties' order of appearance. Moreover, the model contributes to explaining the role of risk aversion. Specifically, the model demonstrates that the effect of risk aversion depends on the initial prior of the plaintiff and the way it affects the distortion of probability. These results are promising because they contribute to explaining that the beliefs about the result of the trial are not sufficient in themselves to understand the behaviors of litigants. As suggested by legal theory, the confidence the parties have in their beliefs is probably more important. The reason is that litigants develop opinions about the interpretation of the law to be applied to their case by the judges. In doing this, they necessarily express an attitude towards the risk of committing a "mistake". This is the reason why the question of confidence is so important.

Our conclusion also has normative implications. If the problem of confidence is negligible, the conclusions of the "optimistic" models developed by Posner or Landes are fully verified and the legal policies built on these hypotheses are probably well designed. However, if the problem of confidence and the aversion to mistakes are important, the conclusions of the model are more ambiguous and the normative problems are more complicated to solve.

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