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Leveraged Buy Out: Dynamic agency model with write-off option.

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Abstract

We present a dynamic agency model in which the LBO fund may write the entrepreneur's project off at the end of the starting stage to invest in a competitive project. The two partners provide unobservable efforts in both stages to enhance the productivity of the acquired company.

We show that under restrictive conditions, the debt-equity contracts induce the entrepreneur and the LBO fund to provide the first best efforts under restrictive conditions in the two stages.

Moreover, the write-off threat boosts the incentives of the entrepreneur and the LBO fund such that they provide high efforts.

If the compensation cost is exogenous, the sharing rule of this cost depends on the quality of the competitive project. The entrepreneur and the bank share the amount of compensation if it is not very profitable. Otherwise, the whole amount of compensation is pledged to the entrepreneur.

If the compensation's amount is endogenous, in order to induce the entrepreneur to provide high effort, the optimal financial contracts must give her the entire compensation's revenue.

Keywords: *Leveraged Buy Out*, incentives, exit, write-off option, double moral hazard.

JEL classification: D82, D92, G32, G33.

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1 Introduction

1.1 Motivation

Despite the fact that the *Leveraged Buy Out* (LBO) acquisitions accounts for a significant part of the private equity investment, many topics in this area are not explored. In many countries most notably in the United States, these projects are financed mostly with debt and a small amount of equity, hence the term *leveraged*: these projects are typically financed with anywhere from 60% to 90% debt (Jensen, 1986, 1989 and Kaplan and Strömberg, 2008).

The present paper deals with two facts in LBO finance:

First, the financial capital structure, particularly the excessive use of debt in these acquisitions: this paper shows that the optimal financial contracts are consistent with this feature in a dynamic framework with double sided moral hazard. We characterize the optimal financial capital structure that results from the solution to an optimal contract problem.

We work out the optimal solution to the financing problem of an entrepreneur (hereafter she) who would like to acquire a company: she is a manager but she has no business experience to manage the acquired company alone. She asks first for money and for advice from the LBO fund (hereafter he). The latter contributes technically and financially into the project and in exchange he gets a share of the project's outcome. The partners sign the holding contract and can ask for additional funds from the bank (hereafter he). Therefore, they sign another contract: the debt contract. The optimal sharing rule of the benefit may change at the end of the first period. The optimal solution and the optimal sharing rule are constrained by the characteristics of the environment such as the information available to both parties. The entrepreneur and the fund have to provide costly efforts which improve the results of the project. This information is unobservable which creates a moral hazard problem.

Second, we focus on the decision to exit, particularly on the write-off route. The LBO fund is expecting for a high return, his aim is to get back his money and to exit as soon as possible in order to invest this money in a new project. The exit of the LBO fund is quite likely if there are good projects in the LBO market looking for finance and for advice. The exit date/route may induce agency conflicts between the entrepreneur and the LBO fund. If the company is going public the entrepreneur keeps the control and may get private benefits. This is no longer true when the buyout is going private such in the trade sale: if she stays in the company, she must share the control with a new partner. In the current paper, we do not consider the issue of the optimal timing/route of exit but we focus on the impact of the write-off threat on the efforts made by the entrepreneur and the LBO fund. To our knowledge, the current paper is the first theoretical one that addresses this issue in buyout acquisitions.

1.2 Related literature

The present model is related to two lines of literature.

First, various papers consider a dynamic agency model with information asymmetry. They analyze the interaction between the financial capital structure and incentives when there is an information asymmetry, due to unobservable efforts. For instance, Bergemann and Hege (1998), Cornelli and Yosha (2003), Repullo and Suarez (2004), Schmidt (2003)...These papers highlighted the importance of adequate incentive-rewarding schemes, the role of the *stage financing* and convertible securities to mitigate the moral hazard problem.

Bergemann and Hege (1998) consider a dynamic agency model in the presence of learning and moral hazard problems. They show that short-term refinancing[1] is never optimal but long-term contract allowing for intertemporal risk-sharing such as the stage financing is optimal: it induces the entrepreneur to provide optimal effort so that the private equity fund will invest further funds in the following stage. If not, the private equity fund writes the project off. However, the *stage financing* may create a "*window dressing*" problem in order to induce the private equity fund to finance the second stage of the project. They conclude that with a convertible debt contract, this entrepreneur's behavior becomes non profitable; the private equity fund will convert his debt into equity if the project looks too profitable. Consequently, he will reduce the entrepreneur's profit (Cornelli and Yosha, 2003). Schmidt (1999, 2003) shows that there is no debt-equity contract that induces both parties to invest efficiently and argues that the use of convertible securities mitigates the double sided moral hazard problem. Moreover, these securities outperform any mixture of debt and equity and they induce both parties to provide optimal efforts. His result is robust to renegotiation and to changes in the timing of investments and information flows.

Repullo and Suarez (2004) consider a wealth-constrained entrepreneur asking for advice and for money from two private equity funds. One of them does not provide effort so he may be considered as a pure or passive financier. The entrepreneur and the other fund have to exert non observable efforts. They conclude that the entrepreneur must ask for advice and fund from the partner who provides both money and advice. The return of the project in case of success is unobservable when they sign the contract, they will learn more about it at the end of the first period. If this return is verifiable, the entrepreneur and the private equity fund sign an initial long-term contract contingent on the project's return (*contingent financing contract*). They show that the private equity fund must get a constant share of the outcome only when the project is profitable. However, when the return is not verifiable, they sign a start-up contract which is negotiated in the expansion stage. They show that the venture capitalist should get no compensation for his initial investment in the lower tail and high compensation in the upper tail of the distribution of returns. They point out that this sharing rule can be approximated by the use of warrants.

Second, another branch of the literature focuses on the various exit vehicles in private equity investments but few papers, mostly empirical studies, focus on the topic of buyouts particularly the write-off route. To our knowledge, most of the theoretical papers study the exit in the venture capital. Most of these papers argue that the project is abandoned when the quality is bad or mediocre in the sense it is only able to return the initial investment.

For instance, Schwienbacher (2002) analyzes the relationship between the level of innovation of the project and the exit decision. He shows that going public is more profitable than trade sale when the project is very innovative. In an IPO strategy, the entrepreneur remains in the firm, keeps its control and can get private benefit. Consequently, she is tempted to distort the innovation strategy so that the IPO looks the preferred exit route.

Giot and Schwienbacher (2007) argue that the exit decision depends on the type of exit strategy and on the timing. For instance, biotechnology and Internet projects are the fastest in exiting through IPO. Unprofitable Internet firms are abandoned quickly via write-offs.

[1] At the end of each period, the entrepreneur sign a new contract with a new venture capital fund. The next period, she is looking for another one; the VC market is supposed to be competitive.

Schmidt, Steffen and Szabó (2008) focus on buyout exit strategies in Europe and the United States. They consider a sample of 666 buyouts between 1990 and 2005. They analyze the determinants influencing the choice of the exit option. Their results show strong support for signaling effect. If the return is very poor, the LBO fund writes off the project early instead of holding it in his portfolio as *living-dead* buyout: he is able to differentiate between good and bad investments quickly. Only the most profitable projects are taken public through an IPO. Nikoskelian and Wright (2005) consider a sample of 321 UK buyouts, exited between 1995 and 2004. They find a positive relationship between the value increase and the management ownership. Das, Jagannathan and Sarin (2002) analyze the options of exit of venture and LBO funds in the US market. They estimate the probability of various exit options and they conclude that the probability of an exit via sales is the highest. Groh and Gottschalg (2008) point out that the US buyouts investments clearly outperform the market benchmark. Ick (2006) investigates the risk and return relationship of private equity relative to public market equity and finds that the private equity returns depend on the stage of the investment. Later stage investments achieve higher risk adjusted returns.

Cumming and MacIntosh (2003) show in an empirical analysis in Canada and USA that IPO are the most profitable followed by secondary sales, buybacks and write-offs for the less profitable projects.

In all these papers, the exit decision depends only on the performance of the project, but they do not consider the characteristics of the company's market: the private equity fund invests in the project in the hope that he will get a high return in a short period of time and he exits as soon as possible in order to invest his money in a new deal. The LBO fund looks continuously for "*LBO stars*". This issue is still pending.

1.3 Results

The model allows to derive the following results:

First, if there is no competitive project, the entrepreneur and the LBO fund provide the first best efforts only when the project is not very risky and the debt's payments are decreasing with the project's revenues. The presence of the bank induces the entrepreneur and the LBO fund to make high efforts because of the threat of liquidation. This result is in line with those of Meckling and Jensen (1976) and Jensen (1986, 1989). In this case, the whole surplus value of the project is retained by the entrepreneur. If the project is very risky, they make the second best efforts and the social value of the project is not optimal. Whether the project is very risky or not, the optimal financial contracts should reward the entrepreneur and the LBO fund only in the good state of the nature. The bank receives no payment in success states of the world. The optimal debt contract exhibits the features of a "*live or die*" contract.

When the payments of the bank are non-decreasing with the project's outcome, the bank payments are fixed: the success payments are equal to the liquidation values whether the project succeeds or fails. Besides, the efforts are not optimal. The agents provide higher efforts when the project is financed only with equity than when the entrepreneur asks for advice and for money from the LBO fund and the bank. The debt has no effects on the agent's incentives.

Second, the threat of exit induces the entrepreneur and the LBO fund to provide the high efforts when all the agents contribute financially into the acquisition. These efforts are not optimal.

Third, under the write-off threat, the LBO fund issues more equity but the bank's investment depends on the write-off cost. If the cost of violation contracts is high, then the threat of write-off becomes incredible even if the LBO fund exits, the latter will pay high compensation to the bank and to the entrepreneur. This is why the bank accepts to lend more debt than when the fund keeps the entrepreneur's project. When the violation cost is low, the bank invests less money into the project.

Fourth, if the compensation cost is fixed by the legislature, the optimal sharing rule depends on the quality of the competitive buyout: if it is not very profitable, it is shared between the bank and the entrepreneur. Otherwise, the whole amount is pledged to the entrepreneur which boosts her incentives to provide high effort in the first period even when the LBO fund will probably abandon the project. When this cost is to be determined in the optimal financial contracts, it is specified such that the entrepreneur captures the entire amount of compensation whether the new project is very profitable or not. The amount of compensation is significantly: it is equal to the half of the revenue of the competitive project.

Fifth, optimal sharing rule does not depend on financial capital structure of the project but it depends on the incentives to provide efforts. The optimal contracts should take into account the countervailing effects of inducing one agent to work, and reflect both incentives.

The paper is structured as follows. The model and the assumptions are presented in section 2. The optimal financial contracts are characterized in section 3. The section 4 analyzes the optimal financial contracts when the project is financed only through equity. We consider the threat of *write-off* of the LBO fund and study its influences on the agents' incentives in section 5. Empirical implications are presented in section 6. The section 7 concludes the paper. All proofs are presented in the appendix.

2 The model

Consider a dynamic agency model with two periods: the starting stage and the productive stage. There are three agents: the entrepreneur E, the LBO fund A and the bank B. The entrepreneur wants to acquire a company. She asks for money and advice from the LBO fund. He issues the amount of equity i and he gets in exchange a part of the revenues of the acquired firm.

The acquisition of the firm requires a fixed initial investment K at time 0, K is significantly high. Let W, i and I denote respectively the capital issued by E, A and B such that $K = W + i + I$. At the end of the period t, the entrepreneur (respectively the LBO fund) gets the part of the benefit β_t (respectively $1 - \beta_t$) such that $0 \leq \beta_t \leq 1$, $t = 1, 2$.

First, the entrepreneur and the LBO fund sign the holding contract in order to establish the holding company[2]. Then, in order to issue further funds, the new company and the bank sign the debt contract.

The entrepreneur and the LBO fund provide respectively the unobservable efforts e and a. These efforts enhance the productivity of the project. The entrepreneur's effort is related

[2] The holding company acquires another company called the *Op Co*, using mostly debt and a small amount of equity. The debt is secured by the *Op Co* assets. The acquiring company uses these assets as collateral for the debt in hopes that the future cash-flows will cover the debt's payments.

to her technical skills and/or to her past experience as a manager. The LBO fund provides a managerial and/or control effort to run properly the project.[3]

2.1 The revenues

The project yields the revenue $\tilde{X}_1$ ($\tilde{X}_2$) at the end of the starting stage (the productive stage) such that:

$$\tilde{X}_t = \begin{cases} X_t & \text{with the probability } p_t = \min\{e_t + a_t, 1\} \\ \gamma^{t-1}C & \text{with the probability } (1-p_t) = \max\{0, 1-(e_t+a_t)\} \end{cases}$$

C is the liquidation value of the project at the end of the first period. $1-\gamma$ $(0 \leq \gamma \leq 1)$ is the depreciation rate. The value of the equipment and machines decreases due to the usage, the passage of time and the outdated technologies. We assume that $X_1 \geq C$ and $X_2 \geq \gamma C$.

e_t and $a_t \in [0, 1]$ are the efforts provided at the stage t, $t = 1, 2$. These efforts are perfect substitutes. If one agent provides unitary effort, the project will succeed with probability 1: $p_t(0,1) = p_t(1,0) = 1$, $t = 1, 2$.

The entrepreneur and the LBO fund choose their efforts simultaneously. When efforts are unobservable, they may be tempted to not provide sufficient efforts. The entrepreneur will rely on the effort of the LBO fund in hopes that the latter will provide unitary effort. This situation creates an asymmetric information problem, specifically it induces a double sided moral hazard problem.

Efforts are costly. The cost functions are given by:

$$c_E(e_t) = \frac{\lambda}{2}e_t^2 \quad \text{and} \quad c_A(a_t) = \frac{\lambda}{2}a_t^2,\ t = 1, 2$$

where $\lambda > 0$ is assumed to be significantly high. Efforts have equal costs when the entrepreneur and the LBO fund make the same levels of efforts. Given the fact that they are perfect substitutes, these efforts have the same impact on the performance of the project. Besides, we assume $C \leq \frac{\lambda}{2\gamma}$.

2.2 The sequence of events

The sequence of events is presented in figure 1:

- At date $t = 0$, the entrepreneur E and the LBO fund A sign the holding contract. Then, they may ask for additional funds from the bank B. They sign the debt contract.
- At date $t = 1$, the agents E and A provide simultaneously and respectively the efforts e_1 and a_1.
  - If $\tilde{X}_1 = X_1$, the bank perceives the payment D_1. The entrepreneur and the LBO fund get respectively $(1-\beta_1)(X_1 - D_1)$ and $\beta_1(X_1 - D_1)$. The LBO fund may write the entrepreneur's project off even if it succeeded, to invest in a competitive buyout which gives him net revenue $R \geq 0$.

[3] The LBO fund provides the same kind of effort as the entrepreneur. He learns these skills from his past investments.

    - ∗ If there is exit, the entrepreneur's project is stopped at date 1. The LBO fund must pay a compensation cost L to the other partners because he violates the contracts. The revenue of the competitive project must be superior to the compensation's cost, otherwise, the LBO fund has no reason to abandon the entrepreneur's project: $R \geq L$. Let η (respectively $1-\eta$) denotes the share of the compensation paid to the entrepreneur (respectively the bank) such that $0 \leq \eta \leq 1$. The variable η is an endogenous variable to be determined in the optimal financial contracts. We suppose that the LBO fund will write the entrepreneur's project off with the probability ζ, $0 \leq \zeta \leq 1$.
    - ∗ Otherwise, the entrepreneur's project is continued and the entrepreneur and the LBO fund choose respectively the efforts e_2 and a_2.
  - If $\tilde{X}_1 = C$, the entrepreneur's project is supposed to be liquidated and the bank perceives the collateral C.

- At date $t = 2$, the project is completed.

  - If $\tilde{X}_2 = X_2$, the bank, the entrepreneur and the LBO funds get respectively the payments D_2, $\beta_2 (X_2 - D_2)$ and $(1-\beta_2)(X_2 - D_2)$ with the probability p_2.
  - If $\tilde{X}_2 = \gamma C$, the project fails and it is liquidated. The bank is paid the whole failure's revenue γC.

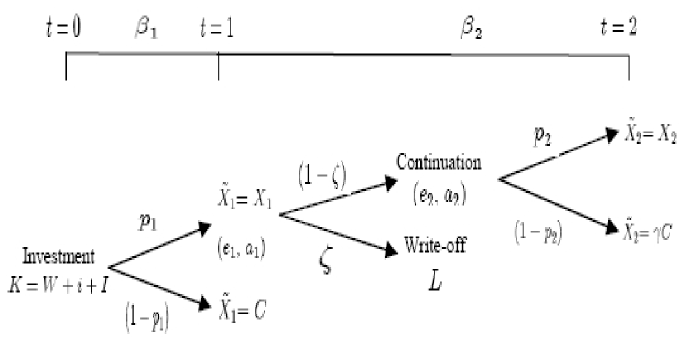

Figure 1- Time line of the game.

The riskless interest rate is normalized to 0. All agents are risk neutral and protected by limited liability: they can only share the outcome of the project.

2.3 The contracts

The agents sign two financial contracts.

a) The debt contract must specify:

- The amount of the debt I issued by the bank.
- The bank payments at the starting stage and the productive stages D_1 and D_2.

The bank lends the money only if he gets a positive expected gain:

$$E(\pi_B) = p_1 \{D_1 - C + (1-\zeta)[p_2(D_2 - \gamma C) + \gamma C] + \zeta(1-\eta)L\} \\ +C - I \geq 0 \tag{CP_B}$$

(CP_B) is his participation constraint. The banks compete for the right to fund the project, then we expect the debt contract to be the best possible for the entrepreneur. Accordingly, the bank B earns no profits.

We suppose that:

$$0 \leq D_t \leq X_t \ , t = 1, 2. \tag{1}$$

b) The holding contract which specifies

- The amount of funds i issued by the LBO fund.
- The fraction of the benefit β_t, $t = 1, 2$ (respectively $1 - \beta_t$) of the entrepreneur (respectively the LBO fund).
- The compensation cost L to be shared between the entrepreneur and the bank.
- The share of compensation η (respectively $1-\eta$) paid to the entrepreneur (respectively the bank) if the LBO fund chooses to exit at the date 1.

The LBO fund issues i only if his expected gain is positive. This condition is written:

$$E(\pi_A) = p_1 \{(1-\beta_1)(X_1 - D_1) + \zeta(R - L) \\ + (1-\zeta)[(1-\beta_2)p_2(X_2 - D_2) - c_A(a_2)]\} \\ -c_A(a_1) - i \geq 0 \tag{CP_A}$$

(CP_A) is his participation constraint. Competition between the investment funds implies that this constraint is binding.

Because of the competition among the LBO funds and the banks, both agents A and B are induced to propose contracts maximizing the expected gain of the entrepreneur which is given by:

$$E(\pi_E) = p_1 \{\beta_1(X_1 - D_1) + (1-\zeta)[\beta_2 p_2(X_2 - D_2) - c_E(e_2)] + \zeta\eta L\} \\ - c_E(e_1) - W.$$

Substituting W for $K - i - I$ into $E(\pi_E)$ gives:

$$E(\pi_E) = p_1 \{\beta_1(X_1 - D_1) + (1-\zeta)[\beta_2 p_2(X_2 - D_2) - c_E(e_2)] + \zeta\eta L\} \\ - c_E(e_1) - K + i + I. \tag{2}$$

2.4 The first best solution

Before solving the game, let us compute the social value of the project without double sided moral hazard and exit problems. The social value of the project is given by:

$$V = p_1[X_1 - (1-\gamma)C + p_2(X_2 - \gamma C) - c_E(e_2) - c_A(a_2)] \\ +C - K - c_E(e_1) - c_A(a_1).$$

Given the first order conditions of V, the first best efforts are written:

$$e_1^{FB} = a_1^{FB} = \frac{1}{\lambda}\left[X_1 - (1-\gamma)C + \frac{1}{\lambda}(X_2 - \gamma C)^2\right] \quad (3)$$

$$e_2^{FB} = a_2^{FB} = \frac{1}{\lambda}\left(X_2 - \gamma C\right). \quad (4)$$

These efforts induce the following success probabilities:

$$p_1^{FB} = \tfrac{2}{\lambda}\left[X_1 - (1-\gamma)C + \tfrac{1}{\lambda}(X_2 - \gamma C)^2\right] \text{ and } p_2^{FB} = \tfrac{2}{\lambda}\left(X_2 - \gamma C\right) . \quad (5)$$

In the following, we assume that:

$$X_1 < \tfrac{\lambda}{2} + (1-\gamma)C \text{ and } X_2 < \tfrac{\lambda}{2} + \gamma C . \quad (6)$$

to ensure that $p_t^{FB} < 1$, $t = 1, 2$.

The equations (3) and (4) show that the entrepreneur and the LBO efforts must make strictly positive efforts during the starting and the productive stages. Note that efforts of the first period depend on revenues of both stages. But, those of the second period depend only on the revenues of the productive stage. They increase with the revenues of the project and decrease with the cost parameter λ and/or the amount of the collateral C.

Moreover, it is straightforward to see that the efforts of the second period are decreasing when γ increases. In the starting stage, the impact of γ on the efforts of the first period is less intuitive:

If γ is high enough $\left(\gamma\epsilon\left[\frac{2X_2-\lambda}{2C}, 1\right]\right)$, e_1^{FB} and a_1^{FB} are increasing with γ. Note that when the depreciation rate is low, the failure revenue γC is high. In contrast, when the depreciation rate is high $\left(\gamma\epsilon\left[0 , \frac{2X_2-\lambda}{2C}\right]\right)$, a small failure revenue will induce the entrepreneur and the LBO fund to provide low levels of efforts $e_1^{FB}$and a_1^{FB}.

Furthermore, the entrepreneur and the LBO fund provide equal levels of efforts which is not surprising because the efforts have equal impacts on the success probabilities and the agents have the same cost function.

Consequently, the optimal social value of the project is given by:

$$V^{FB} = \frac{1}{\lambda}\left[X_1 - (1-\gamma)C + \frac{1}{\lambda}(X_2 - \gamma C)^2\right]^2 + C - K. \quad (7)$$

We assume that V^{FB} is strictly positive. There are many ways to implement the first best solution: the identities of the agents providing advice and financial investments are irrelevant as long as the LBO fund and the bank are not wealth-constrained and there are no transaction costs.

2.5 The efforts in equilibrium

In order to maximize their expected gain, each agent will take into account the levels of effort chosen by the other agent. These strategies are described by their reaction functions.

We solve for optimal financial contracts using a dynamic programming approach (*the backward induction process*). We consider the subgame that begins at the end of the productive stage: first, we determine the reaction functions of the second period. Then, we substitute them into the expected gain of each agent so that we can deduce those of the first period.

The efforts of the second period in equilibrium

Each agent chooses the level of effort that maximizes his expected gain of the second period. The LBO fund provides the effort a_2^* solution of:

$$a_2^* \in \arg\max_{a_2} E(\pi_A^2 | \tilde{X}_1 = X_1)$$

where $E(\pi_A^2 | \tilde{X}_1 = X_1) = p_2 (1 - \beta_2) (X_2 - D_2) - c_A (a_2)$ is his expected gain of the second period.

The entrepreneur effort is given by:

$$e_2^* \in \arg\max_{e_2} E(\pi_E^2 | \tilde{X}_1 = X_1)$$

where $E(\pi_E^2 | \tilde{X}_1 = X_1) = p_2 \beta_2 (X_2 - D_2) - c_E (e_2)$ is her expected gain of the second period.

Lemma 1 *The efforts of the second period of the entrepreneur and the LBO fund e_2^* and a_2^* are given respectively by:*

$$e_2^* = \tfrac{1}{\lambda} \beta_2 (X_2 - D_2) \quad \textit{and} \quad a_2^* = \tfrac{1}{\lambda} (1 - \beta_2) (X_2 - D_2) \tag{8}$$

The success probability is then given by:

$$p_2^* = \frac{1}{\lambda} (X_2 - D_2) \tag{9}$$

If the fraction of benefit of one agent increases, his effort will increase: e_2^* (respectively a_2^*) is an increasing function of β_2 (respectively $1 - \beta_2$). When the difference between the success revenue and the bank's payment is large, the levels of efforts are high. Note that these efforts do not depend on the write-off threat.

Given the fact that the revenue of the second period is fixed, if the optimal financial contracts give powerful incentives to one agent, they will reduce the incentives of the other agent. The optimal financial contracts must boost simultaneously the two agents' incentives.

The efforts of the first period in equilibrium

The entrepreneur and the LBO fund provide respectively the efforts e_1^* and a_1^* given by:

$$e_1^* \in \arg\max_{e_1} E(\pi_E | e_2^*, a_2^*) \quad \text{and} \quad a_1^* \in \arg\max_{a_1} E(\pi_A | e_2^*, a_2^*)$$

We substitute the optimal efforts deduced in the previous paragraph into their expected gains. The first order conditions of $E(\pi_E | e_2^*, a_2^*)$ and $E(\pi_A | e_2^*, a_2^*)$ enable us to deduce the following lemma:

Lemma 2 *The efforts of the first period e_1^* and a_1^* are written:*

$$e_1^* = \frac{1}{\lambda} \left\{ \beta_1 (X_1 - D_1) + \frac{1}{2\lambda} \beta_2 (2 - \beta_2) (1 - \zeta) (X_2 - D_2)^2 + \zeta \eta L \right\} \tag{10}$$

$$a_1^* = \frac{1}{\lambda} \left\{ (1 - \beta_1) (X_1 - D_1) + \zeta (R - L) + \frac{1}{2\lambda} \left(1 - \beta_2^2\right) (1 - \zeta) (X_2 - D_2)^2 \right\} \tag{11}$$

The success probability of the first period:

$$p_1^* = \frac{1}{\lambda}\left\{X_1 - D_1 + \zeta\left[R - (1-\eta)L\right] + \frac{1}{2\lambda}\left(1 + 2\beta_2 - 2\beta_2^2\right)(1-\zeta)(X_2 - D_2)^2\right\} \tag{12}$$

The more the exit probability is high, the more the entrepreneur and the LBO fund will be induced to provide efforts in the starting stage: given λ high, the efforts (10) and (11) are increasing with ζ. If the project fails the LBO fund will not get his money back. The entrepreneur is better off when the project succeeds: if his project is abandoned, he will be paid ηL, otherwise, he gets nothing. Moreover, the effort of the entrepreneur (respectively the LBO fund) is increasing (respectively decreasing) with the compensation cost. Given high fraction of compensation to the entrepreneur will induce her to provide more effort in the starting stage. Notice that the net revenue of the competitive buyout R has an impact only on the fund's effort which is very intuitive.

As in the previous lemma, we find that the entrepreneur's effort e_1^* (respectively the LBO's effort a_1^*) increases with his part of benefit β_1 and/or β_2 (respectively $1-\beta_1$ and/or $1-\beta_2$). Besides, these efforts decrease with the debt payments D_t, $t = 1, 2$.

The lemmas 1 and 2 show that the efforts of the entrepreneur and the LBO fund are independents: they play a strictly dominant strategies.

The probability p_1^* does not depend on β_1 and p_2^* does not depend neither on β_1 nor on β_2. This result is closely related to the assumptions of the model because the efforts are perfect substitutes and the agents have the same function of cost.

The entrepreneur's objective is to maximize her expected gain given the participation constraints of the LBO fund and the bank and the incentive constraints:

$$\begin{aligned}\max_{i,\, I,\, \beta_t,\, D_t,\;\, e_t,\, a_t,\, t=1,\,2} E(\pi_E) = \;& p_1\left\{\beta_1(X_1 - D_1) + (1-\zeta)\left[\beta_2 p_2(X_2 - D_2) - c_E(e_2)\right] + \zeta\eta L\right\} \\ & - c_E(e_1) - K + i + I.\end{aligned}$$

$$\text{s.t.} \qquad (CP_A)\,,\ (CP_B)\,,\ (8),\ (10) \text{ and } (11)$$

with the following conditions:

$$(1) \text{ and } 0 \leq \beta_t \leq 1, \quad t = 1, 2.$$

3 The optimal financial contracts without the *write-off* option

In this section, we consider first the case where the LBO fund cannot abandon the entrepreneur's project because there is no competitive buyout looking for advice and for money.

If $\zeta = 0$, the equations (10) and (11) are therefore written:

$$e_1^* = \frac{1}{\lambda}\left[\beta_1(X_1 - D_1) + \frac{1}{2\lambda}\beta_2(2-\beta_2)(X_2 - D_2)^2\right] \tag{13}$$

$$a_1^* = \frac{1}{\lambda}\left[(1-\beta_1)(X_1 - D_1) + \frac{1}{2\lambda}(1-\beta_2^2)(X_2 - D_2)^2\right] \tag{14}$$

Then, the success probability of the starting stage is given:

$$p_1^* = \frac{1}{\lambda}\left[X_1 - D_1 + \frac{1}{2\lambda}(1 + 2\beta_2 - 2\beta_2^2)\,(X_2 - D_2)^2\right] \tag{15}$$

The efforts of the entrepreneur and the LBO fund and the success probability of the productive stage do not change: they are given respectively by (8) and (9).

The participation constraints of the LBO fund and the bank enable us to write:

$$E(\pi_A) = \quad p_1\left\{(1-\beta_1)\,(X_1 - D_1) + (1-\beta_2)\,p_2\,(X_2 - D_2) - c_A\,(a_2)\right\} - c_A\,(a_1) - i \geq 0 \tag{16}$$

$$E(\pi_B) = \quad p_1\left\{D_1 - (1-\gamma)\,C + p_2\,(D_2 - \gamma C)\right\} + C - I \geq 0 \tag{17}$$

Given the previous results, we are able to solve the entrepreneur's program. Her aim is to maximize her expected gain under the incentive and the participation constraints. The program to be solved is the following:

$$\begin{aligned} \max_{\beta_t,\ i,\ I,\ D_t,\ e_t,\ a_t,\ t=1,\ 2} E(\pi_E) = \quad & p_1\left[\beta_1(X_1 - D_1) + \beta_2 p_2(X_2 - D_2) - c_E(e_2)\right] \\ & -c_E(e_1) - K + i + I \\ \text{s.t.} \qquad & (8),\ (13),\ (14),\ (16) \text{ and, } (17) \end{aligned}$$

with the following conditions:

$$(1) \text{ and } 0 \leq \beta_t \leq 1,\ t = 1,\ 2.$$

Because of the competition among the LBO funds and the banks, the participation constraints (CP_B) and (CP_A) are binding. So we can write:

$$i = p_1\left\{(1-\beta_1)(X_1 - D_1) + p_2(1-\beta_2)(X_2 - D_2) - c_A(a_2)\right\} - c_A(a_1), \tag{18}$$

$$I = p_1\left\{D_1 - (1-\gamma)\,C + p_2\,(D_2 - \gamma C)\right\} + C. \tag{19}$$

We substitute (18) and (19) into the objective function of the entrepreneur's program. Consequently, the optimal financial contracts induce her to maximize the expected social value of the project under the incentive constraints such that:

$$\begin{aligned} \max_{\beta_t,\ D_t,\ e_t,\ a_t,\ t=1,\ 2} V = \quad & p_1[X_1 - (1-\gamma)\,C + p_2\,(X_2 - \gamma C) - c_E(e_2) - c_A(a_2)] \\ & +C - K - c_E(e_1) - c_A(a_1) \\ \text{s.t.} \qquad & (8),\ (13) \text{ and } (14) \end{aligned}$$

with the following conditions:

$$(1) \text{ and } 0 \leq \beta_t \leq 1, \quad t = 1, 2.$$

The following proposition summarizes the properties of the optimal financial contracts:

Proposition 1 *The optimal financial contracts are given by:*

- *If the project is not very risky, in the sense* $X_1 \leq 2(1-\gamma)C$ *and* $X_2 \leq 2\gamma C$*:*

$$D_1^* = 2(1-\gamma)C - X_1 + \frac{1}{\lambda}(X_2 - \gamma C)^2$$
$$D_2^* = 2\gamma C - X_2$$
$$I^* = C - \frac{2}{\lambda}\left[X_1 - (1-\gamma)C + \frac{1}{\lambda}(X_2 - \gamma C)^2\right]^2$$
$$i^* = \frac{3}{2\lambda}\left[X_1 - (1-\gamma)C + \frac{1}{\lambda}(X_2 - \gamma C)^2\right]^2$$

- *If the project is very risky, in the sense* $X_1 > 2(1-\gamma)C$ *and* $X_2 > 2\gamma C$*:*

$$D_1^* = D_2^* = 0$$
$$I^* = C - \frac{1}{\lambda}\left[(1-\gamma)C + \frac{1}{\lambda}X_2\gamma C\right]\left[X_1 + \frac{3}{4\lambda}(X_2)^2\right]$$
$$i^* = \frac{3}{8\lambda}\left[X_1 + \frac{3}{4\lambda}(X_2)^2\right]^2$$

The amount of equity issued by the entrepreneur is given by: $W^* = K - i^* - I^*$.

Whether the project is highly risky or not, the entrepreneur and the LBO fund have equal payoffs: $\beta_t^* = \frac{1}{2}, \quad t = 1, 2$.

See appendix A.

This proposition states that whether the project is very risky or not, the acquisition must be funded jointly by the entrepreneur, the LBO fund and the bank: all agents have to invest strictly positive amount of money in the entrepreneur's project. Besides, the entrepreneur and the LBO fund have equal benefit's shares. If the optimal contracts attribute high share to one of them, they will induce him to provide high effort but simultaneously, they will reduce the effort of the other one. Hence, the optimal financial contracts have to induce both parties to provide optimal efforts.

Note that the debt's payments are decreasing with the outcome of the project in the starting and the productive stages. Consequently, the entrepreneur and the LBO fund are tempted to announce a success to the bank whatever happens. In case of failure, they are better off if they sell the project's assets and pay D_t to the bank and share the difference between them.

When the project is not very risky, the entrepreneur and the LBO fund provide the first best efforts. The levels of these efforts are not very high since the difference between the revenues of success and failure is small, it is easy to induce them to provide the first best efforts. The presence of the bank, more specifically the threat of liquidation induces them to make efficient investment decisions. This is no longer true when the differences between the revenues of failure and success are large. The efforts provided by the entrepreneur and the LBO fund are lower than the optimal efforts (5), despite the fact that we use a powerful incentive scheme where the optimal debt contract exhibits the features of a "*live or die*" contract: the entrepreneur and the LBO fund share equally the revenues of success and pledge the entire revenue of failure to the bank. So, they provide the following second best efforts:

$$e_1^* = a_1^* = \frac{1}{2\lambda}\left[X_1 + \frac{3}{4\lambda}(X_2)^2\right] \quad \text{and} \quad e_2^* = a_2^* = \frac{1}{2\lambda}X_2 \tag{20}$$

These efforts induce the success of the project with the probabilities p_1^* and p_2^* given respectively by:

$$p_1^* = \frac{1}{\lambda}\left[X_1 + \frac{3}{4\lambda}(X_2)^2\right] \quad \text{and} \quad p_2^* = \frac{1}{\lambda}X_2 \tag{21}$$

The expected gain of the entrepreneur is therefore given by:

$$E\left(\pi_E^*\right) = \frac{3}{8\lambda}\left[X_1 + \frac{3}{4\lambda}(X_2)^2\right]^2 - W.$$

The entrepreneur does not capture the social value of the project.

Whether the project is very risky or not, given λ high, the amount of debt I is larger than the amount of equity i issued by the LBO fund. This is consistent with the excessive use of debt noticed in the buyout acquisitions.

The LBO fund issues high equity when the project is not very risky. Consequently, the bank invests higher amount of debt when it is very risky. This may be explained as follows: more the collateral is large, more the bank will be induced to lend money. Moreover, the amount of debt I (respectively equity i) is decreasing (respectively increasing) with the revenues of success of the two stages.

3.1 The bank's payments are non-decreasing with the project's revenues

Consider now that the entrepreneur and the LBO fund cannot lie about the project's result to the bank: the financial contracts are specified such that the bank's payments are non-decreasing with the revenues of the project. We add the following conditions $D_1 \geq C$ and $D_2 \geq \gamma C$ to the entrepreneur's program.

Proposition 2 *The optimal financial contracts are given by:*

$$D_1^c = C \quad \textit{and} \quad D_2^c = \gamma C \tag{22}$$

$$I^c = C\left\{1 + \frac{\gamma}{\lambda}\left[X_1 - C + \frac{3}{4\lambda}(X_2 - \gamma C)^2\right]\right\} \tag{23}$$

and

$$\beta_t^c = \frac{1}{2},\, t = 1,\, 2 \tag{24}$$

$$i^c = \frac{3}{8\lambda}\left[X_1 - C + \frac{3}{4\lambda}(X_2 - \gamma C)^2\right]^2 \tag{25}$$

$$W^c = K - i^c - I^c \tag{26}$$

See appendix B.

Note that the bank's payments do not depend on the quality of the project. In this case, the entrepreneur and the LBO fund provide the following efforts:

$$e_1^c = a_1^c = \frac{1}{2\lambda}\left[X_1 - C + \frac{3}{4\lambda}(X_2 - \gamma C)^2\right] \tag{27}$$

$$e_2^c = a_2^c = \frac{1}{2\lambda}(X_2 - \gamma C) \tag{28}$$

These efforts are not optimal and they are inferior to the ones given by (20). Considering that the payments of the bank are non-decreasing reduces the agents' incentive to provide efforts.

The efforts (27) induce the success of the project with the probabilities:

$$p_1^c = \tfrac{1}{\lambda}\left[X_1 - C + \tfrac{3}{4\lambda}\left(X_2 - \gamma C\right)^2\right] \quad \text{and} \quad p_2^c = \tfrac{1}{\lambda}\left(X_2 - \gamma C\right) \tag{29}$$

In contrast with the previous propositions, the amount of debt is higher than the collateral of the starting stage C. The clause of non-decreasing payments ensures to the bank to get higher success payments D_t if the project is not very risky, and a strictly positive payment if the project is very risky. Given λ high, the amount of additional debt is not very large.

4 What is achieved without debt?

Hereafter, we consider that the bank does not contribute financially into the project. The optimal holding contract must determine endogenously the fractions of benefit given to the entrepreneur and to the LBO fund at the end of each stage.

The entrepreneur and the LBO fund choose the levels of efforts $\tilde{e}_2$ and $\tilde{a}_2$ maximizing their expected gains given by:

$$\begin{aligned} E(\pi_E^2/\tilde{X}_1 = X_1) &= \beta_2\left[p_2\left(X_2 - \gamma C\right) + \gamma C\right] - c_E\left(e_2\right) \\ E(\pi_A^2/\tilde{X}_1 = X_1) &= \left(1 - \beta_2\right)\left[p_2\left(X_2 - \gamma C\right) + \gamma C\right] - c_A\left(a_2\right) \end{aligned}$$

According to the first order conditions of $E(\pi_E^2/\tilde{X}_1 = X_1)$ and $E(\pi_A^2/\tilde{X}_1 = X_1)$, their reaction functions in the productive stage are written:

$$\tilde{e}_2 = \tfrac{1}{\lambda}\beta_2\left(X_2 - \gamma C\right) \quad \text{and} \quad \tilde{a}_2 = \tfrac{1}{\lambda}(1 - \beta_2)\left(X_2 - \gamma C\right) \tag{30}$$

Consequently, the success probability of the second period is given by

$$\tilde{p}_2 = \frac{1}{\lambda}\left(X_2 - \gamma C\right) \tag{31}$$

We substitute $\tilde{e}_2$, $\tilde{a}_2$ and $\tilde{p}_2$ for their expressions given by (30) and (31) into the expected gains of the entrepreneur and the LBO fund:

$$\begin{aligned} E(\pi_E/\tilde{e}_2, \tilde{a}_2) = {} & p_1\left\{\beta_1\left(X_1 - C\right) + \beta_2\left[p_2\left(X_2 - \gamma C\right) + \gamma C\right] - c_E\left(e_2\right)\right\} \\ & + \beta_1 C - c_E\left(e_1\right) - W \\ E(\pi_A/\tilde{e}_2, \tilde{a}_2) = {} & p_1\left\{\left(1 - \beta_1\right)\left(X_1 - C\right) + \left(1 - \beta_2\right)\left[p_2\left(X_2 - \gamma C\right) + \gamma C\right] - c_A\left(a_2\right)\right\} \\ & + (1 - \beta_1) C - c_A\left(a_1\right) - K + W \end{aligned}$$

The first order conditions of $E(\pi_E/\tilde{e}_2, \tilde{a}_2)$ and $E(\pi_A/\tilde{e}_2, \tilde{a}_2)$ enable us to deduce their reaction functions in the starting stage:

$$\tilde{e}_1 = \frac{1}{\lambda}\left[\beta_1(X_1 - C) + \beta_2\gamma C + \frac{1}{2\lambda}\beta_2(2 - \beta_2)\left(X_2 - \gamma C\right)^2\right] \tag{32}$$

$$\tilde{a}_1 = \frac{1}{\lambda}\left[(1 - \beta_1)(X_1 - C) + \left(1 - \beta_2\right)\gamma C + \frac{1}{2\lambda}(1 - \beta_2^2)\left(X_2 - \gamma C\right)^2\right] \tag{33}$$

Accordingly, the success probability of the first period is given by:

$$\tilde{p}_1 = \frac{1}{\lambda}\left[X_1 - (1-\gamma)\,C + \frac{1}{2\lambda}(1 + 2\beta_2 - 2\beta_2^2)\,(X_2 - \gamma C)^2\right] \tag{34}$$

The optimal financial contracts induce the entrepreneur to maximize the social value of the project under the incentive constraints (30), (32) and (33) such that:

$$\begin{aligned} \max_{\beta_t,\ i,\ e_t,\ a_t,\ t=1,2} V = \quad & p_1[X_1 - (1-\gamma)\,C + p_2\,(X_2 - \gamma C) - c_E(e_2) - c_A(a_2)] \\ & + C - K - c_E(e_1) - c_A(a_1) \\ \text{s.t.} \quad & (30)\,,\ (32)\ \text{ and }\ (33) \end{aligned}$$

with the following conditions:

$$0 \leq \beta_t \leq 1,\ t = 1,\, 2$$

The properties of the optimal holding contract are presented in the proposition 3:

Proposition 3 *When the LBO fund issues $i = K - W$, the entrepreneur and the LBO fund must get equal revenues in both periods: $\tilde{\beta}_t = \frac{1}{2}$, $t = 1,\, 2$. The entrepreneur and the LBO fund issue the following amounts of equity:*

$$\begin{aligned} \tilde{W} &= K - \frac{1}{2}C - \frac{3}{8\lambda}\left[X_1 - (1-\gamma)\,C + \frac{3}{4\lambda}\,(X_2 - \gamma C)^2\right] \\ \tilde{\imath} &= \frac{1}{2}C + \frac{3}{8\lambda}\left[X_1 - (1-\gamma)\,C + \frac{3}{4\lambda}\,(X_2 - \gamma C)^2\right] \end{aligned}$$

See appendix B.

This proposition states that the shares of benefit given to the entrepreneur and to the LBO fund do not depend on project's revenues: whether the result of the project in the first period is a success or failure, they must perceive equal parts of the benefit.

Given the incentive constraints (30), (32) and (33), the optimal efforts are written:

$$\tilde{e}_1 = \tilde{a}_1 = \tfrac{1}{2\lambda}\left[X_1 - (1-\gamma)\,C + \tfrac{3}{4\lambda}\,(X_2 - \gamma C)^2\right] \quad \text{and} \quad \tilde{e}_2 = \tilde{a}_2 = \tfrac{1}{2\lambda}\,(X_2 - \gamma C) \tag{35}$$

These efforts are inferior to the first best efforts and even to the efforts (20) provided by the entrepreneur and the LBO fund if the project is highly risky. When the debt's payments are non-decreasing with the outcome of the project, in the productive stage, they provide the same levels of efforts whether the project is financed with debt and equity or just with equity. But, in the starting stage, $\tilde{e}_1$ and $\tilde{a}_1$ are superior to (27). We conclude that the presence of a passive financier such as the bank induces the entrepreneur and the LBO fund to make low efforts which reduces the performance of the project. This implies that the efforts are the highest when the project is financed solely through equity.

The sharing rule does not depend on financial capital structure of the project, but it depends on the incentives to efforts. As explained in the previous sections, the agents have the same cost function and their efforts have equal impacts on the success probabilities, the optimal financial contracts must boost simultaneously the agent's incentives to provide the required efforts. If one of them has the highest part of the payoff so that he makes more effort, the other one will make less effort.

5 The optimal financial contracts under the *write-off* threat

Consider now that the LBO fund may leave the project if there is a success at the end of the starting stage to invest in a new acquisition with a strictly positive probability of exit: $\zeta > 0$. Hereafter, we consider that the bank's payments are non-decreasing.

The competition among the LBO funds and the banks induce A and B to offer the entrepreneur the best contracts possible such that the participation constraints (PC_A) and (PC_B) are binding which enables us to write

$$\begin{aligned} I = \; & p_1 \left\{ D_1 - C + (1-\zeta) \left[p_2 \left(D_2 - \gamma C \right) + \gamma C \right] + \zeta \left(1 - \eta \right) L \right\} \\ & + C \end{aligned} \tag{36}$$

$$\begin{aligned} i = \; & p_1 \left\{ \left(1 - \beta_1 \right) \left(X_1 - D_1 \right) + \zeta \left(R - L \right) \right. \\ & \left. + (1-\zeta) \left[\left(1 - \beta_2 \right) p_2 \left(X_2 - D_2 \right) - c_A \left(a_2 \right) \right] \right\} \\ & - c_A \left(a_1 \right) \end{aligned} \tag{37}$$

Substituting (36) and (37) into the objective function of the entrepreneur gives the following program:

$$\begin{aligned} \max_{\beta_t,\, D_t,\, e_t,\, a_t,\, t=1,\,2} E\left(\pi_E\right) = \; & p_1 \left\{ X_1 - \left[1 - (1-\zeta)\gamma \right] C + \zeta R \right. \\ & \left. + (1-\zeta) \left[p_2 \left(X_2 - \gamma C \right) - c_E \left(e_2 \right) - c_A \left(a_2 \right) \right] \right\} \\ & + C - K - c_E \left(e_1 \right) - c_A \left(a_1 \right) \end{aligned} \tag{38}$$

$$\text{s.t.} \qquad (8),\ (10) \text{ and } (11)$$

with the following conditions:

$$(1),\ D \geq H \text{ and } 0 \leq \beta_t \leq 1, \quad t = 1, 2. \tag{39}$$

5.1 The financial structure when the information is perfect

If there is no double moral hazard problem, the expected gain of the entrepreneur is given by (38). It is surprising that $E\left(\pi_E\right)$ does not depend neither on η nor on L but it depends on R. Hereafter, we show that if the compensation cost is exogenous, the entrepreneur's share of compensation η is an increasing function of R. But, if it is endogenous, L increases with R and the whole amount will be paid to the entrepreneur.

When the information is perfect, the efforts that maximize the expected gain of the entrepreneur are given by the first order conditions of $E\left(\pi_E\right)$. They are written

$$e_1^{PI} = a_1^{PI} = \frac{1}{\lambda} \left\{ X_1 - \left[1 - (1-\zeta)\gamma \right] C + \zeta R + \frac{1}{\lambda} (1-\zeta) \left(X_2 - \gamma C \right)^2 \right\} \tag{40}$$

$$e_2^{PI} = a_2^{PI} = \frac{1}{\lambda} \left(X_2 - \gamma C \right) \tag{41}$$

When the information is perfect, both agents provide the first best efforts in the productive stage.

If the net revenue of the competitive buyout is high ($R > \gamma C$), they provide high efforts in the starting stage: (40) are superior to the first best. In the opposite, the entrepreneur and the LBO fund are induced to provide low efforts because the LBO fund is not tempted to exit: if he does, his expected gain -after the payment of the compensation cost- is low.

Then, the project succeeds with the probabilities

$$p_1^{PI} = \frac{2}{\lambda}\left\{X_1 - [1-(1-\zeta)\gamma]C + \zeta R + \frac{1}{\lambda}(1-\zeta)(X_2-\gamma C)^2\right\} \tag{42}$$

$$p_2^{PI} = \frac{2}{\lambda}(X_2-\gamma C) \tag{43}$$

The expected gain of the entrepreneur is

$$E\left(\pi_E^{PI}\right) = C - K + \frac{1}{\lambda}\left\{X_1 - [1-(1-\zeta)\gamma]C + \zeta R + \frac{1}{\lambda}(1-\zeta)(X_2-\gamma C)^2\right\}^2 \tag{44}$$

This gain is reached if the efforts (8), (10) and (11) satisfy the conditions (40) and (41). In this case, the financial capital structure are given by:

$$\eta = \frac{R}{L} - 1 \text{ and } \beta_t = \frac{1}{2},\ t = 1, 2$$
$$D_2 = 2\gamma C - X_2$$
$$D_1 = 2[1-(1-\zeta)\gamma]C - X_1 - 2\zeta L + \frac{1}{\lambda}(1-\zeta)(X_2-\gamma C)^2$$

These contracts are signed only if the project is not very risky in the sense $X_2 < 2\gamma C$ and $X_1 < 2[1-(1-\zeta)\gamma]C - 2\zeta L$. Moreover, we assume $L \le R \le 2L$ so that $0 \le \eta \le 1$. Note that the debt's payments are decreasing with the revenues of the project.

If the efforts are observable, the LBO fund and the bank issue the following amounts of money

$$i^{PI} = \frac{3}{2\lambda}\left\{X_1 - [1-(1-\zeta)\gamma]C + \zeta R + \frac{1}{\lambda}(1-\zeta)(X_2-\gamma C)^2\right\}^2 \tag{45}$$

$$I^{PI} = C - \frac{2}{\lambda}\left\{X_1 - [1-(1-\zeta)\gamma]C + \zeta R + \frac{1}{\lambda}(1-\zeta)(X_2-\gamma C)^2\right\}^2 \tag{46}$$

$$W^{PI} = K - i^{PI} - I^{PI} \tag{47}$$

5.2 The *write-off* exit and the agents' incentives

5.2.1 L is exogenous

Hereafter, we assume that L given by the legislature and focus on the particular case where the payments of the debt do not decrease when the project's revenues increase. The following proposition characterizes the optimal financial contracts.

Proposition 4 *When the LBO fund has the option to write off the entrepreneur's project at the end of the starting stage, the optimal financial contracts are given by:*

- *If* $R \leq 2L$*, the competitive buyout is not very profitable and*

$$\hat{\eta} = \frac{R}{L} - 1$$
$$\hat{\imath} = \frac{3}{8\lambda}(l)^2$$
$$\hat{I} = C + \frac{1}{\lambda} l \{(1-\zeta)\gamma C - \zeta(R-2L)\}$$

- *If* $R > 2L$*, the competitive buyout is very profitable and*

$$\hat{\eta} = 1$$
$$\hat{\imath} = \frac{3}{8\lambda} l \left\{ X_1 - C + \frac{2}{3}\zeta(R+L) + \frac{3}{4\lambda}(1-\zeta)(X_2 - \gamma C)^2 \right\}$$
$$\hat{I} = C + \frac{1}{\lambda}(1-\zeta)\gamma C \left\{ X_1 - C + \zeta R + \frac{3}{4\lambda}(1-\zeta)(X_2 - \gamma C)^2 \right\}$$

where $l = X_1 - C + 2\zeta(R-L) + \frac{3}{4\lambda}(1-\zeta)(X_2 - \gamma C)^2$.

The entrepreneur issues the amount of equity: $\hat{W} = K - \hat{\imath} - \hat{I}$.

Whether the competitive buyout is very profitable or not, the bank's payments of success are equal to the liquidation values of the project $\hat{D}_1 = C$ *and* $\hat{D}_2 = \gamma C$*. The entrepreneur and the LBO fund have equal payoffs:* $\hat{\beta}_t = \frac{1}{2}$*,* $t = 1, 2$.

The proof of the proposition 4 is presented in the appendix D.

These contracts exhibit the features of the optimal financial contracts discussed in proposition 2: the payments of the bank in case of success are equal to the liquidation values of the project. The optimal sharing rule of the compensation cost depends on the quality of the competitive buyout and on the compensation cost.

When the competitive buyout is not very profitable, the compensation cost is paid to the entrepreneur and to the bank such that:

* If the net revenue of the competitive buyout is high in the sense $\frac{3}{2}L < R \leq 2L$, the entrepreneur perceives higher compensation than the bank.
* In the opposite, the bank gets the highest compensation if $L \leq R \leq \frac{3}{2}L$.

The efforts of the productive stage do not change: they are given by (28). These efforts do not depend on the write-off threat. But, in the starting stage, the entrepreneur and the LBO fund provide the following efforts

$$\hat{e}_1 = \hat{a}_1 = \frac{1}{2\lambda} l \tag{48}$$

The threat of write-off induces the entrepreneur and the LBO fund to provide lower efforts than in the first best in the two stages. The increase of the write-off probability induces them to increase their efforts. If the violation cost is high, the exit becomes costly to the LBO fund. Consequently, the threat of write-off is non credible so they make less efforts in the starting stage. In this case, the share of compensation paid to the entrepreneur (respectively the bank) decreases (respectively increases) if this cost becomes expensive and increases (respectively decreases) if the revenue of the competitive buyout is high.

When $R > L$, the alternative project becomes more attractive for the LBO fund but he must provide high effort in order to succeed the entrepreneur's project so that he gets his money back. Despite the fact that the LBO fund will abandon the project, the entrepreneur prefers working hard in order to reach the success so that she gets a positive payment.

The project succeeds with the probabilities

$$\hat{p}_1 = \tfrac{1}{\lambda} l \quad \text{and} \quad \hat{p}_2 = \tfrac{1}{\lambda}(X_2 - \gamma C) \tag{49}$$

In the presence of the write-off threat, the LBO funds issues higher equity than in the case where he cannot abandon the entrepreneur's project. In the opposite, the bank's investment depends on the write-off cost: if this cost is significantly high, in the sense $L \in \left[\frac{R+\gamma C}{2}\ , R\right]$, the LBO fund is not tempted to exit otherwise, the banks will perceive a high compensation. This is why he accepts to lend more money to the entrepreneur and the LBO fund. When the amount of compensation is low, we need more restrictive conditions in order to compare $\hat{I}$ and I^c.

The expected gain of the entrepreneur is therefore given by

$$\begin{aligned} E(\pi_E) &= \frac{3}{8\lambda}(l)^2 - W \\ &= \hat{\imath} - W \end{aligned}$$

When the competitive buyout is very profitable, the efforts of the productive stage are still given by (28). But, in the starting stage, the entrepreneur provides higher effort than the LBO fund such that

$$\hat{e}_1 = \frac{1}{2\lambda}\left\{X_1 - C + 2\zeta L + \frac{3}{4\lambda}(1-\zeta)(X_2 - \gamma C)^2\right\} > \hat{a}_1 = \frac{1}{2\lambda} l \tag{50}$$

The higher the revenue of the competitive buyout, the more the entrepreneur will be induced to make effort: the LBO fund will probably leave the entrepreneur's project because the compensation cost becomes not significant and she will get the whole amount of compensation. So she is better off in case of success. This is result is not very intuitive: despite the fact that the bank has equal payments in cases of success and failure, he gets a compensation's share only if the competitive project is not very profitable. But, notice that in this case the LBO fund issues a low amount of equity. So in order to induce the bank to lend them more money, he must give him a strictly positive compensation's share in case of write-off.

This is no longer true if the competitive project is very profitable, the LBO fund issues more equity than before; there is no need of additional funds from the bank. To induce the entrepreneur to invest efficient effort, he must get the entire amount of compensation. In this case, the entrepreneur's effort is higher than the LBO's effort despite the fact that e and a are perfect substitutes and both agents have the same functions of cost. The sharing rule of the compensation depends on the financial structure and the agents' incentives.

At the opposite, whether the new acquisition is very profitable or not, the LBO fund provides the same level of effort.

Then, the project succeeds at the end of the first period with the probability :

$$\hat{p}_1 = \frac{1}{\lambda}\left\{X_1 - C + \zeta R + \frac{3}{4\lambda}(1-\zeta)(X_2 - \gamma C)^2\right\}$$

higher than $\frac{1}{\lambda}l$. This is an interesting fact because the LBO fund may use the write off decision as a powerful incentive given to the entrepreneur to make him working hard. This result is consistent with what is noticed in LBO acquisitions: if the project does not meet his short term objectives, the LBO fund threatens to leave the project. Note that when the competitive project is very profitable, in contrast with the bank, the LBO fund issues lower equity than when it is not very profitable.

Note that in both cases, the levels of efforts (48) and (50) are inferior to those provided when the efforts are observable (in perfect information) but superior to (27); when the debt payments are non-decreasing with the revenues of the project and there is no competitive buyout.

Suppose now that the project is implemented without debt. In this case, the entrepreneur and the LBO fund will share equally the revenues (as in the proposition 3). If the fund leaves the entrepreneur's project, he must pledge the entire amount of compensation to the entrepreneur.

Then, the entrepreneur and the LBO fund provide the following efforts:

$$e_1 = \frac{1}{2\lambda}\left\{X_1 - C + 2\zeta L + (1-\zeta)\left[\frac{3}{4\lambda}(X_2 - \gamma C)^2 + \gamma C\right]\right\} \tag{51}$$

$$a_1 = \frac{1}{2\lambda}l + \frac{1}{2\lambda}(1-\zeta)\gamma C \tag{52}$$

In the second period, the efforts are given by (35). It is easy to check that financing the acquisition through the LBO fund induces the agents to provide the highest efforts even if there is a write-off threat. The efforts (51) and (52) do not depend on the quality of the competitive project.

We conclude that it is optimal to issue equity to finance the entrepreneur's project. Otherwise, they ask for debt. To induce the entrepreneur to provide high efforts, the LBO fund must threat her with the exit before the date 2.

5.2.2 L is endogenous

Hereafter, the compensation cost L is a variable to be determined endogenously in the financial contracts.

Proposition 5 *The optimal financial contracts are written*

$$D_1^* = C \quad \textit{and} \quad D_2^* = \gamma C$$

$$I^* = C + \frac{1}{\lambda}\phi(1-\zeta)\gamma C$$

and

$$L^* = \tfrac{1}{2}R \ , \quad \eta^* = 1 \quad \textit{and} \quad \beta_t^* = \tfrac{1}{2},\ t = 1,\, 2$$

$$i^* = \frac{3}{8\lambda}\phi^2$$

$$W^* = K - i^* - I^*$$

where $\phi = X_1 - C + \zeta R + \frac{3}{4\lambda}\left(1-\zeta\right)\left(X_2 - \gamma C\right)^2$.

The proof of this proposition is presented in the appendix E.

The optimal financial contracts have the same properties of those analyzed in the propositions 2, 4 and the lemma 3. The proposition 5 states that the entrepreneur must perceive the whole compensation cost because the bank perceives the same revenue whatever happens.

The efforts of the second period does not depend on the type of the variable L: they are given by (28), but those of the first period become

$$e_1^* = a_1^* = \frac{1}{2\lambda}\phi \tag{53}$$

Notice that these efforts are higher than the efforts (50) but still lower than the efforts provided when the information is perfect.

It is easy to check that the amount of compensation is specified such that the competitive buyout looks not very profitable to the LBO fund. Hence, the threat of write-off becomes incredible. As explained before, if the project is financed through the LBO fund, both agents are induced to provide the highest efforts.

6 Conclusion

The aim of this paper was to study the financial structure in LBO acquisition and to take into account the active role of the LBO fund. We focused on the debt's influence on the agent's incentives in the presence of a double sided moral hazard problem: the entrepreneur and the LBO fund provide unobservable efforts which improve the project's performance. We considered a dynamic agency model with two periods: the starting and productive periods.

Under the condition that the debt's payments are decreasing with the project's outcome, we show that if the project is not very risky, the presence of the bank boosts the agents' incentives so that they provide the first best efforts. This no longer true when the acquired company is very profitable, we need powerful incentive mechanisms: despite the fact that the debt contract exhibits the features of a "*live or die*" contract, the entrepreneur and the LBO fund provide the second best efforts.

This research contributes also to the financial literature that links the exit route and the agents' incentives. It analyzes some characteristics of the LBO finance, namely the *write-off* exit in LBO. This exit route induces the partners to provide high efforts when the project is financed through a mixture of debt and equity under specific conditions. However, these efforts are not optimal: the entrepreneur and the LBO fund provide higher efforts if the acquisition is financed only with equity.

The current paper studied the characteristics of the standard debt-equity contracts and the pure-equity contracts. However, the use of convertible securities such as the convertible bonds ans the convertible preferred stocks becomes prevelant in the buyout projects. This is surprising because these securities are rarely issued in the presence of a passive financiers such as the banks and other outside equity holders. In practice, the LBO fund can convert his debt/stocks into equity in order to increase his controle of the company which reduces the profit and controle of the entrepreneur. It would be intresting to examine how the use of these securities influences the agents' incentives as well as the financial structure, namely the debt to equity ratio.

The decision concerning the exit route can induce agency conflits between the entrepreneur and the LBO fund. The former prefers the IPO exit such that the acquired company is listed on a perfectly competitive stock market. Consequently, she keeps her position and the controle of the company and can get private benefits because of the information asymetry: the shares are sold to a wide spectrum of outside investors. In contrast with the entrepreneur, the LBO fund prefers quick exit routes to get back his money and to invest in a new deal. He takes usually full exit decision of the buyout investments such as sale. Emprically, the probability of an exit via sale was the highest in the buyout stage. To my knowledge no theoritical paper adresses the topic of exit routes in LBO finance. It would be intresting to answer to the following question: how the financial structure in LBO investments solve the agency conflicts due to the choice of the exit route?

In future research it would be insightful to study the other exit routes such as IPO, trade sale and buybacks and their influences on the agents' incentives. Research in this direction is still pending.

Although the investors and financiers point out that the *strip financing* and the *debt syndication* play a major role in mitigating the problems of agency conflicts and information asymmetry, there are no academic papers analyzing how they solve for such problems in buyout investments.

Appendix

A Proof of proposition 1

A.1 The entrepreneur's project is not very risky

The question raised now is the following: what are the parameters' values that the incentive constraints (8), (13) and (14) must satisfy in order to implement the first best solution?

Technically, this is possible if the first best efforts given by (3) and (4) satisfy the conditions (8), (13) and (14). We deduce that:

$$\beta_1^* = \beta_2^* = \frac{1}{2},\ D_1^* = 2(1-\gamma)C - X_1 + \frac{1}{\lambda}(X_2 - \gamma C)^2 \text{ and } D_2^* = 2\gamma C - X_2$$

Note that these solutions exist under the conditions:

$$X_2 \leq 2\gamma C \tag{54}$$

$$(1-\gamma)\, C + \frac{1}{2\lambda}(X_2 - \gamma C)^2 \leq X_1 \leq 2(1-\gamma)C + \frac{1}{\lambda}(X_2 - \gamma C)^2 \tag{55}$$

The first term of (55) is always satisfied given λ high and $X_1 \geq C$ but the second term is fulfilled if $X_1 \leq 2(1-\gamma)C$.

We conclude that when the project is not very risky, in the sense $X_1 \leq 2(1-\gamma)C$ and $X_2 \leq 2\gamma C$, the entrepreneur and the LBO fund provide the first best efforts in the two periods.

Substituting $\beta_t = \frac{1}{2}$ and D_t^*, $t = 1, 2$ into the equations (18) and (19) gives the optimal financial investments of the LBO fund and the bank so that we get the results of the first part of the proposition 1:

$$i^* = \frac{3}{2\lambda}\left[X_1 - (1-\gamma)\, C + \frac{1}{\lambda}\,(X_2 - \gamma C)^2\right]^2$$

$$I^* = C - \frac{2}{\lambda}\left[X_1 - (1-\gamma)\, C + \frac{1}{\lambda}\,(X_2 - \gamma C)^2\right]^2$$

Given λ high, check that I^* is strictly positive and significantly high.

A.2 The entrepreneur's project is very risky

When the project is very risky in the sense $X_1 > 2(1-\gamma)C$ and $X_2 > 2\gamma C$, we have to write the Lagrangian program with all the constraints given by:

$$\begin{aligned}\mathcal{L}= \ & \tfrac{1}{\lambda}\left\{X_1 - D_1 + \tfrac{1}{2\lambda}(1+2\beta_2-2\beta_2^2)\,(X_2{-}D_2)^2\right\}\{X_1-(1-\gamma)C \\ & \quad + \tfrac{1}{\lambda}(X_2 - \ D_2)(X_2-\gamma C) - \tfrac{1}{2\lambda}(1-2\beta_2+2\beta_2^2)(X_2 - \ D_2)^2\Big\} \\ & -\tfrac{1}{2\lambda}\left\{\beta_1(X_1{-}D_1){+}\tfrac{1}{2\lambda}\beta_2(2-\beta_2)\,(X_2{-}D_2)^2\right\}^2 \\ & -\tfrac{1}{2\lambda}\left\{(1-\beta_1)\,(X_1-D_1) + \tfrac{1}{2\lambda}(1-\beta_2^2)\,(X_2{-}D_2)^2\right\}^2 \\ & +C-K+\mu_1\beta_1+\mu_2\beta_2+\mu_3(1-\beta_1)+\mu_4(1-\beta_2) \\ & +\mu_5 D_1+\mu_6 D_2+\mu_7\,(X_1-D_1)+\mu_8\,(X_2-D_2)\end{aligned}$$

Where μ_j, $j = 1..8$ are the Kuhn-Tucker multipliers.

The Kuhn-Tucker conditions give:

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial\beta_1} = \ & \tfrac{1}{\lambda}(X_1{-}D_1)\left[(1-2\beta_1)(X_1{-}D_1){+}\tfrac{1}{2\lambda}(1-2\beta_2)\,(X_2{-}D_2)^2\right] \\ & +\mu_1-\mu_3 \leq 0\end{aligned} \tag{56}$$

$$\beta_1 \geq 0 \tag{57}$$

$$\beta_1\frac{\partial\mathcal{L}}{\partial\beta_1} = 0 \tag{58}$$

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial\beta_2} = \ & \tfrac{1}{\lambda^2}\,(X_2{-}D_2)^2\,\{(X_1-C)+(1-\beta_1)\,(X_1-D_1) \\ & +\tfrac{1}{\lambda}X_2(X_2 - \ D_2) + \tfrac{1}{2\lambda}\beta_2(2-3\beta_2)(X_2 - \ D_2)^2\} \\ & -\tfrac{1}{\lambda^2}\beta_2\,(X_2{-}D_2)^2\,\{2\,(X_1-C)+(X_1-D_1) \\ & +\tfrac{2}{\lambda}X_2(X_2 - \ D_2) - \tfrac{1}{2\lambda}(1-6\beta_2+6\beta_2^2)(X_2 - \ D_2)^2\} \\ & +\mu_2-\mu_4 \leq 0\end{aligned} \tag{59}$$

$$\beta_2 \geq 0 \tag{60}$$

$$\beta_2\frac{\partial\mathcal{L}}{\partial\beta_2} = 0 \tag{61}$$

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial D_1} = \ & -\tfrac{1}{\lambda}\,\{X_1-(1-\gamma)\,C-(1-\beta_1)\,(X_1-D_1) \\ & +\tfrac{1}{\lambda}\,(X_2-\gamma C)\,(X_2 - \ D_2) - \ \tfrac{1}{2\lambda}(2-2\beta_2+\beta_2^2)(X_2 - \ D_2)^2\} \\ & -\tfrac{1}{\lambda}\beta_1\left\{(1-2\beta_1)\,(X_1-D_1)+\tfrac{1}{2\lambda}(1-2\beta_2)\,(X_2{-}D_2)^2\right\} \\ & +\mu_5-\mu_7 \leq 0\end{aligned} \tag{62}$$

$$D_1 \geq 0 \tag{63}$$

$$D_1\frac{\partial\mathcal{L}}{\partial D_1} = 0 \tag{64}$$

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial D_2} = \ & -\tfrac{1}{\lambda^2}(1+2\beta_2-2\beta_2^2)\,(X_2{-}D_2)\,\{X_1-(1-\gamma)\,C \\ & +\tfrac{1}{\lambda}\,(X_2-\gamma C)\,(X_2 - \ D_2) - \ \tfrac{1}{2\lambda}(1-2\beta_2+2\beta_2^2)(X_2 - \ D_2)^2\} \\ & -\tfrac{1}{\lambda^2}\,\{(X_2-\gamma C)-(1-2\beta_2+2\beta_2^2)(X_2 - \ D_2)\} \\ & \quad \left\{X_1-D_1+\tfrac{1}{2\lambda}(1+2\beta_2-2\beta_2^2)\,(X_2{-}D_2)^2\right\} \\ & +\tfrac{1}{\lambda^2}\beta_2(2-\beta_2)\,(X_2{-}D_2)\left\{\beta_1(X_1{-}D_1){+}\tfrac{1}{2\lambda}\beta_2(2-\beta_2)\,(X_2{-}D_2)^2\right\} \\ & +\tfrac{1}{\lambda^2}(1-\beta_2^2)\,(X_2{-}D_2)\left\{(1-\beta_1)\,(X_1-D_1)+\tfrac{1}{2\lambda}(1-\beta_2^2)\,(X_2{-}D_2)^2\right\} \\ & +\mu_6-\mu_8 \leq 0\end{aligned} \tag{65}$$

$$D_2 \geq 0 \tag{66}$$

$$D_2 \frac{\partial \mathcal{L}}{\partial D_2} = 0 \tag{67}$$

The complementary slackness conditions are written:

$$\frac{\partial \mathcal{L}}{\partial \mu_1} = \beta_1 \geq 0 \ , \quad \mu_1 \geq 0 \ , \quad \mu_1 \beta_1 = 0 \tag{68}$$
$$\frac{\partial \mathcal{L}}{\partial \mu_2} = \beta_2 \geq 0 \ , \quad \mu_2 \geq 0 \ , \quad \mu_2 \beta_2 = 0 \tag{69}$$
$$\frac{\partial \mathcal{L}}{\partial \mu_3} = 1 - \beta_1 \geq 0 \ , \quad \mu_3 \geq 0 \ , \quad \mu_3 (1 - \beta_1) = 0 \tag{70}$$
$$\frac{\partial \mathcal{L}}{\partial \mu_4} = 1 - \beta_2 \geq 0 \ , \quad \mu_4 \geq 0 \ , \quad \mu_4 (1 - \beta_2) = 0 \tag{71}$$
$$\frac{\partial \mathcal{L}}{\partial \mu_5} = D_1 \geq 0 \ , \quad \mu_5 \geq 0 \ , \quad \mu_5 D_1 = 0 \tag{72}$$
$$\frac{\partial \mathcal{L}}{\partial \mu_6} = D_2 \geq 0 \ , \quad \mu_6 \geq 0 \ , \quad \mu_6 D_2 = 0 \tag{73}$$
$$\frac{\partial \mathcal{L}}{\partial \mu_7} = X_1 - D_1 \geq 0 \ , \quad \mu_7 \geq 0 \ , \quad \mu_7 (X_1 - D_1) = 0 \tag{74}$$
$$\frac{\partial \mathcal{L}}{\partial \mu_8} = X_2 - D_2 \geq 0 \ , \quad \mu_8 \geq 0 \ , \quad \mu_8 (X_2 - D_2) = 0 \tag{75}$$

It is straightforward to see that $D_2 \neq X_2$ otherwise, the project will fail with probability one in the second period. Hereafter, we will discuss the following cases:

1. $D_1 = X_1$, $0 < D_2 < X_2$ and $0 < \beta_t < 1$, $t = 1, 2$

   According to (68), (69), (70), (71), (72), (73) and (75), the Kuhn-Tucker multipliers $\mu_1 = \mu_3 = \mu_2 = \mu_4 = \mu_5 = \mu_6 = \mu_8 = 0$. We substitute $D_1 = X_1$ and $\mu_1 = \mu_3 = \mu_2 = \mu_4 = \mu_5 = \mu_6 = \mu_8 = 0$ into the first order condition (56) such that:

$$(1 - 2\beta_2) \left\{ (X_1 - C) + \frac{1}{\lambda} X_2 (X_2 - D_2) + \frac{3}{2\lambda} \beta_2 (1 - \beta_2) (X_2 - D_2)^2 \right\} = 0 \tag{76}$$

   The latter condition is satisfied only if $\beta_2 = \frac{1}{2}$. Given $D_1 = X_1$, (64) is satisfied only if (62) is binding:

$$\begin{aligned} \mu_7 = \ & -\tfrac{1}{\lambda} \left\{ X_1 - (1-\gamma) C + \tfrac{1}{\lambda} (X_2 - \gamma C)(X_2 - D_2) \right\} \\ & + \tfrac{1}{2\lambda^2} \left[2 - 2\beta_2 + \beta_2^2 - \beta_1 (1 - 2\beta_2) \right] (X_2 - D_2)^2 \end{aligned} \tag{77}$$

   Substituting $\beta_2 = \frac{1}{2}$ in (65) gives:

$$\begin{aligned} & \tfrac{3}{2} \left\{ X_1 - (1-\gamma) C + \tfrac{1}{\lambda} (X_2 - \gamma C)(X_2 - D_2) - \tfrac{1}{4\lambda} (X_2 - D_2)^2 \right\} \\ & + \tfrac{3}{4\lambda} (X_2 - D_2) \left\{ (X_2 - \gamma C) - \tfrac{1}{2} (X_2 - D_2) \right\} + \tfrac{9}{16\lambda} (X_2 - D_2)^2 = 0 \end{aligned} \tag{78}$$

   Let $Y = (X_2 - D_2)$, the equation (78) becomes therefore:

$$\frac{21}{16\lambda} Y^2 - \frac{9}{4\lambda} (X_2 - \gamma C) Y - \frac{3}{2} [X_1 - (1-\gamma) C] = 0$$

   The latter equation has two solutions:

   - $Y_1 = \frac{6}{7} (X_2 - \gamma C) - \frac{2}{7} \sqrt{9 (X_2 - \gamma C)^2 + 14\lambda [X_1 - (1-\gamma) C]} < 0$
   - $Y_2 = \frac{6}{7} (X_2 - \gamma C) + \frac{2}{7} \sqrt{9 (X_2 - \gamma C)^2 + 14\lambda [X_1 - (1-\gamma) C]}$

If $Y = Y_2$, we get:

$$D_2 = \frac{1}{7}X_2 + \frac{6}{7}\gamma C - \frac{2}{7}\sqrt{9\left(X_2 - \gamma C\right)^2 + 14\lambda\left[X_1 - (1-\gamma)\,C\right]} < 0$$

This leads to a contradiction with our starting hypothesis.

2. $D_1 = 0$, $0 < D_2 < X_2$ and $0 < \beta_t < 1$, $t = 1, 2$

   According to the conditions (68), (69), (70), (71), (73), (74) and (75), $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_6 = \mu_7 = \mu_8 = 0$. Consequently, the conditions (56) and (59) are written:

$$\frac{1}{\lambda}X_1\left[(1-2\beta_1)(X_1) + \frac{1}{2\lambda}(1-2\beta_2)\left(X_2 - D_2\right)^2\right] = 0 \qquad (79)$$

$$\begin{aligned}(1-2\beta_2)\left[(X_1 - C) + \tfrac{1}{\lambda}X_2(X_2 - D_2) + \tfrac{1}{2\lambda}\beta_2\left(1-\beta_2\right)\left(X_2 - D_2\right)^2\right]& \\ +\left(1-\beta_1-\beta_2\right)X_1 = 0&\end{aligned} \qquad (80)$$

   (79) and (80) are satisfied only if $\beta_t = \frac{1}{2}$, $t = 1, 2$. Substituting $D_1 = 0$, $\beta_t = \frac{1}{2}$, $t = 1, 2$ and $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_6 = \mu_7 = \mu_8 = 0$ into the equation (65) gives:

$$\begin{aligned}-\tfrac{3}{2}\left(X_2 - D_2\right)\left\{\tfrac{1}{2}X_1 - (1-\gamma)\,C + \tfrac{1}{\lambda}\left(X_2 - \gamma C\right)\left(X_2 - D_2\right) - \tfrac{5}{8\lambda}\left(X_2 - D_2\right)^2\right\}& \\ -\left\{(X_2 - \gamma C) - \tfrac{1}{2}(X_2 - D_2)\right\}\left\{X_1 + \tfrac{3}{4\lambda}\left(X_2 - D_2\right)^2\right\} = 0&\end{aligned} \qquad (81)$$

   Let $Y = (X_2 - D_2)$, (81) becomes:

$$-\frac{21}{16\lambda}Y^3 + \frac{9}{4\lambda}\left(X_2 - \gamma C\right)Y^2 + Y\left[\frac{1}{4}X_1 - \frac{3}{2}\left(1-\gamma\right)C\right] + X_1\left(X_2 - \gamma C\right) = 0$$

   The latter equation has no real solution.

3. $D_t = 0$ and $\beta_t = 0$, $t = 1, 2$

   According to the equations (70), (71), (74) and (75), the Kuhn-Tucker multipliers $\mu_3 = \mu_4 = \mu_7 = \mu_8 = 0$. Substituting $D_t = 0$, $\beta_t = 0$, $t = 1, 2$ and $\mu_3 = \mu_4 = \mu_7 = \mu_8 = 0$ into the first order condition (56) gives:

$$\mu_1 \le -\frac{1}{\lambda}X_1\left[X_1 + \frac{1}{2\lambda}\left(X_2\right)^2\right] < 0$$

   which does not satisfy (68).

4. $D_t = 0$ and $\beta_t = 1$, $t = 1, 2$

   According to the conditions (68), (69), (74) and (75), $\mu_1 = \mu_2 = \mu_7 = \mu_8 = 0$. We substitute $D_t = 0$, $t = 1, 2$, $\beta_1 = 1$ and $\mu_1 = \mu_2 = \mu_7 = \mu_8 = 0$ into (56) gives:

$$\mu_3 = -\frac{1}{\lambda}X_1\left[X_1 + \frac{1}{2\lambda}\left(X_2\right)^2\right] < 0$$

5. $D_1 = X_1$, $D_2 = 0$ and $0 < \beta_t < 1$, $t = 1, 2$

   According to the equations (68), (69), (70), (71), (72) and (75), the Kuhn-Tucker multipliers $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_8 = 0$. Then, the first order condition (62) implies:

$$\begin{aligned}\mu_7 = \; & -\tfrac{1}{\lambda}\left\{X_1 - (1-\gamma)\,C + \tfrac{1}{\lambda}X_2\left(X_2 - \gamma C\right) + \tfrac{1}{2\lambda}\left[\beta_2(2-\beta_2) + \beta_1(1-2\beta_2)\right]\left(X_2\right)^2\right\} \\ & < 0\end{aligned} \qquad (82)$$

6. $\underline{0 < D_t < X_t \text{ and } 0 < \beta_t < 1,\ t = 1, 2}$

According to (68), (69), (70), (71), (72), (73), (74) and (75), $\mu_j = 0$, $j = 1..8$. We substitute these multipliers into the first order conditions (56), (59), (62) and (65) such that we obtain the following system:

$$(1-2\beta_1)(X_1-D_1)+\frac{1}{2\lambda}(1-2\beta_2)(X_2-D_2)^2 = 0 \tag{83}$$

$$\begin{aligned}&\left\{(X_1-C)+(1-\beta_1)(X_1-D_1)+\tfrac{1}{\lambda}X_2(X_2-D_2)+\tfrac{1}{2\lambda}\beta_2(2-3\beta_2)(X_2-D_2)^2\right\}\\ &-\beta_2\left\{2(X_1-C)+(X_1-D_1)+\tfrac{2}{\lambda}X_2(X_2-D_2)-\tfrac{1}{2\lambda}(1-6\beta_2+6\beta_2^2)(X_2-D_2)^2\right\}=0\end{aligned} \tag{84}$$

$$\begin{aligned}&-\left\{X_1-(1-\gamma)C-(1-\beta_1)(X_1-D_1)-\tfrac{1}{2\lambda}(2-2\beta_2+\beta_2^2)(X_2-D_2)^2\right.\\ &\left.+\tfrac{1}{\lambda}(X_2-\gamma C)(X_2-D_2)\right\}-\beta_1\left\{(1-2\beta_1)(X_1-D_1)+\tfrac{1}{2\lambda}(1-2\beta_2)(X_2-D_2)^2\right\}=0\end{aligned} \tag{85}$$

$$\begin{aligned}&-(1+2\beta_2-2\beta_2^2)(X_2-D_2)\left\{X_1-(1-\gamma)C+\tfrac{1}{\lambda}(X_2-\gamma C)(X_2-D_2)\right.\\ &\quad\left.-\tfrac{1}{2\lambda}(1-2\beta_2+2\beta_2^2)(X_2-D_2)^2\right\}\\ &-\left\{(X_2-\gamma C)-(1-2\beta_2+2\beta_2^2)(X_2-D_2)\right\}\left\{X_1-D_1+\tfrac{1}{2\lambda}(1+2\beta_2-2\beta_2^2)(X_2-D_2)^2\right\}\\ &+\beta_2(2-\beta_2)(X_2-D_2)\left\{\beta_1(X_1-D_1)+\tfrac{1}{2\lambda}\beta_2(2-\beta_2)(X_2-D_2)^2\right\}\\ &+(1-\beta_2^2)(X_2-D_2)\left\{(1-\beta_1)(X_1-D_1)+\tfrac{1}{2\lambda}(1-\beta_2^2)(X_2-D_2)^2\right\}=0\end{aligned} \tag{86}$$

Note that the equations (83) and (84) are satisfied only if $\beta_t = \frac{1}{2}$, $t = 1, 2$. Then, (85) and (86) are written

$$X_1-(1-\gamma)C-\frac{1}{2}(X_1-D_1)+\frac{1}{\lambda}(X_2-\gamma C)(X_2-D_2)-\frac{5}{8\lambda}(X_2-D_2)^2=0 \tag{87}$$

$$\begin{aligned}&-\tfrac{3}{2}(X_2-D_2)\left\{X_1-(1-\gamma)C+\tfrac{1}{\lambda}(X_2-\gamma C)(X_2-D_2)-\tfrac{1}{4\lambda}(X_2-D_2)^2\right\}\\ &-\left\{(X_2-\gamma C)-\tfrac{5}{4}(X_2-D_2)\right\}\left\{X_1-D_1+\tfrac{3}{4\lambda}(X_2-D_2)^2\right\}=0\end{aligned} \tag{88}$$

The equation (87) enables us to deduce:

$$D_1 = 2(1-\gamma)C - X_1 - \frac{2}{\lambda}(X_2-\gamma C)(X_2-D_2)+\frac{5}{4\lambda}(X_2-D_2)^2 \tag{89}$$

Substituting D_1 for its expression given by (89) into (88) gives:

$$\begin{aligned}&\tfrac{3}{2}(X_2-D_2)\left\{X_1-(1-\gamma)C+\tfrac{1}{\lambda}(X_2-\gamma C)(X_2-D_2)-\tfrac{1}{4\lambda}(X_2-D_2)^2\right\}\\ &+\left\{(X_2-\gamma C)-\tfrac{5}{4}(X_2-D_2)\right\}\\ &\left\{2\left[X_1-(1-\gamma)C\right]+\tfrac{2}{\lambda}(X_2-\gamma C)(X_2-D_2)-\tfrac{1}{2\lambda}(X_2-D_2)^2\right\}=0\end{aligned} \tag{90}$$

Let $Y = (X_2 - D_2)$, the equation (90) becomes:

$$\begin{aligned}\tfrac{1}{4\lambda}Y^3-\tfrac{3}{2\lambda}(X_2-\gamma C)Y^2+\left\{\tfrac{2}{\lambda}(X_2-\gamma C)^2-\left[X_1-(1-\gamma)C\right]\right\}Y&\\ +2(X_2-\gamma C)\left[X_1-(1-\gamma)C\right]=0&\end{aligned}$$

This equation has three solutions:

- $Y_1 = 2(X_2-\gamma C) \Rightarrow$ if $Y = Y_1$, $D_2 = 2\gamma C - X_2 < 0$

- $Y_2 = 2(X_2 - \gamma C) + 2\sqrt{(X_2 - \gamma C)^2 + \lambda[X_1 - (1-\gamma)C]} \Rightarrow$ if $Y = Y_2$, $D_2 = 2\gamma C - X_2 - 2\sqrt{(X_2 - \gamma C)^2 + \lambda[X_1 - (1-\gamma)C]} < 0$
- $Y_3 = 2(X_2 - \gamma C) - 2\sqrt{(X_2 - \gamma C)^2 + \lambda[X_1 - (1-\gamma)C]} < 0.$

7. $\underline{0 < D_1 < X_1,\ D_2 = 0 \text{ and } 0 < \beta_t < 1,\ t = 1,\ 2}$.

   According to (68), (69), (70), (71) (72), (74) and (75), $\mu_j = 0, \forall j \in \{1..8\} / \{6\}$. Then the condition (56) is written:

$$(1-2\beta_1)(X_1 - D_1) + \frac{1}{2\lambda}(1-2\beta_2)(X_2)^2 = 0$$

   It is satisfied only if $\beta_t = \frac{1}{2}$, $t = 1,\ 2$. Substituting $D_2 = 0$, $\mu_j = 0$, $j = 1..8/\{6\}$ and $\beta_t = \frac{1}{2}$ into (62) and (65) gives the following system:

$$X_1 - (1-\gamma)C - \frac{1}{2}(X_1 - D_1) + \frac{1}{2\lambda}X_2\left(\frac{3}{4}X_2 - 2\gamma C\right) = 0 \tag{91}$$

$$\begin{aligned}\mu_6 \leq\ & \tfrac{3}{2\lambda^2}X_2\left\{X_1 - (1-\gamma)C + \tfrac{1}{\lambda}X_2\left(\tfrac{3}{8}X_2 - \gamma C\right) - \tfrac{1}{2}(X_1 - D_1)\right\} \\ & + \tfrac{1}{2\lambda^2}(X_2 - 2\gamma C)\left\{X_1 - D_1 + \tfrac{3}{4\lambda}(X_2)^2\right\}\end{aligned} \tag{92}$$

   According to (91), if there is success, the bank's payment in the starting stage is written:

$$D_1 = 2(1-\gamma)C - X_1 - \frac{1}{\lambda}X_2(\frac{3}{4}X_2 - 2\gamma C) \tag{93}$$

   It is straightforward to see that $D_1 < 0$ when $X_1 \geq 2(1-\gamma)C$.

8. $\underline{D_t = 0 \text{ and } 0 < \beta_t < 1,\ t = 1, 2}$

   The conditions (68), (69), (70), (71), (74) and (75) give $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_7 = \mu_8 = 0$. Substituting $D_t = 0$, $t = 1, 2$ and $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_7 = \mu_8 = 0$ into the conditions (56) and (59) gives:

$$(1-2\beta_1)X_1 + \frac{1}{2\lambda}(1-2\beta_2)(X_2)^2 = 0 \tag{94}$$

$$\begin{aligned}&\left\{(2-\beta_1)X_1 - C + \tfrac{1}{2\lambda}(2 + 2\beta_2 - 3\beta_2^2)(X_2)^2\right\} \\ &-\beta_2\left\{(3X_1 - 2C) + \tfrac{1}{2\lambda}(3 + 6\beta_2 - 6\beta_2^2)(X_2)^2\right\} = 0\end{aligned} \tag{95}$$

   The equation (94) and (95) are satisfied only if $\beta_t = \frac{1}{2}$, $t = 1,\ 2$.

   Then, we substitute $D_t = 0$, $\beta_t = \frac{1}{2}$, $t = 1, 2$ and $\mu_7 = \mu_8 = 0$ into the (62) and (65) such that:

$$\mu_5 \leq \frac{1}{2\lambda}\left\{X_1 - 2(1-\gamma)C + \frac{1}{\lambda}X_2\left(\frac{3}{4}X_2 - 2\gamma C\right)\right\}$$

$$\begin{aligned}\mu_6 \leq\ & \tfrac{3}{4\lambda^2}X_2\left\{X_1 - 2(1-\gamma)C + \tfrac{1}{\lambda}X_2\left(\tfrac{3}{4}X_2 - 2\gamma C\right)\right\} \\ & + \tfrac{1}{2\lambda^2}(X_2 - 2\gamma C)\left[X_1 + \tfrac{3}{4\lambda}(X_2)^2\right]\end{aligned}$$

   If μ_5 and μ_6 are Kuhn-Tucker multipliers, we should get the following conditions:

$$X_1 \geq 2(1-\gamma)C - \frac{1}{\lambda}X_2\left(\frac{3}{4}X_2 - 2\gamma C\right) \tag{96}$$

$$X_2 \geq 2\gamma C$$

Given λ high, the condition $X_1 > 2(1-\gamma)C$ enables us to meet the inequality (96). Consequently, the entrepreneur and the LBO fund provide the following efforts:

$$e_1^* = a_1^* = \frac{1}{2\lambda}\left[X_1 + \frac{3}{4\lambda}(X_2)^2\right] \tag{97}$$

$$e_2^* = a_2^* = \frac{1}{2\lambda}X_2 \tag{98}$$

To get the results of the second part of the proposition 1, we substitute (97) and (98) into the participation constraints of the LBO fund and the bank. The optimal financial investments made by the LBO fund and the bank are given respectively by:

$$i^* = \frac{3}{8\lambda}\left[X_1 + \frac{3}{4\lambda}(X_2)^2\right]^2$$

$$I^* = C\left\{1 - \frac{1}{\lambda}\left(1-\gamma+\frac{1}{\lambda}X_2\right)\left[X_1 + \frac{3}{4\lambda}(X_2)^2\right]\right\}$$

We have not presented all the discussed cases but just the most interesting ones, more detailed proof is available under request.

B Proof of the proposition 2

The Lagrangian is written

$$\begin{aligned}
\mathcal{L} = \; & \tfrac{1}{\lambda}\left\{X_1 - D_1 + \tfrac{1}{2\lambda}(1+2\beta_2-2\beta_2^2)(X_2-D_2)^2\right\} \\
& \left\{X_1 - (1-\gamma)C + \tfrac{1}{\lambda}(X_2 - D_2)(X_2-\gamma C) - \tfrac{1}{2\lambda}(1-2\beta_2+2\beta_2^2)(X_2 - D_2)^2\right\} \\
& -\tfrac{1}{2\lambda}\left\{\beta_1(X_1-D_1)+\tfrac{1}{2\lambda}\beta_2(2-\beta_2)(X_2-D_2)^2\right\}^2 \\
& -\tfrac{1}{2\lambda}\left\{(1-\beta_1)(X_1-D_1)+\tfrac{1}{2\lambda}(1-\beta_2^2)(X_2-D_2)^2\right\}^2 \\
& +C-K+\mu_1\beta_1+\mu_2\beta_2+\mu_3(1-\beta_1)+\mu_4(1-\beta_2)+\mu_5 D_1 \\
& +\mu_6 D_2+\mu_7(X_1-D_1)+\mu_8(X_2-D_2)+\mu_9(D_1-C)+\mu_{10}(D_2-\gamma C)
\end{aligned}$$

Where μ_j, $j = 1..10$ are the Kuhn-Tucker multipliers.

The Kuhn-Tucker conditions give:

$$\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial \beta_1} = \; & \tfrac{1}{\lambda}(X_1-D_1)\left[(1-2\beta_1)(X_1-D_1)+\tfrac{1}{2\lambda}(1-2\beta_2)(X_2-D_2)^2\right] \\
& +\mu_1-\mu_3 \leq 0
\end{aligned} \tag{99}$$

$$\beta_1 \geq 0 \tag{100}$$

$$\beta_1\frac{\partial \mathcal{L}}{\partial \beta_1} = 0 \tag{101}$$

$$\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial \beta_2} = \quad & \tfrac{1}{\lambda^2}\left(X_2{-}D_2\right)^2 \{(X_1 - C) + (1-\beta_1)\,(X_1 - D_1) \\
& + \tfrac{1}{\lambda} X_2 (X_2 - \ D_2) + \tfrac{1}{2\lambda}\beta_2 (2 - 3\beta_2)(X_2 - \ D_2)^2 \big\} \\
& - \tfrac{1}{\lambda^2}\beta_2 \left(X_2{-}D_2\right)^2 \{2\,(X_1 - C) + (X_1 - D_1) \\
& + \tfrac{2}{\lambda} X_2 (X_2 - \ D_2) - \tfrac{1}{2\lambda}(1 - 6\beta_2 + 6\beta_2^2)(X_2 - \ D_2)^2 \big\} \\
& + \mu_2 - \mu_4 \leq 0
\end{aligned} \tag{102}$$

$$\beta_2 \geq 0 \tag{103}$$

$$\beta_2 \frac{\partial \mathcal{L}}{\partial \beta_2} = 0 \tag{104}$$

$$\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial D_1} = \quad & -\tfrac{1}{\lambda}\{X_1 - (1-\gamma)\,C - (1-\beta_1)\,(X_1 - D_1) \\
& + \tfrac{1}{\lambda}\left(X_2 - \gamma C\right)(X_2 - \ D_2) - \ \tfrac{1}{2\lambda}(2 - 2\beta_2 + \beta_2^2)(X_2 - \ D_2)^2 \big\} \\
& - \tfrac{1}{\lambda}\beta_1 \left\{ (1-2\beta_1)\,(X_1 - D_1) + \tfrac{1}{2\lambda}(1 - 2\beta_2)\left(X_2{-}D_2\right)^2 \right\} \\
& + \mu_5 - \mu_7 + \mu_9 \leq 0
\end{aligned} \tag{105}$$

$$D_1 \geq 0 \tag{106}$$

$$D_1 \frac{\partial \mathcal{L}}{\partial D_1} = 0 \tag{107}$$

$$\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial D_2} = \quad & -\tfrac{1}{\lambda^2}(1 + 2\beta_2 - 2\beta_2^2)\left(X_2{-}D_2\right)\{X_1 - (1-\gamma)\,C \\
& + \tfrac{1}{\lambda}\left(X_2 - \gamma C\right)(X_2 - \ D_2) - \ \tfrac{1}{2\lambda}(1 - 2\beta_2 + 2\beta_2^2)(X_2 - \ D_2)^2 \big\} \\
& - \tfrac{1}{\lambda^2}\left\{ (X_2 - \gamma C) - (1 - 2\beta_2 + 2\beta_2^2)(X_2 - \ D_2) \right\} \\
& \left\{ X_1 - D_1 + \tfrac{1}{2\lambda}(1 + 2\beta_2 - 2\beta_2^2)\left(X_2{-}D_2\right)^2 \right\} \\
& + \tfrac{1}{\lambda^2}\beta_2 (2 - \beta_2)\left(X_2{-}D_2\right) \left\{ \beta_1 (X_1{-}D_1) {+} \tfrac{1}{2\lambda}\beta_2 (2-\beta_2)\left(X_2{-}D_2\right)^2 \right\} \\
& + \tfrac{1}{\lambda^2}(1 - \beta_2^2)\left(X_2{-}D_2\right) \left\{ (1-\beta_1)\,(X_1 - D_1) + \tfrac{1}{2\lambda}(1 - \beta_2^2)\left(X_2{-}D_2\right)^2 \right\} \\
& + \mu_6 - \mu_8 + \mu_{10} \leq 0
\end{aligned} \tag{108}$$

$$D_2 \geq 0 \tag{109}$$

$$D_2 \frac{\partial \mathcal{L}}{\partial D_2} = 0 \tag{110}$$

The complementary slackness conditions are written:

$$\tfrac{\partial \mathcal{L}}{\partial \mu_1} = \beta_1 \geq 0 \ , \ \ \mu_1 \geq 0 \ , \ \ \mu_1 \beta_1 = 0 \tag{111}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_2} = \beta_2 \geq 0 \ , \ \ \mu_2 \geq 0 \ , \ \ \mu_2 \beta_2 = 0 \tag{112}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_3} = 1 - \beta_1 \geq 0 \ , \ \ \mu_3 \geq 0 \ , \ \ \mu_3\,(1 - \beta_1) = 0 \tag{113}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_4} = 1 - \beta_2 \geq 0 \ , \ \ \mu_4 \geq 0 \ , \ \ \mu_4\,(1 - \beta_2) = 0 \tag{114}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_5} = D_1 \geq 0 \ , \ \ \mu_5 \geq 0 \ , \ \ \mu_5 D_1 = 0 \tag{115}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_6} = D_2 \geq 0 \ , \ \ \mu_6 \geq 0 \ , \ \ \mu_6 D_2 = 0 \tag{116}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_7} = X_1 - D_1 \geq 0 \ , \ \ \mu_7 \geq 0 \ , \ \ \mu_7\,(X_1 - D_1) = 0 \tag{117}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_8} = X_2 - D_2 \geq 0 \ , \ \ \mu_8 \geq 0 \ , \ \ \mu_8\,(X_2 - D_2) = 0 \tag{118}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_9} = D_1 - C \geq 0 \ , \ \ \mu_9 \geq 0 \ , \ \ \mu_9\,(D_1 - C) = 0 \tag{119}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_{10}} = D_2 - \gamma C \geq 0 \ , \ \ \mu_{10} \geq 0 \ , \ \ \mu_{10}\,(D_2 - \gamma C) = 0 \tag{120}$$

We rule out the case where $D_2 \neq X_2$ otherwise, the project will fail with probability one in the second period.

1. <u>$D_1 = X_1$, $0 < D_2 < X_2$ and $0 < \beta_t < 1$, $t = 1, 2$</u>

   According to (111), (112), (113), (114), (115), (116), (118) and (119), the Kuhn-Tucker multipliers $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_8 = \mu_9 = 0$. We substitute $D_1 = X_1$ and $\mu_1 = \mu_3 = \mu_2 = \mu_4 = \mu_5 = \mu_6 = \mu_8 = \mu_9 = 0$ into the first order condition (99) such that

$$(1-2\beta_2)\left\{(X_1 - C) + \frac{1}{\lambda}X_2(X_2 - D_2) + \frac{3}{2\lambda}\beta_2\,(1-\beta_2)\,(X_2 - D_2)^2\right\} = 0 \tag{121}$$

   The latter condition is satisfied only if $\beta_2 = \frac{1}{2}$.Given $D_1 = X_1$, (107) is properly specified only if (105) is binding. Substituting $\beta_2 = \frac{1}{2}$ into the latter condition implies

$$\mu_7 = -\tfrac{1}{\lambda}\left\{X_1 - (1-\gamma)\,C + \tfrac{1}{\lambda}\,(X_2 - \gamma C)\,(X_2 - D_2) - \tfrac{5}{8\lambda}(X_2 - D_2)^2\right\} \tag{122}$$

   μ_7 is a Kuhn-Tucker multiplier only if

$$X_1 \leq (1-\gamma)\,C - \frac{1}{\lambda}X_2(X_2 - D_2) + \frac{5}{8\lambda}(X_2 - D_2)^2 \tag{123}$$

   Given λ high, (123) is never satisfied.

2. <u>$D_1 = 0$, $0 \leq D_2 \leq X_2$ and $0 < \beta_t < 1$, $t = 1, 2$</u>

   If $D_1 = 0$, the complementary condition (119) cannot be satisfied because $\frac{\partial \mathcal{L}}{\partial \mu_9} < 0$.

3. <u>$0 \leq D_1 \leq X_1$, $D_2 = 0$ and $0 \leq \beta_t \leq 1$, $t = 1, 2$</u>

   If $D_2 = 0$, (120) is not satisfied since $\frac{\partial \mathcal{L}}{\partial \mu_{10}} < 0$.

4. <u>$0 < D_t < X_t$ and $0 < \beta_t < 1$, $t = 1, 2$</u>

   According to (111), (112), (113), (114), (115), (116), (117) and (118), $\mu_j = 0$, $j = 1..8$. We substitute these multipliers into the first order conditions (99), (102), (105) and (108), we obtain the following system:

$$(1-2\beta_1)(X_1 - D_1) + \frac{1}{2\lambda}(1-2\beta_2)\,(X_2 - D_2)^2 = 0 \tag{124}$$

$$\begin{aligned}&\left\{(X_1 - C) + (1-\beta_1)\,(X_1 - D_1) + \tfrac{1}{\lambda}X_2(X_2 - D_2) + \tfrac{1}{2\lambda}\beta_2(2-3\beta_2)(X_2 - D_2)^2\right\}\\ &-\beta_2\left\{2\,(X_1 - C) + (X_1 - D_1) + \tfrac{2}{\lambda}X_2(X_2 - D_2) - \tfrac{1}{2\lambda}(1-6\beta_2+6\beta_2^2)(X_2 - D_2)^2\right\} = 0\end{aligned} \tag{125}$$

$$\begin{aligned}&-\tfrac{1}{\lambda}\left\{X_1 - (1-\gamma)\,C + \tfrac{1}{\lambda}\,(X_2 - \gamma C)\,(X_2 - D_2)\right.\\ &\left.-(1-\beta_1)\,(X_1 - D_1) - \tfrac{1}{2\lambda}(2-2\beta_2+\beta_2^2)(X_2 - D_2)^2\right\}\\ &-\tfrac{1}{\lambda}\beta_1\left\{(1-2\beta_1)\,(X_1 - D_1) + \tfrac{1}{2\lambda}(1-2\beta_2)\,(X_2 - D_2)^2\right\} + \mu_9 = 0\end{aligned} \tag{126}$$

$$\begin{aligned}&-\tfrac{1}{\lambda^2}(1+2\beta_2-2\beta_2^2)\,(X_2 - D_2)\left\{\tfrac{1}{\lambda}\,(X_2 - \gamma C)\,(X_2 - D_2)\right.\\ &\qquad\left.X_1 - (1-\gamma)\,C - \tfrac{1}{2\lambda}(1-2\beta_2+2\beta_2^2)(X_2 - D_2)^2\right\}\\ &-\tfrac{1}{\lambda^2}\left\{(X_2 - \gamma C) - (1-2\beta_2+2\beta_2^2)(X_2 - D_2)\right\}\\ &\qquad\left\{X_1 - D_1 + \tfrac{1}{2\lambda}(1+2\beta_2-2\beta_2^2)\,(X_2 - D_2)^2\right\}\\ &+\tfrac{1}{\lambda^2}\beta_2(2-\beta_2)\,(X_2 - D_2)\left\{\beta_1(X_1 - D_1) + \tfrac{1}{2\lambda}\beta_2(2-\beta_2)\,(X_2 - D_2)^2\right\}\\ &+\tfrac{1}{\lambda^2}(1-\beta_2^2)\,(X_2 - D_2)\left\{(1-\beta_1)\,(X_1 - D_1) + \tfrac{1}{2\lambda}(1-\beta_2^2)\,(X_2 - D_2)^2\right\}\\ &+\mu_{10} = 0\end{aligned} \tag{127}$$

Note that the equations (124) and (125) are satisfied only if $\beta_t = \frac{1}{2}$, $t = 1$, 2. Then, (126) and (127) are written

$$\mu_9 = \frac{1}{\lambda}\left\{X_1 - (1-\gamma)\,C - \frac{1}{2}\,(X_1 - D_1) + \frac{1}{\lambda}\,(X_2 - \gamma C)\,(X_2 - \ D_2) - \ \frac{5}{8\lambda}(X_2 - \ D_2)^2\right\} \tag{128}$$

$$\begin{aligned}\mu_{10} = \ & \tfrac{3}{2\lambda^2}\,(X_2{-}D_2)\left\{X_1 - (1-\gamma)\,C - \tfrac{1}{2}\,(X_1{-}D_1) + \tfrac{1}{\lambda}\,(X_2 - \gamma C)\,(X_2 - \ D_2) - \ \tfrac{5}{8\lambda}(X_2 - \ D_2)^2\right\} \\ & \tfrac{1}{\lambda^2}\left\{(X_2 - \gamma C) - \tfrac{1}{2}(X_2 - \ D_2)\right\}\left\{X_1 - D_1 + \tfrac{3}{4\lambda}\,(X_2{-}D_2)^2\right\}\end{aligned} \tag{129}$$

- If $\mu_9 = \mu_{10} = 0$, the equation (128) enables us to deduce that:

  $$D_1 = 2\,(1-\gamma)\,C - X_1 - \frac{2}{\lambda}\,(X_2 - \gamma C)\,(X_2 - \ D_2) + \frac{5}{4\lambda}(X_2 - \ D_2)^2 \tag{130}$$

  Given λ, $D_1 < C$ which does not satisfy (119) because $\frac{\partial \mathcal{L}}{\partial \mu_9} < 0$.

- If $\mu_9 = 0$ and $\mu_{10} > 0$, according to (120), $D_2 = \gamma C$ and D_1 satisfies (130). Substituting $D_2 = \gamma C$ in (130) gives

  $$D_1 = 2\,(1-\gamma)\,C - X_1 - \frac{3}{4\lambda}(X_2 - \ \gamma C)^2 < C \tag{131}$$

- If $\mu_9 > 0$ and $\mu_{10} = 0$, the condition (119) enables us to deduce that $D_1 = C$ which is substituted into (129) so that we get

  $$\begin{aligned}& \tfrac{3}{2}\,(X_2{-}D_2)\left\{X_1 - (1-\gamma)\,C - \tfrac{1}{2}\,(X_1{-}C) + \tfrac{1}{\lambda}\,(X_2 - \gamma C)\,(X_2 - \ D_2) - \ \tfrac{5}{8\lambda}(X_2 - \ D_2)^2\right\} \\ & \left\{(X_2 - \gamma C) - \tfrac{1}{2}(X_2 - \ D_2)\right\}\left\{X_1 - C_1 + \tfrac{3}{4\lambda}\,(X_2{-}D_2)^2\right\} = 0\end{aligned} \tag{132}$$

  Let $Y = (X_2 - \ D_2)$, (132) gives

  $$\begin{aligned}-\tfrac{21}{16\lambda}Y^3 + \tfrac{9}{4\lambda}\,(X_2 - \gamma C)\,Y^2 + \tfrac{1}{2}\left\{3\,[X_1 - (1-\gamma)\,C] - \tfrac{5}{2}\,(X_1{-}C)\right\}Y \\ + (X_1 - C_1)\,(X_2 - \gamma C) = 0\end{aligned} \tag{133}$$

  which has no real solutions.

- If $\mu_9 > 0$ and $\mu_{10} > 0$, given the conditions (119) and (120), we deduce that $D_1 = C$ and $D_2 = \gamma C$ which satisfy (128) and (129).

Consequently, we conclude that

$$\beta_t = \frac{1}{2}, t = 1, 2,\ D_1 = C \text{ and } D_2 = \gamma C \tag{134}$$

We substitute (134) into the participation constraints of the entrepreneur and the LBO fund which completes the proof of the proposition 2.

C Proof of the proposition 3

We substitute (30), (32) and (33) into the objective function. The entrepreneur's program is written:

$$\begin{aligned}\max_{\beta,\ t=1,\ 2} V = \ & \tfrac{1}{\lambda}\left[X_1-(1-\gamma)\,C+\tfrac{1}{2\lambda}(1+2\beta_2-2\beta_2^2)\,(X_2-\gamma C)^2\right]^2\\ & +C-K\\ & -\tfrac{1}{2\lambda}\left[\beta_1(X_1-C)+\beta_2\gamma C+\tfrac{1}{2\lambda}\beta_2(2-\beta_2)\,(X_2-\gamma C)^2\right]^2\\ & -\tfrac{1}{2\lambda}\left[(1-\beta_1)(X_1-C)+(1-\beta_2)\,\gamma C+\tfrac{1}{2\lambda}(1-\beta_2^2)\,(X_2-\gamma C)^2\right]^2\end{aligned}$$

The first order conditions of V give the following equations system:

$$(1-2\beta_1)\,(X_1-C)+(1-2\beta_2)\left[\gamma C+\frac{1}{2\lambda}\,(X_2-\gamma C)^2\right]=0$$

$$\begin{aligned}(2-3\beta_2)&\left[X_1-(1-\gamma)\,C+\frac{1}{2\lambda}(1+2\beta_2-2\beta_2^2)\,(X_2-\gamma C)^2\right]\\ &-\beta_1(X_1-C)-\beta_2\gamma C-\frac{1}{2\lambda}\beta_2(2-\beta_2)\,(X_2-\gamma C)^2=0\end{aligned}$$

This system has two possible solutions but one of them is real and varies from 0 to 1. This solution is given by:

$$\tilde{\beta}_t=\frac{1}{2},\ t=1,\ 2$$

D Proof of the proposition 4

D.1 The competitive project is not very profitable: $R \leq 2L$

Substituting the efforts $\hat{a}_t$ and $\hat{e}_t$, $t=1$, 2 into the objective function gives

$$\begin{aligned}&\max_{\eta,\ \beta_t,\ D_t,\ t=1,\ 2} E\,(\pi_E)=\\ &\tfrac{1}{\lambda}\left\{X_1-D_1+\zeta\,[R-(1-\eta)\,L]+\tfrac{1}{2\lambda}\,(1-\zeta)\left(1+2\beta_2-2\beta_2^2\right)(X_2-\ D_2)^2\right\}\\ &\left\{X_1-[1-(1-\zeta)\,\gamma]\,C+\zeta R+\tfrac{1}{2\lambda}\,(1-\zeta)\left[\ 2(X_2-\ D_2)\,(X_2-\gamma C)-\left(1-2\beta_2+2\beta_2^2\right)(X_2-\ D_2)^2\right]\right\}\\ &-\ \tfrac{1}{2\lambda}\left\{(1-\beta_1)\,(X_1-D_1)+\zeta\,(R-L)+\tfrac{1}{2\lambda}\left(1-\beta_2^2\right)(1-\zeta)\,(X_2-\ D_2)^2\right\}^2\\ &-\ \tfrac{1}{2\lambda}\left\{\beta_1\,(X_1-D_1)+\zeta\eta L+\tfrac{1}{2\lambda}\beta_2\,(2-\beta_2)\,(1-\zeta)\,(X_2-\ D_2)^2\right\}^2+C-K\end{aligned}$$

with the following conditions

$$(1),\ D_1\geq C,\ D_2\geq\gamma C \text{ and } 0\leq\beta_t\leq 1,\quad t=1,2.$$

The Lagrangian is given by

$$\begin{aligned}\mathcal{L}=&\tfrac{1}{\lambda}\left\{X_1-D_1+\zeta\,[R-(1-\eta)\,L]+\tfrac{1}{2\lambda}\,(1-\zeta)\left(1+2\beta_2-2\beta_2^2\right)(X_2-\ D_2)^2\right\}\\ &\left\{X_1-[1-(1-\zeta)\,\gamma]\,C+\zeta R+\tfrac{1}{2\lambda}\,(1-\zeta)\left[\ 2(X_2-\ D_2)\,(X_2-\gamma C)-\left(1-2\beta_2+2\beta_2^2\right)(X_2-\ D_2)^2\right]\right\}\\ &-\ \tfrac{1}{2\lambda}\left\{(1-\beta_1)\,(X_1-D_1)+\zeta\,(R-L)+\tfrac{1}{2\lambda}\left(1-\beta_2^2\right)(1-\zeta)\,(X_2-\ D_2)^2\right\}^2\\ &-\tfrac{1}{2\lambda}\left\{\beta_1\,(X_1-D_1)+\zeta\eta L+\tfrac{1}{2\lambda}\beta_2\,(2-\beta_2)\,(1-\zeta)\,(X_2-\ D_2)^2\right\}^2+C-K+\mu_1\beta_1+\mu_2\beta_2\\ &+\mu_3(1-\beta_1)+\mu_4(1-\beta_2)+\mu_5 D_1+\mu_6 D_2+\mu_7\,(X_1-D_1)+\mu_8\,(X_2-D_2)+\mu_9\,(D_1-C)+\mu_{10}\,(D_2-\gamma C)\\ &+\mu_{11}\eta+\mu_{12}\,(1-\eta)\end{aligned}$$

The first order conditions are written

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\eta}= \quad & \tfrac{1}{\lambda}\zeta L\left\{X_1-\left[1-(1-\zeta)\gamma\right]C+\zeta\left(R-\eta L\right)-\beta_1\left(X_1-D_1\right)\right.\\ & \left.+\tfrac{1}{\lambda}\left(1-\zeta\right)\ \left(X_2-\ D_2\right)\left(X_2-\gamma C\right)-\tfrac{1}{2\lambda}\left(1-\zeta\right)\left(1+\beta_2^2\right)\left(X_2-\ D_2\right)^2\right\}\\ & +\mu_{11}-\mu_{12}\leq 0\end{aligned}\tag{135}$$

$$\eta\geq 0\tag{136}$$

$$\eta\frac{\partial\mathcal{L}}{\partial\eta}=0\tag{137}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\beta_1}= \quad & \tfrac{1}{\lambda}\left(X_1-D_1\right)\left\{\left(1-2\beta_1\right)\left(X_1-D_1\right)+\zeta\left[R-\left(1+\eta\right)L\right]\right.\\ & \left.+\tfrac{1}{2\lambda}\left(1-2\beta_2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\right\}+\mu_1-\mu_3\leq 0\end{aligned}\tag{138}$$

$$\beta_1\geq 0\tag{139}$$

$$\beta_1\frac{\partial\mathcal{L}}{\partial\beta_1}=0\tag{140}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\beta_2}= \quad & \tfrac{1}{\lambda^2}\left(1-\zeta\right)\left(1-2\beta_2\right)\left(X_2-\ D_2\right)^2\left\{X_1-\left[1-(1-\zeta)\gamma\right]C+\zeta R\right.\\ & \left.+\tfrac{1}{2\lambda}\left(1-\zeta\right)\left[\ 2(X_2-\ D_2)\left(X_2-\gamma C\right)-\left(1-2\beta_2+2\beta_2^2\right)\left(X_2-\ D_2\right)^2\right]\right\}\\ & +\tfrac{1}{\lambda^2}\left(1-\zeta\right)\ \left(1-2\beta_2\right)\left(X_2-\ D_2\right)^2\left\{X_1-D_1+\zeta\left[R-\left(1-\eta\right)L\right]\right.\\ & \left.+\tfrac{1}{2\lambda}\left(1-\zeta\right)\left(1+2\beta_2-2\beta_2^2\right)\left(X_2-\ D_2\right)^2\right\}-\ \tfrac{1}{\lambda^2}\left(1-\beta_2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\\ & \left\{\beta_1\left(X_1-D_1\right)+\tfrac{1}{2\lambda}\beta_2\left(2-\beta_2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)^2+\zeta\eta L\right\}\\ & -\ \tfrac{1}{\lambda^2}\beta_2\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\left\{\left(1-\beta_1\right)\left(X_1-D_1\right)+\zeta\left(R-L\right)\right.\\ & \left.+\tfrac{1}{2\lambda}\left(1-\beta_2^2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\right\}+\mu_2-\mu_4\leq 0\end{aligned}\tag{141}$$

$$\beta_2\geq 0\tag{142}$$

$$\beta_2\frac{\partial\mathcal{L}}{\partial\beta_2}=0\tag{143}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial D_1}= \quad & -\tfrac{1}{\lambda}\left\{X_1-\left[1-(1-\zeta)\gamma\right]C+\tfrac{1}{\lambda}\left(1-\zeta\right)\left(X_2-\ D_2\right)\left(X_2-\gamma C\right)\right.\\ & \left.-\left(1-\beta_1\right)\left(X_1-D_1\right)+\zeta L-\tfrac{1}{2\lambda}\left(1-\zeta\right)\left(2-2\beta_2+\beta_2^2\right)\left(X_2-\ D_2\right)^2\right\}\\ & -\ \tfrac{1}{\lambda}\beta_1\left\{\left(1-2\beta_1\right)\left(X_1-D_1\right)+\zeta\left(R-\left(1+\eta\right)L\right)\right.\\ & \left.+\tfrac{1}{2\lambda}\left(1-2\beta_2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\right\}\\ & +\mu_5-\mu_7+\mu_9\leq 0\end{aligned}\tag{144}$$

$$D_1\geq 0\tag{145}$$

$$D_1\frac{\partial\mathcal{L}}{\partial D_1}=0\tag{146}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial D_2}= \quad & -\tfrac{1}{\lambda^2}\left(1+2\beta_2-2\beta_2^2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)\left\{X_1-\left[1-(1-\zeta)\gamma\right]C+\zeta R\right.\\ & \left.+\tfrac{1}{2\lambda}\left(1-\zeta\right)\left[2(X_2-\ D_2)\left(X_2-\gamma C\right)-\left(1-2\beta_2+2\beta_2^2\right)\left(X_2-\ D_2\right)^2\right]\right\}\\ & +\tfrac{1}{\lambda^2}\left(1-\zeta\right)\left[-\left(X_2-\gamma C\right)+\left(1-2\beta_2+2\beta_2^2\right)\left(X_2-\ D_2\right)\right]\left\{X_1-D_1\right.\\ & \left.+\zeta\left[R-\left(1-\eta\right)L\right]+\tfrac{1}{2\lambda}\left(1+2\beta_2-2\beta_2^2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\right\}\\ & +\tfrac{1}{\lambda^2}\left(1-\beta_2^2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)\left\{\left(1-\beta_1\right)\left(X_1-D_1\right)\right.\\ & \left.+\zeta\left(R-L\right)+\tfrac{1}{2\lambda}\left(1-\beta_2^2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\right\}\\ & +\tfrac{1}{\lambda^2}\beta_2\left(2-\beta_2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)\left\{\beta_1\left(X_1-D_1\right)+\zeta\eta L\right.\\ & \left.+\tfrac{1}{2\lambda}\beta_2\left(2-\beta_2\right)\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\right\}+\mu_6-\mu_8+\mu_{10}\leq 0\end{aligned}\tag{147}$$

$$D_2 \geq 0 \tag{148}$$

$$D_2 \frac{\partial \mathcal{L}}{\partial D_2} = 0 \tag{149}$$

The complementary slackness conditions are written:

$$\frac{\partial \mathcal{L}}{\partial \mu_1} = \beta_1 \geq 0 \ , \quad \mu_1 \geq 0 \ , \quad \mu_1 \beta_1 = 0 \tag{150}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_2} = \beta_2 \geq 0 \ , \quad \mu_2 \geq 0 \ , \quad \mu_2 \beta_2 = 0 \tag{151}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_3} = 1 - \beta_1 \geq 0 \ , \quad \mu_3 \geq 0 \ , \quad \mu_3 (1 - \beta_1) = 0 \tag{152}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_4} = 1 - \beta_2 \geq 0 \ , \quad \mu_4 \geq 0 \ , \quad \mu_4 (1 - \beta_2) = 0 \tag{153}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_5} = D_1 \geq 0 \ , \quad \mu_5 \geq 0 \ , \quad \mu_5 D_1 = 0 \tag{154}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_6} = D_2 \geq 0 \ , \quad \mu_6 \geq 0 \ , \quad \mu_6 D_2 = 0 \tag{155}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_7} = X_1 - D_1 \geq 0 \ , \quad \mu_7 \geq 0 \ , \quad \mu_7 (X_1 - D_1) = 0 \tag{156}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_8} = X_2 - D_2 \geq 0 \ , \quad \mu_8 \geq 0 \ , \quad \mu_8 (X_2 - D_2) = 0 \tag{157}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_9} = D_1 - C \geq 0 \ , \quad \mu_9 \geq 0 \ , \quad \mu_9 (D_1 - C) = 0 \tag{158}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_8} = D_2 - \gamma C \geq 0 \ , \quad \mu_{10} \geq 0 \ , \quad \mu_{10} (D_2 - \gamma C) = 0 \tag{159}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_9} = \eta \geq 0 \ , \quad \mu_{11} \geq 0 \ , \quad \mu_{11} \eta = 0 \tag{160}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_8} = 1 - \eta \geq 0 \ , \quad \mu_{12} \geq 0 \ , \quad \mu_{12} (1 - \eta) = 0 \tag{161}$$

We discuss the most important cases as in the appendix B but we will present only the case solving the entrepreneur's program.

Consider that $0 < \eta < 1$, $0 < D_t < X_t$ and $0 < \beta_t < 1$, $t = 1, 2$, according to (150), (151), (152), (153), (154), (155), (156), (157), (160) and (161), all the Kuhn-Tucker multipliers are null except μ_9 and μ_{10}. Substituting all these multipliers into the first order condition (138) gives the following equation:

$$(1 - 2\beta_1)(X_1 - D_1) + \zeta [R - (1 + \eta) L] + \frac{1}{2\lambda} (1 - 2\beta_2)(1 - \zeta)(X_2 - D_2)^2 = 0 \tag{162}$$

Given $0 < D_t < X_t$, this equation is solved only if

$$\eta = \frac{R}{L} - 1 \text{ and } \beta_t = \frac{1}{2}, \, t = 1, 2 \tag{163}$$

Note that we assumed $L \leq R \leq 2L$ to ensure that $0 < \eta < 1$ which means that the competitive buyout is not be very profitable. In this case, substituting (163) into the equations (144) and (147) gives:

$$\begin{aligned} \mu_9 = \quad & \tfrac{1}{2\lambda} \left\{ X_1 - 2 [1 - (1 - \zeta) \gamma] C - \tfrac{5}{4\lambda} (1 - \zeta)(X_2 - D_2)^2 \right. \\ & \left. + D_1 + 2\zeta L + \tfrac{2}{\lambda} (1 - \zeta)(X_2 - D_2)(X_2 - \gamma C) \right\} \end{aligned} \tag{164}$$

$$\begin{aligned} \mu_{10} = \quad & \tfrac{3}{4\lambda^2} (1 - \zeta)(X_2 - D_2) \{ X_1 - 2 [1 - (1 - \zeta) \gamma] C \\ & + D_1 + 2\zeta L - \tfrac{5}{4\lambda} (1 - \zeta)(X_2 - D_2)^2 \\ & + \tfrac{1}{\lambda} (1 - \zeta)(X_2 - D_2)(X_2 - \gamma C) \} \\ & + \tfrac{1}{2\lambda^2} (1 - \zeta) [2 (X_2 - \gamma C) - (X_2 - D_2)] \{ X_1 - D_1 \\ & + 2\zeta (R - L) + \tfrac{3}{4\lambda} (1 - \zeta)(X_2 - D_2)^2 \} \end{aligned} \tag{165}$$

- If $\mu_9 = 0$, the success payment of the bank can be written

$$\begin{aligned} D_1 = \; & 2\left[1-(1-\zeta)\gamma\right]C - X_1 - 2\zeta L - \tfrac{2}{\lambda}(1-\zeta)\;(X_2 - \;D_2)(X_2-\gamma C) \\ & + \tfrac{5}{4\lambda}(1-\zeta)(X_2 - \;D_2)^2 < C \end{aligned} \tag{166}$$

which leads to a contradiction: $\frac{\partial \mathcal{L}}{\partial \mu_9} < 0$.

- Consider now $\mu_9 > 0$, the condition (158) is satisfied only if $D_1 = C$. Substituting $D_1 = C$ in the equation (165)

$$\begin{aligned} \mu_{10} = \; & \tfrac{3}{4\lambda^2}(1-\zeta)(X_2 - \;D_2)\left\{X_1 - \left[1-2(1-\zeta)\gamma\right]C + 2\zeta L \right. \\ & \left. - \tfrac{5}{4\lambda}(1-\zeta)(X_2 - \;D_2)^2 + \tfrac{1}{\lambda}(1-\zeta)(X_2 - \;D_2)(X_2-\gamma C)\right\} \\ & + \tfrac{1}{2\lambda^2}(1-\zeta)\left[2(X_2-\gamma C) - (X_2 - \;D_2)\right]\left\{X_1 - C \right. \\ & \left. + 2\zeta(R-L) + \tfrac{3}{4\lambda}(1-\zeta)(X_2 - \;D_2)^2\right\} \end{aligned}$$

We show that $\mu_{10} > 0$. Then, (159) implies that $D_2 = \gamma C$.

To get the other results of the proposition 4, we substitute (163), $D_1 = C$ and $D_2 = \gamma C$ into the reaction functions of the entrepreneur and the LBO fund and in the participation constraints of the LBO fund and the bank.

D.2 The competitive project is very profitable: $R > 2L$

A straight application of the Kuhn-Tucker theorem gives the following results:

$$\eta = 1 \text{ and } \beta_t = \frac{1}{2},\; t = 1,\, 2 \tag{167}$$

$$D_1 = C \text{ and } D_2 = \gamma C \tag{168}$$

To get the second part of the proposition 4, we substitute (167) and (168) in the reaction functions of the entrepreneur and the LBO fund. Then, we substitute all in the participation constraints (CP_A) and (CP_B) to deduce the amounts of issued debt and equity.

E The proof of the proposition 5

As in the appendix D, after substituting the efforts into the objective function, we solve:

$$\begin{aligned} & \max_{\eta,\; L,\; \beta_t,\; D_t,\; t=1,\; 2} E(\pi_E) = \\ & \tfrac{1}{\lambda}\left\{X_1 - D_1 + \zeta\left[R-(1-\eta)L\right] + \tfrac{1}{2\lambda}(1-\zeta)\left(1+2\beta_2-2\beta_2^2\right)(X_2 - \;D_2)^2\right\} \\ & \left\{X_1 - \left[1-(1-\zeta)\gamma\right]C + \zeta R + \tfrac{1}{2\lambda}(1-\zeta)\left[\;2(X_2 - \;D_2)(X_2-\gamma C) - \left(1-2\beta_2+2\beta_2^2\right)(X_2 - \;D_2)^2\right]\right\} \\ & - \;\tfrac{1}{2\lambda}\left\{(1-\beta_1)(X_1-D_1) + \zeta(R-L) + \tfrac{1}{2\lambda}\left(1-\beta_2^2\right)(1-\zeta)(X_2 - \;D_2)^2\right\}^2 \\ & - \tfrac{1}{2\lambda}\left\{\beta_1(X_1-D_1) + \zeta\eta L + \tfrac{1}{2\lambda}\beta_2(2-\beta_2)(1-\zeta)(X_2 - \;D_2)^2\right\}^2 + C - K \end{aligned}$$

with the following conditions

$$(1),\; D_1 \geq C,\; D_2 \geq \gamma C,\; 0 \leq L \leq R \text{ and } 0 \leq \beta_t \leq 1, \quad t = 1, 2.$$

The Lagrangian is given by

$$\mathcal{L} = \tfrac{1}{\lambda}\left\{X_1 - D_1 + \zeta\left[R - (1-\eta)L\right] + \tfrac{1}{2\lambda}(1-\zeta)\left(1 + 2\beta_2 - 2\beta_2^2\right)(X_2 - D_2)^2\right\}$$
$$\left\{X_1 - \left[1 - (1-\zeta)\gamma\right]C + \zeta R + \tfrac{1}{2\lambda}(1-\zeta)\left[\,2(X_2 - D_2)(X_2 - \gamma C) - \left(1 - 2\beta_2 + 2\beta_2^2\right)(X_2 - D_2)^2\right]\right\}$$
$$-\ \tfrac{1}{2\lambda}\left\{(1-\beta_1)(X_1 - D_1) + \zeta(R - L) + \tfrac{1}{2\lambda}\left(1 - \beta_2^2\right)(1-\zeta)(X_2 - D_2)^2\right\}^2$$
$$-\ \tfrac{1}{2\lambda}\left\{\beta_1(X_1 - D_1) + \zeta\eta L + \tfrac{1}{2\lambda}\beta_2(2-\beta_2)(1-\zeta)(X_2 - D_2)^2\right\}^2 + C - K + \mu_1\beta_1 + \mu_2\beta_2$$
$$+\mu_3(1-\beta_1) + \mu_4(1-\beta_2) + \mu_5 D_1 + \mu_6 D_2 + \mu_7(X_1 - D_1) + \mu_8(X_2 - D_2) + \mu_9(D_1 - C) + \mu_{10}(D_2 - \gamma C)$$
$$+\mu_{11}\eta + \mu_{12}(1-\eta) + \mu_{13}L + \mu_{14}(R - L)$$

The first order conditions are written

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial L} = \ & -\tfrac{1}{\lambda}\zeta(1-\eta)\left\{X_1 - \left[1-(1-\zeta)\gamma\right]C + \zeta R\right.\\ & \left.+\tfrac{1}{\lambda}(1-\zeta)(X_2 - D_2)(X_2 - \gamma C) - \tfrac{1}{2\lambda}(1-\zeta)\left(1 - 2\beta_2 + 2\beta_2^2\right)(X_2 - D_2)^2\right\}\\ & +\ \tfrac{1}{\lambda}\zeta\left\{(1-\beta_1)(X_1 - D_1) + \zeta(R-L) + \tfrac{1}{2\lambda}\left(1-\beta_2^2\right)(1-\zeta)(X_2 - D_2)^2\right\}\\ & -\ \tfrac{1}{\lambda}\zeta\eta\left\{\beta_1(X_1 - D_1) + \zeta\eta L + \tfrac{1}{2\lambda}\beta_2(2-\beta_2)(1-\zeta)(X_2 - D_2)^2\right\}\\ & +\mu_{13} - \mu_{14} \le 0\end{aligned} \tag{169}$$

$$L \ge 0 \tag{170}$$

$$L\frac{\partial\mathcal{L}}{\partial L} = 0 \tag{171}$$

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial\eta} = \ & \tfrac{1}{\lambda}\zeta L\left\{X_1 - \left[1-(1-\zeta)\gamma\right]C + \zeta(R - \eta L) - \beta_1(X_1 - D_1)\right.\\ & \left.+\tfrac{1}{\lambda}(1-\zeta)\ (X_2 - D_2)(X_2 - \gamma C) - \tfrac{1}{2\lambda}(1-\zeta)\left(1+\beta_2^2\right)(X_2 - D_2)^2\right\}\\ & +\mu_{11} - \mu_{12} \le 0\end{aligned} \tag{172}$$

$$\eta \ge 0 \tag{173}$$

$$\eta\frac{\partial\mathcal{L}}{\partial\eta} = 0 \tag{174}$$

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial\beta_1} = \ & \tfrac{1}{\lambda}(X_1 - D_1)\left\{(1-2\beta_1)(X_1 - D_1) + \zeta\left[R - (1+\eta)L\right]\right.\\ & \left.+\tfrac{1}{2\lambda}(1-2\beta_2)(1-\zeta)(X_2 - D_2)^2\right\} + \mu_1 - \mu_3 \le 0\end{aligned} \tag{175}$$

$$\beta_1 \ge 0 \tag{176}$$

$$\beta_1\frac{\partial\mathcal{L}}{\partial\beta_1} = 0 \tag{177}$$

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial\beta_2} = \ & \tfrac{1}{\lambda^2}(1-\zeta)(1-2\beta_2)(X_2 - D_2)^2\left\{X_1 - \left[1-(1-\zeta)\gamma\right]C + \zeta R\right.\\ & \left.+\tfrac{1}{2\lambda}(1-\zeta)\left[\,2(X_2 - D_2)(X_2 - \gamma C) - \left(1 - 2\beta_2 + 2\beta_2^2\right)(X_2 - D_2)^2\right]\right\}\\ & +\tfrac{1}{\lambda^2}(1-\zeta)\ (1-2\beta_2)(X_2 - D_2)^2\left\{X_1 - D_1 + \zeta\left[R - (1-\eta)L\right]\right.\\ & \left.+\tfrac{1}{2\lambda}(1-\zeta)\left(1 + 2\beta_2 - 2\beta_2^2\right)(X_2 - D_2)^2\right\} -\ \tfrac{1}{\lambda^2}(1-\beta_2)(1-\zeta)(X_2 - D_2)^2\\ & \left\{\beta_1(X_1 - D_1) + \tfrac{1}{2\lambda}\beta_2(2-\beta_2)(1-\zeta)(X_2 - D_2)^2 + \zeta\eta L\right\}\\ & -\ \tfrac{1}{\lambda^2}\beta_2(1-\zeta)(X_2 - D_2)^2\left\{(1-\beta_1)(X_1 - D_1) + \zeta(R - L)\right.\\ & \left.+\tfrac{1}{2\lambda}\left(1-\beta_2^2\right)(1-\zeta)(X_2 - D_2)^2\right\} + \mu_2 - \mu_4 \le 0\end{aligned} \tag{178}$$

$$\beta_2 \ge 0 \tag{179}$$

$$\beta_2\frac{\partial\mathcal{L}}{\partial\beta_2} = 0 \tag{180}$$

$$
\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial D_1} = \quad & -\tfrac{1}{\lambda}\left\{X_1 - \left[1-(1-\zeta)\gamma\right]C + \tfrac{1}{\lambda}(1-\zeta)(X_2 - D_2)(X_2-\gamma C)\right. \\
& -(1-\beta_1)(X_1-D_1) + \zeta L - \tfrac{1}{2\lambda}(1-\zeta)\left(2-2\beta_2+\beta_2^2\right)(X_2 - D_2)^2\Big\} \\
& -\tfrac{1}{\lambda}\beta_1\left\{(1-2\beta_1)(X_1-D_1)+\zeta\left(R-(1+\eta)L\right)\right. \\
& \left.+\tfrac{1}{2\lambda}(1-2\beta_2)(1-\zeta)(X_2 - D_2)^2\right\} + \mu_5 - \mu_7 + \mu_9 \leq 0
\end{aligned} \tag{181}
$$

$$D_1 \geq 0 \tag{182}$$

$$D_1 \frac{\partial \mathcal{L}}{\partial D_1} = 0 \tag{183}$$

$$
\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial D_2} = \quad & -\tfrac{1}{\lambda^2}\left(1+2\beta_2-2\beta_2^2\right)(1-\zeta)(X_2 - D_2)\left\{X_1 - \left[1-(1-\zeta)\gamma\right]C + \zeta R\right. \\
& \left.+\tfrac{1}{2\lambda}(1-\zeta)\left[2(X_2 - D_2)(X_2-\gamma C) - \left(1-2\beta_2+2\beta_2^2\right)(X_2 - D_2)^2\right]\right\} \\
& +\tfrac{1}{\lambda^2}(1-\zeta)\left[-(X_2-\gamma C) + \left(1-2\beta_2+2\beta_2^2\right)(X_2 - D_2)\right]\left\{X_1 - D_1\right. \\
& \left.+\zeta\left[R-(1-\eta)L\right] + \tfrac{1}{2\lambda}\left(1+2\beta_2-2\beta_2^2\right)(1-\zeta)(X_2 - D_2)^2\right\} \\
& +\tfrac{1}{\lambda^2}\left(1-\beta_2^2\right)(1-\zeta)(X_2 - D_2)\left\{(1-\beta_1)(X_1-D_1)\right. \\
& \left.+\zeta(R-L) + \tfrac{1}{2\lambda}\left(1-\beta_2^2\right)(1-\zeta)(X_2 - D_2)^2\right\} \\
& +\tfrac{1}{\lambda^2}\beta_2(2-\beta_2)(1-\zeta)(X_2 - D_2)\left\{\beta_1(X_1-D_1)+\zeta\eta L\right. \\
& \left.+\tfrac{1}{2\lambda}\beta_2(2-\beta_2)(1-\zeta)(X_2 - D_2)^2\right\} + \mu_6 - \mu_8 + \mu_{10} \leq 0
\end{aligned} \tag{184}
$$

$$D_2 \geq 0 \tag{185}$$

$$D_2 \frac{\partial \mathcal{L}}{\partial D_2} = 0 \tag{186}$$

The complementary slackness conditions are written:

$$\tfrac{\partial \mathcal{L}}{\partial \mu_1} = \beta_1 \geq 0 \ , \quad \mu_1 \geq 0 \ , \quad \mu_1\beta_1 = 0 \tag{187}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_2} = \beta_2 \geq 0 \ , \quad \mu_2 \geq 0 \ , \quad \mu_2\beta_2 = 0 \tag{188}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_3} = 1-\beta_1 \geq 0 \ , \quad \mu_3 \geq 0 \ , \quad \mu_3(1-\beta_1) = 0 \tag{189}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_4} = 1-\beta_2 \geq 0 \ , \quad \mu_4 \geq 0 \ , \quad \mu_4(1-\beta_2) = 0 \tag{190}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_5} = D_1 \geq 0 \ , \quad \mu_5 \geq 0 \ , \quad \mu_5 D_1 = 0 \tag{191}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_6} = D_2 \geq 0 \ , \quad \mu_6 \geq 0 \ , \quad \mu_6 D_2 = 0 \tag{192}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_7} = X_1 - D_1 \geq 0 \ , \quad \mu_7 \geq 0 \ , \quad \mu_7(X_1-D_1) = 0 \tag{193}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_8} = X_2 - D_2 \geq 0 \ , \quad \mu_8 \geq 0 \ , \quad \mu_8(X_2-D_2) = 0 \tag{194}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_9} = D_1 - C \geq 0 \ , \quad \mu_9 \geq 0 \ , \quad \mu_9(D_1-C) = 0 \tag{195}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_{10}} = D_2 - \gamma C \geq 0 \ , \quad \mu_{10} \geq 0 \ , \quad \mu_{10}(D_2-\gamma C) = 0 \tag{196}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_{11}} = \eta \geq 0 \ , \quad \mu_{11} \geq 0 \ , \quad \mu_{11}\eta = 0 \tag{197}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_{12}} = 1-\eta \geq 0 \ , \quad \mu_{12} \geq 0 \ , \quad \mu_{12}(1-\eta) = 0 \tag{198}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_{13}} = L \geq 0 \ , \quad \mu_{13} \geq 0 \ , \quad \mu_{13}L = 0 \tag{199}$$

$$\tfrac{\partial \mathcal{L}}{\partial \mu_{14}} = R-L \geq 0 \ , \quad \mu_{14} \geq 0 \ , \quad \mu_{14}(R-L) = 0 \tag{200}$$

Hereafter, we focus only on the most interesting cases.

Consider that $0 < L < R$, $0 < D_t < X_t$ and $0 < \beta_t < 1$, $t = 1,\, 2$. According to (187), (188), (189), (190), (191), (192), (193), (194), (197), (198), (199)and (200), all the Kuhn-Tucker multipliers are null except μ_9, μ_{10}, μ_{11} and μ_{12}. Substituting all these multipliers into the first order condition (175) gives the following equation:

$$(1-2\beta_1)(X_1-D_1)+\zeta[R-(1+\eta)L]+\frac{1}{2\lambda}(1-2\beta_2)(1+\zeta)(1-\zeta)^2(X_2-D_2)^2=0 \tag{201}$$

Given $0<D_t<X_t$, this equation is solved only if

$$\eta=\frac{R}{L}-1 \text{ and } \beta_t=\frac{1}{2},\ t=1,2 \tag{202}$$

We assume $L\leq R\leq 2L$ to ensure that $0\leq\eta\leq 1$.

■ If $\mathbf{0<\boldsymbol{\eta}<1}$, according to (199) and (200), $\mu_{13}=\mu_{14}=0$. As shown in the previous appendix, the equations (181) and (184) imply that:

$$\begin{aligned}\mu_9= & \tfrac{1}{2\lambda}\left\{X_1-2[1-(1-\zeta)\gamma]C-\tfrac{5}{4\lambda}(1-\zeta)(X_2-D_2)^2\right.\\ & \left.+D_1+2\zeta L+\tfrac{2}{\lambda}(1-\zeta)(X_2-D_2)(X_2-\gamma C)\right\}\\ \mu_{10}= & \tfrac{3}{4\lambda^2}(1-\zeta)(X_2-D_2)\left\{X_1-2[1-(1-\zeta)\gamma]C\right.\\ & +D_1+2\zeta L-\tfrac{5}{4\lambda}(1-\zeta)(X_2-D_2)^2\\ & \left.+\tfrac{1}{\lambda}(1-\zeta)(X_2-D_2)(X_2-\gamma C)\right\}\\ & +\tfrac{1}{2\lambda^2}(1-\zeta)[2(X_2-\gamma C)-(X_2-D_2)]\left\{X_1-D_1\right.\\ & \left.+2\zeta(R-L)+\tfrac{3}{4\lambda}(1-\zeta)(X_2-D_2)^2\right\}\end{aligned}$$

The discussion of the signs of μ_9 and μ_{10} leads to the following conclusions: both multipliers are strictly positive and the debt's payments are non-decreasing with the revenues of the project such that $D_1=C$ and $D_2=\gamma C$ (see appendix D for more details). But, substituting $D_1=C$, $D_2=\gamma C$ and $\mu_{13}=\mu_{14}=0$ in the first order equation (169) gives

$$-\frac{1}{2}(2L-R)\left\{X_1-2[1-(1-\zeta)\gamma]C+D_1+2\zeta L+\frac{3}{4\lambda}(1-\zeta)(X_2-\gamma C)^2\right\}=0 \tag{203}$$

The latter equation is satisfied only if:

1. the first term of the equation (203) is null. It implies that $L=\frac{1}{2}R$ which leads to a contradiction: given $0<\eta<1$, substituting $L=\frac{R}{2}$ into (202) gives $\eta=1$.
2. the second term of this equation is null, the compensation cost is strictly negative:

$$L=\frac{1}{2\zeta}\left\{[1-2(1-\zeta)\gamma]C-X_1-\frac{3}{4\lambda}(1-\zeta)(X_2-\gamma C)^2\right\}<0$$

■ If $\boldsymbol{\eta}=\mathbf{1}$, given $0<D_1<X_1$, the equation (175) is written

$$(1-2\beta_1)(X_1-D_1)+\zeta[R-2L]+\frac{1}{2\lambda}(1-2\beta_2)(1-\zeta)(X_2-D_2)^2=0 \tag{204}$$

which gives

$$L=\frac{1}{2}R \text{ and } \beta_t=\frac{1}{2},\quad t=1,2 \tag{205}$$

As a result, the first order conditions (181) and (184) give

$$\begin{aligned}\mu_9= & \tfrac{1}{\lambda}\left\{X_1-[1-(1-\zeta)\gamma]C+\tfrac{1}{\lambda}(1-\zeta)(X_2-D_2)(X_2-\gamma C)\right.\\ & \left.-\tfrac{1}{2}(X_1-D_1)+\zeta L-\tfrac{5}{8\lambda}(1-\zeta)(X_2-D_2)^2\right\}\end{aligned} \tag{206}$$

$$\begin{array}{rl}\mu_{10}= & \frac{3}{2\lambda^2}\left(1-\zeta\right)\left(X_2-\ D_2\right)\{X_1-\left[1-\left(1-\zeta\right)\gamma\right]C\\ & -\frac{1}{2}\left(X_1-D_1\right)+\frac{1}{2}\zeta R-\frac{5}{8\lambda}\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\\ & +\frac{1}{\lambda}\left(1-\zeta\right)\left(X_2-\ D_2\right)\left(X_2-\gamma C\right)\}\\ & +\frac{1}{2\lambda^2}\left(1-\zeta\right)\left[2\left(X_2-\gamma C\right)-\left(X_2-\ D_2\right)\right]\{X_1-D_1\\ & +\zeta R+\frac{3}{4\lambda}\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\}\end{array} \tag{207}$$

If $\mu_9 = 0$, (206) enables us to deduce

$$\begin{array}{rl}D_1= & 2\left[1-\left(1-\zeta\right)\gamma\right]C-X_1-\zeta R-\frac{2}{\lambda}\left(1-\zeta\right)\ \left(X_2-\ D_2\right)\left(X_2-\gamma C\right)\\ & +\frac{5}{4\lambda}\left(1-\zeta\right)\left(X_2-\ D_2\right)^2<C\end{array}$$

but if $D_1 < C$, $\frac{\partial \mathcal{L}}{\partial \mu_9} < 0$.

If $\mu_9 > 0$, according to (195), $D_1 = C$. Substituting $D_1 = C$ in (207) gives:

$$\begin{array}{rl}\mu_{10}= & \frac{3}{4\lambda^2}\left(1-\zeta\right)\left(X_2-\ D_2\right)\{X_1-\left[1-2\left(1-\zeta\right)\gamma\right]C\\ & +\zeta R-\frac{5}{4\lambda}\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\\ & +\frac{2}{\lambda}\left(1-\zeta\right)\left(X_2-\ D_2\right)\left(X_2-\gamma C\right)\}\\ & +\frac{1}{2\lambda^2}\left(1-\zeta\right)\left[2\left(X_2-\gamma C\right)-\left(X_2-\ D_2\right)\right]\{X_1-C\\ & +\zeta R+\frac{3}{4\lambda}\left(1-\zeta\right)\left(X_2-\ D_2\right)^2\}\end{array}$$

Given $\gamma C \leq X_2$ and $D_2 \leq X_2$, it is easy to check that $\mu_{10} > 0$. Then, given (196), $D_2 = \gamma C$ which completes the proof of the proposition 4.

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