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Fraud, Investments and Liability Regimes in Payment Platforms

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Abstract

In this paper, we discuss how fraud liability regimes impact the price structure that is chosen by a monopolistic payment platform, in a setting where merchants can invest in fraud detection technologies. We show that liability allocation rules distort the price structure charged by platforms or banks to consumers and merchants with respect to a case where such a responsibility regime is not implemented. We determine the allocation of fraud losses between the payment platform and the merchants that maximises the platform's profit and we compare it to the allocation that maximises social welfare.

JEL Codes: G21, L31, L42.

Keywords: Payment card systems, interchange fees, two-sided markets, fraud, liability.

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1 Introduction:

The development of electronic data exchange in the banking industry has generated an increase in fraud and cybercrime. For instance, in the United-States, according to the Consumer Sentinel Network (CSN), 1.2 million complaints of consumer fraud have been recorded in 2008.[1] As a consequence, banks can make substantial losses because of fraudulent use of payment cards, which differ across countries and payment systems (See table 1).

Table 1: Loss rate per $100 payment card transaction value in several countries[2]

Country	Spain	Australia	France	UK	US
Losses rate	2.24¢	2.39¢	5¢	9.12¢	9.2¢

Minimizing the occurrence of fraud in electronic payment systems requires costly efforts from all the participants to a transaction: platforms, banks, consumers and merchants.[3] For instance, consumers have to protect their personal data and to report the fraud rapidly once it occurs, whereas platforms, banks and merchants may invest substantial amounts in fraud detection technologies.[4] These efforts in fraud prevention depend on the expected amount of losses and their allocation, which responds to several liability rules, determined either by public laws or by private network rules.

This paper adresses two major issues related to fraud in payment systems: What is the incidence of fraud liability regimes on the price structure that is charged by payment platforms? How do private liability regimes differ from the socially optimal regime that would be implemented by a social planner? In particular, we analyse whether private network rules provide merchants with sufficient incentives to invest in fraud detection technologies and whether these rules generate the socially optimal allocation of fraud losses.

Currently, in most payment card systems, consumers hardly bear meaningful liability for fraudulent use of their payment card, because they are protected both by financial regulations,

[1]Source: Consumer Sentinel Network Data Book for January-December 2008, Federal Trade Commission, February 2009. This report highlights that credit card fraud is the most common form of reported identity theft amounting at 20% of the reported fraudulent transactions.

[2]Source: Richard Sullivan (2010), Federal Reserve Bank of Kansas City, 'The Changing Nature of Payment Card Fraud: Issues for Industry and Public Policy'.

[3]According to the Federal Reserve Board, in the United-States, "On average, by transaction type, issuers incurred 2.2¢ per signature-debit transaction for fraud-prevention and data-security activities and 1.2¢ per PIN-debit transaction. Similarly, networks incurred 0.7¢ per signature-debit transaction for fraud-prevention and data-security activities and 0.6¢ per PIN-debit transaction. Finally, acquirers incurred 0.4¢ per signature-debit transaction for fraud-prevention and data-security activities and 0.3¢ per PIN-debit transaction.". Source: Federal Register / Vol. 75, No. 248 / Tuesday, December 28, 2010 / Proposed Rules.

[4]According to a survey conducted by the Federal Reserve Board in the United-States, issuers engage in various fraud-prevention activities such as "transaction monitoring and fraud risk scoring systems that may trigger an alert or call to the cardholder in order to confirm the legitimacy of a transaction". "Merchants also have fraud-prevention data-security costs, including costs related to compliance with payment card industry data-security standards (PCI-DSS) and other tools to prevent fraud, such as address verification services or internally developped fraud screening models, particularly for card-not-present transactions".

which are public laws (e.g. TILA and regulation Z in the United-States)[5], and by the 'zero liability rule', which has been privately adopted by several payment networks. It follows that, in most payment systems, the burden of fraud losses is shared between banks or platforms and merchants.[6] The allocation of liability between banks and merchants generally depends on private rules that are chosen by payment platforms. Some networks may even use liability rules to provide merchants with incentives to adopt new technologies. For instance, MasterCard and Visa used liability shift measures to induce merchants to adopt fraud prevention technologies on the internet (MasterCard SecureCode^{TM} and Visa 3-D Secure^{TM} respectively).[7] Interestingly, if the merchant implements the 3-D Secure^{TM} technology, the issuer becomes liable for fraud losses for all eCommerce transactions that went through the 3-D Secure^{TM} process. Understanding the impact of fraud losses on payment systems has become a major challenge of the banking industry.

To adress this issue, we consider a monopolistic proprietary payment platform that provides an electronic payment instrument to risk neutral consumers and merchants. Consumers and merchants decide whether or not to adopt the electronic payment instrument based on the price of the payment instrument and on the expected loss that they incur in case of fraudulent transaction. In our setting, we use a broad definition of fraud, which is the use of an electronic payment instrument (or its information) by a person other than its owner, to obtain goods and services without authority for such use.[8] Fraudulent transactions are detected with some probability that is positively related to merchants' investments in fraud prevention technologies. If a fraud is detected, then the participants do not make losses.

Our results highlight the following trade-off for the payment platform. When the level of liability for merchants increases, the number of merchants who accept the electronic payment instrument falls, but merchants tend to invest more in fraud detection technologies, which increases consumers' willingness to use the electronic payment method. The payment platform trades off between increasing the level of liability to minimize the expected loss on fraudulent

[5]For a comparison of consumer protection laws across various countries, see Appendix A.

[6]For instance, in France, according to the "Observatoire de la sécurité des cartes de paiement", fraud losses have been shared in 2009 between banks (41.1%) and merchants (53.5%). Merchants have been held liable mainly for fraud on internet transactions. Consumers were held liable for only 2.3% of the fraud losses. According to Furletti (2005), in the United-States, "consumers of credit cards are shielded from nearly $3 billion in fraud losses each year". According to a more recent survey conducted by the Federal Reserve Board in the US, in 2009, across all types of debit card transactions, 57% of fraud losses were borne by issuers and 43% were borne by merchants. Source: Federal Reserve Register, vol. 75 n°248, 2010.

[7]These services provide Internet merchants with the ability to verify their consumers' true identities through a secure, electronic, non 'face-to-face' authentication process.

[8]Our model does not enable us to distinguish which type of fraud is implemented by the fraudster. We consider any type of fraud that can be empeded by merchant investment. For instance, data breaches and phishing do not depend on merchants' investments (rather on platform's investment). On the contrary, identity theft can be avoided by the merchant's effort to verify the consumer's identity.

transaction and maximizing the transaction volume by encouraging merchants and consumers to accept the electronic payment instrument. In the short term, the existence of a fraud liability regime affects the pricing structure of the payments system. With respect to the standard price structure in two-sided markets (Rochet-Tirole, 2003), the price structure that we obtain takes into account the platform's trade-off between maximising its profit and minimizing the expected loss on fraudulent transactions. If the zero liability rule for consumers applies, the allocation of fraud losses that is chosen by the payment platform maximises social welfare if the detection probability is strictly increasing with the fraud prevention effort. However, in other cases, liability regimes can be used by monopolistic payment platforms to extract rents from merchants, as it enables them to charge higher prices. We also find that our welfare result does not hold if investments are shared between the platform and the merchants. In this case, the payment platform trades off between providing merchants with incentives to invest in fraud detection technologies and choosing to make itself the fraud prevention effort. The payment platform may choose a level of liability for merchants that exceeds the social optimum, so as to extract the rents that the merchants obtain when the platform invests.

We also relax the assumption that merchants are risk neutral. This assumption is critical to obtain that merchants invest more in self prevention when their share of fraud losses increases. We show that our welfare result under the zero liability rule does not hold if merchants are risk averse.

Finally, we determine the incidence of the liability regime on the choice of the interchange fee. We find that, if the issuers are imperfectly competitive, whereas the acquirers are perfectly competitive, the profit maximising interchange fee decreases with the level of liability that is borne by merchants.

The rest of the paper is organized as follows. In Section 2, we summarize the literature related to our study. In Section 3, we develop a theoretical model to analyze the optimal allocation of fraud losses between the payment platform and the merchants. In section 4, we determine the profit maximising allocation of fraud losses. In section 5, we study the welfare maximising allocation of fraud losses. In section 6, we extend the model by studying the optimal allocation of investments between the payment platform and the merchants. In section 7, we relax the assumption that merchants are risk neutral. In section 8, we analyze the role of interchange fees. Finally, we conclude.

2 Related Literature

To our knowledge, this paper is the first attempt to model fraud detection technologies and liability regimes in the literature on payment systems. Our approach thus relies on three different strands of literature: the literature on payment platforms, on investment in two-sided markets, and finally the literature on liability issues in law and economics.

Most papers on payment systems focus on explaining the divergence between the profit maximising price structure that is charged by payment platforms and the price structure that maximises social welfare (see Chakravorti (2010) for a review). In particular, several papers aim at determining whether payment platforms charge excessive interchange fees when they maximise banks' joint profit (as surveyed by Verdier, 2011). Our paper contributes to this literature by extending Rochet-Tirole (2003) to study how the allocation of the expected fraud loss between the platform and the merchants changes the profit-maximising price structure.

The literature on investment in two-sided markets is scarce. For instance, Verdier (2010) determines the optimal price structure of a payment card platform in which monopolistic banks can invest to improve the quality of the payment service. Her model studies how investments should be allocated between monopolistic banks in four-party payment platforms. In particular, she finds that a reduction of interchange fees can improve the allocation of investments by encouraging acquirers to invest, when investments increase consumers' demand. Our model departs from that paper, as we consider a monopolistic proprietary payment platform, and we focus on the optimal allocation of fraud losses between the platform and the merchants. The four-party model is used in section 8 of our paper, where we show that the profit maximizing interchange fee decreases with the level of liability borne by merchants. The only paper that considers merchants' investments in two-sided platforms is the paper by Peitz and Belleflamme (2010), who study the effect of the intermediation mode (for-profit competing platforms versus free access) on sellers' investment, in a model where sellers' investment increase the buyers' utility of belonging to the platform. They show that for-profit intermediation may lead to overinvestment when innovations increase buyers' surplus, because competing intermediaries react by lowering the access fees on the seller side. Our focus is different from theirs, as we take the intermediation mode as given, and focus on the impact of liability rules on sellers' investments incentives.

Our model is also related to the vast literature on tort law whose main goal is to enhance socially optimal decisions on the level of precaution (Brown, 1973). More precisely, our framework shares the same background of ex post liability regimes, while neglecting the problem of non compliance and enforcement of ex ante regulation.[9] In this context, strict liability allocates the losses to the injurer by entitling the victim to compensation, whereas no liability allocates the losses to the victim, by denying the right to compensation (Landes and Posner, 1987). Indeed, liability provides incentives for precaution.[10] We extend this argument to the case of a three party system interrelated through network effects, which is uncommon in law and economics models. In fact, in our framework, the price of a transaction implies not only a choice for a consumer, which generates a loss risk (as also pointed out by law scholars like Cooter and Robin, 1987), but also a pricing strategy by the platform and an incentive for the merchant to invest in fraud detection.

3 The model

We build a model in which a monopolistic payment platform offers an Electronic Payment Instrument (hereafter the EPI) to consumers and merchants. We extend Rochet and Tirole (2003) along several dimensions. We consider that there is an exogenous probability that the EPI is fraudulently used, in a setting where merchants can invest in fraud detection technologies. We define fraud as the use of an electronic payment instrument (or its information) by a person other than its owner, to obtain goods and services without authority for such use. The fraud entails a lump sum loss which does not depend on the transaction value. Our framework enables us to determine how fraud liability should be allocated between the participants to maximise the platform's profit. It also enables us to compare the private optimal allocation to the one that maximises social welfare.

Payment system and allocation of fraud: A monopolistic payment platform provides an electronic payment instrument (e.g. the payment card) to consumers and merchants. The marginal cost of processing a transaction is denoted by c. Consumers and merchants pay transaction fees to the platform, which are denoted by f and m respectively.

When consumers use the EPI, there is an exogenous probability $x \in (0, 1)$ that the payment instrument is intercepted by fraudsters.[11] There is also a probability $q \in [0, 1]$ that the fraud

[9] Ex ante regulation is meant to prevent accidents from occurring through the enforcement of minimum safety standards or compliance restrictions. Ex post liability, exercised after an accident has occurred, is a legal device that enables victims to sue for damages, forcing injurers to internalize part of the harm they cause.

[10] When both parties have to take precaution in order to avoid an accident, strict liability creates no incentives for victim precaution, while no liability would shift the entire residual liability on the victim, inducing optimal victim care. It follows that strict liability and no liability can give incentives to take efficient precaution only to one party, respectively either the injurer or the victim (Dari-Mattiacci, Parisi, 2006).

[11] The assumption that x is exogenous is made for simplicity. Indeed, endogenizing x would introduce another trade-off for the merchant. Higher investments in fraud detection technologies have two effects on hackers' incentives to fraud. On the one hand, higher investments in fraud increase the volume of transactions, which

is detected, which depends on merchants' investments. If the fraud is not detected, all the participants to the transaction make an exogenous loss that we denote by $L > 0$. The loss is allocated between the consumer, the merchant and the payment platform as follows: the consumer (or buyer B) and the merchant (or seller S) bear respectively a share α_B and α_S of the loss, where $\alpha_S + \alpha_B \in [0, 1]$. The rest of the loss, $\alpha_P = 1 - (\alpha_S + \alpha_B)$, is borne by the payment platform. We assume that the parameter α_B is determined by public laws and we consider it as exogenous to the model. In particular, if $\alpha_B = 0$, the zero liability rule applies for consumers. The parameter α_S is privately chosen by the payment platform.[12] We choose to normalize the fraud on cash payments to zero.[13]

Merchants: We consider local monopolist merchants that supply the same good to consumers. The marginal cost of producing the good is denoted by d and the price of the good is denoted by p. The non-discrimination rule holds, such that a merchant cannot not charge a price that depends on the payment method. Merchants are risk neutral[14] and decide whether or not to accept the EPI. If he decides to accept the EPI, a merchant may invest an amount e_S in fraud detection technologies. Investment in "self-protecting" measures to improve fraud detection costs $C_S(e_S)$ to the merchant, where $C_S(e_S)$ is paid per transaction, $C_S'(e_S) \geq 0$, $C_S''(e_S) \geq 0$ and $C_S'''(e_S) \geq 0$.[15] Merchant's investments increase the probability q that a fraudulent transaction is detected, that is, we assume that $dq/de_S > 0$ for all $e_S > 0$. We also assume that $d^2q/d^2e_S \leq 0$ for all $e_S \geq 0$ and that $d^3q/d^3e_S \leq 0$.[16] The amount invested in fraud detection technologies is common knowledge, such that banks and consumers are aware of the security measures implemented by the merchants.[17]

By accepting the EPI, each merchant obtains a transaction benefit $b_S > 0$. As in Rochet and

increases the hackers' incentives to commit fraud. On the other hand, higher investments increase the probability that a fraud is detected, which may discourage hackers to commit fraud.

[12] In our model, we do not study how the losses are allocated between banks and the payment platform. In practice, payment platforms design rules to allocate the losses between issuing and acquiring banks and also to allocate the losses between banks and the platform itself. This issue would deserve a separate study. We reintroduce banks in section 4 and choose to focus on the role of interchange fees in fraud prevention issues.

[13] Introducing the probability that a fraudulent payment is made by cash would not change the trade-offs that we highlight in our model. We would only have to modify assumption (A2) to take into account the losses that are due to fraud on cash payments.

[14] The assumption that merchants are risk neutral is discussed in section 7, where we assume that merchants are risk averse.

[15] The literature on insurance markets makes a distinction between "self insurance" and "self protection". In our setting, merchants invest in "self protecting" measures but cannot reduce the amount of the expected loss. Under the assumption that merchants are risk neutral, both investments are equivalent as L and $(1-q)$ play the same role in the merchant's profit. This is not the case if merchants are risk averse.

[16] The assumption that $d^2q/d^2e_S \leq 0$ ensures that the second-order condition is verified when the merchant chooses its level of investment. The assumption that $d^3q/d^3e_S \leq 0$ ensures that the second-order condition is verified when the platform maximises its profit.

[17] Merchants can inform consumers about their efforts to fight fraud. For instance, online sellers can communicate on the use of a software or a specific technology that improves consumer authentication.

Tirole (2003), merchants are heterogenous over their transaction benefit b_S which is distributed over $\left[\underline{b_S}, \overline{b_S}\right]$ according to the probability density h_S and the cumulative H_S. We assume that $h'_S \geq 0$ to ensure demand (quasi) concavity. We normalize the benefit of accepting cash to zero. The merchant pays a fee m to the payment platform each time a consumer pays with the EPI and bears the cost of investing in fraud detection technologies.

Consumers: Consumers obtain a surplus $v > 0$ if they buy the good that is supplied by the merchants. Each consumer is randomly matched to one merchant and may choose between paying cash or paying with the EPI, if the merchant accepts the EPI. We assume that consumers are risk neutral and that they can observe merchants' investment in fraud detection technologies before deciding whether or not to use the EPI.[18]

If he pays with the EPI, the consumer obtains a transaction benefit b_B which is distributed over $\left[\underline{b_B}, \overline{b_B}\right]$ according to the probability density h_B and the cumulative H_B. We assume that $h'_B \geq 0$ for concavity to hold. The consumer pays a fee f to the payment platform, and anticipates that, with some probability $x(1-q)$, he bears a share α_B of the loss L, because the EPI is fraudulently used without being detected. The benefit of paying cash is normalized to zero. It follows that, if a consumer can choose between cash and the EPI, under the non-discrimination rule, a consumer wishes to use the EPI if and only if

$$b_B - f - \alpha_B x(1-q)L \geq 0, \tag{1}$$

that is, if his transaction benefit is higher than the cost of the transaction fee and the expected fraud loss.

Additional assumptions:

(A1) The hazard rate $\dfrac{h_B(x)}{1 - H_B(x)}$ is increasing.

(A2) In equilibrium, $\min\left\{v - d, \dfrac{1}{xL\alpha_B}\right\} \geq \dfrac{h_B(f + \alpha_B(1-q)xL)}{1 - H_B(f + \alpha_B(1-q)xL)}$.

Assumptions (A1) is similar to Wright (2002) and standard in the literature on payment cards. Assumption (A2) ensures that:

[18] Our parameter α_B could be also used to understand the impact of these assumptions. First, assume that consumers are risk averse, and that their utility function takes the form of a constant risk aversion function (CARA). An increase in consumers' aversion for risk implies a reduction of the elasticity of the consumer demand to the fraud prevention effort. An approximation of this effect in our model would be to reduce α_B. Second, if consumers do not observe the merchants' investments in fraud detection technologies, their demand is inelastic to their investment effort. This situation is captured in our model by setting $\alpha_B = 0$. Therefore, more generally, α_B could be interpreted as a parameter that impacts the consumer's demand sensitivity to the fraud prevention effort.

(i) consumers obtain a much higher surplus from buying the good that from making a transaction with the Electronic Payment Instrument.[19]

(ii) the amount of the expected share of the fraud loss for consumers is not too high, such that it does not exceed the surplus that consumers obtain from making a transaction.[20]

Timing:

The timing of the game is as follows:

1. The platform chooses the liability level α_S and the transaction fees f and m.

2. The merchants decide whether or not to accept the EPI and how much to invest in fraud detection technologies. They also choose the price of the good p.

3. Each consumer is matched randomly to one merchant. Consumers decide on whether or not to buy the good and how to pay for the good (either by cash or with the EPI).

In the following section, we look for the subgame perfect equilibrium and solve the game by backward induction.

4 The equilibrium:

4.1 Stage 3: consumer payment decisions

We start by determining the probability that a consumer wishes to use the EPI. Consider a consumer whose transaction benefit is $b_B \in \left[\underline{b_B}, \overline{b_B}\right]$. This consumer is randomly matched to one merchant, who may or may not accept the EPI. If the merchant accepts the EPI, the consumer chooses his payment method by comparing his expected utility if he pays cash and if he pays with the EPI.

Let us start by the case in which the merchant does not accept the EPI. If the merchant sets $p \leq v$, the consumer wishes to buy the good by paying cash, as his surplus $v - p$ is positive. Otherwise, he does not buy the good.

Now consider the case in which the merchant accepts the EPI. If the merchant sets $p \leq v$, the consumer wishes to buy the good, as he obtains at least a positive surplus if he pays cash. He decides to use the EPI if his expected utility is higher than if he pays cash. It follows that,

[19] Part (i) of Assumption (A2) is standard in the literature (see Wright (2002)). Formally, this corresponds to the Assumption that $v - d \geq h_B(f + \alpha_B(1-q)xL)/(1 - H_B(f + \alpha_B(1-q)xL)$.

[20] Part (ii) of Assumption (A2) is new, as our paper is the first to model the incidence of fraud losses on consumers and merchants' payment choices and platform prices. Formally, this corresponds to the Assumption that: $(1/xL\alpha_B) \geq h_B(f + \alpha_B(1-q)xL)/(1 - H_B(f + \alpha_B(1-q)xL)$.

if $p \leq v$, a consumer wishes to use the EPI if and only if:

$$v - p + b_B - f - \alpha_B(1-q)xL \geq v - p,$$

that is, if and only if

$$b_B - f - \alpha_B(1-q)xL \geq 0.$$

If the merchant sets $p > v$, the consumer never uses cash. The consumer buys the good and pays with the EPI if and only if

$$v - p + b_B - f - \alpha_B(1-q)xL \geq 0.$$

We denote by D_B the probability that a consumer wishes to use the EPI. Considering consumers' heterogeneity, it follows from the previous analysis that

$$D_B = \begin{cases} 1 - H_B(f + \alpha_B(1-q)xL) & \text{if } p \leq v \\ 1 - H_B(f + \alpha_B(1-q)xL + p - v) & \text{if } p > v. \end{cases}$$

Note that the probability that the consumer wishes to use the EPI decreases with the transaction fee, the consumer's liability, the expected amount of fraud loss, but increases with the probability that the fraud is detected.

4.2 Stage 2: EPI acceptance and investments in fraud detection

4.2.1 Prices and card acceptance condition

We now determine the price that is chosen by each merchant, along with the decision to accept the EPI and invest in fraud detection technologies. We start by showing that, because of assumptions (A1) and (A2), the profit of a merchant who accepts the EPI is maximised when he sets a price such that cash-users are not excluded from the market. It follows that merchants who accept the EPI and merchants who do not accept the EPI choose the same price. This enables us to derive the EPI acceptance condition.

Lemma 1 *Each monopolistic merchant maximises its profit by setting $p^* = v$.*

Proof. See Appendix B. ■

We are now able to derive the condition under which a merchant accepts the electronic payment instrument. A merchant accepts the EPI if he makes more profit by doing so, that is if

$$v - d + D_B(f + \alpha_B(1-q)xL)(b_S - \alpha_S x(1-q)L - m - C_S(e_S)) \geq v - d.$$

Since $D_B(f+\alpha_B(1-q)xL) \geq 0$, this condition is equivalent to

$$b_S - \alpha_S x(1-q)L - m - C_S(e_S) \geq 0. \tag{2}$$

Note that a merchant does not accept the EPI if the merchant fee is high or if the amount of the expected fraud loss is high.

4.2.2 Investment in fraud detection technologies

A merchant that accepts the EPI can invest in fraud detection technologies. The amount of investment in fraud detection technologies, which we denote by e_S^*, maximises the merchant's profit under the constraint that the merchant accepts the EPI.

Lemma 2 *If the merchant fee is not too high, all merchants such that* $b_S \geq \widehat{b_S}(\alpha_S, \alpha_B, x, L, m, f)$ *accept the electronic payment instrument, where* $\widehat{b_S}(\alpha_S, \alpha_B, x, L, m, f) \in \left[\underline{b_S}, \overline{b_S}\right]$. *The profit maximising investment for a merchant who accepts the EPI solves:*

$$\alpha_S x L \left.\frac{dq}{de_S}\right|_{e_S^*} - C_S'(e_S^*) = [b_S - \alpha_S x(1-q)L - m - C_S(e_S^*)] \frac{\zeta_B|_{e_S^*}}{e_S^*}, \tag{3}$$

where $\zeta_B = \dfrac{-dD_B/de_S}{D_B/e_S}$ *denotes the elasticity of the consumer's demand to the investment effort.*

Proof. See Appendix C. ■

The merchant chooses its fraud prevention effort so as to equalize the marginal benefits of investments in fraud detection technologies and the marginal cost of investments. The marginal benefits of investments are equal to the marginal gains from lower expected fraud losses (term $\alpha_S xL(dq/de_S)$ in (3)), and to the marginal benefits that are due to an increase in the volume of electronic transactions (term $[b_S - \alpha_S x(1-q)L - m - C_S(e_S^*)]\,(\zeta_B|_{e_S^*}/e_S^*)$ in (3)).

Let us detail each of the two effects that will be referred to as the *expected loss effect* and the *transaction volume effect.* First, if the merchant invests in fraud detection technologies, this increases the probability that a fraudulent transaction is detected, and therefore, this reduces the amount of the expected loss that he has to bear when he accepts the EPI. The expected loss effect has a positive impact on merchant's investments. Second, if consumers bear a positive share of fraud losses, the probability that a consumer wishes to use the EPI is impacted positively by the merchant's investments, as the expected loss decreases. The transaction volume effect has also a positive impact on merchant's investments.

Remark that, because of the transaction volume effect (if $\alpha_B \neq 0$), the merchants invest in fraud prevention technologies even if they bear no liability for fraud, that is if $\alpha_S = 0$. In two-sided markets, the liability regime is not the only incentive that can be used to encourage merchant investment, as merchants care about the transaction volume, which is related to consumer demand. This effect is not present in the literature on law and economics that we mentioned in section 2.

Note also that merchants exert a positive externality on the payment platform and on consumers if $\alpha_B \neq 0$, because their investment in fraud detection technologies reduces the amount of their expected fraud loss.

If $\alpha_B = 0$, the zero liability rule applies for consumers. In this case, all merchants who accept the EPI invest the same amount in fraud detection technologies, which is implicitely defined by

$$\alpha_S x L \left. \frac{dq}{de_S} \right|_{e_S^*} = C_S'(e_S^*). \tag{4}$$

As investments do not impact consumer demand, the transaction volume effect is null under the zero liability rule. A merchant who obtains a higher transaction benefit does not have higher investment incentives, as the marginal benefits obtained through a higher transaction volume are equal to zero.

4.2.3 Comparative statics

In Lemma 3, we give some comparative statics to explain how a merchant's investment in fraud detection technologies vary with the transaction fees, the liability levels and the benefit that a merchant obtains of being paid with the electronic payment instrument.

Lemma 3 *If $\alpha_B > 0$, the merchant's investments in fraud detection technologies increase with the consumer liability, the consumer transaction fee, the merchant's transactional benefit, and the merchant's liability, but they decrease with the merchant fee.*

Proof. See Appendix D. ■

We proved in Lemma 2 that a merchant's investments in fraud detection technologies are chosen such that the marginal benefits are equal to the marginal costs of investments. If the merchant fee increases (resp. if the merchant's transactional benefit increases), all other things being equal, the marginal benefits from investment decrease, because of a reduction of the transaction volume effect. The merchant reacts by reducing its investments in fraud detection technologies.

If the merchant's liability increases, this increases the expected loss effect, because the merchant has more to save when a fraud is detected, whereas this decreases the transaction volume effect, as the merchant's margin per transaction is reduced. Under Assumption (A2), the first effect dominates and the merchant reacts by increasing its investments in fraud detection technologies.

Moreover, if the consumer liability increases or if the consumer fee increases, this increases the transaction volume effect, because the impact of merchant's investments on consumer demand increase. Therefore, the merchant's investments increase.

If the zero liability rule applies, from (4), the merchant's investments in fraud detection technologies do not depend on the transaction fees that are chosen by the payment platform. They only depend on the merchant's liability and the expected loss. As when $\alpha_B > 0$, they decrease with the merchant's liability and it can be shown that they decrease with the expected fraud loss.

In Lemma 4, we determine how the transaction fees and the liability levels impact the probability that a merchant accepts the electronic payment instrument.

Lemma 4 *The probability that a merchant accepts the EPI decreases with the merchant fee, with the consumer fee and with the level of liability that is borne by merchants or by consumers.*

Proof. See Appendix E. ■

A higher merchant fee lowers the transaction margin that the merchant obtains if he accepts the EPI, whereas it reduces the merchant's incentives to accept the EPI, which is a standard effect in the literature on payment cards. Moreover, in our model, the probability that a merchant accepts the EPI also depends on the consumer fee, because merchants exert a positive externality on consumers when they choose to invest in fraud detection technologies. Indeed, this interaction, which is novel in the literature on payment platforms, arises when $\alpha_B \neq 0$ and this is specific to our model setting. Finally, a higher consumer fee decreases the probability that a consumer wishes to use the EPI, which reduces the marginal benefits of investing in fraud detection technologies and the benefits of accepting the EPI for the merchant. Therefore, the probability that a merchant accepts the EPI decreases with the consumer fee.

Most importantly, our model is the first to highlight the impact of liability regimes on merchants' acceptance of payment media. We show in Appendix E that the level of liability has an ambiguous impact on merchants' choice to accept the electronic payment instrument. On the one hand, a higher liability level increases the loss in case of a fraudulent use of the EPI, which discourages merchants to accept the EPI. On the other hand, it increases the level of effort made by merchants, which reduces the probability that the EPI is fraudulently used

- and thus increases the probability that a consumer wishes to use the EPI. From assumption (A2), the first effect dominates in our framework, and therefore, the probability that a merchant accepts the EPI decreases with his liability level.

4.3 Stage 1: Prices and liability levels

At the first stage, the payment platform choses the prices that maximise its profit,

$$\pi_P = (f + m - c)V_P - EL_P,$$

where V_P denotes the transaction volume, as follows:

$$V_P = \int_{\widehat{b_S}}^{\overline{b_s}} h(b_S)(1 - H_B(f + \alpha_B xL(1 - q^*))db_S, \tag{5}$$

EL_P denotes the average expected loss, or:

$$EL_P = \alpha_P xL \int_{\widehat{b_S}}^{\overline{b_s}} (1 - q^*)h(b_S)(1 - H_B(f + \alpha_B xL(1 - q^*)))db_S, \tag{6}$$

and

$$q^* = q(e_S^*).$$

If $\alpha_B = 0$, as q^* does not depend on b_S, we have

$$EL_P = \alpha_P xL(1 - q^*)V_P. \tag{7}$$

For all $\alpha_B \in [0, 1]$, the transaction volume decreases with the consumer transaction fee and with the merchant fee. While this effect is standard in the literature, another question arises in our framework, that is the impact of the transaction prices and the merchants' liability on the expected fraud loss that is borne by the payment platform.

4.3.1 Variations of the expected loss with the prices

We start by determining how the expected fraud loss is impacted by the choice of transaction fees and by the level of liability that is borne by merchants.

Proposition 1 *The expected loss incurred by the payment platform on fraudulent transactions (EL_P) decreases with the consumer transaction fee and with the level of liability that is borne by merchants. EL_P decreases with the merchant fee only if the elasticity of the merchant's effort* *to the merchant fee is small or if the elasticity of the merchant's demand to the merchant fee is high.*

Proof. See Appendix F. ■

An increase in the consumer fee decreases the number of merchants who accept the EPI, whereas it increases merchants' investments in fraud detection technologies. It follows that a higher consumer fee decreases the expected loss that is incurred by the payment platform.

Moreover, a higher level of liability for merchants decreases the expected loss that is borne by the payment platform, as it decreases merchants' acceptance of the EPI, whereas it increases merchants' investment in fraud detection technologies.

An increase in the merchant fee has two effects on the expected loss that is incurred by the payment platform. The higher the merchant fee, the lower the number of merchants who accept the EPI, and the lower the transaction volume. This effect reduces the expected loss that is incurred by the payment platform. At the same time, a higher merchant fee decreases the merchants' investment in fraud detection technologies, which increases the expected loss borne by the payment platform. The impact of an increase in the merchant fee on the expected loss depends on how both effects compensate each other.

4.3.2 The profit maximising price structure under exogenous liability regime

Proposition 2 gives the profit maximising price structure for a given level of merchants' liability.

Proposition 2 *The profit maximising price structure reflects the platform's trade-off between balancing profits between both sides of the market and minimizing the expected loss on fraudulent transactions. The total price is implicitely defined by*

$$\frac{f^* + m^* - c}{f^*} = \frac{1}{\varepsilon_B^V(f^*)} + \frac{\partial EL_P/\partial f}{f^* \partial V_P/\partial f},$$

and the price structure verifies

$$\frac{f^*}{m^*} = \frac{\frac{1}{\varepsilon_S^V(m^*)} + \frac{\partial EL_P/\partial m}{m^* \partial V_P/\partial M}}{\frac{1}{\varepsilon_B^V(f^*)} + \frac{\partial EL_P/\partial f}{f^* \partial V_P/\partial f}},$$

where $\varepsilon_B^V(f) = -(\partial V_P/\partial f)(f/V_P)$ and $\varepsilon_S^V(m) = -(\partial V_P/\partial m)(m/V_P)$ denote the elasticity of the transaction volume to the consumer fee and the merchant fee respectively.

Proof. We denote by $M_P = f + m - c$ the payment platform's gross margin. Assume that there is an interior solution. Solving for the first-order conditions of profit maximisation yields

$$\frac{\partial \pi_P}{\partial f} = M_P \frac{\partial V_P}{\partial f} + V_P - \frac{\partial EL_P}{\partial f} = 0,$$

and

$$\frac{\partial \pi_P}{\partial m} = M_P \frac{\partial V_P}{\partial m} + V_P - \frac{\partial EL_P}{\partial m} = 0.$$

These equations can be rewritten as

$$\frac{f + m - c}{f} = \frac{-V_P}{f \partial V_P / \partial f} + \frac{\partial EL_P / \partial f}{f \partial V / \partial f}, \tag{8}$$

and

$$\frac{f + m - c}{m} = \frac{-V_P}{m \partial V_P / \partial m} + \frac{\partial EL_P / \partial m}{m \partial V_P / \partial m}. \tag{9}$$

Introducing the elasticities $\varepsilon_B^V(f) = -(\partial V_P / \partial f)(f / V_P)$ and $\varepsilon_S^V(m) = -(\partial V_P / \partial m)(m / V_P)$ and dividing the first equation by the second equation yields the result of Proposition 2. In Appendix G-A, we show that the second-order conditions of profit maximisation are verified if $\alpha_B = 0$ and we assume that they hold if $\alpha_B \neq 0$. ■

It is interesting to compare the prices that we find in an interior solution with the prices obtained in the standard two-sided market monopoly pricing formula obtained by Rochet and Tirole (2003). Equations (8) and (9) show that with respect to the standard price structure in two-sided markets, the price structure that we obtain encompasses an additional term that takes into account the platform's trade-off between maximising its profit and minimizing the expected loss on fraudulent transactions.

Notice that if the zero liability rule applies for consumers (that is if $\alpha_B = 0$), from (7), the expected loss only depends on the transaction prices through the transaction volume. It follows that, in this case, the price structure is the same as the one obtained by Rochet and Tirole (2003), that is:

$$\frac{f}{m} = \frac{\varepsilon_B^V(f)}{\varepsilon_S^V(m)},$$

and the total price is implicitely defined by:

$$\frac{f + m - c - (1 - \alpha_S) x L (1 - q^*)}{f} = \frac{1}{\varepsilon_B^V(f)}.$$

4.3.3 An Example under the zero liability rule for consumers

We consider for example the case of uniforms distribution on $[0,1]$ for b_B and b_S, with a cost function $C_S(e_S) = k_S(e_S)^2/2$, a detection probability $q(e_S) = \gamma e_S$, where $\gamma > 0$. We also assume that the zero liability rule applies for consumers. We prove in Appendix H that, if $\alpha_S > 0$, the profit maximising transaction fees are

$$m^* = \frac{1}{3}(1 + c + xL(1 - 3\alpha_S)) + \frac{k_S}{3\alpha_S}\left(-1 + 2\alpha_S\right)(e_S^*)^2, \tag{10}$$

and

$$f^* = \frac{1}{3}(1 + c + xL) - \frac{k_S}{6\alpha_S}\left(2 - \alpha_S\right)(e_S^*)^2, \tag{11}$$

where $e_S^* = \alpha_S x \gamma L / k_S$. From (10) and (11), the consumer fee is higher than the merchant fee if $\alpha_S \neq 0$, as we have

$$f - m = \alpha_S x L(1 - q^*) + \frac{k_S}{2}(e_S^*)^2 \geq 0. \tag{12}$$

If the demands are uniform and symmetric, in Rochet and Tirole (2003)'s model, the profit maximising transaction fees are such that $f = m$. Equation (12) shows that, if $\alpha_S > 0$, the payment platform tends to lower the merchant fee to provide merchants with incentives to invest in fraud detection technologies. The price structure changes in favor of merchants. This is not necessarily the case if demands are not symmetric, or if $\alpha_B \neq 0$. If the zero liability rule does not apply for consumers, the payment platform can use the transaction prices on both sides of the market to encourage merchants to invest in fraud detection technologies, because of the transaction volume effect that we highlighted in Lemma 2.

4.3.4 The profit maximising level of liability

We proceed by assuming that the payment platform has the opportunity to choose the merchant's level of liability at the same stage as the transaction prices. Thus, we start by determining how the merchant's level of liability impacts the plaform's profit. We know from Proposition 1 that the expected loss that is borne by the payment platform decreases with the level of liability borne by merchants. It remains to study how the level of liability borne by merchants impacts the transaction volume. We have

$$\frac{\partial V_P}{\partial \alpha_S} = \underbrace{\frac{-\partial \widehat{b_S}}{\partial \alpha_S} h_S(\widehat{b_S})(1 - H_B(f + \alpha_B x L(1 - q^*)))}_{\text{Term I}} + \underbrace{\int_{\widehat{b_S}}^{\overline{b_s}} h_S(b_S) \frac{\partial D_B(f + \alpha_B x L(1 - q^*))}{\partial \alpha_S} db_S}_{\text{Term II}}. \tag{13}$$

The first term of (13) is negative. It reflects the fact that fewer merchants accept the EPI when the level of liability that is borne by merchants increases. The second term of (13) is positive. It shows that more consumers wish to pay with the EPI when merchants invest in fraud detection technologies. It follows that a higher level of liability for merchants has an ambiguous impact on the transaction volume. Note that if the elasticity of the merchants' demand to their liability level is small (that is, if term I is small), the transaction volume may increase with the merchants' level of liability. Moreover, if the zero liability rule applies for consumers, the second term of (13) is null, and the transaction volume decreases with the merchant's level of liability. Proposition 3 gives the profit maximising level of liability for merchants.

Proposition 3 *A monopolistic payment platform chooses a level of liability for merchants that reflects a trade-off between minimizing the expected loss on fraudulent transactions and maximising the transaction volume. The interior solution for the profit maximising level of liability for merchants solves*

$$(f^* + m^* - c)\left.\frac{\partial V_P}{\partial \alpha_S}\right|_{(f^*,m^*)} = \left.\frac{\partial EL_P}{\partial \alpha_S}\right|_{(f^*,m^*)},$$

where (f^*, m^*) denote the profit-maximizing prices of Proposition 1. If the transaction volume increases with the liability level that is borne by merchants, there is a corner solution such that the payment platform lets the merchants bear all the losses.

Proof. The payment platform chooses the level of liability that maximises its profit. Solving for the first-order condition of profit maximisation yields

$$\frac{\partial \pi_P}{\partial \alpha_S} = (f + m - c)\frac{\partial V_P}{\partial \alpha_S} - \frac{\partial EL_P}{\partial \alpha_S}.$$

In an interior solution, we have

$$(f + m - c)\frac{\partial V_P}{\partial \alpha_S} = \frac{\partial EL_P}{\partial \alpha_S}.$$

From Proposition 1, we know that the expected loss decreases with the level of liability that is borne by merchants. It follows that, if the transaction volume increases with the level of liability borne by merchants, the profit maximising liability level is a corner solution, with the merchants bearing the maximum share of the loss.

In Appendix G-B, we show that the second-order conditions of profit maximisation are verified if $\alpha_B = 0$, and we assume that they hold if $\alpha_B \neq 0$. ■

Proposition 3 shows that the payment platform has an incentive to share the losses on fraudulent transactions with the merchants, as this encourages merchants to accept the electronic payment instrument, unless merchants' demand is inelastic to the level of liability. However, the choice of a liability regime is also a means for the payment platform to extract rents from merchants if the elasticity of the merchants' demand to the liability level is small.

Proposition 4 explains how fraud losses are allocated by a profit maximising monopolistic plaform under the zero liability rule for consumers.

Proposition 4 *Under the zero liability rule for consumers, if the detection probability is strictly increasing with α_S, the platform lets the merchants bear the maximum share of fraud losses.*

Proof. See Appendix G-B. ■

Under the zero liability rule for consumers, the payment platform chooses a level of liability that maximises the probability of fraud detection, if there is an interior solution (See Appendix G-B). However, under our assumptions, the probability to detect a fraudulent transaction is strictly increasing with the merchants' investment effort. Since the merchants' effort is strictly increasing with their share of fraud losses, α_S, the probability of fraud detection is strictly increasing with α_S. Therefore, in our model, there is a corner solution under the zero liability rule, such that the payment platform lets the merchants bear all the losses. This result does not hold if the merchants' effort vary non monotonically with α_S.

In our example with uniform distributions and quadratic cost functions, the transaction fees under full merchant liability ($\alpha_S = 1$) are $f^* = (1 + c + xL - (x\gamma L)^2/2k_S)/3$ and $m^* = (1 + c - 2xL + (x\gamma L)^2/k_S)/3$. The total price is $f^* + m^* = 2(1 + c - xL)/3 + (x\gamma L)^2/6k_S$, and the price difference is $f^* - m^* = (xL - (x\gamma L)^2/2k_S)/3$. Even if consumers bear zero liability for fraud, they pay a share of fraud losses through the transaction fees, which is not explicitely defined through a liability regime. This example also shows how transaction prices vary with the fraud rate. If $\alpha_S = 1$, from (10) and (11), we have

$$\left.\frac{\partial m^*}{\partial x}\right|_{\alpha_S=1} = \frac{-2L}{3}(1 - q^*) \leq 0,$$

and

$$\left.\frac{\partial f^*}{\partial x}\right|_{\alpha_S=1} = \frac{L}{3}(1 - q^*) \geq 0.$$

The merchant fee decreases with the fraud rate in our example when the merchants' share of fraud losses is high, whereas the consumer fee increases with the fraud rate. The total price decreases with the cost of fraud[21], whereas the distortion in the price structure increases with

[21] Indeed, we have $\partial(f^* + m^*)/\partial x = (-2L/3)(1 - q^*/2) < 0$.

the fraud rate and the fraud losses. Proposition 5 shows that this result is very general under the zero liability rule for consumers.

Proposition 5 *Under the zero liability rule, if the detection probability is strictly increasing with α_S, the merchant fee and the consumer fee vary in opposite directions with the fraud rate.*

Proof. See Appendix G-C. ■

5 Welfare maximising liability levels

To study welfare maximizing liability levels, we assume that the merchant's level of liability is decided by a social planner at the first stage, who maximises the sum of the platform's profit, the consumer surplus and the merchant surplus. Then, the payment platform chooses the transaction fees at the following stage. Our aim is to compare the profit maximising level of liability for merchants, which is chosen by the payment platform, to the welfare maximising level of liability for merchants.

5.1 The welfare maximising liability level under the zero liability rule for consumers

We start by analyzing the simple case in which consumers bear zero liability on fraudulent transactions. For this purpose, we need to determine how the liability level borne by merchants impacts the transaction fees that are chosen by the payment platform.

Lemma 5 *If the zero liability rule applies for consumers, the transaction fees chosen by the payment platform decrease with the level of liability that is borne by merchants.*

Proof. See Appendix I. ■

A higher level of liability for merchants provides the payment platform with incentives to lower its prices on both sides of the market, if the zero liability rule applies for consumers. The payment platform's pricing strategy reflects a trade-off between increasing its margin and increasing the transaction volume. A higher level of liability for merchants has two effects on this trade-off. First, it decreases the expected loss on fraudulent transactions, which amounts to a reduction of the platform's marginal cost. The platform bears a lower share of fraud losses, while merchants obtain higher investment incentives. This marginal cost reduction increases the platform's incentives to lower its prices. Second, a higher level of liability for merchants reduces the transaction volume, as fewer merchants adopt the electronic payment instrument, which reduces the platform's incentives to increase its prices. Therefore, when the level of liability for merchants increases, the transaction fees paid by consumers and merchants fall. The payment platform loses some rents on both sides of the market, but this loss is compensated by higher rent extraction through the liability regime, which provides merchants with higher investment incentives.

We are now able to compare the profit maximising level of liability and the welfare maximising level of liability for merchants if consumers do not bear any liability for fraudulent transactions. We assume that social welfare is a concave function of the transaction fees.[22]

Proposition 6 *Under the zero liability rule for consumers, if social welfare is a concave function of α_S, the profit maximising level of liability for merchants is lower than (or equal to) the welfare maximising level of liability.*

Proof. See Appendix J-B. ■

Corollary 1 *If the detection probability is strictly increasing in α_S, the profit maximizing and the welfare maximizing levels of liability are equal under the zero liability rule for consumers.*

We showed in Lemma 5 that the transaction fees paid by the users decrease with the level of liability that is borne by merchants. A direct consequence of Lemma 5 is that consumer and merchant surplus increase when merchants' liability increase. It follows that, from the point of view of total user surplus maximisation, it is socially optimal to let the merchants bear the maximum liability on fraudulent transactions. However, if the regulator takes into account the payment platform's profit, the welfare maximising level of liability for merchants is not necessarily equal to one, except if consumers are held liable for fraud.

The payment platform does not place enough liability on merchants to maximise social welfare, except in the case where it is maximises its profit by letting the merchants bear the maximum liability on fraudulent transactions. This is because the payment platform internalizes imperfectly the impact of the liability regimes on consumer and merchant surplus. In our model, since our assumptions imply that the detection probability is strictly increasing with α_S, the platform lets the merchant bear all the liability for fraud under the zero liability rule. Therefore, in this case, the welfare maximising level of liability is equal to the profit maximising level of liability. However, the welfare maximising level of liability could be strictly higher than the profit maximising level of liability if the merchants' effort could vary non monotonically with

[22] W is concave in α_S for instance if b_S and b_B are uniformely distributed on $[0, 1]$ under some assumptions about the cost of fraud prevention and the sensitivity of the detection probability which are precised in Appendix J. In general, it is possible to prove that π_P is concave in α_S, however, the total user surplus is not necessarily concave in α_S.

the share of fraud losses (that is, if Assumption A2 was lifted). Our result is also driven by the assumption that the probability to detect a fraudulent transaction only depends on merchants' investment. In section 6, we prove that the prices may increase with the level of liability borne by merchants if the investments are shared between the payment platform and the merchants.

5.2 The welfare maximising level of liability if consumers bear some liability for fraud

If consumers bear some liability for fraud, the plaform's prices do not necessarily decrease with the level of liability borne by merchants. The platform can now use the transaction fees to impact merchants' incentives to invest in fraud detection technologies.

6 Platform's investments under the zero liability rule for consumers

We analyze if our welfare result under the zero liability rule holds in an extension of the model that allows the payment platform to invest an amount e_P in fraud detection technologies. The platform incurs a cost $C_P(e_P)$ per transaction, where C_P is a convex cost function. The probability to detect a fraudulent transaction, which we now denote by $q(e_S, e_P)$, increases with the platform's investments, that is $\partial q/\partial e_P \geq 0$. The platform chooses its level of investment at the same stage as the prices, and merchants are able to observe this decision before deciding whether or not to accept the electronic payment instrument. In a supplementary note, which is available upon authors' request, we show that the welfare result obtained under the zero liability rule does not hold when the platform's investments are taken into account.[23] In this situation, the prices chosen by the payment platform do not necessarily decrease with the level of liability borne by merchants.

The intuition of this result is the following. The platform now trades off between providing merchants with incentives to invest in fraud detection technologies and choosing to make itself the fraud prevention effort. The result of this trade-off is impacted by the relative cost of investment for the platform and the merchants, and by the fact that their technological choices may be either independent or may influence each other.

[23] Except in the case where the platform's cost function is linear and if the detection probability is linear in the platform's investment effort.

6.1 An example with independent investments

We start by analyzing the case in which the merchants' investments and the platform's investment are independent. This case can be illustrated by assuming for instance that $q(e_S, e_P)$ is linear and separable in e_S and e_P, that is

$$q(e_S, e_P) = de_S + he_P,$$

where $d \geq 0$ and $h \geq 0$. To understand better the impact of the platform's investments on our welfare result under the zero liability rule for consumers, we specify quadratic investment cost functions for the merchants and the platform, such that $C_S(e_S) = k_S(e_S)^2/2$ and $C_P(e_P) = k_P(e_P)^2/2$, where $k_S > 0$ and $k_P > 0$. We also assume uniform distributions on $[0,1]$ for b_S and b_B. Under these assumptions, at the equilibrium of stage 2, each merchant invests an amount $e_S^* = dLpx\alpha_S/k_S$ in fraud detection technologies. At stage 1, if $\alpha_S > 0$, the prices chosen by the platform are

$$\begin{aligned} f^* &= \frac{1}{3}(1 + c + Lx) - \frac{k_P}{6}(e_P^*)^2 - \frac{k_S}{6}(e_S^*)^2 \frac{2 - \alpha_S}{\alpha_S}, \\ m^* &= \frac{1}{3}(1 + c + Lx(1 - 3\alpha_S)) - \frac{k_P}{6}(e_P^*)^2(1 - 6\alpha_S) + \frac{k_S}{3}(e_S^*)^2 \frac{2\alpha_S - 1}{\alpha_S}, \end{aligned}$$

where $(e_P)^* = hLp/k_P$ denotes the profit maximising investment of the platform.[24] This illustration shows that the consumer fee is not necessarily higher than the merchant fee, unlike our previous example with uniform distributions under the zero liability rule. We have

$$\frac{\partial f^*}{\partial \alpha_S} = \frac{-d^2L^2x^2(1 - \alpha_S)}{3k_S}, \tag{14}$$

and

$$\frac{\partial m^*}{\partial \alpha_S} = -Lx + \frac{(xL)^2}{3k_Pk_S}\left[-d^2k_P(4\alpha_S - 1) + 3h^2k_S\right]. \tag{15}$$

From (14), the consumer fee decreases with the level of liability borne by merchants, whereas from (15), the merchant fee increases with the level of liability borne by merchants if the investment cost of the platform is sufficiently low and if the platform's contribution to increase the detection probability is high enough (through the parameter h). This result can be explained as follows. A higher level of liability for merchants has two effects on the platform's incentives to invest in fraud detection technologies. First, it decreases merchants' acceptance, which reduces

[24] If $e_P^* = 0$, we obtain the prices of our previous example, which did not take into account the platform's investments (See (10) and (11)).

the marginal benefits of investing in fraud detection technologies for the payment platform. Second, it reduces fraud losses, which amounts to a reduction of the platform's marginal cost. This effect impacts the platform's investments in two opposite directions. On the one hand, it decreases the platform's incentives to invest, as the platform bears a lower share of fraud losses. On the other hand, it increases the platform's margin per transaction, which can result in higher investment incentives.

From the point of view of merchants, an increase in their liability raises the value of the platform's investments, as this improves the quality of service provided by the platform. The payment platform trades off between extracting this additional surplus from the merchants through the merchant fee and increasing the transaction volume through lower fees. The variation of the merchant fee with the merchants' share of fraud losses reflects this trade-off, which is not present on the consumer side.

6.2 Related investments

We now analyze the case in which the platform's decision to invest in fraud detection technologies impacts positively the merchant's investment effort. This case can be illustrated by assuming for instance that $q(e_S, e_P)$ is a product of the merchant's investment effort and the platform's investment effort, that is

$$q(e_S, e_P) = de_S e_P + he_P,$$

where $d \geq 0$ and $h \geq 0$. At the equilibrium of stage 2, the merchant's investments in fraud detection technologies are positively related to the platform's prevention effort, and we have $e_S^* = de_P xL\alpha_S/k_S$. Therefore, the platform takes into account this effect in its trade-off between providing merchants with investments incentives and choosing to bear itself the fraud prevention effort. At stage 1, the platform chooses the transaction fees

$$\begin{aligned} f^* &= \frac{1}{3}(1 + c + Lx) - \frac{hLxe_P^*}{6}, \\ m^* &= \frac{1}{3}(1 + c + Lx(1 - 3\alpha_S)) + \frac{k_S}{2}(e_S^*)^2 - \frac{hLx}{3}e_P^*(1 - 3\alpha_S) + \frac{hLxe_P^*}{6}, \end{aligned}$$

where $e_P^* = hk_S Lx/(k_P k_S - d^2 L^2 x^2 \alpha_S(2-\alpha_S))$ if $k_P k_S - d^2 L^2 x^2 \alpha_S(2-\alpha_S) > 0$. The platform's level of investment increases with the share of liability borne by merchants. This result can be explained as follows. An increase in the level of liability borne by merchants amounts to a reduction of the platform's marginal cost, which results in higher investment incentives for the platform. A higher level of liability also raises the impact of the platform's effort on merchants' investment incentives, which provides the platform with an additional incentive to increase its level of prevention effort. As in our previous example, the consumer fee decreases with the level of liability borne by merchants, whereas the merchant fee varies non-monotonically with the share of fraud losses.

7 Risk averse merchants

If merchants are risk averse, they do not necessarily invest more in fraud prevention, because they trade off between reducing the probability that a fraud occurs and reducing their investment cost. The result that risk averse agents do not exert more effort for self protection is standard in the literature on insurance markets (see for instance Dionne and Eeckhoudt, 1985).[25] On the contrary, risk averse agents who invest in self insurance increase their level of effort. However, we do not study this possibility in our setting, as merchants cannot reduce the amount of fraud losses.

Lifting the assumption that merchants are risk neutral impacts our results under the zero liability rule for consumers. As in the case where $\alpha_B \neq 0$, merchants care about the volume of transactions paid with the EPI. Therefore, merchant's investment are related to the price paid by consumers. Furthermore, merchants' investments may either increase or decrease with the share of fraud losses. For instance, with a CARA utility function, merchant's investments increase with their share of fraud losses if their coefficient of absolute risk aversion is sufficiently small, whereas they may decrease with their share of fraud losses if their coefficient of absolute risk aversion is high, and if the fraud prevention effort has a limited impact on the probability to detect a fraudulent transaction (See Appendix K). However, with other utility functions, the incentive to invest in fraud prevention may be as strong for risk preferrers as for risk avoiders.[26]

Therefore, our welfare result under the zero liability rule for consumers does not hold if we assume that merchants are risk averse. The payment platform does not necessarily reduce the probability that a fraud occurs by increasing merchants' share of fraud losses, as merchants may react by reducing their investment effort. In this situation, as in Proposition 3, the profit maximising level of liability reflects a trade-off between maximising the transaction volume and minimising the cost of fraud. However, unlike in Proposition 1, the expected loss varies non monotonically with the share of fraud losses borne by merchants.

[25] As Becker and Ehrlich (1972) note: "Self insurance and market insurance both redistribute income toward hazardous states, whereas self-protection reduces the probability of these states. Unlike insurance, self protection does not redistribute income because the amount spent reducing the probability of a loss decreases income in all states equally, leaving unchanged the absolute size of the loss".

[26] Bryis and Schlesinger (1990) show that it is not possible to find a subclass of risk-averse utility functions to yield a monotonic relationship between risk aversion and the level of self-protection.

8 The role of interchange fees

In this section, we examine an important regulatory challenge, which is the impact of merchant liability on the level of interchange fees.[27] This issue has been examined in the United-States after the vote of the Dodd-Frank act in July 2010, which gives to the Federal Reserve Board the power to regulate interchange fees on debit card transactions. Among the regulatory rules, the "fraud adjustment rulemaking" provides the Board with the opportunity to assess how card networks' authorization choices and fraud procedures may burden the merchant community and potentially increase the volume of debit card fraud. The rulemaking also gives the Board the opportunity to promote the use of the fraud adjustment mechanism as a means of creating incentives for banks and merchants to migrate to more effective fraud detection technologies.

To study this issue, we modify our model setting, by making the standard assumption that the payment platform is now composed of imperfectly competitive issuers and perfectly competitive acquirers.[28] We also assume for simplicity of the model that consumers bear no liability on fraudulent transactions ($\alpha_B = 0$). The issuers charge a fee $f^*(c_I - a)$ to the consumers, whereas the acquirers charge merchants with their perceived marginal cost, that is $m^* = a + c_A$. As in the literature, we make the standard assumption that f^* is decreasing with a, and that the pass-through rate is lower than one, that is $\partial f^* / \partial a \leq 1$. At the first stage of the game, the payment platform chooses the level of interchange fee that maximises banks' joint profit. Then banks choose the transaction prices, merchants invest in fraud detection technologies and consumers make their payments decisions. We denote the profit maximising interchange fee by a^P, and study how the profit maximising interchange fee is impacted by the level of liability that is borne by merchants.

Proposition 7 *If the issuers are imperfectly competitive and if the acquirers are perfectly competitive, the profit maximising interchange fee decreases with the level of liability that is borne by merchants.*

Proof. See Appendix L. ■

Proposition 4 has important implications for regulatory decisions about interchange fees. It means that, if merchants bear a higher share of the loss on fraudulent transactions, the profit maximising interchange fee becomes lower. The result of Proposition 7 may change if consumers are held liable for fraudulent transactions. In this case, merchants' investments are impacted by the transaction fees and by the interchange fee that is chosen by the payment

[27] Interchange fees are paid by the acquiring bank to the issuing bank each time a consumer makes a transaction.
[28] For instance, this assumption is also made in Rochet and Tirole (2002).

platform. The payment platform may decide either to lower or to increase the interchange fee to provide merchants with incentives to increase their investment in fraud detection technologies, depending on the relative importance of the expected loss effect and the transaction volume effect that we highlighted in Lemma 2.

Another interesting aspect of the problem is that regulators may wish to fix a maximum level for the interchange fee, but the payment platform can react by adjusting the level of liability that is borne by merchants for fraudulent transactions. In Appendix L, we show in a simple example that, if the regulator chooses a low level for the interchange fee, the payment platform reacts by choosing a high level of liability for merchants, which may not be desirable from the point of view of social welfare.

9 Conclusion and discussion

Our results highlight the fact that liability regimes can be used by monopolistic payment platforms to extract rents from merchants. From the point of view of a social planner, payment platforms do not place enough liability on merchants for investments that only depend on the merchants' side under the zero liability rule. This result changes if the platform shares the cost of investments with merchants.

Another issue that deserves further research is the problem of compliance in payment systems. This paper has considered only prices and liability regimes as an incentive to encourage merchant investment. However, we think that it would be interesting to compare the impact of different measures on investments and fraud losses such as compliance rules, price incentives and liability shifts.

10 References

References

[1] ARANGO, C. & TAYLOR, V. (2008): "Merchants Acceptance, Costs and Perception of Retail Payments: A Canadian Survey," Bank of Canada, Discussion Paper 2008-12.

[2] BEDRE-DEFOLIE O. & CALVANO E. (2009): "Pricing Payment Cards", ECB Working Paper No 119.

[3] BELLEFLAMME, P. & PEITZ, M. (2010): "Platform Competition and Seller Investment Incentives", Forthcoming in European Economic Review.

[4] BRYIS, E. & SCHLESINGER, H. (1990): "Risk Aversion and the Propensities for Self-Insurance and Self-Protection", Southern Economic Journal, 57, pp. 458-467.

[5] BROWN, J. P. (1973) "Toward an Economic Theory of Liability," The Journal of Legal Studies, 2,2: 323-349

[6] COOTER R. & ROBIN E. (1987), A Theory of Loss Allocation for Consumer Payments, Texas Law Review, 63-129

[7] CHAKRAVORTI (2010): "Externalities in Payment Card Networks: Theory and Evidence," Review of Network Economics, vol.9, Issue 2, Article 3.

[8] DARI-MATTIACCI G. & PARISI F. (2006), The Economics of Tort Law: A précis. The Elgar Companion to Law and Economics (2nd ed.), Edward Elgar Publishing.

[9] DIONNE, G. & EECKHOUDT, L. (1985): "Self-Insurance, Self-Protection and Increased Risk-Aversion," Economic Letters, 17, pp.39-42.

[10] DOUGLAS D.B. (2009): "An examination of the fraud liability shift in consumer card-based payment systems", Federal Reserve Bank of Chicago, Economic Perspectives, First Quarter 2009.

[11] EHLRICH, I. & BECKER, G. (1972): "Market Insurance, Self-Insurance & Self-Protection", The Journal of Political Economy, 80(4), pp. 623-648.

[12] FEDERAL RESERVE REGISTER: "Debit Card Interchange Fees and Routing: Proposed Rule", Vol. 75, No. 248, Tuesday, December 28, 2010.

[13] FURLETTI M. (2005): "The Laws, Regulation and Industry Practices that Protect Consumers who use Electronic Payment Systems: Policy Considerations", Discussion Paper, Payment Cards Center, Federal Reserve Bank of Philadelphia.

[14] GATES, T. & JACOB K. (2009): "Payments Fraud: Perception Versus Reality – A conference Summary", Tiffany Gates and Katy Jacob, 1st Quarter 2009, Economic Perspectives, Federal Reserve Bank of Chicago.

[15] LANDES, W.M. and POSNER, R.A. (1987), The Economic Structure of Tort Law, Cambridge (MA): Harvard University Press.

[16] Observatoire de la sécurité des cartes de paiement, Rapport Annuel 2009.

[17] ROCHET J-C. (2003): "The Theory of Interchange Fees: A synthesis of recent contributions," Review of Network Economics, vol. 2, no. 2, June, pp. 97-124.

[18] ROCHET J-C. & TIROLE J. (2002): "Cooperation Among Competitors: The Economics of Payment Card Associations," The RAND Journal of Economics, vol. 33, no. 4, winter, pp. 549-570.

[19] ROCHET J-C & TIROLE (2003): "Platform Competition in Two-Sided Markets," Journal of the European Economic Association, vol. 1, no. 4, pp. 990-1029.

[20] ROCHET J-C. & TIROLE J. (2006): "Two-sided markets: a progress report," RAND Journal of Economics, vol. 37, no. 3, March, pp. 645-667.

[21] SULLIVAN, R. J. (2010): "The Changing Nature of U.S Card Payment Fraud: Industry and Public Policy Options', Economic Review, Second Quarter 2010, Federal Reserve Bank of Kansas City.

[22] VERDIER M. (2010): "Interchange Fees and Incentives to Invest in Quality of a Payment Card System" (2010), International Journal of Industrial Organization, vol.28, pp. 539-554.

[23] VERDIER M. (2011): "Interchange Fees in Payment Card Systems: a Survey of the literature", Journal of Economic Surveys, Vol.25, Issue 2, pp.273-297.

[24] WRIGHT J. (2002): "Optimal Payment Card Systems," European Economic Review, vol. 47, no. 4, August, pp. 587-612.

[25] WRIGHT J. (2004): "Determinants of Optimal Interchange Fees in Payment Systems," Journal of Industrial Economics, vol. 52, no. 1, March, pp. 1-26.

11 Appendix

Appendix A: Consumer Protection Laws in Various Countries. The following table provides some examples of consumer protection laws in various countries. The common feature of consumer protection laws is that consumer bear hardly meaningful responsibility for fraudulent use of cards in all countries.

Country	Name of the Law	Consumer Protection
USA	TILA/Reg Z for credit cards	Capped at \$50 for all unauthorized transactions.
	Debit Cards	If the cardholder fails to notify the card issuer within 2 days, the cardholder's maximum liability is \$500, of which only \$50 can be attributed to fraud occurring during the first 2 days after the cardholder learnt the loss or theft.
Europe	Payment Service Directive	The cardholder has 13 months to contest an unauthorized transaction. The cardholder's liability is capped at 150 euros if he has failed to keep the personnalized security measures safe. If the cardholder was a victim from an identity theft, he cannot be held liable. No liability in all cases after the fraud is reported. Right for payment service users to enjoy immediate refund of unauthorized transactions following the establishment of the proof.

Appendix B: Proof of Lemma 1. We prove in Lemma 1 than the merchants who accept the EPI and the merchants who do not accept the EPI set the same price $p^* = v$. There are two cases: either a merchant refuses the EPI or he accepts it. Let us start by the first case. If a merchant refuses the EPI, all consumers pay cash, and he makes profit

$$\pi = p - d.$$

In this case, the merchant's profit is maximised when he sets $p = v$, and we have that

$$\pi = v - d.$$

In the second case, the merchant accepts the EPI. If he sets $p \leq v$, he attracts both EPI and cash users. In this case, he makes profit

$$\pi = p - d + D_B(f + \alpha_B(1-q)xL)(b_S - \alpha_S x(1-q)L - m - C_S(e_S)).$$

This profit is maximised at $p^* = v$.

If he sets $p > v$, the merchant attracts only EPI users. In this case, he makes profit

$$\pi = (p - d + b_S - \alpha_S x(1-q)L - m - C_S(e_S))D_B(f + p - v + \alpha_B(1-q)xL). \qquad (16)$$

We now show that the merchant always makes more profit by setting $p^* = v$. For this purpose, we prove that

$$\lim_{p \longrightarrow v} \frac{d\pi}{dp} < 0,$$

and that for any $p > v$, we have

$$\frac{d\pi}{dp} < 0.$$

From (16), we have:

$$\frac{d\pi}{dp} = D_B(f+p-v+\alpha_B(1-q)xL) - h_B(f+p-v+\alpha_B(1-q)xL)(p-d+b_S-\alpha_S x(1-q)L-m-C_S(e_S)).$$

We also have

$$\lim_{p \longrightarrow v} \frac{d\pi}{dp} = D_B(f+\alpha_B(1-q)xL) - h_B(f+\alpha_B(1-q)xL)(v-d+b_S-\alpha_S x(1-q)L-m-C_S(e_S)).$$

This quantity is negative if and only if

$$v - d + b_S - \alpha_S x(1-q)L - m - C_S(e_S) \geq \frac{1 - H_B(f + \alpha_B(1-q)xL)}{h_B(f + \alpha_B(1-q)xL)}. \qquad (17)$$

As the merchant accepts the EPI, we have that $b_S - \alpha_S x(1-q)L - m - C_S(e_S) \geq 0$. It follows that (A2) is a sufficient condition for (17) to hold.

We can now prove that for any $p > v$, $\frac{d\pi}{dp} < 0$. To simplify the notations, we denote by $\widetilde{D_B} = D_B(f + p - v + \alpha_B(1-q)xL)$. We have

$$\begin{aligned}
\frac{d\pi}{dp} &= \widetilde{D_B} - h_B(f + p - v + \alpha_B(1-q)xL)(p - d + b_S - \alpha_S x(1-q)L - m - C_S(e_S)) \\
&< \widetilde{D_B} - h_B(f + p - v + \alpha_B(1-q)xL)(v - d + b_S - \alpha_S x(1-q)L - m - C_S(e_S)) \\
&= \widetilde{D_B}\left[1 - \frac{h_B(f + p - v + \alpha_B(1-q)xL)}{1 - H_B(f + p - v + \alpha_B(1-q)xL)}(v - d + b_S - \alpha_S x(1-q)L - m - C_S(e_S))\right].
\end{aligned}$$

We have $\widetilde{D_B} \geq 0$. Therefore, a sufficient condition for $\frac{d\pi}{dp} < 0$ to hold is that the term into bracket is negative. The term into brackets is negative if and only if

$$v - d + b_S - \alpha_S x(1-q)L - m - C_S(e_S) \geq \frac{1 - H_B(f + p - v + \alpha_B(1-q)xL)}{h_B(f + p - v + \alpha_B(1-q)xL)}.$$

As by assumption (A1) the hazard rate is increasing, we have that, for any $p > v$,

$$\frac{1 - H_B(f + p - v + \alpha_B(1-q)xL)}{h_B(f + p - v + \alpha_B(1-q)xL)} \leq \frac{1 - H_B(f + \alpha_B(1-q)xL)}{h_B(f + \alpha_B(1-q)xL)}.$$

From assumption (A2), we have that

$$v - d + b_S - \alpha_S x(1-q)L - m - C_S(e_S) \geq \frac{1 - H_B(f + \alpha_B(1-q)xL)}{h_B(f + \alpha_B(1-q)xL)}.$$

It follows that

$$v - d + b_S - \alpha_S x(1-q)L - m - C_S(e_S) \geq \frac{1 - H_B(f + p - v + \alpha_B(1-q)xL)}{h_B(f + p - v + \alpha_B(1-q)xL)}.$$

Therefore, we have that, for any $p > v$, $\frac{d\pi}{dp} < 0$. It follows that the merchant makes more profit by setting $p^* = v$, which enables him to attract cash-users and EPI users. We can conclude that all merchants choose a price such that $p^* = v$.

Appendix C: proof of Lemma 2. We proceed in two steps. First, we determine the profit maximising level of investment of a merchant who accepts the EPI. Second, we prove that, if the merchant fee is not too high, some merchants accept the EPI. We start by the first step. A merchant who accepts the EPI chooses the level of investment in fraud detection technologies that maximises its profit,

$$\pi = p - d + D_B(f + \alpha_B(1-q)xL)(b_S - \alpha_S x(1-q)L - m - C_S(e_S)).$$

Solving for the first-order condition of profit-maximisation yields

$$\left[\alpha_S xL \left.\frac{dq}{de_S}\right|_{e_S^*} - C_S'(e_S^*)\right] D_B + [b_S - \alpha_S x(1-q)L - m - C_S(e_S^*)] \left.\frac{dD_B}{de_S}\right|_{e_S^*} = 0. \tag{18}$$

We define $\zeta_B = \frac{-dD_B/de_S}{D_B/e_S}$ the elasticity of the consumers' demand to the fraud detection prevention effort. The merchant's investment in fraud detection technologies is implicitely defined by

$$\alpha_S xL \left.\frac{dq}{de_S}\right|_{e_S^*} - C_S'(e_S^*) = [b_S - \alpha_S x(1-q)L - m - C_S(e_S^*)] \frac{\zeta_B|_{e_S^*}}{e_S^*}.$$

The second-order condition must be verified at e_S^*, that is,

$$\left[-C_S^{"}(e_S^*) + \alpha_S x L \left.\frac{d^2q}{d^2e_S}\right|_{e_S^*}\right] D_B + 2\left[\alpha_S x L \left.\frac{dq}{de_S}\right|_{e_S^*} - C_S^{'}(e_S^*)\right] \left.\frac{dD_B}{de_S}\right|_{e_S^*} + M_S \left.\frac{d^2D_B}{d^2e_S}\right|_{e_S^*} \leq 0, \tag{19}$$

where $M_S = b_S - \alpha_S x(1-q)L - m - C_S(e_S^*)$.

Under the assumption that C_S is convex and q is concave, the first term of this inequality is negative. To determine the sign of the second term, we use equation (18). Since $\zeta_B|_{e_S^*}$ is negative and since the merchant's margin is positive if he accepts the EPI, we conclude that $\alpha_S x L \left.\frac{dq}{de_S}\right|_{e_S^*} - C_S^{'}(e_S^*) \leq 0$ at the profit maximising level of investment. We have

$$\frac{\partial D_B}{\partial e_S} = h_B(\alpha_B x L(1-q^*) + f)\alpha_B x L \frac{dq}{de_S}, \tag{20}$$

and

$$\frac{\partial^2 D_B}{\partial e_S^2} = -h_B'(\alpha_B x L(1-q^*) + f)(\alpha_B x L \frac{dq}{de_S})^2 + h_B(\alpha_B x L(1-q^*) + f)\alpha_B x L \frac{d^2q}{de_S^2}. \tag{21}$$

From (20), we have $\left.\frac{dD_B}{de_S}\right|_{e_S^*} \geq 0$. It follows that the second term of (19) is negative. Finally, since $M_S \geq 0$ and since $\left.\frac{d^2D_B}{d^2e_S}\right|_{e_S^*} \leq 0$ from (21), the last term of (19) is negative. It follows that the second-order condition is always verified at e_S^*.

We now show that merchants accept the EPI if their transactional benefit b_S is such that $b_S \geq \widehat{b_S}(\alpha_S, \alpha_B, x, L, m, f)$, which is the second step of our proof. A merchant accepts the EPI if and only if

$$b_S - m - \alpha_S x(1-q)L - C_S(e_S^*) \geq 0. \tag{22}$$

Let us consider the function $M_S(y) = y - \alpha_S x(1-q)L - C_S(e_S^*)$, where $y = b_S - m$. Note that (22) does not hold if $y < 0$, which happens if the merchant fee is too high. We have that

$$M_S'(y) = 1 + \left.\frac{de_S}{dy}\right|_{e_S^*} \left(\alpha_S x L \left.\frac{dq}{de_S}\right|_{e_S^*} - C^{'}(e_S^*)\right).$$

From (18), we have that $\alpha_S x L \left.\frac{dq}{de_S}\right|_{e_S^*} - C^{'}(e_S^*) \geq 0$. We can also prove, using (18) and the envelop theorem that $\left.\frac{de_S}{dy}\right|_{e_S^*} \geq 0$. It follows that M_S is increasing in y for all $y \geq 0$. Note that $M_S(0) \leq 0$ and that the sign of $M_S(\overline{y})$, where $\overline{y} = \overline{b_S} - m$, depends on m. There are three cases.

Let us start by the first case, in which the merchant fee m is sufficiently high, such that

$M_S(\overline{y}) < 0$. As M_S is increasing in y, for all $b_S \in \left[\underline{b_S}, \overline{b_S}\right]$ and for all $y = b_S - m$, we have $M_S(y) < 0$. It follows that no merchant accepts the EPI.

In the second case, the merchant fee m is sufficiently low, such that $\underline{b_S} - m > 0$ and $M_S(\underline{y}) \geq 0$, where $\underline{y} = \underline{b_S} - m$. As M_S is increasing in y, for all $b_S \in \left[\underline{b_S}, \overline{b_S}\right]$ and for all $y = b_S - m$, we have $M_S(y) \geq 0$. It follows that all merchants accept the EPI.

In the third case, the merchant fee is such that $M_S(\overline{y}) > 0$ and $M_S(\underline{y}) < 0$. As M_S is increasing in y, from the bijection theorem, there exists a threshold that we denote by $\widehat{b_S}(\alpha_S, \alpha_B, x, L, m, f)$ such that merchants accept the EPI for all $b_S \geq \widehat{b_S}(\alpha_S, \alpha_B, x, L, m, f)$.

Appendix D: proof of Lemma 3. From the envelop theorem, we have that, for any $z \in \{\alpha_B, \alpha_S, f, m, b_S\}$

$$\frac{\partial e_S^*}{\partial z} = -\left(\left.\frac{\partial^2 \pi}{\partial^2 e_S}\right|_{e_S^*}\right)^{-1}\left(\left.\frac{\partial^2 \pi}{\partial e_S \partial z}\right|_{e_S^*}\right).$$

As from the second-order condition $\partial^2\pi/\partial^2 e_S \leq 0$, it follows that $\partial e_S^*/\partial z$ has the same sign as $\left.\frac{\partial^2 \pi}{\partial e_S \partial z}\right|_{e_S^*}$.

Let us study the variation of the merchant's investments with the merchant fee. We have that

$$\frac{\partial^2 \pi}{\partial e_S \partial m} = -h_B(f + \alpha_B(1-q)xL)\alpha_B xL\frac{dq}{de_S} \leq 0.$$

From the envelop theorem, $\partial e_S^*/\partial m$ has the same sign as $\partial^2\pi/\partial e_S\partial m$. It follows that the merchant's investment always decreases with the merchant fee. Similarly, we have that

$$\frac{\partial^2 \pi}{\partial e_S \partial b_S} = h_B(f + \alpha_B(1-q)xL)\alpha_B xL\frac{dq}{de_S} \geq 0. \tag{23}$$

It follows that the merchant's investments in fraud detection technologies increase with the merchant's transactional benefit.

We now study the variation of the merchant's investments with the consumer transaction fee. Using the same reasoning, we know that $\partial e_S^*/\partial f$ has the same sign as $\partial^2\pi/\partial e_S\partial f$. We have

$$\frac{\partial^2 \pi}{\partial e_S \partial f} = \left[\alpha_S xL\frac{dq}{de_S} - C_S'(e_S^*)\right]\frac{dD_B}{df} + \left[b_S - \alpha_S x(1-q)L - m - C_S(e_S^*)\right]\alpha_B xL\left.\frac{dq}{de_S}\right|_{e_S^*} h_B'(f + \alpha_B(1-q)xL).$$

From the first-order condition, we have that $\left[\alpha_S xL\left.\frac{dq}{de_S}\right|_{e_S^*} - C_S'(e_S^*)\right] \leq 0$. We also have $dD_B/df \leq 0$. It follows that $\partial^2\pi/\partial e_S\partial f \geq 0$ since h_B' is positive. We can conclude that the merchant's investment increases with the transaction fee that is paid by the consumer.

We determine the variation of the merchant's investments with the consumer liability. Using the same reasoning, $\partial e_S^*/\partial \alpha_B$ has the same sign as $\partial^2\pi/\partial e_S \partial \alpha_B$. We have

$$\frac{\partial^2 \pi}{\partial e_S \partial \alpha_B} = \left[\alpha_S x L \frac{dq}{de_S} - C_S'(e_S^*)\right] \frac{dD_B}{d\alpha_B} + \left[b_S - \alpha_S x(1-q)L - m - C_S(e_S^*)\right] \alpha_B (1-q) x^2 L^2 \left.\frac{dq}{de_S}\right|_{e_S^*} h_B'.$$

Exactly like in the previous proof, we have that $\partial^2\pi/\partial e_S \partial \alpha_B \geq 0$ since h_B' is positive. It follows that the merchant's investment increases with the consumer liability.

Let us study the variation of the merchant's investments with his level of liability. From the reasoning above, $\partial e_S^*/\partial \alpha_S$ has the same sign as $\partial^2\pi/\partial e_S \partial \alpha_S$. We have that

$$\begin{aligned} \left.\frac{\partial^2 \pi}{\partial e_S \partial \alpha_S}\right|_{e_S^*} &= xL \left.\frac{dq}{de_S}\right|_{e_S^*} D_B|_{e_S^*} - x^2 L^2 \alpha_B (1 - q(e_S^*)) \left.\frac{dq}{de_S}\right|_{e_S^*} h_B(f + \alpha_B(1 - q(e_S^*))xL) \\ &= xL\, D_B|_{e_S^*} \left.\frac{dq}{de_S}\right|_{e_S^*} \left[1 - xL\alpha_B(1 - q(e_S^*)) \frac{h_B(f + \alpha_B(1 - q(e_S^*))xL)}{D_B|_{e_S^*}}\right]. \end{aligned}$$

From assumption (A2), we have that

$$\frac{h_B(f + \alpha_B(1 - q(e_S^*))xL)}{D_B|_{e_S^*}} \leq \frac{1}{xL\alpha_B}.$$

As $1 - q(e_S^*) \in [0, 1]$, it follows that

$$\frac{h_B(f + \alpha_B(1 - q(e_S^*))xL)}{D_B|_{e_S^*}} \leq \frac{1}{xL\alpha_B(1 - q(e_S^*))}.$$

Therefore, we have that

$$1 - xL\alpha_B(1 - q(e_S^*)) \frac{h_B(f + \alpha_B(1 - q(e_S^*))xL)}{D_B|_{e_S^*}} \geq 0.$$

As $\left.\frac{dq}{de_S}\right|_{e_S^*} \geq 0$ and $xL\, D_B|_{e_S^*} \geq 0$, we can conclude that

$$\left.\frac{\partial^2 \pi}{\partial e_S \partial \alpha_S}\right|_{e_S^*} \geq 0.$$

It follows that, from assumption (A2), the merchant's investments in fraud detection technologies increase with his liability level.

Appendix E: Proof of Lemma 4.

Impact of the level of liability borne by merchants on EPI acceptance: From (2), the threshold above which merchants accept the EPI solves

$$\widehat{b_S} - m - \alpha_S x L(1-q^*) - C_S(e_S^*) = 0.$$

Differentiating this equation with respect to α_S, we obtain that

$$\frac{\partial \widehat{b_S}}{\partial \alpha_S}\left[1 + \left(\alpha_S x L \left.\frac{dq^*}{de_S}\right|_{e_S^*} - C_S^{'}(e_S^*)\right)\frac{de_S^*}{db_S}\right] = xL(1-q^*) + \frac{de_S^*}{d\alpha_S}\left(C_S^{'}(e_S^*) - \alpha_S x L \left.\frac{dq^*}{de_S}\right|_{e_S^*}\right). \tag{E-1}$$

From (18), we have that $C_S^{'}(e_S^*) - \alpha_S x L \left.\frac{dq^*}{de_S}\right|_{e_S^*} \geq 0$. From Lemma 3, we know that $de_S^*/d\alpha_S \geq 0$. It follows that the right-hand side of the equality is positive.

Let us now determine the sign of the left-hand side of the equality. From Lemma 3, we know that

$$\frac{de_S^*}{db_S} = -\left(\left.\frac{\partial^2 \pi}{\partial^2 e_S}\right|_{e_S^*}\right)^{-1}\left(\left.\frac{\partial^2 \pi}{\partial e_S \partial b_S}\right|_{e_S^*}\right). \tag{E-2}$$

Replacing for de_S^*/db_S in (E-1), we obtain that

$$1+\left(\alpha_S x L \left.\frac{dq^*}{de_S}\right|_{e_S^*} - C_S^{'}(e_S^*)\right)\frac{de_S^*}{db_S} = \left(\left.\frac{\partial^2 \pi}{\partial^2 e_S}\right|_{e_S^*}\right)^{-1}\left[\left.\frac{\partial^2 \pi}{\partial^2 e_S}\right|_{e_S^*} + \left(\alpha_S x L \left.\frac{dq^*}{de_S}\right|_{e_S^*} - C_S^{'}(e_S^*)\right)\left.\frac{\partial^2 \pi}{\partial e_S \partial b_S}\right|_{e_S^*}\right]. \tag{E-3}$$

Replacing for $\left.\frac{\partial^2 \pi}{\partial^2 e_S}\right|_{e_S^*}$ (from (19)) and $\left.\frac{\partial^2 \pi}{\partial e_S \partial b_S}\right|_{e_S^*}$ (from (23)), we obtain that the term into brackets in (E-3) is equal to

$$\left[-C_S^{''}(e_S^*) + \alpha_S x L \left.\frac{d^2 q}{d^2 e_S}\right|_{e_S^*}\right] D_B + 3\left[\alpha_S x L \left.\frac{dq}{de_S}\right|_{e_S^*} - C_S^{'}(e_S^*)\right]\left.\frac{dD_B}{de_S}\right|_{e_S^*} + M_S \left.\frac{d^2 D_B}{d^2 e_S}\right|_{e_S^*},$$

where

$$M_S = b_S - m - \alpha_S x L(1-q^*) - C_S(e_S^*).$$

As $\alpha_S x L \left.\frac{dq}{de_S}\right|_{e_S^*} - C_S^{'}(e_S^*) \leq 0$, $\left.\frac{dD_B}{de_S}\right|_{e_S^*} \geq 0$ from (20) and $\left.\frac{d^2 D_B}{d^2 e_S}\right|_{e_S^*} \leq 0$ from (21), it follows that the term into brackets in (E-3) is negative. As $\left.\frac{\partial^2 \pi}{\partial^2 e_S}\right|_{e_S^*} \leq 0$, we can conclude that

$$1 + \left(\alpha_S x L \left.\frac{dq^*}{de_S}\right|_{e_S^*} - C_S^{'}(e_S^*)\right)\frac{de_S^*}{db_S} \geq 0.$$

It follows from (E-2) that

$$\frac{\partial \widehat{b_S}}{\partial \alpha_S} \geq 0.$$

Therefore, the probability that a merchant accepts the EPI decreases with the level of liability that is borne by the merchants.

Impact of the transaction benefit received by merchants on EPI acceptance: Similarly, we have that

$$\frac{\partial \widehat{b_S}}{\partial f}\left[1+\left(\alpha_S xL\left.\frac{dq^*}{de_S}\right|_{e_S^*}-C_S^{'}(e_S^*)\right)\frac{de_S^*}{db_S}\right]=\frac{de_S^*}{df}\left(C_S^{'}(e_S^*)-\alpha_S xL\left.\frac{dq^*}{de_S}\right|_{e_S^*}\right).$$

As $de_S^*/df \geq 0$, it follows that $\partial\widehat{b_S}/\partial f \geq 0$. It can be also proved in a similar way that $\partial\widehat{b_S}/\partial\alpha_B \geq 0$.

Impact of the transaction fees on EPI acceptance: We also have that

$$\begin{aligned}\frac{\partial \widehat{b_S}}{\partial m}\left[1+\left(\alpha_S xL\left.\frac{dq^*}{de_S}\right|_{e_S^*}-C_S^{'}(e_S^*)\right)\frac{de_S^*}{db_S}\right] &= 1+\frac{de_S^*}{dm}\left(C_S^{'}(e_S^*)-\alpha_S xL\left.\frac{dq^*}{de_S}\right|_{e_S^*}\right)\\ &= \frac{\left.\frac{\partial^2\pi}{\partial^2 e_S}\right|_{e_S^*}+\frac{dD_B}{de_S}\left(C_S^{'}(e_S^*)-\alpha_S xL\left.\frac{dq^*}{de_S}\right|_{e_S^*}\right)}{\left.\frac{\partial^2\pi}{\partial^2 e_S}\right|_{e_S^*}}\geq 0.\end{aligned}$$

It follows that $\partial\widehat{b_S}/\partial m \geq 0$.

Appendix F: Proof of Proposition 1. We start by determining the variation of the expected loss with the consumer fee. To that end, we denote by $\mu(b_S, f, m, \alpha_S, \alpha_B)$ the function defined by

$$\mu(b_S, f, m, \alpha_S, \alpha_B) = (1-q^*)h(b_S)(1-H_B(f+\alpha_B xL(1-q^*)).$$

From (6),

$$EL_P = \alpha_P xL\int_{\widehat{b_S}}^{\overline{b_s}}\mu(b_S, f, m, \alpha_S, \alpha_B)db_S.$$

Hence, we have that[29]

$$\frac{\partial EL_P}{\partial f}=\alpha_P xL\left[\frac{-\partial\widehat{b_S}}{\partial f}\mu(\widehat{b_S}, f, m, \alpha_S, \alpha_B)+\int_{\widehat{b_S}}^{\overline{b_s}}\frac{\partial\mu(b_S, f, m, \alpha_S, \alpha_B)}{\partial f}db_S\right],$$

[29] The conditions to use the Leibniz rule apply.

where

$$\begin{aligned}\frac{\partial\mu(b_S,f,m,\alpha_S,\alpha_B)}{\partial f} &= \frac{-dq}{de_S}\frac{\partial e_S^*}{\partial f}(1-H_B(f+\alpha_B xL(1-q^*))h(b_S)\left[1-\frac{(1-q^*)\alpha_B xLh_B}{1-H_B}\right]+\\ &\quad -(1-q^*)h(b_S)h_B(f+\alpha_B xL(1-q^*)).\end{aligned}$$

From Lemma 3, we have $\partial e_S^*/\partial f \geq 0$. We also have that $dq/de_S \geq 0$. From assumption (A2), we have

$$1-\frac{(1-q^*)\alpha_B xLh_B}{1-H_B}\geq 0.$$

It follows that $\partial\mu(b_S,f,m,\alpha_S,\alpha_B)/\partial f \leq 0$. Since, from Lemma 4, $\partial\widehat{b_S}/\partial f \geq 0$, we conclude that $\partial EL/\partial f \leq 0$.

We now determine the variations of the expected loss with the merchant fee. We have

$$\frac{\partial EL_P}{\partial m}=\alpha_P xL\left[\underbrace{\frac{-\partial\widehat{b_S}}{\partial m}\mu(\widehat{b_S},f,m,\alpha_S,\alpha_B)}_{TermA}+\underbrace{\int_{\widehat{b_S}}^{\overline{b_s}}\frac{\partial\mu(b_S,f,m,\alpha_S,\alpha_B)}{\partial m}db_S}_{TermB}\right],$$

where

$$\frac{\partial\mu(b_S,f,m,\alpha_S,\alpha_B)}{\partial m}=\frac{-dq}{de_S}\frac{\partial e_S^*}{\partial m}(1-H_B(f+\alpha_B xL(1-q^*))h(b_S)\left[1-\frac{(1-q^*)\alpha_B xLh_B}{1-H_B}\right]\geq 0.$$

From Lemma 3, we have $\partial e_S^*/\partial m \leq 0$. From Lemma 4, $\partial\widehat{b_S}/\partial m \geq 0$. From Assumption (A2), we have

$$1-\frac{(1-q^*)\alpha_B xLh_B}{1-H_B}\geq 0.$$

It follows that term A is negative, whereas term B is positive. Therefore, an increase in the merchant fee has an ambiguous impact on the expected loss that is borne by the payment platform.

Let us now study how the level of liability that is borne by merchants impacts the expected loss. We have

$$\frac{\partial EL_P}{\partial\alpha_S}=-\frac{EL_P}{\alpha_P}+\alpha_P xL\left[\underbrace{\frac{-\partial\widehat{b_S}}{\partial\alpha_S}\mu(\widehat{b_S},f,m,\alpha_S,\alpha_B)}_{TermC}+\underbrace{\int_{\widehat{b_S}}^{\overline{b_s}}\frac{\partial\mu(b_S,f,m,\alpha_S,\alpha_B)}{\partial\alpha_S}db_S}_{TermD}\right],$$

where

$$\frac{\partial\mu(b_S,f,m,\alpha_S,\alpha_B)}{\partial\alpha_S}=\frac{-dq}{de_S}\frac{\partial e_S^*}{\partial\alpha_S}(1-H_B(f+\alpha_B xL(1-q^*))h(b_S)\left[1-\frac{(1-q^*)\alpha_B xLh_B}{1-H_B}\right]\leq 0.$$

From Assumption (A2),

$$1-\frac{(1-q^*)\alpha_B x L h_B}{1-H_B}\geq 0.$$

As $dq/de_S \geq 0$, and since $\partial \widehat{b_S}/\partial \alpha_S \geq 0$ from Lemma 4, it follows that

$$\frac{\partial EL_P}{\partial \alpha_S}\leq 0.$$

Appendix G: Second-order conditions if $\alpha_B = 0$.

Appendix G-A: second-order conditions if the payment platform chooses the transaction prices. We provide here the second-order conditions of profit maximisation if $\alpha_B = 0$. The first-order conditions of profit maximisation are

$$\frac{\partial \pi_P}{\partial m}=D_B(f)\left[D_S(b_S^m)-M_P h_S(b_S^m)\right]=0, \tag{24}$$

and

$$\frac{\partial \pi_P}{\partial f}=D_S(b_S^m)\left[D_B(f)-M_P h_B(f)\right]=0. \tag{25}$$

The second derivatives of the platform's profit with respect to the prices and the liability level are

$$\begin{aligned}
\frac{\partial^2 \pi_P}{\partial m^2} &= -2h_S D_B - h_S' D_B M_P, \qquad (26)\\
\frac{\partial^2 \pi_P}{\partial f^2} &= -2h_B D_S - h_B' D_S M_P,\\
\frac{\partial^2 \pi_P}{\partial m \partial f} &= -h_B D_S - h_S D_B + M_P h_S h_B,\\
\frac{\partial^2 \pi_P}{\partial m \partial \alpha_S} &= xL(1-q^*)\frac{\partial^2 \pi_P}{\partial m^2}-(1-\alpha_S)xL\frac{\partial q^*}{\partial \alpha_S}h_S D_B,\\
\frac{\partial^2 \pi_P}{\partial f \partial \alpha_S} &= xL(1-q^*)\frac{\partial^2 \pi_P}{\partial m \partial f}-(1-\alpha_S)xL\frac{\partial q^*}{\partial \alpha_S}h_B D_S,\\
\frac{\partial^2 \pi_P}{\partial \alpha_S^2} &= -2xLD_B h_S xL(1-q^*)\left[1-q^*+(1-\alpha_S)\frac{\partial q^*}{\partial \alpha_S}\right]+M_P D_B\left[xLh_S\frac{\partial q^*}{\partial \alpha_S}-(xL(1-q^*))^2 h_S'\right]\\
&\quad +xLV_P\left[-2\frac{\partial q^*}{\partial \alpha_S}+(1-\alpha_S)\left\{\frac{\partial^2 q}{\partial e_S^2}\left(\frac{\partial e_S}{\partial \alpha_S}\right)^2+\frac{\partial q}{\partial e_S}\frac{\partial^2 e_S}{\partial \alpha_S^2}\right\}\right].
\end{aligned}$$

We denote by $\det M$ the determinant of the Hessian matrix at the profit maximising transaction fees. It can be checked that the second-order conditions of profit maximisation are verified as $h_S' \geq 0$ and $h_B' \geq 0$. From (24) and (25), we have that, at the profit maximising prices,

$D_S = M_P h_S$ and $D_B = M_P h_B$. Therefore, we have

$$\det M|_{(f^*,m^*)} = 2h_S h_B D_S D_B + 2h'_B D_B D_S^2 + 2h'_S D_S D_B^2 + h'_S h'_B D_B D_S M_P^2 + h_S^2 D_B^2 > 0, \quad (27)$$

and

$$\left.\frac{\partial^2 \pi}{\partial m^2}\right|_{(f^*,m^*)} < 0,$$

which proves that the conditions for a maximum to exist at (f^*, m^*) hold.

Appendix G-B: second-order conditions if the payment platform chooses the transaction prices and the level of liability for merchants. We provide here the conditions under which the second-order conditions are verified at $y^* = (f^*; m^*; \alpha_S^*)$ by computing the coefficients of the Hessian matrix.

Denoting the Hessian matrix at $y^* = (f^*; m^*; \alpha_S^*)$ by $H = \begin{pmatrix} a_1 & b & c \\ b & a_2 & d \\ c & d & a_3 \end{pmatrix}$, the second-order conditions are verified if $a_1 \leq 0$, $a_2 \leq 0$, $a_1 a_2 - b^2 \geq 0$, $a_1 a_3 - c^2 \geq 0$, $a_3 a_2 - d^2 \geq 0$ and $\det H \leq 0$ (See hereafter). If these conditions are verified, this proves that the Hessian matrix is semi-definite negative at $y^* = (f^*; m^*; \alpha_S^*)$.

Let us start by the case in which there is an interior solution. From (26), as h'_S and h'_B are positive, we have that $a_1 \leq 0$ and $a_2 \leq 0$. We already proved in Appendix G-A that $a_1 a_2 - b^2 \geq 0$. We now prove that $a_1 a_3 - c^2 \geq 0$ and that $a_3 a_2 - d^2 \geq 0$.

At $y^* = (f^*; m^*; \alpha_S^*)$, if the solution is interior, from (24) and (25), we have that $D_S = M_P h_S$ and $D_B = M_P h_B$. The first-order condition of profit maximisation with respect to α_S is

$$\frac{\partial \pi_P}{\partial \alpha_S} = (f + m - c)\frac{\partial V_P}{\partial \alpha_S} - \frac{\partial EL_P}{\partial \alpha_S} = 0.$$

From (7), we have that

$$\frac{\partial EL_P}{\partial \alpha_S} = -xL(1 - q^*)V_P + (1 - \alpha_S)xLq^* \frac{\partial V_P}{\partial \alpha_S}.$$

From (5), we have that $-\partial V / \partial \alpha_S = D_B(f) h_S(\widehat{b_S}) xL(1 - q^*)$. Therefore, the first-order condition with respect to α_S writes

$$M_P h_S D_B xL(1 - q^*) = xLD_B D_S \left(1 - q^* + (1 - \alpha_S)\frac{dq}{d\alpha_S}\right).$$

As at $y^* = (f^*; m^*; \alpha_S^*)$ we have $D_S = M_P h_S$, in interior solution, we have that

$$xL(1-q^*) = xL\left(1 - q^* + (1-\alpha_S)\frac{dq}{d\alpha_S}\right),$$

that is

$$\left.\frac{dq}{d\alpha_S}\right|_{x^*} = 0.$$

It follows that, in an interior solution, the payment platform chooses the level of liability that maximises the probability of fraud detection.

We denote by

$$\mu = -2\frac{\partial q^*}{\partial \alpha_S} + (1-\alpha_S)\left\{\frac{\partial^2 q}{\partial e_S^2}\left(\frac{\partial e_S}{\partial \alpha_S}\right)^2 + \frac{\partial q}{\partial e_S}\frac{\partial^2 e_S}{\partial \alpha_S^2}\right\}. \tag{GB-1}$$

Lemma 6 *We have $\partial^2 e_S^*/\partial \alpha_S^2 \leq 0$.*

Proof. From the implicit function theorem, we have

$$\begin{aligned}
\frac{\partial e_S^*}{\partial \alpha_S} &= -\left(\left.\frac{\partial^2 \pi}{\partial^2 e_S}\right|_{e_S^*}\right)^{-1}\left(\left.\frac{\partial^2 \pi}{\partial e_S \partial \alpha_S}\right|_{e_S^*}\right) \\
&= \frac{xL(\partial q/\partial e_S)D_B}{D_B(\alpha_S xL \left.\frac{d^2 q}{de_S^2}\right|_{e_S^*} - C_S^{''}(e_S^*))} \\
&= \frac{xL(\partial q/\partial e_S)}{(\alpha_S xL \left.\frac{d^2 q}{de_S^2}\right|_{e_S^*} - C_S^{''}(e_S^*))}.
\end{aligned}$$

It follows that

$$\frac{\partial^2 e_S^*}{\partial \alpha_S^2} = \frac{xLN}{(\alpha_S xL \left.\frac{d^2 q}{de_S^2}\right|_{e_S^*} - C_S^{''}(e_S^*))^2},$$

where

$$\begin{aligned}
N &= xL\left(-\alpha_S \left.\frac{d^2 q}{de_S^2}\right|_{e_S^*} + C_S^{''}(e_S^*)\right)\left.\frac{d^2 q}{de_S^2}\right|_{e_S^*}\frac{\partial e_S}{\partial \alpha_S} \\
&\quad - \left[-\alpha_S xL \left.\frac{d^3 q}{de_S^3}\right|_{e_S^*}\frac{\partial e_S}{\partial \alpha_S} + C_S^{'''}(e_S^*)\right] xL \left.\frac{dq}{de_S}\right|_{e_S^*}.
\end{aligned}$$

Since $\left.\frac{d^3 q}{de_S^3}\right|_{e_S^*} \leq 0$, $\left.\frac{d^2 q}{de_S^2}\right|_{e_S^*} \leq 0$, and $C_S^{'''}(e_S^*) \geq 0$, we have $N \leq 0$. Therefore, we can conclude that $\partial^2 e_S/\partial \alpha_S^2 \leq 0$. ■

Since $\partial^2 e_S/\partial \alpha_S^2 \leq 0$, $\partial q/\partial e_S \geq 0$ and $\partial^2 q/\partial e_S^2 \leq 0$, from (GB-1), we have $\mu \leq 0$. As

$\left.\frac{dq}{d\alpha_S}\right|_{x^*} = 0$ and from (26), we have that

$$\begin{aligned}
\left.\frac{\partial^2 \pi_P}{\partial \alpha_S^2}\right|_{x^*} &= -(xL(1-q^*))^2 \left[2h_S + M_P h_S'\right] D_B + xLV_P\mu, \\
\left.\frac{\partial^2 \pi_P}{\partial \alpha_S \partial f}\right|_{x^*} &= -xL(1-q^*)h_S D_B,
\end{aligned}$$

and

$$\left.\frac{\partial^2 \pi_P}{\partial \alpha_S \partial m}\right|_{x^*} = -xL(1-q^*)D_B(2h_S + h_S' M_P).$$

We now compute $a_1a_3 - c^2$ and $a_3a_2 - d^2$ at $y^* = (f^*; m^*; \alpha_S^*)$. We have

$$\begin{aligned}
a_1a_3 - c^2 &= \left(\left.\frac{\partial^2 \pi_P}{\partial f^2}\right|_{x^*}\right)\left(\left.\frac{\partial^2 \pi_P}{\partial \alpha_S^2}\right|_{x^*}\right) - \left(\left.\frac{\partial^2 \pi_P}{\partial \alpha_S \partial f}\right|_{x^*}\right)^2 \\
&= D_B^2 \left[xL(1-q^*)\right]^2 (3h_S^2 + (M_P h_S')^2 + 4M_P h_S' h_S) - D_B^2 D_S \mu xL \left(2h_S + M_P h_S'\right) \geq 0.
\end{aligned}$$

We also have

$$\begin{aligned}
a_3a_2 - d^2 &= \left(\left.\frac{\partial^2 \pi_P}{\partial m^2}\right|_{x^*}\right)\left(\left.\frac{\partial^2 \pi_P}{\partial \alpha_S^2}\right|_{x^*}\right) - \left(\left.\frac{\partial^2 \pi_P}{\partial \alpha_S \partial m}\right|_{x^*}\right)^2 \\
&= -D_B^2 D_S \mu xL \left(2h_S + M_P h_S'\right) \geq 0.
\end{aligned}$$

We now show that $\det H \leq 0$ at $y^* = (f^*; m^*; \alpha_S^*)$. From the rule of Sarrus, we have

$$\begin{aligned}
\det H &= a_1a_2a_3 + 2bdc - c^2a_2 - b^2a_3 - d^2a_1 \\
&= a_1(a_3a_2 - d^2) + 2bdc - c^2a_2 - b^2a_3.
\end{aligned}$$

At $y^* = (f^*; m^*; \alpha_S^*)$, since $a_1 = -D_S\left(2h_B + M_P h_B'\right)$, we have

$$a_1(a_3a_2 - d^2) = D_B^2 D_S^2 \mu xL \left(2h_S + M_P h_S'\right)\left(2h_B + M_P h_B'\right).$$

We also have

$$2bdc = -2(xL)^2(1-q^*)^2 h_S^2 D_B^3 \left(2h_S + M_P h_S'\right),$$

and

$$-c^2a_2 - b^2a_3 = 2(xL)^2(1-q^*)^2 h_S^2 D_B^3 \left(2h_S + M_P h_S'\right) - \mu xL D_B^3 D_S h_S^2.$$

Using the fact that, at $y^* = (f^*; m^*; \alpha_S^*)$, we have $M_P h_S = D_S$ and $M_P h_B = D_B$, we obtain that

$$\det H = D_B^2 D_S \mu xL M_P \left[3h_B h_S^2 + h_S' h_B' M_P^2 h_S + 2h_S h_B h_S' M_P + 2h_S^2 h_B' M_P\right]. \tag{28}$$

Since $\mu \leq 0$, we can conclude that $\det H \leq 0$ at $y^* = (f^*; m^*; \alpha_S^*)$. Therefore, the Hessian matrix is semi-definite negative at $y^* = (f^*; m^*; \alpha_S^*)$ and the second-order conditions are verified at $y^* = (f^*; m^*; \alpha_S^*)$.

Appendix G-C: Variation of the equilibrium prices with the fraud rate. We differentiate the first-order conditions of profit maximisation with respect to f and m, which are evaluated at $\alpha_S^* = 1$. Since $\partial b_S^m / \partial x = L(1 - q^*)$, we have

$$-h_B(f) D_S(b_S^m) \frac{\partial m^*}{\partial x} + \frac{\partial f^*}{\partial x} D_S(b_S^m)(-2h_B(f) - M_P h_B^{'}) = 0, \qquad \text{(G-C-1)}$$

and

$$-L(1 - q^*)(h_S(b_S^m) + M_P h_S^{'}(b_S^m)) \frac{\partial m^*}{\partial x} + \frac{\partial f^*}{\partial x}(-h_S(b_S^m) D_B(f)) = 0. \qquad \text{(G-C-2)}$$

From (G-C-2),

$$\frac{\partial f^*}{\partial x} h_S(b_S^m) = -\frac{\partial m^*}{\partial x} L(1 - q^*)(h_S(b_S^m) + M_P h_S^{'}(b_S^m)).$$

Since $h_S(b_S^m) \geq 0$, $L(1 - q^*) \geq 0$ and $h_S^{'}(b_S^m) \geq 0$, $\partial f^* / \partial x$ and $\partial m^* / \partial x$ have opposite signs. Therefore, the consumer fee and the merchant fee vary in opposite directions with the fraud rate.

Appendix H: An illustration of Proposition 2. We make the following assumptions: $C_S(e_S) = k(e_S)^2/2$, $q(e_S) = \gamma e_S$, uniform distributions on $[0, 1]$ for b_S and b_B. In this case, from equation (4), we have

$$e_S^* = \frac{\alpha_S x L \gamma}{k},$$

where $\alpha_S x L \gamma^2 / k \leq 1$. The merchant's effort increases with the liability level, the probability that there is a fraudulent transaction, the losses borne by the participants, and the marginal impact of investments on the probability of fraud detection. The probability that a merchant detects a fraudulent transaction is implicitely defined by

$$q(e_S^*) = (\alpha_S x L \gamma^2)/k.$$

In this case, the demands are $D_B = 1 - f$, and $D_S = 1 - m - \alpha_S x L + \dfrac{(\alpha_S x L \gamma)^2}{2k}$. Using the standard price structure/ratio formula, we find that the prices verify

$$f = m + \alpha_S xL - \frac{(\alpha_S xL\gamma)^2}{2k},$$

and

$$\frac{f + m - c - (1-\alpha_S)xL(1-q)}{f} = \frac{1-f}{f}.^{30}$$

Solving for f and m, we obtain that

$$m = \frac{1 + c + xL(1-3\alpha_S) + \frac{(xL\gamma)^2}{k}(2\alpha_S^2 - \alpha_S)}{3},$$

and

$$f = \frac{1 + c + xL + \frac{(\alpha_S^2 - 2\alpha_S)(xL\gamma)^2}{2k}}{3}.$$

We can compute the marginal merchant

$$\begin{aligned}\widehat{b_S} &= m + \alpha_S xL - \frac{(\alpha_S xL\gamma)^2}{2k} \\ &= \frac{1 + c + xL + \frac{(\alpha_S^2 - 2\alpha_S)(xL\gamma)^2}{2k}}{3}.\end{aligned}$$

The merchant demand is

$$D_S(\widehat{b_S}) = \frac{2(1+c) - xL - \frac{(\alpha_S^2 - 2\alpha_S)(xL\gamma)^2}{2k}}{3}.$$

If α_S is chosen by the payment platform (at the same stage as the prices), we have that

$$\frac{\partial \pi_P}{\partial \alpha_S} = M_P \left[\frac{\partial D_S(\widehat{b_S})}{\partial \alpha_S} D_B(f)\right] + \frac{\partial M_P}{\partial \alpha_S} D_B(f) D_S(\widehat{b_S}).$$

As $M_P = 1 - f = D_B = D_S$ at the optimal prices, we have

$$\frac{\partial \pi_P}{\partial \alpha_S} = 2M_P D_B(f) \frac{\partial D_S(\widehat{b_S})}{\partial \alpha_S} \geq 0.$$

In this case, we find that the platform's profit is maximised by choosing $\alpha_S = 1$.

Appendix I: Impact of the merchants' liability on transaction prices under the zero liability rule In this Appendix, we examine how the level of liability borne by merchants impacts the transaction fees that are chosen by the payment platform, if the zero liability rule applies for consumers. By differentiating equations (24) and (25) that define the first-order conditions with respect to α_S, we obtain that

$$\frac{\partial^2\pi_P}{\partial m^2}\frac{\partial m^*}{\partial \alpha_S}+\frac{\partial^2\pi_P}{\partial m\partial f}\frac{\partial f^*}{\partial \alpha_S}+\frac{\partial^2\pi_P}{\partial m\partial \alpha_S}=0, \tag{29}$$

and

$$\frac{\partial^2\pi_P}{\partial f^2}\frac{\partial f^*}{\partial \alpha_S}+\frac{\partial^2\pi_P}{\partial m\partial f}\frac{\partial m^*}{\partial \alpha_S}+\frac{\partial^2\pi_P}{\partial f\partial \alpha_S}=0. \tag{30}$$

Solving for $\partial m^*/\partial\alpha_S$ and $\partial f^*/\partial\alpha_S$ in (29) and (30), we obtain that

$$\frac{\partial m^*}{\partial \alpha_S}=\frac{1}{\det M}\left[xL(1-q^*)(-\det M)-(1-\alpha_S)xL\frac{\partial q^*}{\partial \alpha_S}R\right],$$

and

$$\frac{\partial f^*}{\partial \alpha_S}=\frac{1}{\det M}\left[-(1-\alpha_S)xL\frac{\partial q^*}{\partial \alpha_S}T\right],$$

where

$$R=h_BD_S\frac{\partial^2\pi_P}{\partial m\partial f}-h_SD_B\frac{\partial^2\pi}{\partial f^2},$$

and

$$T=h_SD_B\frac{\partial^2\pi_P}{\partial m\partial f}-h_BD_S\frac{\partial^2\pi_P}{\partial m^2}.$$

We proved in Appendix G-A that $\det M\geq 0$. We now prove that $R\geq 0$ and $T\geq 0$. We have

$$R=-h_B^2D_S^2+h_Bh_SD_BD_S+M_Ph_B^2h_SD_S+h_Sh_B'D_SD_BM_P,$$

and

$$T=h_Bh_SD_BD_S-h_S^2D_B^2+M_Ph_S^2h_BD_B+h_Bh_S'D_BD_SM_P$$

Using the first-order condition, we have that, at the profit maximising prices, $M_Ph_S=D_S$ and $M_Ph_B=D_B$. It follows that, at the profit maximising prices,

$$R=h_Bh_SD_BD_S+h_Sh_B'D_SD_BM_P,$$

and

$$T=h_Bh_SD_BD_S+h_Bh_S'D_BD_SM_P.$$

Since h_S' and h_B' are positive, we have $R\geq 0$ and $T\geq 0$. As $\partial q^*/\partial\alpha_S\geq 0$ and $DetM>0$, it follows that $\partial m^*/\partial\alpha_S\leq 0$ and $\partial f^*/\partial\alpha_S\leq 0$. Hence, Lemma 5 is verified.

Note that, since $\widehat{b_S} = m + \alpha_S xL(1-q^*) + C_S(e_S^*)$, we have

$$\begin{aligned} \frac{d\widehat{b_S}}{d\alpha_S} &= \frac{\partial \widehat{b_S}}{\partial \alpha_S} + \frac{\partial m^*}{\partial \alpha_S} \\ \frac{d\widehat{b_S}}{d\alpha_S} &= \frac{1}{\det M}\left[-(1-\alpha_S)xL\frac{\partial q^*}{\partial \alpha_S}R\right] \leq 0, \end{aligned} \tag{31}$$

as $\partial\widehat{b_S}/\partial\alpha_S = xL(1-q^*)$.

If b_B and b_S are uniformely distributed on $[0,1]$, from the first order conditions, at the profit maximising prices, we have $D_B = D_S = M_P$. In this case, we have $R = T = D_B^2$. Therefore, from (27), we have $\det M = 3D_B^2$. It follows that, in this example, we have

$$\frac{d\widehat{b_S}}{d\alpha_S} = \frac{df}{d\alpha_S} = \frac{-(1-\alpha_S)}{3}xL\frac{\partial q^*}{\partial \alpha_S}. \tag{32}$$

It follows from (32) that

$$\frac{d^2\widehat{b_S}}{d\alpha_S^2} = \frac{d^2 f}{d\alpha_S^2} = \frac{xL}{3}\left[\frac{\partial q^*}{\partial \alpha_S} - (1-\alpha_S)\frac{\partial^2 q^*}{\partial^2 \alpha_S}\right] \geq 0. \tag{33}$$

From Lemma 6, we have $\partial^2 q^*/\partial^2\alpha_S \leq 0$. Therefore, we can conclude that $d^2\widehat{b_S}/d\alpha_S^2 \geq 0$ in the case of uniform distributions on $[0,1]$ for the transactional benefits.

Appendix J: Social welfare analysis.

Appendix J-A:Variation of the consumer and the merchant surplus with the level of liability borne by merchants. We start by computing the consumer surplus. Consumers who pay cash do not obtain any surplus from making a transaction, as a monopolistic merchant sets a price $p^* = v$. A consumer of transactional benefit b_B who pays with the EPI obtains a surplus

$$b_B - f - \alpha_B xL(1-q^*).$$

Agregating this expression over all $b_B \in \left[f + \alpha_B xL(1-q^*), \overline{b_B}\right]$ and over all $b_S \in \left[\widehat{b_S}, \overline{b_S}\right]$, we obtain the agregate consumer surplus, that is

$$S_B = \int_{\widehat{b_S}}^{\overline{b_s}} h(b_S)E(b_B - f - \alpha_B xL(1-q^*)/b_B \geq f + \alpha_B xL(1-q^*))db_S,$$

where $E(b_B - f - \alpha_B xL(1-q^*)/b_B \geq f + \alpha_B xL(1-q^*))$ denotes the mathematical expectancy conditional on $b_B \geq f + \alpha_B xL(1-q^*)$. We have

$$\frac{\partial S_B}{\partial \alpha_S} = X + Y,$$

where

$$\begin{aligned}
X &= \frac{-d\widehat{b_S}}{d\alpha_S} h(\widehat{b_S}) E(b_B - f - \alpha_B xL(1-q^*)/b_B \geq f + \alpha_B xL(1-q^*)), \\
Y &= \int_{\widehat{b_S}}^{\overline{b_s}} h(b_S) \frac{\partial}{\partial \alpha_S} E(b_B - f - \alpha_B xL(1-q^*)/b_B \geq f + \alpha_B xL(1-q^*)) db_S,
\end{aligned}$$

where from the Leibniz rule,

$$\begin{aligned}
\frac{\partial}{\partial \alpha_S} E(b_B - f - \alpha_B xL(1-q^*)/b_B \geq \widehat{b_B}) &= \frac{\partial}{\partial \alpha_S} \int_{\widehat{b_B}}^{\overline{b_B}} (b_B - f - \alpha_B xL(1-q^*)) h_B(b_B) db_B \\
&= \int_{\widehat{b_B}}^{\overline{b_B}} \left(-\frac{\partial f^*}{\partial \alpha_S} + \alpha_B xL \frac{dq}{de_S} \frac{\partial e_S}{\partial \alpha_S} \right) h_B(b_B) db_B \geq 0.
\end{aligned}$$

First case $\alpha_B = 0$. In this case, from proposition 5, the transaction fees decrease with the level of liability that is borne by merchants. Therefore, Y is positive. Term X is also positive, since $d\widehat{b_S}/d\alpha_S \leq 0$ from (31). It follows that the consumer surplus increases with the level of liability that is borne by the merchants.

Second case: $\alpha_B \neq 0$ [TO DO]

Similarly, we compute the agregate merchant surplus by agregating the merchants' profit for all $b_S \in \left[\underline{b_S}, \overline{b_S}\right]$. We have

$$S_S = (v-d)(\overline{b_S} - \underline{b_S}) + \int_{\widehat{b_S}}^{\overline{b_s}} h(b_S)(b_S - m - \alpha_S xL(1-q^*) - C_S(e_S^*))(1 - H_B(f + \alpha_B xL(1-q^*)))db_S.$$

From the Leibniz rule, we have

$$\begin{aligned}
\frac{\partial S_S}{\partial \alpha_S} &= \int_{\widehat{b_S}}^{\overline{b_B}} h(b_S) \frac{\partial}{\partial \alpha_S} \left\{ (b_S - m - \alpha_S xL(1-q^*) - C_S(e_S^*))(1 - H_B(f + \alpha_B xL(1-q^*))) \right\} db_S \\
&= \int_{\widehat{b_S}}^{\overline{b_S}} h(b_S) \left\{ -\left[\frac{\partial m^*}{\partial \alpha_S} + xL(1-q^*) \right] D_B(\widehat{b_B}) + (b_S - m - \alpha_S xL(1-q^*) - C_S(e_S^*)) h_B'(\widehat{b_B}) \frac{dq^*}{d\alpha_S} \right\} db_S.
\end{aligned}$$ [3]

First case $\alpha_B = 0$. In this case, from proposition 5, the transaction fees decrease with the level of liability that is borne by merchants. From (31), we have $\partial m^*/\partial \alpha_S + xL(1-q^*) \leq 0$. It follows that the merchant surplus increases with the level of liability that is borne by merchants.

Second case: $\alpha_B \neq 0$ [TO DO]

Appendix J-B: The social welfare maximising level of liability if $\alpha_B = 0$. We start by proving that the payment platform' profit is concave in α_S at the profit maximising prices (f^*, m^*), which are chosen at stage 2 (after a benevolent social planer chooses the liability level for merchants). From (26), we have

$$\begin{aligned}\frac{\partial^2 \pi_P}{\partial \alpha_S^2} &= -2xLD_B h_S xL(1-q^*)\left[1-q^*+(1-\alpha_S)\frac{\partial q^*}{\partial \alpha_S}\right] + M_P D_B\left[xLh_S\frac{\partial q^*}{\partial \alpha_S} - (xL(1-q^*))^2 h_S'\right] \\ &\quad + xLV_P\left[-2\frac{\partial q^*}{\partial \alpha_S} + (1-\alpha_S)\left\{\frac{\partial^2 q}{\partial e_S^2}\left(\frac{\partial e_S}{\partial \alpha_S}\right)^2 + \frac{\partial q}{\partial e_S}\frac{\partial^2 e_S}{\partial \alpha_S^2}\right\}\right].\end{aligned}$$

From Appendix J-A, at the profit maximising prices, we have that $M_P h_B = D_B$ and $M_P h_S = D_S$. It follows that

$$\begin{aligned}\left.\frac{\partial^2 \pi_P}{\partial \alpha_S^2}\right|_{(f^*,m^*)} &= -2(xL)^2 D_B h_S(1-q^*)\left[1-q^*+(1-\alpha_S)\frac{\partial q^*}{\partial \alpha_S}\right] - M_P D_B (xL(1-q^*))^2 h_S' \\ &\quad + xLV_P\left[-\frac{\partial q^*}{\partial \alpha_S} + (1-\alpha_S)\left\{\frac{\partial^2 q}{\partial e_S^2}\left(\frac{\partial e_S}{\partial \alpha_S}\right)^2 + \frac{\partial q}{\partial e_S}\frac{\partial^2 e_S^*}{\partial \alpha_S^2}\right\}\right].\end{aligned}$$

Since $h_S' \geq 0$, $\partial^2 q/\partial e_S^2 \leq 0$, and $\partial^2 e_S^*/\partial \alpha_S^2 \leq 0$ from Lemma 6, we conclude that

$$\left.\frac{\partial^2 \pi_P}{\partial \alpha_S^2}\right|_{(f^*,m^*)} \leq 0.$$

We now study the concavity of the total user surplus. For this purpose we need to determine the sign of $\partial^2 f^*/\partial \alpha_S^2$ and $\partial^2 \widehat{b_S}/\partial \alpha_S^2$. With uniform distributions for b_B and b_S on $[0,1]$, this sign is positive, and, from Appendix J-B, we have

$$\frac{\partial^2 f^*}{\partial \alpha_S^2} = \frac{\partial^2 \widehat{b_S}}{\partial \alpha_S^2} = \frac{xL}{3}\left[\frac{\partial q^*}{\partial \alpha_S} - (1-\alpha_S)\frac{\partial^2 q^*}{\partial \alpha_S^2}\right] \geq 0.$$

In general, the total user surplus is not necessarily a concave function of α_S. We have

$$\begin{aligned}\frac{\partial^2 S_B}{\partial \alpha_S^2} &= \left[-\frac{\partial^2 \widehat{b_S}}{\partial \alpha_S^2} h_S(\widehat{b_S}) - \left(\frac{\partial \widehat{b_S}}{\partial \alpha_S}\right)^2 h_S'(\widehat{b_S})\right] E(b_B - f - \alpha_B xL(1-q^*)/b_B \geq \widehat{b_B}) \\ &\quad + 2\frac{\partial \widehat{b_S}}{\partial \alpha_S}\frac{\partial f}{\partial \alpha_S} h_S(\widehat{b_S}) D_B(f) - \frac{\partial^2 f^*}{\partial \alpha_S^2} D_B(f) D_S(\widehat{b_S}),\end{aligned}$$

and

$$\frac{\partial^2 S_S}{\partial \alpha_S^2} = -\frac{\partial^2 \widehat{b_S}}{\partial \alpha_S^2} D_B(f) D_S(\widehat{b_S}) + \left(\frac{\partial \widehat{b_S}}{\partial \alpha_S}\right)^2 h_S(\widehat{b_S}) D_B(f).$$

With uniform distributions on $[0,1]$ for b_B and b_S, since $D_B(f) = D_S(\widehat{b_S})$, we have that

$$\frac{\partial^2 S_B}{\partial \alpha_S^2} = -\frac{3}{2}\frac{\partial^2 \widehat{b_S}}{\partial \alpha_S^2} D_B^2(f) + 2\left(\frac{\partial \widehat{b_S}}{\partial \alpha_S}\right)^2 D_B(f),$$

and

$$\frac{\partial^2 S_S}{\partial \alpha_S^2} = -\frac{\partial^2 \widehat{b_S}}{\partial \alpha_S^2} D_B^2(f) + \left(\frac{\partial \widehat{b_S}}{\partial \alpha_S}\right)^2 D_B(f).$$

We now prove that a sufficient condition for total user surplus to be concave in α_S is that

$$C_S''(e_S) \geq \frac{(xL)^2}{3}\left(\frac{dq}{de_S}\right)^2 + xL\frac{d^2 q}{d^2 e_S}.$$

The total user surplus is a concave function of α_S if and only if

$$\frac{\partial^2 S_S}{\partial \alpha_S^2} + \frac{\partial^2 S_B}{\partial \alpha_S^2} \leq 0.$$

As at the profit maximising prices, with uniform distributions, $D_B = D_S$, we have that, at the profit maximising prices,

$$\frac{\partial^2 S_S}{\partial \alpha_S^2} + \frac{\partial^2 S_B}{\partial \alpha_S^2} = -\frac{5}{2}\frac{\partial^2 \widehat{b_S}}{\partial \alpha_S^2} D_B^2(f) + 3\left(\frac{\partial \widehat{b_S}}{\partial \alpha_S}\right)^2 D_B(f).$$

It follows that the total user surplus is a concave function of α_S if and only if

$$\frac{\partial^2 \widehat{b_S}}{\partial \alpha_S^2} D_B(f) \geq \frac{6}{5}\left(\frac{\partial \widehat{b_S}}{\partial \alpha_S}\right)^2.$$

As D_B belongs to $[0,1]$, a sufficient condition for the total surplus to be concave in α_S is that

$$\frac{\partial^2 \widehat{b_S}}{\partial \alpha_S^2} \geq \frac{6}{5}\left(\frac{\partial \widehat{b_S}}{\partial \alpha_S}\right)^2.$$

From (32) and (33), this condition is equivalent to

$$\left[\frac{\partial q^*}{\partial \alpha_S} - (1-\alpha_S)\frac{\partial^2 q^*}{\partial^2 \alpha_S}\right] \geq \frac{(1-\alpha_S)^2}{3} xL\left(\frac{\partial q^*}{\partial \alpha_S}\right)^2,$$

that is

$$\frac{\partial q^*}{\partial \alpha_S}\left[1 - \frac{(1-\alpha_S)^2}{3} xL\left(\frac{\partial q^*}{\partial \alpha_S}\right)\right] - (1-\alpha_S)\frac{\partial^2 q^*}{\partial^2 \alpha_S} \geq 0.$$

As $\partial^2 q^*/\partial^2\alpha_S \leq 0$ from Lemma 6, a sufficient condition for this inequality to hold is that

$$1 - \frac{(1-\alpha_S)^2}{3} xL \left(\frac{\partial q^*}{\partial \alpha_S}\right) \geq 0. \tag{34}$$

If $\alpha_B = 0$, we have

$$\frac{\partial q^*}{\partial \alpha_S} = \frac{xLD_B \left(\frac{\partial q^*}{\partial \alpha_S}\right)^2}{-xL\frac{\partial^2 q^*}{\partial e_S^2} + C_S''(e_S)}.$$

It follows that (34) holds if

$$C_S''(e_S) \geq \frac{(xL)^2}{3}\left(\frac{\partial q^*}{\partial \alpha_S}\right)^2 + xL\frac{\partial^2 q^*}{\partial e_S^2}.$$

We denote by α_S^P the level of liability that maximises the platform's profit and by α_S^W the level of liability that maximises social welfare. We have

$$\left.\frac{\partial W}{\partial \alpha_S}\right|_{\alpha_S^P} = \left.\frac{\partial S_S}{\partial \alpha_S}\right|_{\alpha_S^P} + \left.\frac{\partial S_B}{\partial \alpha_S}\right|_{\alpha_S^P} \geq 0 = \left.\frac{\partial W}{\partial \alpha_S}\right|_{\alpha_S^W}$$

If W is concave in α_S, it follows that $\alpha_S^P \leq \alpha_S^W$.

Appendix L: risk averse merchants We assume that merchants are risk averse and that their expected profit takes the form of a von Neumann Morgenstern utility function which we denote by u_S. We assume that u_S is concave and that the zero liability rule applies for consumers. The probability that a fraud occurs is denoted by $\lambda = x(1-q(e_S))$. We denote by

$$\pi_F = (b_S - m - \alpha_S L - C_S(e_S))D_B + v,$$

the merchant's profit if a fraud occurs (provided that the merchant accepts the EPI), and by

$$\pi_{NF} = (b_S - m - C_S(e_S))D_B + v,$$

the merchant's profit if a fraud does not occur. The merchant's expected profit is

$$E(\Pi) = \lambda u_S(\pi_F) + (1-\lambda)u_S(\pi_{NF}).$$

If the merchant accepts the EPI, he invests an amount e_S^* in self protection such that $\partial E(\Pi)/\partial e_S = 0$, that is

$$-C'(e_S^*)D_B\left[\lambda u_S'(\pi_F) + (1-\lambda)u_S'(\pi_{NF})\right] + \lambda'(e_S^*)\left[u_S(\pi_F) - u_S(\pi_{NF})\right] = 0.$$

The second-order condition holds as u_S is concave. This equation explains how Lemma 4 changes when merchants are risk averse. Except in special cases, the merchant's effort now depends on b_S, m, and f, unlike the situation where the merchants are risk neutral and where the zero liability rule applies for consumers. From the implicit function theorem, $\partial e_S^*/\partial \alpha_S$ has the same sign as

$$\left.\frac{\partial^2 E(\Pi)}{\partial \alpha_S \partial e_S}\right|_{e_S=e_S^*} = \underbrace{C'(e_S^*)D_B^2\lambda L u_S''(\pi_F)}_{(-)} - \underbrace{\lambda'(e_S^*)LD_B u_S'(\pi_F)}_{(+)}.$$

Therefore, unlike in the risk neutral case, the merchant's investment effort does not necessarily increase with his share of fraud losses α_S.

For instance, assume that the merchant's utility function u_S is a $CARA$ function, and that his risk aversion index is denoted by ρ. Let $q(e_S) = \gamma e_S$, where $\gamma > 0$ and $C_S(e_S) = ke_S$. Since $\lambda'(e_S^*) = -\gamma x$, we have

$$\left.\frac{\partial^2 E(\Pi)}{\partial \alpha_S \partial e_S}\right|_{e_S=e_S^*} = LD_B u_S'(\pi_F)\left[\gamma x D_B - \rho k \lambda(e_S^*)\right].$$

If the merchant's risk aversion index is sufficiently low compared to the impact of the merchant's investments on fraud detection, the merchant's investments increase with its level of liability. If the merchant's aversion index is high and the impact on fraud detection is small (γ), the merchant's investments may decrease with its level of liability. Compared to the risk neutral case, the merchant is reluctant to invest in fraud detection because investments can be assimilated to a loss in case a fraud does not occur.

In our CARA example, if $f \neq 1$, with uniform distributions on $[0,1]$ for b_B and b_S, the merchant's effort is

$$e_S^* = \frac{1}{\gamma}\left[1 + \frac{\phi}{k} - \frac{\left[(1-f)k\rho(x+\phi)^2 - 4\right]^{1/2}}{(1-f)kx\rho}\right],$$

where

$$\phi = \frac{\exp(fL\alpha_S\rho)}{\exp(L\alpha_S\rho) - \exp(fL\alpha_S\rho)}.$$

In this special case, the merchant's effort does not depend on b_S or m, but it depends on f.

Appendix K: The role of interchange fees. In this section, we look at the impact of merchants' liability on profit-maximising and welfare maximising interchange fees. As consumers bear no liability on fraudulent transactions, merchants' investments in fraud detection technologies do not depend on the transaction fees that are paid by the users. We have

$$\widehat{b_S} = m + \alpha_S x L(1-q^*) + C_S(e_S^*),$$

where e_S^* solves (4). As $m = a + c_A$, and from the envelop theorem, we have

$$\frac{d\widehat{b_S}}{da} = 1.$$

As the acquirers make zero profit, banks' joint profit is equal to the issuers' profit,

$$\pi_I = (f^*(c_I - a) + a - c_I - (1-\alpha_S)xL(1-q^*))D_B(f)D_S(\widehat{b_S}).$$

Note that the level of investment that is chosen by the merchants depends neither on the transaction fees nor on the interchange fee. From the envelop theorem, as $\left.\frac{\partial \pi_I}{\partial f}\right|_{f^*} = 0$, we have

$$\frac{d\pi_I}{da} = \frac{\partial \pi_I}{\partial a} + \frac{\partial \pi_I}{\partial m}\frac{\partial m}{\partial a}$$

Solving for the first-order condition of profit maximisation yields

$$\frac{d\pi_I}{da} = \left[\frac{df^*}{da} + 1\right] D_B(f)D_S(\widehat{b_S}) - (f^*(c_I - a) + a - c_I - (1-\alpha_S)xL(1-q^*))D_B(f)h_S(\widehat{b_S}).$$

The second-order condition is

$$\frac{d^2\pi_I}{da^2} = D_B\left[\frac{d^2 f^*}{da^2}D_S - 2\left[\frac{df^*}{da} + 1\right] h_S(\widehat{b_S}) - (f^*(c_I - a) + a - c_I - (1-\alpha_S)xL(1-q^*))h_S'(\widehat{b_S})\right] \leq 0.$$

Since $h_S'(\widehat{b_S}) \geq 0$, a sufficient condition for the second-order condition to hold is that $d^2 f^*/da^2 \leq 0$. For instance, the second-order condition holds with uniforms distributions on $[0,1]$ for b_B and b_S and if the issuer is a monopolist, as in this case $f^* = (1 - a + c_I)/2$.

In an interior solution, the profit maximising interchange fee is implicitely defined by

$$(f^*(c_I - a^P) + a^P - c_I - (1-\alpha_S)xL(1-q^*)) = \frac{D_S(\widehat{b_S})}{h_S(\widehat{b_S})}\left[\left.\frac{df^*}{da}\right|_{a^P} + 1\right].$$

The profit maximising interchange fee reflects a trade-off between increasing the transaction volume by encouraging merchants to accept the EPI and maximising the margin per transaction. For instance, with uniforms distributions on $[0,1]$ for b_B and b_S and if the issuer is a monopolist, we have

$$a^P = \frac{c_I - c_A - \alpha_S xL(1-q^*) - C_S(e_S^*)}{2} + (1-\alpha_S)xL(1-q^*).$$

From the implicit function theorem, we have

$$\frac{da^P}{d\alpha_S} = -\left(\frac{d^2\pi_I}{da^2}\right)^{-1}\left(\frac{\partial^2\pi_I}{\partial a\partial\alpha_S}\right),$$

where

$$\begin{aligned}\frac{\partial^2\pi_I}{\partial a\partial\alpha_S} &= -D_B xL(1-q^*)\left[\left(\left.\frac{df^*}{da}\right|_{a^P}+1\right)h_S(\widehat{b_S}) + (f^*+a^P-c_I-(1-\alpha_S)xL(1-q^*))h_S'(\widehat{b_S})\right] \\ &\quad -D_B(f)h_S(\widehat{b_S})\left[xL(1-q^*)+(1-\alpha_S)\frac{dq^*}{de_S}\frac{de_S^*}{d\alpha_S}\right].\end{aligned}$$

As $h_S'(\widehat{b_S}) \geq 0$, $\partial f/\partial a \leq 0$, and $\partial e_S/\partial\alpha_S \leq 0$, we have

$$\frac{\partial^2\pi_I}{\partial a\partial\alpha_S} \leq 0.$$

It follows that the interchange fee decreases with the level of liability that is borne by merchants.

If consumers could be held liable for fraudulent transactions, the merchants' fraud prevention effort could depend on the interchange fee. In that case, the payment platform could decide to lower the interchange fee to encourage merchants to invest in fraud prevention technologies.

We now show in an example that if the interchange fee is chosen by a regulator at stage 1, the payment platform can react at stage 2 by adjusting the level of merchant liability. For instance, assume that b_B and b_S are uniformely distributed on $[0,1]$ and that the issuer is a monopolist. In this case, we have that $f^* = (1-a+c_I)/2$. The payment plaform chooses the level of liability for merchants that maximises the issuer' profit,

$$\pi_I = (\frac{1+a-c_I}{2} - (1-\alpha_S)xL(1-q^*))(\frac{1+a-c_I}{2})(1-a-c_A-\alpha_S xL(1-q^*)-C_S(e_S^*)).$$

With a linear probability such that $q(e_S) = \gamma e_S$, where $0 < \gamma \leq 1$, with a cost fonction such that $C_S(e_S) = ke_S^2/2$, we have

$$e_S^* = \frac{\alpha_S\gamma xL}{k}.$$

Solving for the first-order condition of profit maximisation with respect to α_S, from the envelop theorem, we obtain that

$$(xLr^* + (1-\alpha_S)xL\gamma)\left[1 - a - c_A - \alpha_S xLr^* - C_S(e_S^*)\right] = \left[\frac{1+a-c_I}{2} - (1-\alpha_S)xLr^*\right] xLr^*,$$

where $r^* = 1 - q^*$. The second-order condition writes

$$-\gamma(1+xL)D_S(b_S^m) - 2(xL)^2(1-q^*)(1-q^*+\gamma(1-\alpha_S)) - (1-\alpha_S)(xL)^2\gamma(1-q^*) \leq 0.$$

From the implicit function theorem, $\partial\alpha_S/\partial a$ has the same sign as $\partial^2\pi_I/\partial a\partial\alpha_S$. We have

$$\frac{\partial^2\pi_I}{\partial a\partial\alpha_S} = -2xL(1-q^*) - (1-\alpha_S)xL\gamma \leq 0.$$

We find that the liability level that is chosen by the payment platform decreases with the level of interchange fee. This example shows that if a regulator chooses a level of interchange fee that is quite low, the platform can react by increasing the level of liability that is borne by merchants.