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# Enforcement vs Deterrence in Merger Control: Can Remedies Lead to Lower Welfare?<sup>\*</sup>

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#### Abstract

This paper deals with the enforcement of merger policy, and aims to identify situations where the introduction of remedies can lead to a lower welfare. For this we study how merger remedies affect the deterrence accomplished by controlling mergers, and determine the optimal frequency of investigations launched by the agency. We find that when conditional approvals are possible, it may be harder to deter the most welfare-detrimental mergers, and the agency might have to investigate mergers more often. The resulting welfare from merger control can indeed be lower than without remedies.

JEL classification: K21, L41

Keywords: merger control, merger remedies, enforcement, deterrence

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#### 1 Introduction

In many countries we observe a quite active merger control. However, an outright ban of a merger is rather seldom the action taken by the antitrust authorities. Instead we see that the antitrust authorities quite often require that the merging parties modify the merger, either through structural remedies (for example divestiture of assets) or behavioral remedies (for example specific contractual arrangements).<sup>1</sup> Unfortunately, most theoretical studies of merger control do not allow for remedies. The purpose of this article is to help fill this gap in the literature. We investigate the welfare effect of introducing merger remedies in the presence of possible mistakes by the antitrust authorities and possible deterrence effects of merger control. It is found that an introduction of merger remedies can make it less likely that the worst mergers are deterred, and that allowing for remedies might lead to lower welfare even if the worst mergers are deterred.

Introducing remedies as an option might signal a "soft" merger policy and thereby encourage firms to merge.<sup>2</sup> There are empirical studies that investigate how remedies impact the number of proposed mergers. Seldeslachts et al. (2009) use cross-section data from 28 countries of the number of mergers (frequency) and conclude that prohibition decisions deter, whereas conditional approvals through the application of merger remedies do not. Clougherty and Seldeslachts (2013) look at US merger deterrence using a similar method. They examine composition-based and frequency-based deterrence in the US, and find that launching an investigation and challenging a merger have significant deterrent effects, but prohibitions do not significantly involve more deterrence than remedies.

Even if the empirical studies do help us understand how allowing for remedies may affect the number of proposed mergers, they do not help us to understand the welfare effect of remedies.<sup>3</sup> Could it be that consumers would be better off without remedies as a policy

<sup>&</sup>lt;sup>1</sup>Leveque and Shelanski (2003) provide an overview of the use of merger remedies in the US and EU. For a more recent review of the use of merger remedies in EU, see Motta et al. (2007).

<sup>&</sup>lt;sup>2</sup>According to Neven et al. (1993, p.7) "lawyers in particular are aware that this may give them significant bargaining power with the (European) Commission even in doubtful cases". Note that this clearly goes against the original expectation about the possibility of a remedial action - according to Baer and Redcay (2003), the requirement to file a pre-merger notification and wait pending the agency's review was reckoned to increase the negotiation power of the agency, because an eventual litigation over the remedy involved supplementary delay, so firms were expected to become more inclined to accept the settlement terms requested by the agency.

<sup>&</sup>lt;sup>3</sup>Assessing the overall impact of merger control requires to assess both the magnitude of type I and II errors (banning pro-competitive mergers and clearing anticompetitive ones, respectively) and the degree of

alternative? To pinpoint the mechanisms at play, we apply a theoretical model. We allow for both type I and type II errors by antitrust authorities, as well as the potential for deterrence of mergers. As a benchmark we follow Sørgard (2009), and let the antitrust authorities either ban or clear any merger (remedy is not an option). It is shown that if the quality of merger control is sufficiently high, the worst mergers are deterred. However, the merger investigations as such can have a detrimental effect on welfare: the reason is that those mergers that are investigated are chosen among those that are not deterred. Since the mergers that have the largest anti-competitive effects are already deterred, this leads to a large risk of type I errors (prohibiting welfare enhancing mergers). We complement Sørgard (2009) by allowing for remedies, and comparing the outcome with the benchmark where remedies are not an option. The purpose is to pin down the situations which make remedies welfare-reducing, so as to enable a policy discussion on what the antitrust authorities should bear in mind when allowing for remedies.

It turns out that to address the welfare effect of introducing remedies, it is important to understand (i) how merger remedies will affect the unconditional clearance rate and (ii) the change in profits from unconditional to conditional approval (with remedies). At one extreme, all conditional clearances are replacing unconditional clearances that are present in a no-remedy regime, and at the other extreme all conditional clearances are replacing bans that would be present in the no remedy regime. If most of the remedies are replacing unconditional bans that would be present in a no-remedy regime, and the profit from clearance with remedies is close to the profit with unconditional clearance, then it is obvious that introducing remedies as an option will make mergers in general more profitable for the firms. For a given activity level by the agency, this will lead to more mergers being proposed.

We show that introducing remedies as an option can make it more difficult to deter the worst mergers, those that are most detrimental to welfare. If the firms know that remedies would be an option they have more incentives to propose the worst mergers, because there is a chance that they may be cleared with remedies. It might be that introducing remedies tips the balance, and suddenly some of the worst mergers are proposed. Then it is obvious that the overall effect of allowing for remedies is probably negative.

deterrence achieved. While the former may be easier to capture (see for instance Duso et al. (2007) for an estimation in the European Commission's case), the latter is hard to measure, although the academic literature agrees on the necessity to take it into account (see for instance Joskow (2002), Crandall and Whinston (2003) and Baker (2003)).

If the agency enjoys a sufficiently high quality for its merger control activity, it will succeed in deterring the worst mergers even if the conditional approval is allowed for. However, it is still a question how active the agency should be, and for this the agency has to take into account both the direct (enforcement) and the indirect (deterrence) effects. The former is the welfare outcome of bans and conditional approvals applied instead of clearances, whereas the latter stands for the welfare impact of a change in the population of submitted merger projects due to increased control activity by the agency. We show the introduction of remedies as an option will modify both these effects. For instance, although the worst mergers are deterred, allowing for remedies might lead to more mergers being proposed and then less deterrence on the margin for a given activity level by the agency. Also in this case it is of importance how the introduction of remedies will affect the unconditional clearance rate of mergers. If there is only a limited reduction in the rate of unconditional clearance, then it is likely that more mergers are proposed and thereby fewer mergers with a negative impact of welfare are deterred. On the other hand, the introduction of remedies will have an ambiguous effect on the enforcement. Some beneficial mergers that initially would have been banned will now be cleared with remedies, while some beneficial mergers that would have been cleared unconditionally are now solved with remedies.

Finally, we discuss the agency's optimal activity level. Again, it is found that the change in the unconditional clearance rate following the introduction of remedies as an option is crucial. If the unconditional clearance rate drops only marginally, that would encourage more firms to merge. If the worst mergers are deterred initially, then this makes it more likely that the activity level is higher after the introduction of a merger remedy option. We show that the change in the unconditional clearance rate must be compared with the difference in welfare between clearing mergers unconditionally and clearing them with remedies. When considering to launch one more merger investigation after allowing for merger remedies, we show that it is crucial how large effect the marginal investigation has on the number of additionally deterred merger in a no-remedy regime versus a regime with remedies.

Although merger remedies are widely used by competition authorities, there are only a limited number of theoretical studies of this policy instrument. These studies typically consider structural remedies, where the merging parties are forced to sell out assets or brands. They find that introducing remedies might lead to lower consumer welfare.<sup>4</sup> In contrast to

 $<sup>^{4}</sup>$ Cabral (2003) shows that any divestiture to an entrant may lead to lower consumer welfare, because the entrant then might be prevented from introducing its own brand (which in the retail market can be interpreted

those studies, we allow for type I and type II errors in addition to the possible deterrence effect of merger control.<sup>5</sup> This allows us to investigate how an active merger control will have both a direct effect (on enforcement) and an indirect effect (on deterrence), and how remedies will influence the trade-off between the direct and indirect effects.

In the next section we present our basic model. In section 3 we identify the conditions ensuring that the most harmful mergers are actually abandoned. For the rest of the paper we assume that the worst mergers are deterred. In section 4 we analyze the trade-off between the enforcement and the deterrence effect, then section 5 identifies the optimal activity level for the antitrust authorities with and without remedies. The final section provides some concluding remarks, and relates to the empirical findings in the existing literature.

### 2 Basic assumptions and notations

Consider the set of potential profitable mergers of an economy, denoted Y. A given project  $y \in Y$  may be more or less detrimental to welfare, so one can rank them according to their welfare effect from 0, the least anticompetitive one, to  $\overline{y}$ , the most anticompetitive one. Denote  $W^M(y)$  the net welfare impact of merger y, where  $W^M$  is decreasing in y, and call  $y^0$  the "neutral" merger, i.e. such that  $W^M(y^0) = 0$ . Then  $W^M(0) > 0$  and  $W^M(\overline{y}) < 0$  as long as  $y^0 \in (0, \overline{y})$ .

As before mentioned, we only consider privately profitable mergers, meaning in the absence of any merger control. Denote  $\Pi^M(y)$  the joint profit from merger. In order to merge, the firms need to incur a fixed sunk cost C, the same for all, where  $\Pi^M(y) > C$ ,  $\forall y$ .

The competition agency (CA henceforth) conducts merger control with probability  $N \in [0, 1]$ , which stands for the probability of investigating any given merger y. Normalize the cost of merger enforcement to zero. If investigated, the merger project may be either cleared

as a new store). Vergé (2010) applies a Cournot model, and shows that reallocation of assets to existing rivals through remedies is detrimental to consumer welfare unless there are sufficiently large synergies. Vasconcelos (2010) also applies a Cournot model, and shows that the potential for remedies can influence which mergers are proposed and in some cases lead to lower consumer welfare.

 $<sup>{}^{5}</sup>$ Cosnita-Langlais and Tropeano (2012) also allow for decision errors due to asymmetric information, but instead focus on how the potential for remedies will influence the merging firms' incentives to invest in efficiency gains. See also Barros et al. (2010), that also allow for private information. In contrast to us, in the latter model they assume that antitrust authorities are better informed about the effect of the merger than the merging parties.

or banned. We assume that a ban has a zero welfare impact. The decision to clear a merger however may be of two types: unconditional or subject to remedies. As a result, we are going to consider and compare throughout the paper two possible regimes, the "strict" one, not allowing for remedies, and the "remedy" regime, in which the merger approval may involve remedies.

Thus, let  $g^S$  denote the probability of approval following investigation in the "strict", noremedy regime. As a result, a merger will be banned with probability  $1-g^S$ . Furthermore, let  $g^S = g + \gamma h$ , where g stands for the unconditional approval probability and h stands for the probability of clearing the merger subject to remedies in the so-called "remedy" regime. Let us assume from now on that  $\gamma \in [0, 1]$ . Thus,  $g^S = g$  for  $\gamma = 0$ , i.e. the probability of clearance is the same, regardless of the possibility of remedies, but  $g^S = g + h$  for  $\gamma = 1$ , i.e. some "former" approvals become conditional clearances when remedies are used. In other words, parameter  $\gamma = \frac{g^S - g}{h}$  measures to what extent the difference in unconditional approval rates between the two possible regimes is due to the presence of conditional clearances. Finally, let  $g^S, g$  and h be all strictly decreasing in y: the more anti-competitive a merger, the less likely the clearance decision, be it unconditional or not.

As compared with the unconditional approval, the conditional clearance leads to different profit and welfare effects. Let  $\Pi^R(y)$  denote the joint profit from merger when remedies apply, with  $\Pi^R \leq \Pi^M, \forall y$ . In other words, we assume that the remedies are costly for the merging firms. The net welfare effect when the merger is conditionally accepted will be denoted  $W^R(y)$ . As for  $W^M(y)$ , we assume that  $W^R(y)$  is decreasing in y.

The timing of actions will be the following:

Stage 1: the Competition Agency (CA) determines the probability N of launching an investigation.

Stage 2: the merging firms (or insiders) decide whether to merge or not.

Stage 3: the CA investigates submitted mergers and each investigated merger is cleared (possibly under conditions) or banned. Merger control is imperfect: the CA makes both types of errors.<sup>6</sup>

The game will be solved by backward induction. The paper aims to compare the outcomes of the merger policies allowing or not for remedies.

<sup>&</sup>lt;sup>6</sup>To justify this, one can think of the CA receiving a signal imperfectly correlated with the true welfare effect of the merger.

#### 3 Impact of remedies on merger profitability and incentives

Given the exogenous probability to clear a merger, conditionally or not, we start by looking into the outcome of the firms' decision at the second stage. The insiders will merge only if it is profitable to do so, i.e. if the expected profit is positive given the cost of merging and the probability to see their merger banned.

In the "strict", no-remedies regime, the expected profit writes:

$$E^{S}\Pi(y) = (1 - N)\Pi^{M}(y) + N\Pi^{M}(y)g^{S}(y) - C$$
  
=  $\Pi^{M}(y) [1 - N + Ng^{S}(y)] - C,$  (1)

where the square bracket stands for the total probability to see the merger materialize, i.e. of not being investigated (1 - N) or of being cleared if investigated  $(Ng^S)$ .

In contrast, the merger policy allowing for remedies leads to an expected profit of:

$$E^{R}\Pi(y) = (1-N)\Pi^{M}(y) + N \left[\Pi^{M}(y)g(y) + \Pi^{R}(y)h(y)\right] - C$$
  
=  $\Pi^{M}(y) \left[1 - N + Ng(y)\right] + \Pi^{R}(y)Nh(y) - C,$  (2)

where the square bracket stands for the total probability to see the merger materialize under the exact form that it was submitted, and the term Nh(y) stands for the probability of conditional approval in case of investigation.

Comparing the two above expressions enables us to establish the impact of remedies on the merger profitability, and hence on the private decision to merge:

**Proposition 1** Allowing for remedies increases expected merger profitability iff  $\frac{\Pi^R(y)}{\Pi^M(y)} > \gamma$ .

**Proof.** Recall that  $g^S = g + \gamma h$ . Thus  $E^R \Pi(y) - E^S \Pi(y) = \left(\Pi^M(y) \left[1 - N + Ng(y)\right] + \Pi^R(y)Nh(y) - C\right)$   $- \left(\Pi^M(y) \left[1 - N + Ng(y)\right] + N\Pi^M(y)\gamma h(y) - C\right) = Nh(y) \left[\Pi^R(y) - \gamma \Pi^M(y)\right].$ Therefore  $E^R \Pi(y) > E^S \Pi(y)$  iff  $\frac{\Pi^R(y)}{\Pi^M(y)} > \gamma$ .

The condition identified in Proposition 1 may be rewritten as  $\frac{g^S-g}{h} < \frac{\Pi^R(y)}{\Pi^M(y)}$ , or, equivalently,  $h\Pi^R(y) > (g^S-g)\Pi^M(y)$ . In other words, the profitability of mergers and the incentive to submit them increase when remedies are possible as long as the relative ratio between net unconditional and conditional approval rates is lower than the relative profit ratio between the remedy and the strict regimes, or, equivalently, the profit from merger weighted by the net probability of approval is higher with rather than without remedies.

Note that if there is no change in the probability of an unconditional clearance ( $\gamma = 0$ ), then those mergers that would have been cleared with remedies will be banned. Then it is no surprise that as long as the increase in the probability of a clearance is sufficiently low, abolishing remedies will make a merger less profitable. Note also that the larger the profit with remedies relative to that from an unconditional approval, the less likely that the scenario with no remedies would make mergers more profitable.

Two additional conclusions may be drawn from the expected profitability comparison between the two merger policy regimes, with and without remedies.

On the one hand, for a given number of merger investigations, it is straightforward to see that allowing for remedies may trigger more mergers being submitted. Denote  $y^*(N)$ and  $y^{**}(N)$  the "critical" or "marginal" mergers in the "strict" and the "remedy" regimes respectively, i.e.  $y^*(N)$  such that  $E^S\Pi(y^*, N) = 0$  and  $y^{**}(N)$  such that  $E^R\Pi(y^{**}, N) = 0$ . Then for a given  $N, y^{**}(N) \ge y^*(N)$  as long as  $\frac{\Pi^R(y)}{\Pi^M(y)} \ge \gamma$ .

On the other hand, one may equally draw a comparison of the CA's activity ensuring the same number of mergers being submitted in the two regimes (i.e. compare  $N^S$ and  $N^R$  such that  $y^*(N^S) = y^{**}(N^R)$ ). Let  $y^*(N^S)$  be such that  $E^S\Pi(y^*, N^S) = 0 \Leftrightarrow \Pi^M(y) \left[1 - N^S + N^S g(y)\right] + N^S \Pi^M(y) \gamma h(y) = C$ ; this yields  $N^S = \frac{\Pi^M(1-g) - \gamma h \Pi^M}{\Pi^M - C}$  as the activity level leading to  $y^*$  in the strict, no-remedy regime. Let now  $y^{**}(N^R)$  be such that  $E^R \Pi(y^{**}, N^R) = 0 \Leftrightarrow$ 

 $\Pi^{M}(y) \left[1 - N^{R} + N^{R}g(y)\right] + N^{R}\Pi^{R}(y)h(y) = C; \text{ then } N^{R} = \frac{\Pi^{M}(1-g) - h\Pi^{R}}{\Pi^{M} - C} \text{ is the agency's investigation frequency leading to } y^{**} \text{ in the merger policy regime allowing for remedies.}$ Assuming that  $y^{*} = y^{**}$ , one gets that  $N^{R} > N^{S} \Leftrightarrow \gamma < \frac{\Pi^{R}(y)}{\Pi^{M}(y)}$ . Summing up yields the following:

**Corollary 1** Iff  $\gamma < \frac{\Pi^R(y)}{\Pi^M(y)}$ , then:

(i)  $y^{**}(N) > y^{*}(N)$ , for a given N: the remedy regime leads to more mergers being submitted for a given level of activity on behalf of the CA;

(ii)  $N^R > N^S$  if  $y^* = y^{**}$ : in order to keep constant the number of mergers being submitted, the CA must be "more active"/investigate more often when remedies are possible.

Following the above discussion it is clear that when facing a non-zero probability of investigation, some merger projects will not be submitted by the firms because they expect a ban. The question we tackle now is precisely which merger projects will be thus deterred. For this, one needs to consider how the expected profit from merger depends on the merger type y.

Consider the "strict", no-remedy regime. The worst, most anticompetitive mergers will not be submitted if  $\frac{\partial E^S \Pi(y)}{\partial y} < 0 \Leftrightarrow \frac{\partial \Pi^M}{\partial y} \left[1 - N + Ng^S(y)\right] + \Pi^M(y) N \frac{\partial g^S(y)}{\partial y} < 0$ . Re-writing this expression yields  $\frac{\partial \Pi^M}{\Pi^M(y)} < \frac{-\frac{\partial g^S(y)}{\partial y}N}{1 - N + Ng^S}$ , which we can interpret as follows: the relative profit increase from submitting a more anticompetitive merger (the LHS term) must be lower than the relative change in the probability to see the merger accepted (the RHS term). This leads to the Proposition 1 in Sørgard (2009), according to which in the strict, no-remedy regime, under the assumption that mergers with a larger negative welfare impact face a higher probability of ban  $(\frac{\partial g^S}{\partial y} < 0)$ , a sufficient condition for the "right deterrence" to occur, i.e. the worst mergers not being submitted, is that  $\frac{\partial \Pi^M}{\partial y} \leq 0$ .

In other words, as long as the screening performed by the agency is good enough (i.e. the more anticompetitive the merger, the higher the likelihood of a ban), then the right deterrence is achieved as soon as there is a negative relationship between the merger's type y and its profit (meaning that the more pro-competitive the merger, the more profitable it is). In short, as long as the worst mergers have lower chances of approval, they will be deterred if they are also relatively less profitable. As emphasized by Sørgard (2009), empirical studies do not necessarily support this negative relationship, but the important conclusion to reach is that the quality or performance of merger control depends on which merger projects are actually deterred.

The possibility of a conditional approval is likely to impact on the factors enabling the "right" deterrence, and we discuss this next.

**Proposition 2** When allowing for remedies, the "right" deterrence obtains, i.e. the most welfare-detrimental mergers are abandoned, if  $\begin{pmatrix} \frac{\partial \Pi^M}{\partial y} \\ \frac{\Pi^M(y)}{\Pi^M(y)} + \frac{N \frac{\partial g(y)}{\partial y}}{1-N+Ng} < 0 \end{pmatrix} \cap \begin{pmatrix} \frac{\partial \Pi^R}{\partial y} + \frac{\partial h(y)}{\partial y} \\ \frac{\partial W}{\partial y} \\ \frac{\partial W}{$ 

**Proof.** Under the remedy regime, the worst mergers will be deterred if  $\frac{\partial E^R \Pi(y)}{\partial y} < 0 \Leftrightarrow \frac{\partial \Pi^M}{\partial y} [1 - N + Ng] + \Pi^M(y) N \frac{\partial g(y)}{\partial y} + \frac{\partial \Pi^R}{\partial y} Nh + \Pi^R(y) N \frac{\partial h(y)}{\partial y} < 0$ . Equivalently,  $\Pi^M(y) [1 - N + Ng] \left( \frac{\frac{\partial \Pi^M}{\partial y}}{\Pi^M(y)} + \frac{N \frac{\partial g(y)}{\partial y}}{1 - N + Ng} \right) + \Pi^R(y) Nh \left( \frac{\frac{\partial \Pi^R}{\partial y}}{\Pi^R(y)} + \frac{\frac{\partial h(y)}{\partial y}}{h} \right) < 0, \text{ where } \frac{\partial g(y)}{\partial y} < 0 \text{ and } \frac{\partial h(y)}{\partial y} < 0 \text{ as well.} \quad \blacksquare$  In short, it all comes down to the same negative relationship between the merger type and its profit, but extended to take into account the profit made in case of conditional approval.

Let us now finally compare the conditions ensuring the "right" deterrence between the two merger policy regimes: under the strict, no-remedy regime, this condition writes

 $\left(\frac{\partial \Pi^{M}}{\partial y}\left[1-N+Ng\right]+\Pi^{M}(y)N\frac{\partial g(y)}{\partial y}\right)+N\left(\frac{\partial \Pi^{M}}{\partial y}\gamma h+\Pi^{M}(y)\gamma\frac{\partial h}{\partial y}\right)<0, \text{ whereas when remedies are possible it writes } \left(\frac{\partial \Pi^{M}}{\partial y}\left[1-N+Ng\right]+\Pi^{M}(y)N\frac{\partial g(y)}{\partial y}\right)+N\left(\frac{\partial \Pi^{R}}{\partial y}h+\Pi^{R}(y)\frac{\partial h(y)}{\partial y}\right)<0. \text{ It is easy to see the sufficient condition for the latter, } \left(\frac{\partial \Pi^{M}}{\partial y}<0\right)\cap\left(\frac{\partial \Pi^{R}}{\partial y}<0\right), \text{ implies the sufficient condition for the former } \left(\frac{\partial \Pi^{M}}{\partial y}<0\right). \text{ Thus, the following holds:}$ 

**Corollary 2** If the worst mergers are abandoned when the remedies are allowed, then they are necessarily deterred when the conditional approval is not available.

One way to interpret Corollary 2 is to say that under the previously identified sufficient conditions, giving up remedies is not costly in terms of achieving the "right" deterrence. However, the opposite is obviously not true. Moreover, this tells nothing about the welfare impact of switching from one merger policy regime to the other, and this is what we tackle next.

#### 4 Impact of remedies on welfare

At the first stage of the game, the CA determines its activity level or frequency of investigation N by maximizing its objective function. Let us detail below the expression of the CA's objective in each regime.

In the "strict", no-remedy regime, the CA maximizes an expected welfare equal to:<sup>7</sup>

$$E^{S}W = \int_{0}^{y^{*}(N)} \left[ (1-N) W^{M}(y) + Ng^{S}(y)W^{M}(y) \right] dy.$$
(3)

The integral sums up the gain from enforcing the merger policy, namely the welfare effect from both mergers that are submitted and not investigated and those that are investigated and cleared.<sup>8</sup>

<sup>&</sup>lt;sup>7</sup>Remember that we assume that a ban has a zero welfare impact.

<sup>&</sup>lt;sup>8</sup>Note that we can leave out from the social welfare function a second integral,  $\int_{y^*(N)}^{\overline{y}} W^M(y) dy$ , corresponding to the expected welfare from the indirect, deterrent effect of merger policy, since all mergers between  $y^*(N)$  and  $\overline{y}$  are abandoned.

When remedies are allowed, the CA's objective becomes:

$$E^{R}W = \int_{0}^{y^{**}(N)} \left[ (1-N) W^{M}(y) + N \left( W^{M}(y)g(y) + W^{R}(y)h(y) \right) \right] dy.$$
(4)

The interpretation of the two integrals is roughly the same, except that part of the welfare gain from enforcing the merger policy comes now from the merger projects that are conditionally cleared when investigated. This possibility also modifies the marginally deterred merger (hence  $y^{**}$  instead of  $y^{*}$ ).

Before discussing the maximization and its outcome as such, the mere comparison of the CA's objective function in the two possible regimes provides some interesting insights:

**Lemma 1** Assume that the same number of mergers are submitted in both regimes, i.e.  $y^* = y^{**}$ . Then  $E^R W \ge E^S W$  iff  $\gamma \le \frac{W^R}{W^M}$ .

**Proof.** Rewrite  $E^S W$  as  $\int_0^{y^*(N)} W^M(y) \left[1 - N + Ng(y)\right] dy + \int_0^{y^*(N)} W^M(y) \gamma Nh(y) dy$ . Then for  $y^* = y^{**}$  it is straightforward to see that  $E^R W \ge E^S W$  iff  $\gamma \le \frac{W^R}{W^M}$ .

In other words, in the case of equal deterrence, Lemma 1 states that the expected welfare comparison between both regimes simply comes down to the comparison between  $\gamma = \frac{g^S - g}{h}$  and the relative welfare threshold  $\frac{W^R}{W^M}$ , or equivalently, between the ratio of net unconditional/conditional approval rates and the ratio of relative social gain from merger. In particular, the possibility of remedies lowers the expected welfare iff the social gain from merger weighted by the net probability of clearance is higher under the strict regime  $(E^RW < E^SW \Leftrightarrow (g^S - g)W^M > hW^R)$ . This is quite intuitive: given the assumption of equal number of mergers submitted under both regimes, the difference between the two expected welfare functions only comes from the change in approval rates, or more precisely, the transformation of unconditional into conditional clearances that occurs for some of the submitted mergers. So as soon as the welfare change from this can be signed, the expected welfare comparison is straightforward.

Furthermore, the same comparison of objective functions between the two regimes equally yields the following:

**Lemma 2** If  $\gamma \geq \frac{\Pi^R}{\Pi^M}$ , then a sufficient condition for  $E^R W \leq E^S W$  is that  $\gamma \geq \frac{W^R}{W^M}$ .

**Proof.** Recall that  $\gamma \geq \frac{\Pi^R}{\Pi^M} \Leftrightarrow y^{**}(N) \leq y^*(N)$  for a given N, and in this case

 $\int_{0}^{y^{**}(N)} \left[ W^{M}(y)(1-N+Ng(y)) \right] dy \leq \int_{0}^{y^{*}(N)} W^{M}(y) \left[ 1-N+Ng(y) \right] dy.$  Then in order for  $E^{R}W \leq E^{S}W$ , it is enough to have  $\int_{0}^{y^{**}(N)} W^{R}(y)Nh(y)dy \leq \int_{0}^{y^{*}(N)} W^{M}(y)\gamma Nh(y)dy$ , and for this a sufficient condition is  $\gamma \geq \frac{W^{R}}{W^{M}}$ .

Assume no longer the same deterrence between the two regimes. Then Lemma 2 states that the comparison between  $\gamma$  and the relative welfare threshold  $\frac{W^R}{W^M}$  suffices to compare the two regimes in terms of expected welfare. For instance, if one considers the limit case  $y^{**}(N) = y^*(N) \Leftrightarrow \gamma = \frac{\Pi^R}{\Pi^M}$ , it is straightforward to check that  $E^R W > E^S W$  if  $\gamma < \frac{W^R}{W^M}$ , as shown in Lemma 1. Therefore Lemma 2 provides the outcome of the expected welfare comparison in a more general case as compared with Lemma 1, i.e. without assuming equal deterrence between the two regimes, but at the cost of lessening the condition enabling this comparison (only a sufficient condition instead of the necessary and sufficient condition of Lemma 1). Nonetheless, the same reasoning as above holds: as long as one can rank the degree of deterrence achieved under each regime, i.e.  $y^{**}(N) \leq y^*(N)$ , then the expected welfare comparison will be basically dictated by the comparison between the welfare from merger weighted by the respective net probability to see the merger materialize, with or without remedies  $((g^S - g)W^M \leq hW^R)$ .

Incidentally, Lemma 2 identifies a precise situation in which remedies do lead to lower welfare: for this it is enough that  $\gamma > \frac{\Pi^R}{\Pi^M}$  and  $\gamma > \frac{W^R}{W^M}$ . In other words, this is he case when remedies are quite costly both from the private and the public point of view, since both the private and the public gain from mergers is higher in case of unconditional merger decisions. Note moreover the intuition behind the relative high value of  $\gamma$  that enables this situation: the conditional clearances replace many of the previous unconditional ones, as well as a lot of former bans, which basically explains the high social cost of remedies and their subsequent suboptimality. Finally, recall that this result of remedies lowering the welfare is obtained despite neglecting the cost of public enforcement of merger control, as well as based on the assumption that the worst merger do get deterred by the public intervention. With a positive enforcement cost and without deterring the most welfare-detrimental mergers, the application of remedies is highly likely to lead to even lower welfare.

Finally, note that if  $E^R W \ge E^S W$ , then necessarily  $E^R W(N^{**}) \ge E^S W(N^*)$ , where  $N^{**}$ and  $N^*$  denote the CA's optimal choices of activity levels or frequency of investigation with and without remedies respectively. However, this tells us nothing on the comparison between these optimal choices, i.e.  $N^{**} \ge N^*$ . For this we shall examine next the outcome of the maximization problem for the CA.

### 5 Impact of remedies on the optimal activity level

#### 5.1 Enforcement and deterrence effects

In the case of the "strict", no-remedy regime, the maximization of the CA's objective function requires a FOC that writes as follows:

$$\frac{\partial E^S W}{\partial N} = \int_0^{y^*(N)} \left[ \left( g^S(y) - 1 \right) W^M(y) \right] dy + W^M(y^*) \left[ 1 - N + N g^S(y^*) \right] \frac{dy^*}{dN} = 0.$$
(5)

The optimal choice  $N^*$  in terms of activity level (or frequency of investigation) for the CA strikes the balance between the expected marginal gain of increasing the investigation activity (the LHS term of the FOC) and the corresponding marginal cost, normalized here to zero (the RHS term). Following Sørgard (2009), the marginal gain from launching one more merger investigation is itself composed of two distinct parts, called the enforcement and the deterrence effects. These are the first and second term of the above FOC respectively. Let us recall their interpretation. The enforcement effect, equal to  $\int_0^{y^*(N)} \left[ \left( g^S(y) - 1 \right) W^M(y) \right] dy$ , represents the net welfare impact of a ban following an investigation. The deterrence effect, which is equal to  $W^M(y^*) \left[1 - N + Ng^S(y^*)\right] \frac{dy^*}{dN}$ , represents the outcome of the increased frequency of investigation for the number of mergers actually submitted, and thereby on expected welfare. Again, following Sørgard (2009), in particular Proposition 2, it is easy to check that if the worst mergers are abandoned in presence of merger control (i.e. the "right deterrence" is achieved), then in equilibrium the deterrence effect is positive while the enforcement effect is negative provided that  $W^M(y^*) < 0$ . In other words, the CA faces a trade-off when deciding how many mergers to investigate: a higher frequency deters more mergers, which is welfare improving if the deterred mergers are detrimental to welfare, but one more investigation may lead to the ban of a welfare-increasing merger as well. The cost of increasing the frequency of investigation, in the hope of avoiding type II errors (i.e. clearing detrimental mergers) is the possibility of making type I errors (banning pro-competitive ones). The equilibrium is met when the last deterred merger is detrimental to welfare,  $W^M(y^*) < 0$ , leading to a positive deterrence effect and a negative enforcement effect in equilibrium.<sup>9</sup>

<sup>&</sup>lt;sup>9</sup>The latter stays negative as long as the marginal cost of investigations is sufficiently low.

When allowing for remedies, the FOC on the CA's objective function writes as follows:

$$\frac{\partial E^R W}{\partial N} = \int_0^{y^{**}(N)} \left[ (g(y) - 1) W^M(y) + h(y) W^R(y) \right] dy + \left[ W^M(y^{**}) \left( 1 - N + Ng(y^{**}) \right) + Nh(y^{**}) W^R(y^{**}) \right] \frac{dy^{**}}{dN} = 0.$$
(6)

It is easy to note that when allowing for remedies, both the enforcement and the deterrence effects are affected. Part of the enforcement effect is now due to the welfare impact of a conditional approval instead of an unconditional approval (the term  $\int_0^{y^{**}(N)} h(y) W^R(y) dy$ ), while part of the deterrence effect comes now from the welfare impact of some mergers being abandoned although the conditional approval was available (the term  $Nh(y^{**})W^R(y^{**})\frac{dy^{**}}{dN}$ ).

Let us assume throughout the rest of the paper a unique interior solution in terms of optimal number of investigations launched by the agency.<sup>10</sup> The CA's optimal choice of an activity level when remedies are available,  $N^{**}$ , will obviously depend on the sign of the two above-mentioned effects. The latter are in turn determined by the characteristics of the last merger to be submitted/deterred. Thus the following result holds:

**Lemma 3** Assume the "right" deterrence under the remedies regime. Then the enforcement and the deterrence effect have opposite signs at the interior optimal activity level of the agency. In particular:

(i) a sufficient condition for the enforcement effect to be negative and the deterrence effect to be positive in equilibrium is that the last merger to be deterred is detrimental, with or without remedies;

(ii) a sufficient condition for the enforcement effect to be positive and the deterrence effect to be negative in equilibrium is that the last merger to be deterred is welfare-improving, with or without remedies.

**Proof.** Merger profitability is decreasing in N, so from the expected profit function  $E^R \Pi(y)$  one can easily derive that the "right" deterrence, i.e.  $\frac{\partial E^R \Pi(y)}{\partial y} < 0$ , implies  $\frac{dy^{**}}{dN} < 0$ .

<sup>&</sup>lt;sup>10</sup>We thus focus on the more interesting case, to the extent that the possible corner solutions where the agency never controls mergers or controls every submitted project are hardly realistic. Moreover, should there be more than one interior candidate activity level, it would suffice to retain the one corresponding to the global maximum. Finally, it can be checked that the concavity of the agency's objective function is compatible with all the subsequent sufficient conditions that we single out in the remaining of the paper.

Rewriting (6):  $\underbrace{\int_{0}^{y^{**}(N)} \left[ (g(y) - 1) W^{M}(y) + h(y) W^{R}(y) \right] dy}_{\text{enforcement effect}} = \underbrace{-\frac{dy^{**}}{dN}}_{>0} \left[ \begin{array}{c} W^{M}(y^{**}) \left[ 1 - N + Ng(y^{**}) \right] \\ + Nh(y^{**}) W^{R}(y^{**}) \end{array} \right]$ 

indicates that in equilibrium sign (enforcement effect)

 $= sign \left[ \left[ 1 - N + Ng(y^{**}) \right] W^M(y^{**}) + Nh(y^{**}) W^R(y^{**}) \right].$  A sufficient condition for signing the enforcement effect in equilibrium, and thereby the determine effect as well, is that  $signW^M(y^{**}) = signW^R(y^{**}).$ 

According to point (i) of Lemma 3, as long as the "right" deterrence is achieved by investigating mergers, the deterrence effect when remedies are available is positive in equilibrium iff  $W^M(y^{**}) [1 - N + Ng(y^{**})] + Nh(y^{**})W^R(y^{**}) < 0$ . It is straightforward to see that a sufficient condition for the deterrence effect to be positive is that  $W^M(y^{**}) < 0$  and  $W^R(y^{**}) < 0$ , meaning the last deterred merger would have been detrimental to welfare if cleared, with or without remedies. In this case, given the normalization of the marginal investigation cost to zero, the enforcement effect is negative in equilibrium - thus we provide here an extension of Sørgard (2009) to the case of conditional approvals.

The detailed expression of the enforcement effect reflects the trade-off leading to an interior solution in terms of merger investigation rate when  $W^M(y^{**}) < 0$  and  $W^R(y^{**}) < 0$ :  $\int_0^{y^{**}(N)} \left[ (g(y) - 1) W^M(y) + h(y) W^R(y) \right] dy$   $= \int_0^{y^0} (g(y) - 1) W^M(y) dy + \int_{y^{0}}^{y^{**}} (g(y) - 1) W^M(y) dy + \int_0^{y^{00}} h(y) W^R(y) dy + \int_{y^{00}}^{y^{**}} h(y) W^R(y) dy,$ where  $y^0$  is such that  $W^M(y^0) = 0$  and  $y^{00}$  is such that  $W^R(y^{00}) = 0$ . A negative enforcement effect is due to the substantial wrongful bans  $(\int_0^{y^0} (g(y) - 1) W^M(y) dy < 0)$  and wrongful conditional approvals  $(\int_{y^{00}}^{y^{**}} h(y) W^R(y) dy < 0)$ . This is the opportunity cost of further increasing on the margin the control rate N, whose "benefit" is the positive deterrence effect (since all deterred mergers  $y \ge y^{**}$  were harmful, with or without remedies:  $W^M(y^{**}) < 0$  and  $W^R(y^{**}) < 0$ .<sup>11</sup>

Point (ii) in Lemma 3 deals with the opposite case: still assuming the "right" deterrence, the enforcement effect is positive and the deterrence effect is negative in equilibrium

<sup>&</sup>lt;sup>11</sup>Alternatively, the intuition is easy to grasp by considering the limit case where  $W^R(y^{**}) = 0$ . In that case a remedy will fix the harm for the marginal merger, which will therefore imply for it a zero impact on welfare. The resulting situation is analogous to Sørgard (2009): the conditional approval will have exactly the same deterrence effect on welfare as a ban, and thus the last merger being deterred leads to a welfare improvement because it is a detrimental merger.

iff  $W^M(y^{**}) [1 - N + Ng(y^{**})] + Nh(y^{**})W^R(y^{**}) > 0$ . Again, a sufficient condition for this is that  $W^M(y^{**}) > 0$  and  $W^R(y^{**}) > 0$ , meaning the last deterred merger would have been welfare-improving, with or without remedies. In other words, some of the deterred mergers were actually pro-competitive, and the CA makes type I errors when controlling the remaining/actually submitted projects, to the extent that they are not unconditionally accepted. The negative deterrence effect in equilibrium requires in turn the enforcement effect to be positive in equilibrium.

To see the intuition for this, it is useful to recall that at the optimum, the interior solution necessarily strikes the balance between opposite-sign effects. Here the deterrence effect is negative in equilibrium, because increasing marginally the frequency of investigation will trigger even more mergers to be abandoned, although they would have been increased welfare if cleared (since  $W^M(y^{**}) > 0$ ). But the enforcement effect is positive in equilibrium, hence the trade-off, because one more investigation launched will also improve welfare through the conditional acceptance it will entail, since all submitted mergers to the last are welfareimproving if conditionally accepted ( $W^R(y^{**}) > 0$ ).

It is thus important to note that the remedy regime modifies the composition of both the enforcement and deterrence effects as compared with the strict, no-remedy regime. Incidentally, this enables a more complete definition of the CA's optimal activity level. The mere fact that conditional approvals are available indicates that the CA's decision errors also apply to them: some mergers are deterred although they might have been conditionally cleared, whereas other mergers are no longer banned but conditionally approved. The welfare impact of these decisions now enters the CA's trade-off, and Lemma 3 identifies two polar cases that are compatible with an interior solution in terms of the CA's optimal activity level.

The most important remark deals however with the interpretation of the two cases, (i) and (ii). Consider for instance the "neutral" merger defined by  $W^M(y^0) = 0$ . Then case (i), where  $W^M(y^{**}) < 0$ , corresponds to all mergers  $y \le y^{**}$  being submitted, although part of them are welfare-decreasing  $(y^0 \le y < y^{**})$ . In other words, not all submitted mergers are welfare-improving, but all the deterred mergers were welfare-decreasing, and therefore the CA's optimal activity level  $N^{**}$  is compatible with under-deterrence, to the extent the imperfect merger screening allows some of the anti-competitive mergers to be submitted. In turn, case (ii), for which  $W^M(y^{**}) > 0$ , has all the mergers such that  $y \in (y^{**}, y^0]$  abandoned. Therefore all submitted mergers are welfare-improving, but all deterred mergers are welfare-improving.

were not welfare-decreasing. The CA's optimal activity level  $N^{**}$  is now compatible with over-deterrence, since the imperfect merger screening deters some welfare-improving mergers.

At any rate, and as before mentioned, the possibility of a conditional approval modifies both sides of the trade-off that the CA faces, i.e. the enforcement and the deterrence effects. As a result, it is likely that the optimal choice of an activity level will differ between the two regimes.

#### 5.2 Optimal activity levels with and without remedies

Let us start by assuming that the same number of mergers is submitted in both regimes. For the sake of the comparison, let us rewrite below the FOC in the case of the "strict", no-remedy regimes, by using  $g^S = g + \gamma h$ :

$$\frac{\partial E^{S}W}{\partial N} = \int_{0}^{y^{*}(N)} \left[ (g(y) - 1) W^{M}(y) + \gamma h(y) W^{M}(y) \right] dy + \left[ W^{M}(y^{*}) \left( 1 - N + Ng(y^{*}) \right) + W^{M}(y^{*}) N \gamma h(y^{*}) \right] \frac{dy^{*}}{dN} = 0.$$
(7)

Then, based on (6) and (7), the following holds:

**Lemma 4** For  $y^* = y^{**}$ , then  $N^* \ge N^{**}$  iff  $\gamma \ge \frac{W^R}{W^M}$ .

**Proof.** Denote 
$$\Delta$$
 the difference between the two FOCs when  $y^*(N) = y^{**}(N)$ . Then  

$$\Delta = \frac{\partial E^S W}{\partial N}\Big|_{y^* = y^{**}} - \frac{\partial E^R W}{\partial N}\Big|_{y^* = y^{**}} = \left(\int_0^{y^*(N)} \gamma h(y) W^M(y) dy + W^M(y^*) N \gamma h(y^*) \frac{dy^*}{dN}\right)\Big|_{y^* = y^{**}} - \left(\int_0^{y^{**}(N)} h(y) W^R(y) dy + N h(y^{**}) W^R(y^{**}) \frac{dy^{**}}{dN}\right)\Big|_{y^* = y^{**}}.$$
If  $\gamma > \frac{W^R}{W^M}$ , then  $\Delta > 0 \Leftrightarrow N^{**} < N^*$  since the the expected welfare functions are concave

in N.

If  $\gamma < \frac{W^R}{W^M}$ , then  $\Delta < 0 \Leftrightarrow N^{**} > N^*$  following the same argument.

In other words, we have identified a sufficient condition to rank the CA's optimal activity levels with and without remedies available, provided that the same deterrence is achieved under both regimes. According to Lemma 4, this sufficient condition is, again, the comparison between  $\gamma$ , the change rate from net unconditional approvals into conditional ones, and the relative welfare ratio  $\frac{W^R}{W^M}$ . To grasp the intuition, it is useful to follow the proof and recall that the comparison of optimal activity levels between the two regimes results from that of the first order conditions on the respective expected welfare functions. Leaving aside the difference in the number of mergers submitted under each regime, expressions (6) and (7) differ in as much as part of the enforcement effect is due to the welfare impact of mergers no longer banned but conditionally accepted,  $\gamma W^M(y) \ge W^R(y)$ , and by the same token, part of the deterrence effect is due to those mergers that are abandoned but might have been conditionally, instead of unconditionally, cleared  $(W^R(y^{**}) \ge \gamma W^M(y^*))$ . When both regimes yield the same expected profitability for merger projects, i.e.  $y^* = y^{**}$ , the remaining relevant comparison is the one between the welfare gains from merger, weighted by the respective net approval rates:  $(g^S - g)W^M \le hW^R$ .

Finally, taking into account both Lemma 1 and Lemma 4, the following obtains:

**Corollary 3** Assume that the same number of mergers is submitted under both regimes. Iff  $\gamma \geq \frac{W^R}{W^M}$ , then  $N^* \geq N^{**}$  and  $E^S W \geq E^R W$  as well.

Equivalently, in the particular case of identical expected merger profitability  $(y^* = y^{**})$ , meaning  $\gamma = \frac{\Pi^R(y)}{\Pi^M(y)}$  following Proposition 1 and the discussion preceding Corollary 1, the sign of the difference between the relative increases in profits and welfare levels respectively  $(\frac{\Pi^R(y)}{\Pi^M(y)} - \frac{W^R(y)}{W^M(y)} \ge 0)$  directly indicates the ranking of the optimal activity levels for the CA and the resulting expected welfare levels as well. The intuition is simple: recall that identical expected merger profitability also means identical deterrence, and therefore the relative gain from enforcing a given regime only comes from minimizing the type II errors, or false approvals. This explains the direct relationship between the optimal activity level and the expected welfare  $(N^* \ge N^{**}$  and  $E^SW \ge E^RW$  as well): a more intense control activity prevents more anticompetitive mergers and thereby yields a higher welfare.

However, for a general comparison of the optimal activity levels, it is necessary to relax the assumption of equal deterrence under both regimes. This leads to the following:

Proposition 3 Assume that the most detrimental mergers are deterred. Then:

(i) for  $\gamma < \frac{\Pi^R}{\Pi^M}$  and  $\gamma < \frac{W^R}{W^M}$ ,  $\frac{d(y^* - y^{**})}{dN}\Big|_{N^{**}} < 0$  is a sufficient condition to have  $N^* < N^{**}$ ; (ii) for  $\gamma > \frac{\Pi^R}{\Pi^M}$  and  $\gamma > \frac{W^R}{W^M}$ ,  $\frac{d(y^* - y^{**})}{dN}\Big|_{N^{**}} > 0$  is a sufficient condition to have  $N^* > N^{**}$ ;

(iii) for  $\gamma > \frac{\Pi^R}{\Pi^M}$  and  $\gamma < \frac{W^R}{W^M}$ ,  $W^M(y^*) > 0$  and  $\frac{d(y^* - y^{**})}{dN}\Big|_{N^{**}} > 0$  are sufficient conditions to have  $N^* < N^{**}$ ;

(iv) for  $\gamma < \frac{\Pi^R}{\Pi^M}$  and  $\gamma > \frac{W^R}{W^M}$ ,  $W^M(y^*) < 0$  and  $\frac{d(y^* - y^{**})}{dN}\Big|_{N^{**}} < 0$  are sufficient conditions to have  $N^* > N^{**}$ .

See proof in the Appendix.

Proposition 3 provides sufficient conditions to rank the optimal activity levels between both merger control regimes when merger investigations deter the most welfare-detrimental merger projects, but the two regimes do not equally deter.

The cases displayed in Proposition 3 differ in terms of conditions enabling the comparison of optimal activity levels. The first two cases identified correspond to the situations where the private and public incentives due to merger control are each time compatible: higher merger profitability and higher social gain from controlling merger for the remedy regime in case (i), and the opposite in case (ii). This lack of conflict between private and public incentives goes along with a unique and quite simple sufficient condition for comparing the optimal investigation rates between regimes: the local monotonicity of the deterrence gap, or, alternatively, the impact of an infinitesimal increase in the investigation frequency on the deterrence or merger profitability differential in the vicinity of the optimal activity level of the remedy regime. Equivalently, we call this the marginal deterrence gap. In turn, the two remaining cases deal with situations where the private and public incentives regarding mergers are not aligned: lower merger profitability but higher social welfare from controlling mergers and allowing for remedies in case (iii), and higher merger incentives but lower social gain for the remedy regime in case (iv). In order to compare the optimal investigation rates between regimes, such conflicting incentives require a further sufficient condition, beyond signing the marginal determence gap. Formally, this additional sufficient condition deals with the type of the marginal merger in the strict, no-remedy regime, but it basically comes down to the occurrence of either under- or over-deterrence in equilibrium.

Let us provide the intuition for the results displayed in Proposition 3.

Consider case (i): it identifies the remedy regime as the one where mergers are less deterred in absolute terms ( $\gamma < \frac{\Pi^R}{\Pi^M} \Leftrightarrow y^{**} > y^*$ ), and also as the regime yielding a higher social gain from controlling mergers ( $\gamma W^M < W^R \Leftrightarrow (g^S - g)W^M < hW^R$  indicates that the merger welfare effect, weighted by the net approval rate, is higher in the remedy regime). As before mentioned, this is a case of aligned incentives, with the remedy regimes being "preferred" by both the merging firms and the competition authority. We find that the optimal investigation rate will be higher with remedies,  $N^{**} > N^*$ , provided that the marginal deterrence goes the same way as the absolute deterrence ( $\frac{d(y^* - y^{**})}{dN}\Big|_{N^{**}} < 0$  means that  $y^*$  diminishes faster than  $y^{**}$  when one more investigation is launched, indicating a "slower" deterrence when remedies are available). In other words, the competition agency can afford to conduct a more active merger policy in the remedy regime, given the lower reactivity of firms to its intervention (i.e. the lower, both absolute and marginal, deterrence, and hence the lower opportunity cost induced), and will optimally choose to do so, since the social gain from controlling mergers is higher.

The same type of argument goes for case (ii), which deals with the symmetrically opposite situation in terms of aligned incentives.

Let us turn now to cases (iii) and (iv), which exhibit in contrast conflicting incentives between the firms and the agency. For instance, in case (iii), when remedies are available the merger profitability is lower (or, equivalently, the absolute deterrence is higher), but the social welfare from controlling mergers is higher:  $\gamma > \frac{\Pi^R}{\Pi^M} \Leftrightarrow y^{**} < y^*$  but  $\gamma W^M < W^R \Leftrightarrow (g^S - y^*)$  $g W^M < h W^R$ . We find that the CA will optimally be more active in controlling mergers when remedies are allowed,  $N^{**} > N^*$ , provided that the marginal deterrence goes the same way as the absolute deterrence  $\left(\frac{d(y^*-y^{**})}{dN}\right|_{N^{**}} > 0$  means that  $y^{**}$  diminishes faster than  $y^*$  when one more investigation is launched, indicating a "quicker" deterrence when remedies are available), and also provided that the optimal investigation rates induce over-deterrence  $(W^M(y^*) > 0)$ leads to  $W^R(y^*) > 0$ ,  $W^M(y^{**}) > 0$  and  $W^R(y^{**}) > 0$ ). The intuition is the following: thanks to the higher "reactivity" of firms when remedies are available (higher absolute and marginal deterrence), the CA will be more active in controlling mergers as compared with the strict, noremedy regime, because the social gain from its public intervention is higher ( $\gamma W^M < W^R$ ). This holds as long as allowing for remedies does not lead to more wrongful approvals, or type II errors:  $W^{M}(y^{*}) > 0$ ,  $W^{R}(y^{*}) > 0$ ,  $W^{M}(y^{**}) > 0$  and  $W^{R}(y^{**}) > 0$  indicate that the remedy regime "replicates", precisely through the higher "reactivity" of firms, the outcome of over-deterrence in equilibrium obtained under the strict regime, meaning that all submitted mergers in equilibrium (i.e. for  $N^*$  and  $N^{**}$  respectively) are welfare-improving, therefore no type II errors are possible.

In contrast, case (iv) has the CA optimally control fewer mergers in the remedy regime  $(N^* > N^{**})$  because the gain from its intervention is lower:  $\gamma W^M > W^R \Leftrightarrow (g^S - g)W^M > hW^R$  indicates that the merger welfare gain, weighted by the net approval rate, is lower in the remedy regime. This holds however whenever the strict regime exhibits higher deterrence, both absolute ( $\gamma < \frac{\Pi^R}{\Pi^M} \Leftrightarrow y^{**} > y^*$ ), and marginal  $(\frac{d(y^* - y^{**})}{dN}\Big|_{N^{**}} < 0$  means that  $y^*$  diminishes faster than  $y^{**}$  when one more investigation is launched), although the optimal investiga-

tion rates actually induce under-deterrence  $(W^M(y^*) < 0 \text{ leads to } W^R(y^*) < 0, W^M(y^{**}) < 0$ and  $W^R(y^{**}) < 0$  as well). Basically, the under-deterrence outcome indicates that all deterred mergers were welfare-decreasing, but all submitted mergers are not welfare-improving, therefore the imperfect merger control leads to type II errors (wrongful clearances, with or without remedies). As a result, the CA will optimally be less active when allowing for remedies as long as the remedy regime replicates the under-deterrence obtained under the strict regime, given the lower reactivity of firms to public intervention and the lower gain obtained from controlling mergers when conditional approvals are possible.

# 6 Concluding remarks

The purpose of this article has been to discuss the possible welfare effects of merger remedies. For this, we highlight the crucial role played by the remedies for the change in unconditional clearance rate. We show that allowing for merger remedies has a non-trivial effect on the incentives to merge, the agency's merger control activity level as well as the welfare effect of merger control. In particular, our analysis indicates that when conditional approvals are possible, it may be harder to deter the most welfare-detrimental mergers, the agency might have to investigate mergers more often (hence an additional cost of conditional approvals), and the final welfare from merger control might be lower than without remedies.

Let us relate our results to the empirical findings in the literature. Although the empirical findings are limited, it is shown in Seldeslachts et al. (2009) that more clearances conditional on remedies tends to increase the number of proposed mergers. Although remedies might be good for enforcement - for example solving a merger with remedies can be better than banning it - our analysis indicates that their impact on deterrence might be crucial. First, introducing remedies might imply that some of the worst mergers would suddenly be profitable to propose. If so, we are no longer deterring the right mergers. Second, a regime with merger remedies can - as the empirical study indicates - lead to more mergers being proposed. This will lead to less deterrence on the margin, unless there is a sufficiently large increase in the agency's merger control activity.

Our analysis has important implications for how we should test empirically the effect of remedies. There are several empirical studies that question the welfare effects of imposing remedies. However, all these studies consider only the enforcement effect - either of behavioral or structural remedies applied in specific merger control cases. We point out that this might not capture the potentially most important problem associated with remedies. It might lead to less deterrence of mergers that on the margin are detrimental to welfare, and even a shift in direction of the worst mergers no longer being deterred.

### References

- Baer, W.J. and R.C. Redcay (2003) "Solving Competition Problems in Merger Control: The Requirements for an Effective Divestiture Remedy," in F. Leveque and H. Shelanski, *Merger Remedies in American and European Union Competition Law*, London: Edward Elgar Publishers.
- [2] Baker, J. B. (2003) "The case for antitrust enforcement", Journal of Economic Perspectives 17(4), p.27-50.
- [3] Barros, P.P., J. Clougherty and J. Seldeslachts (2010) "How to measure the deterrence effects of merger policy: frequency or composition?", International Journal of the Economics of Business 17(1), p. 1-8.
- [4] Cabral, L. (2003) "Horizontal Mergers with Free-Entry: Why Cost Efficiency May Be a Weak Defense and Asset Sales a Poor Remedy", International Journal of Industrial Organization 21(5), p. 607-623.
- [5] Clougherty, J.A., and J. Seldeslachts (2013) "The Deterrence Effects of U.S. Merger Policy Instruments," Journal of Law, Economics and Organization 29, p. 1114-1144.
- [6] Cosnita-Langlais, A. and J.-P. Tropeano (2012) "Do Remedies Affect the Efficiency Defense? An Optimal Merger-Control Analysis", International Journal of Industrial Organization, 30(1), p. 58-66.
- [7] Crandall, R. W. and C. Winston (2003) "Does antitrust policy improve consumer welfare? Assessing the evidence", Journal of Economic Perspectives 17(4), p. 3-26.
- [8] Duso, T., D. Neven et L-H. Röller (2007) "The Political Economy of European Merger Control: Evidence Using Stock Market Data", Journal of Law and Economics 50(3), p.455-489.

- [9] Joskow, P. (2002) "Transaction Cost Economics, Antitrust Rules and Remedies", Journal of Law, Economics and Organization 18(1), p.95-116.
- [10] Leveque, F. and H. Shelanski (eds.) (2003) "Merger Remedies in American and European Union Competition Law", Edwar Elgar, Cheltenham, England.
- [11] Motta, M., M. Polo and H. Vasconcelos (2007) "Merger Remedies in the EU: An Overview", Antitrust Bulletin, 52, p. 603-631.
- [12] Neven, D., Nuttal, R. and P. Seabright (1993), Merger in Daylight: The Economics and Politics of European Merger Control, London CEPR.
- [13] Seldeslachts, J., J.A. Clougherty, and P.P. Barros (2009) "Settle for Now but Prevent for Tomorrow: The Deterrence Effects of Merger Policy Tools", Journal of Law and Economics Vol. 52(3), August 2009, p.607-634.
- [14] Sørgard, L. (2009) "Optimal Merger Policy: Enforcement vs. Deterrence", Journal of Industrial Economics 57(3), p. 438-456.
- [15] Vasconcelos, H. (2010) "Efficiency Gains and Structural Remedies in Merger Control", The Journal of Industrial Economics 58(4), p. 742-766.
- [16] Vergé, T. (2010) "Horizontal Mergers, Stuctural Remedies and Consumer Welfare in a Cournot Oligopoly with Assets", The Journal of Industrial Economics 58(4), p. 723-741.

# 7 Appendix

#### Proof of Proposition 3.

In order to compare the two FOCs when  $y^* \neq y^{**}$ , one may use their monotonicity and consider evaluating one of them at the optimal activity level corresponding to the other regime, since we focus on the case of a unique interior optimum. For instance, if  $\frac{\partial E^S W}{\partial N}\Big|_{N^{**}} \geq$ 0, then  $N^{**} \leq N^*$ .

Recall that  $\frac{\partial E^S W}{\partial N} = \int_0^{y^*(N)} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^M(y) \right] dy$ 

$$\begin{split} &+ \left[ W^{M}(y^{*}) \left[ 1 - N + Ng(y^{*}) \right] + W^{M}(y^{*}) N\gamma h(y^{*}) \right] \frac{dy^{*}}{dN} \\ &\text{and that } N^{**} \text{ is such that } \frac{\partial E^{R}W}{\partial N} = \int_{0}^{y^{**}(N)} \left[ (g(y) - 1) W^{M}(y) + h(y) W^{R}(y) \right] dy \\ &+ \left[ W^{M}(y^{**}) \left[ 1 - N + Ng(y^{**}) \right] + Nh(y^{**}) W^{R}(y^{**}) \right] \frac{dy^{**}}{dN} = 0. \\ &\text{Therefore } \left. \frac{\partial E^{S}W}{\partial N} \right|_{N^{**}} = \int_{0}^{y^{*}(N^{**})} (g(y) - 1) W^{M}(y) dy + \int_{0}^{y^{*}(N^{**})} \gamma h(y) W^{M}(y) dy \\ &+ \left. \frac{dy^{*}}{dN} \right|_{N^{**}} W^{M}(y^{*}) \left[ 1 - N^{**} + N^{**}g(y^{*}) \right] + \left. \frac{dy^{*}}{dN} \right|_{N^{**}} N^{**}h(y^{*}) \gamma W^{M}(y^{*}). \end{split}$$

$$\begin{split} & (i) \ \text{let} \ \gamma < \frac{\Pi^R}{\Pi^M} \ (\text{i.e.} \ y^{**} > y^*) \ \text{and} \ \gamma < \frac{W^R}{W^M} \\ & \text{Then} \ \int_0^{y^{*}(N^{**})} (g(y) - 1) \ W^M(y) dy + \int_0^{y^{*}(N^{**})} \gamma h(y) W^M(y) dy \\ & < \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) \ W^M(y) + h(y) W^R(y) \right] dy \\ & = - \left[ W^M(y^{**}) \left[ 1 - N^{**} + N^{**}g(y^{**}) \right] + N^{**}h(y^{**}) W^R(y^{**}) \right] \frac{dy^{**}}{dN} \\ & \text{therefore} \ \frac{\partial E^S W}{\partial N} \Big|_{N^{**}} < \frac{dy^*}{dN} \Big|_{N^{**}} W^M(y^*) \left[ 1 - N^{**} + N^{**}g(y^*) + N^{**}h(y^*) W^R(y^*) \right] \\ & - \frac{dy^{**}}{dN} \Big|_{N^{**}} \left[ W^M(y^{**}) \left( 1 - N^{**} + N^{**}g(y^{**}) \right) + N^{**}h(y^{**}) W^R(y^{**}) \right] \frac{dy^{**}}{dN} \Big|_{N^{**}} \\ & < \left[ (1 - N^{**}) + N^{**}(g(y^{**}) + \gamma h(y^{**})) \right] \left( W^M(y^*) \frac{dy^*}{dN} \Big|_{N^{**}} - \frac{W^R(y^*)}{\gamma} \frac{dy^{**}}{dN} \Big|_{N^{**}} \right), \\ & \text{thanks to} \ \gamma < \frac{W^R}{W^M}, \ \frac{dy^*}{dN} \Big|_{N^{**}} < 0, \ - \frac{dy^{**}}{dN} \Big|_{N^{**}} > 0, \ \text{and the monotonicity of} \ W^M, W^R, g, \end{split}$$

and h.

In order for 
$$\frac{\partial E^S W}{\partial N}\Big|_{N^{**}} < 0$$
, leading to  $N^{**} > N^*$ , it is enough to have  
 $\left(W^M(y^*) \left. \frac{dy^*}{dN} \right|_{N^{**}} - \frac{W^R(y^*)}{\gamma} \left. \frac{dy^{**}}{dN} \right|_{N^{**}} \right) < 0 \Leftrightarrow \frac{\left. \frac{dy^*}{dN} \right|_{N^{**}}}{\left. \frac{dy^{**}}{dN} \right|_{N^{**}}} < \frac{W^R(y^*)}{\gamma W^M(y^*)}$ , and since  $\gamma < \frac{W^R}{W^M}$ , a sufficient condition for this is  $\frac{\left. \frac{dy^*}{dN} \right|_{N^{**}}}{\left. \frac{dy^{**}}{dN} \right|_{N^{**}}} < 1$ .

(ii) let  $\gamma > \frac{\Pi^R}{\Pi^M}$  and  $\gamma > \frac{W^R}{W^M}$ Thanks to  $\gamma > \frac{\Pi^R}{\Pi^M}$ , i.e.  $y^{**} < y^*$ ,  $\frac{dy^*}{dN}\Big|_{N^{**}} < 0$ ,  $-\frac{dy^{**}}{dN}\Big|_{N^{**}} > 0$ , and the monotonicity of  $W^M, W^R, g$ , and h, one has that:

$$\begin{split} \frac{\partial E^{S}W}{\partial N}\Big|_{N^{**}} &> \frac{dy^{*}}{dN}\Big|_{N^{**}} \left(W^{M}(y^{*})\left[1-N^{**}+N^{**}g(y^{*})\right]+N^{**}h(y^{*})\gamma W^{M}(y^{*})\right) \\ &-\left[W^{M}(y^{**})\left(1-N^{**}+N^{**}g(y^{**})\right)+N^{**}h(y^{**})W^{R}(y^{**})\right]\frac{dy^{**}}{dN}\Big|_{N^{**}} \\ &> \frac{dy^{*}}{dN}\Big|_{N^{**}} W^{M}(y^{**})\left(1-N^{**}+N^{**}g(y^{**})+N^{**}h(y^{**})\gamma\right) \\ &-\frac{dy^{**}}{dN}\Big|_{N^{**}}\frac{W^{R}(y^{**})}{\gamma}\left(1-N^{**}+N^{**}g(y^{**})+N^{**}h(y^{**})\gamma\right) \\ &= \left(1-N^{**}+N^{**}g(y^{**})+N^{**}h(y^{**})\gamma\right)\left(\frac{dy^{*}}{dN}\Big|_{N^{**}} W^{M}(y^{**})-\frac{dy^{**}}{dN}\Big|_{N^{**}}\frac{W^{R}(y^{**})}{\gamma}\right). \\ &\text{Then } \left.\frac{\partial E^{S}W}{\partial N}\Big|_{N^{**}} > 0 \text{ if } \left.\frac{dy^{*}}{dN}\Big|_{N^{**}} W^{M}(y^{**})-\frac{dy^{**}}{dN}\Big|_{N^{**}}\frac{W^{R}(y^{**})}{\gamma} > 0, \text{ and for this a sufficient condition is that } \left.\frac{dy^{*}}{dN}\Big|_{N^{**}} > \frac{dy^{**}}{dN}\Big|_{N^{**}} \operatorname{since } \gamma > \frac{W^{R}}{W^{M}}. \end{split}$$

(iii) let  $\gamma > \frac{\Pi^R}{\Pi^M}$  and  $\gamma < \frac{W^R}{W^M}$ Thanks to  $\gamma > \frac{\Pi^R}{\Pi^M}$ , i.e.  $y^{**} < y^*$  and the monotonicity of  $W^M, W^R, g$ , and h, one has

that:

$$\begin{split} \frac{\partial E^S W}{\partial N} \Big|_{N^{**}} &< \int_0^{y^*(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^M(y) \right] dy \\ &+ \frac{dy^*}{dN} \Big|_{N^{**}} W^M(y^{**}) \left[ 1 - N^{**} + N^{**}g(y^{**}) \right] + \frac{dy^*}{dN} \Big|_{N^{**}} N^{**}h(y^{**}) W^R(y^{**}) \\ &= \int_0^{y^{*}(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^M(y) \right] dy \\ &- \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + h(y) W^R(y) \right] dy \frac{\frac{dy^*}{dN} \Big|_{N^{**}}}{\frac{dy^{**}}{dN} \Big|_{N^{**}}} . \\ & \text{Then } \frac{\partial E^S W}{\partial N} \Big|_{N^{**}} < 0 \text{ (i.e. } N^{**} > N^* \text{ ) if } \int_0^{y^*(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^R(y) \right] dy \\ &- \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + h(y) W^R(y) \right] dy \frac{\frac{dy^*}{dN} \Big|_{N^{**}}}{\frac{dy^{**}}{dN} \Big|_{N^{**}}} < 0 \\ & \Leftrightarrow \frac{\int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + \eta(y) W^R(y) \right] dy \\ & = \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + \eta(y) W^R(y) \right] dy \\ & > \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^R(y) \right] dy \\ & > \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^R(y) \right] dy \\ & > \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^R(y) \right] dy \\ & > \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^M(y) \right] dy \\ & > \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^M(y) \right] dy \\ & > \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^M(y) \right] dy \\ & > \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^M(y) \right] dy > 0, \text{ for which } W^M(y) \right] dy \\ & > \int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + \gamma h(y) W^M(y) \right] dy > 0, \text{ for which } W^M(y^*) > 0 \text{ is a sufficient condition. Then it is enough to have } 1 < \frac{\frac{dy^*}{dN}}{\frac{dy^*}{N^{**}}}{\frac{dy^*}{N^{**}}} \text{ for } \frac{\partial E^S W}{\partial N} \Big|_{N^{**}} < 0 \text{ (i.e. } N^{**} > N^*). \end{aligned}$$

$$\begin{array}{l} (\mathrm{iv}) \ \mathrm{let} \ \gamma < \frac{\Pi^R}{\Pi^M} \ (\mathrm{i.e.} \ y^{**} > y^*) \ \mathrm{and} \ \gamma > \frac{W^R}{W^M} \\ \mathrm{Then} \ \frac{dy^*}{dN} \Big|_{N^{**}} W^M(y^*) \ [1 - N^{**} + N^{**}g(y^*)] + \frac{dy^*}{dN} \Big|_{N^{**}} N^{**}h(y^*)\gamma W^M(y^*) \\ > \ \frac{dy^*}{dN} \Big|_{N^{**}} W^M(y^{**}) \ [1 - N^{**} + N^{**}g(y^{**})] + \frac{dy^*}{dN} \Big|_{N^{**}} N^{**}h(y^{**})W^R(y^{**}) \\ \mathrm{thanks} \ \mathrm{to} \ \gamma > \frac{W^R}{W^M} \ \mathrm{and} \ \mathrm{the} \ \mathrm{monotonicity} \ \mathrm{of} \ W^M, W^R, g, \ \mathrm{and} \ h; \\ \mathrm{but} \ W^M(y^{**}) \ [1 - N^{**} + N^{**}g(y^{**})] + N^{**}h(y^{**})W^R(y^{**}) = \frac{-\int_0^{y^{**}(N^{**})} \left[(g(y) - 1)W^M(y) + h(y)W^R(y)\right]dy}{\frac{dy^{**}}{dN^{**}}} \\ \mathrm{therefore} \ \frac{\partial E^S W}{\partial N} \Big|_{N^{**}} > \int_0^{y^{*}(N^{**})} \left[(g(y) - 1)W^M(y) + \gamma h(y)W^M(y)\right]dy \\ - \frac{\frac{dy^*}{dN}}{\frac{dy^{**}}{dN}} \Big|_{N^{**}} \int_0^{y^{**}(N^{**})} \left[(g(y) - 1)W^M(y) + h(y)W^R(y)\right]dy. \\ \mathrm{Then} \ \frac{\partial E^S W}{\partial N} \Big|_{N^{**}} > 0, \ \mathrm{i.e.} \ N^{**} < N^*, \ \mathrm{if} \ \frac{\int_0^{y^{*}(N^{**})} \left[(g(y) - 1)W^M(y) + \gamma h(y)W^M(y)\right]dy}{\int_0^{y^{**}(N^{**})} \left[(g(y) - 1)W^M(y) + h(y)W^R(y)\right]dy} > \frac{\frac{dy^*}{dN}}{\frac{dy^{**}}{dN^{**}}}. \\ \mathrm{Assume} \ W^M(y^*) < 0. \ \mathrm{This} \ \mathrm{leads} \ \mathrm{to} \ W^R(y^*) < 0 \ \mathrm{as} \ \mathrm{well}, \ \mathrm{since} \ \gamma > \frac{W^R}{M}, \ \mathrm{but} \ \mathrm{also} \end{aligned}$$

 $W^M(y^{**}) < 0$  and  $W^R(y^{**}) < 0$  since  $y^{**} > y^*$  and  $W^M$  and  $W^R$  are decreasing in y. Consequently, given  $y^{**} > y^*$ , one has that  $\int_0^{y^{**}(N^{**})} \left[ (g(y) - 1) W^M(y) + h(y) W^R(y) \right] dy$ 

$$= \int_{0}^{y^{*}(N^{**})} \left[ (g(y) - 1) W^{M}(y) + h(y) W^{R}(y) \right] dy + \underbrace{\int_{y^{*}(N^{**})}^{y^{**}(N^{**})} \left[ (g(y) - 1) W^{M}(y) + h(y) W^{R}(y) \right] dy}_{<0} \\ < \int_{0}^{y^{*}(N^{**})} \left[ (g(y) - 1) W^{M}(y) + h(y) W^{R}(y) \right] dy. \\ \text{Thus } \frac{\int_{0}^{y^{*}(N^{**})} \left[ (g(y) - 1) W^{M}(y) + \gamma h(y) W^{M}(y) \right] dy}{\int_{0}^{y^{*}(N^{**})} \left[ (g(y) - 1) W^{M}(y) + h(y) W^{R}(y) \right] dy} > \underbrace{\int_{0}^{y^{*}(N^{**})} \left[ (g(y) - 1) W^{M}(y) + h(y) W^{R}(y) \right] dy}_{\int_{0}^{y^{*}(N^{**})} \left[ (g(y) - 1) W^{M}(y) + h(y) W^{R}(y) \right] dy} > 1 \text{ since } \\ \gamma > \frac{W^{R}}{W^{M}}. \\ \text{A further sufficient condition to eventually have } \frac{\partial E^{S}W}{\partial N} \Big|_{N^{**}} > 0, \text{ i.e. } N^{**} < N^{*}, \text{ is } \\ 1 > \frac{\frac{dy^{*}}{dN} \Big|_{N^{**}}}{dN^{*}}. \end{aligned}$$

$$1 > \frac{\frac{dy^*}{dN}\Big|_{N^{**}}}{\frac{dy^{**}}{dN}\Big|_{N^{**}}}.$$