On the Complementarity of Money and Credit

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Abstract

I propose a model where agents optimally choose to conduct their business using two payment instruments, money and bilateral credit. A friction in the timing of transactions rationalizes the use of both instruments and makes it optimal for agents to use money as a means of settlement for credit. Money and credit complement each other. With anticipated inflation, complementarity implies that the credit to money ratio decreases with inflation.

Keywords: Coexistence of Money and Credit.

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1 Introduction

This paper is a theoretical inquiry into the relationship between money and bilateral credit as means to facilitate the exchange of commodities. It is well known that the walrasian framework is ill-suited to generate a role for money as a means of exchange. Models where money has an essential role as a means of exchange typically depart from the walrasian model in several dimensions. The walrasian framework on the other hand is perfectly at ease with credit being used to transact. The present paper asks which features would an environment - which is structured so as to induce agents to trade goods bilaterally, but it is otherwise fairly close to a walrasian world- have to display in order to make both money and credit circulate together as means of exchange in a way that is socially beneficial.

The world I am going to describe is one where buyers and sellers choose to trade goods using both money and a bilateral credit instrument: cash and promissory notes. Since lenders might be reluctant to lend - being worried about the possibility that borrowers will not repay their debt- notes are guaranteed by collateral. Lenders repay their debt with money which will emerge as the means of settlement. Both payment instruments - cash and credit- are nominal and risk-free. Agents are not confronted with any source of uncertainty. Cash and credit transactions differ only with respect to the time at which an obligation is discharged: the former type of transaction involving an immediate cash payment, the latter a delayed cash payment.

It is natural to ask why would anyone accept a promise of later cash in a world where agents are impatient. A seller surely prefers to sell for immediate cash rather than a promise of later cash, since the second option can only postpone the time at which he will be able to spend it[1]. Suppose, though, that the seller is not going to be able to spend cash immediately the following period, having to hold it for the entire period - I will call it the period of rest- and getting a chance to spend it only in two periods time. If that is the case, a promise of next period money has a chance to be accepted and a

[1] One might think that a sufficiently high interest rate could make a seller willing to lend. However the interest rate can be positive only if there is an incentive to lend resources.

positive interest rate can emerge. Provided agents have to keep idle balances for some time, an incentive to accept such promises exists. "If we mean by hoarding the holding of idle balances, then my theory of the rate of interest might be expressed by saying that the rate of interest serves to equate the demand and supply of hoards" in the words of John M. Keynes[2]. Credit is beneficial for creditors since it allows them to partly compensate the time cost of holding idle balances and for debtors since it allows them to increase consumption - although at the cost of an interest payment-. An interest rate reflecting purely the elapse of time and balancing the incentives of creditors and debtors will emerge endogenously.

I capture the former idea assuming that all agents have to spend an entire period resting after producing. I will argue that in the absence of a period of rest, not only promises of future money are not exchanged, but money itself is not going to be essential in this environment. A superior allocation can be reached trading goods on promissory notes and then having all creditors and debtors meet and clear promises against each other[3]. The period of rest prevents a clearing system to work properly: an agent who has just produced and is ready for settlement cannot do it right away, since he is in his period of rest. Given that centralized clearing cannot be organized smoothly, agents will try to make the best out of the situation, using money itself to settle debt. Cash and promissory notes emerge together as means of exchange and cash serves for settlement. Moreover, the combination of two cash-based means of exchange leads to an allocation which is superior to the allocation the economy would reach with just one of the two. This is the sense in which money and credit are complementary.

Interestingly, complementarity implies that the credit to money ratio decreases with anticipated inflation, since inflation increases the opportunity cost of holding a promise of future money more than the opportunity cost of holding immediate money. In a world with cash and a non-cash credit instrument, agents would substitute away from cash in order to avoid the inflation tax. Here, with cash and a cash-based credit instrument the reverse happens. Boyd, Levine and Smith (2001) highlight the fact that in high inflation

[2] In "Alternative Theories of the Rate of Interest".

[3] Such promissory notes would clearly not be promises of money.

countries credit has been severely harmed by inflation. An accurate historical account of the dramatic impact the German hyperinflation had on credit relationships can be found in Holtfrerich (1986).

1.1 Related Literature

The model is related to several papers in the literature. The structure is similar to Banerjee and Maskin (1996), with bilateral meetings and walrasian markets. Here, however, I do not consider any asymmetry of information about the quality of goods. The spatial separation literature a' la Townsend (1980) considers a similar environment, but where the itineraries are fixed. Here agents are free to pick their itineraries.

The random matching literature a' la Kiyotaki and Wright (1993) has addressed the issue of the coexistence of money and credit. The typical credit arrangement examined in this literature doesn't involve the use of cash for settlement and it is socially beneficial since it provides better risk-sharing than monetary trade. The main theoretical contribution is Kocherlakota and Wallace (1998), where a risk-sharing credit arrangement is operated using a communication technology which suffers interruptions at random times. Agents carry cash in order to purchase commodities when communication is interrupted. The credit arrangement itself does not involve cash payments. One can think of an institution providing transaction services, which substitutes for money rather than complement it. Here I focus instead on a bilateral form of credit which complements monetary trade. No doubt, both risk-sharing credit arrangements and bilateral credit instruments exist in reality. The reasons why they are adopted, the theories of the interest rate and the predictions for the effects of monetary policy on macroeconomic variables they generate, are however quite different. A way to see the difference between the two approaches is to use the language of commitment and monitoring. Kocherlakota and Wallace (1998) consider monitoring of transactions the crucial feature for a credit system to work. In a complementary effort, here I consider instead a world where monitoring is not feasible and credit arises thanks to the ability of agents to bilaterally commit to future actions with collateral. In the random matching tradition Berentsen, Camera and Waller (2006) show that the Lagos and Wright (2005) model of monetary exchange can accommodate bank credit. Here neither random matching nor banks are considered. Crucially, the resting period captures idleness of resources - a concept they also use- in quite a different way[4]. Also, in the present paper, unlike theirs, both lenders and borrowers have a strict incentive to participate in the credit system. He, Huang and Wright (2005) find a safe keeping role for banks in a random matching model. Here the focus is not on safekeeping.

A paper dealing with the coexistence of money and bilateral credit in a similar vein - i.e. focusing on the issue of commitment rather than on monitoring- is Shi (1996). The present paper and his have two main features in common: the use of collateral as a commitment technology and the resting period. The former is fairly standard: if monitoring is impossible the alternative is to guarantee repayment with collateral. I use a stylized form of collateral in the model, similar to the proverbial "pound of flesh" Shylock asks for in William Shakespeare's *Merchant of Venice:* "If you repay me not on such a day,/ In such a place, such sum or sums as are/ Express'd in the condition, let the forfeit/ Be nominated for an equal pound/ Of your fair flesh, to be cut off and taken/ In what part of your body pleaseth me."[5] More generally the reader can think of assets used as collateral. The latter issue is more subtle. In the paper by Shi (1996) the period of inaction agents have to undergo after entering a credit contract has only the purpose of making repayment feasible. In the present environment, which does not feature random matching, the existence of a resting period is responsible for the fact that both money and credit are essential and money serves to repay promissory notes, since it prevents a clearing system for promissory notes to work smoothly.

Let me summarize the discussion so far going back to the initial question about the walrasian framework. We know that some bilateral trade, some restrictions on the meetings pattern, on the communication and monitoring possibilities and on the ability of agents to commit is needed for money to have value in a walrasian world. In order to have a role for credit when monitoring is not feasible though, agents should be able to

[4] A symptom of the difference is the effect of inflation on the nominal interest rate, which is one to one in their model, while it is not one to one in the present paper.

[5] *Merchant of Venice*, Act I, Scene III.

at least partially commit to future actions- which is where collateral plays a role-. This is also known. But in the present model - given the possibility for agents to freely move around (no random matching or predetermined itineraries are assumed)- frictions could be undone in a way that wouldn't leave any role for money. The resting period provides the crucial assumption which makes money and credit stand and stand together. It limits the possibilities of multilateral meetings, which in turn limits the possibilities of communication - in the terminology of Townsend (1987)- in an environment where travel and communication would be too easy for money to have any role. Townsend (1989) explores the coexistence of money and credit in a world where the former is used to trade with strangers and the latter is used in enduring relationships. Here such a form of credit is not feasible since agents' relationships are not enduring enough.

The period of rest in the model is literally the night-time or - changing the time scale- the Jewish Shabbat or the Christian Sunday. The overlapping generation model by Freeman (1996) features a similar assumption: creditors and debtors do not arrive on the central island - where the clearing of debt happens- all at the same time. Here, unlike in Freeman (1996), there is not going to be any centralized clearing: settlement in equilibrium will happen in a bilateral way. More generally, rest could be thought of as a Baumol-Tobin type of friction, as in Grossman and Weiss (1983), where agents cannot visit their bank and withdraw every period, but they have to do it every other period. In their paper such a friction induces short-run non-neutralities of monetary policy. Here the resting period induces non-neutralities even in steady state. The fact that inflation harms credit in this economy, squares with Green (2002) who argues that inflation may destroy the acceptability of money in a model where debt is not settled with money, but not in a model where money serves as the means of settlement.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 derives the equilibrium. Section 4 concludes.

Day (Morning – Evening)

	Evening	Morning	Afternoon	Evening	Morning
Type 1	Rest	**Consume**	**Produce**	**Rest**	Consume
Type 2	Consume	**Produce**	**Rest**	**Consume**	Produce
Type 3	Produce	**Rest**	**Consume**	**Produce**	Rest

Figure 1: A Day: Height Hours Shifts

2 The Model

Time is discrete and continues for ever. Agents are infinitely lived. They can consume, produce and rest. There are $N > 4$[6] islands arranged on a circle, indexed by $j = 1, ..., N$. Each island is inhabited by a continuum of mass three of agents. On each island j one and only one type of perishable commodity (j) can be produced: agents living on island j consume commodity j and produce commodity $j + 1$ (modulo N). After producing individuals need to rest for one period, during which they will not be able to consume and produce[7]. Some of agents j will be consuming in the morning, producing in the afternoon and resting in the evening (type 1), some others producing in the morning, resting in the afternoon and consuming in the evening (type 2) and finally some resting in the morning, consuming in the afternoon and producing in the evening (type 3). There will therefore be eight hours periods in a day and agents will do either the morning, afternoon or evening shift. (See Figure 1).

Every ordered pair of islands is connected by ships. Agents are free to choose their itineraries[8]. On each island competitive markets for the exchange of the local commodity

[6] $N > 4$ guarantees that promises cannot be simply swapped instead of being repayed.

[7] For simplicity I assume that the disutility of consuming or producing without resting is infinite. A finite disutility would be enough.

[8] In equilibrium agents only visit the islands where their buyers and their sellers live. Potentially however they can travel to any island.

open each period, closing at the end of the period. When agents arrive on the island they are matched to a trading partner in a way that avoids the possibility for two agents to meet each other twice and also to meet some agents who met either of the two in the past. In the terminology of Aliprantis, Camera and Puzzello (2005), such a matching function is strongly anonymous. Since I do not consider in this paper credit systems based on enduring relationships, this assumption is meant to rule them out by making relationships short-lived. Let me stress that this form of matching is quite different from the Kiyotaki and Wright (1993) type of matching. In the matching literature meetings between types of agents are random: in the present context, there would be random matching in that sense if agents were assigned randomly to islands. Here instead agents are matched according to some exogenous process - which might, but doesn't have to, be random- only once they are on the island they have chosen to visit. Agents can be re-matched without cost to another trading partner should they choose to. This assumption -together with the fact that there is a continuum of agents- keeps the price competitive. At the beginning of time, on each island, there is an amount M of fiat money, in the form of durable and worthless pieces of paper. Agents have also the option of issuing their own promises. There is no central record-keeping technology to monitor and enforce promises. To induce agents to repay their promises in an environment without societal enforcement is a fairly complicated business, unless collateral is available. Each agent has an individual specific durable collateral without which he is unable to consume[9]. This is just meant to represent the perfect form of collateral: only the owner cares about it and he desperately needs it. Collateral allows borrowers to commit to repay. In order to allow the lender to commit to return the collateral without extorting from his borrower more money than stipulated in the original contract, I assume that on the ships there are lockers where the collateral can be stored. When issuing a promise, each agent surrenders his collateral to his trading partner in exchange for the commodity he wants to purchase. The collateral is then stored in the locker and the only key of the locker is held by the borrower. Repayment takes time: the issuer of a promise needs to

[9] For simplicity I assume that the disutility of consuming without the collateral is infinite. A finite disutility would be enough.

go back to his island, produce and sell in order to gain the money needed to make final payment. When the issuer has got money he goes back to the ship, inserts the money in the locker, opens it with the key and gets his collateral back. The lockers can be opened only after money is inserted and the key is unique. This detailed description should make clear that such a commitment technology is bilateral, being quite different from societal enforcement technologies. The fact that agents have to rest after producing, prevents multilateral meetings where borrowers and lenders could simultaneously clear all promises. Societal enforcement is limited. At all times agents can simply decide not to participate in the exchange process. On each island there are competitive markets for the exchange of goods for money (the money market) and for the exchange of goods for promises (the credit market).

Agents are characterized by a linear utility function $u(x_{t,j}) = x_{t,j}$ where $x_{t,j}$ is the quantity bought at time t on island j and a quadratic cost function -in terms of utils- $c(y_{t,j}) = \frac{1}{2}(y_{t,j})^2$ where $y_{t,j}$ is the quantity produced by producer j.[10] The objective of an agent of type j is to maximize

$$\sum_{t\in T}\beta^t \left[x_{t,j}^M + x_{t,j}^C - \beta\frac{1}{2}\left(y_{t+1,j+1}^M + y_{t+1,j+1}^C\right)^2\right],$$

where $T = \{0, 3, 6, ...\}$ in order to take into account the resting period and $0 < \beta < 1$ is the time discount rate. $x_{t,j}^M$ and $x_{t,j}^C$ are the quantities of good j bought at time t by agent j on the money market and on the credit market respectively, while $y_{t+1,j+1}^M$ and $y_{t+1,j+1}^C$ the quantities of good $j+1$ sold for money and for credit. Define the price on the money market for good j at time t as $p_{t,j}$ and the price on the credit market as $q_{t,j}$. The former is the amount of current period dollars needed to buy a unit of good j, the latter the amount of next period dollars needed to buy a unit of good j, since the good traded on the credit market is sold now and paid with cash next period. The amount of next period dollars an agent can obtain selling a unit of the good on the credit market divided by the amount of current period dollars he can obtain selling a unit of the good on the money market is the opportunity cost of accepting immediate cash and defines

[10] Quasi-linearity is crucial. The specific functional forms are not. For instance a strictly concave $u(x)$ and linear $c(y)$ would equally do.

the gross nominal interest rate at time t: $1 + i_t = \frac{q_t}{p_t}$.

2.1 Sequence of Events

I will first describe the sequence of events informally. For the sake of the exposition, suppose there are 8 islands. Figure 2 summarizes the sequence of events. In the morning, agent 1 is on island 1. He spends money to buy his consumption good from agent 8. He then increases his purchases issuing bilateral promises secured by collateral. At the end of the morning agent 1 and 8 travel together to island 2[11]. Meanwhile agent 2 is also coming to island 2. In the afternoon, agent 1 sells goods to agent 2 for money. With the money he received, he repays agent 8 and gets his collateral back. He then sells goods to agent 2 for promises, becoming a creditor. At the end of the afternoon he travels with agent 2 to island 3. Meanwhile agent 8 travels to his consumption island. In the evening, agent 1 will be resting and waiting for repayment. The next morning he will restart the cycle.

Formally the sequence of events is as follows. Agent j at time t uses part or possibly all of the money he accumulated in the past (m^j_{t-1}) to buy goods on the money market $p_{t,j}x^M_{t,j} \leq m^j_{t-1}$; he then proceeds to buy on the credit market borrowing an amount $d^j_t = q_{t,j}x^C_{t,j}$; at $t+1$ he produces for money in order to pay off his debt $p_{t+1,j+1}y^M_{t+1,j+1} = m^j_{t+1} \geq d^j_t$; he then lends an amount $b^j_{t+1} = q_{t+1,j+1}y^C_{t+1,j+1}$; at $t+2$, he follows his debtor on island $j+2$ and obtains, as a repayment, an amount of money $m^j_{t+2} = b^j_{t+1}$. At time $t+2$ he also receives a lump-sum money transfer $\tau_{t+2,j}$ from the government or pays a lump-sum tax[12]. The previous sequence of exchanges gives rise to the following three constraints: a cash-in-advance constraint

$$p_{t,j}x^M_{t,j} \leq m^j_{t-1},$$

[11] Once they arrive on an island, agents are matched according to a strongly anonymous matching function. A debtor and a creditor will not meet each other again in the future. To overcome this problem debtors and creditors travel together.

[12] I assume that agents receive transfers only when they are resting in order to rule out possible redistributive effects of monetary policy.

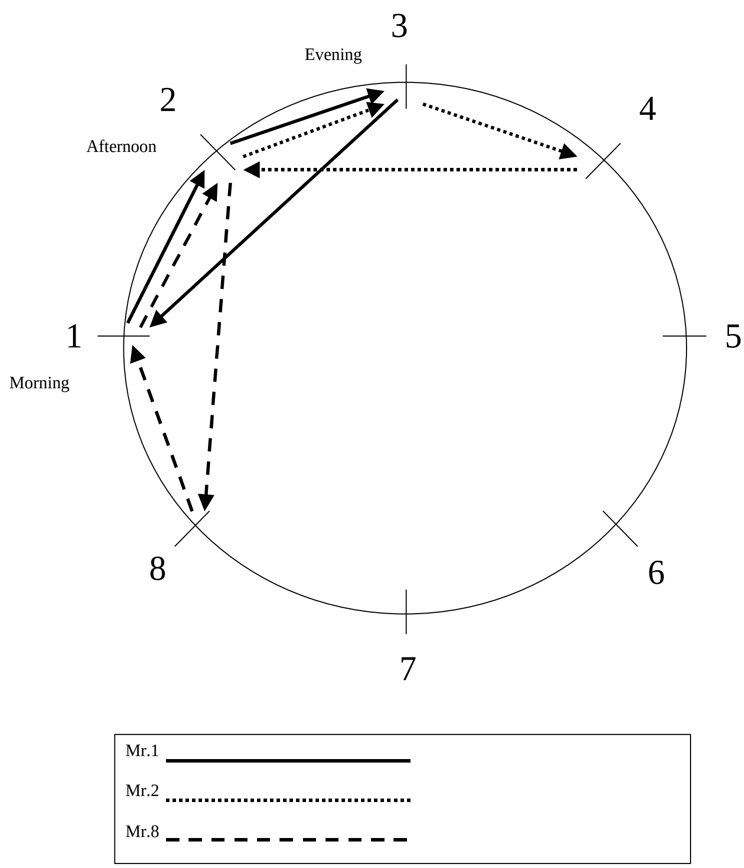

Figure 2: Sequence of Events

a repayment constraint

$$p_{t+1,j+1}y^M_{t+1,j+1} \geq q_{t,j}x^C_{t,j},$$

which states that the amount of money obtained producing has to be at least enough to repay the debt; the budget constraint

$$m^j_{t+2} = q_{t+1,j+1}y^C_{t+1,j+1} + p_{t+1,j+1}y^M_{t+1,j+1} - q_{t,j}x^C_{t,j} + m^j_{t-1} - p_{t,j}x^M_{t,j} + \tau_{t+2,j},$$

which says that the amount of money an agent carries into the following three period cycle is equal to the amount he obtains as a repayment of credit, the amount left after repaying and after purchasing with money and the lump-sum transfer from the government.

The economy features limited enforcement. Agents can always secure themselves at least zero, not participating in the exchange process. Agents should be willing to repay their debts. The relevant incentive constraint states that producing tomorrow for promises and consuming in three periods must give non-negative utility:

$$\beta^{t+3}\left(x^M_{t+3,j} + x^C_{t+3,j}\right) - \beta^{t+1}\frac{1}{2}\left(y^M_{t+1,j+1} + y^C_{t+1,j+1}\right)^2 \geq 0.$$

Observe that this constraint reflects the choice to repay debts, since the option of not repaying implies the impossibility of consuming in the future.

2.2 Discussion of the Main Assumptions

The model features four main assumptions:

1. complete absence of double coincidence of wants rules barter out;

2. absence of a record keeping technology and of societal enforcement rules multilateral credit contracts out;

3. collateral makes the exchange of promises possible, giving borrowers a limited form of bilateral commitment power; lockers allow lenders to commit not to hold up their borrower;

4. agents do not always participate in the exchange process: they have to rest after producing. Rest is crucial to generate an essential role for cash and promises of future cash.

The main differences with the most popular frameworks in the literature - random matching and spatial separation models- is that agents can choose their itineraries. In this sense the paper is closer to Banerjee and Maskin (1996).

Since this paper is not about barter, I rule it out assuming an extreme form of absence of double coincidence of wants. Milder forms would suffice. Absence of double coincidence of wants alone wouldn't be enough to induce a role for money as a medium of exchange if agents could meet in the same market place and strike multilateral deals. Spatial separation of markets is also not enough when agents can choose to travel to the same island and thus overcome the spatial separation. As I am going to argue below the resting period is the crucial assumption preventing multilateral deals. In particular the resting period prevents the possibility to organize a multilateral clearing system for promises which would make money redundant in this economy.

The paper focuses on the interaction of money and bilateral credit rather than multilateral credit. I therefore rule out all devices allowing agents to exchange multilateral credit contracts. In fact I assume that there isn't any technology allowing agents to keep track of each other trading histories. I also rule out societal enforcement technologies and any technology allowing agents to commit to future actions other than collateral. Kocherlakota (1998) shows that in a world with full enforcement and commitment money would not be needed as a medium of exchange. The present paper therefore abstracts from the type of risk-sharing arrangement proposed by Kocherlakota and Wallace (1998) and Wallace (2000) where a centralized monitoring technology allows to overcome the difficulties associated with random trading opportunities.

Collateral in the model is individual specific, has no public value and is necessary to consume. It is the perfect collateral: the owner desperately wants it back and the creditor doesn't care about it, since it has no value for anybody else except its owner- and thus it has no resale value-. In the present model the borrower leaves the collateral with the lender until the debt is repaid. Such a form of collateralized borrowing is known as a repurchase agreement (Repo). The fact that collateral is necessary to consume makes default an unattractive option -which gives zero for ever- reflected in the incentive constraint. Lockers are designed in order to make it impossible for the lender to extract more than what had been agreed at the time at which the transaction occurred. Notice that this technology is bilateral and weak enough so as not to conflict with the lack of societal enforcement assumed throughout.

Agents need to rest for one period after producing. To understand the role of this assumption in the present paper, consider for a moment the possibility that agents don't have to rest after producing. In such a case, agents would go trough a cycle of consumption and production. A back of the envelope calculation, considering just steady states and ignoring inflation, shows that trading with money, output would be $y = \beta$. The alternative possibility would be for agents to buy on promissory notes, sell for promissory notes, meeting all on the same island and swapping promissory notes at the end of the cycle (still a period two cycle of consumption and production). This alternative would entail $y = 1$. Without the resting period, a superior allocation would be obtained without money, which would then be not essential. Therefore, the resting period - preventing debtors to re-enter immediately after they sold goods- doesn't allow the clearing system to work effectively. Of course agents could try to implement the same clearing system even when they do have to rest, simply waiting for each other. That would require wasting even more time (at least two periods), inducing a waste of resources. Once a clearing system cannot function properly, agents will try to settle their debts in alternative ways. The rest of the paper shows that they will use money to settle debt.

2.3 Individuals

Since I will consider a symmetric equilibrium, I drop the index for agents and islands. Individuals choose $\left(x_t^M, x_t^C, y_{t+1}^M, y_{t+1}^C, m_{t+2}\right)$ to solve

$$\max \quad \sum_{t=0,3,\ldots} \beta^t \left[x_t^M + x_t^C - \beta \frac{1}{2} \left(y_{t+1}^M + y_{t+1}^C \right)^2 \right]$$

$$s.t. \quad p_t x_t^M \leq m_{t-1} \qquad [\lambda_t]$$

$$p_{t+1} y_{t+1}^M \geq q_t x_t^C \qquad [\mu_t]$$

$$m_{t+2} = p_{t+1} y_{t+1}^M + q_{t+1} y_{t+1}^C + m_{t-1} - p_t x_t^M - q_t x_t^C + \tau_{t+2} \qquad [\gamma_t]$$

$$\beta^{t+3}\left(x_{t+3}^M + x_{t+3}^C\right) \geq \beta^{t+1} \frac{1}{2}\left(y_{t+1}^M + y_{t+1}^C\right)^2 \qquad [\phi_t]$$

$$x_t^C \geq 0 \qquad [\psi_t]$$

$$y_{t+1}^C \geq 0 \qquad [\rho_t]$$

together with the transversality condition on money holdings. In brackets are the multipliers of the constraints. Whenever $\frac{p_{t+1}}{p_t} > \beta$, the multiplier $\lambda_t > 0$, for all t. Remember that the gross nominal interest rate is $(1+i_t) = \frac{q_t}{p_t}$ for all t.

The Euler equations are

$$1 + \psi_t = \frac{p_t}{p_{t+1}} \beta (1+i_t) \left(y_{t+1}^M + y_{t+1}^C\right)(1+\phi_t), \tag{1}$$

which equates the marginal benefit of consuming on credit to the marginal cost of producing tomorrow for money in order to repay and

$$\beta^3 \frac{p_{t+1}}{p_{t+3}} (1+i_{t+1}) = \beta \left(y_{t+1}^M + y_{t+1}^C\right) - \frac{\rho_t}{(1+\phi_t)}, \tag{2}$$

which equates the marginal benefit of consuming with money in three periods time to the marginal cost of producing for a promise tomorrow. Then, the complementary slackness condition for the repayment constraint is

$$\mu_t \left[y_{t+1}^M - (1+i_t) \frac{p_t}{p_{t+1}} x_t^C \right] = 0, \tag{3}$$

and for the incentive constraint is

$$\phi_t \left[\beta^3 \left(x_{t+3}^M + x_{t+3}^C\right) - \beta \frac{1}{2} \left(y_{t+1}^M + y_{t+1}^C\right)^2 \right] = 0, \tag{4}$$

and

$$\rho_t y_{t+1}^C = 0, \tag{5}$$

$$\psi_t x_t^C = 0, \tag{6}$$

with $\psi_t, \mu_t, \phi_t, \rho_t \geq 0$.

Below, I solve system (1)-(6) in a stationary equilibrium for the one period interest rate, the credit to money ratio, total output and the multipliers.

Finally as a benchmark consider an economy where credit cannot be used. In this case output is

$$\widetilde{y}_{t+1} = \beta^2 \frac{p_{t+1}}{p_{t+3}}.$$

2.4 The Government

On each island, the government issues money every period and gives lump-sum transfers to agents in their rest period. The government budget constraint equates the increase (decrease) in the money supply on each island to the total transfers (taxes) to agents:

$$M_t - M_{t-1} = T_t,$$

where I dropped the index for the islands since I solve for a symmetric equilibrium. The money supply grows at a rate z_t :

$$M_t = (1 + z_t) M_{t-1}.$$

3 Equilibrium

From the start I have normalized the price of money to one, in order to concentrate on monetary equilibria and avoid the issue -well known and extensively discussed in the literature- of equilibria where money is not valued.

Definition 1 *In a stationary symmetric competitive monetary equilibrium with credit, agents maximize utility subject to the budget constraint, the cash-in-advance constraint, the repayment constraint, the incentive constraint; all markets clear at all times: the goods-for-money market* $x_t^M = y_t^M$*, the goods-for-credit market* $x_t^C = y_t^C$*, the market for money* $m_t = M_t$*; finally, the Government sets a constant growth rate of money and fulfills its budget constraint.*

The following Proposition characterizes equilibrium. The proof is in the Appendix. Credit is constrained if the incentive constraint is binding and unconstrained otherwise.

Proposition 1 *For* $\beta \geq \left(\frac{1}{2}\right)^{\frac{1}{2}}$*, there exist a unique equilibrium with unconstrained credit. For* $\beta < \left(\frac{1}{2}\right)^{\frac{1}{2}}$*, there exist a unique equilibrium with unconstrained credit if* $1+\pi \geq \frac{1}{4\beta^3}$*. If* $1+\pi < \frac{1}{4\beta^3}$ *there exists a unique equilibrium with constrained credit. Money and credit are essential as media of exchange and money is the unique means of settlement.*

In equilibrium, each period one third of the population is consuming using both money and credit, one third is producing for money and credit and one third is resting while repayment takes place. One third of agents of, say, type j in the morning - period t- are on island j and trade money for goods. They also issue promises secured by the collateral to increase their consumption of good j. Then they travel to island $j+1$. In the afternoon - period $t+1$- they produce for money and for promises. They repay their promises in the afternoon, handing fiat money to their creditors (agents $j-1$) who have travelled with them and are waiting on island $j+1$. The collateral is returned and promises destroyed. In the evening they leave to island $j+2$ where they rest and wait for repayment to take place. Finally they travel to island j to restart the cycle.

If agents are sufficiently patient, the incentive constraint is slack: the long run benefit arising from being able to keep consuming outweighs the short run benefit from not producing, not repaying and loosing the vital collateral. Also agents are always willing to lend and borrow and thus the non-negativity constraints are not binding. When agents are sufficiently impatient, the incentive constraint is binding for low levels of the inflation rate and an equilibrium with constrained credit emerges, where agents are producing the exact amount that keeps them indifferent between repaying and not repaying. In this constrained credit equilibrium the amount produced is independent of inflation.

The two media of exchange coexist in a fundamental sense: each agent uses both at every stage to trade and cash is used also to settle debt. A comparison of output in two economies, with and without credit, reveals that the economy with credit performs better than the corresponding economy without credit, except when agents are very impatient and deflation is close to the Friedman Rule, in which case the two economies perform equally. Credit allows agents who cannot use liquidity presently to earn interest lending resources to agents who are keen on obtaining extra liquidity in order to increase consumption: credit allows to achieve a socially preferred outcome. The combination of the two means of exchange moves the economy closer to the first best frontier. Money helps credit to work providing the means of payment and the use of credit improves output and welfare[13]. This gives substance to the claim that the two instruments are complementary.

3.1 The Effect of Inflation on Credit

I will focus on the case where credit is unconstrained. The Euler equations are

$$1 = \beta \frac{1+i}{1+\pi} y \tag{7}$$

which describes the behavior of a debtor who increases his utility in the afternoon borrowing an extra unit (LHS) and will have to repay his debt with interest producing an extra unit in the evening (RHS); and

$$y = \beta^2 \frac{1+i}{(1+\pi)^2}, \tag{8}$$

which describes the behavior of a creditor who is lending an extra unit in the afternoon (LHS) and will get repayment with interest in the evening but will be able to spend it only the next morning since he has to rest during the night (RHS). There is a time wedge between the borrower -who cares about one period ahead- and the lender -who cares about two periods ahead, because of the resting period-. The creditor is holding money overnight, before being able to spend it. Figure 3 (where MB stands for marginal benefit, MC for marginal cost and $y = y^M + y^C$) depicts the behavior of a borrower and a lender.

The repayment constraint gives the ratio of the amount produced in the economy for credit and the amount produced for money -the credit to money ratio- as the inverse of the gross real interest rate

$$\frac{y^C}{y^M} = \frac{1+\pi}{1+i}, \tag{9}$$

[13] Using the sum of utilitites of the three types of agents in equilibrium as a measure of welfare, the same result would be obtained.

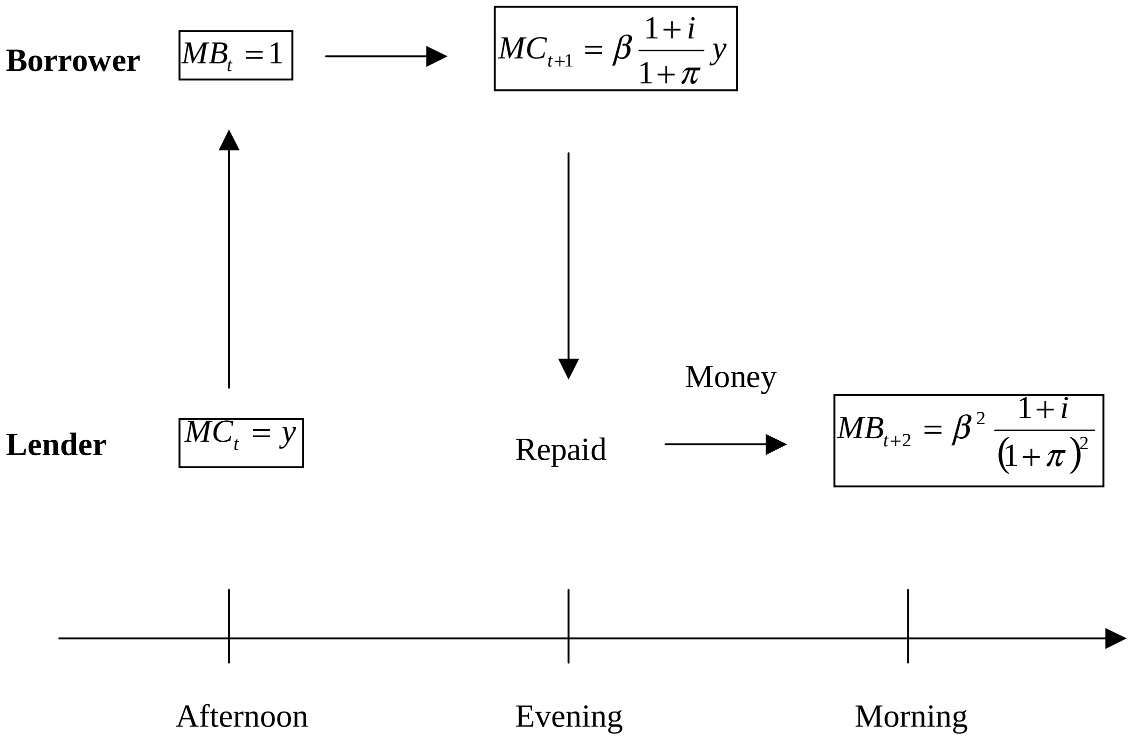

Figure 3: Euler Equations

Equations (7) and (8) together give the gross nominal interest rate and output. The gross nominal interest rate is

$$1 + i = \left(\frac{1 + \pi}{\beta} \right)^{\frac{3}{2}}.$$

In equilibrium there is a positive interest rate. This is necessary for a promise of money-later to be accepted instead of money-now.

Total output, which is given by

$$y = \left(\frac{\beta}{1 + \pi} \right)^{\frac{1}{2}},$$

is decreasing in inflation. When agents are impatient, if inflation is low, output is constant. Interestingly, Bullard and Keating (1995) find evidence that output is relatively stable for low inflation rates and decreases for higher inflation rates.

The ratio of credit to money is given by (9):

$$\frac{y^C}{y^M} = \frac{\beta^{\frac{3}{2}}}{(1 + \pi)^{\frac{1}{2}}},$$

which is decreasing in inflation. With inflation, the opportunity cost of holding a claim to future money is higher than the opportunity cost of holding money itself. In a model where credit is not cash-based the reverse would be true. The increase in the nominal interest rate would make more costly to hold money. Agents thus would switch to the alternative -inflation free- medium of exchange and the credit to money ratio would increase with inflation. In the present world where credit is cash-based, inflation has the opposite effect on credit.

4 Conclusion

I addressed the issue of the coexistence of money and credit, considering an environment where money and bilateral credit can both be used as media of exchange. I showed that it is socially beneficial to have both means of exchange, that money serves as a means of settlement and credit allows both agents who would like to increase their consumption to obtain a loan and agents who hold idle resources for some time to lend them out at an interest. Anticipated inflation affects macroeconomic variables, harming output and credit.

A complementarity proposition can be tested with microdata. Since money serves to settle, debtors will have to obtain cash in the future in order to repay their debts. Attanasio, Guiso and Jappelli (2002) estimate a money demand function based on the Baumol-Tobin model with time, interest on checking accounts and consumption as explanatory variables using microdata from the Bank of Italy. The model suggests that debt might also have a role in determining agents' demand for cash. One could use a similar specification with debt as an additional explanatory variable. The present paper suggests that the sign of the coefficient for debt should be positive.

5 Appendix

Proof of Proposition 1. First, I will take for granted that (1)-(6) describe individual behavior appropriately. Then after deriving the equilibrium I will give an argument for why this has to be so. Using equations (1)-(6) and the equilibrium conditions on the money and credit markets, the equilibrium system is

$$1 = \frac{\beta}{1+\pi}(1+i)\left(y^C + y^M\right)(1+\phi),$$

$$\frac{\beta^2}{(1+\pi)^2}(1+i)(1+\phi) + \frac{\rho}{\beta} = \left(y^C + y^M\right)(1+\phi),$$

$$\mu\left[y^M - \frac{(1+i)}{(1+\pi)}y^C\right] = 0,$$

$$\phi\left[\beta^2\left(y^C + y^M\right) - \frac{1}{2}\left(y^C + y^M\right)^2\right] = 0,$$

$$\rho y^C = 0,$$

with $\mu, \phi, \rho \geq 0$. Notice that $\psi = 0$ always. When $\beta \geq \left(\frac{1}{2}\right)^{\frac{1}{2}}$, $\phi = \rho = 0$ and $\mu > 0$ for all $1+\pi \geq \beta$. The nominal interest rate is

$$(1+i) = \left(\frac{1+\pi}{\beta}\right)^{\frac{3}{2}}.$$

The credit to money ratio is

$$\frac{y^C}{y^M} = \frac{\beta^{\frac{3}{2}}}{(1+\pi)^{\frac{1}{2}}}.$$

Finally total output is given by

$$y^C + y^M = \frac{\beta^{\frac{1}{2}}}{(1+\pi)^{\frac{1}{2}}}.$$

When $\beta < \left(\frac{1}{2}\right)^{\frac{1}{2}}$, for $1+\pi \geq \frac{1}{4\beta^3}$, $\mu > 0$, $\phi = \rho = 0$, output and the interest rate are the same as in the unconstrained money and credit equilibrium: $y = \left(\frac{\beta}{1+\pi}\right)^{\frac{1}{2}}$ and $(1+i) = \left(\frac{1+\pi}{\beta}\right)^{\frac{3}{2}}$. For $1+\pi < \frac{1}{4\beta^3}$, $\phi = 1 - 4\beta^3(1+\pi) > 0$. For $\left(\frac{1}{2}\right)^{\frac{1}{2}} \leq 1+\pi < \frac{1}{4\beta^3}$, $\rho = 0$ and $\mu > 0$ and the nominal interest rate is

$$(1+i) = 2(1+\pi)^2,$$

and the real interest rate

$$1 + r = 2(1+\pi),$$

the credit to money ratio is

$$\frac{y^C}{y^M} = \frac{1}{2(1+\pi)},$$

and output is constant

$$y^C + y^M = 2\beta^2.$$

For $\beta < 1+\pi < \left(\frac{1}{2}\right)^{\frac{1}{2}}$, lenders don't have a strict incentive to lend, output is $y = \frac{\beta^2}{(1+\pi)^2}$. Finally observe that output is

$$y = y^C + y^M = \frac{\beta^{\frac{1}{2}}}{(1+\pi)^{\frac{1}{2}}},$$

when $\beta \geq \left(\frac{1}{2}\right)^{\frac{1}{2}}$ or $\beta < \left(\frac{1}{2}\right)^{\frac{1}{2}}$and $1+\pi \geq \frac{1}{4\beta^3}$;

$$y = 2\beta^2,$$

when $\beta < \left(\frac{1}{2}\right)^{\frac{1}{2}}$and $\left(\frac{1}{2}\right)^{\frac{1}{2}} \leq 1+\pi < \frac{1}{4\beta^3}$;

$$y = \frac{\beta^2}{(1+\pi)^2},$$

when $\beta < \left(\frac{1}{2}\right)^{\frac{1}{2}}$and $\beta < 1+\pi < \left(\frac{1}{2}\right)^{\frac{1}{2}}$. Output in the economy with only money is

$$\widetilde{y} = \frac{\beta^2}{(1+\pi)^2}.$$

It is easy to check that output in the economy with credit is always higher except when there is no incentive to lend, in which case output is the same. Notice that First Best output is $y^{FB} = 1$. For this to be an equilibrium, agents shouldn't want to deviate. When an agent is consuming, it is fairly intuitive that he wants to use both money and credit since they allow him to transact and consume more. When an agent is producing, several possible deviations have to be considered. First, he may refuse to repay his debt. The short term gain is given by the decrease in the productive effort. In this case, though, he would loose his collateral, would be unable to consume in the future and would thus get zero for ever, which represents the long term loss. This possibility is captured by the incentive constraint: when agents are patient enough the option to avoid repayment is strictly dominated, while when they are impatient the option to default becomes more attractive and the constraint becomes binding. Second, he could repay his debt, but then produce for money instead of lending. This deviation would be particularly attractive if the agent could spend his cash immediately and consume. The resting period makes it attractive to lend in order to partly compensate for the time cost of holding balances for a period. The interest rate should be high enough so as to induce lenders to lend but not too high otherwise borrowers wouldn't want to borrow. In equilibrium the rate of interest exactly balances the incentives of the two groups. Consider then the possibility to repay debts with someone else's promise instead of money. In this case the original issuer of the promise will not meet the holder of his collateral again since agents, once they are on an island, are matched in a way that precludes meeting past trading partners and trading partners of past trading partners. Agents wouldn't issue promises in the first place and this is enough to rule out the deviation. Observe that even if an agent could somehow meet in the future a third party holding his collateral - for instance because the matching function is not strongly anonymous-, such a credit chain can only slow agents down. Since agents cannot consume without collateral, they would like to get it back as soon as possible. If a promise circulates in the economy the original issuer would have to starve until he manages to repay it. Agents could wait for each other and meet all on the same island, clear promissory notes simultaneously and restart their life. This would induce a waste of at least one extra period (since an agent's creditor and debtor rest in different periods) and is thus a dominated alternative. Notice that the complete lack of double coincidence of wants excludes repayment with commodities. Essentially it excludes barter and thus also inter-temporal barter. ■

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