Incentives and competition under moral hazard

A. Attar[1] and A. Chassagnon[3,4]

December 2006

Abstract

The work introduces a simple framework to study the relationship between competition and incentives under non-exclusivity. We characterize the equilibria of an insurance market where intermediaries compete over the contracts they offer to a single consumer in the presence of moral hazard. Non-exclusivity is responsible for under-insurance and positive profits in an otherwise competitive set-up. A relevant notion of optimality is also introduced and it is argued that market equilibria fail to be constrained-efficient.
Keywords: Non-exclusivity, Common Agency, Linear Prices, Insurance, Moral Hazard.
JEL Classification: D43, D82, G22.

.

1 Introduction

This paper contributes to analyze the relationship between incentives and competition, when asymmetric information is explicitly taken into account.

The analysis of the aggregate implications of asymmetric information in competitive markets has become an important element of contemporary economic theory. The need for a theoretical understanding of the relationship between markets and contracts is at the centre-stage of many recent developments in contract theory as well as in researches on General Equilibrium. Furthermore, it constitutes a crucial step to extend traditional macroeconomic schemes to markets where agents are heterogeneous with respect to their access to information.

We consider insurance and credit markets. We think of these markets as a setting where multiple intermediaries compete over the contract offers they are making to a representative agent. Our focus is on markets where contracts are non-exclusive: every single consumer can simultaneously accept the proposals of many financiers, giving rise to several contractual externalities among competitors.

[1]IDEI, Université de Toulouse I.
[3]University of Toulouse (GREMAQ)
[4]PSE (Ecole Normale Supérieure, Paris)

Several financial markets seem not to operate (even implicitly) under exclusive assumptions. A first example is given by the US credit card market: consumers typically hold several cards and are often given incentives to open new accounts.

One should also observe that courts can enforce exclusive relationships only if they have access to a form of centralized information over agents' characteristics. Such an institutional framework is lacking in several contexts. In particular, Jappelli and Pagano (2002) document that information sharing among lenders (in the form of credit bureaus or public credit registers) is not usually observed in most underdeveloped countries, especially for small loans. Sovereign debt relationships constitute another example of non-exclusive dealings (Bolton and Jeanne (2005)).

More generally, the relevance and the geographical extension of multi-banking relationship is now an established fact. For example, Ongena and Smith (2000b) emphasize that only less than 15 percent of European firms maintain a single relationship. Even though multi-banking is typically regarded as a relevant source of financing for big companies, several studies document its relevance for small firms for countries like the US, Italy and Portugal (see Detragiache, Garella, and Guiso (2000) and Petersen and Rajan (1994)). The rise of multiple lending phenomena is frequently associated to the high cost of implementing exclusive, single-lending relationships. The argument is supported by a number of empirical studies emphasizing that only in a few cases debt covenants explicitly include exclusivity clauses (Asquith and Wizman (1990) and Smith and Warner (1979)).

As for insurance markets both life insurance and annuity contracts are typically non-exclusive. In particular, the UK legislation explicitly requires every pension provider to allow the consumer to purchase annuities from other companies and this right is known as "Open Market Option" (OMO).

At the same time, several evidence suggests that competition in non-exclusive markets can hardly be reconciled with competitive outcomes. In an empirical survey on the US credit card market in the period $1981 - 1989$ Ausubel (1991) found clear evidence of extra-profit for intermediaries: the rate of return to credit card issuers is at least three times higher than the average of the banking sector. Looking at the insurance scenario, the 2006 British Survey of Annuity Pricing reports that there is strong evidence that both life insurance instruments and annuities are priced higher than what is suggested by actuarial considerations, despite the presence of a fairly high number of intermediaries. In addition, it is an established empirical fact that, in the US as well as in Europe, very few individuals voluntarily purchase life annuities, despite the fact that they look a valuable asset for risk-averse consumers in standard insurance set-ups.[1]

This paper provides a simple characterization of equilibria in non-exclusive markets where many intermediaries strategically compete in the presence of moral hazard on the agent's side. We emphasize how under-insurance and positive extra-profits are a typical implication of this sort of competition. In addition, we identify a rationale for regulatory interventions in a non-exclusive context. More precisely, we emphasize that market equilibria fail to be constrained efficient: a social planner who could not control agents' choices may indeed perform better than markets as long as he retains the power of controlling trades. In a next step we consider the situation where the planner cannot modify the structure of markets either. That is, he *cannot* restrict financial intermediaries from offering further trade opportunities (i.e. side contracts). This seems a natural scenario to investigate in all those situations where a social planner is in the impossibility of enforcing exclusivity clauses. Interestingly, we show that even in this constrained scenario there is room for regulatory issues: public insurance provision turns out to be always welfare improving.

Many important researches have tried to provide a theoretical assessment of these situations. In particular, the analysis of non-exclusive relationships under moral hazard has been initiated by the influential works of Pauly (1974), Arnott and Stiglitz (1991), Arnott and Stiglitz (1993) and Hellwig (1983) and reformulated by Bizer and DeMarzo (1992) and Kahn and Mookherjee (1998) and Bisin and Guaitoli (2004).

The works closest to ours are Parlour and Rajan (2001) and Bisin and Guaitoli (2004) who explicitly modeled the strategic interactions among intermediaries who simultaneously offer contracts to a single borrower-consumer. Both these researches stress that the crucial role of non-exclusivity is to support positive-profit equilibria and to induce a distorted distribution of surplus. This paper studies the same economy, where a binary effort choice is considered. We generalize their analysis so to make fully explicit the relationship between equilibrium allocations and equilibrium contracts, which are issued but not bought at equilibrium. The role of these latent offers is crucial to prevent standard Bertrand-like deviations which result in offering a higher quantity at a lower price. More precisely, the presence of such contracts allows the agent to act as a coordinating device. The threat of buying the deviating contract together with the latent ones and punish the deviator by shirking allows one to induce non-standard outcomes.

Interestingly, we are able to show that this sort of allocations can be as well supported in a simpler game where two competitors (principals) offer non-linear schedules to a single agent. The strategic role of the agent will then be to select her desired item in both menus; since a complete information scenario is considered, there will be offers that are not accepted by any type at equilibrium. The corresponding equilibria

[1] This is typically regarded as the Annuity Puzzle Brown (2001).

constitute an instance of the failure of the Revelation principle in games with multiple principals.[2] Our characterization results allow one to get simple intuitions about the welfare properties of the equilibria we exhibit. Differently from Parlour and Rajan (2001) , none of our equilibrium allocations will satisfy a constrained-efficiency property.[3] In addition, we provide a class of examples showing how pure equilibria may fail to be efficient even if the planner is subject to non-exclusive clauses, which was neglected in Bisin and Guaitoli (2004).

The paper is organized as follows. Section 2 presents the general two-state, two-effort insurance economy we study. Section 3 analyzes the properties of the exclusive game, where the consumer is allowed to accept only one contract at a time. Section 4 presents general results about equilibrium characterization. Section 5 and 6 examine the properties of high effort equilibria in the situation where consumers' preferences are homogeneous, which allows us to provide a stronger foundation for our strategic approach and to relate our results to the Common Agency literature. Finally, section 7 discusses the welfare implications of our analysis.

2 The economy

We consider an insurance economy lasting two periods. It is populated by a single representative consumer and by a countable infinite set of intermediaries. To simplify the analysis, only the two idiosyncratic states 1 and 2 are considered (we take 1 to be the "bad" state and 2 to be the "good" one). The probability distribution over the set $\{1, 2\}$ depends on an unobservable effort $e = \{a, b\}$, with $a > b$. We take $\pi_a(\pi_b)$ to be the probability of occurrence of state $2(1)$, and it is assumed that $\pi_a > \pi_b$.

The agent's utility from consuming c_s, with $s = \{1, 2\}$, is state-independent and it is represented by the function $u : \mathbb{R}_+ \to \mathbb{R}_+$ that is continuous, increasing and concave. In addition, we denote agents' endowment by $W = (w_1, w_2) \in \mathbb{R}^2_+$.

Trades are represented by insurance contracts offered by intermediaries to consumer.

We consider J intermediaries; the set of feasible offers for the $i-th$ intermediary is referred to as $\mathcal{D}^i$ and $\mathcal{D} = \times_{i \in N} \mathcal{D}^i$ is the collection of all the $\mathcal{D}^i$ sets. Every intermediary is allowed to offer a continuum of insurance contracts. Offers are restricted to be linear: insurance companies propose a benefit-premium ratio

[2] See Martimort and Stole (2002) and Peters (2001). In particular, the relationship between the failure of the Revelation principle and the existence of latent contract offers in games with complete information is stressed in Martimort and Stole (2003).

[3] Attar, Campioni, and Piaser (2006) show that all market equilibria studied by Parlour and Rajan (2001) indeed turn out to be constrained-efficient.

and fix a maximum quantity to be sold; the consumer is then free to choose her desired amount of insurance from every intermediary. To represent this sort of interaction, we assume that each intermediary i offers a contract $d^i = (d_1^i, d_2^i)$ consisting in a pair of state-contingent transfers. The agent could buy a fraction $\lambda_i \in [0,1]$ of this contract.

Insurance relationships are non-exclusive: every single consumer chooses a subset of intermediaries to interact with and her trade decisions cannot be contracted upon. A pure strategy for the consumer is hence described by the map

$$s_c : \mathcal{D} \ \rightarrow \{a, b\} \times [0,1]^J,$$

that associates to every array of offered contracts a level of effort selects the desired amount of insurance from the (linear) menu offered by every single principal. We also take S_c to be collection of all the s_c maps. The payoff to intermediary i is given by:

$$V^i = -((1-\pi_e)d_1^i + \pi_e d_2^i)\lambda_i,$$

when the effort e is chosen and the fraction λ_i of the offer d^i is bought. We denote $\vec{\pi}_e = ((1-\pi_e), \pi_e)$, and we then write $V^i = -\vec{\pi}_e \cdot d^i \lambda_i$.

The agent-consumer is risk-averse. Her preferences are represented by the VNM utility

$$\tilde{U}(c_1, c_2, e) = (1-\pi_e)u(c_1) + \pi_e u(c_2) - e,$$

where e denotes the cost of effort and $c_s = w_s + \sum_{i \in J} \lambda_i d^i$, for $s = \{1,2\}$ is the contingent consumption. In what follows, we will always refer to $U(C) = \tilde{U}(C, e(C))$, with $e(C) \in \arg\max_e \tilde{U}(C, e)$. That is, we represent preferences through to the consumer's indirect utility $U(C)$, as it is standard in moral hazard economies. In addition, we take $\mathcal{A} = \{C \in \mathbb{R}_+^2 \ / \ e(C) = a\}$ to be the set of ex-post consumption profiles inducing the choice $e = a$ and $\mathcal{B} = \{C \in \mathbb{R}_+^2 \ / \ e(C) = b\}$ to be that inducing $e = b$.

The frontier of the $\mathcal{A}$ set will be referred to as the Incentive frontier. We denote it by $\mathcal{G}$:

$$\mathcal{G} = \{(c_1, c_2) \in \mathbb{R}_+^2 : \tilde{U}(c_1, c_2, a) = \tilde{U}(c_1, c_2, b)\}$$

In the insurance market multiple intermediaries compete over the contractual offers they are making to a risk-averse consumer. The strategic relationships are hence represented by the complete information game:

$$\Gamma = \{\tilde{U}(.), \left(V^i(.)\right)_{(i\in J)}, \mathcal{D}, S_c\}.$$

The game Γ can hence be thought as a Bertrand-Edgeworth game, where a strategic (representative) consumer plays the role of the traditional demand curve. We will be focusing our attention on pure strategy Subgame Perfect Equilibria (SPE). At any SPE of the game Γ:

i) The agent optimally chooses her effort level together with the desired composition of her portfolio. That is, for every offered array $D = (d^1, d^2, \ldots, d^J)$:

$$((\lambda_i(D))_{i\in J}, e(D)) \in \arg\max_{e,\lambda} \, (1-\pi_e)u(c_1) + \pi_e u(c_2) - e \tag{1}$$

s.t.

$$c_s = w_s + \sum_{i\in J} \lambda_i d_s^i, \quad for \; s = \{1, 2\}$$

ii) For every array $D^{-i} = (d^1, \ldots, d^{i-1}, d^{i+1}, \ldots, d^J)$ of menu offers proposed by his competitors, the choice of d^i by intermediary i should maximize V^i subject to equation (1) that takes into account the agent's behavior and to the feasibility requirement $V^i \geq 0$.

3 Exclusivity

When exclusive dealings are considered, the agent can at most accept one contract at a time. We hence regard this as a benchmark case, since our concern is the analysis of situations where trade decisions cannot be restricted. For this reason, we devote the next paragraphs to investigate the properties of the insurance market under exclusivity.

The equilibrium outcome of the exclusive game can be represented as the solution of the program maximizing the utility of the agent under a non negative profit constraint (see Chassagnon and Chiappori (1997)). We will therefore consider two possible consumption profiles:

- The allocation $E = (h_1, h_2)$ such that:

$$E = \left((1-\pi_b)\, w_1 + \pi_b\, w_1, (1-\pi_b)\, w_2 + \pi_b w_1\right),$$

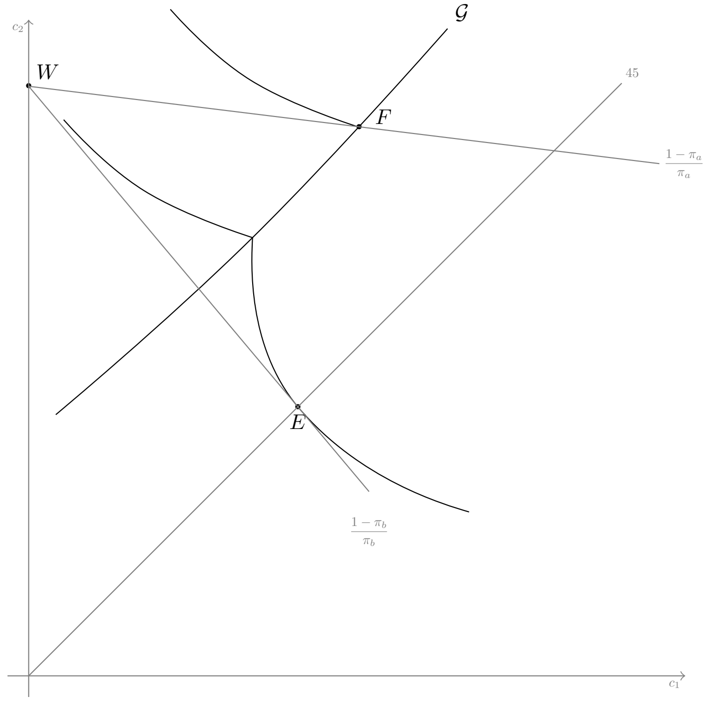


Figure 1: Equilibrium consumption under exclusivity

which lies at the intersection of the 45 degree line with the zero-profit line of slope $\dfrac{1-\pi_b}{\pi_b}$.

- The allocation $F = (f_1, f_2)$, which is defined by:

$$\begin{cases} u(f_2) - u(f_1) &= \dfrac{a-b}{\pi_a - \pi_b} \\ (1-\pi_a)\, f_1 + \pi_a\, f_2 &= (1-\pi_a) w_1 + \pi_a\, w_2 \end{cases}$$

That is, F is located at the intersection of the incentive frontier $\mathcal{G}$ with the zero-profit line of slope $\dfrac{1-\pi_a}{\pi_a}$.
One should notice that $E \in \mathcal{B}$ corresponds to the consumption level appropriated by the single agent when $e = b$ is chosen, whereas $F \in \mathcal{A}$ is that corresponding to $e = a$.
It is hence straightforward to provide a characterization of (pure strategy) equilibrium allocations under exclusivity in terms of the relative cost of high effort $\Delta = a - b$.

Lemma 1 *The nature of the exclusive equilibrium depends on the relative cost of implementing the high level of effort Δ. More precisely, there exists a cut-off level $\tilde{\Delta}$ such that whenever $\Delta > \tilde{\Delta}$ the effort $e = b$ will be always selected at equilibrium.*

Proof. Consider the contract $\tilde{d} = (\tilde{d}_1, \tilde{d}_2)$ defined by

$$\begin{cases} (1-\pi_b)\, u(\tilde{c}_1) + \pi_b\, u(\tilde{c}_2) & = \quad u((1-\pi_b)\, \tilde{c}_1 + \pi_b\, \tilde{c}_2) \\ \qquad\qquad -\vec{\pi}_a \cdot \tilde{d} & = 0, \end{cases}$$

where $\tilde{C} = W + \tilde{d}$ is the consumption allocation corresponding to the choice of the additional insurance $\tilde{d}$. Importantly, $\tilde{d}$ is defined independently of Δ.

For every $\tilde{d}$, we let $\tilde{\Delta}$ be the cost differential guaranteeing that the corresponding $\tilde{C}$ lies on the incentive frontier $\mathcal{G}$:

$$\tilde{\Delta} = (\pi_a - \pi_b)\,\big(u(\tilde{c}_2) - u(\tilde{c}_1)\big).$$

One should observe that if $\Delta = \tilde{\Delta}$, then $\tilde{C} = F$ and the single agent will be indifferent between the consumption allocations E and F.

In addition:

- For all $\Delta > \tilde{\Delta}$, the allocation $F(w)$ will lie between W and $\tilde{C}$. It follows that $U(\tilde{C}) = U\big(E\big) > U(F)$; that is, the effort e=b will be selected, and the equilibrium allocation will be E.

With a similar reasoning one can establish that the consumer will strictly prefer F to E for all $\Delta < \tilde{\Delta}$, and will therefore take the high level of effort. ■

We are now ready to discuss the properties of non-exclusive environments.

4 Non-exclusivity

This section investigates the features of insurance relationships where exclusivity clauses are not enforceable. If contracts are exclusive, then every single intermediary can offer allocations which are contingent on the consumer's trades with his competitors. In addition, a court can enforce exclusive agreements only if trades are observable and verifiable at no cost. Among other things, this requires a very particular institutional setting in the form of centralized information about agents' trades.

When contracts are non-exclusive, a form of contractual externality arises. The decisions of the $j-th$ intermediary affect intermediary's i payoff through its effects on the agents' behavior. In such a strategic context, the single agent is therefore acting as a coordinating device amongst competitors. In particular, her portfolio choices crucially determine the structure of markets. We hence start this section by remarking a basic feature of the agent's choices in our non-exclusive context.

For every array of offered contracts $d = (d^1,\, d^2,\, \ldots,\, d^J)$, the single agent is allowed to choose her preferred consumption level by buying a fraction λ^j of principal j's proposal. Importantly, the set of feasible allocations

$\mathcal{F} = \{(c_1, c_2) \in \mathbb{R}^2_+ : (c_1, c_2) = W + \sum_{i \in J} \lambda_i d^i\}$ is convex.

The agent's behavior crucially depends on the prices of all received offers. We denote p^i the price of the contract offered by the $i-th$ intermediary: $p^i = \left|\dfrac{d_2^i}{d_1^i}\right|$. Given the effort choice e, we also let $\tau^C = \dfrac{1-\pi_e}{\pi_e}\dfrac{u'(c_1)}{u'(c_2)}$ be the marginal rate of substitution evaluated at $C \in \mathcal{F}$.

For every offers' array, we will henceforth order the prices of contracts in accordance with the index of principals. That is, without loss of generality, we consider:

$$p^1 \leq p^2 \leq \cdots \leq p^J.$$

In particular, one can show the following:

Lemma 2 *Let $d = (d^1, d^2, \ldots, d^J)$ be an array of insurance contract offers such that the corresponding optimal consumption choice of the agent C does not lie on the Incentive Compatibility frontier $\mathcal{G}$. Then:*

i) If intermediary i proposes a positive insurance contract, then his offer will be entirely *bought (i.e. $\lambda^i = 1$) if $p^i < \tau^C$. Furthermore, whenever its offer is bought ($\lambda_i \in (0, 1]$), one gets $p^i \leq \tau^C$.*

ii) If intermediary i proposes a negative insurance contract, then his offer will be entirely bought if $p^i > \tau^C$. Furthermore, whenever its offer is bought ($\lambda_i \in (0, 1]$), one gets $p^i \geq \tau^C$.

iii) If $p^i = \tau^C$, then any insurance contract d^i proposed by any intermediary i will be partially bought (i.e. $0 < \lambda^i < 1$).

Proof.

We start by establishing the following statement. (A): consider the consumption allocation C; if d^i is a positive (negative) insurance contract and the consumer accepts it (i.e. $\lambda_i > 0$), then $\tau^C \geq p^i$ ($\tau^C \leq p^i$). To prove it, one has to observe that if d^i is a positive insurance contract, then, since $\lambda_i > 0$, the allocation $C - \lambda\, d_i$ is achievable by the consumer for every $\lambda < \lambda_i$. It follows that $U(C - \lambda\, d_i) \leq U(C)$, which indeed implies $\tau^C \geq p^i$. Following the same reasoning one can prove (A) for the case of d^i being a negative insurance contract.

In a next step, we prove the statement (B): if d^i is a positive (negative) insurance contract which is not entirely accepted (i.e. $\lambda_i < 1$), then then $\tau^C \leq p^i$ ($\tau^C \geq p^i$). We only refer, for simplicity reasons, to the positive insurance case. Hence, if d^i is not fully accepted, then $C + \lambda\, d_i$ belongs to $\mathcal{F}$, for every $\lambda < 1 - \lambda_i$. Since d^i a positive insurance contract, one gets that $U(C + \lambda\, d_i) \leq U(C)$, which indeed implies $\tau^C \leq p^i$. Once again, one can use a similar argument to establish (B) for the case of d^i being a negative insurance contract.

One should notice that if the d^i is a positive insurance contract, then (A) implies the second part of equivalence (i). To contradict the first part of (i), it should be $p^i < \tau^C$ and $\lambda_i = 0$. However, such a situation is impossible because it would directly violate (B).

Point (ii) can be established along the same lines.

Finally, (iii) is an immediate implication of point (A) and point (B). ■

Notice that this statement by nature applies to any local optimal allocation of $\mathcal{F}$.

The consumer will hence have a clear incentive to first buy those contracts which are offered at a lower price.

Importantly, if we take C to be the equilibrium consumption allocation, then any contract with a price (strictly) higher than τ^C will never be bought, while contracts offered at a price strictly lower than τ^C will be entirely accepted.

The remaining of this section is devoted to analyze the general properties of market equilibria under non-exclusivity. We re-establish, in a general context, several results on moral hazard economies with non-exclusive contracts. As long as the incentive problem is binding, competition among an arbitrarily large number of intermediaries typically delivers non-competitive outcomes in the form of under-insurance and positive extra-profits. As a starting point, it is useful to remark that whenever the incentive problem is very mild, then the presence of non-exclusive contracts will be of no aggregate impact. In other words, we can prove in our context the following standard result:

Lemma 3 *If $U(C) < U\big(\pi_b c_2 + (1 - \pi_b) c_1\big)$ for all $C \in \mathcal{F}$, then there exists a unique (pure strategy) equilibrium allocation characterized by: $c_2 = c_1 = \pi_b w_2 + (1 - \pi_b) w_1$.*

Proof. See Arnott and Stiglitz (1993) and Bisin and Guaitoli (2004). ■

If the cost of high effort is sufficiently high, then this choice does not represent a credible threat and a standard Bertrand argument will apply.[4]

[4] A similar argument is suggested in Proposition 1 and 2 in Parlour and Rajan (2001).

Our analysis will therefore be devoted to evaluate and discuss the structure of high effort equilibria. Even though our reference framework is quite abstract, the results presented in this section identify several properties that should be satisfied by any pure strategy equilibrium of the game Γ when the high effort is selected. Throughout this section, we let $C \in \mathcal{A}$ denote a pure strategy equilibrium outcome.

In a first step, we show that negative insurance will never be issued at equilibrium. The role of this sort of contracts will therefore only be relevant to analyze the deviation stage.

Lemma 4 *Let $C \in \mathcal{A}$ be a pure strategy equilibrium outcome of the game Γ. Then, every accepted contract involves positive insurance.*

Proof. Suppose not. Then, we let j be the intermediary offering the negative insurance contract d_j, with the agent buying a fraction λ_j of it. Let p^j be the price of this contract; it follows from Lemma 2 that:

$$p^j \geq \tau^j > \frac{1-\pi_a}{\pi_a}.$$

If the high effort is selected, the $j-th$ intermediary is hence earning a negative profit which implies that d^j cannot be part of an equilibrium. ■

The main results of this section are presented in the next Proposition. The properties of high effort equilibria are identified depending on whether the corresponding allocations belong or not to the incentive frontier $\mathcal{G}$. In the former case, equilibria will be supported by having all intermediaries fully active, while a typical feature of equilibrium allocations outside $\mathcal{G}$ is that at least one intermediary will be rationed. There will be, in other words, some available insurance which the consumer will not find convenient to buy.

Proposition 1 *The following statements are equivalent:*

1. *The equilibrium outcome $C \in \mathcal{A}$ belongs to the Incentive Compatibility frontier $\mathcal{G}$.*
2. *C is supported by all intermediaries being active. In particular, the offer of each intermediary is fully accepted, i.e. $\lambda_i = 1\ \forall i \in J$.*
3. *The consumer strictly prefers C to any other feasible outcome: $U(C) > U(C)\ \forall C \in \mathcal{F} \cup \mathcal{B}$.*

Proof. The proof is developed in three steps.

The first step consists in showing that if the outcome C belongs to the frontier $\mathcal{G}$, then it must be that all contracts are accepted, i.e. $\lambda_i > 0\ \forall i$. By contradiction, let us suppose that at equilibrium there is one inactive intermediary, say the $i-th$ one. Then, he could profitably deviate by offering a positive insurance contract $d^{i\prime}$ with a price arbitrarily close to $\dfrac{1-\pi_b}{\pi_b}$ and strictly higher than it, so that $-\vec{\pi}_b \cdot d^{i\prime} > 0$. It is immediate to verify that the agent will always have the incentive to accept it and to select $e = b$.

In addition, one can also show that the consumer will entirely buy all offered contracts:

$$C = W + \sum_{j \in J} d^j.$$

If, indeed, there was (at least) one positive λ_i strictly lower than 1, then, for every $\lambda < \min(\lambda_i, 1-\lambda_i)$, both $C - \lambda\, d_i$ and $C + \lambda\, d_i$ should be feasible consumption choices. Thus, $U(C - \lambda\, d_i) \leq U(C)$ and $U(C + \lambda\, d_i) \leq U(C)$. This contradicts the fact that the marginal rate of substitution under high effort evaluated at C is different from that calculated under low effort.[5]

The second step is to show that at any (pure strategy) equilibrium where all intermediaries are active, C will be strictly preferred to any other feasible outcome. To prove that, it is enough to remark that if at equilibrium $\lambda_i = i \;\; \forall i \in J$, then the set of feasible consumptions for the agent will entirely be contained in $\mathcal{A}$. This indeed follows from the fact that all contracts offered at equilibrium are positive insurance ones. Using a standard convexity argument one hence gets $U(C) > U(C) \; \forall C \in \mathcal{F}$.

Finally, we have to show that if there is no consumption allocation that guarantees the consumer the same utility as C, then it must be that C belongs to the frontier $\mathcal{G}$. By contradiction, we suppose that $C \notin \mathcal{G}$. Two cases should be considered depending on whether the active intermediaries charge the same price or not. In the latter case, it must be $p^1 < \tau^C$, by Lemma 2. Then, let us take a $p \in (p^1, \tau^C)$ and denote $\mathcal{Z} \subset \mathcal{A}$ a neighborhood of C such that $\tau^C > p$ for all $C \in Z$. Take now any contract d which price is smaller than τ^C, and such that $-\vec{\pi}_a \cdot d > 0$; then, consider the deviation $d^{1\prime} = d^1 + \epsilon d$ for intermediary 1, and let $C(\epsilon)$ be the optimal consumption choice of the agent, given the new array of offers. If ϵ is small enough, then the price of $d^{1\prime}$ will be strictly below p and we will also have $C(\epsilon) \in Z$. The contract $d^{1\prime}$ will hence be entirely bought (i.e. $\lambda_1 = 1$), which guarantees, by Lemma 2, that the deviation is profitable.

We now have to consider the case where at equilibrium all contracts are offered at the same price, i.e. $p^i = p \;\forall i$. Once again, we develop an argument by contradiction. If $C \notin \mathcal{G}$, then any intermediary

[5] Indeed, if $\tau = \left| \dfrac{d_2^i}{d_1^i} \right|$ is the price of d_i, then, inequality $U(C - \lambda\, d_i) \leq U(C)$ would imply that $\dfrac{1-\pi_a}{\pi_a}\dfrac{u'(c_1)}{u'(c_2)} \geq \tau$ and inequality $U(C + \lambda\, d_i) \leq U(C)$ would imply that $\dfrac{1-\pi_b}{\pi_b}\dfrac{u'(c_1)}{u'(c_2)} \leq \tau$, with the consequence that $\dfrac{1-\pi_a}{\pi_a} \geq \dfrac{1-\pi_b}{\pi_b}$, a contradiction.

necessarily earns positive profits, i.e. $\tau > \dfrac{1-\pi_a}{\pi_a}$. If this was not the case, then a single intermediary could always gain by offering a negative insurance contract at a price strictly smaller (but arbitrarily close to) $\dfrac{1-\pi_b}{\pi_b}$, which the consumer will have an incentive to accept selecting the effort $e = b$.

We now take intermediary 1 to be the one earning the smallest (positive) profit at equilibrium and consider a small positive insurance contract d such that $\vec{\pi}_a \cdot d > 0$ and $U(C + d) > U(C)$. If intermediary 1 lowers his price deviating to $d^{1\prime} = d^1 + d$, the agent will accept it. If she accepts $d^{1\prime}$ together with other contracts, then the deviating contract will be entirely bought (see Lemma 2), and the deviation will be profitable as long as $e = a$ is selected. If the agent chooses the high level of effort, then the deviation will be profitable even in the extreme case when $d^{1\prime}$ is the only accepted contract at the deviation stage. To stress this point, it is enough to remark that $\vec{\pi}_a \cdot d^1 < \vec{\pi}_a \cdot \big(C - W\big)$, since at C all intermediaries earn a positive profit. Then, take any $\epsilon < \vec{\pi}_a \cdot \big(C - W\big) - \vec{\pi}_a \cdot d^1$. If ϵ is small enough to get $\pi_a \cdot \big(C'(\epsilon) - W\big) > \pi_a \cdot \big(C - W\big) - \epsilon$, where $C'(\epsilon)$ is the optimal consumption choice at the deviation stage, then one gets $\pi_a \cdot \big(C'(\epsilon) - W\big) > \pi_a \cdot d^1$, which makes the deviation profitable.

Since the consumer could gain by accepting $d^{1\prime}$ and choosing $e = a$, for $C \in \mathcal{A}$ to be supported at equilibrium, it must be that the effort $e = b$ is chosen at the deviation stage, so to make the deviation not profitable. This remains true if d is arbitrarily close to $(0, 0)$. For $C \notin \mathcal{G}$ to be an equilibrium outcome, there must hence exist an outcome $L \in \mathcal{B}$ such that $U(C) = U(L)$, which generates a contradiction. ■

Equilibrium allocations outside $\mathcal{G}$ therefore involve the presence of latent outcomes. That is, there are offers not accepted at equilibrium which role is crucial to block deviations from incumbent as well as entrant intermediaries. The structure of "latent contracts equilibria" turns out to be significantly different from that of the equilibria on $\mathcal{G}$. The following pages explore this issue in greater detail.

4.1 High effort equilibria on $\mathcal{G}$: general properties

Every equilibrium outcome on $\mathcal{G}$ is supported by all contracts being fully accepted. That is, the single agent finds optimal not to ration any of the intermediaries. In addition, one can show that every competitor must be charging the same price, so that the equilibrium will exhibit a fairly symmetric structure.

Proposition 2 *If the equilibrium outcome C belongs to the frontier G, then all intermediaries must set the same price: $p^1 = p^2 = ... = p^J = \tau^C$*

Proof. The proof is developed by contradiction. Let us consider the situation $p^1 < p^J$. Since all contracts have been (entirely) accepted, Lemma 2 guarantees that $p^J \leq \tau^C$. In addition every contract will involve positive insurance (see Lemma 4). Let us denote $\overline{C}$ and $\underline{C}$ as the optimal consumption choices of the agent when contracts with price p^J and p^1 have been respectively removed. At equilibrium, it should be $U(C) > U(\underline{C})$, and $U(C) > U(\overline{C})$. We then consider a neighborhood of C such that the the corresponding agent's utility is strictly grater than both $U(\underline{C})$ and $U(\overline{C})$ in any element of the neighborhood.

Let us assume that intermediary 1 is deviating to $d^{1\prime} = d^1 + \epsilon d$, where d is a (small) negative insurance contract which price is lower than $\dfrac{1-\pi_a}{\pi_a}$. If ϵ is taken to be small enough, a continuity argument guarantees that the offer $d^{1\prime}$ will always be at least partially bought. Importantly, if $d^{1\prime}$ is entirely bought $(i.e.\ \lambda_i = 1)$, then the deviation is unambiguously profitable.

Given the offers' array $\{d^{1\prime}, d^2, ..., d^J\}$, we let $C(\epsilon)$ be the corresponding optimal consumption choice. In addition, we denote $\overline{C}(\epsilon)$ the outcome that would be selected at the deviation stage if all contracts of price p^J were removed. One should notice that $U\big(\overline{C}(0)\big) = U(\overline{C}) < U\big(C(0)\big) = U(C)$. A standard continuity argument hence guarantees that $U\big(\overline{C}(\epsilon) < U\big(C(\epsilon)\big)$. This implies that, at the deviation stage, the agent will have an incentive to buy some of the contracts offered at a price p^J. Then, from Lemma 2 it follows that $\tau^{C(\epsilon)} \geq p^J$; if ϵ is small enough, the price of $d^{1\prime}$ will be strictly smaller than p^J. Summarizing, we have that $\tau^{C(\epsilon)} > |\dfrac{d_2^{1\prime}}{d_1^{1\prime}}|$, so that $d^{1\prime}$ will be fully accepted. This guarantees that the deviation is profitable.
■

If an equilibrium on $\mathcal{G}$ exists, then it cannot be a zero-profit one. That is, the exclusive allocation F of Figure 1 cannot be supported at equilibrium under non-exclusivity. If this was the case, indeed, a single intermediary would always have an incentive to offer a small additional amount of insurance at a price $p \in \big(\dfrac{1-\pi_b}{\pi_b}, \tau^C\big)$. This would increase his profit and induce the consumer to buy this additional amount of insurance and to select the low effort.

Equilibria on $\mathcal{G}$ are hence identified by Cournot-like outcomes, with all intermediaries earning a strictly positive profit.

Several recent researches emphasized the relevance of these situations. With reference to a moral hazard scenario, the works of Parlour and Rajan (2001), Attar, Campioni, and Piaser (2006), Martimort (2004) and others characterize pure strategy equilibria where all intermediaries are treated symmetrically. That is, they are equally sharing the surplus, leaving zero extra rent to the single agent.[6] These authors argue that

[6]The related work of Bizer and DeMarzo (1992) also characterizes a situation where all contracts are take up at equilibrium. Given the different distribution of bargaining powers, though, it happens that any single intermediary is earning zero-profit at equilibrium.

a crucial condition to support this sort of equilibrium outcomes is to make the single consumer indifferent between two alternative consumption profiles in the high effort subspace. More precisely, it is assumed that the consumer can get the same utility by buying all contracts and by buying only $J-1$ of them. This guarantees that deviations trying to reduce the agent's payoff will always be blocked. In other words, the equilibrium payoff will be available at the deviation stage.

The main difference between our set-up and the above mentioned ones consists in the fact that, for every given offers' array, the set of feasible consumption choices is always convex. This enables us to greatly simplify equilibrium analysis. In particular, one can prove that there will not exist any pure strategy equilibrium where an allocation $C \in \mathcal{G}$ is implemented. Given the convexity of the set $\mathcal{F}$, it turns out that there will always be unilateral profitable deviations implying a raising price. To develop this argument, we introduce some additional notation. For $\alpha \in [0,1]$, let $C_\alpha = \big(c_1 + \pi_a\, \alpha, c_2 - (1-\pi_a)\alpha\big)$ be any allocation lying on the iso-profit line of slope $\dfrac{1-\pi_a}{\pi_a}$ passing through C; importantly, if $\alpha = 0$, then $C_\alpha = C$. We focus here on the case $\alpha < 0$, which implies that the corresponding C_α is at the north-west of C. Then, for every given α, we take $W_\alpha = \left(c_1 + \alpha\pi_a \dfrac{\tau^{C_\alpha} - \tau^a}{\tau^{C_\alpha} - \tau^C}, c_2 - \tau^C \alpha\pi_a \dfrac{\tau^{C_\alpha} - \tau^a}{\tau^{C_\alpha} - \tau^C}\right)$ to be the point on the equilibrium price line which is connected to C_α by a line of slope τ^{C_α}. That is, if the single consumer was offered the insurance contract $C_\alpha - W_\alpha$, then she would always have an incentive to entirely accept it (see Figure 2).[7]

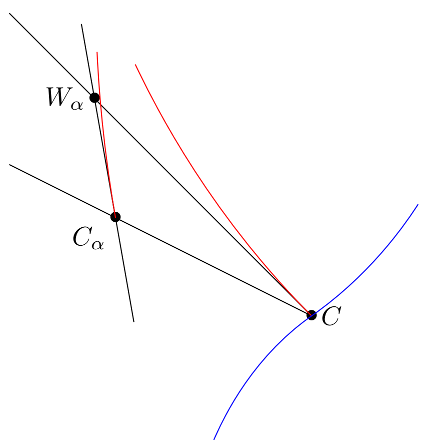


Figure 2: Relationship between W_α and C_α

The following proposition can now be established.

[7]One has to observe that the sequence of points W_α converges to C when $\alpha \to 0$. This is indeed implied by the two inequalities $0 \le W_{2\alpha} - C_2 \le -\alpha\pi_a\tau^C$ and $0 \le C_1 - W_{1\alpha} \le -\alpha\pi_a$.

Proposition 3 *No allocation $C \in \mathcal{G}$ can be supported at a pure strategy equilibrium in the game Γ.*

Proof. We show here that there always exists a profitable deviation for a single intermediary, say the $J-th$ one involving a price higher than the equilibrium price $p = \tau^C$. We now consider any α small enough to guarantee that the corresponding W_α lies between $W^{-J} = W + \sum_{i=1}^{i=J-1} d^i$ (see Figure 3).

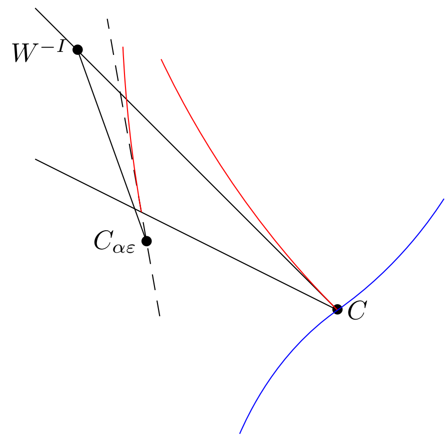


Figure 3: Existence of a profitable deviation for any equilibrium on $\mathcal{G}$

By construction, we have that:

$$\tau^{C_\alpha} > \frac{W_2^{-J} - C_{2\alpha}}{C_{1\alpha} - W_1^{-J}}.$$

Consider then the following deviation for intermediary J: instead of offering d^J, he proposes $d_\varepsilon^{J\prime} = (1 + \varepsilon)(C_\alpha - W^{-J})$.

Observe that if the agent decided to entirely accept all contracts at the deviation stage, then she would achieve the allocation:

$$C_{\alpha\varepsilon} = C_\alpha + \varepsilon\big(C_\alpha - W^{-J}\big),$$

which is depicted in Figure 3. In particular, it is always possible to select an ε small enough to guarantee that the corresponding $C_{\alpha\varepsilon}$ belongs to $\mathcal{A}$ and to satisfy the following inequality:

$$\tau^{C_{\alpha\varepsilon}} > \frac{W_2^{-J} - C_{2\alpha\varepsilon}}{C_{1\alpha\varepsilon} - W_1^{-J}}, \tag{2}$$

which exactly guarantees that the agent will have an incentive to accept all contracts.[8]. In addition, the deviating contract guarantees a profit higher than the original one:

$$-\vec{\pi}_a \cdot (C - W^{-J}) = -\vec{\pi}_a \cdot (C_\alpha - W^{-J}),$$

In other words, the original (positive) profit has been multiplied by $(1+\varepsilon)$. ■

4.2 High effort equilibria outside $\mathcal{G}$: general properties

The previous discussion stressed that allocations outside $\mathcal{G}$ may either be supported as linear price equilibria, i.e. all intermediaries are offering the same price, or through non-linear prices. In both cases some latent contracts will be issued at equilibrium. Given the array of contracts offered at equilibrium $d = (d^1, d^2, \ldots, d^J)$, we take $I = \{i \in J : \lambda_i(d) > 0\}$ to be the set of active principals and its complement I^c to be the set of inactive ones. As a consequence, we can introduce the following:

Definition 1 *For every equilibrium outcome $C \in \mathcal{A}$, a latent outcome is a feasible point $L \in \mathcal{B}$ lying on the equilibrium indifference curve. The outcome L can be in principle supported by many alternative profiles of offered contracts. We say that an inactive principal $k \in I^c$ is "latent" if his equilibrium offer d^k could support a latent outcome L.*

We identify in what follows a set of necessary conditions guaranteeing that the equilibrium is robust to deviations of active and latent intermediaries.

One has first to observe that from Proposition 1 (see point (iii))it follows that if $C \notin \mathcal{G}$ is a (pure strategy) equilibrium outcome, then there must exist an allocation L such that $U(C) = U(L)$.

The next paragraphs provide a further characterization of the structure of the latent consumption L.

A first lemma shows that even if all latent contracts taken together could support L, then the same latent outcome would remain available even if any of the latent contracts is removed.

Lemma 5 *Let $C \in \mathcal{A}$ be a pure strategy equilibrium outcome of the game Γ. Then, the latent consumption L remains available to the consumer if any of the latent contracts d^k is removed.*

Proof. Suppose not. In this case, whenever (the latent) principal k removes his offer, the highest utility that the agent could earn by choosing $e = b$ will be strictly lower than $U(C)$. Then, as in point (iii) of

[8]This is an immediate implication of our Lemma 2

Proposition 1, intermediary k could propose (instead of d^k) a small positive insurance contract that the agent will accept, together with the effort choice $e = a$. This deviation guarantees a positive profit to the latent intermediary k, a contradiction. ■

This allows us to prove our main Proposition:

Proposition 4 *If the equilibrium outcome C does not belong to the frontier G, then:*

i) All latent contracts will be offered at the same price $p^k = p^L$ and it will always be $\tau^L = p^L$.

ii) The two points $C \in \mathcal{A}$ and $L \in \mathcal{B}$ are connected by a line of slope p^L.

iii) The two vectors $\big(\nabla U(C) - \nabla U(L)\big)$ and $\vec{\pi}_a = \big(\pi_a, (1-\pi_a)\big)$ are collinear.

Proof. To prove (i), we consider the latent offer d^k and we let $\lambda_k \in (0, 1]$ be the fraction of that offer which the consumer is willing to buy when the low effort is chosen. That is: $L = W + \sum_{i \neq k} \big(\lambda_i d^i\big) + \lambda_k d^k$. For every $\epsilon \in (0, \lambda_k)$ the outcome $L - \epsilon d^k$ is hence a feasible choice for the consumer. It follows that $U(L) \geq U(L - \epsilon d^k)$. One should also observe that, by Lemma 5, the outcome $L + \epsilon d^k$ is feasible for any $\epsilon \in [0, 1]$. As a consequence, one gets:

$$U(L) \geq max\{U(L - \epsilon d^k), U(L + \epsilon d^k)\}$$

for every k. This directly implies $p^k = p^L = \tau^L \; \forall k$.

For convenience, we choose arbitrarily some positive contract d^L of price p^L.

In a next step we establish (ii). To this extent, we show that all active contracts which price is lower than p^L will be fully bought at equilibrium. If there are such contracts, we denote by I the last active intermediary I whose offered price is lower than p^L. Then, for $k > I$, we let α^k be the (unique) real number (positive or negative) such that $d^k = \alpha^k \, d^L$. Since L is the consumer's optimal choice inside $\mathcal{B}$, we know from Lemma 2 that all contracts of price lower than τ^L should be bought in order to reach L. That is:

$$L = W + \sum_{i=1,\ldots,I} d^i + \big(\sum_{k>I} \lambda_k \alpha^k\big) d^L, \qquad (3)$$

Let suppose by contradiction that for some $i^0 \leq I$, $\lambda_{i^0} < 1$. Then, the consumer can still achieve the outcome C with the new offers' array $\{d^1, \ldots, \lambda_{i^0} d^{i^0}, \ldots, d^I, d^{I+1}, ..., d^J\}$. One should observe that given these offers, the allocation L will still be available, otherwise the i^0– th intermediary could profitably decrease his price.

This could be written:

$$L = W + \Big(\sum_{i=1,\dots,I} d^i\Big) - (1-\lambda_{i^0})d^{i^0} + \Big(\sum_{k>I} \lambda'_k \alpha^k\Big) d^L, \tag{4}$$

which contradict the preceding remark unless $\lambda_{i^0} = 1$.

All active contract being entirely bought, it follows that $C = W + \big(\sum_{i=1,\dots,I} d^i\big)$, and that equation (3) can be rewritten as:

$$L - C = \Big(\sum_{k>I} \lambda_k \alpha^k\Big) d^L, \tag{5}$$

which guarantees that the line connecting L and C has slope τ^L.

Finally, we remark that from (i) and (ii) it follows that $\dfrac{l_2 - c_2}{c_1 - l_1} = \dfrac{1-\pi_b}{\pi_b}\dfrac{u'(l_1)}{u'(l_2)}$, which corresponds to:

$$\tau^L l_1 + l_2 = \tau^L c_1 + c_2. \tag{6}$$

To prove(iii) one has to show that there exists a $\theta \in \mathbb{R}$ such that:

$$\nabla U_L - \nabla U_C = \theta \vec{\pi}_a, \tag{7}$$

with $\nabla U_C = \big((1-\pi_e)u'(x_1), \pi_e u'(x_2)\big)$ for $C = \{C, L\}$.

Let $D = (d^1, d^2, \dots, d^J)$ be the array of contracts offered at equilibrium and consider a unilateral deviation of one of the active intermediaries, say the $i-th$ one, offering $d^i + d$ instead of d^i, where d is taken to be small enough. In particular, $d^i + d$ is taken to be a positive insurance contract which price is lower than $p^L = \tau^k$.

Consider first the agent's behavior. At the deviation stage, she can achieve the consumption outcome $C + d \in \mathcal{A}$, as well as all the allocations on a line of slope τ^L passing through $C + d$. In particular, we let $L'(d) = L + \delta$ be the corresponding optimal consumption outcome of the agent in the set $\mathcal{B}$. We hence have:

$$\tau^L(l_1 + \delta_1) + (l_2 + \delta_2) = \tau^L(c_1 + d_1) + (c_2 + d_2) \tag{8}$$

Using (6), equation (8) can be rewritten as:

$$\tau^L \delta_1 + \delta_2 = \tau^L d_1 + d_2,$$

which corresponds to:

$$\nabla U_L \cdot \delta = \nabla U_L \cdot d.$$

For the deviation $d^{i\prime} = d^i + d$ to be profitable, it should be $-\vec{\pi}_a \cdot d > 0$. Given the new offers' array, the maximum utility that the agent could receive in the low effort region is given by $U(L) + \nabla U_L \cdot \delta$. Since $C \in \mathcal{A}$ is an equilibrium outcome, deviations like $d^{i\prime}$ should be blocked. For this to be guaranteed, $U(C+d)$ must necessarily be (strictly) lower than the maximum utility that the agent could reach, at the deviation stage, selecting a consumption profile in $\mathcal{B}$. Since we are restricting to deviations that take place in a neighborhood of C, one should observe that the agent could always achieve (at least) the payoff $U(C+d) = U(C) + \nabla U_C \cdot d$, by selecting the high effort. We should hence have:

$$-\vec{\pi}_a \cdot d > 0 \implies U(L) + \nabla U_L \cdot d > U(C) + \nabla U_C \cdot d.$$

Since $U(C) = U(L)$, one gets:

$$-\vec{\pi}_a \cdot d > 0 \implies (\nabla U_L - \nabla U_C) \cdot d > 0. \tag{9}$$

However, this last inequality shows that the two operators $-\vec{\pi}_a$ and $\nabla U_L - \nabla U_C$ are collinear, so that there exist a real number θ satisfying:

$$\nabla U_L - \nabla U_C = \theta\, \vec{\pi}_a.$$

■

5 Existence and characterization: the homogeneous case

This section is devoted to discuss existence and characterization of high effort equilibria which corresponding allocation do not belong to the incentive frontier $\mathcal{G}$. Most of the analysis will be developed for the specific class of preferences $u(c) = c^\gamma$, with $\gamma \in (0, 1)$, and we leave for future investigations the development of a general investigation.

Taking the agent's utility to be of the form $u(c) = c^\gamma$ has the advantage of greatly simplifying equilibrium characterization leading to sharper welfare predictions, as will be clarified in the following pages.

A main feature of the context we are considering is that it will never be possible for an allocation $C \notin \mathcal{G}$ to be supported at a pure strategy equilibrium of the game Γ by having all intermediaries charging the same price. This is indeed a consequence of the following:

Lemma 6 *Consider two points $C \in \mathcal{A}$ and $L \in \mathcal{B}$ with the same MRS $\tau^C = \tau^L = p$ and such that:*

$$pc_1 + c_2 = pl_1 + l_2 \tag{10}$$

$$c_1 \leq l_1. \tag{11}$$

Then, it must be that:

$$\pi_a u'(c_2) \geq \pi_b u'(l_2)$$

Proof. Since $\tau^C = \tau^L = p$, one gets:

$$\pi_a = \frac{u'(c_1)}{u'(c_1) + pu'(c_2)} \qquad\qquad \pi_b = \frac{u'(l_1)}{u'(l_1) + pu'(l_2)}$$

We hence have to prove that:

$$\frac{1}{\dfrac{p}{u'(c_1)} + \dfrac{1}{u'(c_2)}} > \frac{1}{\dfrac{p}{u'(l_1)} + \dfrac{1}{u'(l_2)}}.$$

A sufficient condition for the result is that the function $(c_1, c_2) \mapsto \dfrac{p}{u'(c_1)} + \dfrac{1}{u'(c_2)}$ is increasing along the line connecting C and L. Consider $f(x) = \dfrac{p}{u'(c_1 + x)} + \dfrac{1}{u'(c_2 - px)}$, which derivative is such that $f'(x)/p = \dfrac{-u''(c_1 + x)}{(u'(c_1 + x))^2} - \dfrac{-u''(c_2 + x)}{(u'(c_2 + x))^2} = (1 - \gamma)(\dfrac{1}{c_1 + x} - \dfrac{1}{c_2 + x} = \dfrac{(1 - \gamma)(c_2 - c_1)}{(c_1 + x)(c_2 + x)} > 0$, which completes the proof ■

Lemma 7 *Consider a particular game Γ where consumer's preferences are represented by $u(c) = c^\gamma$ with $\gamma \in (0, 1)$. If the allocation $C \notin \mathcal{G}$ is a (pure strategy) equilibrium outcome where the high level of effort is* *selected, then at least two contracts must be offered at different prices.*

Proof. The proof can be established by contradiction. Let us then assume $C \notin \mathcal{G}$: we first recall that at equilibrium every intermediary necessarily earns positive profits, i.e. $p = \tau^C > \frac{1-\pi_a}{\pi_a}$, where p is the equilibrium price (see Proposition 1).

We now take intermediary 1 to be the one earning the smallest (positive) profit at equilibrium and consider a small positive insurance contract d such that $\vec{\pi}_a \cdot d > 0$ and $U(C+d) > U(C)$. If intermediary 1 deviates to $d^{1\prime} = d^1 + d$, the agent will accept it. To generate the contradiction it would then be enough to show that the deviation $d^{1\prime}$ induces the effort choice $e = a$. Since we have considered a small d, the agent's optimal consumption choices in the subspaces $\mathcal{A}$ and $\mathcal{B}$ will be on a line of slope $p = \tau^C$. We denote the corresponding allocations $C' = C + (c_1, c_2)$ and $L' = L + (y_1, y_2)$, respectively. The relevant utilities can be hence approximated by:

$$U(C') = U(C) + (1-\pi_a)u'(c_1)c_1 + \pi_a u'(c_2)c_2 = \pi_a u'(c_2)(\tau^C c_1 + c_2)$$

and

$$U(L') = U(L) + (1-\pi_b)u'(l_1)y_1 + \pi_b u'(l_2)y_2 = \pi_b u'(l_2)(\tau^C y_1 + y_2).$$

We remark that at equilibrium $U(C) = U(L)$. In addition, since C' and L' are connected by a line of slope τ, one gets: $\tau^C c_1 + c_2 = \tau^C y_1 + y_2$. Thus, given that $\pi_a u'(c_2) > \pi_b u'(l_2)$, $U\big(C'\big)$ will be strictly greater than $U\big(L'\big)$, and this proves that the agent's optimal effort choice will be $e = a$. ■

Given that every equilibrium outcome $C \notin \mathcal{G}$ will be supported by non-linear prices, it is possible to identify some restrictions on the consumer's initial wealth such that an equilibrium exists. More precisely, we consider the allocations C and L identified by the following set of conditions:

$$U(C) = U(L) \tag{12}$$

$$\tau^L c_1 + c_2 = \tau^L l_1 + l_2 \tag{13}$$

$$\pi_a(1-\pi_a)\big(u'(c_1) - u'(c_2)\big) = \pi_a\,(1-\pi_b)\,u'(l_1) - \pi_b\,(1-\pi_a)\,u'(l_2) \tag{14}$$

$$\frac{(1-\pi_b)}{\pi_b} \geq \tau^L > \tau^C > \frac{w_2 - c_2}{c_1 - w_1} > \frac{(1-\pi_a)}{\pi_a}, \tag{15}$$

$$w_1 + 2(l_1 - c_1) = w_2 + 2(l_2 - c_2), \tag{16}$$

$$\frac{w_2 + 3(l_2 - c_2)}{w_1 + 3(l_1 - c_1)} < \frac{l_2}{l_1}, \qquad -\vec{\pi}_a \cdot \tilde{d} < -\vec{\pi}_a \cdot \big(C - W\big). \tag{17}$$

Relationships (12)-(17) have a straightforward interpretation: (12) requires the single agent to be indifferent between the outcome C and the outcome L, and (13) states that C and L are connected by a line with slope τ^L in the space (c_1, c_2). That is, the price of the contracts offered by intermediaries 2 and 3 is τ^L. The consumer is hence indifferent between buying only d^1 (and choosing $e = a$) and buying it together with either d^2 or d^3 so to achieve $L \in \mathcal{B}$. As already clarified, (14) requires that the gradient of the utility U evaluated in C is parallel to that evaluated in L. The second and the third inequality in (15) guarantee that accepting the offer $d^1 = C - W$ is an optimal choice for the consumer when she selects $e = a$. The last inequality implies that the only active intermediary earns a positive profit offering d^1, at a price $|\frac{w_2 - c_2}{w_1 - c_1}|$. Given (16), buying only the latent offers d^2 and d^3 will be sufficient to get full insurance for the consumer. Finally, (17) provides a clearer characterization of the amount of latent insurance needed to support the equilibrium consumption. More precisely, the first inequality in (17) guarantees that there is enough latent insurance to make the low-effort threat credible, while the second inequality puts a lower bound on that amount, which is sufficient to block deviations involving negative insurance.[9]

If $u(c) = c^\gamma$, it is then possible to show that there always exists a pair of consumption allocations $\big(C, L\big)$ which satisfies $(12) - (17)$.[10] Importantly, any allocation $\big(C, L\big)$ identified by $(12) - (17)$ can be supported at equilibrium in the game Γ.

Proposition 5 *If $u(c) = c^\gamma$, then there exists a pure strategy equilibrium of the game Γ with the following players' behaviors:*

[9] The amount $\tilde{d}$ defines exactly the positive insurance contract offered at a price τ^L which is necessary to exactly achieve full insurance starting from the consumption point $\big(\frac{c_1 + w_1}{2}, \frac{c_2 + w_2}{2}\big)$. The role of $\tilde{d}$ will be carefully examinated in the next section.

[10] See Lemma 1 in Attar and Chassagnon (2006).

i) $d^1 = C - W, \quad \Lambda^1 = \{0,1\}$,

ii) $d^2 = d^3 = L - C, \quad \Lambda^2 = \Lambda^3 = [0,1]$,

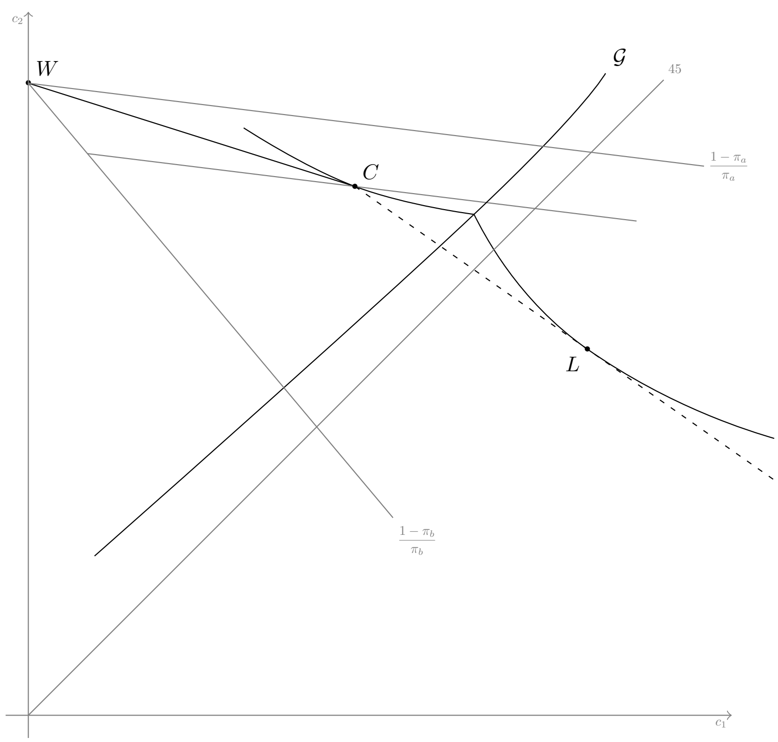
iii) $d^4 = d^5 = \cdots = d^N = (0,0)$,

iv) $\lambda_1 = 1, \lambda_2 = \lambda_2 = \cdots = \lambda_N = 0$, *and* $e = a$,

where C *and* L *are determined through* $(12) - (17)$.

Proof. See Attar and Chassagnon (2006). ■

The geometric structure of the equilibrium is depicted in the following Figure.



6 Take-it or leave-it offers and menus

The aim of this section is to clarify the strategic foundations of our approach to the study of non-exclusive markets. Throughout the paper, we refer to a scenario where intermediaries compete à la Bertrand-Edgeworth. That is, each of them is selecting one price and fixing the maximal amount of insurance that the consumer can buy. A natural question to raise would therefore be: could a single intermediary gain by unilaterally deviating to a simple take-it or leave-it offer? Since there does not seem to be any reason to assume that singleton contract offers are not feasible in insurance and financial markets, the robustness of our equilibrium characterization to such an enlargement of intermediaries' strategy spaces would be an important property to be satisfied.

When an intermediary deviates to a take-it or leave-it contract, there is a reduced strategic role for the agent, since she is not anymore allowed to select her favorite amount of insurance at the given price. In other words, this sort of deviations may constitute a severe form of punishment for consumers and therefore break the conjectured equilibrium.

We show in the following paragraphs that each of our (pure strategy) equilibria indeed satisfies this property. In particular, the allocation characterized in Proposition 5 can be supported at equilibrium of a simpler game where two competitors (principals) are allowed to offer menus of alternatives to the single agent. As will become clearer in the following, this result enables us to relate our findings with those of the recent literature on Common Agency games under moral hazard.

We will therefore consider the following game Γ'. Two principals compete through the menu offers they are simultaneously making to a single agent. Given the set $\mathcal{D}^i$ of offers available to the $i-th$ principal, we let $\mathcal{P}(\mathcal{D}^i)$ be the set of available menus.[11] A strategy for the $i-th$ principal will hence be a map from $\mathcal{P}(\mathcal{D}^i)$ to itself. For every array of offered menus, the agent will choose one item from the menu of each competitor and take her (binary) effort decision.

In the next paragraphs, we show how the equilibrium outcome $C \notin \mathcal{G}$ of the game Γ can indeed be supported in a symmetric equilibrium in the menu game Γ' where only 2 principals are considered.

Given the outcome $C \notin \mathcal{G}$ defined in Proposition 5, let us consider a situation where any of the two competitors offer the following menu of insurance contracts to the single agent:

$$\mathcal{M}^i = \{\mu^i \frac{d^1}{2} + \lambda^i \tilde{d}\}, \qquad \text{for } \ i \in \{1, 2\}$$

where $\mu^i \in [0, 1]$ and $\lambda^i \in [0, 1]$ define the consumer's selected item and the insurance contracts d^1 and $\tilde{d}$ have been defined in Proposition 5. Importantly, every menu $\mathcal{M}^i$ is convex. In addition, by selecting $\mu = 1$ and $\lambda = 0$ in the menu of both principals the agent can still achieve the outcome C. In a similar way, the choice $\mu = 0$ and $\lambda = 1$ will enable the consumer to get to the consumption point L. This follows from the fact that the maximum amount of latent insurance that the agent could buy when the menu $\mathcal{M}$ is offered by both competitors is given by $2\tilde{d}$, which is always greater than $2d$.[12]

One should observe that the strategy we are following to construct the equilibrium menus is to allow every

[11] That is, we take $\mathcal{P}(\mathcal{D}^i)$ to be the Power Set of $\mathcal{D}^i$.

[12] This can indeed by directly checked by manipulating the system $(12) - (17)$.

single principal to offer the amount of latent insurance, say $\tilde{d}$, that correspond to that available off equilibrium in the game Γ. More precisely, when a single principal removes his offer, the frontier of the agent's set of feasible consumptions is that represented in the following Figure 4, where is has been also introduced the iso-profit of slope $\dfrac{1-\pi_a}{\pi_a}$ passing through C.

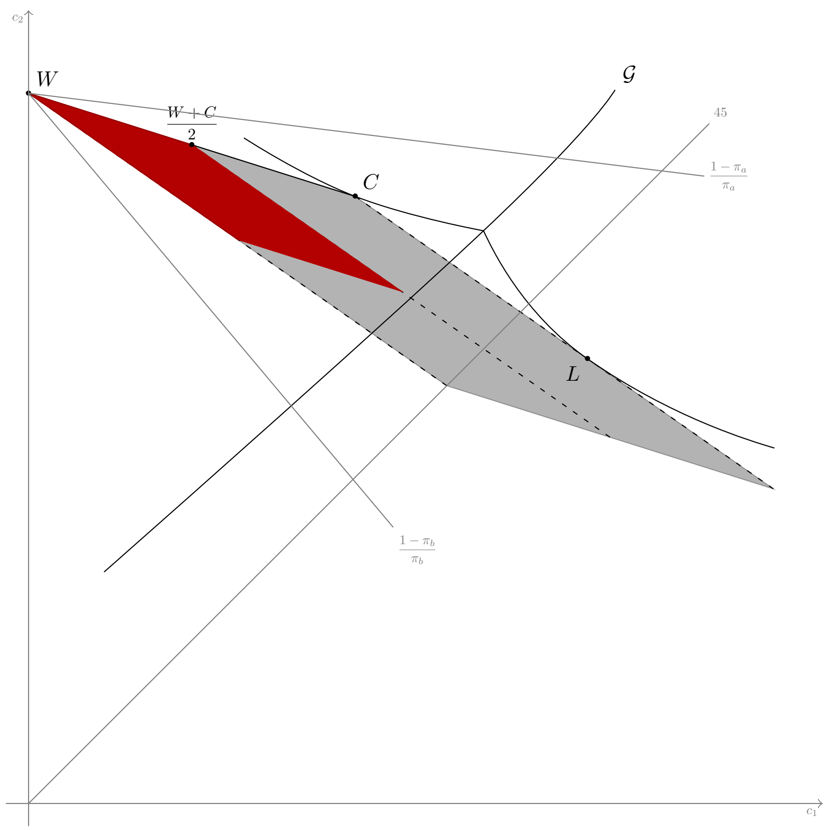


Figure 4: Available consumption choices at the deviation stage

That is, the agent can achieve $P = (\dfrac{c_1+w_1}{2}, \dfrac{c_2+w_2}{2}) \in \mathcal{A}$ as well as the corresponding allocation Q on the $45^{\cdot}$, which is connected to P by a line of slope τ^L. In particular, one gets:

$$Q = (\frac{\tau^L(c_1+w_1)+(w_2+c_2)}{\tau^L+1}, \frac{\tau^L(c_1+w_1)+(w_2+c_2)}{\tau^L+1}) \in \mathcal{B}.$$

To show that the outcome C can be supported at a symmetric equilibrium through the menu offers $\mathcal{M}$, one has to fully describe the consumer's behavior. Given that we are studying the homogeneous case $u(c) = c^{\gamma}$, we know that the agent's optimal consumption choices will be on linear expansion paths. That is, these choices can be identified along the lines $\dfrac{c_2}{c_1}$ and $\dfrac{l_2}{l_1}$ starting from the origin. To identify the main

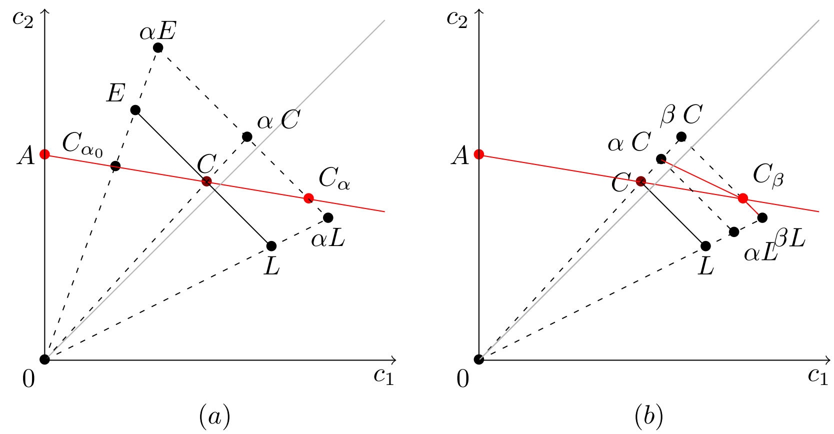


Figure 5: Properties of the optimal consumer's choices

features of consumer's behavior, we find convenient to introduce the the outcome C_α. For every $\alpha \in \mathbb{R}$, C_α is taken to be at the intersection between the (iso-profit) line with slope $\frac{1-\pi_a}{\pi_a}$ passing through C and the line connecting αC and αL. Hence: $C_\alpha = (c_1 + \pi_a f(\alpha), c_2 - (1-\pi_a) f(\alpha))$, with $f'(\alpha) > 0$ and $f(1) = 0$. We also consider the outcome $E = (e_1, e_2)$ such that $\frac{1-\pi_a}{\pi_a} \frac{u'(e_1)}{u'(e_2)} = \tau^L$, i.e. E is the optimal choice of the agent in the set $\mathcal{A}$ if she was offered divisible contracts at a price τ^L. For every $\alpha \in \mathbb{R}$, αE, αC, C_α and αL will all lie on the same line. A main feature of our construction turns out to be that any unilateral deviation to positive insurance contracts will induce the choice of the low effort $e = b$. Conversely, it will not be possible for a single intermediary to profitably propose a negative insurance contract which induces the agent to select $e = b$. These results are indeed implied by the following Lemma.

Lemma 8 *If $u(c) = c^\gamma$ and the outcomes C and L satisfy $(12) - (17)$, then:*

1. $U(C_\alpha) < U(\alpha L) \quad \forall \alpha \neq 1$ *such that* $C_\alpha \in \mathcal{A}$,
2. $U(K) < U(\alpha L) \quad \forall K \in \mathcal{A}$ *lying between* C_α *and* αL,
3. $U(\alpha C) < U(\beta L) \quad \forall (\alpha, \beta)$ *with* $\beta > \alpha > 1$ *and such that the line connecting* αC *and* C_β *has slope* τ^C.

Points 1,2, and 3 refer to Figure $2(a)$ and point 4 refers to Figure $2(b)$.

Proof. See Attar and Chassagnon (2006). ■

It is now possible to show that a single principal, say principal 1 (P1), cannot profitably deviate to any menu different from $\mathcal{M}$. To show this, it will be enough to consider simple deviations $d^{1\prime}$ in take-it or leave-it offers.[13]

We will first consider the deviations $d^{1\prime}$ to a negative insurance contract, which are easier to be handled.

If $d^{1\prime}$ is a negative insurance contract, to guarantee positive profits and when the agent is taking the high effort $e = a$, its price should be below $\frac{1-\pi_a}{\pi_a}$. Then, whenever $d^{1\prime}$ is bought, the corresponding K will lie to the left of the line connecting P and Q. But there will always be an element of this line which the consumer will strictly prefer to any point inside the triangle WPQ. If If P1 deviates to a negative insurance contract, followed by the agent choosing $e = b$, the relevant price should be higher than τ^L, for the agent to have an incentive to buy it, and lower than $\frac{(1-\pi_b)}{\pi_b}$, to be profitable for the deviator. If the high level of effort was indeed selected, any negative insurance contract involving a price higher than τ^L will earn negative profits. However, by buying $\tilde{d}$ from the non-deviating principal, the agent could achieve an allocation like H on the 45 degree line, and there is no contract, which price is between τ^L and $\frac{(1-\pi_b)}{\pi_b}$, that could increase the agent's utility.

We consider now deviations to a positive insurance contract $d^{1\prime}$ which induce the agent to select the high effort $e = a$ at the deviation stage. In particular, we first analyze deviations involving a price $p' = |\dfrac{d_2^{1\prime}}{d_1^{1\prime}}| \in (p^C, pL]$, where $p^C = \dfrac{w_2 - c_2}{c_1 - w_1}$ is the price of the contract d^1. If such a deviation is chosen, then by Lemma 2 the agent will still select $\beta = 1$ in the menu of the non-deviating principal, so to get the outcome P. One should observe that $d^{1\prime}$ should be such that: $-\vec{\pi}_a \cdot d^{1\prime} > -\vec{\pi}_a \cdot \tilde{d}$. This last requirement follows from the fact that the deviation should be profitable, i.e. $-\vec{\pi}_a \cdot d^{1\prime} > -\vec{\pi}_a \cdot d^1 = -\vec{\pi}_a \cdot (C - W)$, together with (17). This, in turn, implies that $d^{1\prime} = \tilde{d} + d^{1\prime\prime}$, with $d^{1\prime\prime}$ being a positive insurance contract which price is lower than τ^L. Then, the terminal point on the frontier of feasible consumption corresponding to the offers $(d^{1\prime}, d^2, ..., d^N)$, that is, the point $W + d^{1\prime\prime} + 2\tilde{d}$, will fall below the line passing through the origin and L, as it is the case for the point $W + 3d^2$.[14] Hence, at the deviation stage, the agent can achieve her optimal choice in the subset $\mathcal{B}$, since it lies on the line passing through the origin and L.

In a next step we look at deviations $d^{1\prime}$ which induce the agent to select $\beta < 1$ from the non-deviating principal menu and to take the effort $e = a$. This sort of deviations should involve a price $p' \leq p^C$. In addition, since P is available, it must be $U(K) > U(P)$ where K is the agent's utility at the deviation stage. The corresponding feasible consumption choices are depicted in Figure 4. By definition, the agent will not have any incentive to select the allocation $M = K + d^1$ and to choose $e = a$. In addition, for every

[13] This is true since the consumer is allowed to select at most one allocation from every menu.

[14] See the equilibrium condition (17).

$K \in \mathcal{A}$ she can always get an outcome like γL that is on the ray connecting the origin to L.[15] To prove that there will not be any profitable $d^{2\prime}$ inducing $e = a$, we now introduce some further notation. We take P_β to be the intersection between the line of slope $\frac{1-\pi_a}{\pi_a}$ passing through P and the line connecting K and M. In addition, we denote we denote βL (with $\beta < \gamma$) the projection of P_β through a line of slope τ^L on the ray connecting 0 and L. Finally, we let $\alpha P \in \mathcal{A}$ be the optimal choice of the agent along the line of slope τ^P passing through P_β. For every $K \in \mathcal{A}$ one has that:

$$
\begin{aligned}
U(K) &\leq U(\alpha P) \qquad (18)\\
U(\beta L) &\leq U(\gamma L) \qquad (19)
\end{aligned}
$$

One should observe that the first inequality is satisfied by construction, since $|\frac{d_1^1}{d_2^1}| \leq \tau^C$. To establish the second one, we remark that $\vec{\tau}^L \cdot K \leq \vec{\tau}^L \cdot M$, and, since C_β is between K and M, we have $\vec{\tau}^L \cdot C_\beta \leq \vec{\tau}^L \cdot M$, with $\vec{\tau}^L = \left(\tau^L, 1\right)$. That is, βL will fall at the left of γL. This indeed implies $U(\beta L) \leq U(\gamma L)$.
Hence, one can apply the last point of lemma 2 so to get $U(\beta L) > U(\alpha C)$, which, given the inequalities above, implies that $U(\gamma L) > U(K)$. This shows that there is no profitable deviation inducing $e = a$.

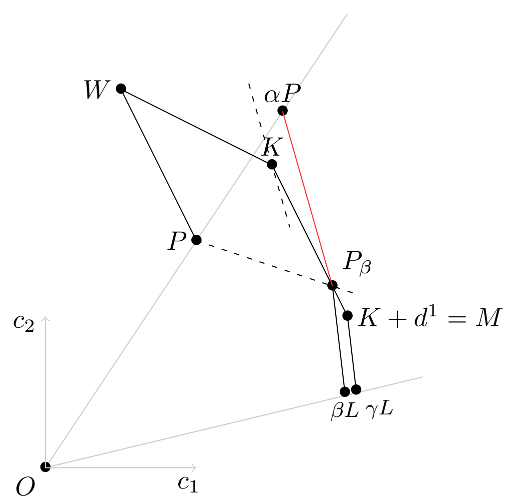


Figure 6: A deviation of intermediary 2

Finally, we have to show that there cannot be any profitable deviation $d^{1\prime}$ inducing low effort either. Such

[15] The availability of γL is guaranteed by construction.

a deviation could only be profitable if its price is higher than $\frac{1-\pi_b}{\pi_b}$. The agent, though, will never have any incentive to buy such a contract since she can already achieve the allocation Q on the 45 degree line at a price $\tau^L \leq \frac{1-\pi_b}{\pi_b}$.[16]

Several recent developments in oligopoly theory stressed the relevance of situations where many principals are strategically competing over the contract offers they are making to a single agent. In particular, those contexts where the agent is allowed to contract with any subset of principals are referred to as delegated Common Agency. When Common Agency games of complete information are considered (i.e. when the agent's type space is a singleton), it is now well established that the set of equilibrium outcomes supportable when principals are allowed to compete by offering menus of alternatives to the single agent is in general larger than the set of equilibria associated to "simple" competition in take-it or leave-it offers. More precisely, let consider a game where two principals interact in the presence of a single agent who is taking a non-contractible effort choice. Then, the particular equilibrium allocation we supported could not have been implemented by letting the two principals just compete over singleton contracts.[17]. It should also be remarked that, when Common Agency games of moral hazard are considered, there is a very weak support for the restriction to take-it or leave-it offers (Attar, Piaser, and Porteiro (2006)).

7 Welfare analysis

Defining efficiency in the class of our model is not an easy task. The large debate on the concept of third best efficiency is a signal of that. In a context with non-exclusive contracting the strategic interaction among principals becomes relevant for the planner.

Let us first remark that in the standard Second Best analysis the planner retains control on the consumption/insurance allocations, but is subject to the same informational problems about agent's decisions of the players. In other words, there is a correspondence between second-best efficiency and exclusivity on contracts. Hence, the planner directly controls (or can contract upon) the mechanisms that the principals of the game will use. This direct control implies that the social planner can always implement the desired allocation by means of point contracts (take-it-or-leave-it offers), hence the agent's choice on λ_is is *de facto* limited to $\{0, 1\}$.

[16]The availability of this outcome is guaranteed by (17).

[17]This is an instance of the so-called failure of the Revelation principle in games of multiple principals $\big($see Martimort and Stole (2002) and Peters (2001)$\big)$

Our equilibrium analysis has already emphasized that the externalities induced by competition under moral hazard are sufficient to guarantee that every equilibrium allocation of the game Γ will fail to be Second Best efficient.

In a non-exclusive interaction, the planner looses control on the mechanisms of the principals, hence indirectly on the total insurance the agent has available for purchases. As a consequence, the social planner does not control the hidden action (effort) of the agent, as well as the agent's choice on the number/quantity of insurance contracts she will accept (λ_is). Hence, the optimal allocation must be robust to side-contracting. In the definition of the relevant efficiency concept, we must take into account the strategic behavior of the principals as well as the agent's one.

The planner can use/design transfers to all the players, as long as these transfers satisfy aggregate feasibility. Note that a transfer to the agent effectively changes the endowment point w, and hence may equivalently be thought of as public insurance. Let $d^0 = (d_1^0, d_2^0)$ be the state-contingent transfer offered by the planner. That is, we let the planner be denoted as firm 0.

The set of allocations which are feasible for a planner who is subject to non-exclusivity is indeed given by:

- The incentive compatibility on the agent's side:

$$(e, \{\lambda_i\}_{i=1}^N) = \arg \max_{e', \{\lambda_i'\}_{i=1}^N} \; (1 - \pi_{e'}) u(w_1 + \sum_{i=1}^N \lambda_i' d_1^i) + \pi_{e'} \; u(w_2 + \sum_{i=1}^N \lambda_i' d_2^i) - e' \tag{20}$$

- Robustness of the optimal allocation: $\left(d^i, d^{-i}\right)$ has to be such that:

$$V^i\left(d^i, e(d^i, d^{-i}, W), \lambda_i(d^i, d^{-i}, W)\right) \geq -\tilde{\lambda}_i[(1 - \pi_{\tilde{e}})\tilde{d}_1^i + \pi_{\tilde{e}}\tilde{d}_2^i] \;\; \forall i \;\text{ and }\; \forall \tilde{d}^i \in \mathbb{R}^2. \tag{21}$$

One should notice that offering a public insurance contract to the single agent indeed corresponds to control the endowment allocation $W = (w_1, w_2)$. The planner must hence take into account the optimal behavior of all players given alternative public insurance offers.

To show that market equilibria may fail to be constrained efficient, we present our discussion in the context of the Example of Proposition 5.

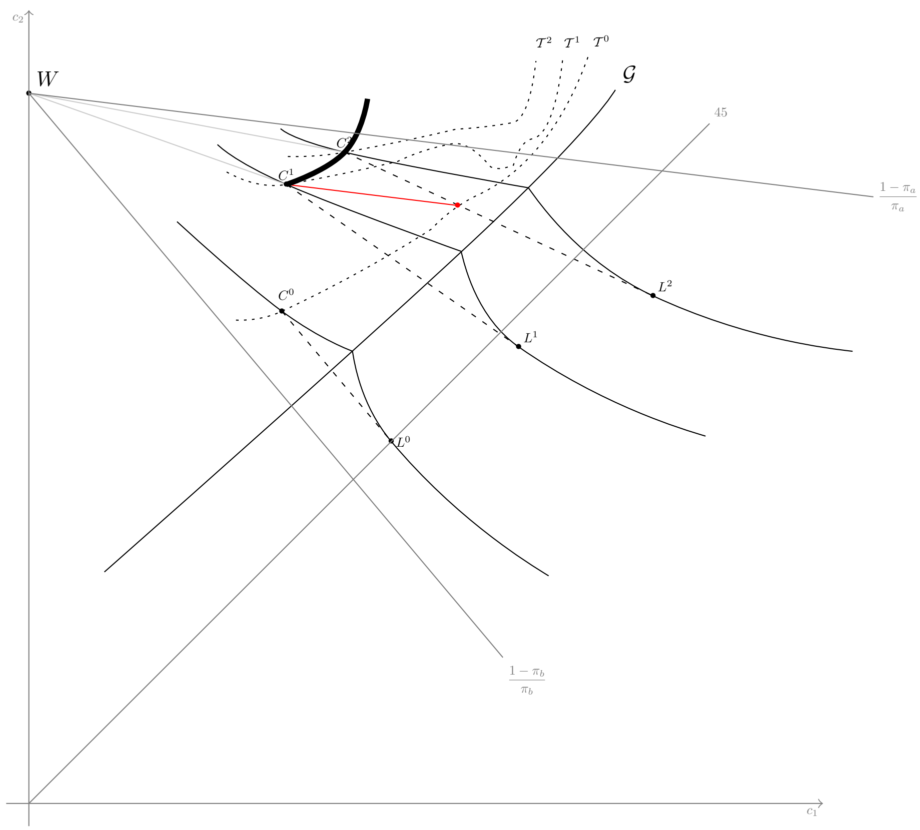
Let us take any $\tau < \dfrac{1 - \pi_b}{\pi_b}$ which is supported at equilibrium. Given τ, at every equilibrium allocation $C = (c_1, c_2)$ it must be the that:

$$U(C) = U(L) \tag{22}$$

$$\tau^L c_1 + c_2 = \tau^L l_1 + l_2 \tag{23}$$

$$\tau^L = \frac{(1-\pi_b)u'(l_1)}{\pi_b u'(l_2)} \tag{24}$$

We let $T(\tau)$ be the relationship between c_2 and c_1 which is induced by equations (22)-(24). The equilibrium allocation corresponding to the latent price τ will belong to $T(\tau)$ but, clearly, only one point of $T(\tau)$ constitutes an equilibrium allocation. The geometry of the $T(\tau)$ curves is described in the following Figure.



Importantly, one can show that T curves which correspond to different τ will not intersect. In addition, the curve $T^2 = T(\tau^2)$ will always lie to the left of a curve like $T^2 = T(\tau^2)$ for $\tau^2 < \tau^1$.

In other words, we can think of T^0 as the frontier of the set of feasible allocations in our problem.

Let us first show that in the particular case $u(c) = c^\gamma$, the curve $\mathcal{T}^0$ is increasing, and hence it cuts the relevant iso-profit line passing through C. In this case, the candidate high-effort equilibrium at point C is not robust to individually profitable deviations of some active insurer. Hence, in the power utility case there cannot be an equilibrium with $e = a$ and $\tau^L = \frac{1-\pi_b}{\pi_b}$.

Lemma 9 *In the c^γ-case conditions* (22) *and* (23) *together with the additional requirement* $\tau^L = \frac{1-\pi_b}{\pi_b}$ *identify an increasing curve* $\mathcal{T}^0$.

Proof. Given the requirement that $\tau^L = \frac{1-\pi_b}{\pi_b}$, L belongs to the 45 degree line and equations (22) and (23) can be written in terms of the only two variables (c_1, c_2). In fact, by equation (23), $L = (l, l)$ with $l = (1-\pi_b)c_1 + \pi_b c_2$. The equation we have to consider is then:

$$(1-\pi_a)u(c_1) + \pi_a u(c_2) = u(l) \quad \Longleftrightarrow$$

$$\mathcal{T}^0 = (1-\pi_a)u(c_1) + \pi_a u(c_2) - u\left((1-\pi_b)c_1 + \pi_b c_2\right) = 0 \tag{25}$$

It should be first noted that it cannot be that both conditions $\dfrac{\partial \mathcal{T}^0}{\partial c_1} < 0$ and $\dfrac{\partial \mathcal{T}^0}{\partial c_2} < 0$ hold together. Assume they hold together, the corresponding first-order inequalities would be:

$$\begin{cases} (1-\pi_a)u'(c_1) & < & (1-\pi_b)u'(l) \\ \pi_a\, u'(c_2) & < & \pi_b u'(l) \end{cases}$$

Since the marginal utility in convex, using $u''' \geq 0$, we can majorate $u'(l)$ with $(1-\pi_b)u'(c_1) + \pi_b u'(c_2)$ and then

$$\frac{\partial \mathcal{T}^0}{\partial c_2} \geq \pi_a\, u'(c_2) - \pi_b\big((1-\pi_b)u'(c_1) + \pi_b u'(c_2)\big) \tag{26}$$

$$= (\pi_a - \pi_b)u'(c_1) + \pi_b^2\big(u'(c_1) - u'(c_2)\big) > 0 \tag{27}$$

Hence, $\dfrac{\partial \mathcal{T}^0}{\partial c_2} > 0$. ■

If the locus $\mathcal{T}^0$ is increasing in the consumption space, there is room for deviations. In particular, those deviations that reduce the utility of the agent without altering her effort choice become profitable. The area identified by the triangle ZCR in the following figure is the region of profitable deviations.

Notice that for the points in the triangle ZCR in Figure 7 Lemma 8 does not hold, hence it is possible to find state-contingent consumption levels that increase the profit of the insurance companies without inducing changes in the choice of effort of the agent.

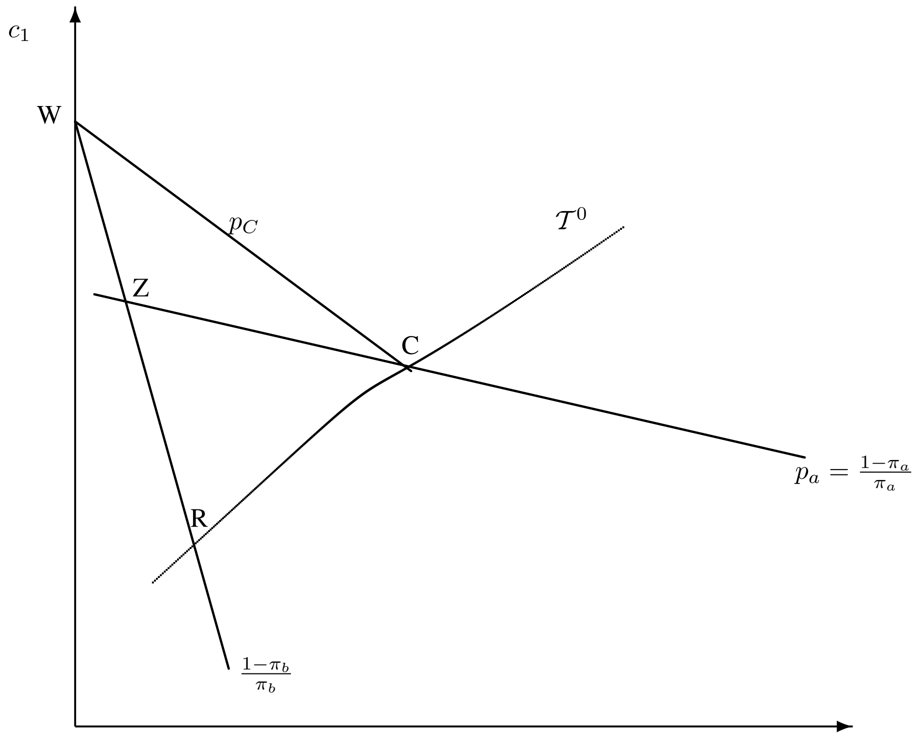


Figure 7: Profitable deviations from C when $\mathcal{T}$ is increasing.

Hence, any high-effort equilibrium for which $\tau^L = \frac{1-\pi_b}{\pi_b}$ can be destroyed by profitable deviations of an active principal. This type of deviations are non-profitable when $\mathcal{T}^0$ does not intersect the iso-profit line passing through the candidate equilibrium point C. As argued in section **??**, whenever $\tau^L < \frac{1-\pi_b}{\pi_b}$, $\mathcal{T}$ is tangent to the iso-profit line with slope $-\frac{1-\pi_a}{\pi_a}$ exactly at point C. High-effort equilibria in this set-up exhibit positive profits for the active principals, and latent contracts issued at negative latent-prices.

In the c^γ-case, every equilibrium is therefore inefficient with respect to the third-best outcome.

Is that possible that the action of the planner be such that none of the private insurers wants to enter the market for insurance? The answer is yes.

We shall argue, in particular, that it is possible to implement any allocation located on the frontier represented by equation:

$$(1-\pi_a)u(c_1) + \pi_a u(c_2) - u((1-\pi_b)c_1 + \pi_b c_2) - (a-b) = 0 \tag{28}$$

Let's indeed consider a consumption bundle $C^0 = (c_1^0, c_2^0)$ lying on on the frontier T^0 and verifying (28); we also let $L^0 = (l^0, l^0)$ be the corresponding latent allocation. One should notice that $l^0 = (1-\pi_b)c_1^0 +$

$\pi_b c_2^0$.

We then consider the following behavior for the public planner (which we can also call intermediary 0) and the private intermediaries $1, 2, \dots, J$:

- The planner proposes the following non-linear menu

$$\mathcal{M}^0 = (d^0 + \lambda \tilde{d}^0)$$

  where $d^0 = C - W$ and it is offered at a price $p^0 > \frac{1-\pi_a}{\pi_a}$, $\tilde{d}^0 = L - C$ and it is offered at a price $p_L = \frac{1-\pi_b}{\pi_b}$, and $\lambda \in [0, 1]$;

- the private insurance companies propose the null contract, $d^1 = d^2 = \cdots = d^J = (0, 0)$.

These behaviors support the equilibrium allocation C^0 which belongs to the T^0 curve. The following proposition emphasizes that this allocation can indeed be implemented by the planner:

Proposition 6 *When the planner provides public insurance by means of the piece-wise linear menu $\mathcal{M}^0$, then it is a best reply for the single agent to accept the element $d^0 \in \mathcal{M}'$ (i.e. to set $\lambda = 0$) and for every private insurer to offer the null insurance contract.*

Proof. We should verify that intermediary $i \in \{1, 2, \dots, J\}$ has no interest in deviating. Take intermediary 1 as a candidate deviator. If he proposes positive insurance, it should be at a price lower than $\dfrac{1-\pi_a}{\pi_a}$ to be accepted by the agent. Then, he could only make profit in the case the agent remains in the high effort area. Let us take $K = (k_1, k_2) \in \mathcal{A}$ as the optimal consumption allocation selected by the agent at the deviation stage. It hence follows from Lemma 8 that:

$$(1-\pi_a)u(k_1) + \pi_a u(k_2) - u((1-\pi_b)k_1 + \pi_b k_2) - (a - b) < 0,$$

which implies in turn that the agent will prefer the feasible allocation $(1-\pi_b)k_1 + \pi_b k_2$ to K, that is a contradiction.

If he proposes negative insurance, the only way for the agent to accept the offer is to remain in $\mathcal{A}$, making the deviation non profitable. A contradiction. ∎

In particular, we can state the following:

Proposition 7 *Given any positive-profit high-effort equilibrium $C \in \mathcal{A}$ satisfying (22), (23), (24), there exists a system of transfers and of public insurance that Pareto dominates it.*

Proof. Let us now construct a system of transfers, that satisfies aggregate feasibility and allow to redistribute the planner's profits to the private insurers.

The system of public insurance is a non-linear menu of the type $\mathcal{M}^0 = (\alpha d^0 + \beta \tilde{d^0})$ as defined above, where the single agent has to choose α and β, appropriately. The system of transfers to the players in the economy is the following: each private insurer receives $\frac{1}{J}$-th of the total profit from the purchase of the insurance (V).

Given the offered menu $\mathcal{M}^0$, it is a best reply for the agent to choose $\alpha = 1$ and $\beta = 0$. In fact, since $U(C) = U(L)$ the agent is indifferent between C and L. Choosing $\alpha < 1$ would allow to reach lower indifference curves, and even supplementing with $\beta \geq 1$ would still keep the agent below point L.

For the private insurers, we have previously shown that offering the null contract is a best reply when m^0 is available. Receiving the transfer $\frac{1}{J}\ V$, does not change the main argument of the proof. Each individual insurer takes into account that in the aggregate C and L are available, and that there is no profitable deviation for him. ■

At the equilibrium with transfers, the social planner proposes a piece-wise linear menu of insurance. The private insurers offer the null contract, the agent buys the insurance proposed by the planner and exerts high effort. Finally, the planner redistributes by means of un-contingent transfers the profits to the private insurers according to his objective function.

References

ARNOTT, R., AND J. STIGLITZ (1991): "Price Equilibrium, Efficiency, and Decentralizability in Insurance Markets," NBER DP 3642.

——— (1993): "Equilibrium in Competitive Insurance Markets with Moral Hazard," mimeo, Boston College.

ASQUITH, P., AND T. WIZMAN (1990): "Event Risk, Covenants, and Bondholders Returns in Leveraged Buyouts.," *Journal of Financial Economics*, 27, 195–213.

ATTAR, A., E. CAMPIONI, AND G. PIASER (2006): "Multiple lending and constrained efficiency in the credit market," *Contributions to Theoretical Economics*, 6(1).

ATTAR, A., AND A. CHASSAGNON (2006): "On moral hazard and nonexclusive contracts," mimeo IDEI-PSE.

ATTAR, A., G. PIASER, AND N. PORTEIRO (2006): "A note on Common Agency models of moral hazard," *Economics Letters*, forthcoming.

AUSUBEL, L. M. (1991): "The Failure of Competition in the Credit Card Market," *American Economic Review*, 81(1), 50–81.

BISIN, A., AND D. GUAITOLI (2004): "Moral hazard with non-exclusive contracts," *Rand Journal of Economics*, 2, 306–328.

BIZER, D., AND P. DEMARZO (1992): "Sequential Banking," *Journal of Political Economy*, 100(1), 41–61.

BOLTON, P., AND O. JEANNE (2005): "Structuring and Restructuring Sovereign Debt: The Role of Seniority," NBER Working Paper No. 11071.

BROWN, J. (2001): "Private pensions, mortality risk, and the decision to annuitize," *Journal of Public Economics*, 82.

CHASSAGNON, A., AND P. CHIAPPORI (1997): "Insurance under Moral Hazard and Adverse Selection: the case of Pure Competition," Mimeo, PSE, Paris.

DETRAGIACHE, P., P. GARELLA, AND L. GUISO (2000): "Multiple versus single banking relationships: Theory and Evidence," *Journal of Finance*, 55, 1385–1403.

HELLWIG, M. (1983): "On moral hazard and non-price equilibria in competitive insurance markets," mimeo, Boston College.

JAPPELLI, T., AND M. PAGANO (2002): "Information Sharing, Lending and Defaults: Cross-Country Evidence," (26).

KAHN, C. M., AND D. MOOKHERJEE (1998): "Competition and Incentives with Nonexclusive Contracts," *RAND Journal of Economics*, 29(3), 443–465.

MARTIMORT, D. (2004): "Delegated Common Agency under Moral Hazard and the Formation of Interest Group," mimeo, IDEI.

MARTIMORT, D., AND L. A. STOLE (2002): "The revelation and delegation principles in common agency games," *Econometrica*, 70(4), 1659–1673.

——— (2003): "Contractual externalities and common agency equilibria," *Advances in Theoretical Economics*, 3(1).

PARLOUR, C. A., AND U. RAJAN (2001): "Competition in Loan Contracts," *American Economic Review*, 91(5), 1311–1328.

PAULY, M. (1974): "Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection," *Quarterly Journal of Economics*, 88, 44–62.

PETERS, M. (2001): "Common Agency and the Revelation Principle," *Econometrica*, 69(5), 1349–1372.

PETERSEN, M., AND R. RAJAN (1994): "The Benefit of Lending Relationships: Evidence from Small Business Data," *Journal of Finance*, 49, 1367–1400.

SMITH, C., AND J. WARNER (1979): "On Financial Contracting: An Analysis of Bond Covenants," *Journal of Financial Economics*, (7), 117–161.