Pareto Optima and Competitive Equilibria with Moral Hazard and Financial Markets*

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Abstract

In this paper, we study a two-period pure exchange economy with idiosyncratic uncertainty, moral hazard and multiple consumption goods. We consider two different market structures: contingent commodity markets on the one hand, and financial plus spot commodity markets on the other hand. We propose a competitive equilibrium concept for each market structure. We first verify that it is possible to decentralize constrained efficient allocations as equilibria with contingent markets. Subsequently, we characterize the conditions which prevent constrained efficient allocations to be decentralized as equilibria with financial markets.

Keywords: Hidden action, multiple goods, constrained efficiency.

JEL Classification: D53, D61, D82.

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1. Introduction

The analysis of decentralization of efficient allocations through competitive equilibria when asymmetric information is present has been an active area since long time ago. A seminal result in this area of research is due to Prescott and Townsend (1984), who show that under very general conditions on fundamentals of the economy, it is possible to decentralize efficient allocations through competitive equilibria in the case of both adverse selection and moral hazard.

In that context, decentralization occurs when agents make their optimal choices on exogenously given markets by taking as given suitably defined prices for exchangeable commodities.

It is well known that Prescott and Townsend's result hinges on the assumption that consumers make their choices on markets which are open only before uncertainty is resolved, and then simply fulfil the endorsed obligations. No trading is allowed after the resolution of uncertainty.

The efficiency criterion related to this assumption hence implies that the planner can verify all actual trades consumers make. In an economy with moral hazard as the one we are interested in, this means that there is only one hidden action consumer can take: choosing their preferred level of effort.

Aware of the importance of such an assumption on the market structure, Prescott and Townsend argue explicitly that it is the most natural one, if the competitive market paradigm has to be reconciled with an appropriate constrained efficient criterion.[1] Yet, given the strong enforcement power this assumption implies, it is interesting to study if decentralization of efficient allocations is still possible when such an assumption is removed

Indeed, doubts have been raised that in the case of moral hazard the result of Prescott and Townend holds if the planner cannot control all trades consumers make. In particular, Stiglitz and his co-authors have argued that to decentralize efficient allocations when the planner does not have enough enforcement power taxes and subsidies are required.[2]

On the other hand, Lisboa (2001) has recently showed that decentralization occurs in a model with financial and spot markets if consumers have very specific preferences for consumption goods and effort.[3]

Following these lines of research, and their somehow conflicting intuitions, in this paper we propose a unified framework to answer the following question: under which hypothesis on market structure and fundamentals of the economy (preferences and number of consumption goods), it is possible to decentralize an efficient allocation as a competitive equilibrium when moral hazard is present?

[1] Prescott and Townsend (1984), p. 42.

[2] See Greenwald and Stiglitz (1986), Arnott and Stiglitz (1986), Arnott et al. (1992).

[3] In particular, preferences are separable in the cost of effort.

We analyze a two-period pure exchange economy, populated by a large numbers of consumers subject to idiosyncratic uncertainty on their endowments. Given the existing market structure, they choose a bundle of consumption goods, possibly a financial asset, and an hidden action, which alters the probability of idiosyncratic states.

We consider two different market structures, and two consistently different equilibrium concepts. On the one hand, we assume there exist contingent markets for delivery of commodities conditional on realization of the idiosyncratic state. In this case, consumers are not allowed to trade after the resolution of uncertainty. On the other hand, we let consumers to trade on financial markets before the resolution of uncertainty, so as to redistribute their income across idiosyncratic states. Once uncertainty is realized, consumers trade on spot markets for consumption goods, so as to maximize their ex-post utility.

The main result of the paper is to show that, while in the first case decentralization of efficient allocation always occurs, in the second case it occurs provided that restrictive hypotheses are satisfied, such as that of a single consumption good or that of additive separability of ex-post utility function between consumption goods and action.

The intuition for this result follows from the observation that an (ex-ante) efficient and incentive compatible bundle of action and consumption goods may fail to be optimal ex-post. In particular, it may happen that an allocation of consumption goods is not an optimal choice when trade on spot markets occurs, given prices and the action chosen in the first period. If this is the case, an efficient allocation cannot be decentralized as an equilibrium with spot markets, for in this case consumers would rather choose a different consumption bundle.

Indeed, this problem does *not* arise when there is a single consumption good, as in this case there are no spot markets, nor when utility is additively separable, as in this case ex-post utility from consumption is not affected by the choice of action. When these assumptions are not satisfied, this problem does arise under conditions that we explicitly characterize.

It is of interest to notice that in proving the above result we provide an explicit example where the traditional equivalence between equilibrium allocations in economies with complete contingent markets and in economies with complete financial and commodity spot markets breaks down because of asymmetric information.

The paper is organized as follows: in section 2, the economy is described, while the appropriate efficient allocation is characterized in section 3. In section 4, the competitive equilibrium with contingent markets is introduced, and decentralization of efficient allocation is studied for this case. In section 5, the competitive equilibrium with financial markets is introduced, and decentralization for this case is analyzed in section 6. Finally, the relevant related literature is discussed in section 7.

2. The economy

We consider a pure exchange economy with $L \geqslant 1$ consumption goods. The economy is populated by a large number of consumers all ex ante equal, and lasts two periods $t = 0, 1$. There is no consumption at $t = 0$, when consumers face an idiosyncratic uncertainty which realizes in the second period. At $t = 1$, each consumer may be in one out two states $s \in \mathcal{S} = \{1, 2\}$.

Uncertainty affects only the amount of each good consumers are endowment with at the beginning of the second period. Individual endowments are denoted by $w = (w_1, w_2) \in \mathbb{R}^{2L}_{+}$.[4] It is assumed that $w_1 \gg w_2$, so that state $s = 1$ will be referred to as the good state.[5]

Consumers can choose an action $a \in \mathcal{A} = [0, 1]$ affecting, on the one hand, the probabilities of the idiosyncratic states, and, on the other hand, the utility consumers get from consumption.

Preferences are represented by the following expected utility function $v : \mathbb{R}^{2L}_{+} \times \mathcal{A} \to \mathbb{R}$:

$$v(x_1, x_2, a) := \pi_1(a)\, u(x_1, a) + \pi_2(a)\, u(x_2, a)\,,$$

where $\pi_s(a) : \mathcal{A} \to (0, 1)$ is a function giving the probability of state s when action a is chosen.

According to the above general formulation, a is assumed to affect the utility of every consumption good once the state is realized. This case will be referred to as non-separable preferences.

A polar case arises when a does not affects the utility of any consumption good once the state is realized. This case will be referred to as separable preferences and it is formally stated in the following:

Assumption S. (Separable preferences) *There exist utility indexes $b : \mathbb{R}^{L}_{+} \to \mathbb{R}$ and $c : \mathcal{A} \to \mathbb{R}$ such that $u(x_s, a) := b(x_s) - c(a)$.*

Intermediate cases are possible, as a may affect the utility of some, but not all consumption goods. As this hypothesis will be relevant for the main result, we formalize it in a particularly simple configuration:

[4] $\mathbb{R}^{L}_{+}$ denotes the positive orthant of dimension L.

[5] Given two vectors $x, y \in \mathbb{R}^{L}_{+}$, $x \gg y$ means $x_l > y_l$ for all l, $x > y$ means $x_l \geqslant y_l$ for all l and $x \neq y$, and $x \geqslant y$ means $x_l \geqslant y_l$ for all l.

Assumption H. (Hemi-Non-separable preferences) *There exist utility indexes* $f : \mathbb{R}_+ \to \mathbb{R}$ *and* $g : \mathbb{R}_+^{L-1} \times \mathcal{A} \to \mathbb{R}$ *such that* $u(x_s, a) := f(x_{sj}) + g(x_s^{-j}, a)$ *for some consumption good* j *,*

where $x_s^{-j} \in \mathbb{R}_{++}^{L-1}$ is a bundle of all consumption goods but good sj.

It is assumed that the action chosen by a consumer remains unknown to anyone else in the economy, so that information is asymmetric. On the other hand, it is assumed that endowments are verifiable at $t = 1$, so that it is possible to determine in which idiosyncratic state consumers are.

3. Constrained efficient allocation

Suppose there exists a benevolent planner perfectly informed on the fundamentals of the economy, in particular on preferences and endowments. Suppose in addition that he is subject to the same limits on information as any other economic agent, so that he cannot verify the level of action chosen by consumers.

The role of the planner is to assign a feasible distribution of consumption goods and to prescribe an action so as to maximize consumers' utility. Because of limits on information, the planner can only prescribe actions that are optimal given the assigned consumption bundle. In the standard jargon, this means that the planner can only choose among the set of incentive compatible actions.

Implicit in this reasoning is the fact that the planner observes actual consumption at $t = 1$, so that he can enforce a given distribution of consumption bundles. It is therefore excluded that consumers re-trade after having received their prescribed bundles. This in turn implies that the choice of a is the only hidden action consumers take.

From the above discussion, it follows that a constrained efficient (CE) allocation is the solution of the following:

$$\max_{x_1, x_2, a} \quad v(x_1, x_2, a)$$

$$\text{s.t.} \quad \pi_1(a)(x_1 - w_1) + \pi_2(a)(x_2 - w_2) \leqslant 0\,, \tag{1a}$$

$$a = \arg\max\, v(x_1, x_2, a)\,. \tag{1b}$$

In the above problem, (1a) is the feasibility constraint, which holds in expected value thanks to assumption of large number of consumers, while (1b) is the incentive compatibility constraint. It states that the only admissible actions in the planner's choice set are those maximizing the consumers' utility given the prescribed consumption bundle.

In what follows, $\mathcal{B}$ will denote the set of (x, a) satisfying (1a) and (1b).

In the rest of the paper, we will be interested in answering the following question: under which hypotheses on market structure and fundamentals of the economy (preferences and number of consumption goods), it is possible to decentralize a CE allocation as a competitive equilibrium?

As anticipated in the introduction, we are interested in studying the above question in the context of two different market structures. On the one hand, we will consider contingent markets for delivery of commodities conditional on the realization of uncertainty. In this case, consumers will only be allowed to make their choices at $t = 0$. This implies in particular that trading after the realization of uncertainty is not permitted. On the other hand, we will consider a market structure where consumers at $t = 0$ can access financial markets so as to redistribute their income across idiosyncratic states. Once uncertainty is realized, spot markets for consumption good open, and consumers choose their optimal bundles given price and all choices made at $t = 0$.

4. The contingent market (CM) model

In this section, we suppose there exists a complete set of markets for commodities with delivery contingent on idiosyncratic states. Consumers trade in these markets, which are open at $t = 0$, by selling their state-contingent endowments and buying a bundle of consumption goods to be delivered only if the corresponding state is realized.

Once bundles have been chosen, markets close. Consumers then choose their preferred action a. At $t = 1$, the uncertainty is resolved, and consumption goods are eventually delivered according to purchases.

In order to define an equilibrium concept for this economy, we have to introduce an appropriate price system. Given the complete set of contingent-commodity markets, SL prices have to be quoted. Suppose for a moment that there is no hidden action. In this case, we know from Malinvaud (1972) that there exist restrictions on the price system such that a well defined equilibrium concept displaying satisfactory efficiency properties can be defined.

These restrictions essentially require the price of one unit of good l delivered in state s to be properly related to the actual probability of that state. In particular, suppose that p_l is the price of one unit of good l with delivery at $t = 1$ no matter what the state is. Following Malinvaud (1972), we may call it the price for *sure delivery* of good l.[6]

[6] For economies with symmetric information, Magill and Shafer (1992) provide a formal argument justifying the fact that commodity prices are state independent when there is only idiosyncratic uncertainty.

Given the price for sure delivery, and given that state s has probability π_s, we let the price at $t = 0$ of one unit of good l with delivery at $t = 1$ in state s be equal to $\pi_s p_l$.

The introduction of hidden action does not alter the above reasoning in an essential way. In this case π_s depends on a. Hence we shall naturally assume that the price of one unit of good l with delivery contingent on state s when action a is chosen is equal to $\pi_s(a) p_l$.

To complete the reasoning it is only required to take into account the appropriate level of action. The only possible candidate is the incentive compatible one, which satisfies:

$$a = \arg\max v(x_1, x_2, a) .$$

From the above expression it is apparent that the incentive compatible level of a is well defined only once the whole consumption bundle (x_1, x_2) is considered. Therefore the price of one unit of good l in state s cannot be quoted unless the consumption level of all the other goods is specified.

Moreover, the price one unit of good l in state s may well be different when consumed in conjunction with different quantities of other consumption goods. For this reason, the price system just constructed is sometime referred to as non-linear.

4.1. CM consumers' problem

From the above discussion, it follows that at $t = 0$ – taking $p \in \mathbb{R}^L_{++}$ as given – consumers choose (x_1, x_2, a) so as to solve the following:

$$\max_{x_1, x_2, a} \quad v(x_1, x_2, a)$$

$$\text{s.t.} \quad \pi_1(a)\,(p \cdot (x_1 - w_1)) + \pi_2(a)\,(p \cdot (x_2 - w_2)) \leqslant 0\,, \tag{2a}$$

$$a = \arg\max\, v(x_1, x_2, a)\,. \tag{2b}$$

In the above problem, (2a) is the budget constraint – a single inequality because of the market structure, while (2b) is the incentive compatibility constraint.

In what follows, $\mathcal{C}(p)$ will denote the set of (x_1, x_2, a) that satisfy (2a)–(2b).

4.2. CM Equilibrium

In a contingent market (CM) equilibrium, consumers optimize and markets clear. These properties are collected in the following:

Definition 4.1. *A CM equilibrium is (x, a, p) such that:*

1. $(x, a) = \arg\max \{v(x, a) \mid (x, a) \in \mathcal{C}(p)\}$,

2. $\pi_1(a)(x_1 - w_1) + \pi_2(a)(x_2 - w_2) = 0$.

The market clearing condition in the above definition is expressed in expected value because of the assumption of large number of consumers.

4.3. Decentralization in CM equilibrium

Given the extremely simplified economy we are considering, not only decentralization of CE allocations as CM equilibrium is not a problem, but actually more can be proved, as the following proposition shows:

Proposition 1. *Every CE allocation is a CM equilibrium allocation for some price vector p. Moreover, Every CM equilibrium allocation is a CE allocation.*

Proof. See the Appendix ∎

It should be apparent that the price vector referred to in the above proposition is nothing but the vector of multipliers for the feasibility constraint in the planner's problem.

The above proposition is particularly useful for our purpose because it implies that the set of CE allocations and that of CM equilibrium allocations coincide.

5. The financial market (FM) model

In this section, we assume there exists at $t = 0$ a complete set of financial markets, together with a complete set of spot markets opened at $t = 1$.

The hypothesis on the market structure implies that, at $t = 0$, instead of trading on contingent markets, consumers trade on financial markets so as to transfer income from one state to the other.

The financial asset traded at $t = 0$ may be equivalently interpreted as an insurance contract, or as a portfolio of two Arrow-securities, each paying one unit of account if and only if state s happens.

In what follows, $\tau = (\tau_1, \tau_2) \in \mathbb{R}^2$ will denote the state-contingent payoffs of the financial asset chosen by consumers.

After having bought the asset, consumers choose their preferred action. At $t = 1$, uncertainty resolves, and trading on spot markets takes place. Consumers use income obtained from endowments and asset payoff to buy goods with immediate delivery.

To define a suitable equilibrium concept for the economy just described, $S + L$ prices must be considered, respectively for the financial asset and the consumption goods.

Given the structure of trades, asset prices depend on the action chosen by consumers, which in turn depends on optimal consumption at $t = 1$. The incentive compatibility constraint must then be adapted to take into account the trading on spot markets.

5.1. FM consumers' problem

From the above discussion, it follows that at $t = 0$ – taking $p \in \mathbb{R}^L_{++}$ as given – consumers choose (x, a, τ) so as to solve:

$$\max_{x_1, x_2, a, \tau_1, \tau_2} \quad v(x_1, x_2, a)$$

$$\text{s.t.} \quad \pi_1(a)\tau_1 + \pi_2(a)\tau_2 \leqslant 0\,, \tag{3a}$$

$$p \cdot (x_1 - w_1) \leqslant \tau_1\,, \tag{3b}$$

$$p \cdot (x_2 - w_2) \leqslant \tau_2\,, \tag{3c}$$

$$a = \arg\max \{v(x_1, x_2, a)\,, \tag{3d}$$

$$x_1 = \arg\max \{u(x_1, a) \,|\, p \cdot (x_1 - w_1) \leqslant \tau_1\}\,, \tag{3e}$$

$$x_2 = \arg\max \{u(x_2, a) \,|\, p \cdot (x_2 - w_2) \leqslant \tau_2\}\,. \tag{3f}$$

In the above problem, (3a) is the budget constraint for the financial asset, with prices actuarially fair given the action chosen, while (3b) and (3c) are the budget constraints for the consumption goods. As for (3d)–(3f), they constitute the (extended) incentive compatibility constraint. They imply, in particular, that the action a must be optimal given that consumption bundles are themselves optimal at $t = 1$.

In what follows, $\mathcal{D}(p)$ will denote the set of (x, a, τ) that satisfy (3a)–(3f).

5.2. FM equilibrium

In an FM equilibrium consumers optimize and markets clear. These properties are summarized in the following:

Definition 5.1. *An FM equilibrium is* (x, a, τ, p) *such that:*

1. $(x, a, \tau) = \arg\max \{v(x, a) \,|\, (x, a, \tau) \in \mathcal{D}(p)\}$,

2. $\pi_1(a)(x_1 - w_1) + \pi_2(a)(x_2 - w_2) = 0$.

We remark that the market clearing condition in above definition is expressed in expected value because of the assumption of large number of consumers.[7]

5.3. Decentralization in FM equilibrium

In this section we tackle our main question, for we study whether it is possible to decentralize a CE allocation as a FM equilibrium. Given proposition 1, we can answer this question using CM equilibria.

Indeed – given a CM equilibrium (x, a, p) – we can construct transfers $(\tau_1, \tau_2) := (p \cdot (x_1 - e_1), p \cdot (x_2 - e_2))$ and let the tuple (x, a, τ, p) be an *extended contingent markets* (ECM) equilibrium. The problem then amounts to understanding if an ECM equilibrium is also an FM equilibrium.

At first glance, there seems to be an easy way to conclude that this is indeed the case. Inspection of choice sets $\mathcal{C}(p)$ and $\mathcal{D}(p)$ reveals that $(x, a, \tau) \in \mathcal{D}(p)$ implies $(x, a) \in \mathcal{C}(p)$. Therefore, one may reason as follows: take an ECM $(\hat{x}, \hat{a}, \hat{\tau}, \hat{p})$. Provided that $(\hat{x}, \hat{a}, \hat{\tau}) \in \mathcal{D}(\hat{p})$, if $(\hat{x}, \hat{a}, \hat{\tau})$ were *not* an FM equilibrium, there would exist $(\bar{x}, \bar{a}, \bar{\tau}) \in \mathcal{D}(\hat{p})$ such that $v(\bar{x}, \bar{a}) > v(\hat{x}, \hat{a})$. Since $(\bar{x}, \bar{a}) \in \mathcal{C}(\hat{p})$, it follows that $(\hat{x}, \hat{a})$ cannot be a CM equilibrium. Hence we shall conclude that every ECM is also an FM equilibrium.

Yet this reasoning holds provided that $(\hat{x}, \hat{a}, \hat{\tau}) \in \mathcal{D}(\hat{p})$. The relevant question then is whether an ECM equilibrium does indeed belong to the choice set of the FM consumers' problem.

Inspection of $\mathcal{D}(p)$ reveals that it is possible that this condition is *not* satisfied. Equations (3e) and (3f) are the crux of the problem. They require $\hat{x}$ be optimal at $t = 1$ – when trading on consumption goods takes place – given prices $\hat{p}$, transfers $\hat{\tau}$ and the action $\hat{a}$ chosen at $t = 0$.

If there is only one consumption good, these equations do not add any relevant constraint on FM equilibrium, since there is no trading at $t = 1$. If preferences are separable, the hidden action does not affect ex post utility, hence even in this case we may suspect that these equations do not impose any further constraint on FM equilibrium. In this two cases, we therefore expect the above inclusion to be satisfied.

When preferences are not separable, consumers who are given enough income to buy $\hat{x}$ may still want to choose a different consumption bundle. This happens when $\hat{x}$ does not satisfy (3e)–(3f), where a is taken as fixed. In this case, there exists a budget-feasible $\tilde{x}$ that gives higher utility in at least one state s. Yet there

[7] In the interpretation of τ as a portfolio of Arrow-securities, the market clearing condition on the asset market is automatically satisfied at the optimum.

is no guarantee that $(\tilde{x}, \hat{a})$ satisfies the incentive compatibility constraint in the CM consumer's problem.

While these observations highlight the main intuition behind the impossibility for an ECM equilibrium to also be an FM equilibrium, in order to verify it we shall compare the first order conditions characterizing the two type of equilibria.

6. Characterization of CM and FM equilibrium

In order to characterize the CM and the FM equilibrium we first derive an appropriate reduced-form consumers' problem for each type of equilibrium. In this way, we can get simpler and easily comparable first order conditions.

We begin with the CM consumer's problem. In this case we simply replace the incentive compatibility constraint (2b) with the corresponding first order condition:

$$\partial_a \, v(x_1, x_2, a) = 0 \, .$$

It follows that the reduced-form consumers' problem ($\mathrm{CM}_{\mathtt{rf}}$) is given by the following:

$$\begin{aligned} \max_{x_1,\, x_2,\, a} \quad & v(x_1, x_2, a) \\ \text{s.t.} \quad \pi_1(a)\,(p \cdot (x_1 - w_1)) + \pi_2(a)\,(p \cdot (x_2 - w_2)) \; & \leqslant \; 0\,, && \text{(4a)} \\ \partial_a \, v(x_1, x_2, a) \; & = \; 0\,. && \text{(4b)} \end{aligned}$$

The derivation of the reduced form consumers' problem for the FM equilibrium is more involved, and it is presented in detail in the Appendix. Here we simply sketch the main idea to get it, which amounts to solving the problem backwards, starting from $t = 1$. At this date, constraints (3e) and (3f) require (x_1, x_2) to be optimal given prices p and the choices of a and τ made at $t = 0$. We let $x_s(\tau, a)$ for $s = 1, 2$ denote the optimal choices of consumption goods.[8] Given these demands, it is possible to show that the reduced form consumers' problem ($\mathrm{FM}_{\mathtt{rf}}$) is given by:

$$\begin{aligned} \max_{\tau, a} \quad & v(x_1(\tau, a), x_2(\tau, a), a) \\ \text{s.t.} \quad \pi_1(a)\,(p \cdot (x_1(\tau, a) - w_1)) + \pi_2(a)\,(p \cdot (x_2(\tau, a) - w_2)) \; & \leqslant \; 0\,, && \text{(5a)} \\ \partial_a v(x_1(\tau, a), x_2(\tau, a), a) \; & = \; 0\,. && \text{(5b)} \end{aligned}$$

[8] For simplicity, we neglect dependance on prices, which are not choice variables.

This problem has a similar structure to the previous one, the main differences being that admissible consumption bundles satisfy (3e)−(3f) and that choice variables are reduced to a and τ .

6.1. First order conditions for CM equilibrium

In this section we derive the equations that characterize the CM equilibrium. From definition 4.1 it is clear that they amount to the first order conditions for the CM consumers' problem augmented by market clearing conditions.

Using the $\mathrm{CM}_{\mathtt{rf}}$ consumers' problem, we can easily derive the former set of equations, which is given by:[9]

$$\partial_{x_1} v - \gamma \pi_1 p - \mu \partial_{a x_1} v \;=\; 0\,, \tag{6a}$$

$$\partial_{x_2} v - \gamma \pi_2 p - \mu \partial_{a x_2} v \;=\; 0\,, \tag{6b}$$

$$\gamma \sigma + \mu \partial_{aa} v \;=\; 0\,, \tag{6c}$$

where γ is the multiplier for constraint (4a), μ is the multiplier for constraint (4b) and:[10]

$$\sigma \;:=\; \partial_a \pi_1 \left(p \cdot (x_1 - w_1) \right) + \partial_a \pi_2 \left(p \cdot (x_2 - w_2) \right).$$

We remark that (6a)−(6b) are $2L$ equations, while (6c) is a single equation. Together with market clearing conditions and the constraints in the $\mathrm{CM}_{\mathtt{rf}}$ consumers' problem, they characterize the CM equilibrium.

6.2. First order conditions for FM equilibrium

In this section we derive the equations that characterize the FM equilibrium. From definition 5.1 it is clear that they amount to the first order conditions for the FM consumers' problem augmented by market clearing conditions.

As before, we shall use the $\mathrm{FM}_{\mathtt{rf}}$ consumers' problem to derive the relevant set of first order conditions. At this point, there is an important *caveat*: the $\mathrm{FM}_{\mathtt{rf}}$ problem is expressed in terms of a and τ, and so are the corresponding first order conditions, while the set of first order condition for the $\mathrm{CM}_{\mathtt{rf}}$ problem is expressed in terms of a and x .

[9] To save on notation, we write $\partial_{ax_1} v$ for $\partial_{x_1} \partial_a v$, and π_s for $\pi_s(a)$. Moreover, we omit the first order conditions relative to the multipliers.

[10] In deriving (6c) we have used (4b).

To make the two sets comparable, we need to express the former in terms of the same variables as the latter. This transformation is indeed possible, as shown in Appendix, and it implies that the set of first order conditions is given by:[11]

$$\partial_{x_1} v - \gamma \pi_1 p - \mu \partial_{ax_1} v + \psi_1 = 0\,, \tag{7a}$$

$$\partial_{x_2} v - \gamma \pi_2 p - \mu \partial_{ax_2} v + \psi_2 = 0\,, \tag{7b}$$

$$\gamma \sigma + \mu \partial_{aa} v + \psi_a = 0\,, \tag{7c}$$

where γ is the multiplier for constraint (5a), μ is the multiplier for constraint (5b), σ is defined as before, and the ψ-terms are defined as follows:

$$\psi_s := \mu \pi_s \left(\partial_{ax_s} u - \left(\partial_{ax_s} u \cdot \partial_{\tau_s} x_s \right) p \right)\,, \tag{8a}$$

$$\psi_a := \mu \left(\pi_1 \left(\partial_{ax_1} u \cdot \partial_a x_1 \right) + \pi_2 \left(\partial_{ax_2} u \cdot \partial_a x_2 \right) \right)\,. \tag{8b}$$

We observe that (7a)$-$(7b) are $2L$ equations, while (7c) is a single equation. Together with market clearing conditions and the constraints in the $\mathrm{FM}_{\mathtt{rf}}$ consumers' problem, they characterize the FM equilibrium.

6.3. Comparing first order conditions

In this section, we investigate when (7a)$-$(7c) are equal to (6a)$-$(6c), for when this is the case, we can conclude that an ECM equilibrium $(\hat{x}, \hat{a}, \hat{\tau}, \hat{p})$ is also a FM equilibrium.

Since the two sets of equations differ only in the ψ-terms, it is necessary to study when $\psi_s = 0$ and $\psi_a = 0$ when evaluated at $(\hat{x}, \hat{a}, \hat{\tau}, \hat{p})$. From (8a) and (8b), we know this happens if and only if the following system of $2L + 1$ equations is satisfied:

$$\partial_{ax_1} u - \left(\partial_{ax_1} u \cdot \partial_{\tau_1} x_1 \right) p = 0\,, \tag{9a}$$

$$\partial_{ax_2} u - \left(\partial_{ax_2} u \cdot \partial_{\tau_2} x_2 \right) p = 0\,, \tag{9b}$$

$$\pi_1 \left(\partial_{ax_1} u \cdot \partial_a x_1 \right) + \pi_2 \left(\partial_{ax_2} u \cdot \partial_a x_2 \right) = 0\,. \tag{9c}$$

In what follows, we let $\Gamma(x, a, \tau, p)$ denote the above system. Our objective is to study the condition for which $\Gamma(x, a, \tau, p) = 0$.

[11] As before, we omit the first order conditions relative to the multipliers.

When there is a single consumption good or when preferences are separable, we easily verify that $\Gamma(x, a, \tau, p) \equiv 0$.

Indeed, if there is a single consumption good, p can be normalized to one, $\partial_{\tau_s} x_s \equiv 1$ and $\partial_a x_s \equiv 0$, so that the above system is identically satisfied.[12]

If preferences are separable, then $\partial_{ax_s} u \equiv 0$, so that the above system is again identically satisfied.

In these two cases, the equations characterizing the CM and the FM equilibrium are identical. Hence, we shall conclude that every ECM equilibrium is also a FM equilibrium, thus verifying the intuition in section 5.3.

The following proposition collects these findings:

Proposition 2. *Suppose there is single consumption good or assumption S holds. Then every ECM equilibrium is an FM equilibrium.*

We now consider the possibility that $\Gamma(\hat{x}, \hat{a}, \hat{\tau}, \hat{p}) \neq 0$, so that an ECM equilibrium is *not* an FM equilibrium. If we let $\zeta_s := (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s)$, we notice that (9a)−(9b) imply that:

$$\partial_{ax_1} u(x_1, a) \quad = \quad \zeta_1 p\,, \tag{10a}$$

$$\partial_{ax_2} u(x_2, a) \quad = \quad \zeta_2 p\,. \tag{10b}$$

According to (10a)−(10b), for the system (9a)−(9c) to be satisfied it is necessary that $\partial_{ax_s} u$ and p are collinear, with scale factor $\zeta_s \neq 0$, when evaluated at the ECM equilibrium. If this condition is violated, the system cannot be satisfied, and an ECM equilibrium cannot be an FM equilibrium. The following proposition formalizes this intuition:

Proposition 3. *Suppose there are multiple consumption goods and preferences are non-separable. Provided that $\partial_{ax_s} u(\hat{x}_s, \hat{a}) \neq \zeta_s \hat{p}$ for $\zeta_s \neq 0$, then an ECM equilibrium $(\hat{x}, \hat{a}, \hat{\tau}, \hat{p})$ is not an FM equilibrium.*

As a final step, we verify that the condition in the above proposition is not empty:

Proposition 4. *Suppose there are multiple consumption goods and preferences satisfy assumption H. Then an ECM equilibrium $(\hat{x}, \hat{a}, \hat{\tau}, \hat{p})$ is not an FM equilibrium.*

Proof. The proof consists in verifying that $\partial_{ax_s} u(\hat{x}_s, \hat{a}) \neq \zeta_s \hat{p}$ for $\zeta_s \neq 0$. When assumption H holds, the (Bernoulli) utility function satisfies:

[12] Notice that in this case $\partial_{ax_s} v$ is a scalar.

$$\partial_{ax_{sj}} u \equiv 0\,, \quad \text{and} \quad \partial_{ax_{si}} u = \partial_{ax_{si}} g(x_s^{-j}, a) \quad \text{for} \quad i \neq j\,.$$

Hence $\partial_{ax_s} u(\hat{x}, \hat{a}) = \zeta_s \hat{p}$ for $\zeta_s \neq 0$ implies:

$$0 = \zeta_1 \hat{p}_j\,, \tag{11a}$$

$$\partial_{ax_{1i}} g(\hat{x}_1^{-j}, \hat{a}) = \zeta_1 \hat{p}_i \quad \text{for} \quad i \neq j\,, \tag{11b}$$

$$0 = \zeta_2 \hat{p}_j\,, \tag{11c}$$

$$\partial_{ax_{2i}} g(\hat{x}_2^{-j}, \hat{a}) = \zeta_2 \hat{p}_i \quad \text{for} \quad i \neq j\,. \tag{11d}$$

As the above system is inconsistent, we conclude that $\partial_{ax_s} u(\hat{x}, \hat{a}) \neq \zeta_s \hat{p}$. ■

A further intuition for the result in proposition 3 is obtained by comparing the restrictions on the gradient of the (Bernoulli) utility index u both in the FM and in the CM equilibrium. According to (3e) and (3f), in an FM equilibrium it is required that $\partial_{x_s} u$ and p are collinear. On the other hand, we show in the Appendix that, in a CM equilibrium, $\partial_{x_s} u$ must satisfy the following equation:

$$\left(\frac{\pi_s - \mu \partial_a \pi_s}{\pi_s} \right) \partial_{x_s} u = \gamma p + \mu \partial_{ax_s} u\,. \tag{12}$$

According to (12), the same collinearity restriction is satisfied in a CM equilibrium provided that assumption S holds, or that preferences are non-separable and $\partial_{ax_s} u = \zeta_s p$ for $\zeta_s \neq 0$.

Finally, we shall comment on the possibility of extending the result in proposition 3 beyond a specific assumption in proposition 4. Indeed, the result in proposition 3 holds whenever the system of equations defining the CM equilibrium augmented by the equations $\partial_{ax_s} u(x, a) = \zeta_s p$ is *not* satisfied. Since the original system has as many equations as unknowns, by adding $2L$ equations, but only 2 unknowns (ζ_1, ζ_2) , we obtain a system with more equations than unknowns. Hence, we expect that in general it will not be satisfied.

7. Related literature

The idea of the CM equilibrium dates back to the seminal paper of Prescott and Townsend (1984). Formally, their model is rather different from ours. To avoid non-convexities both in the utility functions − due to a discrete action set − and in the budget constraints, lotteries on consumption goods are introduced, and consumers are allowed to choose among them. While the objects of trades are different, the market structure and the nature of the price system are analogous to those considered here.

Rustichini and Siconolfi (2003) have recently revised the model of Prescott and Townsend, generalizing the conditions for existence and optimality of equilibrium.

Closely related to the above ones is the model of Jerez (2005), who imposes the relevant incentive compatibility constraint on firm offering insurance contracts, instead of consumers, and obtains a result analogous to proposition 1.

In the above models, the only hidden action consumers make is choosing a. The relevance of this assumption for decentralizing constrained efficient allocations has been stressed in a series of papers by Arnott and Stiglitz (1986), Greenwald and Stiglitz (1986) and Arnott et al. (1992). Using a model of optimal taxation, with the planner redistributing income through insurance contracts assigned at $t = 0$ and taxing trades on spot commodity markets, these authors show that there are cases in which taxes and/or subsidies are actually required at the optimum. In particular, Arnott and Stiglitz (1983) provide an example showing that one of these cases arises if there are multiple consumption goods and preferences satisfy assumption H.

Yet differences in modeling choices made the connection between those two strands of literature non-straightforward. As a matter of fact, the explicit comparison of the CM with the FM equilibrium is an attempt to clarify that connection, and in particular the importance of differences in trade structures.

When there are multiple consumption goods, the FM model presented here is closely related to that of Lisboa (2001), who studies a case with separable preferences and obtains an efficiency result that can be reconciled with proposition 2.

When there is a single consumption good, differences between the CM and the FM model fade out, and so does the relevance of controlling ex post trades. Yet for decentralization to occur it is still required that ex ante trades are observable. Under this assumption, Helpman and Laffont (1975) and Kocherlakota (1998) obtain an efficiency result that can be reconciled with proposition 2.

The assumption of observability of ex ante trades is related to the possibility of enforcing exclusivity clauses in insurance contracts. Incidentally, we always assume that ex ante trades are observable, and that exclusivity covenants can be enforced at no cost. For when this assumption is removed, it is well known that in general it is not possible to decentralize constrained efficient allocations (see for example Arnott and Stiglitz (1988), Arnott and Stiglitz (1991a), Arnott and Stiglitz (1991b), Bisin and Guaitoli (2004), Bizer and DeMarzo (1992), Hellwig (1983) and Kahn and Mookherjee (1998)).

Finally, we notice that both the CM and the FM model have a common predecessor in Malinvaud (1972), who considers only the case of symmetric information, and generalizes the fair-price system to the multi-commodity.

Appendix

• Proof of Proposition 1

For the first part of the proposition, take a CE allocation (x, a). If it not a CM equilibrium allocation, then there exists another feasible and incentive compatible $(\bar{x}, \bar{a}) \in \mathcal{B}$ such that $v(\bar{x}, \bar{a}) > v(x, a)$. But this contradicts the fact that (x, a) was a CE allocation.[13] For the second part of the proposition, take a CM equilibrium allocation (x, a). If it is not a CE allocation, then there exists another (budget) feasible and incentive compatible $(\bar{x}, \bar{a}) \in \mathcal{C}(p)$, such that $v(\bar{x}, \bar{a}) > v(x, a)$. But this contradicts the fact that (x, a) was a CM equilibrium allocation.

• Reduced-form FM consumers' problem

Starting with the constraints relevant at $t = 1$, we notice that (3e) and (3f) imply that an admissible x_s satisfies:

$$\max_{x_s} u(x_s, a) \quad \text{s.t.} \quad p \cdot (x_s - e_s) - \tau_s \leqslant 0\,.$$

Let $x_s(\tau, a)$ denote the solution of the above problem.[14] It is characterized by the following equations:

$$\partial_{x_s} u - \lambda_s p \;=\; 0\,, \tag{A.1a}$$

$$p \cdot (x_s - e_s) - \tau_s \;=\; 0\,. \tag{A.1b}$$

Substitute $x_s(\tau, a)$ in the FM consumers' problem and rewrite it as follows:[15]

$$\begin{aligned} \max_{\tau,a} \quad & v(x_1(\tau, a), x_2(\tau, a), a) \\ \text{s.t.} \quad & \pi_1(a)\,(p \cdot (x_1(\tau, a) - w_1)) + \pi_2(a)\,(p \cdot (x_2(\tau, a) - w_2)) \leqslant 0\,, \\ & a = \arg\max v(x_1(\tau, a), x_2(\tau, a), a)\,. \end{aligned} \tag{A.2a}$$

Replace (A.2a) with the corresponding first order condition:

$$\partial_{x_1} v \cdot \partial_a x_1 + \partial_{x_2} v \cdot \partial_a x_2 + \partial_a v = 0\,. \tag{A.3}$$

[13] This argument assumes that a CM equilibrium exists.

[14] It is apparent from $(A.1)$ that consumption in state s depends only on the transfer in state s. To save on notation, we shall write $x_s(\tau, a)$ instead of $x_s(\tau_s, a)$. Moreover, since prices are always taken as given, we do not explicitly write them as an argument of the demand function.

[15] Here (A.1b) has been used to substitute for τ_s in (3a).

Since (A.1a)−(A.1b) imply that:

$$\partial_{x_s} v \cdot \partial_a x_s = \pi_s \partial_{x_s} u \cdot \partial_a x_s = \pi_s \lambda_s (p \cdot \partial_a x_s) = 0\,,$$

(A.3) reduces to:

$$\partial_a v(x_1(\tau, a), x_2(\tau, a), a) = 0\,, \tag{A.4}$$

so that the $\mathrm{FM}_{\mathtt{rf}}$ consumer's problem is given by the following:

$$\begin{aligned} \max_{\tau, a} \quad & v(x_1(\tau, a), x_2(\tau, a), a) \\ \text{s.t.} \quad \pi_1(a)\,(p \cdot (x_1(\tau, a) - w_1)) + \pi_2(a)\,(p \cdot (x_2(\tau, a) - w_2)) & \leqslant 0\,, \\ \partial_a v(x_1(\tau, a), x_2(\tau, a), a) & = 0\,. \end{aligned}$$

• Derivation of equations (7a)−(7c)

An interior solution of the $\mathrm{FM}_{\mathtt{rf}}$ problem is characterized by the following equations:[16]

$$\pi_1 \lambda_1 - \gamma \pi_1 - \mu\,(\partial_{a x_1} v \cdot \partial_{\tau_1} x_1) = 0\,, \tag{A.6a}$$

$$\pi_2 \lambda_2 - \gamma \pi_2 - \mu\,(\partial_{a x_2} v \cdot \partial_{\tau_2} x_2) = 0\,, \tag{A.6b}$$

$$\gamma \sigma + \mu \partial_{aa} v + \mu\,(\partial_{a x_1} v \cdot \partial_a x_1 + \partial_{a x_2} v \cdot \partial_a x_2) = 0\,, \tag{A.6c}$$

$$\pi_1\,(p \cdot (x_1 - w_1)) + \pi_2\,(p \cdot (x_2 - w_2)) = 0\,, \tag{A.6d}$$

$$\partial_a\, v(x_1, x_2, a) = 0\,, \tag{A.6e}$$

where γ is the multiplier for the first constraint, μ is the multiplier for the second constraint and:

$$\sigma \ := \ \partial_a \pi_1\,(p \cdot (x_1 - w_1)) + \partial_a \pi_2\,(p \cdot (x_2 - w_2))\,.$$

Multiply (A.6a)−(A.6b) by p and use (A.1a) to get the following system of equations:

[16] In deriving (A.6a)−(A.6b) we used (A.1a) and the fact that $\partial_{\tau_2} x_1 = \partial_{\tau_1} x_2 = 0$, while in deriving (A.6c) we used the fact that $p \cdot \partial_a x_s = 0$.

$$\partial_{x_1} v - \gamma \pi_1 p - \mu \left(\partial_{ax_1} v \cdot \partial_{\tau_1} x_1\right) p \;=\; 0\,, \tag{A.7a}$$

$$\partial_{x_2} v - \gamma \pi_2 p - \mu \left(\partial_{ax_2} v \cdot \partial_{\tau_2} x_2\right) p \;=\; 0\,, \tag{A.7b}$$

$$\gamma \sigma + \mu \partial_{aa} v + \mu \left(\partial_{ax_1} v \cdot \partial_a x_1 + \partial_{ax_2} v \cdot \partial_a x_2\right) \;=\; 0\,, \tag{A.7c}$$

$$\pi_1 \left(p \cdot (x_1 - w_1)\right) + \pi_2 \left(p \cdot (x_2 - w_2)\right) \;=\; 0\,, \tag{A.7d}$$

$$\partial_a\, v(x_1, x_2, a) \;=\; 0\,. \tag{A.7e}$$

We now manipulate the last addends in (A.7a)−(A.7c) so as to get the expressions in the text. Recall that:

$$\partial_{ax_s} v = \partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u\,.$$

Since $p \cdot \partial_{\tau_s} x_s = 1$, we get the following chain of equalities:

$$\begin{aligned}
\left(\partial_{ax_s} v \cdot \partial_{\tau_s} x_s\right) p &= \left(\left(\partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u\right) \cdot \partial_{\tau_s} x_s\right) p \\
&= \left(\partial_a \pi_s \left(\partial_{x_s} u \cdot \partial_{\tau_s} x_s\right) + \pi_s \left(\partial_{ax_s} u \cdot \partial_{\tau_s} x_s\right)\right) p \\
&= \left(\partial_a \pi_s \lambda_s \left(p \cdot \partial_{\tau_s} x_s\right) + \pi_s \left(\partial_{ax_s} u \cdot \partial_{\tau_s} x_s\right)\right) p \\
&= \partial_a \pi_s \lambda_s p + \pi_s \left(\partial_{ax_s} u \cdot \partial_{\tau_s} x_s\right) p \\
&= \partial_a \pi_s \partial_{x_s} u + \pi_s \left(\partial_{ax_s} u \cdot \partial_{\tau_s} x_s\right) p \\
&= \partial_{ax_s} v - \pi_s \partial_{ax_s} u + \pi_s \left(\partial_{ax_s} u \cdot \partial_{\tau_s} x_s\right) p \\
&= \partial_{ax_s} v - \pi_s \left(\partial_{ax_s} u - \left(\partial_{ax_s} u \cdot \partial_{\tau_s} x_s\right) p\right)\,.
\end{aligned} \tag{A.8}$$

Since $p \cdot \partial_a x_s = 0$, we also get the following chain of equalities:

$$\begin{aligned}
\left(\partial_{ax_s} v \cdot \partial_a x_s\right) &= \left(\partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u\right) \cdot \partial_a x_s \\
&= \partial_a \pi_s \left(\partial_{x_s} u \cdot \partial_a x_s\right) + \pi_s \left(\partial_{ax_s} u \cdot \partial_a x_s\right) \\
&= \partial_a \pi_s \lambda_s \left(p \cdot \partial_a x_s\right) + \pi_s \left(\partial_{ax_s} u \cdot \partial_a x_s\right) \\
&= \pi_s \left(\partial_{ax_s} u \cdot \partial_a x_s\right)\,.
\end{aligned} \tag{A.9}$$

Substituting (A.8) and (A.9) in (A.7a)−(A.7c) we get equations (7a)−(7c).

• Derivation of equation (12)

Using (4b), direct calculations give:

$$\partial_{ax_s} v = \partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u \, .$$

Substitute the above equation in (6a)−(6b) and collect terms to get:

$$\left(\frac{\pi_s - \mu \partial_a \pi_s}{\pi_s} \right) \partial_{x_s} u = \gamma p + \mu \partial_{ax_s} u \, , \tag{A.10}$$

which is equation (12) in the text.

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