

# Why does overnight liquidity cost more than intraday liquidity? \*

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March 20, 2007

## Abstract

In this paper, we argue that the observed difference in the cost of intraday and overnight liquidity is part of an optimal payments system design. In our environment, the interest charged on overnight liquidity affects output while the cost of intraday liquidity only affects the distribution of resources between money holders and non-money holders. The low cost of intraday liquidity follows from the Friedman rule and it is optimal to deviate from the the Friedman rule with respect to overnight liquidity. The cost differential simultaneously reduces the incentive to overuse money and encourages risk sharing.

**Keywords:** Friedman rule; Overnight; Intraday; Liquidity.

**JEL classification:** E31; E51; E58.

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\*Iowa State University, University of Missouri, and Federal Reserve Bank of New York, respectively. This work was inspired by discussions with Narayana Kocherlakota. We also thank seminar participants at University College-Dublin and University of Reading. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of New York or the Federal Reserve System.

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# 1 Introduction

Banks routinely face mismatches between the timing of receipts and payments in a day. To smooth out the flow of payments between banks, central banks often provide intraday credit to banks.<sup>1</sup> They also offer overnight liquidity to banks who end a day with inadequate reserves to cover their payments. A key puzzle in the economics of payments is the large difference between the cost of intraday and overnight liquidity. Typically, the cost of intraday liquidity is very close to zero while the interest rate charged on overnight liquidity is in the 3-6% range. In this paper, we provide one possible argument for why such a cost-spread may be part of an optimal money and payments system design. Indeed, we argue that intraday (within a period) liquidity should come at a low cost because it has a low social opportunity cost, an idea originally due to Friedman (1969). In contrast, overnight (across periods) liquidity should be expensive because its overuse causes a reduction in output.

We study a random-relocation economy similar to Champ, Smith, and Williamson (1996) but augmented to include features from Freeman (1996) and Mills (2004, forthcoming). The setting is a two-period lived overlapping generations model where limited communication and random relocation create an endogenous transactions role for fiat money.<sup>2</sup> Agents are *ex ante* identical. Before the end of their first period of life, a fraction of agents receives a ‘liquidity shock’. Agents who receive the shock are relocated (henceforth “early movers”) and the only asset they can carry with them is money. Agents who are not relocated when young may possibly be relocated at the beginning of their second period of life (henceforth “late movers”). Agents can exert costly effort to reduce the probability of being an early mover but the fraction of late movers is exogenously given. Money competes as an asset with a linear storage technology that transforms period  $t$  goods into

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<sup>1</sup>See Ennis and Weinberg (2007) for a survey of the issues.

<sup>2</sup>Economies with spatial separation and limited communication were first studied by Townsend (1980, 1987).

period  $t + 1$  goods. The return on storage is negatively affected by the fraction of early movers. In contrast, the fraction of late movers does not affect the return on storage. This is the key distinction between intraday and overnight liquidity. It captures the idea that intraday liquidity is used mainly for settlement while central banks often use the overnight rate to steer the economy.

In this environment a planner would like to distort the consumption of early movers but not of late movers (who are similar to non-movers from the perspective of the planner). The planner wants to provide incentives for young agents to exert effort so as to reduce the fraction of early movers thereby improving the return to storage and hence output. It is possible to decentralize the planner's allocation by setting a high cost of liquidity for early movers and no cost of liquidity for late movers.

The main insight is that the cost of liquidity should be related to its role in payments or as a substitute for other assets that contribute to output. If liquidity is used only to make payments, then it should have a very low cost so as to share risk between ex-ante identical agents. If liquidity can be used as a substitute for other assets, then it should have a high cost to reduce the incentive to overuse it.

The payments literature has provided several explanations for the low cost of intraday liquidity. Angelini (1998), and Bech and Garratt (2003) argue that a high cost of intraday liquidity can lead to costly delay of payments. Zhou (2000) and Martin (2004) argue that a low cost of liquidity provides risk-sharing when payments needs are random. In all of these models, the focus is on the cost of intraday liquidity rather than on the relationship between intraday and overnight liquidity.

In contrast to these earlier papers, we consider an environment in which costly overnight liquidity is necessary to obtain the planner's allocation. In such an environment, the fact that intraday liquidity should have a low cost naturally arises as an application of the Friedman rule; simply put, at a zero rate the social cost of using money is equal to the opportunity cost of holding money. For this reason, our paper is also related to the

literature concerned with the optimality of the Friedman rule.<sup>3</sup> In our model, deviating from the Friedman rule is desirable because money is overused if it is “too good an asset”. Specifically, excessive use of money (“too many movers”) reduces output.<sup>4</sup>

To highlight the role of the different elements of our model, we introduce them one at a time. Section 2 describes the basic features of the physical environment. In section 3 we assume that the effort exerted by agents is observable. In this case, the planner does not need to distort consumption to achieve the desired level of effort. The planner’s allocation can be decentralized with a low cost for overnight liquidity. Section 4 considers the case where effort is unobservable. With hidden effort, the planner chooses to distort consumption to provide incentives for agents to exert effort. The planner’s allocation can be decentralized if the cost of overnight liquidity is high. Finally, in section 5 we introduce intraday liquidity in the model and show that its cost should be zero.

## 2 The environment

We consider a random relocation model in the style of Townsend (1987) augmented to include an effort decision. There is an infinite sequence of time periods. Dates are indexed by  $t = \dots - 1, 0, 1, \dots$ <sup>5</sup> The world is divided into two spatially separated locations. Each location is populated by a continuum of overlapping generations of agents of unit mass. Agents live for two periods. There is a single, perishable consumption good.

The economy’s key friction is a physical restriction preventing the consumption good from moving between the two locations. We further assume that claims on one island’s consumption good are not transferable to the other island. In contrast, some agents may

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<sup>3</sup>See Bhattacharya, Haslag, and Martin (2005) and the references therein.

<sup>4</sup>Our paper is also related to recent attempts to study optimal monetary policy in environments with hidden effort such as da Costa and Werning (2003) and Golosov, Kocherlakota, and Tsyvinski (2003).

<sup>5</sup>It is standard in this literature to have an initial period and an initial old generation but to ignore the welfare of that initial generation. To simplify the exposition we choose instead to have no initial period. Bhattacharya, Haslag, and Martin (2006) contrast economies with or without an initial date.

have to move between islands. Young agents are all identical *ex ante* but face uncertainty regarding where they will spend their old age. Let  $\pi_i$  denote the probability that the  $i$ th young agent will be relocated to the other island. We denote by  $\Pi$  the aggregate fraction of young agents who are relocated. Since all agents  $i$  are *ex ante* identical, we can write  $\pi_i = \pi \forall i$ , provided all agents exert the same amount of effort. Under certain regularity conditions, the law of large numbers applies and the measure of agents relocated will be equal to the probability that one agent will be relocated; that is,  $\pi = \Pi$ . The identity of those who are relocated is revealed before the end of the period. The two islands are symmetric so that the flow of relocated agents to and from an island offset each other, leaving the population unchanged. We refer to relocated agents as movers and the others as non-movers.

## 2.1 Endowments and technology

At each date a young agent is endowed with  $\omega$  units of the single consumption good. In addition, each young agent is endowed with one unit of time. Old agents receive no endowment. The young agent can divide their time between leisure and effort, denoted by  $l_t$  and  $e_t$ , respectively, and  $l_t + e_t = 1$ .

Effort by agent  $i$  affects the probability that a person will be relocated; we posit the function  $\pi_i(e)$  satisfies  $\pi'_i < 0$ ,  $\pi''_i > 0$ ,  $\pi_i(0) = \bar{\pi} < 1$ , and  $\pi_i(1) = \underline{\pi} \geq 0$ . Also,  $\lim_{e_i \rightarrow 0} \pi'_i = -\infty$  and  $\lim_{e_i \rightarrow 1} \pi'_i = 0$ . Hence, as effort by the  $i$ th agent approaches zero, the marginal effect on the probability of being relocated is greatest. As effort approaches its maximum level, the effect on that probability vanishes.

There is a storage technology on each island transforming date  $t$  goods into date  $t + 1$  goods. We assume that the storage technology requires monitoring by young agents. Monitoring is costless but needs to occur later than the time at which early movers are relocated. Hence, the return on storage is negatively related to the fraction of movers. If no one moves, the storage technology yields a fixed real gross return of  $x > 1$ . More generally

though, we assume this return is given by  $x [1 - \Pi(1 - \phi)] > 1$  where  $\phi$  is the fraction of  $x$  lost because some agents had to move. The return of each individual project thus depends on the fraction of movers on the island. The higher the mass of movers, the lower is the island's average return.<sup>6</sup>

It is important to note that any individual agent behaves as if he can influence (via effort) his own probability of relocation but not the return to the storage technology (which depends on the aggregate value  $\Pi$ ). So, the representative agents takes  $\Pi$  as given, ignoring the impact that the agent, who is atomistic, has on the social return.

As an alternative to the standard relocation shock assumption, we could have assumed that agents are subject to enforcement shocks which force them to use cash. See Kahn and Roberds (forthcoming) for an environment in which the use of cash is motivated by enforcement shocks.

## 2.2 Preferences

Agents derive utility from old-age consumption and from enjoying leisure when young. Utility is additively separable in consumption and leisure. Let  $c_t$  denote old-age consumption enjoyed by members of the generation born at date  $t$ ; utility derived from the consumption good is represented by the function  $U(c_t)$ . We assume that  $U(\cdot)$  is twice continuously differentiable and strictly concave; formally,  $U' > 0$  and  $U'' < 0$ . Let  $V(1 - e_t)$  characterize the function mapping leisure into utility, where we apply the constraint on time with  $l_t = 1 - e_t$ . We assume  $V(\cdot)$  is increasing and strictly concave. In other words  $V' \geq 0$ ,  $V'' < 0$ ,  $\lim_{l \rightarrow 0} V' = \infty$ , and  $\lim_{l \rightarrow 1} V' = 0$ .

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<sup>6</sup>A potentially useful parable is that the endowment is in units of apples. Let the storage technology be interpreted as the apple trees. So a young agent can take apple seeds to turn into apples next period as the mature tree yields fruit when old. The social return owes to each agent maintaining bees to pollinate the trees. Movers cannot maintain their beehives. Consequently, a measure of beehives fail to pollinate the island's trees, lowering the aggregate return per tree. In this way, the return per unit of the consumption good has a social aspect. We simply assume that the island's trees are identical for a given measure of functioning beehives.

The time line is as follows: An agent is born, receiving an endowment. Next, the storage and effort decisions must be made followed by the relocation announcement. Movers are permitted to create claims on the deeds to their storage and are relocated. The period ends. In the following period, agents who are now old consume.

### 3 An economy with observable effort

As a benchmark we study the case where effort is observable. We show that in this case the planner wants to equalize consumption between movers and non-movers. Then we show how the planner's allocation can be decentralized.

#### 3.1 The planning problem

We derive the allocation chosen by a planner who can observe the effort exerted by agents. Since all generations are identical, the planner seeks to maximize the expected utility of a young agent in a representative generation.

Since goods cannot move between islands, the planner must distribute the goods available on an island to the agents present. At the planner's disposal are the goods stored on behalf of the date  $t$  generation, denoted by  $s_t$ , and the goods available from the date  $t + 1$  generation's endowment. We focus on stationary allocations so the amount of goods of the date  $t + 1$  generation's endowment that is stored,  $s_{t+1}$ , must be equal to  $s_t$ .

The planner takes into account the fact that agents' efforts affect the probability with which they are relocated and, indirectly, the return on investment. Since effort is observable, the planner can punish agents who do not exert the desired amount of effort. We assume that the punishment can be severe enough so that no agent will unilaterally deviate from the effort prescribed by the planner.

The planner's problem is given by

$$\max_{c_t^m, c_t^n, e_t, s_t} \Pi(e)U(c_t^m) + [1 - \Pi(e)]U(c_t^n) + V(1 - e_t), \quad (1)$$

subject to

$$\Pi(e_t)c_t^m + [1 - \Pi(e_t)]c_t^n = x [1 - \Pi(e_t)(1 - \phi)] s_t + \omega_{t+1} - s_{t+1}, \quad (2)$$

$$s_t \leq \omega_t, \quad (3)$$

where  $c_t^m$  denotes the quantity of the consumption good allocated to movers,  $c_t^n$  denotes the quantity of the consumption good allocated to non-movers. We let  $\lambda_t$  and  $\mu_t$  denote the Lagrange multipliers associated with the first and second constraints, respectively.

The first-order conditions for the planner's problem are given by

$$\Pi(e_t)U'(c_t^m) - \lambda_t\Pi(e_t) = 0, \quad (4)$$

$$[1 - \Pi(e_t)]U'(c_t^n) - \lambda_t[1 - \Pi(e_t)] = 0, \quad (5)$$

$$\Pi'(e_t)[U(c_t^m) - U(c_t^n)] - V'(1 - e_t) - \lambda_t\{\Pi'(e_t)[x(1 - \phi)s_t - c_t^m + c_t^n]\} = 0, \quad (6)$$

$$\lambda_t[x(1 - \Pi(e_t)(1 - \phi))] - \mu_t - \lambda_{t+1} = 0. \quad (7)$$

All constraints hold with equality because the planner chooses positive levels of consumption, effort, and storage. In a stationary allocation, since  $s_{t+1} = s_t$  and  $x[1 - \Pi(e_t)(1 - \phi)] > 1$ , the planner wants to store as many goods as possible. It follows that  $s_{t+1} = s_t = \omega_t$ . In effect, the planner stores all the endowment goods of the young, and distributes the stored goods from the previous generation between old movers and old non-movers on the island.

Equations (4) and (5) imply that  $U'(c_t^m) = \lambda_t = U'(c_t^n)$ . This implies  $c_t^m = c_t^n = c_t$ . In other words, the planner provides the same amount of consumption to movers and non-movers and the risk of relocation is shared perfectly.

With  $c_t^m = c_t^n = c_t$  and  $s_t = \omega$ , equation (6) simplifies to

$$V'(1 - e_t) = -U'(x[1 - \Pi(e_t)(1 - \phi)]\omega)\Pi'(e_t)[x(1 - \phi)\omega]. \quad (8)$$

The LHS of equation (8) is positive and increases from 0 to infinity as effort goes from 0 to 1. Since  $\Pi' < 0$ , the RHS of equation (8) is positive. Hence there will be an interior solution for the effort level. A sufficient condition for the effort level to be unique is that

the RHS of equation (8) be decreasing. The planner trades off between the marginal cost of the effort to the individual agent versus the marginal benefit in terms of additional resources available to the society.

Let  $\{c^{m*}, c^{n*}, e^*, s^*\}$  denote the planner's allocation. We can summarize these results in the following proposition.

**Proposition 1** *The planner stores all available goods,  $s^* = \omega$ , and gives the same consumption to movers and non-movers,  $c^{m*} = c^{n*}$ . The planner's choice of effort,  $e^*$ , is determined by equation (8).*

### 3.2 Decentralizing the planning solution

In this section, we study *one* possible way in which the planner's allocation can be decentralized. We assume the existence of a public sector, a joint government and central bank, which we call the CB. The CB can issue fiat money at no cost and can choose prices at which it buys and sells goods using that money. We associate the liquidity provided by the CB in this case with overnight liquidity. The CB redistributes any profits it makes in a lump-sum fashion. Since effort is observable, we also assume that the CB can punish agents who do not choose the level of effort consistent with the planner's allocation. The CB is also subject to the physical restriction that prevents goods from moving across islands even as cash remains transportable across the two islands.

The timing goes as follows: First, young agents receive their endowment and invest it in the storage technology. They must also decide how much effort to exert. After the effort decision is made, agents learn whether they must relocate. The CB opens and agents can exchange fiat money for the claims on stored goods they hold. Recall that claims cannot be accepted on the other island. Then movers relocate. In their second period of life, nonmovers consume the goods to which they have a claim. Movers buy goods from the CB with the money they hold.

Note that the CB does not need to keep track of agents from one island to another.

While the CB operates on both islands, the branch on one island does not need to know whom the branch on the other island has traded with. Also, in contrast to the standard random relocation model, there is no need for banks in this economy. It is enough for the central bank to buy and sell goods, as we show below.<sup>7</sup>

Let  $\bar{m}_t$  denote the amount of money given to agents in exchange for  $q_t$  goods. Let  $v_t$  denote the units of goods per unit of money (the inverse of the price level). It follows that  $q_t = v_t \bar{m}_t$  where  $\bar{m}_t$  and  $v_t$  are both choice variables of the CB. We assume that the CB buys and sells goods at the same price. Formally, at each date, the CB in any island receives goods from young movers leaving that island worth  $v_t \bar{m}_t$ . The CB also repays goods to old movers (those who have moved from the other island) worth  $v_t \bar{m}_{t-1}$ . The difference, denoted by  $T_t$  is distributed in lump-sum fashion to young agents:

$$v_t \bar{m}_t - v_t \bar{m}_{t-1} = T_t. \tag{9}$$

Each unit of good held by the CB at date  $t - 1$  turns into  $x [1 - \Pi(1 - \phi)]$  units of goods at date  $t$ .

An equilibrium in this economy is a choice of consumption, effort, and storage that maximizes the agent's problem, described below, taking the policy of the CB as given. The objective of the CB is to maximize agents' expected utility.

Young non-movers cannot consume any more by obtaining cash from the CB. For this reason, we assume that non-movers do not exchange claims for money at the CB even if they are indifferent between doing so or not. Young movers cannot do worse than getting money from the CB since they would consume nothing in the absence of the cash.<sup>8</sup>

All agents choose effort  $e_i = e^*$  since the CB can punish agents who do not choose the level or effort consistent with the planner's allocation. Let  $\Pi^* \equiv \Pi(e^*)$ ; also note

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<sup>7</sup>See Schreft and Smith (1997) for an exposition of a standard random relocation model. See Haslag and Martin (forthcoming) for more details on the role of the central bank in providing liquidity.

<sup>8</sup>One interpretation for this arrangement is that the CB makes discount window loans to agents, as in Antinolfi and Keister (2006) or Haslag and Martin (forthcoming). However, our arrangement requires less information as the CB does not need to keep track of moving agents, as noted above.

that  $\pi_i(e_i) = \Pi(e^*)$ . The representative young person born at date  $t$  solves the following problem:

$$\max_{c_t^m, c_t^n, s_t} \Pi^* U(c_t^m) + (1 - \Pi^*) U(c_t^n) + V(1 - e_t^*), \quad (10)$$

subject to

$$s_t \leq \omega + T_t, \quad (11)$$

$$c_t^n \leq x [1 - \Pi^*(1 - \phi)] s_t, \quad (12)$$

$$v_t \bar{m}_t \leq s_t, \quad (13)$$

$$c_t^m \leq v_{t+1} \bar{m}_t. \quad (14)$$

First note that any agent takes the aggregate probability of relocation as given and not something they can influence. Equation (11) states that agents cannot store more than their endowment and the transfer from the CB. Equation (12) indicates that the consumption of non-movers cannot exceed the return on their storage. Equation (13) tells us that the real money balances received from the CB cannot exceed the goods that are offered in exchange. Finally, equation (14) states that the consumption of movers is bounded by the amount of goods obtained in exchange for money from the CB.

**Proposition 2** *An equilibrium with perfect risk sharing—that is,  $c^{m*} = c^{n**}$ —can be implemented by the CB setting the return to money equal to the return to storage.*

**Proof.** If the money supply is held constant,  $T$  must be equal to zero. Since consumption increases in  $s_t$ , agents choose  $s_t = \omega$ . Consumption of movers and non-movers will thus be equal if and only if

$$\frac{v_{t+1}}{v_t} = x [1 - \Pi(1 - \phi)].$$

This means that the return on money between periods is equal to the return on storage.

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In other words, this is the Friedman rule. By implementing the Friedman rule, the CB is able to obtain the efficient allocation. In this economy,  $v_t$  is a policy variable and not a market determined price level. This is because money does not circulate between agents. We thus have a model of “intermediated money” since money only circulates between the CB and the agents. In particular, the gross steady-state return on money is not equal to one. Martin and Haslag (forthcoming) show in a related framework, that this economy is the limiting case of an economy in which the money circulate between agents and in which the CB implements the Friedman rule.

Thus far, we have shown that when effort choice is observable and enforceable, a central bank can replicate the first best by implementing the Friedman rule (setting a zero opportunity cost for overnight liquidity). To foreshadow, below we show that when effort choice is unobservable, deviating from the Friedman rule will be called for.

## 4 An economy with hidden effort

In the remainder of this paper, we assume that an agent’s choice of effort is unobservable. First, we study an agent’s effort choice. Next, we characterize the planner’s allocation. Finally, we show how the planner’s allocation can be decentralized.

### 4.1 Effort choice

First, we consider the choice of effort for an agent who takes  $c^m$  and  $c^n$  parametrically. The individual  $i$ ’s problem can be written as

$$\max_e \pi(e)U(c^m) + [1 - \pi(e)]U(c^n) + V(1 - e). \quad (15)$$

Taking the partial derivative of the objective function with respect to  $e$  and setting it to zero yields

$$\pi'(e) [U(c^m) - U(c^n)] - V'(1 - e) = 0. \quad (16)$$

Since  $V'(1) = 0$ , it is apparent from equation (16) that when  $c^n = c^m$ , the optimal choice of effort is  $e = 0$ . That is, an agent who is completely insured against the risk of relocation exerts no effort to reduce this risk. Effort will be positive only if  $c^m < c^n$ . Intuitively, only if movers receive less consumption than non-movers will agents try to avoid being relocated.

Since the effort level chosen by an agent depends on  $c^n$  and  $c^m$ , we can write  $e = e(c^m, c^n)$  by invoking the implicit function theorem.

**Lemma 1** *An agent's optimum effort is (i) decreasing in the quantity of consumption good allocated to movers; and (ii) increasing in the quantity of the consumption good allocated to non-movers.*

**Proof.** After substituting the  $e(c^m, c^n)$  into the first-order condition and differentiating, we get

$$\pi''[U(c^m) - U(c^n)]de + V''de + \pi'U'dc^m = 0.$$

After collecting terms and rearranging, we can write

$$\frac{de}{dc^m} = -\frac{\pi'U'}{\pi''[U(c^m) - U(c^n)] + V''}.$$

As usual, the denominator is the second-order condition. If  $\pi'' > 0$  and  $V'' < 0$ , the sign of the denominator is negative, ensuring that the second-order condition is satisfied. In other words, the value of  $e$  that satisfies the first-order condition is indeed a maximum.<sup>9</sup> With  $\pi' < 0$  and positive marginal utility, the implication is that  $\frac{\partial e}{\partial c^m} < 0$ . An increase in the quantity of the consumption given to movers will result in less effort by the consumer. As

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<sup>9</sup>So,  $\pi'' > 0$  corresponds to a case in which the impact of effort on the probability of moving is getting algebraically bigger; that is, a smaller negative number.

movers get more of the consumption, holding everything else constant, there is less need to use effort to try to indirectly insure themselves against moving. It is straightforward to show that  $\frac{\partial e}{\partial c^n} > 0$  along the same lines as above. ■

## 4.2 The planner's problem

The planner chooses how much of the available goods to save and how much to allocate between movers and nonmovers so as to maximize the expected utility of a representative generation. Physically, the planner collects endowments from the young on each island. These goods can be consumed by agents alive in that period or invested in the storage technology. The planner also collects goods that were invested last period. The planner is aware of the effect movers have on the return to the technology. Since the planner can identify movers and non-movers, each type may receive different quantities of goods.

Recall that the planner's problem is

$$\max_{c_t^m, c_t^n, s_t} \Pi(e_t)U(c_t^m) + [1 - \Pi(e_t)]U(c_t^n) + V(1 - e_t),$$

subject to

$$\begin{aligned} \Pi(e_t)c_t^m + [1 - \Pi(e_t)]c_t^n &= x [1 - \Pi(e_t)(1 - \phi)] s_t + \omega_{t+1} - s_{t+1}, \\ s_t &\leq \omega_t. \end{aligned}$$

In a steady state,  $s_t = s_{t+1} = s \leq \omega$ . The constraints can thus be combined into

$$\Pi(e)c^m + [1 - \Pi(e)]c^n = x [1 - \Pi(e)(1 - \phi)]\omega. \quad (17)$$

We can substitute the effort function from the previous section,  $e_t = e(c_t^m, c_t^n)$ , and take the partial derivatives with respect to  $c^m$  and  $c^n$  to get two first-order conditions:

$$e_m \Pi' [U(c^m) - U(c^n) - \lambda(c^m - c^n + x(1 - \phi)\omega)] - e_m V' + \Pi(e) [U'(c^m) - \lambda] = 0,$$

and

$$e_n \Pi' [U(c^m) - U(c^n) - \lambda(c^m - c^n + x(1 - \phi)\omega)] - e_n V' + (1 - \Pi(e)) [U'(c^n) - \lambda] = 0,$$

where  $\lambda$  is the Lagrange multiplier on constraint (17) and  $e_n \equiv \frac{\partial e(c^m, c^n)}{\partial c^m}$  and  $e_m \equiv \frac{\partial e(c^m, c^n)}{\partial c^n}$ .

Solve both equations for  $\lambda$  to get

$$\frac{e_m \Pi' [U(c^m) - U(c^n)] - e_m V' + \Pi(e) U'(c^m)}{e_m \Pi' [c^m - c^n + x(1 - \phi)\omega] + \Pi(e)} = \lambda,$$

and

$$\frac{e_n \Pi' [U(c^m) - U(c^n)] - e_n V' + (1 - \Pi(e)) U'(c^n)}{e_n \Pi' [c^m - c^n + x(1 - \phi)\omega] + (1 - \Pi(e))} = \lambda.$$

We can eliminate  $\lambda$  and obtain one equation in  $c^n$  and  $c^m$ . Let  $\Delta_1 \equiv e_m \Pi' [U(c^m) - U(c^n)] - e_m V' + \Pi(e) U'(c^m)$ ,  $\Delta_2 \equiv e_m \Pi' [c^m - c^n + x(1 - \phi)\omega] + \Pi(e)$ ,  $\Gamma_1 \equiv e_n \Pi' [U(c^m) - U(c^n)] - e_n V' + (1 - \Pi(e)) U'(c^n)$ , and  $\Gamma_2 \equiv e_n \Pi' [c^m - c^n + x(1 - \phi)\omega] + (1 - \Pi(e))$ , then it is easily checked that  $\frac{\Delta_1}{\Delta_2} = \frac{\Gamma_1}{\Gamma_2}$ . The planner's resource constraint gives us a second equation in  $c^n$  and  $c^m$  so we can solve for these two unknowns. Let  $c^{m**}$  and  $c^{n**}$  denote the level of consumption chosen by the planner. Next we can solve for the effort level:  $e^{**} = e(c^{m**}, c^{n**})$ .

**Proposition 3** *The planner gives less consumption to movers than non-movers:  $c^{m**} < c^{n**}$ .*

**Proof.** Suppose, instead, that the planner chooses  $c^{m**} = c^{n**} = c$ . We can rearrange the expression  $\frac{\Delta_1}{\Delta_2} = \frac{\Gamma_1}{\Gamma_2}$ , to get

$$[x(1 - \phi)\omega] U'(c) \Pi' = V'. \tag{18}$$

The left hand side of this expression is strictly positive. If  $c^{m**} = c^{n**} = c$ , consumers exert no effort and  $V'(1) = 0$ , so equation (18) cannot hold. We know from lemma 1 that some effort will be exerted if  $c^{n**}$  is increased,  $c^{m**}$  is decreased, or both. ■

The intuition is that the planner wants agents to exert some effort to reduce their chance of relocation. To incentivize agents, he makes relocation a costly activity.

Note that the level of effort desired by the planner is different when effort is hidden than when effort is observable; in fact,  $e^{**} < e^*$ . When effort is hidden, the planner chooses

lower effort because the benefit from increasing effort must be traded-off with the cost of distorting consumption. In other words, if the planner were to choose  $e^{**} = e^*$ , the cost of a marginal decrease in effort would be of second order while the benefit of improving risk sharing between movers and nonmovers would be of first order.

It is clear from our specification of the planner's problem that hidden effort and the cost imposed by movers in terms of lost return to storage are *both* important features of the model economy. Together, these features can account for why the planner does not want to offer perfect insurance to agents in this economy. If movers did not impose a cost on the economy, the planner would not be concerned with reducing the mass of movers. It would then be optimal to provide equal consumption to all agents since they are risk-averse and ex-ante identical. Instead, if movers imposed a cost but the probability of moving was exogenous, then the planner would again choose to provide equal consumption to all agents since it would not be possible to mitigate the cost imposed by movers.

### 4.3 The decentralized economy with hidden effort

With hidden effort, the CB can no longer punish agents directly for not choosing the appropriate level of effort. Agents will now need to be provided the right incentives before they exert the level of effort chosen by the planner,  $e^{**}$ .

The representative young agent born at date  $t$  solves the following problem:

$$\max_{c^m, c^n, s} \pi(e(c^m, c^n))U(c_t^m) + (1 - \pi(e(c^m, c^n)))U(c^n) + V[1 - e(c^m, c^n)], \quad (19)$$

subject to constraints (11), (12), (13), and (14). Combining the last three equations, we can write

$$\frac{c^m}{c^n} = \frac{v_{t+1}}{v_t} \frac{1}{x[1 - \Pi(1 - \phi)]}.$$

The CB can obtain any ratio  $c^m/c^n$  it desires by choosing  $v_{t+1}$  and  $v_t$  appropriately. Clearly, for  $v_t/v_{t+1} = x[1 - \Pi(1 - \phi)]$ , consumption smoothing arises in the decentralized

economy. As  $v_t/v_{t+1}$  increases, the ratio  $c^m/c^n$  decreases, since the rate of return of money decreases. Creating a wedge between the returns to the two assets and making relocation more costly stimulates agents to provide more effort. To replicate the planning solution, the CB chooses  $v_t/v_{t+1}$  in order to set

$$\frac{c^m}{c^n} = \frac{c^{m**}}{c^{n**}},$$

which in turn is consistent with agents choosing effort equal to  $e^{**}$ .

Deviating from the Friedman rule works particularly well in this environment because only movers use money. However, we can show that our results extend to an environment where it is difficult to distinguish different types of agents.

Our model explains why a central bank would want to deviate from the Friedman rule. In order to implement the efficient allocation, the central bank equates the social cost of using money with opportunity cost of holding money. In this setup, both are positive. Thus, we can account for why the overnight rate on borrowing money is positive. Next, we account for why intraday rates will be zero.

## 5 Late movers and intraday liquidity

It is well documented that the liquidity provided by central banks overnight is considerably more costly than the liquidity provided intraday.<sup>10</sup> Our model can help shed some light on this pattern. Indeed, as we argue below, such a pattern may well be part of an optimal payments system design.

In order to have both intraday and overnight liquidity provision, we need to modify the pattern of relocations slightly in a manner analogous to Freeman (1996) and Mills (2004, forthcoming). As before, there is a probability  $\Pi$  that a young agent must relocate, and  $\Pi$  is still a function of the agent's effort. We call such agents *early-movers*. Agents who are not relocated when young may be relocated at the beginning of their *second* period of

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<sup>10</sup>See Bech and Garratt (2003) and Zhou (2000) for a discussion.

life with probability  $\alpha$ . We call such agents *late-movers*. As above,  $\alpha$  is also the fraction of relocated old agents. For simplicity, we make  $\alpha$  exogenous implying that effort has no effect on  $\alpha$ .

We assume that late-movers have to leave their island before previously stored goods in that island pay off. This is to motivate a need for money on their part. (Recall that in any case, goods cannot move across locations.) In contrast to early-movers, however, late-movers do not affect the return on the storage technology as the benefits from their presence when young have already realized. This distinction between intraday and overnight liquidity is key. It can also be related to the fact that overnight liquidity is used to transfer resources intertemporally while intraday liquidity is not.

The expected utility of a young agent in this economy can be written as

$$\pi(e)U(c_t^{em}) + [1 - \pi(e)] \left[ \alpha U(c_t^{lm}) + (1 - \alpha) U(c_t^n) \right] + V(1 - e_t), \quad (20)$$

where  $c_t^{em}$  is the quantity of the good consumed by the early mover,  $c_t^{lm}$  is the quantity of the good consumed by a late mover, and  $c_t^n$  is the quantity consumed by a non-mover. Henceforth, the superscripts *em* and *lm* will stand for early and late movers respectively.

## 5.1 The case of observable effort

When effort is observable, it is straightforward to see that the planner will choose  $c_t^{em} = c_t^{lm} = c_t^n$ . The amount of storage and even the effort are the same as those chosen in section 3.1. With observable effort, the planner does not need to distort the agents' consumption, and hence chooses to insure them perfectly against the risk of relocation.

As in section 4.3, we assume that the CB can impose a punishment high enough so that agents choose effort  $e^*$ . The representative young agent solves

$$\max_{c_t^{em}, c_t^{lm}, c_t^n, s_t} \Pi^* U(c_t^{em}) + (1 - \Pi^*) \left[ \alpha U(c_t^{lm}) + (1 - \alpha) U(c_t^n) \right] + V(1 - e^*),$$

subject to

$$s_t \leq \omega + T_t, \tag{21}$$

$$c_t^n \leq x [1 - \Pi^*(1 - \phi)] s_t, \tag{22}$$

$$v_t \bar{m}_t^{em} \leq s_t, \tag{23}$$

$$c_t^{em} \leq v_{t+1} \bar{m}_t^{em}, \tag{24}$$

$$v_{t+1} \bar{m}_t^{lm} \leq x [1 - \Pi^*(1 - \phi)] s_t, \tag{25}$$

$$c_t^{lm} \leq v_{t+1} \bar{m}_t^{lm}. \tag{26}$$

All constraints hold with equality. The last two constraints combined, together with constraint (22), show that  $c_t^{lm} = c_t^n$ . Then by the same argument as in section 3.2, the CB can set  $c_t^{lm} = c_t^{em} = c_t^n$  by choosing

$$\frac{v_{t+1}}{v_t} = x [1 - \Pi^*(1 - \phi)].$$

The Friedman rule can replicate the planner's allocation in this environment. Under the Friedman rule, the real cost of liquidity to agents is zero, whether money is needed intraday or overnight.

## 5.2 The case of hidden effort

From the perspective of the planner, late-movers and non-movers are equivalent. Neither type has a negative effect on the return to the storage technology and, conditional on not being an early-mover, the probability of being a non-mover is exogenous. Hence, inspection of the expected utility function given by expression (20) reveals that the planner will choose to set  $c_t^{lm} = c_t^n$ . With  $c_t^{lm} = c_t^n$  the planner's problem is the same as it was in section 4. In particular, lemma 1 and proposition 3 both hold.

We can now turn to the problem faced by a CB in a decentralized economy. An representative agent chooses  $c_t^n, c_t^{em}, c_t^{lm}, e_t$ , and  $s_t$  to maximize

$$\pi U(c_t^{em}) + (1 - \pi) \left[ \alpha U(c_t^{lm}) + (1 - \alpha) U(c_t^n) \right] + V(1 - e),$$

subject to constraints (21),(22), (23), (24), (25), and (26).

**Proposition 4** *The planner gives less consumption to early movers than to late movers and non-movers:  $c^{em**} < c^{n**} = c^{lm**}$ .*

**Proof.** Combining equations (25) and (26), at equality, we get  $c^{lm} = x [1 - \Pi(1 - \phi)] s_t = c^n$ . With  $c^{lm} = c^n$ , this problem is equivalent to the problem of section 4.3. The CB can achieve the planner's allocation by deviating from the Friedman rule. ■

In contrast to the previous section, the real cost of liquidity is not the same intraday and overnight. Intraday, the cost of liquidity is zero since late-movers and non-movers enjoy the same consumption. The real cost of overnight liquidity is strictly positive since  $c_t^{em} < c_t^n$ .<sup>11</sup>

Note that the CB could make  $c^{lm} \neq c^n$ . For example, if the price at which goods are sold is different from the price at which goods are bought in the same period. But the CB prefers to set  $c^{lm} = c^n$ . By buying and selling goods at the same price, the CB's interactions with late-movers resemble a loan at an interest rate of zero.

This model suggests an efficiency reason for why central banks provide intraday and overnight liquidity at very different costs. Intraday, the need for liquidity is only related to making payments and has no effect on aggregate real activity. Its only role is to allocate available resources. In contrast, the cost of overnight liquidity has implications for aggregate real activity in our model.

From the perspective of young agents, money and storage are two ways of transferring resources between periods. When money is as good an asset as storage, agents have no

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<sup>11</sup>It is possible to show that the main insight of this section does not depend on the planner and the CB being able to perfectly discriminate between early and late movers. In Bhattacharya, Haslag and Martin (2007), we extend the analysis to consider a case in which movers are indistinguishable. The impact on the risk-sharing allocation is indeterminate. On the one hand the planner may not want to impose as much consumption inequality because it hurts late movers. On the other hand, because a given amount of consumption inequality provides fewer incentives, the planner may want to impose more consumption inequality. This resembles the standard trade-off between a 'substitution' and an 'income' effect.

incentives to avoid holding money. In our environment, this reduces the return on storage because there are too many movers. The CB can make money a less attractive asset by deviating from the Friedman rule. As money becomes a worse alternative to storage, agents have more incentives to avoid having to hold money. This is beneficial as it increases the return on storage.

Since the planner wants overnight liquidity to be more costly because early movers reduce the return on storage while late movers do not, one may think that our results could be overturned by simply assuming that late movers reduce the return on storage while early movers do not. This is not the case, however, because any agent can pretend to be early movers while early movers cannot pretend to be late movers. If the CB sets the cost of overnight liquidity at zero and attempts to set a positive price for intraday liquidity, all agents have an incentive to pretend to be early movers.

## 6 Conclusion

In this paper, we propose a new explanation for the puzzling difference between the costs of overnight and intraday liquidity. We argue that the low cost of intraday liquidity is simply an application of the Friedman rule in an environment where a deviation of the Friedman rule is optimal with respect to overnight liquidity.

Our model has a number of desirable features: The cost of overnight liquidity can affect output so there is a role for a high cost of overnight liquidity. Agents can choose the composition of their asset portfolio between money and storage and, if money is ‘too good an asset,’ money is overused. Intraday liquidity is not used to transfer goods intertemporally, but only to make payments. With these features, we show that a central bank can implement a planner’s allocation by setting a high cost to overnight liquidity and a low cost to intraday liquidity.

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