On multiple-principal multiple-agent models of moral hazard *

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Abstract

In multiple-principal multiple-agent models of moral hazard, we provide sufficient conditions for the outcomes of pure-strategy equilibria in direct mechanisms to be preserved when principals can offer indirect communication schemes. The conditions include strong robustness in the direct mechanism game, as developed in the literature on competing mechanisms by Peters (2001) and Han (2007a), and a no-correlation property we define. We provide a rationale for restricting attention to take-it or leave-it offers, as is typically done in applications. We show via examples that it is necessary to allow direct mechanisms to be stochastic and to include private recommendations from principals to agents to preserve the corresponding equilibrium outcomes, and that the no-correlation condition is tight.

Key words: Moral hazard, multiple principal, multiple agent, direct mechanisms.
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1 Introduction

We consider multiple-principal, multiple-agent models of pure moral hazard. That is, principals compete through mechanisms in a scenario where there is complete information about the types of agents, but agents' effort is not contractible. Our goal is to establish conditions under which equilibria in which principals offer direct mechanisms are robust to the possibility that any principal may deviate to a richer communication scheme (i.e., an indirect mechanism) to interact with agents.

With multiple principals, it is well-established that there is a loss of generality in focusing on simple incentive compatible direct mechanisms (see, for example, Peck (1997), Martimort and Stole (2002), and Peters (2001)). That is, there exist equilibrium outcomes with richer communication schemes that are not replicable in direct mechanisms. In such contexts, it is important to understand whether there can be some rationale for the restriction to simple mechanisms which has typically been postulated in most of the competing principals literature.[1] It turns out (see Theorems 1 and 2 in Peters (2003)) that, whenever multiple principals interact in the presence of a single agent, pure strategy equilibria in direct mechanisms remain equilibria when richer communication schemes are feasible. We aim to provide a similar result for multiple-principal, multiple-agent games.

The papers cited in the previous paragraph all allow for the general case of incomplete information, where an important role of communication schemes is to allow an agent to convey private information to the principal. Our focus in this paper is only on complete information with non-contractible effort.

We extend the original Myerson (1982) framework to multiple-principal, multiple-agent models of moral hazard. We hence develop a model of competing mechanisms in the presence of many agents, where two-sided communication is considered. That is, principals' decisions are contingent on the messages sent by agents, and agents' choices depend on the information conveyed by the private recommendations each of them is receiving. At first glance, the idea of a communication scheme in a complete information setting may seem strange. It turns out that, by communicating privately

[1] See Peters (2003) and Han (2007a) for a discussion. If there is complete information and the effort is contractible, this restriction requires that each principal offers a single pay-for-effort contract to each agent.

with an agent, a principal can create asymmetric information across agents, before they choose effort.

Much applied work has been done on moral hazard with common agency (i.e., with multiple principals and a single agent).[2] More recently, the analysis of games where principals compete in the presence of many agents who take unobservable actions has been developed in several contexts. For example, the literature on two-sided markets analyzes a framework where two groups of agents interact through a finite number of platforms (principals). A relevant issue is given by those situations where each single agent can trade with more than one platform at a time, which is referred to as multi-homing (see, for example, Caillaud and Jullien (2003) and Armstrong (2006)). These models can be seen as applications of multiple-principal multiple-agent schemes of moral hazard by interpreting the agents' participation choices as non-contractible actions. This literature typically postulates the existence of a continuum of agents in every market, whereas we focus on games with a finite number of agents.

A model of competing principals with a finite number of agents who take a non-contractible action is developed by Ishiguro (2005). In his paper the principals offer take-it or leave-it contracts incorporating exclusivity clauses. Our setting is more general, since we do not restrict the participation decisions of the agents nor the communication strategies of the principals. Other applications of multi-principal multi-agent models of moral hazard are found in areas such as the analysis of network industries (Cremer, Rey, and Tirole (2000), Dogan (2005)) and political economy (see the survey work of Tabellini (2000)).

These papers restrict attention to a situation in which no form of communication between principals and agents is involved; in addition, principals propose take-it or leave-it offers. To provide a foundation for such a choice, we extend the the notion of strong robustness, introduced in Peters (2001) and Han (2007a), to the context of non-contractible effort.

An equilibrium of a given multiple-principal multiple-agent game is said to be strongly robust if there does not exist a continuation equilibrium in the agents' game that would induce a principal to deviate. In Section 3 we show that any pure strategy strongly robust equilibrium of a simple direct mechanisms game will survive the intro-

[2] See, for example, Kahn and Mookherjee (1998), Parlour and Rajan (2001), Bisin and Guaitoli (2004) and Attar, Campioni, and Piaser (2006).

duction of indirect communication schemes. In particular, it will also be strongly robust in the more complex game where principals can choose any indirect mechanism.

Much of the literature on competing mechanisms restricts attention to equilibria in which principals play pure strategies. Typically, attention is also focused on strongly robust equilibria. See Han (2007a) for a discussion on the multiple agent case, and Han (2007b) on the common agency (i.e., single agent) case.

We prove our result for the specific case where principals do not correlate their allocation decisions with the recommendations they send at equilibrium. This setting, though limited, is sufficient to establish the robustness of pure strategy equilibria in games without communication, that are mostly used in applications.[3] A corollary to our theorem provides a rationale for considering take-it or leave-it offers in multi-principal multi-agent games of moral hazard.

In Section 4, we show via an example that private communication with the agents allows a principal to achieve outcomes which would not be available otherwise. Hence, stochastic mechanisms and private recommendations are indispensable elements of our construction. We also provide an example to show that the no-correlation property is necessary for the theorem to go through. If a principal offers recommendations correlated with allocations, then another principal can strictly benefit from deviating to an indirect mechanism.

As yet, little is known about multiple-principal, multiple-agent models. The menu theorems of Martimort and Stole (2002) and Peters (2001) do not extend straightforwardly to a general multiple-principal setting.[4] The methodology proposed by Pavan and Calzolari (2006) has also not yet been extended to multiple-principal multiple-agent games. Our theorem represents one step forward toward a general characterization of equilibria in this framework. Our result, based on non-contractible effort, complements the work of Han (2007a), who considers complete information with contractible effort in a model similar to that of Prat and Rustichini (2003).

[3]For the most part, the theoretical literature on competing mechanisms also does not explicitly consider recommendations. Exceptions include Epstein and Peters (1999) who introduce private recommendations in a model without moral hazard, and Peters (2001) who argues that recommendations might have a role even in models with a single agent.

[4]Han (2006) extends the menu theorems to a restricted class of multiple-principal multiple-agent games, in which the contract between a principal and agent is essentially bilateral, and separate from the contract with any other principal or agent.

2 The Model

There are n principals dealing with k agents, where $n \geq 2$ and $k \geq 2$. Let Y_j be a set of deterministic allocations available to principal j, with typical element $y_j \in Y_j$. An allocation can be, for example, monetary transfers, tax rates, prices, or quantities, depending on the particular interpretation of the model. In our moral hazard framework, y_j can be interpreted as an incentive scheme, i.e. as a function of (say) profit or output, which is randomly distributed according to a distribution that depends on effort. In the game we study, each principal j chooses an allocation in the set $\Delta\left(Y_j\right)$, the set of lotteries that can be generated over the set of deterministic allocations Y_j.

There is complete information about agent types. Each agent i chooses an unobservable effort $e^i \in E^i$, where E^i is a finite set. Therefore, the model is one of pure moral hazard. We denote the vector of efforts as $e = \left(e^1, e^2, ..., e^k\right) \in E = \times_{i=1}^k E^i$.

We use the general communication structure for principal-agent models introduced by Myerson (1982). Each principal j chooses a message space M_j^i and a recommendation space R_j^i for each agent. To avoid measure-theoretic issues that arise with continuum spaces, we restrict M_j^i and R_j^i to be finite (possibly empty) for each i and j. We allow each principal to choose E^i as a possible recommendation space to communicate with agent i. Let $R_j = \times_{i=1}^k R_j^i$ denote the set of recommendations principal j can make, and $M_j = \times_{i=1}^k M_j^i$ the set of messages he can receive. The allocations and recommendations chosen by principal j depend on the messages received from the agents.

As in Myerson (1982), principal j's behavior is described by the choice rule $\pi_j : M_j \to \Delta\left(Y_j \times R_j\right)$. That is, principal j may choose a stochastic mechanism, which provides a lottery over allocations and recommendations for some message array m_j. When the choice rule π_j is stochastic, principal j chooses a realization from the lottery π_j, and communicates the realized recommendations r_j to the agents. Conditional on observing r_j^i, agent i updates her belief about the allocation y_j, but need not know the actual realization. Since recommendations are private, two agents i and i' may have different posterior beliefs about principal j's chosen allocation, y_j. Potentially, this allows a principal to induce a correlated equilibrium in the continuation game in which agents choose efforts.

A mechanism offered by principal j is thus given by $\gamma_j = \left(M_j, R_j, \pi_j\right)$. Mechanisms are publicly observed, but the message from agent i to principal j, and the recommendation from principal j to agent i, are observed only by i and j. As is usual in the literature, principals commit to their mechanisms before agents send messages.

There are two stages at which agent i moves in the game. First, she sends a message array $m^i = \left(m^i_1, \ldots, m^i_n\right)$ to the principals. Then, after observing only her private recommendations $r^i = \left(r^i_1, \ldots, r^i_n\right)$, she chooses an effort $e^i \in E^i$. Given the offered mechanisms, let $\mu^i \in \Delta\left(M^i\right)$ denote the message strategy of agent i, and let $\delta^i : M^i \times R^i \to \Delta\left(E^i\right)$ be her strategy in the effort game, where $M^i = \times_{j=1}^n M^i_j$ and where $R^i = \times_{j=1}^n R^i_j$.

Since each principal j commits to his mechanisms (including his strategy π_j) at the start of the game, agents' best responses will depend on the array of offered mechanisms $\gamma = (\gamma_1, \ldots, \gamma_n)$. Let $\beta^i = (\mu^i, \delta^i)$ represent agent i's strategy, with $\beta = (\beta^1, \ldots, \beta^k)$ denoting the joint strategy of the agents.

The time structure of the interaction is provided in Figure 1.

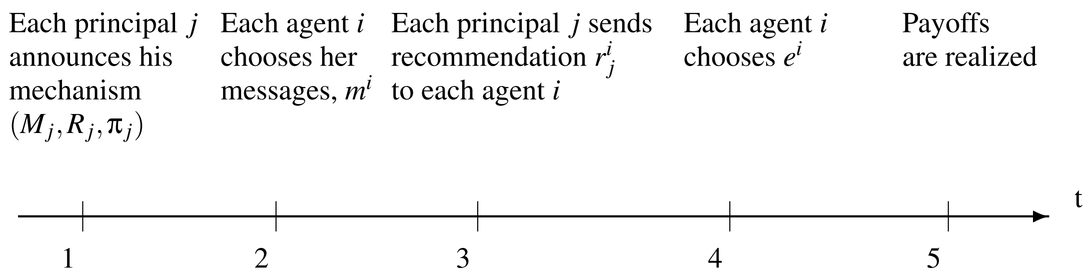

Figure 1: Timing of the generalized communication game

Agent i's payoff from a final outcome (y, e) is given by the von Neumann–Morgenstern utility function $U^i(y, e)$ and principal j's payoff is given by $V_j(y, e)$.[5] The mechanisms offered by principals, γ, and strategies played by agents, β, induce a distribution over the outcome space $Y \times E$. With a standard abuse of notation, let $U^i(\gamma, \beta)$ denote agent i's expected utility given γ and β, and let $V_j(\gamma, \beta)$ be principal j's expected utility.

In this complete information framework, a direct mechanism is defined as follows. Principals do not solicit messages from agents, and directly suggest the actions they should take. That is, $M^i_j = \emptyset$ and $R^i_j = E^i$ for every $j = 1, \ldots, n$ and for every $i = 1, \ldots, k$. Finally, $\pi_j \in \Delta\left(Y_j \times E\right)$. A mixed strategy for an agent in a direct mechanism is given

[5] If the allocation is an incentive scheme, hence y is a random variable with constant domain and a distribution that varies with effort, $U^i(y, e)$ and $V_j(y, e)$ must be thought of as expected utilities.

by $\delta^i : (E^i)^n \to \Delta(E^i)$. Any mechanism in which, for any principal j and any agent i, either $M^i_j \neq \emptyset$ or $R^i_j \neq E^i$, or both, is an indirect mechanism.[6]

3 Robustness

This section provides our result on the robustness of pure strategy equilibria of direct mechanism games to the introduction of communication. In our setting, principals are playing a game with each other, and their choices of mechanisms must correspond to a Nash equilibrium of this game. Further, agents' choices of messages and efforts must represent continuation equilibria, given the mechanisms chosen by the principals and recommendations received by the agents.

We first observe that, with multiple principals and stochastic mechanisms, incentive compatibility (in particular, agents obeying principals' recommendations) is a troublesome notion. An agent may be recommended different actions by different principals. For example, if two principals are both randomizing over recommendations, since principals choose their strategies independently, there is a strictly positive probability that an agent will receive different recommendations from the principals. Given this difficulty, we bypass the issue of agents obeying recommendations received from principals, and require only that, given the strategies of principals and other agents, agents play an equilibrium of the effort game.

Ex-ante, this is a complete information game: no participant has a non-trivial type. However, since agents receive private recommendations from principals, agents may have private information when they play the effort game. Hence, in the spirit of perfect Bayesian equilibrium, we require that each agent i plays a best response following any recommendation array $r^i = (r^i_1, \ldots, r^i_n)$ she may receive.

Recall that a mechanism offered by principal j is defined by (M_j, R_j, π_j). A direct mechanism is defined by $(\emptyset, E, \pi_j)$, where $\pi_j \in \Delta(Y_j \times E)$. Let $\Gamma_{\mathcal{D}}$ be the direct mechanism game among the principals. In this game, each principal j chooses a direct mechanism $\pi_j \in \Delta(Y_j \times E)$ at stage 1 (see Figure 1), and each agent i plays a strategy δ^i.

[6] Our definition of a direct mechanism is that employed by Myerson (1982), and matches the one employed in most of the literature on competing mechanisms. A different route is suggested in Epstein and Peters (1999), who include the communication about other principals' mechanisms in the set of messages available to each single agent. This general formulation, though, does not provide relevant insights on equilibrium characterization. For this reason, we adopt a more standard formulation.

Let $\Gamma_{\mathcal{G}}$ be the indirect mechanism game, in which each principal j chooses (M_j, R_j, π_j), where (with a slight abuse of notation) $\pi_j : M_j \to \Delta(Y_j \times R_j)$, and each agent i plays a strategy $\beta^i = (\mu^i, \delta^i)$.

In an equilibrium of either $\Gamma_{\mathcal{D}}$ or $\Gamma_{\mathcal{G}}$, we require that (i) each principal plays a best response, given other principals' strategies and agents' strategies, and (ii) each agent i plays a best response for every recommendation array r^i she may receive, given principals' strategies and other agents' strategies. As is usual in the literature, a mechanism (M_j, R_j, π_j) is referred to as a pure strategy for principal j (even though the choice rule π_j may provide a lottery over allocations and recommendations). A mixed strategy for principal j is then defined as a probability distribution over mechanisms. For convenience, we refer to an equilibrium of $\Gamma^{\mathcal{D}}$ or $\Gamma^{\mathcal{G}}$ in which principals play pure strategies as a pure strategy equilibrium.

The question we answer is the following: under what conditions does an equilibrium outcome of $\Gamma_{\mathcal{D}}$ survive the introduction of more complex communication mechanisms? In a related context, Han (2007a) identifies a set of pure strategy equilibria which satisfy this property. In particular, he introduces the notion of "strong" robustness arguing that for every equilibrium of Γ_D satisfying such a property there is an outcome equivalent equilibrium of Γ_M which also satisfies it. Importantly, he proves this statement in a scenario where agents' actions are contractible. We establish a similar result in a moral hazard context, where the impossibility to control agents' actions introduces and additional form of externalities.

Following Peters (2001) and Han (2007a), we define robustness of equilibria in a competing mechanism game with moral hazard as follows.

Definition 1 *(i) Let* (π^*, δ^*) *be an equilibrium of the direct mechanism game* $\Gamma^{\mathcal{D}}$. *The equilibrium is strongly robust if, for every principal j, every direct mechanism* $\tilde{\pi}_j$ *and every continuation equilibrium* $\tilde{\delta}$, $V_j(\pi^*, \delta^*) \geq V_j((\tilde{\pi}_j, \pi^*_{-j}), \tilde{\delta})$.
(ii) Let (γ^*, β^*) *be an equilibrium of the indirect mechanism game* $\Gamma^{\mathcal{G}}$. *The equilibrium is strongly robust if, for every principal j, every indirect mechanism* $\tilde{\gamma}_j$ *and every continuation equilibrium* $\tilde{\beta}$, $V_j(\gamma^*, \beta^*) \geq V_j((\tilde{\gamma}_j, \gamma^*_{-j}), \tilde{\beta})$.

In other words, an equilibrium of the direct (indirect) mechanism game is strongly robust if, regardless of the continuation equilibrium of the agents' effort game that follows, no principal j can improve his own payoff by unilaterally deviating to some other direct (indirect) mechanism.

When will an equilibrium of the direct mechanism game be strongly robust? First, suppose there is a single principal. Then, the optimal incentive compatible direct mechanism, and the associated continuation equilibrium in the agents' effort game, constitute a strongly robust equilibrium: by definition, there does not exist another incentive compatible direct mechanism and continuation equilibrium that leave the principal strictly better off. In this case, incentive compatibility on the agents' part can be characterized as agents obeying the recommendations received from the principal. With multiple principals, the concept of obedience is troublesome, since agents may receive different recommendations from different principals. Instead, we just require that agents play some continuation equilibrium in the effort game. Strong robustness requires that, even by inducing agents to play a continuation equilibrium most favorable to herself, no principal can gain from a unilateral deviation.

To prove our theorem, we introduce a further restriction on mechanisms: the private recommendations sent by a principal to agents must be uncorrelated with the corresponding allocations. We refer to this as the no-correlation property.

Definition 2 *In a direct mechanism, a strategy π_j of principal j has no correlation between recommendations and allocations if there exist marginal densities $\pi_{y_j} \in \Delta(Y_j)$ and $\pi_{e_j} \in \Delta(E)$ such that $\pi_j(y_j, e) = \pi_{y_j}(y_j)\, \pi_{e_j}(e)$ for each $y_j \in Y_j$ and $e \in E$.*

A special case of recommendations uncorrelated with allocations is when recommendations are deterministic rather than stochastic. For example, assume that each agent can put in a binary effort, say high or low. In addition, suppose that in equilibrium, each principal recommends that each agent should choose high effort. Then, recommendations are deterministic, and regardless of allocation strategies, satisfy our definition of being uncorrelated with allocations.

Note that even when recommendations are uncorrelated with allocations, principals can still induce a correlated equilibrium in the agents' effort game. Although agents have symmetric information about allocations, the recommendations serve the role of a private randomization device, as in Aumann (1974).

Using the no-correlation property, we can now state our theorem about the robustness of pure strategy equilibria in the direct mechanism game $\Gamma_{\mathcal{D}}$. The theorem provides sufficient conditions for an equilibrium outcome of a direct mechanism game to remain an equilibrium outcome of an indirect mechanism game. Formally:

Theorem 1 *Suppose the direct mechanism game $\Gamma_{\mathcal{D}}$ has a strongly robust equilibrium (π^*, δ^*) in which, for each principal j, π_j^* is a pure strategy that satisfies the no-correlation property. Then, in the indirect mechanism game $\Gamma_{\mathcal{G}}$, it remains a strongly robust equilibrium for each principal j to offer the mechanism $(\emptyset, E, \pi_j^*)$ and for each agent i to play δ^{i*}. Thus, the joint distribution over allocations and efforts that obtains in the equilibrium of the direct mechanism game remains an equilibrium outcome of the indirect mechanism game.*

Proof.

Consider the indirect mechanism game $\Gamma_{\mathcal{G}}$. Suppose that, in this game, every principal j offers a mechanism $\left(M_j, R_j, \pi_j\right) = (\emptyset, E, \pi_j^*)$, where π_j^* is his equilibrium pure strategy in the direct mechanism game $\Gamma_{\mathcal{D}}$. It is immediate that $\delta^* = (\delta^{1*}, \cdots, \delta^{k*})$ must remain a continuation equilibrium in the agents' efforts game.

Thus, if the equilibrium (π^*, δ^*) is not strongly robust in the game $\Gamma^{\mathcal{G}}$, it must be that there exists a deviation by some principal j' to a mechanism $\tilde{\gamma}_{j'} = \left(\tilde{M}_{j'}, \tilde{R}_{j'}, \tilde{\pi}_{j'}\right) \neq \left(\emptyset, E, \pi_{j'}^*\right)$, and a continuation equilibrium of the agents' effort game, $\tilde{\beta}$, such that principal j' earns a strictly greater utility than in the equilibrium (π^*, δ^*). The mechanisms and the agents' effort strategies $\tilde{\delta}$ induce a (possibly correlated) distribution over allocations y and efforts e. Let $\tilde{\nu}(y, e)$ denote this distribution. Then, it must be that $V_{j'}((\tilde{\gamma}_{j'}, \pi_{-j}^*), \tilde{\beta}) > V_{j'}(\pi^*, \delta^*)$.

Now, every principal $j \neq j'$ is using recommendations uncorrelated with allocations. Since each agent i observes only the mechanisms, his own message $m_{j'}^i$ to principal j', and his own recommendation array $r^i = (e_1^i, \ldots, r_{j'}^i, \ldots, e_n^i)$, the efforts chosen must remain uncorrelated with the allocations of principals $j \neq j'$. Hence, we can write $\tilde{\nu}(y, e) = \tilde{\nu}_{j'}(y_{j'}, e) \cdot \prod_{j \neq j'} \pi_{y_j}^*(y_j)$, where $\pi_{y_j}^*(\cdot)$ is the marginal distribution over the allocations of a principal $j \neq j'$, given his strategy π_j^*.

It is now straightforward for principal j' to induce the same joint distribution in the direct mechanism game. Rather than playing the strategy $\pi_{j'}^*$, he now plays the strategy $\tilde{\nu}_{j'}$. Since this strategy induces the same joint distribution over efforts and allocations as in the continuation equilibrium of the indirect mechanism game, it must be a best response for each agent i to obey the recommendation of principal j', and to ignore the recommendations of the others (else agent i would have a profitable deviation in the indirect mechanism game, rather than playing $\tilde{\delta}$). But if every agent i obeys the recommendation of principal j', and principal j' plays $\tilde{\nu}_{j'}(y_{j'}, e)$ in the direct mechanism game, the same joint distribution over allocations and efforts is induced as in the indirect mechanism game. Hence, if principal j' has a profitable deviation in the indirect mechanism game, he has a profitable deviation in the direct mechanism game as well, contradicting the assumption that (π^*, δ^*) is a strongly robust equilibrium of Γ_D. ■

Observe that the theorem cannot be extended to mixed strategies for principals. The reason is that whenever a principal plays a mixed strategy the agent can observe the realization of that mixed strategy. The realization of the mixed strategy, in turn, constitutes relevant information for the other principals. This intuition applies to the single-agent case as well (Peters (2003)), and an example in this direction is provided in Han (2007a) for the case with contractible effort.

The no correlation properties plays a critical role in our construction. Suppose that, in some equilibrium of $\Gamma_{\mathcal{D}}$, a principal $\tilde{j}$ offers recommendations correlated with his allocations. If some other principal j were to deviate, the resulting distribution over allocations and efforts may continue to exhibit such correlation. Thus, principal j alone cannot replicate the correlation with his own strategy. We show the necessity of this condition in Section 4.

Standard models of multiple principals with complete information on the agents' side typically do not consider communication. Instead, principals offer just allocations, and agents take a non-contractible effort. We call such a game a "game without recommendations." If the equilibrium outcomes generated in a game without recommendations were replicable as strongly robust equilibria in direct mechanism games with recommendations, with strategies that satisfied the conditions of our theorem, we would be confident that no principal could gain by a unilateral deviation to an indirect mechanism. The result is hence a direct implication of our main theorem.

In a game without recommendations, let $\sigma_j \in \Delta(Y_j)$ denote the (mixed) strategy of principal j, and let $\rho^i : \prod_{j=1}^{k} \Delta(Y_j^i) \to \Delta(E^i)$ denote the strategy of agent i. Let $\sigma = (\sigma_1, \ldots, \sigma_n)$, and $\rho = \left(\rho^1, \ldots, \rho^k\right)$.

Corollary 1.1 *Let (σ^*, ρ^*) be a pure strategy equilibrium in the game without communication. If the corresponding equilibrium outcome can be supported as a strongly* *robust equilibrium in $\Gamma_{\mathcal{D}}$, then it remains an equilibrium outcome in the indirect mechanism game $\Gamma_{\mathcal{G}}$.*

Proof. Consider a pure strategy equilibrium (σ^*, ρ^*) of the game without recommendations. We can construct strategies (π, δ) in the game $\Gamma_{\mathcal{D}}$ that replicate the outcome of the equilibrium in the game with no recommendations. For example, let $\pi_j = \sigma_j^* \times e_1^1 \times \cdot \times e_1^k$ for each principal j. That is, each principal offers the allocation lottery σ_j and the recommendation array $(e_1^i, \cdots, e_1^k)$ to the agents. Set $\delta^i(\pi_1^i, \ldots, \pi_n^i) = \rho^{i*}(\sigma_j^i, \ldots, \sigma_j^n)$ for each i.

By construction, π satisfies the property that recommendations are uncorrelated with allocations for each principal j. Notice that (π, δ) induces the same distribution over terminal payoffs as (σ^*, ρ^*).

Therefore, from Theorem 1, if it is a strongly robust equilibrium in $\Gamma_{\mathcal{D}}$ for each principal j to offer π_j and for each agent i to play δ^i, it remains a strongly robust equilibrium in $\Gamma_{\mathcal{G}}$ for each principal j to offer $\gamma_j = (\emptyset, E, \pi_j)$, and for each agent to play δ^i. ■

Corollary 1.1 shows that pure strategy equilibria of games without communication can survive the introduction of complex forms of communication if they can be supported as strongly robust equilibria in games with communication. Example 1 in Section 4 shows that the latter restriction is non-trivial: Introducing recommendations in a game without communication allows a principal to induce a correlated (rather than a Nash) equilibrium in the agents' effort game, and may lead to her being strictly better off.

One implication of our analysis is that, in the game $\Gamma_{\mathcal{D}}$, it is sufficient to analyze incentive compatible (i.e., obedient) behavior by the agents at the deviation stage, since only one principal deviates to a mechanism with recommendations. In other words, we only need to consider deviations where the principal's recommendations are followed by agents in the continuation game. If those deviations are not profitable, then none is and the equilibrium will be strongly robust. Even in a multiple-principal context where the notion of obedience is not helpful to characterize equilibria, there is a rationale for considering incentive compatibility at the deviation stage in games without recommendations.

Our results cannot be straightforwardly extended to games with incomplete information. The intuition is the following: even if recommendations are uncorrelated with allocations, a recommendation from principal j to agent i may communicate information about the type of some other agent i'. This may lead to a correlation between agents' efforts and principals' allocations, which is difficult for a single principal to replicate in a direct mechanism.

In a recent paper, Peters (2004) provides a thought-provoking example in a setting with two principals and two agents who are taking some non contractible effort. His Example 1 suggests that "In a multiple agency environment [...] pure strategy equilibria are not robust against the possibility that principals might deviate to more complex indirect mechanisms".[7] Peters restricts attention to deterministic allocations with no recommendations. By introducing lotteries over allocations in his example, one can recover the robustness of direct mechanism equilibria.[8]

4 Stochastic mechanisms, recommendations and no correlation

As mentioned, the existing literature on multiple-principal games of complete information does not consider any form of communication, and restricts the analysis to deterministic take-it or leave-it offers. In this section, we provide two examples that illustrate the features of our construction. Each example has two principals and two agents. Example 1 shows that stochastic mechanisms and private recommendations are necessary features of the framework. In the absence of either of these features, it may be possible for a principal to profitably deviate toward an indirect mechanism involving communication from agents. Example 2 shows that the no-correlation property is critical. If it is violated, a principal may gain from deviating to an indirect mechanism.

Example 1

Let $n = 2$ and $k = 2$, with the space of deterministic allocations being $Y_1 = \{y_{11}, y_{12}\}$ and $Y_2 = \{y_{21}, y_{22}\}$ for principal 1 and 2 respectively, and the effort spaces being $E^1 = \{a_1, a_2\}$ and $E^2 = \{b_1, b_2\}$, for agent 1 and 2. The payoffs of the game are given in the following matrix. The first payoff is that of principal 1 (P1), who chooses the row in

[7] Peters (2004, p. 184).

[8] Details are available from the authors on request.

the table; the second payoff is that of principal 2 (P2), who chooses the column, and the last two payoffs are those of agent 1 and 2, respectively.

		y_{21}		y_{22}	
		b_1	b_2	b_1	b_2
y_{11}	a_1	$(0,1,0,10)$	$(50,1,6,6)$	$(1,0,-1,2)$	$(0,0,4,1)$
	a_2	$(0,1,-10,-10)$	$(-10,1,0,10)$	$(2,0,3,2)$	$(1,0,2,-1)$
y_{12}	a_1	$(0,1,0,-10)$	$(-200,1,0,0)$	$(3,0,-1,-1)$	$(1,0,1,1)$
	a_2	$(0,1,10,0)$	$(4,1,1,6)$	$(1,0,1,1)$	$(0,0,-1,-1)$

Table 1: Payoffs in Example 1

In equilibrium, P2 must play y_{21}, which strictly dominates y_{22}. Hence, for the rest of the example, we focus on P1 and the two agents. In particular, the payoff matrices we show for the remainder of the example ignore the payoff of P2.

Given that P2 plays y_{21}, the relevant payoffs in the game are provided below, where the first element in each cell is the payoff of P1, and the remaining two numbers are the payoffs to the two agents.

$y = y_{11}$

	b_1	b_2
a_1	$(0,0,10)$	$(50,6,6)$
a_2	$(0,-10,-10)$	$(-10,0,10)$

$y = y_{12}$

	b_1	b_2
a_1	$(0,0,-10)$	$(-200,0,0)$
a_2	$(0,10,0)$	$(4,1,6)$

First, suppose P1 offers no recommendations (so that $R_1^1 = R_1^2 = \emptyset$), but can choose a lottery over allocations, so that $y = py_{11} + (1-p)y_{12}$. We show that the optimal mechanism in this setting has P1 offering the allocation y_{12}. Agents then play (a_2, b_2) in the effort game, so that principal 1 obtains a utility of 4.

The corresponding payoffs in the agents' effort game are (again, the first element in each cell is the payoff of P1, and the remaining two numbers are the agents' payoffs):

	b_1	b_2
a_1	$(0, 0, 20p-10)$	$(250p-200, 6p, 6p)$
a_2	$(0, 10-20p, -10p)$	$(4-14p, 1-p, 4p+6)$

For $p < \frac{5}{7}$, b_2 strictly dominates b_1. We first analyze the equilibrium possibilities that result if P1 chooses p in this range. For $p \leq \frac{1}{7}$, the action a_2 of agent 1 is a best response to b_2, and results in a utility of $(4-14p)$ for P1. Thus, if P1 induces agent 1 to play a_2, his utility is maximized at $p = 0$ at a value of 4. For $p \in [\frac{1}{7}, \frac{5}{7})$, the action a_1 of agent 1 is a best response to b_2, and results in a utility of $(250p-200)$ for the principal. This has a supremum in this range of p at $p = \frac{5}{7}$, and a value of $-\frac{150}{7}$.

Thus, over all $p < \frac{5}{7}$, the maximal payoff P1 achieves in equilibrium is 4, obtained when $p = 0$.

When $p = \frac{5}{7}$, for agent 1, a_1 strictly dominates a_2, and agent 2 is indifferent over b_1, b_2. The maximal utility P1 can obtain is 0, when agent 2 plays b_1.

Finally, for $p > \frac{5}{7}$, the unique equilibrium of the agents' subgame is (a_1, b_1), with P1's utility being 0.

Hence, the optimal allocation for P1 is y_{12} (i.e., choosing $p = 0$), with resultant equilibrium (a_2, b_2) in the agents' game. Principal 1 achieves a utility of 4.

Since the mechanism is deterministic, it is clearly also the optimal mechanism with deterministic take-it-or-leave-it offers.

Now, consider the following indirect mechanism. P1 communicates with agent 1, with the message space being $M_1^1 = \{m_1, m_2\}$. Set $M_1^2 = R_1^1 = R_1^2 = \emptyset$. The allocation rule of P1 is $\tilde{\pi}_1(m_k) = y_{1k}$ for $k = 1, 2$. Note that we specify a deterministic allocation rule in the indirect mechanism.

In the indirect mechanism, a simultaneous-move game is induced between the agents, and can be represented as follows.

	b_1	b_2
(m_1, a_1)	$(0, 0, 10)$	$(50, 6, 6)$
(m_2, a_1)	$(0, 0, -10)$	$(-200, 0, 0)$
(m_1, a_2)	$(0, -10, -10)$	$(-10, 0, 10)$
(m_2, a_2)	$(0, 10, 0)$	$(4, 1, 6)$

Notice that (m_2, a_1) and (m_1, a_2) are both strictly dominated by (m_2, a_2). Further, there is no pure strategy equilibrium in this game. The game has the following unique Nash equilibrium:

- Agent 1 mixes between (m_1, a_1) and (m_2, a_2), with probabilities $3/5$ and $2/5$.
- Agent 2 mixes between b_1 and b_2 with probabilities, $1/3$ and $2/3$.

Thus, P1's expected payoff from the indirect mechanism is $316/15 > 4$. That is, this principal has a higher payoff from the indirect mechanism than is achievable in a direct mechanism.

Allowing for recommendations and stochastic allocations, we can resurrect the equilibrium of the indirect mechanism in a direct mechanism. A direct mechanism with recommendations in this example may be characterized as a function $\pi : Y_1 \times E^1 \times E^2 \to [0, 1]$, where $\pi(y, a, b)$ is the probability that P1 chooses allocation y and recommends effort a to agent 1 and b to agent 2.

In the equilibrium of the indirect mechanism above, the resultant distribution over allocations and efforts is: $\pi(y_{11}, a_1, b_1) = 1/5$, $\pi(y_{11}, a_1, b_2) = 2/5$, $\pi(y_{12}, a_2, b_1) = 2/15$ and $\pi(y_{12}, a_2, b_2) = 4/15$. Suppose P1 plays this strategy in the direct mechanism. That is, P1 chooses allocations and efforts according to $\pi(\cdot)$, and announces the resulting recommendations to the agents.

It is straightforward to check that neither agent has an incentive to deviate, so the mechanism is incentive compatible. For example, when agent 2 is told "b_2", his posterior beliefs place probability $3/5$ on (y_{11}, a_1) and $2/5$ on (y_{12}, a_2). Given these beliefs, b_2 is a (weak) best response. Principal 1 obtains the utility $\frac{316}{15}$, as before. ∎

In Example 1, principal 1 uses an indirect mechanism to communicate privately with agent 1, thereby sustaining a correlated outcome over allocations and efforts. In an indirect mechanism, this correlation is feasible even with deterministic allocations. However, in the direct mechanism, such a correlation can be replicated only if the principal sends private recommendations, and only if allocations are stochastic.

Thus, even in a pure moral hazard setting, if there are multiple agents, private communication between a principal and an agent can allow the principal to achieve superior outcomes. Therefore, to characterize the mechanisms selected at equilibrium in a setting with complete information and pure moral hazard, it is important to allow for such communication.

Example 1 also illustrates the condition in Corollary 1.1 that, for an equilibrium of the game without communication to be supportable as an equilibrium in the indirect mechanism game, it should be supportable as a strongly robust equilibrium in the direct mechanism game. As mentioned in the example, in the game without communication, agents must play a Nash equilibrium of their continuation game, and principal 1 achieves a maximum payoff of 4. Once recommendations are introduced, the principal can induce agents to play a correlated equilibrium that results in a higher payoff for herself.

Finally, observe that, in Example 1, principal 2 is essentially passive, since she has a strictly dominant strategy. Thus, the example can be re-interpreted as one with a single principal and two agents. As such, it points out that stochastic allocations and recommendations, two important features adopted by Myerson (1982), are necessary for the revelation principle to go through in this setting. Our example thus complements the work of Strausz (2003), who provides an example in a setting of pure adverse selection with one principal and two agents. He shows that it is no longer true that any payoff implementable by a deterministic indirect mechanism can be replicated by a deterministic direct mechanism. In his example, an agent with veto power may veto a direct deterministic mechanism and prefer an indirect one.

The intuition of our example can thus be potentially applied to institutional contexts where a single principal selects his preferred incentive scheme in the presence of multiple agents and moral hazard. Consider for instance the literature on moral hazard in teams. In a situation where first best efficiency is not implementable, this literature has traditionally suggested several devices to improve efficiency: monitoring, yardstick competition, or repetition.[9] Our example suggests that, in such a context, explicitly allowing the single principal to communicate with the agents may be welfare-enhancing.

Next, we provide an example to show that the no-correlation assumption is necessary for our theorem to go through.

Example 2

As in Example 1, there are two principals and two agents. The payoffs of the game are shown in Table 2. Principal 1 (P1) has only one allocation, represented by a single row in the table. Principal 2 (P2) chooses the column. In each cell, the first payoff is that of P1, the second payoff is that of P2, and the last two payoffs are those of agents 1 and 2, respectively.

		y_{21}			y_{22}	
		b_1	b_2		b_1	b_2
y_1	a_1	$(12,1,5,5)$	$(0,1,2,7)$	a_1	$(12,1,6,6)$	$(0,1,2,7)$
	a_2	$(0,1,7,2)$	$(0,1,0,0)$	a_2	$(0,1,7,2)$	$(0,1,0,0)$

Table 2: Payoffs in Example 2

First, we identify a particular strongly robust equilibrium in direct mechanisms. Consider the following strategy for P2:

$$\pi_2 = \begin{cases} (y_{21}, a_1, b_1) & \text{with probability } \frac{1}{2} \\ (y_{22}, a_2, b_2) & \text{with probability } \frac{1}{2}, \end{cases}$$

where y_{21} and y_{22} are the allocations and (a_1, b_1) and (a_2, b_2) the recommendations to the two agents. Since P2 is indifferent across all outcomes, to find a strongly robust equilibrium in which P2 plays the strategy π_2, it is sufficient to focus on continuation equilibria (in the agents' game) that maximize the expected payoff of P1. Since P1 only earns a positive payoff from the outcome following the play of (a_1, b_1) in the agents' game, her payoff is maximized by maximizing the probability of this outcome.

[9] See d'Aspremont and Gérard-Varet (1998) for an overview.

It is straightforward to show that when P2 offers y_{21}, the continuation equilibrium that maximizes the expected payoff of P1 is the correlated equilibrium that places probability $\frac{1}{3}$ on each of (a_1,b_1), (a_2,b_1), and (a_1,b_2), and yields P1 an expected payoff of 4. Similarly, when P2 offers y_{22}, the expected payoff of P1 is maximized by the correlated equilibrium that places probability $\frac{1}{2}$ on (a_1,b_1), and probability $\frac{1}{4}$ each on (a_1,b_2) and (a_2,b_1).

In a direct mechanism, P1 cannot offer recommendations that are contingent on the recommendations offered by P2. Thus, to maximize P1's expected payoff, we must consider the same correlated equilibrium in the agents' game, regardless of whether P2 offers y_{21} or y_{22}. Notice that the correlated equilibrium which places probability $\frac{1}{3}$ on each of (a_1,b_1), (a_2,b_1), and (a_1,b_2) continues to be a correlated equilibrium when P2 offers y_{22}. This correlated equilibrium can be induced by a strategy π_1 of P1, such that:

$$\pi_1 = \begin{cases} (y_1,a_1,b_1) & \text{with probability} \quad \frac{1}{3} \\ (y_1,a_2,b_1) & \text{with probability} \quad \frac{1}{3} \\ (y_1,a_1,b_2) & \text{with probability} \quad \frac{1}{3}. \end{cases}$$

The continuation equilibrium has both agents obeying the recommendation of P1. Now, π_1 and π_2, along with the continuation equilibrium in which both agents obey the recommendations of P1, constitute a strongly robust equilibrium of the direct mechanism game: neither principal can gain from a unilateral deviation to another direct mechanism. In this strongly robust equilibrium, the expected payoff of P1 is 4.

Next, suppose that, when P2 offers the direct mechanism π_2, P1 can instead deviate to an indirect mechanism. We show that there exists an indirect mechanism that yields P1 an expected payoff of 5, by inducing the optimal correlations separately for each allocation P2 may choose.

Consider the following indirect mechanism offered by P1. The sets of recommendations for agents 1 and 2 are, respectively $R = \{r_1,r_2,r_3,r_4\}$ and $S = \{s_1,s_2,s_3,s_4\}$. Agent 1 plays the following strategies after he receives the recommendation of P1:
r_1: play a_1 regardless of the recommendation offered by P2.
r_2: play a_1 if P2 sends recommendation a_1 and a_2 if P2 sends recommendation a_2
r_3: play a_1 if P2 sends recommendation a_2 and a_2 if P2 sends recommendation a_1
r_4: play a_2 regardless of the recommendation offered by P2.

Agent 2's strategy is similar (substitute s_i for r_i and b_j for a_j above).

Since P1 has only one allocation to offer (y_1), we specify the allocation rule only in terms of probabilities over recommendations sent to the agents. Consider the following probabilities over R and S.[10]

	s_1	s_2	s_3	s_4
r_1	$1/3$	0	$1/6$	$1/6$
r_2	0	0	0	0
r_3	0	$1/12$	0	0
r_4	$1/4$	0	0	0

Table 3: Probabilities over P1's recommendations in indirect mechanism

It is straightforward to check that, for every pair of recommendations in the support of P2's strategy, the above recommendations and agents' strategies induce the same distribution over outcomes as the optimal (for P1) correlated equilibrium in the agents' continuation game. Thus, the specified strategies constitute a continuation equilibrium in the agents' game. P1 earns an expected payoff of 5 from the indirect mechanism, higher than the 4 she can earn from a direct mechanism. ■

This example shows that, in the absence of the no-correlation property, a principal may wish to deviate to an indirect mechanism even if other principals offer direct mechanisms. Using a larger communication space than in the direct mechanism, P1 can extract the information on the realized allocation of P2's mechanism by communicating with the agents. The correlation among allocations and recommendations in P2's decisions is crucial. From the viewpoint of P1, the recommendation offered by P2 is equivalent to an unknown type for each agent. Thus, the optimal mechanism for P1 must be contingent on this unknown parameter.

In our example, P2 is indifferent across all outcomes. It is easy to see that, if P2 had non-trivial preferences over outcomes, she may, in turn, wish to make recommendations contingent on the (contingent) recommendations of P1, and so on, leading to the infi-

[10]Notice that the distribution shown places zero probability on r_2, so we can construct the example with R consisting of only three elements. The probabilities shown here are illustrative, and constructed for the purpose of showing one example. There are other probability distributions that also produce the same outcomes.

nite regress problem mentioned by McAfee (1993) and the universal message space of Epstein and Peters (1999).

5 Conclusion

The existing literature on multiple-principal, multiple-agent models of complete information typically restricts attention to situations where no communication is allowed. In particular, it does not consider recommendations sent from principals to agents. We have considered a general competing mechanism framework with moral hazard: agents' actions cannot be contracted upon. In such a setting, we have shown that the pure strategy equilibrium allocations of games without communication will survive the introduction of any form of communication, as long as they can be supported as strongly robust equilibria of direct mechanism games.

More broadly, we have shown that, if principals' recommendations are uncorrelated with their allocations, the outcome of a strongly robust pure strategy equilibrium of the direct mechanism game can also be sustained as the outcome of a strongly robust equilibrium in the indirect mechanism game. There is, therefore, a rationale for restricting attention to simple mechanisms in moral hazard settings with multiple principals and multiple agents.

Observe that the no-correlation conditions we introduce in the paper provides for sufficiency, but the intuition we get from Example 2 points also in the direction of necessity. If an equilibrium in direct mechanisms exhibits correlation between allocations and recommendations for a principal, then from the point of view of his rivals the agents would have relevant information to extract.

Of course, strongly robust pure strategy equilibria may not exist in the direct mechanism game. In particular, the competing mechanism game between the principals may have a "matching pennies" flavor, with only a mixed strategy equilibrium. Even if a pure strategy equilibrium exists, it may fail to be strongly robust: a principal may wish to deviate to a mixed strategy, or the payoff to a principal may depend on the particular continuation equilibrium chosen in the agents' game (consider, for example, an agents' game with two pure strategy continuation equilibria, one of which favors one principal and another which favors a second principal).

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