

Motivate and Select – Markets and the Form of Compensation

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Abstract

The paper provides a rationale for the simultaneous use by an employer of termination contracts and bonus payments based on some observable but non verifiable measure of performance. Bonus payments are credible since the continuation of the employment relationship allows the principal to earn a rent. In a setting with unemployment, this could be the case if the employee's performance signals that his productivity exceeds the average productivity of unemployed workers by a sufficient amount. Thus, bonus payments can be used to motivate a worker to perform difficult tasks, while termination contract motivates them performing easy ones. The model also makes predictions about the impact of labor market characteristics, employees' heterogeneity and exogenous turnover, on the optimal incentive mix.

Keywords: Relational contracts, Termination contract, Bonus, Ex post selection

JEL classification: D82, J31, J41

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1 Introduction

Performance-based pay is not the only way to motivate employees, as could be suggested by the traditional principal-agent approach. Indeed, in addition of piece rates, employers use a variety of other economic incentives, such as profit sharing, firing, promotion or bonuses based on informal agreements¹. The use of performance-based pay is limited by the difficulty of measuring individual performance in an objective way. Indeed, in many instances, the available performance measures may be observable for the contracting agents, but too hard to verify by a third party. Yet this information is still useful to motivate employees since informal agreements can be tied on it. Such “relational contracts” are not enforced by a court but by the potential loss of cooperation between the contracting parties. The theory based on relational incentive contracts provides a useful understanding of the existence of some of the incentive tools mentioned above.

The early literature on relational contracts focuses on frameworks with symmetric information (see Shapiro and Stiglitz (1984) and Bull (1987) among others). MacLeod and Malcomson (1989) propose the most complete treatment of the problem. They prove that provided there is a sufficient rent from employment, either a performance-based bonus (workers who perform satisfactory are paid an end of period bonus) or a termination contract (fixed and performance standard to be achieved in order to keep occupation) could be sustained in equilibrium. The rent corresponds to the surplus² generated by the continuation of the employment relationship. If the latter is interrupted each party receives an exogenously given outside opportunity. MacLeod and Malcomson (1998) go further and stress the importance of labor market conditions on the choice between termination contract and bonus. When agents are homogeneous in skills and there is unemployment, the principal can replace an employee with an unemployed agent at no cost. There is no rent for the employer to continue the employment relationship. Then, the bonus is not credible

¹MacLeod and Parent (1997) analyze different incentive schemes used in the US. Hayes and Shaefer (1997) provide evidence for the use of subjective performance measures when boards of directors decide the salary and bonus of chief executives. Cappelli and Chauvin (1991) provide evidence in favor of the use of termination contracts as an instrument to motivate agents to work.

²Since both the principal and employees are risk neutral, the reward is a pure transfer.

and the termination contract is the only way to motivate employees. Conversely, when the latter are in short supply, vacancies cannot be immediately filled, which creates a rent from keeping an agent. Then a performance-based bonus is credible. MacLeod and Malcomson (1998) show that if the cost of unoccupied vacancies is not too high, then an equilibrium with full employment and bonus could emerge.

However, in real world contracts different incentive tools may coexist³. Nevertheless, there are few theoretical explanations⁴ for this coexistence. In this paper we propose a rationale for the simultaneous use of termination contract and bonus when information about employee's production is unverifiable. We examine this issue in a setting with heterogeneous in unobservable ability agents and endogenous, depending on labor market conditions, outside opportunities.

Levin (2003) characterizes the optimal relational contract with hidden information. Since the information asymmetry parameter changes from period to period, his focus is on the restrictions on revelation due to the self-enforcing character of the agreement. In our model, agents are privately informed about their ability which is the same over time. Under type persistence, revelation is a costly strategy for the principal. Indeed, when performance is not verifiable a principal can end the contract in any period. Then an agent is reluctant to reveal his information since it allows the employer to reap his surplus. In the model, we explicitly set aside *ex ante* screening and focus on the effects of *ex post* selection⁵ on the optimal relational contract.

We build our analysis in a framework with a continuum of firm owners proposing homogeneous positions. A job corresponds to the set of tasks an employee is supposed to perform⁶. The task an employee receives in a given period may be more or less difficult. Since the principal observes the difficulty at the end of the period, succeeding a task is a signal about the agent's ability. The firm owner's problem

³See for example Baker, Gibbs and Holmström (1994) for a study of the wage policy of a firm.

⁴Baker, Gibbons and Murphy (1994) show that when a principal has available both verifiable and unverifiable performance measure, it can be in his interest to use both measures thus proposing a formal performance-based pay and an informal bonus.

⁵At the end of a period a principal only keeps agents that have attained the performance standard set in the termination contract.

⁶Our production functions is inspired by the one presented in Garicano (2000), firm's production corresponds to the number of successfully performed problems. The number of problems a firm can solve at each period is limited by its employees' competence and time constraints.

is to motivate agents to perform the largest possible set of tasks. If employees are in short supply the equilibrium contract is still a performance-based bonus. Hence, we focus on situations with unemployment. To give a flavor of the analysis consider that employees have larger average ability than unemployed⁷.

When agents are heterogeneous, there could be a rent for the principal from keeping a successful insider. Indeed, solving a hard task signals high productivity, that may exceed the productivity of the average unemployed agent. Since the gap between the successful employee and the average unemployed should be sufficiently high, a necessary condition for the bonus to be credible, is high heterogeneity in the initial distribution of abilities. Hence, an end of period bonus can be credibly used to motivate agents to perform tasks that reveal their high productivity. Since its use limits to difficult tasks, it can be complemented by a termination contract that guarantees incentives for performing easy tasks. We show that a necessary condition for the simultaneous use of bonus and termination contract is that the expected productivity of insiders is sufficiently larger than the expected productivity of unemployed agents.

If the initial distribution of abilities is not sufficiently heterogeneous, then the rent from keeping an employee is too low. The bonus can not be credibly used and incentives are guaranteed only with a termination contract. However, we show that the set of tasks for which credible incentives are guaranteed is constrained, in the sense that there are tasks for which the firing rule is not credible. Since employees have larger expected productivity than unemployed agents, an insider under-performing a difficult task, could still be more productive than an outsider. Then the principal prefers to keep the agent rather than to fire him, which destroys the latter's motivation to perform difficult tasks.

To give the intuition of our results we considered above that insiders have larger average ability than unemployed agents. In the model this is endogenously obtained and is due to *ex post* selection by the principal. When a firm owner proposes a termination contract he fires under-performing (less skillful) agents over time. Thus, the stationary market equilibrium is characterized by two distributions, inside the

⁷Hereafter we explain that this is an endogenous property of our model.

firm and on the labor market. They satisfy the property we considered above, namely the average ability of insiders is larger than the one of outsiders. The gap between distributions affects the form of the contract and depends on labor market conditions. For example, a lower exogenous turnover raises the gap of average ability between employed and unemployed agents, thus facilitates the use of the bonus for a larger set of tasks.

The rest of the paper is organized as follows. Section 2 describes the framework. In Section 3 we present the incentive compatible termination contract and performance based bonus. In Section 4 we discuss the self-enforceability constraints. In Section 5 we present our main findings. Then we conclude. All proofs are in the Appendix.

2 Framework

Economy We consider an economy composed of a continuum of measure one of firm owners (principals, employers) and a continuum of measure N of potential employees (agents). All parties are endowed with some ability θ . Firm owners are homogeneous and their ability is normalized to 1. Employees are heterogeneous and θ is agent's private information. Let $S(\cdot)$ be the commonly known distribution of workers' ability, $\theta \in [\underline{\theta}, \bar{\theta}]$. We assume that even the most able worker has less ability than a firm owner⁸ ($1 > \bar{\theta} > \underline{\theta} \geq 0$).

An agent is either employed or unemployed at any point in time. The utility of an employee from exerting effort in period t writes as $(W_t - c)$, where W_t is the reward function and c is the effort cost. Any unemployed agent receives an exogenously given unemployment benefit of z . An employer-employee match can be interrupted with probability $(1 - \alpha)$ for exogenous reasons. This exogenous turnover rate is the same whatever agent's ability. It causes an employee to enter the unemployment pool.

All parties are infinitely lived and discount the future at a common rate δ .

⁸We set aside the endogenous decision to become a firm owner or to remain an employee. However, we consider that only the most able agents become firm owners.

Production Firm's production corresponds to the number of successfully performed tasks (problems). They are equally valuable to solve and their price is normalized to one. However, some of them are more difficult than others. The complexity x of a particular task is *ex ante* unknown, but it is drawn from a commonly known distribution. Dealing with a task requires time, succeeding it requires ability and effort. Each agent in the firm has one unit of time by period and spends it completely in dealing with one problem. An employee θ can solve a problem, by expending some effort, only if $x \leq \theta$. If $x > \theta$, he cannot solve it. The effort cost function for an agent θ writes:

$$c(x, \theta) = \begin{cases} c & \text{if } x \leq \theta \\ +\infty & \text{if } x > \theta \end{cases}$$

Since time is limited, a principal hires employees and delegates them the solution of easy tasks. Thus, he spends his time in dealing with problems, that employees are unable to solve. Tasks arrive at the workers' level and it is assumed that any task received by the organization should be solved⁹. Then the number of employees a principal hires and thus the number of problems the organization deals with in each period, is given by the following time constraint:

$$(1) \quad n_t T_t = 1$$

where n_t is the number of employees, and T_t is the proportion of problems, that employees transmit to the principal. T_t is a function of employees' ability and the principal's productivity requirements (*i.e.* the set of problems for which he motivates agents to work), it is determined in Section 5.1.

For the rest of the paper we make the following normalizations. First, problems' complexity is uniformly distributed on the support $[0, 1]$. Hence, θ designs the proportion of problems an agent is able to solve. Second, a principal solves problems without cost¹⁰.

⁹The production function is in the spirit of Garicano (2000). Here production is organized in a two layer knowledge based hierarchy. Each employee receives a problem and solves it if able and supposed to. If not, he transmits it to the principal.

¹⁰Thus, the instantaneous profit of a firm owner that works alone is equal to 1. Indeed, he deals

Output is produced whenever either workers or the firm owner are able to solve a problem, thus the expected output in period t is $y_t = 1 \times n_t$, and a firm owner's expected profit¹¹ in period t writes as:

$$(2) \quad \Pi_t = n_t(1 - E[W_t])$$

The employer-employee relationship. Some features of a contract are clearly defined, measurable by a third party, thus enforceable. Others are observable inside the firm, but not measurable in a way that could be verified in court. We assume that the difficulty of a given task and whether it has been succeeded or not is not verifiable, thus a legally enforceable contract cannot be written on it. However, it is observed by the employee dealing with the task and the employer. So they can agree on a relational contract, enforced by the possibility of future actions for each of them.

1. At the beginning of each period a firm-owner with vacant occupations and unemployed agents are matched.
2. The principal proposes a contract C to the agents. The contract specifies the reward W_t and implicit¹² performance requirement(s). The reward consists of a fixed wage w_t that the principal pays whatever agent's output and a bonus b_t contingent on employee's performance. The fixed wage is verifiable: a principal cannot renege on its payment. The bonus is paid at the end of a period. Since it is contingent on unverifiable performance, it cannot be enforced by a court. If the offer is rejected, the agent returns to the unemployment pool and the vacancy remains unoccupied. If an agent accepts the contract, he receives a task and decides to solve it (if able) or not.
3. At the end of the period the principal observes the difficulty of each problem, which employee received it and whether it was solved or not. He decides to pay

with one task by period, solves it with probability 1 and without effort cost.

¹¹Our results are not related to this specification of firm's production function.

¹²Since the performance is not verifiable, the principal cannot commit on explicit (*i.e.* enforceable by a third party) performance requirements.

or not the bonus and the contracting parties decide whether to stay matched for the next period. Exogenous separations occur.

Assumption 1. *A principal proposes the same contract to any employee.*

There are two agency problems on the employees' side: moral hazard as effort is not observable, and adverse selection since the agent's type is private information. With Assumption 1 we rule out *ex ante* screening and focus on the moral hazard problem. This assumption simplifies the analysis without imposing, in our view, strong restrictions on it. Since contracts contingent on performance are not enforceable, *ex ante* selection is costly for a firm owner. The intuition is that once an agent revealed his type, the principal has the possibility to fire him if he expects a larger profit from an agent of the unemployment pool. Thus, to obtain revelation, the menu of contracts proposed by the principal should be such that he gets the same expected profit from any agent whatever his type. These additional constraints increase importantly the cost of screening for the principal¹³.

Assumption 2. *Labor market is anonymous.*

The reasons why a particular employer-employee match came to end are not observed by any party on the labor market, neither employers, nor employees. We rule out the possibility for firm owners as well as for employees to build external reputation¹⁴.

Assumption 3. *Employees are anonymous in the firm.*

The decision to keep or fire an employee is only based on his performance in the current period. Said differently, a principal does not have the history of employee's previous performance, nor he knows the number of periods the agent spent in the firm. This restriction allows us to focus on the impact of *ex post* selection on the optimal relational contract. However, the assumption is not innocuous and we propose a detailed discussion of its implications in Section 6.

¹³However, here pooling is assumed since this informal analysis is not complete. Thus, we cannot claim that revelation is obviously sub-optimal. For more complete discussion about the optimal revelation contract under adverse selection without commitment we refer the reader to Laffont and Tirole (1993, pp. 437-460).

¹⁴This assumption is in the line of MacLeod and Malcomson (1998).

Thus, the principal proposes the same contract to any employee (assumption 1), and its terms only depend on agent's performance in the current period (assumption 3).

In what follows we begin by separately characterize the incentive compatible termination contract and performance-based bonus. We show that if the principal could write enforceable contracts, he would propose a bonus to motivate agents. Then we present self-enforceability conditions, and discuss the rationale for the simultaneous use of termination contract and bonus.

3 Incentive compatibility

Let $V_t(\theta)$ (*resp.* $V_t^U(\theta)$) represent the expected future utility of an employee (*resp.* unemployed) with type θ . Our analysis focuses on the steady state, thus we drop the t index.

3.1 Termination contract

Under a termination contract the wage paid to an employee is independent of performance. However, at the end of the period the employer can terminate the contract if performance is unsatisfactory. Thus, a termination contract is specified by a wage-performance pair (w, \underline{x}) . Where w is the fixed wage and \underline{x} the performance standard to be achieved in order to keep the job¹⁵.

In a setting with heterogeneous agents, there are two potential reasons for an employee to be fired: shirking and lack of performance. Since unsuccessful agents are fired at each period, the choice of performance standard allows *ex post* selection. We first present the incentive compatible fixed wage, then we discuss the selection effect of the associated performance standard.

¹⁵It is not in the principal's interest to fire an agent which performance is satisfactory. Since the expected productivity of an employee is increasing in the difficulty of the transmitted problem, if the principal fires an agent unable to solve a problem $\bar{x} - \epsilon$, then it is in his interest to fire any agent who fails to solve a problem $x < \bar{x} - \epsilon$.

The incentive compatibility of a termination contract. In each period an employee with ability $\theta < \underline{x}$, is in one of the following situations.

- He receives a problem of difficulty $x \leq \theta$. If he works, he solves it and keeps his job with probability α ; if he shirks, he is fired. Thus, the probability for spending the cost of effort is $Prob(x \leq \theta) = \theta$.
- He receives a problem of difficulty $x > \theta$:
 - if $x < \underline{x}$ the agent is fired for not meeting the standard
 - if $x > \underline{x}$ he keeps his job with probability α .

Then the inter-temporal expected utility of an employee with $\theta < \underline{x}$ who exerts effort is:

$$(3) \quad V(\theta) = w - \theta c + \delta(1 - \alpha)V_U(\theta) + \delta\alpha[(1 - \underline{x} + \theta)V(\theta) + (\underline{x} - \theta)V_U(\theta)]$$

The expected utility of an agent with $\theta \geq \underline{x}$ who exerts effort writes:

$$(4) \quad V(\theta) = w - \underline{x}c + \delta(1 - \alpha)V_U(\theta) + \delta\alpha V(\theta)$$

An unemployed agent receives the unemployment compensation z . At the end of each period, there is an endogenous probability¹⁶ λ to find a new job and quit the unemployment pool. Since labor market is anonymous (assumption 2), this probability is the same for any unemployed agent, whatever his employment history. Thus, λ is independent of θ . The inter-temporal expected utility of an unemployed agent is:

$$V_U(\theta) = z + \delta[\lambda V(\theta) + (1 - \lambda)V_U(\theta)]$$

For a wage to be incentive compatible, the agent's value from working should be larger than his value from shirking.

$$w - c + \delta\alpha V(\theta) + \delta(1 - \alpha)V_U(\theta) \geq w + \delta V_U(\theta)$$

¹⁶It is determined in Section 5.2.

Thus, an employee solves a problem only if the rent from keeping a job is sufficiently high.

$$(5) \quad V(\theta) - V_U(\theta) \geq \frac{c}{\alpha\delta}$$

Since in our setting the cost of effort is independent of θ , and the probability to be re-employed is the same for any unemployed agent (according to assumption 2), we have the following property.

Lemma 1. *The incentive compatible wage does not depend on θ .*

For a formal proof of the lemma, see Appendix 8.1. When the incentive compatibility constraint is binding (*i.e.* $(V(\theta) - V_U(\theta)) = \frac{c}{\alpha\delta}$) and since the re-employment probability is independent of agent's ability level, it is easy to see that the inter-temporal expected utility of an unemployed agent¹⁷ does not depend on θ . Let us now turn to $V(\theta)$. A larger θ (for $\theta < \underline{x}$) corresponds to a higher probability to keep a job (the associated gain is $\delta\alpha(V(\theta) - V_U(\theta))$), but also a higher probability to spend the effort cost c . The marginal gain equals the marginal cost. Hence, the inter-temporal expected utility of an employee is also independent of θ .

Given these results we simplify the notation as follows, V instead of $V(\theta)$ and V_U instead of $V_U(\theta)$.

Thus, the incentive compatible fixed wage¹⁸ that binds constraint (5) is:

$$(6) \quad w = c\underline{x} + \frac{c(1 - \delta\alpha)}{\delta\alpha} + (1 - \delta)V_U$$

Expanding the set of tasks delegated to workers raises the probability that an agent spends effort cost (or the probability to be fired for an agent with type $\theta < \underline{x}$), which reduces V and makes the utility stream when working less attractive for the agent. Thus, a larger performance standard should be combined with a higher wage in order to motivate workers.

¹⁷ $V_U(\theta) = z + \delta V_U(\theta) + \lambda\delta(V(\theta) - V_U(\theta)) \Leftrightarrow V_U(\theta)(1 - \delta) = z + \frac{\lambda c}{\alpha}$

¹⁸This wage motivates any agent to solve any task $x \leq \min\{\theta, \underline{x}\}$.

The selection effect of a performance standard. By choosing a performance standard a principal alters the characteristics of insiders' distribution. Indeed, the choice of \underline{x} sets the proportion of agents who will be fired in each period and replaced by outsiders. Let $F(\theta)$ be the stationary cumulative distribution function of employees' ability. The agents fired in each period are those unable to solve a problem below the performance standard: $v = \int_{\underline{\theta}}^{\underline{x}} F(\theta)d\theta = \int_{\underline{\theta}}^{\underline{x}} (\underline{x} - \theta)f(\theta)d\theta$. Thus, for given¹⁹ distribution of outsiders $Q(\theta)$, the steady state distribution of insiders $F(\theta)$ is determined by the following expression.

$$(7) \quad Q(\theta) = \begin{cases} \frac{(1 - \alpha)F(\theta) + \alpha \int_{\underline{\theta}}^{\theta} (\underline{x} - u)f(u)du}{(1 - \alpha + \alpha v)} & \text{if } \theta \in [\underline{\theta}; \underline{x}] \\ \frac{(1 - \alpha)F(\theta) + \alpha v}{(1 - \alpha + \alpha v)} & \text{if } \theta \in [\underline{x}; \bar{\theta}] \end{cases}$$

The steady state distribution of outsiders is composed of employees that quit their job for exogenous reasons, and employees that are fired because their lack of competence has been detected by the principal.

Lemma 2. *Insiders' distribution of ability dominates the outsiders' distribution, in the sense of first order stochastic dominance.*

3.2 Bonus

A contract is "pure bonus" when the principal motivates agents with a reward based on performance realization. In this case the fixed wage ensures agents' participation (*i.e.* $V(\theta) \geq V_U(\theta)$).

The bonus b is paid to an agent conditionally on performing successfully a task. The contract satisfies the following constraints:

$$\begin{cases} b - c + \alpha\delta V(\theta) + (1 - \alpha)\delta V_U(\theta) \geq \alpha\delta V(\theta) + (1 - \alpha)\delta V_U(\theta) & (IC) \\ V(\theta) \geq V_U(\theta) & (IR) \end{cases}$$

As the principal's profit is decreasing in agents' reward, it is easy to see that he

¹⁹Each principal is "too small" to have an individual impact on market conditions, thus he takes the distribution of outsiders' ability as given.

binds both constraints and we have $b = c$, and w such that²⁰ $V(\theta) = V_U(\theta)$. If such a contract can be enforced, then each employee solves any task he is able to.

Since in the case of termination contract the principal pays rents to the agents, the performance-based pay is the less expensive way to motivate employees. Thus, if the principal could write enforceable contracts, he would choose a performance-based pay to provide incentives. However, since performance is not verifiable the employer-employee relationship is steered by a relational contract, and the bonus could only be used to motivate agents to perform a subset of difficult tasks (above some threshold value \bar{x}). In the next section, we introduce the restrictions imposed on the contract space by the need for the agreement to be self-enforceable.

4 Self-enforceability

4.1 The Contract

The equilibrium self-enforceable contract when there is unemployment is $C^E = (w, b, \underline{x}, \bar{x})$, where w and b are respectively the fixed wage and the bonus, \underline{x} is a minimum performance to be achieved in order to keep a job and \bar{x} defines the lower bound of the set of tasks for which the bonus is used. C^E has the following characteristics.

- For problems $x \geq \bar{x}$, the principal motivates the agents with a performance based bonus.

As we already mentioned it in Section 3, the bonus corresponds to the least costly way for the principal to motivate employees. It is in his interest to adopt this incentive scheme for the largest possible set of tasks. However, the principal pays the bonus only if there is a sufficient rent from continuing the employment relationship. So it can be credibly used only for tasks that signal that the employee's productivity is above the one of the average unemployed. Since the expected productivity of an agent performing $\underline{\theta}$ equals the productivity expected from an unemployed, there is

²⁰It is easy to show that w does not depend on θ , and $w = z$. Recall that z is the unemployment benefit.

no rent from keeping the employee. Then the principal cannot credibly promise the bonus for this task. The bonus is credible only for tasks which difficulty is above a threshold level $\bar{x} \in]\underline{\theta}, \bar{\theta}]$.

- For problems $x \leq \underline{x}$, agents' motivation is guaranteed by the mean of a termination contract.

To motivate agents to perform easy tasks (*i.e.* $x \leq \underline{x}$) the principal can use a termination contract, where \underline{x} is the performance standard an employee must achieve in order to keep his job.

- $\bar{x} \geq \underline{x}$

In equilibrium the principal does not duplicate incentive schemes for a given task. To give the intuition for this, let us consider the contrary, $\bar{x} < \underline{x}$. There is a set of problems $x \leq \underline{x}$ for which an agent makes effort (if able) in order to keep his occupation. Moreover, the principal pays the bonus to a successful agent for the tasks $(\underline{x} - \bar{x})$. This raises the principal's cost without any gain, since the set of solved problems does not change. Thus, the firm owner is strictly better off by increasing \bar{x} .

- If an agent fails to perform a task $x \geq \bar{x}$, he keeps his job.

Consider a situation in which termination contract and bonus are simultaneously used by the principal, and the incentive compatible fixed wage is given by (6). If the principal fires agents who fail to solve tasks $x \geq \bar{x}$, he destroys the incentives provided with the termination contract to any agent with ability $\theta < \bar{x}$. Indeed, the probability that those employees are fired for lack of performance increases, thus their rent from employment is no more sufficient to guarantee effort provision. A more detailed discussion of this point can be found in Appendix 8.7. Note that this condition is only relevant if the contract proposed by the principal is a mix of bonus and termination contract.

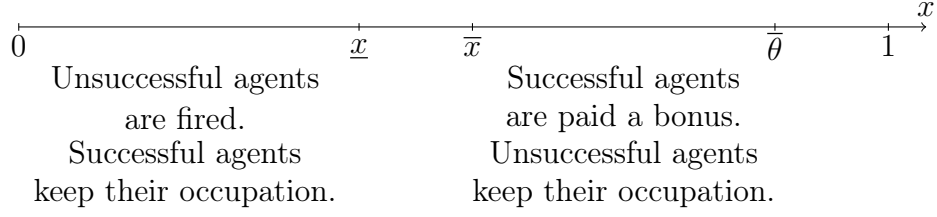


Figure 1: The self-enforceable contract C^E

In the rest of the section, we analyze the self-enforceability constraints for the terms of the contract.

4.2 Self-enforceability of the minimum performance standard

The problem of self-enforceability for a termination contract does not concern the fixed wage (a principal can credibly commit on its amount and payment) but the threat to fire an unsuccessful agent. Suppose the performance standard is set too high. Then, as employees are selected, the profit expected from an insider who has not solved a problem below the performance standard could still be larger than the profit expected from an outsider. Thus, the principal cannot credibly commit to fire the unsuccessful employee. Agents anticipate it and do not solve the problem.

The rule to fire any agent performing below the performance standard is self-enforcing, if the profit expected from an insider that fails to solve \underline{x} ($\pi^{in}(\theta \leq \underline{x})$) is lower than the profit expected from an unemployed agent (π^{out}).

$$(8) \quad \pi^{in}(\theta \leq \underline{x}) \leq \pi^{out}$$

where

$$\begin{aligned} \pi^{in}(\theta \leq \underline{x}) = & \underline{x} - \int_{\underline{\theta}}^{\underline{x}} \frac{(\underline{x} - \theta)f(\theta)}{F(\underline{x})} d\theta - w(\underline{x}) + (1 - \alpha)\delta\pi^{out} + \\ & \alpha\delta \left(\left(1 - \int_{\underline{\theta}}^{\underline{x}} \frac{(\underline{x} - \theta)f(\theta)}{F(\underline{x})} d\theta\right) \pi^{in}(\theta \leq \underline{x}) + \int_{\underline{\theta}}^{\underline{x}} \frac{(\underline{x} - \theta)f(\theta)}{F(\underline{x})} d\theta \pi^{out} \right) \end{aligned}$$

$$\pi^{out} = \underline{x} - \int_{\underline{\theta}}^{\underline{x}} (\underline{x} - \theta)q(\theta)d\theta + \int_{\bar{x}}^{\bar{\theta}} (\theta - \bar{x})q(\theta)d\theta(1 - b) - w(\underline{x}) + \delta\pi^{out}$$

Since for a given performance standard, the conditional expected productivity of an agent is increasing in the difficulty of the problem that has been transmitted, if the threat of firing is credible for an agent transmitting \underline{x} , it will also be credible for an employee who fails to solve $x < \underline{x}$.

After some transformations the self-enforceability condition (8) writes:

$$-(\underline{x} - \int_{\underline{\theta}}^{\underline{x}} \frac{\theta f(\theta)}{F(\underline{x})} d\theta) + Q(\underline{x}) (\underline{x} - \int_{\underline{\theta}}^{\underline{x}} \frac{\theta q(\theta)}{Q(\underline{x})} d\theta) - (1-b)(1-Q(\bar{x})) \left(\frac{\int_{\bar{x}}^{\bar{\theta}} \theta q(\theta) d\theta}{(1-Q(\bar{x}))} - \bar{x} \right) \leq 0$$

In order to simplify the exposition, we adopt the following notation for the rest of the paper: $l_{in} = \left(\underline{x} - \int_{\underline{\theta}}^{\underline{x}} \frac{\theta f(\theta)}{F(\underline{x})} d\theta \right)$, $l_{out} = \left(\underline{x} - \int_{\underline{\theta}}^{\underline{x}} \frac{\theta q(\theta)}{Q(\underline{x})} d\theta \right)$ and

$$h = \left(\frac{\int_{\bar{x}}^{\bar{\theta}} \theta q(\theta) d\theta}{(1-Q(\bar{x}))} - \bar{x} \right) \text{ which is equal}^{21} \text{ to } \left(\frac{\int_{\bar{x}}^{\bar{\theta}} \theta f(\theta) d\theta}{(1-F(\bar{x}))} - \bar{x} \right).$$

With the new notation, the constraint (8) writes:

$$(9) \quad -l_{in} + Q(\underline{x})l_{out} - (1-b)(1-Q(\bar{x}))h \leq 0$$

Since insiders are selected and have a larger expected productivity than outsiders, the threat to fire an agent may not be credible for some difficult tasks. It is easy to see that a performance standard $\underline{x} = \bar{\theta}$ cannot be self-enforced. Indeed, the constraint writes $\int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \theta q(\theta) d\theta \leq 0$. According to Lemma 2 the latter inequality is never satisfied. Therefore, the threat to fire the employee is not credible for this task, which destroys agents' incentives to work on it.

4.3 Self-enforceability of the bonus

As a bonus cannot be enforced by a third party the principal should have the right incentives to pay it. There are two possibilities for a principal who reneges on paying the bonus. First, he can fire a successful agent and replace him by an outsider. Second, he can keep the agent without paying the bonus. In the latter case the employee continues to perform tasks for which motivation is guaranteed

$$^{21} \int_{\bar{x}}^{\bar{\theta}} \frac{\theta f(\theta)}{1-F(\bar{x})} d\theta = \int_{\bar{x}}^{\bar{\theta}} \frac{\theta q(\theta) \frac{(1-\alpha+\alpha v)}{1-\alpha}}{1 - \frac{(1-\alpha+\alpha v)Q(\bar{x})-\alpha v}{1-\alpha}} d\theta = \int_{\bar{x}}^{\bar{\theta}} \frac{\theta q(\theta)}{1-Q(\bar{x})} d\theta$$

by the termination contract. Hence, the bonus can only be credible if the ongoing relationship with the employee provides a sufficient rent to the principal and if the profit expected from the performance of tasks under the bonus contract is sufficiently high to justify its payment. We now successively analyze each of those situations.

The principal pays the bonus rather than fire the agent. A firm owner does not pay the bonus unless the expected future gains from employment exceed its value. For given \bar{x} , the more difficult the problem solved by an agent, the higher his expected productivity. Thus, if the principal pays the bonus when an agent solves a problem \bar{x} , he will obviously pay it for problems with difficulty $x > \bar{x}$. The self-enforceability constraint states that the expected profit from an insider who solved a problem \bar{x} ($\pi^{in}(\theta \geq \bar{x})$) should be sufficiently high in comparison with the profit expected from an outsider:

$$(10) \quad -b + \alpha\delta\pi^{in}(\theta \geq \bar{x}) \geq \alpha\delta\pi^{out}$$

where

$$\pi^{in}(\theta \geq \bar{x}) = \underline{x} + \int_{\bar{x}}^{\bar{\theta}} \frac{(\theta - \bar{x})f(\theta)}{1 - F(\bar{x})} d\theta (1 - b) - w(\underline{x}) + (1 - \alpha)\delta\pi^{out} + \alpha\delta\pi^{in}(\theta \geq \bar{x})$$

The condition (10) becomes:

$$(11) \quad \frac{b(1 - \alpha\delta)}{\alpha\delta} - Q(\underline{x})\left(\underline{x} - \frac{\int_{\underline{\theta}}^{\underline{x}} \theta q(\theta) d\theta}{Q(\underline{x})}\right) - Q(\bar{x})\left(\frac{\int_{\bar{x}}^{\bar{\theta}} \theta q(\theta) d\theta}{1 - Q(\bar{x})} - \bar{x}\right)(1 - b) \leq 0$$

$$\Leftrightarrow \frac{b(1 - \alpha\delta)}{\alpha\delta} - Q(\underline{x})l_{out} - (1 - b)Q(\bar{x})h \leq 0$$

This constraint illustrates why it is impossible to use the bonus to provide incentives to perform “easy” tasks. For the bonus to be credible, outsiders should not be close substitutes for an insider who succeeds a problem. If $\bar{x} = \underline{x} = \underline{\theta}$, this condition is never satisfied. For $\underline{x} = \underline{\theta}$ there is no selection, *i.e.* the distribution of insiders is the same as the distribution of outsiders. Thus, the expected productivity of an

insider who solves a problem close to $\underline{\theta}$ is the same as the expected productivity of an outsider. An agent from the unemployment pool is a perfect substitute for the insider. There is no rent from keeping the successful agent, so the principal cheats on paying the bonus.

The principal pays the bonus rather than renege but keep the agent. As mentioned above, a principal could keep a successful agent without paying him the bonus. The following constraint prevents such situations:

$$(12) \quad -b + \alpha\delta\pi^{in}(\theta \geq \bar{x}) \geq \alpha\delta\pi_{dev}^{in}(\theta \geq \bar{x})$$

where $\pi_{dev}^{in}(\theta \geq \bar{x})$ is the expected profit from an insider who solved a problem $x = \bar{x}$ after principal's deviation on the payment of the bonus. An employee who has been cheated once on the bonus believes²² that the principal will continue to cheat. Hence, he stops solving problems for which motivation is guaranteed by the bonus. However, the employee still solves problems for which incentives are guaranteed with the termination contract²³.

$$\pi_{dev}^{in}(\theta \geq \bar{x}) = \underline{x} - w(\underline{x}) + (1 - \alpha)\delta\pi^{out} + \alpha\delta\pi_{dev}^{in}(\theta \geq \bar{x})$$

After simplifications (12) writes:

$$(13) \quad \frac{b(1 - \delta\alpha)}{\alpha\delta} - (1 - b)\left(\frac{\int_{\bar{x}}^{\bar{\theta}} \theta f(\theta) d\theta}{(1 - F(\bar{x}))} - \bar{x}\right) \leq 0$$

$$\Leftrightarrow \frac{b(1 - \alpha\delta)}{\alpha\delta} - (1 - b)h \leq 0$$

Note that when \bar{x} is close to $\bar{\theta}$, then the expected gain from an employee able to solve these tasks is low, which makes more profitable principal's deviation.

²²There are many patterns of beliefs consistent with equilibrium. We characterize the set of self-enforcing agreements using the most severe punishment.

²³It is in the employee's interest to do so, as far as his expected utility when employed and performing tasks below the performance standard \underline{x} , is larger than his expected utility if unemployed.

4.4 Self-enforceability of the mix of termination contract and bonus

Recall that in an equilibrium with joint use of a termination contract and a bonus it must be credible that the principal keeps agents who fail to solve problems with difficulty $x \geq \bar{x}$. This is self-enforceable if the profit expected from an insider who is unable of solving a problem \bar{x} ($\pi^{in}(\theta \leq \bar{x})$) must exceed the profit expected from an outsider:

$$(14) \quad \pi^{in}(\theta \leq \bar{x}) \geq \pi^{out}$$

where

$$\begin{aligned} \pi^{in}(\theta \leq \bar{x}) = & \underline{x} - \frac{\int_{\underline{\theta}}^{\underline{x}} (\underline{x} - \theta) f(\theta) d\theta}{F(\bar{x})} - w(\underline{x}) + (1 - \alpha)\delta\pi^{out} + \\ & \alpha\delta \left(\left(1 - \frac{\int_{\underline{\theta}}^{\underline{x}} (\underline{x} - \theta) f(\theta) d\theta}{F(\bar{x})}\right) \pi^{in}(\theta \leq \bar{x}) + \frac{\int_{\underline{\theta}}^{\underline{x}} (\underline{x} - \theta) f(\theta) d\theta}{F(\bar{x})} \pi^{out} \right) \end{aligned}$$

After simplifications the condition (14) becomes:

$$\frac{F(\underline{x})}{F(\bar{x})} \left(\underline{x} - \frac{\int_{\underline{\theta}}^{\underline{x}} \theta f(\theta) d\theta}{F(\underline{x})} \right) - Q(\underline{x}) \left(\underline{x} - \frac{\int_{\underline{\theta}}^{\underline{x}} \theta q(\theta) d\theta}{Q(\underline{x})} \right) + (1-b)(1-Q(\bar{x})) \left(\frac{\int_{\bar{x}}^{\bar{\theta}} \theta q(\theta) d\theta}{1-Q(\bar{x})} - \bar{x} \right) \leq 0$$

which is equivalent to:

$$(15) \quad \frac{F(\underline{x})}{F(\bar{x})} l_{in} - Q(\underline{x}) l_{out} + (1-b)(1-Q(\bar{x})) h \leq 0$$

Without the selection effect on insiders distribution ($l_{in} = l_{out} = l$ and $F(\theta) = Q(\theta)$), the left hand side of the constraint writes as $\left(lF(\underline{x}) \left(\frac{1}{F(\bar{x})} - 1 \right) + (1-b)(1-Q(\bar{x})) h \right)$, which is always positive, and the constraint is never satisfied.

Lemma 3. *A mix of termination contract and bonus can be self-enforced only if the expected productivity of the employee is sufficiently larger than the expected productivity of the average unemployed.*

5 Equilibrium

5.1 Principal's program

A principal is “too small” to have an impact on market conditions. Thus, each firm owner maximizes his profit for a given outside option (V_U) and outsiders' distribution ($Q(\theta)$). Then workers' reservation utility and insiders' and outsiders' distributions are determined consistently in market equilibrium.

The proportion of transmitted problems T under the contract C^E is:

$$(16) \quad T = 1 - \underline{x} + \int_{\underline{\theta}}^{\underline{x}} F(\theta) d\theta - \int_{\bar{x}}^{\bar{\theta}} (1 - F(\theta)) d\theta$$

By replacing (1) and (16) in (2), we can write the principal's program:

$$\max_{\underline{x}, \bar{x}} \Pi = \frac{1}{T} (1 - w - b \int_{\bar{x}}^{\bar{\theta}} (1 - F(\theta)) d\theta)$$

with

$$\begin{cases} b = c \\ w = c\underline{x} + \frac{c(1 - \alpha\delta)}{\alpha\delta} + (1 - \delta)V_U \end{cases}$$

and under the self-enforceability (9), (11), (13), (15) constraints.

5.2 The labor market

Employees' outside option. The equilibrium re-employment probability guarantees the equilibrium of flows between the firm and the labor market. The steady state flow into the unemployment pool is $n(1 - \alpha + \alpha v)$. Some agents quit for exogenous reasons $(1 - \alpha)$, others are fired for lack of performance αv . The flow out of the unemployment pool is $\lambda(N - n)$. A proportion λ of the $(N - n)$ unemployed agents²⁴ finds a job. Thus, we obtain: $\lambda = \frac{(1 - U)}{U} (1 - \alpha + \alpha v)$, where $U = \frac{N - n}{N}$ corresponds to the unemployment rate.

²⁴We assume that an agent who quits a firm cannot be immediately reemployed. It is a convention and it does not affect our qualitative results.

Abilities distributions. As discussed above, the initial distribution of ability is split into two stationary distributions - inside the firm and on the labor market. In equilibrium the following condition holds:

$$(17) \quad S(\theta) = (1 - U)F(\theta) + UQ(\theta)$$

The gap between the insiders' and outsiders' productivity plays an important role to characterize the self-enforceable relational contract. As we already explained it this gap is endogenous, from equations (7) and (17) we notice that it depends on labor market conditions, namely the unemployment rate and the exogenous turn-over.

In what follows, we discuss the extend to which agents' heterogeneity affects self-enforceability of termination contracts and performance-based bonus. Furthermore we analyze the extend to which the trade-off between those tools is affected by labor market conditions.

5.3 Pure termination contract

There exist values of the parameters for which a performance-based bonus cannot be self-enforced. Let us denote by m the mean and by σ^2 the variance that characterize the initial distribution of abilities.

Proposition 1. *If $c \geq \min\left\{\frac{\alpha\delta m}{1 - \alpha\delta(1 - m)}; \frac{\alpha\delta}{1 - \alpha\delta}\left(\bar{\theta} - m + \frac{\alpha\sigma^2}{1 - \alpha + \alpha(\bar{\theta} - m)}\right)\right\}$ then the bonus can not be self-enforced.*

We note that it is impossible to commit on a bonus if the expected surplus from paying it (agent's expected production) is not sufficiently high. More interestingly, the bonus cannot be implemented if the initial distribution of employees is not sufficiently heterogeneous. Indeed, a principal is eager to pay the bonus only if the surplus of continuing the relationship with the successful employee is high enough. To say it differently, by solving a difficult task a worker signals that he is more able than the average unemployed agent. This is sufficiently valuable to guarantee the payment of the bonus only if the distribution is not too homogeneous²⁵.

²⁵To illustrate our purpose assume that initially ability is uniformly distributed on the support

When Proposition 1 holds, the firm's owner motivates employees with a pure termination contract. He maximizes his profit under the self-enforceability constraint of \underline{x} :

$$-l_{in} + Q(\underline{x})l_{out} \leq 0$$

In this case the existence of heterogeneity limits the set of self-enforceable termination contracts, in the sense that a principal cannot credibly commit to fire an insider who fails to solve a difficult task. We show in Appendix 8.5 that there exists some threshold level for \underline{x} , denoted by \hat{x} , such that the principal cannot motivate employees to solve tasks $x > \hat{x}$. Since employees are in average more productive than the unemployed, the principal cannot be too demanding with them. Clearly, this restriction on the credible performance requirements is due to the endogenous selection of insiders. The degree of selection depends on the performance standard, but also on labor market conditions, namely the unemployment rate and exogenous turnover.

A higher exogenous turnover raises the incentive cost of the termination contract. Indeed, if agent's probability to quit the firm whatever his performance is larger, it raises the wage the principal should pay to motivate the employee. Thus, in a setting the principal is not constrained by the self-enforceability, a higher turnover leads to a decrease of the performance standard. In our setting a higher turnover may play in the opposite sense.

Proposition 2. *A higher turnover (i.e. lower α) softens the self-enforceability constraint and thus increases \hat{x} .*

A higher turnover reduces the degree of heterogeneity between the distribution of insiders and that of outsiders. First, when the proportion of agents who quit the firm for exogenous reasons is larger, insiders are less selected. The intuition is that among the employees who successfully performed the required tasks, larger proportion quits the firm and is replaced by unemployed agents. Second, in the unemployment pool, the proportion of unemployed for exogenous reasons (rather

$[m-\epsilon, m+\epsilon]$. Then the condition $\frac{\alpha\delta}{1-\alpha\delta} \left(\bar{\theta} - m + \frac{\alpha\sigma^2}{1-\alpha+\alpha(\bar{\theta}-m)} \right)$ writes as $\frac{\epsilon(1-\alpha) + (4/3)\alpha\epsilon^2}{1-\alpha+\alpha\epsilon}$. The latter only depends on ϵ and increases with its value.

than for lack of performance) increases, which improves outsiders' distribution. A lower gap in the expected productivity of insiders and outsiders expands the set of tasks for which the threat to fire an unsuccessful insider is credible.

5.4 Termination contract and bonus

In this section we discuss the characteristics of the incentive mix, when the principal uses both termination contract and performance-based bonus. Performing a difficult task signals high ability, and if outsiders are not close substitutes to insiders, the bonus payment may be credible.

It is in the principal's interest to use the bonus as a motivation tool for the largest possible set of tasks; he should choose the lower \bar{x} , compatible with the self-enforceability constraints. Hereafter we analyze which one of the following constraints is the most stringent, and thus determines the principal's decision.

$$\left\{ \begin{array}{l} -l_{in} + Q(\underline{x})l_{out} - (1-c)(1-Q(\bar{x}))h \leq 0 \quad (9) \\ \frac{c(1-\alpha\delta)}{\alpha\delta} - (1-c)h - Q(\underline{x})l_{out} + (1-c)(1-Q(\bar{x}))h \leq 0 \quad (11) \\ \frac{c(1-\alpha\delta)}{\alpha\delta} - (1-c)h \leq 0 \quad (13) \\ \frac{F(\underline{x})}{F(\bar{x})}l_{in} - Q(\underline{x})l_{out} + (1-c)(1-Q(\bar{x}))h \leq 0 \quad (15) \end{array} \right.$$

Constraint (15) guarantees that the expected productivity of an outsider does not exceed the expected productivity of an insider who is unable to solve \bar{x} . If this constraint is satisfied and if the surplus of tasks under bonus contract is sufficiently high to justify its payment (*i.e.* constraint (13) is satisfied), then it is never in the principal's interest to fire a successful insider and replace him by an outsider. To say it differently, if the principal uses a mix of termination contract and performance-based bonus, constraint (11) never binds.

The constraint (13) rewrites (by using (17)) as:

$$\frac{c(1-\alpha\delta)}{\alpha\delta} - (1-c) \left(\int_{\bar{x}}^{\bar{\theta}} \frac{\theta s(\theta)}{1-S(\bar{x})} d\theta - \bar{x} \right) \leq 0$$

For a large set of initial ability distributions, the left hand side of the constraint is

increasing²⁶ in \bar{x} (*i.e.* larger \bar{x} makes the constraint more difficult to satisfy). When the bonus is used for a small set of tasks, then the expected gain from an agent able to solve these tasks is smaller, which makes more profitable principal's deviation.

Conversely, the left hand side of (15) decreases with \bar{x} . So if there exists a solution for \bar{x} that satisfies both constraints, the principal chooses the lowest possible \bar{x} , constraint (15) obviously binds.

Finally, if the expected productivity of an outsider equals the expected productivity of an insider who is unable to solve \bar{x} , it is obviously larger than the expected productivity of an insider who cannot solve \underline{x} (since $\bar{x} \geq \underline{x}$). So if (15) binds, (9) is satisfied.

The principal maximizes his profit, with respect to \underline{x} , and \bar{x} is given by the constraint:

$$\frac{F(\underline{x})}{F(\bar{x})}l_{in} - Q(\underline{x})l_{out} + (1 - c)(1 - Q(\bar{x}))h = 0$$

When the constraint is satisfied with equality, an increase of \underline{x} raises \bar{x} . Thus, in equilibrium both incentive tools are substitutes. A more extensive use of the bonus reduces the set of tasks for which the principal motivates the agent through the termination contract.

Proposition 3. *A lower turnover (*i.e.* high α) expands the set of tasks for which the bonus is credible (*i.e.* \bar{x} decreases).*

In markets with low turnover, the bonus can be more extensively used by the principal. There are two reasons for that. First, a lower turnover increases the continuation probability of the relationship, thus increasing the cost of reneging on the agreement. Second, it expands the gap between insiders' and outsiders' productivity, which is the effect highlighted in Proposition 3. As already discussed in Section 5.3, a lower turnover improves insiders' distribution and worsens outsiders' distribution, thus increasing the productivity gap between the average employee and

²⁶As shown by Bagnoli and Bergstrom (2005), a sufficient condition for $(\frac{\int_{\bar{x}}^{\bar{\theta}} \theta s(\theta) d\theta}{(1 - S(\bar{x}))} - \bar{x})$ to be monotone decreasing in \bar{x} is that the density function $s(\theta)$ and the reliability function $\int_{\bar{x}}^{\bar{\theta}} (1 - S(\theta)) d\theta$ are log-concave. For a list of the distributions that satisfy these conditions, we refer the reader to Bagnoli and Bergstrom (2005).

unemployed agent, which in turn relaxes constraint (15) and reduces \bar{x} . In fine the positive effect of lower turnover on the utilization of the bonus is twofold. It is used for a larger set of tasks, furthermore insiders are better selected, thus problems for which the bonus is paid are solved more often.

6 Discussion

This article provides some new elements in the understanding of incentive schemes used by managers. We derive conditions for the simultaneous use of termination contracts and bonus payments in order to motivate employees to work. We show that this is possible only if there is a difference in the expected productivity of an employee and an unemployed agent. The gap in expected productivities of employees and unemployed, in our setting, can be driven by: either the impact of *ex post* selection on insiders' and outsiders' distributions or agent's seniority. In the paper we focus on the first effect and set the second one aside (with assumption 3). Over time unsuccessful employees' are fired and replaced, insiders' and outsiders' stationary distributions are endogenously obtained, which generates the difference in the expected productivity of employed and unemployed agents.

Let us now briefly discuss the potential effect of seniority. To expose our intuitions we assume that the pool of unemployed has the same characteristics over time (set aside the endogenous selection). An agent who has been in the firm for a long time has larger expected productivity than an unemployed one. The reason for that is related on the fact that an agent with longer career in the firm is one that succeeded successive tests, *i.e.* an employee that has not been fired for lack of performance. When an agent arrives in the firm the bonus cannot be credibly proposed since his expected productivity is the same as that of an unemployed. However, with employee's seniority bonus could be used to motivate the agent to perform some difficult tasks. The larger the number of periods the agent spends in the firm the larger the set of tasks for which the bonus can be credibly used. We hope that a more precise characterization of the optimal contract with seniority would be the subject of a future work. However, we think that in a model that would take into

account both the endogenous evolution of distributions of insiders and outsiders, and the effect of seniority, the findings of this paper would be still present.

7 References

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8 Appendix

8.1 Proof of Lemma 1

The incentive compatibility constraint for the fixed wage of a termination contract writes:

$$V(\theta) - V_U(\theta) \geq \frac{c}{\alpha\delta}$$

For an agent with type $\theta < \underline{x}$. The difference between the expected utility when employed and the expected utility when unemployed is $V(\theta) - V_U(\theta) = \frac{w - z - \theta c}{1 - \alpha\delta(1 - \underline{x} + \theta) + \lambda\delta}$. Thus, the incentive compatibility constraint corresponds to:

$$\frac{w - z - \theta c}{1 - \alpha\delta(1 - \underline{x} + \theta) + \lambda\delta} \geq \frac{c}{\alpha\delta} \Leftrightarrow w \geq \frac{c(1 - \alpha\delta)}{\alpha\delta} + c\underline{x} + z + \frac{c\lambda}{\alpha}$$

For an agent with type $\theta \geq \underline{x}$. Now we have:

$$V(\theta) - V_U(\theta) = \frac{w - z - \underline{x}c}{1 - \alpha\delta + \lambda\delta}$$

$$\frac{w - z - \underline{x}c}{1 - \alpha\delta + \lambda\delta} \geq \frac{c}{\alpha\delta} \Leftrightarrow w \geq \frac{c(1 - \alpha\delta)}{\alpha\delta} + c\underline{x} + z + \frac{c\lambda}{\alpha}$$

Thus, the incentive compatible wage does not depend on θ .

8.2 Stationary distributions

The distribution of insiders at the beginning of a period is $F(\theta)$. $\tilde{F}(\theta)$ is the modified distribution after departures (for exogenous and endogenous reasons) took place. Vacancies are fulfilled to agents from the unemployment pool, with cumulative distribution function $Q(\theta)$.

Stationarity condition writes:

$$(18) \quad \underbrace{(1 - \alpha + \alpha v)}_{\text{proportion of hired agents}} Q(u) + \underbrace{\alpha(1 - v)}_{\text{proportion of remaining insiders}} \tilde{F}(u) = F(u)$$

Where $\tilde{F}(u)$ is:

$$(19) \quad \tilde{F}(u) = \begin{cases} \frac{\int_{\underline{\theta}}^u (1 - (\underline{x} - \theta))f(\theta)d\theta}{1 - v} & \text{if } u \in [\underline{\theta}; \underline{x}] \\ \frac{F(u) - v}{1 - v} & \text{if } u \in [\underline{x}; \bar{\theta}] \end{cases}$$

Thus, from (18) and (19) we obtain:

$$Q(u) = \begin{cases} \frac{F(u)(1 - \alpha) + \alpha \int_{\underline{\theta}}^u (\underline{x} - \theta)f(\theta)d\theta}{1 - \alpha + \alpha v} & \text{if } u \in [\underline{\theta}; \underline{x}] \\ \frac{F(u)(1 - \alpha) + \alpha v}{1 - \alpha + \alpha v} & \text{if } u \in [\underline{x}; \bar{\theta}] \end{cases}$$

8.3 Proof of Lemma 2

The case of $\theta \geq \underline{x}$:

$$\begin{aligned} Q(\theta) - F(\theta) &= \frac{(1 - \alpha)F(\theta) + \alpha v}{1 - \alpha + \alpha v} - F(\theta) \\ &= \frac{\alpha v(1 - F(\theta))}{1 - \alpha + \alpha v} \end{aligned}$$

It is easy to see that

- $\frac{\alpha v(1 - F(\theta))}{1 - \alpha + \alpha v} > 0$, for any $\theta < \bar{\theta}$
- $\frac{\alpha v(1 - F(\theta))}{1 - \alpha + \alpha v} = 0$ for $\theta = \bar{\theta}$

The case of $\theta < \underline{x}$:

$$\begin{aligned} Q(\theta) - F(\theta) &= \frac{(1 - \alpha)F(\theta) + \alpha \int_{\underline{\theta}}^{\theta} (\underline{x} - u)f(u)du}{1 - \alpha + \alpha v} - F(\theta) \\ &= \frac{\alpha \int_{\underline{\theta}}^{\theta} (\underline{x} - u - v)f(u)du}{1 - \alpha + \alpha v} \end{aligned}$$

$\int_{\underline{\theta}}^{\theta} (\underline{x} - u - v)f(u)du$ increases with θ when $\theta < \underline{x} - v$ and decreases with θ when $\theta > \underline{x} - v$. So if $\int_{\underline{\theta}}^{\theta} (\underline{x} - u - v)f(u)du \geq 0$ for $\theta = \underline{\theta}$ and $\theta = \underline{x}$, it is also positive for any $\theta \in]\underline{\theta}, \underline{x}[$.

- For $\theta = x_w$, $\int_{\underline{\theta}}^{x_w} (x_w - u - v_w)f(u)du = (1 - F(x_w))v_w > 0$.

- For $\theta = \underline{\theta}$, $\int_{\underline{\theta}}^{\bar{\theta}} (x_w - u - v_w) f(u) du = 0$.

8.4 Proof of Proposition 1

The condition $c \geq \frac{\alpha\delta m}{1 - \alpha\delta(1 - m)}$

Constraint (13) writes:

$$\frac{c(1 - \alpha\delta)}{(1 - c)\alpha\delta} - \int_{\bar{x}}^{\bar{\theta}} \frac{(\theta - \bar{x})s(\theta)}{1 - S(\bar{x})} d\theta \leq 0$$

We assume that the initial distribution is such that the left hand side increases with \bar{x} . A sufficient condition for this is that the density function $s(\theta)$ and the reliability function $\int_{\bar{x}}^{\bar{\theta}} (1 - S(\theta)) d\theta$ are log-concave. See Bagnoli and Bergstrom (2005) for a list of distributions that satisfy those conditions. Thus, if the constraint is not satisfied for $\bar{x} = 0$ it will never be satisfied. This we obtain the following condition:

$$\frac{c(1 - \alpha\delta)}{(1 - c)\alpha\delta} - \int_{\underline{\theta}}^{\bar{\theta}} \theta s(\theta) d\theta > 0 \Leftrightarrow c \geq \frac{\alpha\delta m}{1 - \alpha\delta(1 - m)}$$

The condition $c \geq \frac{\alpha\delta}{1 - \alpha\delta} \left(\bar{\theta} - m + \frac{\alpha\sigma^2}{1 - \alpha + \alpha(\bar{\theta} - m)} \right)$

Constraint (11) writes:

$$\frac{c(1 - \alpha\delta)}{\alpha\delta} - Q(\underline{x}) \left(\underline{x} - \frac{\int_{\underline{\theta}}^{\underline{x}} \theta q(\theta) d\theta}{Q(\underline{x})} \right) - (1 - c)Q(\bar{x}) \left(\frac{\int_{\bar{x}}^{\bar{\theta}} \theta q(\theta) d\theta}{1 - Q(\bar{x})} - \bar{x} \right) \leq 0$$

We are interested on the case, this constraint is not satisfied even in the most favorable situation.

First, the left hand side is decreasing in \underline{x} . Thus, for $\underline{x} = \bar{x}$ we have:

$$\frac{c(1 - \alpha\delta)}{\alpha\delta} - (1 - c) \frac{Q(\bar{x})}{1 - Q(\bar{x})} \int_{\bar{x}}^{\bar{\theta}} (1 - Q(\theta)) d\theta - \int_{\underline{\theta}}^{\bar{x}} (\bar{x} - \theta) q(\theta) d\theta \leq 0$$

The latter is the most easy to satisfy for $\bar{x} = \bar{\theta}$:

$$\frac{c(1 - \alpha\delta)}{\alpha\delta} - \int_{\underline{\theta}}^{\bar{\theta}} (\bar{\theta} - \theta) q(\theta) d\theta \leq 0 \Leftrightarrow \frac{c(1 - \alpha\delta)}{\alpha\delta} - \int_{\underline{\theta}}^{\bar{\theta}} \frac{(\bar{\theta} - \theta)s(\theta)(1 - \alpha + \alpha(\bar{\theta} - \theta))}{1 - \alpha + \alpha v + U\alpha(\bar{\theta} - \theta - v)} d\theta \leq 0$$

Finally we can show that the constraint is the easiest to satisfy for $U \rightarrow 0$. Thus,

the constraint becomes:

$$\frac{c(1 - \alpha\delta)}{\alpha\delta} - \int_{\underline{\theta}}^{\bar{\theta}} \frac{(\bar{\theta} - \theta)s(\theta)(1 - \alpha + \alpha(\bar{\theta} - \theta))}{1 - \alpha + \alpha v} d\theta \leq 0$$

where $v = \int_{\underline{\theta}}^{\bar{\theta}} (\bar{\theta} - \theta)s(\theta)d\theta = \bar{\theta} - m$ and $\int_{\underline{\theta}}^{\bar{\theta}} (\bar{\theta} - \theta)^2 s(\theta)d\theta = (\sigma^2 + (\bar{\theta} - m)^2)$. We replace in $\frac{c(1 - \alpha\delta)}{\alpha\delta} - \int_{\underline{\theta}}^{\bar{\theta}} \frac{(\bar{\theta} - \theta)s(\theta)(1 - \alpha + \alpha(\bar{\theta} - \theta))}{1 - \alpha + \alpha v} d\theta \leq 0$, thus after simplifications we obtain $c \leq \frac{\alpha\delta}{1 - \alpha\delta} \left(\bar{\theta} - m + \frac{\alpha\sigma^2}{1 - \alpha + \alpha(\bar{\theta} - m)} \right)$.

So if $c \geq \frac{\alpha\delta}{1 - \alpha\delta} \left(\bar{\theta} - m + \frac{\alpha\sigma^2}{1 - \alpha + \alpha(\bar{\theta} - m)} \right)$, the constraint (11) is never satisfied.

8.5 Proof that some performance standards are not self-enforceable

We show that the self-enforceability constraint for \underline{x} is satisfied for some values and that there exists some \hat{x} above which performance standards are not credible.

We proceed in two steps:

- First we show that the constraint is binding for $\underline{x} = \underline{\theta} \equiv 0$ and not satisfied for $\underline{x} = \bar{\theta}$.
- Second we show that it is decreasing in \underline{x} , for $\underline{x} \rightarrow 0$.

First step: The constraint is:

$$B \equiv -\left(\underline{x} - \int_0^{\underline{x}} \frac{\theta f(\theta)}{F(\underline{x})} d\theta\right) + Q(\underline{x})\left(\underline{x} - \int_0^{\underline{x}} \frac{\theta q(\theta)}{Q(\underline{x})} d\theta\right) \leq 0$$

- For $\underline{x} = \bar{\theta}$, $B = \int_0^{\bar{\theta}} \theta f(\theta) d\theta - \int_0^{\bar{\theta}} \theta q(\theta) d\theta$. Under Lemma 2 we have $B > 0$, the constraint is not satisfied.
- For $\underline{x} = 0$, $B = 0$

Second step: We now show that B is decreasing in \underline{x} , for $\underline{x} \rightarrow 0$.

Let us first assume that in the firm and on the labor market there is the same distribution $F(\theta)$ (there is no specification on the distribution function $F(\cdot)$). In this case the constraint writes:

$$\tilde{B} \equiv -(\underline{x} - \int_0^{\underline{x}} \frac{\theta f(\theta)}{F(\underline{x})} d\theta) + F(\underline{x})(\underline{x} - \int_0^{\underline{x}} \frac{\theta f(\theta)}{F(\underline{x})} d\theta) \leq 0$$

It is satisfied for any \underline{x} . For $\underline{x} = 0$, $\tilde{B} = 0$. So the derivative of \tilde{B} with respect to \underline{x} , for $\underline{x} = 0$ should be negative. We have $\frac{\partial \tilde{B}}{\partial \underline{x}} = -(1 - F(\underline{x})) + \frac{f(\underline{x})}{F(\underline{x})}(\underline{x} - \int_0^{\underline{x}} \frac{\theta f(\theta)}{F(\underline{x})} d\theta) \Big|_{\underline{x}=0} < 0$

Let us now turn to the case of different distributions, one on the market, and one inside the firm. After simplifications the derivative of the constraint writes:

$$\frac{\partial B}{\partial \underline{x}} = -\frac{Q(\underline{x})}{F(\underline{x})} \left(\frac{\frac{\partial v}{\partial \underline{x}}}{F(\underline{x})} - F(\underline{x}) \right) + \frac{f(\underline{x})}{F(\underline{x})} \left(\underline{x} - \int_0^{\underline{x}} \frac{\theta f(\theta)}{F(\underline{x})} d\theta \right)$$

$\frac{\partial v}{\partial \underline{x}} = \int_0^{\underline{x}} \frac{f(\theta)(1 - \alpha + \alpha v)}{(1 - \alpha + \alpha(\underline{x} - \theta))} d\theta \geq \frac{F(\underline{x})(1 - \alpha + \alpha v)}{(1 - \alpha + \alpha \underline{x})}$ and from Lemma 2 we know that $Q(\underline{x}) \geq F(\underline{x})$. It follows that:

$$\frac{\partial B}{\partial \underline{x}} \leq -\left(\frac{1 - \alpha + \alpha v}{1 - \alpha + \alpha \underline{x}} - F(\underline{x}) \right) + \frac{f(\underline{x})}{F(\underline{x})} \left(\underline{x} - \int_0^{\underline{x}} \frac{\theta f(\theta)}{F(\underline{x})} d\theta \right)$$

For $\underline{x} \rightarrow 0$ the latter expression converges to $\frac{\partial \tilde{B}}{\partial \underline{x}} \Big|_{\underline{x}=0}$ so we should have $\frac{\partial B}{\partial \underline{x}} \Big|_{\underline{x}=0} < 0$.

8.6 Proof of Propositions 2 and 3

Before proceeding to the proof of the propositions, we derive the expressions of $f(\theta)$ and $q(\theta)$ as functions of the initial distribution of abilities $s(\theta)$:

$$f(\theta) = \begin{cases} \frac{(1 - \alpha + \alpha v)s(\theta)}{(1 - \alpha + \alpha v) + U\alpha(\underline{x} - \theta - v)} & \text{if } \theta \in [\underline{\theta}; \underline{x}] \\ \frac{(1 - \alpha + \alpha v)s(\theta)}{(1 - \alpha + \alpha v) - U\alpha v} & \text{if } \theta \in [\underline{x}; \bar{\theta}] \end{cases}$$

$$q(\theta) = \begin{cases} \frac{(1 - \alpha + \alpha(\underline{x} - \theta))s(\theta)}{(1 - \alpha + \alpha v) + U\alpha(\underline{x} - \theta - v)} & \text{if } \theta \in [\underline{\theta}; \underline{x}] \\ \frac{(1 - \alpha)s(\theta)}{(1 - \alpha + \alpha v) - U\alpha v} & \text{if } \theta \in [\underline{x}; \bar{\theta}] \end{cases}$$

Equations (7) and (17) are used.

Recall that $v = \int_{\underline{\theta}}^{\underline{x}} F(\theta) d\theta$. It is immediate to see that $v < \underline{x}F(\underline{x}) < \underline{x}$. Hereafter, we consider $\underline{\theta} = 0$.

8.6.1 Proof of Proposition 2

The self-enforceability constraint for termination contract is:

$$-\frac{\int_0^{\underline{x}} (\underline{x} - \theta) f(\theta) d\theta}{F(\underline{x})} + \int_0^{\underline{x}} (\underline{x} - \theta) q(\theta) d\theta \leq 0$$

The derivative with respect to α writes:

$$-\frac{1}{F(\underline{x})} \int_0^{\underline{x}} (\underline{x} - \theta) \frac{df(\theta)}{d\alpha} d\theta + \frac{v}{F(\underline{x})^2} \int_0^{\underline{x}} \frac{df(\theta)}{d\alpha} d\theta + \int_0^{\underline{x}} (\underline{x} - \theta) \frac{dq(\theta)}{d\alpha} d\theta$$

We adopt the following notation:

$$A \equiv -\frac{1}{F(\underline{x})} \int_0^{\underline{x}} (\underline{x} - \theta) \frac{df(\theta)}{d\alpha} d\theta + \frac{v}{F(\underline{x})^2} \int_0^{\underline{x}} \frac{df(\theta)}{d\alpha} d\theta$$

$$B \equiv \int_0^{\underline{x}} (\underline{x} - \theta) \frac{dq(\theta)}{d\alpha} d\theta$$

In what follows we show that each of these terms is positive.

Proof that $B > 0$

$$B = \int_0^{\underline{x}} \frac{(1-U)s(\theta)(\underline{x}-\theta-v)(\underline{x}-\theta)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))^2} d\theta - \frac{\partial v}{\partial \alpha} \int_0^{\underline{x}} \frac{(1-U)s(\theta)\alpha(1-\alpha+\alpha(\underline{x}-\theta))(\underline{x}-\theta)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))^2} d\theta$$

- The first term rewrites as follows: $\frac{(1-U)}{1-\alpha+\alpha v} \int_0^{\underline{x}} \frac{f(\theta)(\underline{x}-\theta-v)(\underline{x}-\theta)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))} d\theta$

We notice that for low (resp high) θ , $f(\theta)(\underline{x}-\theta-v)$ is positive (resp negative).

Since $\frac{(\underline{x}-\theta)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))}$ is decreasing in θ the positive terms are highly weighted. Thus, if the expression is positive when all the terms are equally weighted, it will also be positive in the case of highly weighted positive

terms. With equally weighted terms we have

$$\begin{aligned} \frac{(1-U)k}{1-\alpha+\alpha v} \int_0^{\underline{x}} f(\theta)(\underline{x}-\theta-v)d\theta &= \frac{(1-U)k}{1-\alpha+\alpha v} v(1-F(\underline{x})) \\ &> 0 \end{aligned}$$

where k is a constant. So

$$\frac{(1-U)}{1-\alpha+\alpha v} \int_0^{\underline{x}} \frac{f(\theta)(\underline{x}-\theta-v)(\underline{x}-\theta)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))} d\theta > 0$$

- Now we show that $\frac{\partial v}{\partial \alpha} < 0$.

$$\begin{aligned} \frac{\partial v}{\partial \alpha} \gamma &= - \int_0^{\underline{x}} \frac{U(\underline{x}-\theta)s(\theta)(\underline{x}-\theta-v)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))^2} d\theta \\ &< 0 \end{aligned}$$

where $\gamma = \left(\frac{(1-\alpha)}{1-\alpha+\alpha v} + \int_0^{\underline{x}} \frac{(\underline{x}-\theta)s(\theta)\alpha(1-U)(1-\alpha+\alpha v)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))^2} d\theta \right) \Rightarrow \gamma > 0$.

To show that $-\int_0^{\underline{x}} \frac{U(\underline{x}-\theta)s(\theta)(\underline{x}-\theta-v)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))^2} d\theta < 0$ we apply the same kind of argument as above.

Thus, we have $B > 0$.

Proof that $A > 0$

$$A = \underbrace{-\frac{1}{F(\underline{x})^2} \int_0^{\underline{x}} \frac{\partial f(\theta)}{\partial \alpha} ((\underline{x}-\theta)F(\underline{x})-v)d\theta}_{\equiv a} - \underbrace{\frac{1}{F(\underline{x})^2} \frac{\partial v}{\partial \alpha} \int_0^{\underline{x}} \frac{\partial f(\theta)}{\partial v} ((\underline{x}-\theta)F(\underline{x})-v)d\theta}_{\equiv b}$$

- Proof of $a > 0$

$$\begin{aligned} a &= \frac{1}{F(\underline{x})^2} \int_0^{\underline{x}} \frac{Us(\theta)(\underline{x}-\theta-v)((\underline{x}-\theta)F(\underline{x})-v)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))^2} d\theta \\ &= \frac{1}{F(\underline{x})^2(1-\alpha+\alpha v)} \int_0^{\underline{x}} \frac{Uf(\theta)(\underline{x}-\theta-v)((\underline{x}-\theta)F(\underline{x})-v)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))} d\theta \\ &\geq \frac{U}{F(\underline{x})^2(1-\alpha+\alpha v)} \int_0^{\underline{x}-v} \frac{f(\theta)(\underline{x}-\theta-v)((\underline{x}-\theta)F(\underline{x})-v)}{((1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v))} d\theta \end{aligned}$$

In the last expression the positive terms are highly weighted. So it is sufficient to show that the expression is non negative when all terms are equally weighted:

$$\begin{aligned}
\int_0^{\underline{x}-v} f(\theta)(F(\underline{x})(\underline{x}-\theta)-v)d\theta &= vF(\underline{x})F(\underline{x}-v) + F(\underline{x})(v - \int_{\underline{x}-v}^{\underline{x}} F(\theta)d\theta) - vF(\underline{x}-v) \\
&\geq vF(\underline{x})F(\underline{x}-v) + F(\underline{x})(v - F(\underline{x})v) - vF(\underline{x}-v) \\
&= v(1 - F(\underline{x}))(F(\underline{x}) - F(\underline{x}-v)) \\
&> 0
\end{aligned}$$

Thus, we have $a > 0$.

- Proof of $b > 0$

$$b = -\frac{U\alpha}{F(\underline{x})^2(1-\alpha+\alpha v)} \frac{\partial v}{\partial \alpha} \int_0^{\underline{x}} \frac{f(\theta)(1-\alpha+\alpha(\underline{x}-\theta))(F(\underline{x})(\underline{x}-\theta)-v)}{(1-\alpha+\alpha v) + U\alpha(\underline{x}-\theta-v)} d\theta$$

As we have already shown $\frac{\partial v}{\partial \alpha} < 0$, so the sign of b is given by the expression:

$$\begin{aligned}
\int_0^{\underline{x}} \frac{f(\theta)(1-\alpha+\alpha(\underline{x}-\theta))(F(\underline{x})(\underline{x}-\theta)-v)}{(1-\alpha+\alpha v) + U\alpha(\underline{x}-\theta-v)} d\theta &\geq k \int_0^{\underline{x}} f(\theta)(F(\underline{x})(\underline{x}-\theta)-v)d\theta \\
&\geq 0
\end{aligned}$$

Thus, we have $b > 0$

8.6.2 Proof of Proposition 3

The constraint writes:

$$\frac{\int_0^{\underline{x}} (\underline{x}-\theta)f(\theta)d\theta}{F(\bar{x})} - \int_0^{\underline{x}} (\underline{x}-\theta)q(\theta)d\theta + (1-b) \int_{\bar{x}}^{\bar{\theta}} (1-Q(\theta))d\theta \leq 0$$

The derivative with respect to α is:

$$\underbrace{\frac{1}{F(\bar{x})^2} \left(\int_0^{\underline{x}} (\underline{x}-\theta)F(\bar{x}) \frac{df(\theta)}{d\alpha} d\theta - v \int_0^{\bar{x}} \frac{df(\theta)}{d\alpha} d\theta \right)}_{\equiv J} - \underbrace{\int_0^{\underline{x}} (\underline{x}-\theta) \frac{dq(\theta)}{d\alpha} d\theta}_{\equiv L} - \underbrace{(1-b) \int_{\bar{x}}^{\bar{\theta}} \frac{dQ(\theta)}{d\alpha} d\theta}_{\equiv H}$$

The sign of L . $L = -B < 0$.

The sign of H .

$$H = -(1-b) \left(\int_{\bar{x}}^{\bar{\theta}} \frac{(1-U)(1-S(\theta))v}{(1-\alpha+\alpha v-U\alpha v)^2} - \frac{\partial v}{\partial \alpha} \int_{\bar{x}}^{\bar{\theta}} \frac{(1-U)(1-S(\theta))\alpha(1-\alpha)}{(1-\alpha+\alpha v-U\alpha v)^2} \right)$$

We have already shown that $\frac{\partial v}{\partial \alpha} < 0$, so $H < 0$.

The sign of J . We can rewrite the expression as follows:

$$J = \frac{1}{F(\bar{x})^2} \left(\int_0^{\underline{x}} ((\underline{x}-\theta)F(\bar{x})-v) \frac{df(\theta)}{d\alpha} d\theta - v \int_{\underline{x}}^{\bar{x}} \frac{df(\theta)}{d\alpha} d\theta \right)$$

Note that $\int_0^{\underline{x}} ((\underline{x}-\theta)F(\bar{x})-v) \frac{df(\theta)}{d\alpha} d\theta$, is similar to $-A$, thus by applying the same kind of analysis we can show that it is negative. The last term:

$$\begin{aligned} & - \int_{\underline{x}}^{\bar{x}} \frac{Us(\theta)v^2}{(1-\alpha+\alpha v-U\alpha v)^2} d\theta - \frac{\partial v}{\partial \alpha} \int_{\underline{x}}^{\bar{x}} \frac{Us(\theta)(1-\alpha)\alpha v}{(1-\alpha+\alpha v-U\alpha v)^2} d\theta = \\ & - \int_{\underline{x}}^{\bar{x}} \frac{Us(\theta)v}{(1-\alpha+\alpha v-U\alpha v)^2} d\theta \left(v + \frac{\partial v}{\partial \alpha} \alpha(1-\alpha) \right) \end{aligned}$$

We show that $\left(v + \frac{\partial v}{\partial \alpha} \alpha(1-\alpha) \right)$ is positif. Indeed:

$$\left(v + \frac{\partial v}{\partial \alpha} \alpha(1-\alpha) \right) \geq \int_0^{\underline{x}} (\underline{x}-\theta)f(\theta)d\theta - \int_0^{\underline{x}} \frac{(\underline{x}-\theta)f(\theta)U\alpha(\underline{x}-\theta-v)}{(1-\alpha+\alpha v)+U\alpha(\underline{x}-\theta-v)} d\theta > 0.$$

So $J < 0$.

The left hand side of the constraint decreases with α .

8.7 Characteristics of the equilibrium contract

An agent who fails to perform a task $x > \bar{x}$ keeps his occupation. Let us assume that the principal proposes a contract with termination contract and bonus, and that he announces that agents who fail to perform a task $x \in [\bar{x}, \bar{x} + \Delta]$, will be fired. In this case the inter-temporal expected utility of an agent $\theta < \bar{x}$ writes:

$$V(\theta) = w - \theta c + \delta(1-\alpha)V_U(\theta) + \delta\alpha[(1-\underline{x}-\Delta+\theta)V(\theta) + (\underline{x}+\Delta-\theta)V_U(\theta)]$$

The introduction of the bonus and the threat for an agent to be fired if not performing tasks larger than \bar{x} , reduces the inter-temporal utility of any employee with $\theta < \bar{x}$. Thus, the incentive compatible wage for problems $x \leq \underline{x}$ should be:

$$w = c(\underline{x} + \Delta) + \frac{c(1 - \alpha\delta)}{\alpha\delta} + (1 - \delta)V_U$$

We notice that if the principal increases the efficiency wage by Δ , then the payment of the bonus duplicates²⁷ the incentives and is not optimal.

²⁷Recall that any, task independently of its difficulty, provides the same profit to the principal if solved.