Does product market competition improve the labour market performance? *

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Abstract

In this paper, I construct a model in which the labour market exhibits search frictions, whereas Cournot competition is assumed in the intermediate goods markets. Although all the sectors are identical "ex ante ", at the equilibrium, the level of competition, the real wage, and the amount of hours worked is endogenously distributed. The properties of the long run free-entry equilibrium show that a more competitive product market raises employment, but it has ambiguous effects both on the average real wage and on the utility of the employees. Moreover, from a normative viewpoint, the level of employment and the degree of competition may be inefficiently high. Numerical results based on Belgian data are finally performed.

Keywords: product market competition; search-matching equilibrium; barriers to entry.
JEL codes: E24, J64, L16

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1 Introduction

The interactions between product market (de)regulation and labour market performance have been the objective of many empirical and theoretical papers in recent years. Does tougher competition in the goods market increase the level of employment in the labour market? According to most of the literature, the answer seems to be a qualified yes. At a theoretical level, more agents competing in the product market implies a lower mark-up that can be chosen by each single firm and a larger aggregate quantity produced in equilibrium. This in turn raises labour demand, for any given level of wages. Such a theoretical prediction seems to be confirmed by recent empirical studies. For instance, according to the OECD (2006), liberalization in goods market is one decisive factor that helps to explain why some countries (Ireland, Austria, Scandinavia, and the Netherlands) experience high employment rates even if their labour markets remain very regulated. In this avenue, the most recent analyzes are conducted by Nicoletti and Scarpetta (2005) and Griffith, Harrison, and Macartney (2007), while a detailed survey is in Schiantarelli (2005). Considering a panel of some OECD countries over the past two decades, Nicoletti and Scarpetta reach two important conclusions. First, regulations that curb competition and entry have substantially reduced the employment rates in OECD countries over the past two decades. Second, the negative impact of such product market rigidities on employment is much costlier, the more regulated is the labour market. Therefore, product market reforms should induce larger gains in term of employment in countries whose labour market is more rigid.

Less attention, however, has been devoted to the welfare implications of product marked (de)regulation on the labour market.

The objective of this paper is twofold. First, I analyze the effects of tougher competition in the goods market on employment, wages and hours worked when the labour market present frictions and efficient bargaining is assumed between workers and firms. Second, turning to the normative analysis, I wonder what is the optimal level of competition and employment in such economy.

To perform this task, I construct a general equilibrium model with Cournot competition in the goods sector and matching frictions *à la* Pissarides (2000) in the labour market.

More in detail, I consider an economy with a finite and exogenous number of intermediate sectors, each of them composed by a constant labour force, and only one final consumption good. In the final good sector perfect competition is assumed, whereas firms compete *à la* Cournot in the intermediate sectors. The labour market presents frictions, so that the matching of firms with workers is a process that takes time. The creation and the destruction of jobs in each market follow a continuous-time Markov Chain with a discrete number of states. The probability that one more job is created in one sector is endogenous and depends on the level of unemployment and the number of vacancies posted in that sector. In addition, at a certain exogenous rate, a new intermediate product, replacing an existing one, is invented in the economy and all the jobs present in the "old" sector are destroyed. A free-entry condition imposes that firms post vacancies as long as they earn positive expected profits. At the equilibrium, the level of competition (i.e. the number of firms competing), the real wage, and the amount of hours worked is not the same among the intermediate markets but is endogenously distributed.

The choice of Cournot competition is made for two reasons. First, differently from other papers (for instance Blanchard and Giavazzi, 2003 and Ebell and Haefke, 2006), I am considering a framework in which the number of firms producing in a market varies in equilibrium according to a stochastic process, so that any firm' strategy depends not only on the actual level of competition, but also on the probability that new competitors will enter the market. The properties of the Cournot equilibrium as the number of players varies are well-known (see Frank, 1965), and it seems therefore an appropriate choice for this kind of analysis. Second, this paper focuses on the long run free-entry equilibrium, in which Cournot models are not subject to the critiques sometimes addressed to other settings (for instance, free-entry in a monopolistic competition set-up is modeled as a change in the elasticity of substitution in the utility function, a parameter that should remain fixed).
The main contributions of this paper are the following.

I show that a deregulation in the product market (captured by a decrease in the entry costs) raises employment, the real wages and workers' utility, while a decrease in workers' bargaining power increases employment but has an ambiguous impact on the real wages. The ambiguity comes from the two opposite effects that intervene on the real wage. A reduction in workers' bargaining power implies a lower fraction of the rents accruing to the employees (so depressing the real wages) but also an increase in employment and in turn a larger amount and a lower price of the final consumption good produced (an income effect that tends to raise the real wages).

Blanchard and Giavazzi (2003) conclude that in the long run a reduction in workers' bargaining power leads to more employment and leaves real wages unaffected. This result depends on the two specific features of their model: the assumption of symmetric equilibrium (in which any intermediate firm sets the same price), and the long-run equilibrium condition under which profits per worker are equal to a fixed cost of entry. In a symmetric equilibrium, the relative price (i.e the price of the good produced over the consumption price index) is equal to 1. The real wage is given by the difference between the relative price and the profit of the firm per worker. Hence, in the long run, the real wage is simply equal to one minus the fixed entry cost, unaffected by changes in the bargaining power. Spector (2004) gets a different result in a context of fixed capital in the production function. The reduction in workers' rents may offset the reduction in the consumption good price, so that the final effect on the real wage is negative.

The second contribution of the present paper is a normative one. I show that a free-entry equilibrium in which workers' bargaining power is not strong enough, may deliver an inefficiently high level of employment and competition. Under the standard hypothesis of a Cobb-Douglas matching technology, imposing that the worker's bargaining power is equal to the elasticity of the matching technology leads to an excess of employment and competition. In other terms, the Hosios (1990) condition does not ensure the efficiency of the decentralized equilibrium. This is quite obvious, for the present model presents several departures from a standard matching framework: the law of motion of employment, the bargaining problem, the imperfect competition in the product market.

This result depends on two sources of externalities. Any firm deciding to enter the market lowers both the probability for other firms to fill their vacancy and, by making the market more competitive, their (expected) profits. These two effects are not taken into account by the single firm, so that entry is more desirable to the entrant than it is to society. To limit the incentives firms have to enter the market, worker's bargaining power must be high enough, so that employees capture a large fraction of the total rent.

It must be stressed that such excess of entry result is in line with the conclusion exposed by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) about free entry and social inefficiency. Mankiw and Whinston prove that imperfect competition models with an homogeneous good and a fixed cost of entry deliver an inefficiently high level of competition, because of the "business stealing" effect explained above.

A numerical simulation is finally conducted on the basis of Belgian data. The aim of such exercise is simply to see how the gap between the *laissez faire* and the optimal employment rates can be reduced. A policy aimed at lowering the cost of opening a vacancy does not better the performance of the decentralized economy. Instead, increasing workers' bargaining power allows to bridge the gap between the optimal and the *laissez faire* outcomes.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the decentralized equilibrium. Section 4 studies the policy implications, while Section 5 analyzes the welfare problem. Section 6 shows the quantitative results obtained. Finally, section 7 concludes.

2 The model

2.1 Preferences and technology

I consider an economy with one final consumption good and a large number I of intermediate goods. The final good market is perfectly competitive, whereas Cournot competition is assumed within each intermediate sector. The final good production function takes a CES form:

$$Y = \left[\sum_{i=1}^{I} Q_i^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}} \tag{1}$$

in which Q_i is the amount of intermediate good i used by the production process of the final good and $s > 1$ to ensure decreasing marginal productivity. Cost minimization in the final good sector leads to the inverse demand for each intermediate good i:

$$p(Q_i) \equiv \frac{P_i}{P} = \left(\frac{Q_i}{Y}\right)^{-\frac{1}{s}}$$
$$\text{with } P \equiv \left[\sum_{i=1}^{I} P_i^{1-s}\right]^{\frac{1}{1-s}} \tag{2}$$

P is the price index. Parameter s is the elasticity of the demand for good i.

Time is continuous. In each intermediate sector there are L infinitely-lived and risk-neutral workers; they can be employed only in that industry, so there are I perfectly segmented labour markets. Each firm is made of a (filled or vacant) job. The I labour markets present some unexplained frictions that make the trading process between firms and workers not instantaneous. Therefore, to produce and compete in one sector, a firm has to post a job vacancy, meet a worker and bargain with him about the wage and the number of hours worked. The intermediate firm production function is identical in each sector and is given by l_i, where l_i is the amount of hours worked supplied by the employee in sector i, and $0 \leq l_i \leq 1$[1]. The total amount of good i produced at time t is equal to $Q_{i,t} = \sum_j l_{i,j}(t))$, the subscript j denoting a generic firm operating in sector i at time t.

Workers have homogeneous instantaneous utility functions, denoted by $v_i l_i + \phi(l_i)$, with v_i being the hourly real wage and $\phi(l_i)$ the disutility of work. For simplicity, I assume an iso-elastic function $\phi(l_i) = z - l_i^{\epsilon}/\epsilon$, $\epsilon > 1$. When unemployed, the worker enjoys an instantaneous utility z, the value of devoting all your time to leisure.

2.2 The Stochastic Environment

The creation and destruction of jobs in each intermediate market i follows a continuous time Markov chain that takes values in the set $L = \{0, 1, 2, ...L\}$. I assume that in small interval of time *dt at most* one firm can enter in a sector. So, if x_i is the number of firms active in sector i, the probability that one more firm enters is given by $m_{x_i}\, dt$, while the probability that more that one firm enter is equal to zero. The rate m_{x_i} positively

[1]Considering hours worked in firms' production function is coherent with the empirical evidence recently emphasized by Hall (2006). He shows that in the U.S. economy over the past 60 years hours per worker can account for more than half of cyclical variations in total hours of work.

depends on V_{x_i}, the number of job-vacancies, and $L - x_i$ the number of unemployed workers in sector i. So, $m_{x_i} = m(V_{x_i}, L - x_i)$, with $m(.,.)$ being identical in every sector, homogeneous of degree one, and increasing in both arguments. m_x is a sort of black box, capturing the presence of frictions in the labour market.

Moreover, with a probability $\delta\, dt$ a new intermediate product is invented in the economy, making one existing good obsolete. All the jobs in the "old" intermediate sector are destroyed and massive layoffs occur. To keep the model as simple as possible, I also assume that all the L workers of the sector destroyed start searching for a job in the new one. Such hypothesis about a sector-specific destruction rate wants to be an (admittedly simplified) approximation of a product life-cycle. The economy is subject to a "creative destruction" force that allows the creation of new products but makes the existing ones obsolete. Indeed, as stressed in many marketing studies, the final stage of a product life-cycle does not not necessarily take the form of a slow decline in time[2]. Sometimes, the rise of new goods (often but not always technologically more advanced) makes the decline more steady or even transform it in a "collapse"[3].

The Markov chain just described can be represented by the following σ-matrix $\Sigma \equiv (\sigma_{x,y},\ x, y \in [1, 2, ...L])$:

$$\begin{aligned} \sigma_{x,x+1} &= m_x, & \sigma_{x,y} &= 0, \quad y - x > 1, \\ \sigma_{x,x} &= -(m_x + \delta), & \sigma_{x,0} &= \delta \\ \sigma_{0,1} &= m_1, & \sigma_{0,0} &= -m_1. \end{aligned} \tag{3}$$

Following Karlin and Tavaré (1982) and Van Doorn and Zeifman (2005), I refer to a process of this type as a birth process with killing, with m_x and δ respectively being the birth (i.e. the creation of one more job) and the killing (i.e. the destruction of all the jobs in the sector) rate. Let define the level of tightness in the labor market as $\theta_{x_i} \equiv \frac{V_{x_i}}{L - x_i}$. By the constant returns to scale assumption, the rate at which a single firm fills its vacancy when x firms are already active in market i can be defined

[2]Consider for instance the analysis about "disruptive innovation" pioneered by Christensen (1997).

[3]In a standard matching model, the destruction rate is job-specific, meaning that every match faces a probability of being destroyed. I consider a sector-specific destruction rate for simplicity. A job-specific separation rate would make the asset price equations even more difficult to manage with, since every firm would have to consider both the probability that the sector evolves by one unit and the probability that it decreases by one unit.

as $q(\theta_{x_i}) \equiv m_{x_i}/V_{x_i}$ and the rate at which a single worker finds a job is given by $m_{x_i}/(L - x_i) = \theta_{x_i} q(\theta_{x_i})$. I also define $\eta \equiv \frac{d\,(1/q(\theta))}{d\theta} \cdot \theta\, q(\theta)$, the elasticity of the expected duration of filling a vacancy with respect to tightness [4].

Intermediate sectors are identical *ex-ante*, having the same number of workers L, and the same matching and production technology. So I can remove the subscript i. Let $\pi_{x,t}$ be the probability that a time t there are x active firms in a generic intermediate market. Then:

$$\begin{aligned}
\pi_{x,\,t+dt} &= [\,1 - \delta dt - m_x dt\,] \cdot \pi_{x,t} \;+\; m_{x-1} dt \cdot \pi_{x-1,\,t} \quad \forall x \in [1, 2, ..., L],\\
\pi_{0,\,t+dt} &= [\,1 - m_0 dt\,] \cdot \pi_{0,t} \;+\; \delta dt \cdot \sum_{x=1}^{L} \pi_{x,\,t}.
\end{aligned}$$

One can look for a steady-state probability distribution, where $\pi_{x,\,t+dt} = \pi_{x,t}$, $\forall t$. Expressing π_x in terms of π_{x-1} and knowing that $\sum_{x=1}^{L} \pi_x = 1 - \pi_0$ yields:

$$\begin{aligned}
\pi_x &= \frac{m_{x-1}}{m_x + \delta} \cdot \pi_{x-1} \quad \text{with } x \in [\,1, 2, ., L\,],\\
\pi_0 &= \frac{\delta}{\delta + m_0}.
\end{aligned}$$

Finally, solving backwards, one obtains:

$$\pi_x = \prod_{n=0}^{x-1} \frac{m_n}{m_{n+1} + \delta} \cdot \pi_0 \;=\; \frac{\delta}{m_x + \delta} \cdot \prod_{n=0}^{x-1} \frac{m_n}{m_n + \delta} \tag{4}$$

The probability π_x that in one intermediate sector x firms compete in the market depends on L, δ and the endogenous rates $m_n = (L-n)\theta_n q(\theta_n)$ $\forall n \in [0,\, 1,\, 2, \ldots x]$.

If I is sufficiently large, I can apply the law of large numbers and define the aggregate level of employment

$$E = \sum_{x=0}^{L} x \cdot \pi_x \cdot I. \tag{5}$$

[4]While in a text-book Pissarides model the measure $m = m(V, U)$ represents the *number* of matches produced at each moment in the aggregate economy, in this paper, $m_{x_i} = m(V_{x_i}, L - x_i)$ is the *rate* at which a new match is created in a generic labour market i. Such a stochastic process, where the number of possible entrants in each intermediate market cannot be greater than one in a small interval of time dt, allows to model firms' dynamic decisions (that depend both on the number of competitors present in the market and on the rate at which new players will enter), while keeping the setting as tractable as possible.

Of course, the level of unemployment is given by: $U = \sum_{x=0}^{L} (L - x) \cdot \pi_x \cdot I$.

Lemma 1 *The level of employment E is increasing in* m_x, $\forall x \in [0, 1, 2, ., L-1]$. *More in general,* $\sum_{x=0}^{L} g(x) \cdot \pi_x \cdot I$ *is increasing (decreasing) in* m_x *for any function* $g(.)$ *increasing (decreasing) in* x.

It easy to check that $\frac{d\pi_x}{dm_x} < 0$, $\frac{d\pi_x}{dm_n} > 0$ if $n \in [0, 1, 2, ..., x-1]$, and $\frac{d\pi_x}{dm_n} = 0$ if $n \in [x+1, x+2, ..., L]$. Hence, differentiating (5), one gets:

$$\frac{dE}{dm_x} = x \cdot \frac{d\pi_x}{dm_x} + \sum_{n=x+1}^{L} \frac{d\pi_n}{dm_x} \cdot n$$

The first term at the RHS is negative, while the sum is composed by positive terms. Since $\sum_{n=0}^{L} \pi_n = 1$, then

$$\begin{aligned} -\frac{d\pi_x}{dm_x} &= \sum_{n=x+1}^{L} \frac{d\pi_n}{dm_x} &\Longleftrightarrow \\ -\frac{d\pi_x}{dm_x} \cdot x &= x \cdot \sum_{n=x+1}^{L} \frac{d\pi_n}{dm_x} &\Longleftrightarrow \\ -\frac{d\pi_x}{dm_x} \cdot x &< \sum_{n=x+1}^{L} \frac{d\pi_n}{dm_x} \cdot n & \end{aligned}$$

The last inequality implies that $\frac{dE}{dm_x} > 0, \forall x \in [0, 1, ..L-1]$. It is easy to verify that the same result applies if x is replaced by an increasing transformation of x. Hence, $\sum_{x=0}^{L} g(x) \cdot \pi_x \cdot I$ is increasing (decreasing) in m_x for any function $g(.)$ increasing (decreasing) in x. ■

2.3 Asset price equations

Let r be the discount rate common to all agents. The expected lifetime income for an unemployed worker in a sector with x competitors, $W_U(x)$ solves the following equation:

$$\begin{aligned} rW_U(x) &= z + \theta_x q(\theta_x) [W_E(x+1) - W_U(x)] \\ &+ (L - x - 1) \theta_x q(\theta_x) [W_U(x+1) - W_U(x)] + \delta [W_U(0) - W_U(x)], \end{aligned} \tag{6}$$

with $x \in [0, 1.., L-1]$. Being unemployed when the level of employment is equal to x is like holding an asset that pays you a dividend of z and at a rate $\theta_x q(\theta_x)$ it can be transformed into employment (hence, $x+1$ jobs are active in that market). In addition, the value of the asset can also change because at a rate $(L-x-1)\,\theta_x q(\theta_x)$ some other unemployed worker can find a job. In that case, the value of being unemployed shifts from $W_U(x)$ to $W_U(x+1)$. Finally, at a rate δ that sector can become obsolete in the economy. All the workers employed there lose their job and start their unemployment spell in the new sector. The capital gain will be equal to $W_U(0) - W_U(x)$.

Similarly, the asset price equation for a worker employed in a sector with x competitors is equal to:

$$
\begin{aligned}
rW_E(x) \;&=\; v_x l_x \;+\; z - \frac{l_x^{\epsilon}}{\epsilon} \;+\; \delta\,[W_U(0) - W_E(x)] \\
&+ (L-x)\theta_x q(\theta_x)\,[W_E(x+1) - W_E(x)]\ ,
\end{aligned}
\tag{7}
$$

with $x \in [1, 2, ..L]$.

On the other side of the market, the Bellman equation for a job vacancy is then given by:

$$
\begin{aligned}
rJ_V(x) \;&=\; -h \;+\; q(\theta_x)\,[J_E(x+1) - J_V(x)] \\
&+ [m_x - q(\theta_x)]\,[J_V(x+1) - J_V(x)] \;+\; \delta\,[J_V(0) - J_V(x)]\,,
\end{aligned}
\tag{8}
$$

with $x \in [0, L-1]$. Similarly, the value of an active firm with $x-1$ competitors takes the following form:

$$
\begin{aligned}
rJ_E(x) \;&=\; p(Q_x)\,l_x - v_x l_x \;+\; \delta\,[J_V(0) - J_E(x)] \\
&+ (L-x)\theta_x q(\theta_x)\,[J_E(x+1) - J_E(x)]\ ,
\end{aligned}
\tag{9}
$$

with $x \in [1, 2, ..., L]$. Function $p(Q_x)$ is expressed in (2) and represents the real price of the intermediate good when x firms are competing in the market. $J_V(0)$ is the value of a vacancy when the sector is destroyed.

3 Equilibrium

3.1 Bargaining

Firms and workers bargain over wages and hours worked not only when they match for the first time but every time a change in the demand occurs because a new competitor enters the market. An axiomatic Nash solution is considered. I impose that the threat points for workers and firms in the Nash program are not their options outside the match (respectively, W_U and J_V), but their utilities of remaining together and producing nothing[5]. I make this choice for two reasons.

First, wages and hours worked are bargained not only by workers (respectively, firms) that have justed ended their unemployment (resp. vacancy) spell but also by incumbents that have to change their strategy in the Cournot game. It seems more appropriate, especially for such workers and firms, to assume that in the case of failure of an agreement they decide not to leave. One can think for instance that workers are on strike and nothing is produced.

The second reason is tractability. Assuming, as in a standard Pissarides model, that the threats points are the outside options does not rule out the existence of an equilibrium, but makes the model less tractable (details are available on request).

The threat points for an employee and an employer when the negotiation fails are respectively given by:

$$r\bar{W_E} = z + \delta \left[W_U(0) - \bar{W_E}\right] \tag{10}$$

$$r\bar{J_E} = -\delta \left[\bar{J_E} - J_V(0)\right] \tag{11}$$

If an agreement is not concluded, the worker remains employed, he does not receive any wage and enjoys an instantaneous utility of z. The firm does not produce and does not pay the wage. Still, at a rate δ that sector becomes unproductive.

I define $w \equiv v \cdot l$, the total real wage received by the employee, and solve the Nash maximization problem with respect to $\{w, l\}$ instead of $\{v, l\}$:

$$\begin{aligned} w_x,\, l_x \;&=\; \text{argmax } \left[W_E(x) - \bar{W_E}\right]^{\beta} \left[J_E(x) - \bar{J_E}\right]^{1-\beta} \\ &\text{s.t.} \\ W_E(x) \;&>\; W_U(x-1) \\ J_E(x) \;&>\; J_V(x-1) \quad \text{with}\, x \in [1, 2, ..L]. \end{aligned} \tag{12}$$

$J_V(x-1)$ represents the expected discounted value of a vacancy when $x-1$ firms compete in the market. In Appendix A, I show that the solution of (12) coincides

[5]This kind of bargaining game has been introduced by Rosen (1997) and Hall and Milgrom (2006) arguing that, a disagreement in the negotiation between workers (or unions) and firms usually implies a delay in the production, strikes, not massive lay-offs or quits.

with the equilibrium of an extensive form game in which workers and firms alternate each other in making offers and have only one instant to make their bargain. The constraints imposed in the maximization imply that the worker (the firm) always has the possibility to abandon the negotiation and become unemployed (an idle vacancy) if this choice makes him (it) better off. I assume, as Rosen (1997) and Hall and Milgrom (2006) do, that such constraints are not binding: no player has an incentive to quit the negotiation and this holds for any value of x.

Computing the F.O.C.s yields :

$$\beta \left[J_E(x) - \bar{J_E} \right] = (1-\beta) \left[W_E(x) - \bar{W_E} \right].$$

$$\beta \, \frac{l_x^{\epsilon-1}}{W_E(x) - \bar{W_E}} = (1-\beta) \, \frac{p'(Q_x)\, l_x + p(Q_x)}{J_E(x) - \bar{J_E}},$$

$\forall x \in [1, 2, ..., L]$ and with $p'(Q_x) \equiv \partial p(Q_x)/\partial l_x$. By using equations (7), (9), (11) and (10), I get the following equilibrium equations of wages and hours worked :

$$w_x = \beta\, p(Q_x)\, l_x + (1-\beta)\, \frac{l_x^{\epsilon}}{\epsilon} \tag{13}$$

$$\begin{aligned} l_x^{\epsilon-1} &= \left[p'(Q_x)\, l_x + p(Q_x) \right] \\ &= p(Q_x) \left[1 - \frac{1}{x \cdot s} \right] \end{aligned} \tag{14}$$

$\forall x \in [1, 2, ..., L]$. The second line in (14) is obtained by using equation (2). For every x, equations (13) and (14) define the equilibrium values of l_x and w_x.

Equation (13) has a straightforward interpretation. The wage is a weighted average of the total revenues obtained in the intermediate sector ($p(Q_x) l_x$) and the opportunity cost of employment in terms of hours worked ($z - \phi(l_x) = l_x^{\epsilon}/\epsilon$). The weights are given by the bargaining power of workers and firms, β and $1 - \beta$. If the worker has no bargaining power, he receives an instantaneous utility from being employed exactly equal to z. On the other hand, when $\beta = 1$, all the profits earned in the market accrue to the employee. As we will see, in this limit case, firms cannot recoup the cost h and no firm will post a vacancy.

Equation (14) looks very similar to a standard solution of a $x-$players Cournot game. Each worker-firm pair maximizes its surplus, given the optimal strategy of the other players. In equilibrium, the marginal revenue of a firm must be equal to the marginal utility of leisure for a worker[6]. Notice also that, from a single firm's viewpoint, the total amount of the final good, Y, is given. This is due to the fact that, with I large enough, a single firm's decision has an impact only within each sector but does not affect the price index P and quantity Y.

3.1.1 Partial equilibrium properties of wages and hours worked

I consider now the properties of wages and hours worked as competition increases in a generic sector, while the rest of the economy is considered as given.

From (13), the derivative of w with respect to l is:

$$\frac{dw_x}{dl_x} = \beta\left[p'(Q_x)l_x + p(Q_x)\right] + (1-\beta)l_x^{\epsilon-1} = l_x^{\epsilon-1} > 0.$$

Moreover, some standard properties of Cournot models are fulfilled: As the number of competitors x increases, the quantity produced by a single firm, l_x, decreases, whereas the aggregate quantity Q_x increases[7]. So the total wage decreases as x increases:

$$w_{x+1} - w_x = \beta[p(Q_{x+1})l_{x+1} - p(Q_x)l_x] + \frac{1-\beta}{\epsilon}\left[l_{x+1}^{\epsilon} - l_x^{\epsilon}\right] < 0, \qquad (15)$$

Workers employed in more competitive sectors get lower wages but enjoy more leisure time. If I ignore for simplicity the integer problem, the former effect outweighs the latter: Using (13), the instantaneous utility of an employed worker, $w + \phi(l_x)$, is decreasing in x:

$$\frac{d\,[w_x + \phi(l_x)]}{d\,x} = \beta\left\{\frac{d\,p(Q_x)}{d\,Q_x}\frac{d\,Q_x}{d\,x}l_x + \frac{d\,l_x}{d\,x}\left[p(Q_x) - l_x^{\epsilon-1}\right]\right\} < 0.$$

[6] Differentiating the first line of equation (14) with respect to l_x, I obtain:

$$[p''(Q_x)l_x + 2p'(Q_x)] - (\epsilon-1)l_x^{\epsilon-2}.$$

A sufficient condition for this equation to be negative is $[p''(Q_x)l_x + 2p'(Q_x)] < 0$. Computing the derivatives, this implies $\frac{1+s}{s} < 2x$, that is always true for any $x \geq 1$ and $s > 1$.

[7] The necessary assumptions to prove such properties are satisfied (demand twice differentiable and tending to 0 for Q_x sufficiently large, cost function increasing and twice differentiable, profit function strictly concave). For the complete proof, I refer to Frank (1965).

The first term inside the braces is negative because Q_x is increasing in x; the second term is also negative since l_x decreases in x, while the expression inside the square brackets is positive by (14).

3.2 Free-entry in vacancy creation

To close the model and find the equilibrium values of θ_x, I introduce a free-entry condition in vacancy creation. Firms enter one intermediate market as long as the expected return of posting a vacancy is non negative. This means that:

$$r J_V(x) = 0 \quad \forall x \in [0, 1, ..L - 1] \tag{16}$$

The expected discounted value of a job when $x + 1$ agents are active in a market must be equal to the expected cost of filling a vacancy:

$$J_E(x + 1) = \frac{h}{q(\theta_x)} \quad \forall x \in [0, 1, 2, ..L - 1] \tag{17}$$

Finally, using (9), (16), and (17) one gets:

$$\frac{h}{q(\theta_{x-1})} = \frac{p(Q_x)l_x - w_x + (L - x)\theta_x h}{r + \delta + (L - x)\theta_x q(\theta_x)} \qquad \forall x \in [1, 2, ..L]. \tag{18}$$

The LHS represents the expected duration of filling a vacancy when $x - 1$ firms are already active in the market. On the RHS, the expected profit is made of two terms: profits attained when the firm has $x - 1$ competitors (that is $p(Q_x)l_x - w_x$) and all the profits that can be earned with at least x competitors, weighted by the rate m_x (since $(L - x)\theta_x h = m_x \frac{h}{q(\theta_x)} = m_x \cdot J_E(x + 1)$).

The equations in (18) represent a system of L unknown variables, $[\theta_0, \theta_1, ..., \theta_{L-1}]$. Note that for $x = L$ we have:

$$\frac{h}{q(\theta_{L-1})} = \frac{p(Q_L)l_L - w_L}{r + \delta} \tag{19}$$

Labour market tightness θ_{L-1} does not depend on other values of θ. The endogenous variables l_L and w_L are uniquely defined by the F.O.C.s (13) and (14) evaluated at $x = L$. I can therefore solve the system in (18) "backward", starting from θ_{L-1} and going back to θ_{L-2}, θ_{L-3}, ..., θ_0.

3.2.1 Properties of labour market tightness

I am interested in knowing how the equilibrium value of tightness θ_x changes with x. The following lemma summarizes the results:

Lemma 2 *$\theta_x < \theta_{x-1}$, $\forall x \in [1, 2, ..L-1]$. Hence, $m_x < m_{x-1} \forall x \in [1, 2, ..L-1]$.*

Proof. See Appendix B. ■

Lemma 2 states that the number of vacancies posted decrease as competition gets tougher. This makes sense, since a more competitive product market squeezes firms' profits, dampening the incentives in vacancy creation. Such negative effect on the supply side of the labour market outweighs the reduction in the number of unemployed workers as x goes up, so that $\theta_x \equiv V_x/(L-x)$ is decreasing in x. Equation (16) implies that expected discounted profits are equal to zero for any given level of competition in the goods market. A trade-off arises: in less competitive markets firms can attain higher revenues but stand in a longer queue to fill their vacancies.

3.3 General Equilibrium

Definition 1 *A long-run general equilibrium is defined as a vector $[\, l_x,\, w_x,\, \theta_{x-1},\, P_x \,]$ $\forall x \in [1, 2, ..., L]$, a probability distribution $[\pi_0, \pi_1, \pi_2, ..., \pi_L]$, and a value Y of the final good satisfying:*

1. *the F.O.C.s (13) and (14) of the bargaining problem, $\forall x \in [1, 2, ..., L]$.*
2. *The zero profit condition (18), $\forall x \in [1, 2, ..., L]$.*
3. *The steady-state distribution (4).*
4. *The conditions in the final good sector (1) and (2).*

The F.O.C.s (13), (14), and the demand function (2) determine the values of w_x and l_x as a function of Y $\forall x$. Then, substituting the equilibrium values of w_x and l_x in the system (18), I can express the elements of the vector $[\theta_0,\, \theta_1, ..., \theta_{L-1}]$ in terms of Y. In turn, using (4), I also determine the probabilities $[\pi_0,\, \pi_1, ..., \pi_L]$ as a function of

Y. Finally, equilibrium in the final good sector implies: $Y = \left[\sum_{i=1}^{I}(Q_i(Y))^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}$. Using (14), this equality is equivalent to:

$$Y = Y^{\frac{1}{1+s(\epsilon-1)}} \cdot A,$$
$$\text{with } A \equiv \left\{\sum_{i=1}^{I} x_i^{\frac{s-1}{s}} \cdot \left[\frac{1}{x_i}\left(\frac{x_i s}{x_i s - 1}\right)^s\right]^{\frac{s-1}{s[1+s(\epsilon-1)]}}\right\}^{\frac{s}{s-1}}$$

It is easy to see that this equilibrium has two solutions for Y, one equal to zero and the other positive. As $Y = 0$, nothing is produced in the intermediate sectors, all workers are unemployed and the probability distribution collapses to a mass point $x = 0$. Henceforth, I will concentrate on the positive equilibrium.

4 Competition in Products and Labour Markets

I now assess the impact on average employment, real wage and workers' utility of a change in β and h in every sector of the economy. To simplify the analysis, I assume henceforth a Cobb-Douglas matching function, $m_x = a\,(L-x)^{\eta} \cdot V_x^{1-\eta}$, with $\eta = 0.5$, in line with the findings of Petrongolo and Pissarides (2001). At the general equilibrium level, we have to consider the impact of a change in the parameters on the amount of the final good produced, Y, and, in turn, how this influences vacancy decisions and the negotiation for wage and hours.

4.1 Effects on Employment

The results are summarized in the following Proposition:

Proposition 1 *A decrease in workers' bargaining power β or in the cost of opening a vacancy h raises the aggregate level of employment, E.*

Proof.

A marginal change in β and h has a partial equilibrium and a general equilibrium effect. In Appendix C, I show that θ_x is decreasing in β and h, $\forall x \in [0, 1, ...L-1]$,

conditional on Y. Hence, a lower β or h raises θ_x and, in turn, $m_x = (L-x)\theta_x q(\theta_x)$, *conditional on* Y.
Consider now the effect of a change in β or h on Y. The latter can be written as:

$$Y = \left[\sum_{i=1}^{I} Q_i^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}} \cong \sum_{x=0}^{L} Q_x \pi_x I^{\frac{s}{s-1}} = \sum_{x=0}^{L} x \cdot \left[\left(\frac{Y}{x}\right)\left(1-\frac{1}{xs}\right)^{s}\right]^{\frac{1}{1+s(\epsilon-1)}} \pi_x I^{\frac{s}{s-1}}.$$

Hence:

$$Y = \sum_{x=0}^{L} x\left(1-\frac{1}{xs}\right)^{\frac{s}{s(\epsilon-1)}} \pi_x^{\frac{1+s(\epsilon-1)}{s(\epsilon-1)}} I^{\frac{1+s(\epsilon-1)}{(s-1)(\epsilon-1)}}$$

From Lemma 1, Y is increasing in m_x. Moreover, a higher Y enhances firms' revenues (see equation 14). A reduction in workers' bargaining power or in the cost of a vacancy raises the amount of the final good produced and this in turn has a positive impact on firms' revenues, amplifying the partial equilibrium effect explained above.

■

A lower bargaining power for workers reduces the wage and raises firms' expected profits. So, more competitors will enter the labour market by posting a vacancy. This in turn augments the employment. A similar effect occurs by lowering the cost of opening a vacancy h. Proposition 1 is in line with the empirical findings of Nicoletti and Scarpetta and Griffith, Harrison, and Macartney (2007), and with the theoretical conclusions obtained by Blanchard and Giavazzi (2003) and Ebell and Haefke (2006).

4.2 Effects on the real wage and on workers' utility

A decrease in the cost of opening a vacancy.

From Proposition 1, a decrease in h augments m_x $\forall x$. m_x raises the final good Y and, in turn, this has an impact on the real wage and the hours worked. To analyze the effect of m_x on the production of the consumption good, notice that Y can be written as:

$$Y = \left[\sum_{i=1}^{I} Q_i^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}} = \left[\sum_{x=0}^{L} Q_x^{\frac{s-1}{s}} \pi_x I\right]^{\frac{s}{s-1}}$$

Recall that the intermediate quantity Q_x is increasing in x. Then, from Lemma 1, a higher m_x raises the amount of the final good, Y. From (2), a higher Y enhances $p(Q_x)$.

In turn, this has a positive impact on hours worked: the RHS of the F.O.C. (14) shifts upwards, so hours worked go up. As a consequence, the real wage also increases:

$$\frac{d\,w_x}{dY} = \beta\frac{d\,p(Q_x)}{dY}l_x + \frac{d\,l_x}{dY}\left[\beta\left(p(Q_x) + p'(Q_x)l_x\right) + (1-\beta)l_x^{\epsilon-1}\right] > 0. \;\; \forall x$$

Knowing that the instantaneous utility of workers $w_x + \phi(l_x) = \beta\left(p(Q_x)l_x - \frac{l_x^\epsilon}{\epsilon}\right) + z$, we have:

$$\frac{d\left[w_x + \phi(l_x)\right]}{dY} = \beta\frac{d\,p(Q_x)}{dY}l_x + \beta\frac{d\,l_x}{dY}\left[p(Q_x) + p'(Q_x)l_x - l_x^{\epsilon-1}\right].$$

Such derivative is positive because $p(Q_x)$ and l_x are increasing in x, while the term inside the square brackets is zero for the F.O.C. (14). So, reducing the cost of opening a vacancy raises both the real wage and worker's instantaneous utility *for any given level of competition* x.

Consider now the impact of h on the average real wage,

$$\bar{w} \equiv \frac{1}{E}\sum_{x=1}^{L} w_x\,x\,\pi_x\,I.$$

Two opposite effects are at work. On one hand, a higher Y raises $\bar{w}$, because w_x is increasing in Y for any x. On the other hand, Lemma 1 cannot be used to assess the impact of a higher m_x on $\bar{w}$, because we do not know if $w_x \cdot x$ is an increasing or decreasing function of x. Since a reduction in the vacancy cost raises both m_x and Y, the final effect on the average real wage cannot be signed. In other terms, a lower h enhances the real wage for any given level of competition x, but, by raising tightness, it also changes the distribution $[\pi_0, \pi_1, \pi_2, ..., \pi_L]$. It may be possible that at the new equilibrium, in which the average level of employment is higher, workers are more likely to be in sectors with fiercer competition, where the real wages are lower. The former, income, effect pushes $\bar{w}$ up, whereas the latter, distribution, effect lowers it. Of course, the same reasoning applies to the average workers' utility.

A decrease in workers' bargaining power.

It is easy to verify that a reduction in workers' bargaining power β also augments the final good produced Y and hours worked l_x, for any given x. However, the effect on the real wage w_x is now ambiguous. The reason is that w_x is positively affected by Y and

l_x, as in the case of a reduction in h, but it is also decreasing in β. A lower bargaining power for the workers implies a smaller fraction of the surplus originated by the match and, in turn, a lower wage. That means:

$$\frac{d\,w_x}{d\,\beta} = \frac{\partial\,w_x}{\partial\,\beta} + \frac{\partial\,w_x}{\partial\,Y} \cdot \sum_{x=0}^{L-1} \frac{\partial\,Y}{\partial\,m_x} \frac{\partial\,m_x}{\partial\,\beta}.$$

The first term is positive, while the second one is negative, for a higher β lowers m_x. As consumers, workers benefit of the decrease in β, since a larger amount of the final is produced and consumed. Yet, the employees receive a lower fraction of the surplus. The final effect of β on w_x cannot be signed. *A fortiori*, I cannot assess the impact of a reduction in workers' bargaining power on the average real wage.

Differently from Proposition 1, the conclusions of this sub-section contrast with those obtained by Blanchard and Giavazzi (2003)[8]. The reason of these competing results is twofold: the (a)symmetry of the long-run equilibrium, and the different zero-profits conditions imposed. Blanchard and Giavazzi focus on a symmetric equilibrium, in which all the intermediate firms set the same price. So, the relative price $p(Q_i)$ is equal to 1. Further, in the free-entry equilibrium, profits per worker (i.e. $p(Q) - w$) are equal to a fixed cost of entry. It is then clear that the real wage w increases as the fixed cost decreases, whereas it is unaffected by changes in β. In the present paper, on the contrary, the equilibrium is asymmetric (in the sense that the quantity and price is not the same among the intermediate sectors), and the real wage in the long-run is still a function of β.

5 Optimality

In the decentralized economy, there are two departures from the competitive framework, namely frictions in the labour market and imperfect competition in the goods market. I consider a centralized economy in which a social planner has to choose the optimal number of vacancies and hours worked in any sector. Notice that the interactions *between* intermediate sectors come only from the final good production function. By the constant returns to scale assumption, the latter can be written as $Y = \sum_{i=1}^{I} p(Q_i)Q_i$.

[8]Ebell and Haefke (2006) do not analyse the long-run effect of competition on the real wages.

Moreover, from Euler's formula, $-\frac{dp(Q_i)}{dQ_i}\frac{dQ_i}{dl_i}Q_i = \sum_{i\neq j=1}^{I}\frac{dp(Q_j)}{dQ_i}\frac{dQ_i}{dl_i}Q_j$. Hence, any effect arising between intermediate sectors (the RHS of the equation above) disappears and I can study the social planner problem focusing only on what happens *within* a generic intermediate sector.

Following Shimer (2004), the welfare function can be expressed for any given x in the following recursive form:

$$\begin{aligned} r\,\Omega_x \;=\; & \max_{\theta_x,\,l_x}\; p(Q_x)Q_x \,+\, x\left(z \,-\, \frac{l_x^{\epsilon}}{\epsilon}\right) + (L-x)z \,-\, h(L-x)\theta_x \\ & +\,(L-x)\,a\,\theta_x^{1-\eta}\,[\Omega_{x+1} \,-\, \Omega_x] \,+\, \delta\,[\Omega_0 \,-\, \Omega_x] \end{aligned} \tag{20}$$

$$\text{s.t.}\;\; Q_x \;= x\cdot l_x. \qquad \forall x \in [0,1,2,...L]$$

When x firms are active in a generic intermediate sector, the social planner has to maximize intermediate firms' revenues, the utility of leisure of workers, net to the cost of opening a vacancy. Moreover, at a rate $m_x = (L-x)\,a\,\theta_x^{1-\eta}$ the level of employment increases by one unit, causing a change of the surplus from Ω_x to Ω_{x+1}, and at a rate δ the sector is destroyed and another one is instantaneously created. The constraint in (20) reminds that, differently from the *laissez faire* economy, the social planner considers *ex ante* a symmetric solution, in which every firm uses the same amount of hours worked. Of course at $x = L$, the sector is in full employment and the social planner has only to choose the amount of hours worked. The solutions $(\theta^{\circ}, l^{\circ})$s to problem (20) verify the following F.O.Cs:

$$(1-\eta)\,a(\theta_x^{\circ})^{-\eta}\cdot[\Omega_{x+1} \,-\, \Omega_x] \;=\; h \tag{21}$$

$$p(Q_x^{\circ}) \;=\; (l_x^{\circ})^{\epsilon-1} \tag{22}$$

The intuition of the above equations is the following. At the social optimum, the cost of marginal increase in θ_x, h, must be equal to the marginal gain, given by $(d\,\theta_x q(\theta_x)/d\,\theta_x)\,[\Omega_{x+1} - \Omega_x] = (1-\eta)\,a(\theta_x^{\circ})\,[\Omega_{x+1} - \Omega_x]$. Moreover, the optimal level of hours worked l_x° is such that the increase in production must be equal to the opportunity cost in terms of leisure.

By the Euler's formula, $\frac{dp(Q_x)}{dQ_x}\frac{dQ_x}{dl_x}Q_x$ cancels out with the sum of the derivatives of the prices in the other sector with respect to l_x. Comparing (14) with (22) one obtains

$l_x^\circ > l_x^* \; \forall x$, the superscript * denoting henceforth the decentralized equilibrium values of the endogenous variables. This inequality holds since l_x^ϵ is increasing in l and $p(Q_x)$ is always greater than $p(Q_x) + p'(Q_x)l_x$. So the level of hours worked in equilibrium is always inefficiently low. Notice also that equation (22) would coincide with the outcome of a worker-firm negotiation, were the good market perfectly competitive.

Denote $S_x \equiv p(Q_x)l_x - l_x^\epsilon/\epsilon$. Using (21) and (22) and subtracting the optimal solution Ω_x from Ω_{x+1} yields:

$$\frac{(r+\delta)h}{a(1-\eta)}(\theta_x^\circ)^\eta = (x+1)\,S_{x+1}^\circ - x\,S_x^\circ + \frac{\eta}{1-\eta}\,h\left[(L-x-1)\theta_{x+1}^\circ - (L-x)\theta_x^\circ\right].$$

After some algebra, one gets:

$$\frac{r+\delta}{a}(\theta_x^\circ)^\eta + \eta(L-x)\,\theta_x^\circ = \frac{1-\eta}{h}\left[(x+1)\,S_{x+1}^\circ - x\,S_x^\circ\right] + \eta(L-x-1)\theta_{x+1}^\circ. \quad (23)$$

A comparison of (23) with the free-entry equilibrium condition (18) delivers the following result:

Proposition 2 *If $\beta \leq \eta$, in the decentralized equilibrium the aggregate level of employment is inefficiently high $\forall x$.*

Proof. I first consider the case in which in the decentralized equilibrium $\beta = \eta$ and I prove that the level of employment is inefficiently high. Then, I show that this results holds *a fortiori* if $\beta < \eta$. Using the wage equation (13) and imposing $\eta = \beta$, the decentralized equilibrium condition (18) can be written as:

$$\begin{aligned} &\frac{r+\delta}{a}(\theta_x^*)^\eta + (L-x-1)\,(\theta_{x+1}^*)^{1-\eta}(\theta_x^*)^\eta - (1-\eta)(L-x-1)\,\theta_{x+1}^* \\ &= \frac{1-\eta}{h}\,S_{x+1}^* + \eta(L-x-1)\,\theta_{x+1}^*. \end{aligned} \quad (24)$$

I proceed now in three steps. First, I show that, for all x, S_{x+1}^* is always larger than $(x+1)\,S_{x+1}^\circ - x\,S_x^\circ$. Then I show that $\theta_{L-1}^* > \theta_{L-1}^\circ$. Finally, I prove that $\theta_x^* > \theta_x^\circ$, $\forall x$.

STEP 1 : $S_{x+1}^* > (x+1)\cdot S_{x+1}^\circ - x\cdot S_x^\circ, \;\; \forall x \in [1, 2, ...L]$.

For the proof, see Appendix D.

STEP 2 : $\theta_{L-1}^* > \theta_{L-1}^\circ$.

When $x = L-1$, equations (23) and (24) respectively become:

$$\frac{r+\delta}{a}(\theta^{\circ}_{L-1})^{\eta} + \eta\theta^{\circ}_{L-1} = \frac{1-\eta}{h}\left[L \cdot S^{\circ}_{L} - (L-1)\cdot S^{\circ}_{L-1}\right]$$
$$\frac{r+\delta}{a}(\theta^{*}_{L-1})^{\eta} = \frac{1-\eta}{h} S^{*}_{L}.$$

From Step 1, the RHS in the decentralized equilibrium equation is larger than the RHS in the welfare equation. Then, looking at the LHS, $\theta^{*}_{L-1} > \theta^{\circ}_{L-1}$.

STEP 3 : $\theta^{*}_{x} > \theta^{\circ}_{x} \ \ \forall x \in [0, 1, 2, ...L-2]$.

Having shown that $\theta^{*}_{L-1} > \theta^{\circ}_{L-1}$ I can proceed backward and consider the case $x = L-2$. It is then clear that the RHS in (24) is larger than the RHS in (23), because of the inequality proved in Step 1 and because $\eta(L-x-1)\,\theta^{*}_{L-1} > \eta(L-x-1)\ \theta^{\circ}_{L-1}$ by Step 2. So, the LHS in (24) is larger than the LHS in (23). Consider now the LHS in (24). If:

$$\begin{aligned} &\eta(L-x)\theta^{*}_{x} \geq \\ &(L-x-1)(\theta^{*}_{x+1})^{1-\eta}(\theta^{*}_{x})^{\eta} - (1-\eta)(L-x-1)\theta^{*}_{x+1} \qquad \forall x \end{aligned} \tag{25}$$

then,

$$\frac{r+\delta}{a}(\theta^{*}_{x})^{\eta} + \eta(L-x)\,\theta^{*}_{x} > \frac{r+\delta}{a}(\theta^{\circ}_{x})^{\eta} + \eta(L-x)\ \theta^{\circ}_{x}, \qquad \forall x \tag{26}$$

since the LHS in (26) is larger than the RHS in (24), that in turn is larger than the RHS in (26). But (25) implies:

$$\left(\frac{\theta^{*}_{x}}{\theta^{*}_{x+1}}\right)^{\eta} - \eta\,\frac{L-x}{L-x-1}\cdot\frac{\theta^{*}_{x}}{\theta^{*}_{x+1}} - 1 + \eta \ \leq \ 0.$$

Such inequality is always verified, provided that $\theta^{*}_{x} > \theta^{*}_{x+1}$[9]. Then, with (26) being always true, $\theta^{*}_{x} > \theta^{\circ}_{x} \ \ \forall x \in [0, 1, 2, ...L-1]$. Finally, from Lemma 1, a higher θ_x implies a higher level of employment.
If $\beta < \eta$, the inequality in STEP 1 holds *a fortiori*, so the level of employment is also too high..

■

[9]When $\theta^{*}_{x} = \theta^{*}_{x+1}$, the LHS is negative. Moreover, the function is decreasing in $\theta^{*}_{x}/\theta^{*}_{x+1}$ when $\theta^{*}_{x} > \theta^{*}_{x+1}$, $\forall\, 0 < \eta \leq 1$.

Cournot competition leads to an inefficiently low level of hours worked, as each firm tends to produce a quantity l_x smaller than the optimal one in order to keep the market price higher.

Three features explain why with $\beta \leq \eta$ the optimal level of employment and competition is lower than *laissez faire* one: Search frictions in the labour market, imperfect competition in the product market, and the rent sharing rule *à la* Hall and Milgrom (2006).

The presence of frictions in the labour market makes search externalities emerge, as any firm deciding to post a vacancy fails to consider both the decrease in other firm's vacancy-filling probability and the increase in workers' job-finding probability. Moreover, any firm deciding to enter or not the market also fails to consider the reduction in other firms' profits caused by the increase in competition. Thus, if employers' bargaining power is not low enough, entry is more desirable to the single firm than it is to the social planner, that takes the reduction in incumbent firms's profits into account.

On top of that, the rent-sharing rule I imposed also leads to an excessive level of tightness if worker's bargaining power is not high enough. Indeed, in Appendix E, I show that, even if the product market was perfectly competitive, still the level of employment would be inefficiently high with $\beta \leq \eta$. The reason is the following. In the bargaining process (12), workers do not use the opportunity cost of employment rW_U as threat point. Thus, the wage equation (13) is not affected by tightness. *Ceteris paribus*, firms post more vacancies than under a standard Pissarides (2000) bargaining rule in which the fall-back position is W_U, since an increase in tightness does not push the wage up, squeezing firms' profits.

Hence, there are too few unemployed workers. Since in the *laissez faire* economy the extent of substitution of these two inputs depends on workers' bargaining power, a strong β is needed to limit vacancy posting.

This excess of entry result is in line with the findings of Mankiw and Whinston (1986) and Suzumura and Kiyono (1987). Both papers prove that imperfect competition models in which firms can enter the market paying a fixed cost deliver an inefficiently high level of competition. Indeed, this paper can be framed in the same environment: It assumes an imperfectly competitive good market where firms can enter only by involving in a costly search in the labour market. What for Mankiw and

Whinston is a fixed cost, in this paper corresponds to the expected cost of filling a job vacancy, $h/q(\theta_x)$. Were the labour market perfectly competitive (i.e. a spot market in which entry has no cost), an infinite number of firms would enter and produce, ensuring perfect competition even in the goods market. The social optimum would then coincide with the decentralized outcome.

In a similar model, Ebell and Haefke (2006) reach the opposite conclusion: If the Hosios condition holds, the level of employment is inefficiently low. The source of such conflicting results mainly depends on the different welfare functions considered. In the present paper, the social planner's problem consists in choosing the optimal quantity produced by the single intermediate firm, and the optimal number of intermediate firms that must compete in each sector. Such results are then compared with the free-entry long run equilibrium. In Ebell and Haefke's paper, the social planner has to select the quantity produced by a single firm, but not the number of firms that can be active in the market. This choice is then compared with the short-run decentralized equilibrium, where free-entry is not allowed. In such a case, monopolistic competition induces each firm to produce less than the optimal level, in order to secure a higher mark-up. So, firms hire less workers than in a competitive optimal framework[10].

Simulation results (presented in the next section) try to quantify the order of magnitude in terms of employment of such excess of entry inefficiency.

[10] Actually, in Ebell and Haefke's framework, there is also a hiring externality - opposite in sign. Since the wage is proportional to the marginal revenues, that are decreasing in a monopolistic set-up, firms will be induced to hire more than the optimal level in order to reduce the wage paid to all the workers. Such strategic behaviour has been first studied by Stole and Zwiebel (1996) and extended to matching models by Cahuc and Wasmer (2001). In their model, Ebell and Haefke show that the first, monopolistic effect prevails and firms hire less than in a competitive framework, unless workers' bargaining power is extremely high.

6 Quantitative Results

6.1 Calibration

I take the month as unit of time. Data refer to the 1997-1998 period where the stocks were fairly stable in Belgium. To calibrate the model, I make use of various surveys[11], published statistics[12], the quantitative results obtained in Cardullo and Van der Linden (2007), and results found in the literature. Table 1 presents the results. As in the previous sections, I assume the following Cobb-Douglas matching function $m_x \equiv a(L-x)^{\eta}V_x^{\eta}$. The elasticity η is imposed equal to 0.5, the value mostly adopted in the literature (see Petrongolo and Pissarides, 2001). In Cardullo and Van der Linden (2007), the calibrated value for workers' bargaining power β is 0.5 for the high-skilled sector and 0.56 for the low-skilled one. I set it equal to 0.5. Making use of the zero profit condition in vacancy creation, I calibrate the cost of opening a vacancy h so that the expected duration of unemployment is in line with the findings of Dejemeppe (2005).[13] Parameter a is a scaling factor for h and it is set equal to 0.12, so that the expected duration of filling a vacancy is around 3 months. The discount rate is fixed at 0.004 (5% on an annual basis). The number of workers in each intermediate sector is set equal to 20 in order to have a sufficiently large degrees of product competition. The elasticity of the demand in the intermediate sectors, s, is set equal to 5, in order to have an average wage in the economy of 1235 Euros (a value in accordance with the results obtained in Cardullo and Van der Linden (2007)). I assume that hours worked l are in an interval between 0 and 2. Workers' utility of leisure is given by $2^{\epsilon} - l^{\epsilon}$. The parameter ϵ is set equal to 4, so that on average employees devote to market work around 41 % of their time[14]. A sensitivity analysis is conducted on these parameters.

In absence of precise estimations about the sector specific destruction rate δ, a value of 0.005 is taken.

[11] Simoens, Denys, and Denolf (1998), Denolf, Denys, and Simoens (1999) and Delmotte, Van Hootegem, and Dejonckheere (2001).

[12] Published by national and regional PES in Belgium and by Eurostat (2002a) and Eurostat (2002b).

[13] From her analysis of unemployment dynamics in Belgium, the average unemployment duration in 1992 was equal to 2 years in the South of Belgium and to 1.5 years in the North.

[14] The average wage and the average number of hours worked are defined respectively as: $(1/E) \cdot \sum_{x=1}^{L} w_x \, x \, \pi_x I$ and $(1/E) \cdot \sum_{x=1}^{L} l_x \, x \, \pi_x I$.

6.2 Simulation Results

Figures 3 and 4 show that labour market tightness θ_x is decreasing in x while the steady-state distribution π_x is an increasing function both in the *laissez faire* economy and in the centralized one.

The simulation results are summarized in Table 2 and Table 3. I first evaluate the impact of a decrease in the cost of opening a vacancy h on the average values of the following variables: the wage, the rate of employment ($e = E/L$), the share of hours worked, and the volume of work, defined as the total number of hours worked in the economy over their total potential amount, $H \equiv \sum_{x=0}^{L} l_x \, \pi_x \, / \, L \cdot 2$. The first column of Table 2 shows the main result: the employment rate in the free-entry equilibrium is higher than the optimal one, the difference being around 11 %. In terms of volume of work H such discrepancy is much lower, around 4 %. The other columns of Table 2 show the effects of a decrease in the cost of opening a vacancy h. Such a reduction has almost no impact both on the wage and on the share of time spent working, whereas it slightly raises the employment rate and the volume of work. The discrepancy between the optimal and the decentralized employment level remains fairly stable. A reduction by 20% of the vacancy cost is needed in order to shorten the employment gap only by 1 %. Intervening on the vacancy cost is ineffective for the following reason. A lower h decreases the externality of one more vacancy created, but at the same time induces more firms to post vacancies. In other terms, the negative externality a single firm creates when entering the market has a lower cost for the society, but there are more firms that generate such externality in the new equilibrium. The first effect tends to reduce the gap between the optimal and the *laissez faire* outcomes, the second tends to widen it.

In Table 3, I consider the effects of a change in workers' bargaining power. Keeping the assumption of a matching function elasticity $\eta = 0.5$, I wonder for which value of β the welfare inefficiency can be close to 0. Differently from h, the parameter β does not appear in the welfare function, since the social planner cares only about the total surplus and not about its distribution between workers and firms. So, a higher β, by squeezing firms' profits and making entry less attractive, could (partially) offset the excess of entry inefficiency. Indeed, with $\beta = 0.7$, the difference between optimal and decentralized volume of work is around 2 %.

6.3 Sensitivity analysis

A sensitivity analysis is conducted on some parameters of the model. Tables 4 and 5 list the results. In Table 4, I consider a change in the elasticity of demand s, as well as in the workers' utility parameter ϵ. Such variations do not change the main conclusions of the original model, that is a difference around 11 percent between the optimal and the decentralized employment rate and a difference of 3 percent in terms of total hours worked .

In Table 5, I consider different values for the matching elasticity η. The level of wages and the amount of hours worked barely change, since these variables are chosen via the bargaining process and β is kept equal to 0.5. Employment increases with η. In the present simulation, $0 < \theta_x < 1$, for all x. Hence, by equation (18), a higher η lowers the expected duration of filling a vacancy $(1/q(\theta_x) = a^{-1}\,\theta_x^{\eta})$ but raises the factor at which future profits are discounted $(\theta_x q(\theta_x) = a\,\theta_x^{1-\eta})$. The first effect is stronger: more vacancies are created, raising the employment rate. Keeping workers' bargaining power equal to 0.5, the employment inefficiency gap decreases with η. This is because even the social planner, when η goes up, selects more vacancies for any given level of $L - x$. Such increase is slightly larger than in the *laissez faire* equilibrium. In the last row of Table 5, I compute for any η the value of β such that the difference between the decentralized and the optimal total numbers of hours worked is less than 1 %. Since the ineffficency gap decreases with η, a lower β/η ratio is needed to be close to the optimum. With $\eta = 0.5$, β must be equal to 0.75; with $\eta = 0.7$, β must be set to 0.8, around 14% more.

So, as far as the value of 0.5 can be considered a good proxy of the elasticity in the matching technology, β should be at least 50 per cent larger of η to set to zero the inefficiency gap.

7 Conclusions

In this paper, the two-way relationship between product market competition and labour market performance has been studied both from a positive and from a normative viewpoint. As far the positive analysis is concerned, it is shown that a lower cost of opening a vacancy or a reduction in workers' bargaining power raise aggregate employment, but has ambiguous effect on the average real wage and on employee's instantaneous utility.

Turning to the welfare analysis, however, the conclusion reached is that the decentralized economy may lead to an excessive level of employment and competition. A "business stealing" effect is at work in such framework: Any single firm deciding to enter the market fails to consider the reduction both in other firms' expected profits and in their probability of finding a worker. Simulation results predicts that, in order to be close to the optimal level of employment, workers' bargaining power must be larger than the elasticity η in the matching function. If the latter is imposed to be 0.5, then β must be around 0.75.

Some caveats must be advanced about the model specification. Imposing perfectly segmented labour markets is undoubtedly a major restriction. Workers are locked in their sector unless a new product is invented. Allowing workers to search across sectors would be a more realistic extension. Finally, it would be also interesting to study the dynamic evolution of the model and not focusing only on the steady state distribution. All these extensions are left for future research.

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Appendix A: The bargaining game

The bargaining process I pursue is very close to Hall and Milgrom (2006); their model, in turn, is an adapted version of Binmore, Rubinstein, and Wolinsky (1986). The maximization problem in (12) can be seen as a limit case of an extensive form bargaining game of offers and counter-offers. More precisely, consider a bargaining process that takes place over time and where firms and workers alternate in making proposals about the wage and the numbers of hours worked. After a proposal of the counterpart, a player has three options. He can abandon the bargaining (and so get an utility of either J_V or W_U, the outside options of the employer an the employee), disagree and make a counter-offer, accept the offer. Binmore et al. (1986) show that the subgame perfect equilibria of two bargaining games beginning with a proposal either by the employer or the worker are unique. So the value of rejecting an offer and continuing to bargain is uniquely defined.

When the worker (respectively, the firm) decides to reject the other player's offer and make a counter-proposal, he receives a utility flow equal to z (resp. to zero), his utility of leisure. I also introduce an hazard rate, s, that the agreement is no longer convenient. In this case, the firm-worker pair is broken. Then, the pair starts a new negotiation. The expected discounted values for an employer and an employee in the case the production opportunity disappears and x firms are active, are given respectively by $\bar{W}_E$ and $\bar{J}_E$ (equations (11) and (10)). Consider a negotiation over the wage [15]. The time period separating one offer from the next one is τ. Since the value of rejecting an offer and continuing to bargain is uniquely defined[16], the worker's equilibrium strategy is to accept any offer that makes him at least as well-off than both continuing the bargaining and abandoning it. There exists, therefore, a lowest wage w' that makes the worker indifferent between such options and, symmetrically, there exists a highest wage w'' that makes the firm indifferent. It is then clear that the optimal strategy for a worker is to offer always w'' and for a firm to offer always w'. The equations governing the equilibrium are the following:

$$\begin{aligned} W_E(x,w') &= \max\left\{W_U(x-1),\ z\tau + e^{-r\tau}\left[\left(1-e^{-s\tau}\right)\bar{W}_E + e^{-s\tau}W_E(x,w'')\right]\right\} \\ J_E(x,w'') &= \max\left\{J_V(x-1),\ e^{-r\tau}\left[\left(1-e^{-s\tau}\right)\bar{J}_E + e^{-s\tau}J_E(x,w')\right]\right\} \end{aligned} \tag{27}$$

[15] The case of a negotiation over wages and hours worked is similar.

[16] For the proof, I refer to Binmore et al. (1986).

I assume, as Hall and Milgrom (2006), and Rosen (1997), that neither workers nor firms have an incentive to abandon the negotiation. In other terms, the constraints in (12) are never binding . Therefore, the system (27) becomes:

$$\begin{aligned} W_E(x,w') &= z\tau + e^{-r\tau}\left(1-e^{-s\tau}\right)\bar{W_E} + e^{-(r+s)\tau}W_E(x,w'') \\ J_E(x,w'') &= e^{-r\tau}\left(1-e^{-s\tau}\right)\bar{J_E} + e^{-(r+s)\tau}J_E(x,w') \end{aligned} \tag{28}$$

In equilibrium, $w' = w'' = w$. So, $W_E(x,w') = W_E(x,w'') = W_E(x)$ and $J_E(x,w'') = J_E(x,w') = J_E(x)$. Moreover, letting τ, the period separating one offer from the next, approach 0, I get:

$$(W_E(x) - J_E(x)) = \frac{z}{r+s} + \frac{s}{r+s}\left(\bar{J_E} - \bar{W_E}\right) \tag{29}$$

This equation is very similar to equation (17) in Hall and Milgrom (2006). If I assume $s \to +\infty$, that is the parties have only an instant to make their bargain, the surplus sharing rule will become:

$$W_E(x) - \bar{W_E} = J_E(x) - \bar{J_E}. \tag{30}$$

It coincides with the F.O.C. for w_x of the maximization problem in (12) when $\beta = 0.5$.[17] The threats points for an employer and employee are given respectively by $\bar{J_E}(x)$ and $\bar{W_E}(x)$. Using equations (9) and (7), I get:

$$\begin{aligned} (1-\beta)\left[\frac{w_x^* + \phi(l_x^*) + \delta W_U(0) + (L-x)\theta_x q(\theta_x) W_E(x+1)}{r+\delta+(L-x)\theta_x q(\theta_x)} - \bar{W_E}\right] &= \\ \beta\left[\frac{p(Q_x^*)l_x^* - w_x^* + \delta J_V(0) + (L-x)\theta_x q(\theta_x) J_E(x+1)}{r+\delta+(L-x)\theta_x q(\theta_x)} - \bar{J_E}\right]. \end{aligned} \tag{31}$$

Finally, using (10) and (11), I obtain:

$$w_x^* = \beta p_x l_x^* + (1-\beta)\frac{(l_x^*)^\epsilon}{\epsilon}.$$

[17] Assuming a probability β that Nature selects the worker as first mover in the game yields the generalized Nash solution.

Appendix B: Proof of Lemma 2

Let denote for simplicity $R_x \equiv p(Q_x)l_x - w_x \;\forall x \in [1, 2, ..L]$ and recall that R_x is decreasing in x (firms' revenues decrease with competition). Knowing by (19) that $r + \delta = \frac{R_L q(\theta_{L-1})}{h}$, equation (18) can be written as:

$$\frac{1}{q(\theta_{x-1})} = \frac{R_x + h\,(L-x)\theta_x}{R_L\, q(\theta_{L-1}) + h\,(L-x)\theta_x q(\theta_x)}.$$

Multiplying both sides by $q(\theta_x)$, one gets:

$$\frac{q(\theta_x)}{q(\theta_{x-1})} = \frac{R_x\, q(\theta_x) + h\,(L-x)\theta_x q(\theta_x)}{R_L\, q(\theta_{L-1}) + h\,(L-x)\theta_x q(\theta_x)} \qquad \forall\, x \in [1, ..L]. \tag{32}$$

Consider the case $x = L - 1$. Equation (32) evaluated at $x = L - 1$ implies that $q(\theta_{L-1}) > q(\theta_{L-2})$ if and only if $R_{L-1} > R_L$. This is always the case, since firms' revenues R_x decrease with competition.

Now consider the case $x = L - 2$. Again, equation (32) evaluated at $x = L - 2$ implies that $q(\theta_{L-2}) > q(\theta_{L-3})$ if and only if $R_{L-2}\, q(\theta_{L-2}) > R_L\, q(\theta_{L-1}) = h(r + \delta)$. This yields:

$$\frac{R_{L-2}}{r+\delta} > \frac{h}{q(\theta_{L-2})} = \frac{R_{L-1} + h\theta_{L-1}}{r + \delta + \theta_{L-1} q(\theta_{L-1})} \iff$$
$$(r+\delta)\, R_{L-1} + h\,(r+\delta)\,\theta_{L-1} < (r+\delta)\, R_{L-2} + R_{L-2}\,\theta_{L-1} q(\theta_{L-1})$$

Since $R_{L-2} > R_{L-1}$, a sufficient condition for the last inequality to hold is:

$$h(r+\delta) < R_{L-2}\, q(\theta_{L-1}) \iff$$
$$\frac{h}{q(\theta_{L-1})} < \frac{R_{L-2}}{r+\delta} \iff$$
$$\frac{R_L}{r+\delta} < \frac{R_{L-2}}{r+\delta}$$

The last inequality is always verified since R_x is decreasing in x. So $q(\theta_{L-2}) > q(\theta_{L-3})$ holds.

With $x = L - 3$, by (32), one gets that $q(\theta_{L-3}) > q(\theta_{L-4})$ if and only if $R_{L-3}\, q(\theta_{L-3}) > R_L\, q(\theta_{L-1}) = h(r + \delta)$. Following the same steps, one gets:

$$\frac{R_{L-3}}{r+\delta} > \frac{h}{q(\theta_{L-3})} = \frac{R_{L-2} + 2h\theta_{L-2}}{r + \delta + 2\theta_{L-2} q(\theta_{L-2})} \iff$$
$$(r+\delta)\, R_{L-2} + 2h\,(r+\delta)\,\theta_{L-2} < (r+\delta)\, R_{L-3} + R_{L-3}\, 2\theta_{L-2} q(\theta_{L-2})$$

A sufficient condition for the last inequality to hold is:

$$\begin{aligned} h(r+\delta) &< R_{L-3}\, q(\theta_{L-2}) \iff \\ \frac{h}{q(\theta_{L-2})} &< \frac{R_{L-3}}{r+\delta} \end{aligned}$$

The last inequality always holds, since we have just shown that $\frac{h}{q(\theta_{L-2})} < \frac{R_{L-2}}{r+\delta}$ and $R_{L-3} > R_{L-2}$. Therefore $q(\theta_{L-3}) > q(\theta_{L-4})$.
The same steps can be undertaken for any other value of x. So, $\theta_x < \theta_{x-1}$, $\forall\, x \in [\,1, ..L\,]$.

Appendix C: Proof of Proposition 1

- Comparative statics on β

Consider a Cobb-Douglas matching function: $m_x = a(L-x)^{\eta} V_x^{1-\eta}$ with $\eta = 0.5$ and recall that $R_x \equiv p(Q_x)l_x - w_x = (1-\beta)\left[p(Q_x)l_x - \frac{l_x^{\epsilon}}{\epsilon}\right]$. Equation (18) then becomes:

$$\theta_{x-1} = \left[\frac{a}{h} \cdot \frac{R_x + h(L-x)\theta_x}{r + \delta + m_x}\right]^{\frac{1}{\eta}}. \qquad (33)$$

To show that $\frac{d\theta_{x-1}}{d\beta} < 0 \quad \forall x \in [1, 2, ...L]$, I undertake the following steps:

1. STEP : I show that $\frac{d\theta_{L-1}}{d\beta} < 0$.
2. STEP : I show that $\theta_{L-1} = -(1-\beta)(1-\eta)\frac{d\theta_{L-1}}{d\beta}$.
3. STEP: I show that $\frac{d\theta_{L-2}}{d\beta} < 0$ if $\theta_{L-1} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-1}}{d\beta}$.
4. STEP: I show that $\frac{d\theta_{L-3}}{d\beta} < 0$ if $\theta_{L-2} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-2}}{d\beta}$ and, in turn, a sufficient condition for such inequality is $\theta_{L-1} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-1}}{d\beta}$.
5. STEP: More in general, $\frac{d\theta_{x-1}}{d\beta} < 0$ if $\theta_x \geq -(1-\beta)(1-\eta)\frac{d\theta_x}{d\beta}$ and, in turn, a sufficient condition for such inequality is $\theta_{x+1} \geq -(1-\beta)(1-\eta)\frac{d\theta_{x+1}}{d\beta}$.

From the last step, moving backward, I get $\frac{d\theta_{x-1}}{d\beta} < 0 \quad \forall x \in [1, 2, ...L]$.

1 STEP:

From (33) evaluated at $x = L$, one gets:

$$\frac{d\,\theta_{L-1}}{d\,\beta} = \frac{1}{\eta}\left[\frac{a}{h}\cdot\frac{R_L}{r+\delta}\right]^{\frac{1}{\eta}-1}\cdot\frac{a}{h\cdot(r\,+\,\delta)}\cdot\frac{d\,R_L}{d\,\beta}\;<\;0$$

The derivative is negative because $\frac{d\,R_x}{d\,\beta}\;=\;-\frac{R_x}{1-\beta}\;<\;0,\quad\forall x\in[1,2,...L]$.

2 STEP:
The equality comes directly by imposing $\eta = 0.5$ and using $\frac{d\,R_L}{d\,\beta}\;=\;-R_L/(1-\beta)$.

3 STEP:
Differentiating (33), one gets:

$$\frac{d\,\theta_{x-1}}{d\,\beta} = \frac{\theta_x^{1-\eta}}{\eta}\cdot\frac{a}{h}\cdot\frac{\Gamma_x}{(r\,+\,\delta\,+\,m_x)^2},\tag{34}$$

with

$$\Gamma_x\;\equiv\;\left[\frac{dR_x}{d\beta}\;+h(L-x)\frac{d\,\theta_x}{d\,\beta}\right](r+\delta+m_x)-\frac{d\,m_x}{d\,\beta}\left[R_x+h(L-x)\theta_x\right].$$

The sign of the derivative is equal to the sign of Γ_x. To show that $\Gamma_x\;<\;0$, notice that

$$h(L-x)\frac{d\,\theta_x}{d\,\beta}m_x\;=\;h(L-x)^2\theta_x q(\theta_x)\frac{d\,\theta_x}{d\,\beta}\;<\;\frac{d\,m_x}{d\,\beta}h(L-x)\theta_x\;=\;h(L-x)^2(1-\eta)\theta_x q(\theta_x)\frac{d\,\theta_x}{d\,\beta}.$$

and

$$\frac{d\,R_x}{d\,\beta}m_x\;=\;-\frac{R_x}{1-\beta}\,(L-x)\theta_x q(\theta_x)\;<\;\frac{d\,m_x}{d\,\beta}R_x\;=\;R_x(L-x)(1-\eta)q(\theta_x)\frac{d\,\theta_x}{d\,\beta}.$$

$$\text{if}\;\;\frac{d\,\theta_x}{d\,\beta}\;<\;0\quad\text{and}\quad\theta_x\;\geq\;-(1-\beta)(1-\eta)\frac{d\,\theta_x}{d\,\beta}.\tag{35}$$

If the two conditions in (35) hold, then $\frac{d\,\theta_{x-1}}{d\,\beta}\;<\;0$. From Step 1, we know that $\frac{d\,\theta_{L-1}}{d\,\beta}\;<\;0$; from Step 2, $\theta_{L-1}\;=\;-(1-\beta)(1-\eta)\frac{d\,\theta_{L-1}}{d\,\beta}$. So $\frac{d\,\theta_{L-2}}{d\,\beta}\;<\;0$.

4 and 5 STEP:
The procedure underlying Step 3 and Step 4 is the following.

From (35), $\frac{d\theta_{L-3}}{d\beta} < 0$ if $\theta_{L-2} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-2}}{d\beta}$ and $\frac{d\theta_{L-2}}{d\beta} < 0$. The latter condition has been proved in the previous step, so only the former has to be shown. Once proved that $\frac{d\theta_{L-3}}{d\beta} < 0$, one gets $\frac{d\theta_{L-4}}{d\beta} < 0$ if $\theta_{L-3} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-3}}{d\beta}$. In turn, once proved that $\frac{d\theta_{L-4}}{d\beta} < 0$, to show that θ_{L-5} is decreasing in β, one only needs to prove that $\theta_{L-4} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-4}}{d\beta}$, and so on.
In general, $\frac{d\theta_{x-1}}{d\beta} < 0$ if $\theta_x \geq -(1-\beta)(1-\eta)\frac{d\theta_x}{d\beta}$ $\forall x \in [0, 1, ...L-1]$.
Comparing (33) and (34) with $\eta = 0.5$, one gets that $\theta_x \geq -(1-\beta)(1-\eta)\frac{d\theta_x}{d\beta}$ if

$$-(1-\beta)\cdot\Gamma_{x+1} \;\leq\; [R_{x+1} + h(L-x-1)\theta_{x+1}]\cdot(r+\delta+m_{x+1}) \tag{36}$$

Since:

$$-(1-\beta)\frac{dR_{x+1}}{d\beta}(r+\delta+m_{x+1}) \;=\; R_{x+1}\,(r+\delta+m_{x+1})\,,$$

the inequality (36) can be written in the following way:

$$\begin{aligned}-(1-\beta)\left\{h(L-x-1)\frac{d\,\theta_{x+1}}{d\,\beta}(r+\delta+m_{x+1}) - \frac{d\,m_{x+1}}{d\,\beta}\left[R_{x+1} + h(L-x-1)\theta_{x+1}\right]\right\}\\ \leq\; h(L-x-1)\theta_{x+1}\,(r+\delta+m_{x+1})\end{aligned} \tag{37}$$

So, for $\frac{d\theta_{L-3}}{d\beta} < 0$ to be verified, it is sufficient to show that (37) holds at $x+1 = L-1$. From Step 2, I know that:

$$-(1-\beta)(1-\eta)h\frac{d\,\theta_{L-1}}{d\,\beta}(r+\delta+m_{L-1}) \;\leq\; h\theta_{L-1}(r+\delta+m_{L-1}). \tag{38}$$

So, if the LHS of (37) evaluated at $x+1 = L-1$ is not greater than the LHS of (38), then inequality (37) is verified and $\frac{d\theta_{L-3}}{d\beta} < 0$. Dividing the LHS of (37) by $(r+\delta+m_{L-1})$ and doing some algebra yields:

$$-\left(r+\delta+m_{L-1}\right)\frac{d\theta_{L-1}}{d\beta}h(1-\beta)\eta \;\leq\; \frac{d\theta_{L-1}}{d\beta}(1-\eta)(1-\beta)\left[R_{L-1} + h\theta_{L-1}\right]$$

Simplifying and imposing $\eta = 0.5$:

$$\frac{h}{q(\theta_{L-1})} \quad\leq\quad \frac{R_{L-1} + h\theta_{L-1}}{r+\delta+m_{L-1}} \quad=\quad \frac{h}{q(\theta_{L-2})}$$

This inequality is always verified since, from Lemma 2, $\theta_x < \theta_{x-1}$, $\forall x$. Therefore, $\theta_{L-2} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-2}}{d\beta}$ and, consequently, $\frac{d\theta_{L-3}}{d\beta} < 0$. To show that $\frac{d\theta_{L-4}}{d\beta} < 0$,

one must undertake the same passages: from Step 3, θ_{L-4} is decreasing in β if $\theta_{L-3} \geq -(1-\beta)(1-\eta)\frac{d\,\theta_{L-3}}{d\,\beta}$. In turn, this is equivalent to prove inequality (37) evaluated at $x+1 = L-2$. Using the fact that $\theta_{L-2} \geq -(1-\beta)(1-\eta)\frac{d\,\theta_{L-2}}{d\,\beta}$, inequality (37) holds even at $x+1 = L-2$.
Proceeding backward, we get that $\frac{d\,\theta_x}{d\,\beta} < 0 \;\; \forall x \in [0, 1, 2, ...L-1]$.

- *Comparative statics on h*

The procedure is the same as in the comparative statics for β. I show that:

1. $d\,\theta_{L-1}/d\,h$ is negative and that $\theta_{L-1} = h(1-\eta) \cdot d\,\theta_{L-1}/d\,h$.

2. $d\,\theta_{x-1}/d\,h$ is negative if $\theta_x \geq h(1-\eta) \cdot (d\,\theta_x/d\,h)$ that, in turn, holds if $\theta_{x+1} \geq h(1-\eta) \cdot (d\,\theta_{x+1}/d\,h)$.

Consider point 1:

$$\frac{d\,\theta_{L-1}}{d\,h} = -\frac{\theta_{L-1}}{h\,\eta} < 0.$$

Hence, $\theta_{L-1} = h(1-\eta) \cdot d\,\theta_{L-1}/d\,h$. To show point 2, consider the following derivative:

$$\frac{d\,\theta_{x-1}}{d\,h} = a\theta_{x-1}^{1-\eta} \cdot \frac{T_x}{\eta\,[h\,(r + \delta + m_x)]^2}, \tag{39}$$

with

$$T_x \equiv \left[(L-x)\theta_x + h(L-x)\theta_x\frac{d\,\theta_x}{d\,h}\right] h(r+\delta+m_x) - \left[(r+\delta+m_x) + h\frac{d\,m_x}{d\,h}\right][R_x + h(L-x)\theta_x]\,.$$

The sign of (39) is equal to the sign of T_x. After some algebra, one gets that $T_x < 0$ if $\theta_x \geq -h(1-\eta)\frac{d\,\theta_x}{d\,h}$. In turn, by (33) and (39), a sufficient condition for such inequality to hold is:

$$-h \cdot T_{x+1} \leq [R_{x+1} + h(L-x-1)\theta_{x+1}] \cdot (r + \delta + m_{x+1})\,. \tag{40}$$

So, to show that θ_{L-2} is decreasing in h, I have to show that (40) holds at $x+1 = L-1$. Since $\theta_{L-1} \geq h(1-\eta)\frac{d\,\theta_{L-1}}{d\,h}$, (40) is verified if

$$-h^3(1-\eta)\frac{d\,\theta_{x+1}}{d\,h}a(L-x-1)(r+\delta+m_x) \geq -h \cdot T_{x+1},$$

evaluated at $x+1 = L-1$. After some algebra, such inequality is equivalent to:

$h(1-\eta)(r+\delta+m_{x+1}) + R_{x+1}(1-\eta)q(\theta_{x+1}) \;\geq\; h(r+\delta+m) - h(L-x-1)(1-\eta)\theta_{x+1}q(\theta_{x+1})$,

evaluated at $x+1 = L-1$. Dividing by $1-\eta = 0.5$, and simplifying one gets:

$$R_{x+1}q(\theta_{x+1}) \;\geq\; h(r+\delta) \;=\; R_{L-1}q(\theta_{L-1})$$

Such inequality is always verified, for the LHS of (32) is greater than one, $\forall x$. Hence, $\theta_{L-2} \;\geq\; -h(1-\eta)\frac{d\,\theta_{L-2}}{d\,h}$ and θ_{L-3} is decreasing in h. The same steps can be undertaken for $\theta_{L-4},\; \theta_{L-5},\; ....,\theta_0$.

Appendix D: Details of the proof of Proposition 2

The inequality $S^*_{x+1} \;>\; (x+1)\cdot S^{\circ}_{x+1} \;-\; x\cdot S^{\circ}_x$ is equivalent to:

$$p(Q^*_{x+1})l^*_{x+1} \;-\; \frac{(l^*_{x+1})^{\epsilon}}{\epsilon} \;>\; (x+1)\left[p(Q^{\circ}_{x+1})l^{\circ}_{x+1} \;-\; \frac{(l^{\circ}_{x+1})^{\epsilon}}{\epsilon}\right] - x\left[p(Q^{\circ}_x)l^{\circ}_x \;-\; \frac{(l^{\circ}_x)^{\epsilon}}{\epsilon}\right].$$

Notice that the term at the RHS can be written as:

$$p(Q^{\circ}_{x+1})l^{\circ}_{x+1} - \frac{(l^{\circ}_{x+1})^{\epsilon}}{\epsilon} \;\;+\;\; x\cdot\left[p(Q^{\circ}_{x+1})l^{\circ}_{x+1} - \frac{(l^{\circ}_{x+1})^{\epsilon}}{\epsilon} - p(Q^{\circ}_x)l^{\circ}_x + \frac{(l^{\circ}_x)^{\epsilon}}{\epsilon}\right]. \qquad (41)$$

Consider first the term outside the square brackets. Recall from (22) that the optimal and the decentralized level of hours worked coincide if there is perfect competition. Moreover, revenues are always higher in a Cournot market than in a perfect competition:

$$p(Q^{\circ}_{x+1})l^{\circ}_{x+1} \;-\; \frac{(l^{\circ}_{x+1})^{\epsilon}}{\epsilon} \;<\; p(Q^*_{x+1})l^*_{x+1} \;-\; \frac{(l^{\circ}_{x+1})^{\epsilon}}{\epsilon}.$$

It is then sufficient to show that the term in (41) inside the square graphs is negative to prove the inequality of Step 1. But this is the case if $p(Q^{\circ}_x)l^{\circ}_x + \frac{(l^{\circ}_x)^{\epsilon}}{\epsilon}$ is decreasing in x. Ignoring for simplicity the integer problem, I get:

$$\frac{d\left[p(Q^{\circ}_x)l^{\circ}_x - \frac{(l^{\circ}_x)^{\epsilon}}{\epsilon}\right]}{d\,x} \;=\; \frac{d\,p(Q^{\circ}_x)}{d\,Q^{\circ}_x}l^{\circ}_x\frac{d\,Q^{\circ}_x}{d\,x} \;+\; \frac{d\,l^{\circ}_x}{d\,x}\left[p(Q^{\circ}_x) \;-\; (l^{\circ}_x)^{\epsilon-1}\right] \;<\; 0.$$

The term inside the square bracket is equal to zero, while the first term is negative since $p(Q_x)$ has a negative slope.

Appendix E: Decentralized vs. optimal solution in the case of perfect competition

Consider the same two-tier productive scheme explained in section 2. The only difference is that in each intermediate sector there is a continuum of workers of measure L. Perfect competition prevails in each intermediate market. Firms and workers are price-takers. Thus, in computing their expected lifetime income, $W_E(x) = W_E$, $J_E(x) = J_E$, $J_V(x) = J_V$, and $W_U(x) = W_U$, $\forall x$. Keeping the same bargaining process (12), the F.O.C.s for wage and hours worked become:

$$w^* = \beta p(Q^*) l^* + (1-\beta) \frac{l^{*\,\epsilon}}{\epsilon}$$

$$l^{*\,\epsilon - 1} = p(Q^*)$$

The free-entry condition $J_E = \frac{h}{q(\theta)}$ can be written as:

$$\frac{h}{q(\theta)} = \frac{p(Q^*) - w^*}{r + \delta} = \frac{(1-\beta)\left[p(Q^*) l - \frac{l^{*\,\epsilon}}{\epsilon}\right]}{r + \delta}.$$

The social planner's problem is the same as in (20), with the only difference that x, the level of employment in a given sector, is now a continuous variable. So, $\Omega_{x+1} - \Omega_x$ is replaced by $d\,\Omega_x / d\,x$. Computing the F.O.C.s and applying the envelope theorem yields[18]:

$$\frac{h}{q(\theta)} = \frac{(1-\eta)\left[p(Q^\circ) l^\circ - \frac{l^{\circ\,\epsilon}}{\epsilon}\right] - \eta h \theta}{r + \delta},$$

in which $l^\circ = l^*$ for the F.O.C. (22). A comparison between the *laissez faire* outcome and the social planner's one shows that the Hosios condition $\beta = \eta$ is not sufficient to decentralize the optimum. If $\beta \leq \eta$, the equilibrium level of tightness is inefficiently high. The Hosios condition *and* a tax $\tau = \eta h \theta$ levied on firms' profits are needed to ensure the efficiency in the decentralized economy.

[18] For the existence of a solution, see Shimer (2004).

8 Figures and Tables

Parameters	
r	0.004
δ	0.005
ϵ	4
s	5.5
I	50
h (Euro/month)	24000
β	0.5
η	0.5
L	20
a	0.12
Endogenous var. (average)	
l (%. average)	41.0
$1/\theta q(\theta)$ (months)	19.5
$1/q(\theta)$ (months)	3.4
$\bar{w}$ (Euro/month)	1235

Table 1. Calibration: Parameters and levels of endogenous variables in steady state.

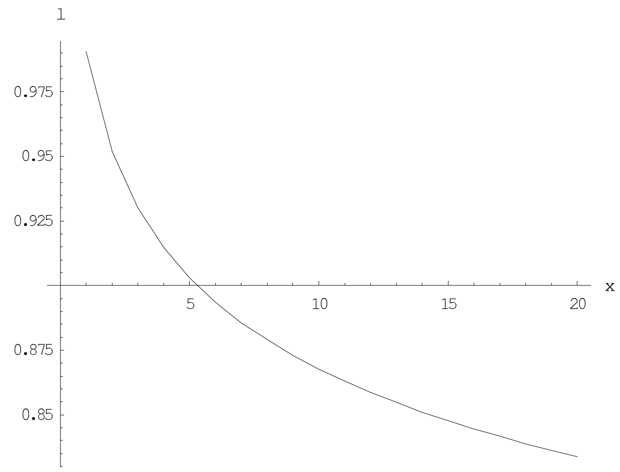

Figure 1: Simulation results: Hours Worked $l \in [0, 2]$.

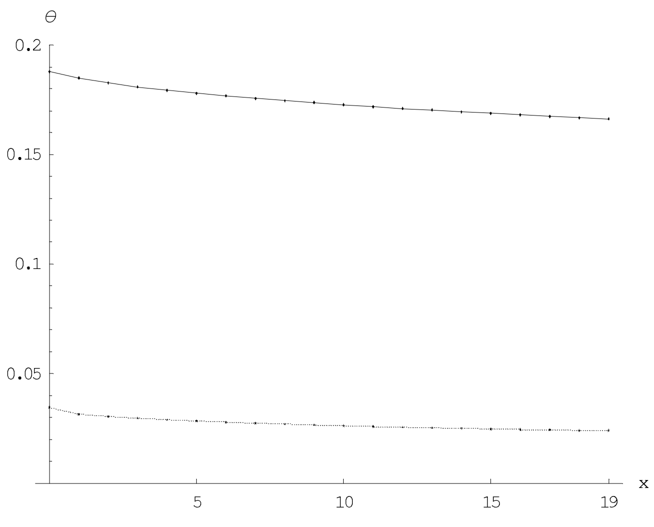

Figure 2: A comparison of the optimal level of labour market tightness (dotted line) with the decentralized one (continuous line).

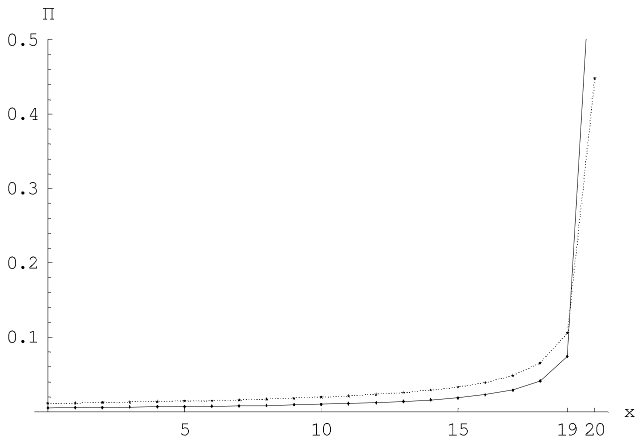

Figure 3: A comparison of the optimal steady state distribution (dotted line) with the decentralized one (continuous line).

Variables	h = 24000	h = 16000	h = 14000
$\bar{w}$ (euros per month)	1235	1232	1231
e^* (per cent)	90.9	92.3	92.6
Share of hours worked	41.0	41.8	41.8
H^* (per cent)	38.0	38.5	38.7
$e^* - e^\circ$	11.1	10.6	10.4
$H^* - H^\circ$	4.3	4.2	4.1

Table 2. Simulation Results. Variation in the cost of opening a vacancy. Superscript * denotes the free-entry equilibrium values, while superscript $^\circ$ the optimal ones.

Variables	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$
Employment rate e^* (per cent)	90.9	88.9	85.9
Volume of work H^* (per cent)	38.0	37.2	36.0
$e^* - e^\circ$ (per cent)	11.1	9.1	6.0
$H^* - H^\circ$ (per cent)	4.3	3.6	2.3

Table 3. Simulation Results. Variation in workers' bargaining power β when $\eta = 0.5$.

Parameters	Benchmark	1° case	2° case	3° case
ϵ	4	2	4	3
s	5.5	5.5	6.5	4
Variables				
e^* (per cent)	90.9	89.5	89.2	94.2
$\bar{w}$ (euros per month)	1235	1877	1015	2404
Share of hours worked (per cent)	41	55.7	39.8	53.0
Volume of work H^* (per cent)	38.0	49.9	35.5	50.0
$H^* - H^\circ$ (per cent)	4.3	5.3	4.2	5.0
$e^* - e^\circ$ (per cent) if $\beta = \eta = 0.5$	11.1	12.4	11.1	10.7
$H^* - H^\circ$ (per cent) if $\beta = 0.7$	2.3	3.1	1.9	3.4

Table 4. Sensitivity analysis.

Parameter	Benchmark	1° case	2° case	3° case
η	0.5	0.4	0.6	0.7
Variables				
e^* (per cent)	90.9	86.8	93.0	94.3
w (euros per month)	1235	1244	1230	1228
Share of hours worked (per cent)	41	41.9	41.8	41.8
Volume of work H^* (per cent)	38.0	36.4	38.9	30.3
$H^* - H^\circ$ (per cent)	4.3	4.6	3.5	2.6
$e^* - e^\circ$ (per cent) if $\beta = \eta$	11.1	11.7	9.1	6.8
β/η s.t $H^* - H^\circ < 1\,\%$	1.5	1.62	1.3	1.14

Table 5. Sensitivity analysis: change in the matching elasticity η.