Product Specification with Differentiation by Attributes

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Abstract

This paper considers a probabilistic duopoly in which consumers follow a random decision rule based on products' attributes. A two-stage game is studied in which firms choose, first, the specific attributes of their product and, then, compete in prices. The existence of a perfect Nash equilibrium is demonstrated under different cost assumptions. When costs are exogenous, firms classically choose the highest attribute indices. When unit or fixed costs are attributes-dependent, firms select heterogeneous attribute indices and equilibrium product differentiation is both horizontal and vertical. This new finding is relevant to describe many market situations. Moreover, the firm selling the less appreciated product makes the highest profit, its margin being preserved by the horizontal differentiation: this result contrasts with the high quality advantage highlighted in pure vertical differentiation models. Finally, this analysis explains observed strategies of systematic product improvement by some benchmark firms.

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1 Introduction

High-quality good producers sometimes face major difficulties on their leading product market, as highlighted by the following examples. In the 16th century, Venetian tradesmen had an impressive hegemony on woolen cloths and textile markets: their reputation was based on a technological advantage over their North European competitors. By their reference status in term of quality, these sellers attracted many consumers who were ready to pay a high price, generating sufficient revenues to cover high production costs (Rapp, 1975, p. 508). However, North European countries successfully improved their products while keeping competitive costs, triggering the decline of Venetian trade at the 17th century. In the same spirit, the American company Caterpillar Tractor faced similar difficulties at the beginning of the 1980's. From 1930 to 1982, this heavy earth-moving equipment manufacturer improved systematically its product by offering new attributes and services, putting aside cost considerations (Miller, 1990, p. 22). Caterpillar's product was perceived as a benchmark by its competitors. As underlined by Peters and Waterman in their book "In Search of Excellence" (1982, p 171), Caterpillar was also reputed for its reliability, making the high prices acceptable for its customers. However, the emergence of a competitor offering fair products at low prices, the Japanese company Komatsu, constrained Caterpillar to a severe cost reduction program of $1.8 billion.

These examples cannot be easily linked with current literature on product specification, which is concerned with the analysis of quality and variety settings. Quality choices have been largely studied in deterministic models with vertical differentiation, when costs are exogenous (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982; Choi and Shin, 1992; Wauthy, 1996) and when costs are quality-dependent (Ronnen, 1991; Motta, 1993; Lehmann-Grube, 1997; Kuhn, 2007). A similar work has been carried out by Anderson, de Palma and Thisse (1992) in the logit oligopoly. Two conclusions emerge from these papers. First, the introduction of quality-dependent unit or fixed costs (instead of exogenous costs) reduces the quality levels chosen by firms. Second, the high-quality firm makes the highest profit. Consequently, current models do not explain the systematic product improvement strategies such as those previously described. In particular, they fail to explain why the high quality firm pays little attention to its costs, even though this firm makes a low profit.

At first sight, it seems that the existence of several characteristics of differentiation on the market could reduce the advantage of the high-quality firm. Yet, the main conclusion derived from standard multi-dimensional models suggests that firms maximize differentiation in one dominant characteristic (for example, the quality difference) and minimize it in all the other characteristics (Irmen and Thisse, 1998).[1] This does not provide a rationale for the above-mentioned firms' behavior.

This paper provides a new and alternative framework to study firms' choices of product specification in the duopoly. It is supposed here that products are differentiated by their attributes, an approach which recalls that of Lancaster (1966). In the "Differentiation by Attributes" (henceforth, DBA) model, consumer's choice among a discrete number of product is described by a probabilistic rule based on specific attributes. This model embodies existing forms of differentiation (Laurent, 2007). When the specific attributes of the two goods provide consumers with the same utilities, differentiation is horizontal. When a single product possesses all the specific attributes available on the market, and the other none, differentiation is vertical. Finally, when each good has some specific attributes but that provide consumers with different utility levels, the above two dimensions of differentiation are simultaneously taken into account. In a classical way, existence of perfect Nash equilibrium is studied in a two-stage game. In the first stage, firms select an attribute index. Cases of exogenous costs, attributes-dependent unit costs and attributes-dependent fixed costs are considered. In the second stage, firms compete in prices.

The main results are derived from the properties of the perfect equilibrium highlighted. When costs are exogenous, firms select the highest attribute indices. When unit costs are convex and individual utilities concave in the attribute index, product specifications have the following properties: one firm chooses the index equating the marginal utility and the marginal cost of its attributes, while its rival selects the highest possible attribute index. This last finding comes from the reference status of the high-index firm and provides a possible explanation of the improvement strategies discussed at the beginning of the paper. When fixed costs depend on attributes, a standard linear utility and quadratic fixed costs setting is used. One firm selects the attribute index equating the marginal utility and the marginal fixed cost by consumer, while its rival chooses a lower index. To sum up, products are differentiated both horizontally and vertically at the equilibrium with endogenous costs, this setting being observed on many

[1] In the duopoly framework, firms' location choices were also analyzed over a one-dimensional space of varieties (d'Aspremont et al., 1979; Salop, 1979). When this space is linear, note that firms' location may affect the vertical differentiation (Gabszewicz and Thisse, 1986). Two-dimensional address models were studied by Neven and Thisse (1990) and Economides (1993).

markets. Moreover, it is shown that the firm selling the less appreciated product makes the highest profit. Indeed, choosing a low attribute index is cost saving while the horizontal differentiation on the market generates a sufficient profit. This result contrasts with the high quality advantage highlighted in vertical differentiation models.

The paper is organized as follows. Existence and uniqueness of a Nash price equilibrium in the DBA duopoly are recalled in Section 2. Section 3 introduces the concept of attribute index and exposes the sequential game considered. Section 4 studies firms' choices of specific attributes at the perfect equilibrium. The last Section concludes and proofs of some propositions are presented in the Appendix.

2 Price equilibrium in the DBA duopoly

At first, this Section introduces the choice probabilities of the DBA model. Then, forms of differentiation described by the corresponding demand system are analyzed. Finally, existence and uniqueness of a Nash price equilibrium are studied.

2.1 Choice probabilities

Consider a market in which two differentiated products are available, each consumer purchasing exactly one unit of one product. It is supposed that consumers follow a probabilistic reasoning based on products' attributes, their behavior being described by a random decision rule model. The framework employed here is called "Differentiation By Attributes" because products are differentiated according to the specific attributes they possess. This model is formally equivalent to the structures with Selection By Aspects (Restle, 1961) and with Elimination By Aspects (Tversky, 1972), these two frameworks being themselves identical when only two options are considered.

According to the conclusions of Payne and Bettman (2001), using such decision rules is time-saving for the consumer because the decision making does not require a complete listening of all the attributes. Moreover, the authors show that the Elimination By Aspects heuristic provides a good trade-off between the deliberation cost and the quality of the decision. This random decision rule model is more relevant than a random utility structure to describe small purchase decisions locking up a limited amount of income (see Fader and McAlister, 1990). Indeed, it is simply unlikely that consumers make a complete examination of all the characteristics and of all the products in this context. This structure is especially appropriate to describe supermarket shopping because consumers make several small decisions in a reduced time. The DBA model is also relevant when the choice of a product is a direct and immediate consequence of a special attribute it possesses: in the examples mentioned in introduction, the purchase is triggered by a brand name or by the additional quality offered.

In this model, specific non-price attributes of product i, with $i = \{1, 2\}$, provide consumers with a utility u_i. These individuals do not take into account the attributes shared by the products because they are useless for the choice among a set of products. In such a framework, it is traditionally easier embodying discrete attributes, such as specific accessories or brand names, than continuous variables, as prices or qualities. However, this difficulty is overcome by Rotondo (1986) who suggests representing the linear price difference as a specific attributes of the least expensive product.[2] For small purchase decisions, the general level of prices on the market does not really matter, whereas a price difference between products is noteworthy. The probability P_i of selecting i rather than j depends on the price hierarchy:

- if $p_i \geq p_j$,

$$P_i^{\overline{p}} = \frac{u_i}{u_i + u_j + p_i - p_j} \qquad P_j^{\underline{p}} = \frac{u_j + p_i - p_j}{u_i + u_j + p_i - p_j} \tag{2.1}$$

- if $p_j \geq p_i$,

$$P_i^{\underline{p}} = \frac{u_i + p_j - p_i}{u_i + u_j + p_j - p_i} \qquad P_j^{\overline{p}} = \frac{u_j}{u_i + u_j + p_j - p_i} \tag{2.2}$$

The numerator of the probabilities represents all the specific attributes of the product considered. For example, if $p_i \geq p_j$, good i has only non-price specific attributes providing the utility u_i, but if $p_j \geq p_i$, good i also possesses a price attribute, the price gap $p_j - p_i$. The denominator represents all the specific attributes of the two products. In a set of homogeneous goods ($u_i = u_j = 0$), the least expensive good is always selected. When prices are identical, choice probabilities simply equal the utility index ratio, as in the Luce model (1959).

[2] Representing a difference of continuous variables as an attribute is consistent with Tversky's works (1977) on similarity. Rotondo (1986) tests empirically several forms of price difference and finds that the linear function provides the more realistic description of consumers' behavior.

These probabilities can be interpreted in two different ways. In the spirit of Restle, P_i is the probability of choosing product i because it possesses a specific attribute. For example, a specific brand name may be chosen immediately by a consumer without considering other attributes. Following Tversky, P_i may also represent the probability of eliminating j after having selected a specific attribute of i as the elimination criterion.[3] For example, all the products not possessing a "fair trade" label could be eliminated by an ethical consumer.

2.2 Forms of differentiation

Consider now a market in which N consumers follow a probabilistic reasoning based on attributes: N is supposed sufficiently high so that risk-neutral firms simply use $X_i = NP_i$ and $X_j = NP_j$ as demand functions. Their detailed properties and the equivalences mentioned in the Section can be found in another paper (Laurent, 2007). Observation of choice probabilities reveals that these functions are defined piecewise and possess a kink in $p_i = p_j$ when $u_i \neq u_j$ (this kink vanishes when $u_i = u_j$). Now and for the rest of the analysis, it is supposed that $p_1 \geq p_2$ leading to the following expressions:

$$X_1 = \frac{Nu_1}{u_1 + u_2 + p_1 - p_2} \quad ; \quad X_2 = \frac{N(u_2 + p_1 - p_2)}{u_1 + u_2 + p_1 - p_2}$$

Obviously, it will be necessary to check that equilibrium prices, if they exist, respect this condition $p_1 \geq p_2$. The type of differentiation on the market depends on the values of utility variables.

Horizontal differentiation. Product differentiation is said purely horizontal in a duopoly framework when market shares equal $P_i = P_j = 1/2$ for non-homogenous products sold at identical prices. This setting is realized in the DBA model when $u_1 = u_2 > 0$: the specific attributes of the two products are appreciated in the same way into the population of consumers and each agent chooses its preferred variety. This special case of the DBA model is formally equivalent to a Quadratic Address Model (d'Aspremont *et al.*, 1979) in which consumers' density of preferences over the space of varieties is not uniform.[4]

[3] For a complete presentation of the Elimination By Aspects model, see Batsell *et al.* (2003).

[4] More precisely, the density linking the two structures is symmetric and reaches its maximum at the middle of the segment: many consumers prefer the central variety.

Vertical differentiation. Differentiation is said purely vertical in the duopoly when market shares equal $P_i = 1$ and $P_j = 0$ for identical prices. In this framework, there exists a preference hierarchy between products. These proportions are obtained in the DBA model when $u_1 > 0$ and $u_2 = 0$: one product possesses all the specific attributes on the market and all consumers prefer this product to its rival. Heterogeneous qualities can be taken into account as a particular form of vertical differentiation in the DBA model. The method of integration used for prices is simply duplicated for qualities: the quality difference is perceived as a specific attribute of the high-quality good. When products are differentiated by their only qualities, utility indices verify $u_1 = q_1 - q_2$ and $u_2 = 0$ (the minimum level of quality q_2 is shared by products 1 and 2 and is not taken into account during the choice process). In this setting, demand functions become:

$$X_1 = \frac{N(q_1 - q_2)}{q_1 - q_2 + p_1 - p_2} \;\; ; \;\; X_2 = \frac{N(p_1 - p_2)}{q_1 - q_2 + p_1 - p_2}$$

This special case of the DBA model is formally equivalent to a duopoly with differentiation by qualities (using the variant of Tirole, 1988) when a specific non-uniform distribution of consumers' intensity of preference for quality is considered.[5]

Horizontal and vertical differentiation. When product differentiation is horizontal and vertical, the market share of the most appreciated product belongs to the interval $]1/2; 1[$ and that of its rival to $]0; 1/2[$. For $u_1 > u_2 > 0$, choice probabilities are given by $P_1 = u_1/(u_1 + u_2) > 1/2$ and $P_2 = u_1/(u_1 + u_2) < 1/2$ when prices are identical. Consequently, differentiation is horizontal up to the level u_2, the specific attributes of the two products providing the same utility, and vertical for a level $u_1 - u_2$, product 1 also possessing additional attributes. There exists an important difference between the DBA model and multi-dimensional structures (Neven and Thisse, 1990; Economides, 1993; Irmen and Thisse, 1998). Utility indices in the DBA model may integrate in a simple and endogenous way many characteristics of differentiation. Demand systems of deterministic models are often more complex and represent a limited number of characteristics, given *a priori*. In these structures, the equilibrium analysis also requires

[5]The density linking this structure with the DBA model is a decreasing function of preference intensity for quality. As many consumers have a low willingness to pay for the quality, this function looks like the observed income distribution into the population of consumers.

knowing which characteristic is "dominant" whereas the DBA model is not subjected to this limitation. Finally, the DBA duopoly is not equivalent to a binomial logit with heterogeneous qualities, whose demand functions have the following expressions (Anderson *et al.*, 1992):

$$X_1 = \frac{N\exp((q_1 - q_2 + p_2 - p_1)/\mu)}{1 + \exp((q_1 - q_2 + p_2 - p_1)/\mu)} \;\; ; \;\; X_2 = \frac{N}{1 + \exp((q_1 - q_2 + p_2 - p_1)/\mu)}$$

where $\mu > 0$ is a finite parameter correlated with the variance of the distribution. For asymmetric and finite levels of quality, note that choice probabilities never equal 1: consequently, the setting of pure vertical differentiation can not be represented in the logit duopoly whereas the DBA model provides a more general framework of differentiation.

2.3 Price equilibrium

Existence and uniqueness of a Nash price equilibrium in pure strategies is studied in the DBA model. Each firm i bears a unit cost c_i, a fixed cost F_i and makes a profit given by $\Pi_i = (p_i - c_i)X_i - F_i$.

Proposition 1 (Laurent, 2007, p 15) *There exists a Nash price equilibrium verifying $p_i \geq p_j$, with $i, j \in \{1, 2\}$ and $i \neq j$, if and only if:*

$$u_i \geq u_j \tag{2.3}$$

and

$$c_i - c_j \geq \sqrt{u_i u_j} - u_i \tag{2.4}$$

Moreover, this equilibrium is unique.

Assume that $u_1 > u_2$. When unit costs are identical, existence of the equilibrium is guaranteed and the firm selling the most appreciated product sets the highest price. Equilibrium values are given by:

$$p_1^* = \frac{u_1 + \sqrt{\Delta}}{2} + c \;\; ; \;\; p_2^* = u_1 + u_2 + c$$

with $\Delta = u_1^2 + 4u_1(u_1 + u_2)$. The high-quality firm does not select systematically the highest price for its product. Suppose that product 2 has a quality advantage such that $u_2 = q_2 - q_1$, whereas product 1 is endowed with a different specific attribute providing the utility u_1: when $u_1 > u_2$, firm 1 sets the highest price for its low-quality product.[6]

When unit costs are asymmetric, there exists an equilibrium if the unit cost of 1 is higher than that of 2, which is intuitive. However, firm 2 may also have a higher unit cost than that of 1 because $\sqrt{u_1 u_2} - u_1 \leq 0$ but the gap of unit costs must be weak. Equilibrium prices in $p_1 \geq p_2$ are given by:

$$p_1^* = \frac{u_1 + \sqrt{\Delta}}{2} + c_1 \tag{2.5}$$

$$p_2^* = u_1 + u_2 + c_1 \tag{2.6}$$

where $\Delta = u_1^2 + 4u_1(u_1 + u_2 + c_1 - c_2)$. Under the equilibrium conditions, firm 1 always selects a higher price than its rival. This Nash equilibrium is weak in the sense that firm 1 could make the same profit by choosing another price belonging to the interval $[p_2; +\infty[$. The analysis of reaction functions shows why p_2^* increases with c_1 but does not depend on c_2. If firm 1 decides to deviate from the equilibrium by changing its price, the local best response function of firm 2 classically incites it to modify its own price in the same direction. However, at the equilibrium, firm 1 is locally insensitive to a small price variation of firm 2. This observation recalls practices of pricing imitation like those described by Lazer (1957 p. 130-131), and particularly the case in which the firm selling the high-quality good sets a *reference price* on the market. In this context, each other firm chooses the reference price minus a certain amount, which depends on the quality gap with the reference firm. It will be shown afterwards that this property has an impact on attributes choices in the DBA model.

Equilibrium profits are given by:

$$\Pi_1^* = Nu_1 - F_1 \text{ and } \Pi_2^* = \frac{N(u_1 + u_2 + c_1 - c_2)(\sqrt{\Delta} - u_1)}{\sqrt{\Delta} + u_1} - F_2$$

When fixed costs are identical $F_1 = F_2$, the firm selling the "most appreciated" product (such that $u_i > u_j$) makes the highest profit when the gap of differentiation is sufficiently high compared to the gap of costs:

[6] On the coffee market, fair trade products have sometimes a lower quality than that of some rivals but are sold at a higher price. This observation can be explained by the DBA model, simply by considering that a fair trade label is more appreciated than a quality difference in the population of consumers.

$$\Pi_i^* > \Pi_j^* \Leftrightarrow c_i - c_j < u_i - u_j \tag{2.7}$$

Such a property is also highlighted by Anderson and Renault (2006) in a general setting of duopoly with vertical differentiation. When $c_1 = c_2$, the firm selling the most appreciated good always makes the highest profit.

3 A two-stage game with attributes choices

This section exposes our assumptions on attribute choices and the stages of a sequential game with price competition.

3.1 Differentiation by attributes and product specification

Standard economic models suppose that products may be differentiated along a vertical dimension, the level of quality, or along a horizontal dimension, the type of variety. However, these representations seem somewhat arbitrarily restrictive to describe firms' behaviors. In a broader framework, product managers have perceived long ago the necessity to develop *specific attributes* for their products in order to build a competitive advantage (for instance, see Nash, 1937, p 256). These attributes take sometimes the form of a quality or a variety difference but could also represent a discrete accessory, the fame of a brand name, *etc.* Moreover, the type of differentiation affected by a particular attribute depends on the global setting of all the specific attributes. Consider the example of a quality difference. In the DBA model, if products differ only by their qualities such that $u_1 = q_1 - q_2$ and $u_2 = 0$, an increase in the quality gap rises the vertical dimension on the market.[7] Conversely, when the less appreciated product has a quality advantage given by $u_2 = q_2 - q_1$ (product 1 possessing specific attributes such that $u_1 > u_2$), a rise in the quality gap reduces vertical differentiation and increases the horizontal dimension.

This paper assumes that the decision problem of a marketing service may be better described by a choice made into a set of specific attributes than by a choice of "varieties" or "qualities". A given product i is endowed with some specific attributes k_i chosen by its producer among a set of available attributes

[7] In this case, comparative statics properties are similar to that obtained in deterministic models with vertical differentiation. See Laurent (2007, p 21).

noted K_i ($k_i \subset K_i$). Each firm is perfectly informed if a given attribute is specific to its product or not.[8] Consequently, available attribute sets are disjoint: $K_i \cap K_j = \emptyset$. Moreover, it is supposed that any set of specific attributes can be represented by a synthetic *attribute index (or "index")*, a continuous positive variable $a_i : k_i \to [0; A_i[$ with $A_i = a(K_i)$. Several sets of specific attributes may be associated with the same index a. This index is viewed as a measure of the level of "accessories" of the considered product. Comparison between firms is easier when attributes are represented by indices rather than by vectors: if a firm i realizes a product innovation, its attribute index a_i is simply assumed to increase. Such an index represents choices of product specification in a compact way but does not provide informations on the type of attributes selected to reach the specification retained.

Two types of variables are affected by attribute indices: the utilities obtained by consumers and the costs beard by firms (assumptions on endogenous costs are detailed in section 4). A given index a provides consumers with the utility $u : a \to \mathbb{R}^+$. Thus, a consumer purchasing a good i with a set of attributes k_i obtains the utility $u_i = u(a_i)$ (with the normalization $u(0) = 0$). If firms choose sets k_1 and k_2 verifying $a_1 = a_2$, their products provide consumers with the same utility. The function u also verifies $u'(a_i) \geq 0$: any increase in the attribute index generates an additional utility for the consumer. Finally, it is assumed in a standard way that the marginal utility of the attributes decreases with the index: $u''(a_i) \leq 0$. The following Inada conditions are supposed to hold $\lim_{a_i \to +\infty} u'(a_i) = 0$ and $\lim_{a_i \to 0} u'(a_i) \to +\infty$.

Competition between firms is represented in a two-stage game in pure strategies. In the first stage, firms choose simultaneously an attribute index for their products, depicting the new attributes offered.[9] Imitation is assumed impossible: for example, the fabrication process of specific attributes is secret. In the second stage, the indices previously chosen are of a common knowledge and firms compete in prices. The game is solved by backward induction. A subgame perfect Nash equilibrium is defined by a vector of equilibrium indices $(a_1^*; a_2^*)$ and a vector of equilibrium prices $(p_1^*(a_1, a_2), p_2^*(a_1, a_2))$ for all indices (a_1, a_2).

[8] The only "stationary" choices of specific attributes are investigated. The equilibrium specification is computed here under the assumption that firms' belief in the specific nature of an attribute are not wrong.

[9] A simultaneous choice of product specification is more appropriate than a sequential one if the new generations of products are developed by multiple firms (Aoki and Prusa, 1997, p 104). Such a perspective is followed here.

4 Product innovation and equilibrium attribute indices

This section solves the two-stage game for different costs assumptions: exogenous costs, attribute-dependent unit costs and attribute-dependent fixed costs.

4.1 Attributes choices with exogenous costs

In this section, unit and fixed costs are given by $c_1 = c_2$ and $F_1 = F_2$ whatever the attribute indices chosen. At the end of the first stage, suppose that attributes choices verify conditions (2.3) and (2.4). In this case, equilibrium prices are given by (2.5) and (2.6).

Each firm's profit strictly increases with its attribute index. Consequently, the highest indices are chosen when costs are exogenous. The existence of a perfect Nash equilibrium is guaranteed only if equilibrium indices effectively meet condition (2.3), implying that $A_1 \geq A_2$. These conclusions are summarized here:

Proposition 2 *When costs are independent of the attributes, there exists two subgame perfect Nash equilibria differing only by the identity of the firms and verifying $p_i \geq p_j$ with $i, j \in \{1, 2\}$ and $i \neq j$ if and only if $A_i \geq A_j$. The highest possible indices are chosen.*

The DBA model respects the "principle of maximum differentiation", each firm having an interest in pushing as far as possible its advantage. This finding is similar to that obtained in a deterministic model of vertical differentiation with complete coverage of the market (Tirole, 1988) or in a logit oligopoly (Anderson *et al.*, 1992, p 237). Differentiation is horizontal for a level A_j and vertical for a level $A_i - A_j$. However, this result depends crucially on the assumption of costs independence.

4.2 Attributes choices with endogenous unit costs

Whereas models embodying horizontal and vertical differentiation generally assume zero costs, choice of product specification with endogenous unit or fixed costs can be studied in the DBA framework. This section supposes that unit costs increase with attribute indices whereas exogenous fixed costs are identical. Thus, each firm possesses a set of available specific attributes, discovered previously by its applied research service, and selects the attributes which will be finally added to the product. Assumption of endogenous unit costs is especially relevant when product's specificity come from costly raw materials, what is true both for the textile market of Venetian tradesmen and for Caterpillar's tractor market.

Each firm's unit cost is represented by the same function $c : q \to \mathbb{R}^+$ with $c(0) = 0$. For firm i, this unit cost is invariant with the quantity produced but increasing and convex with the index: $c'(a_i) > 0$ and $c''(a_i) \geq 0$. This assumption means that the attributes realizing the best trade-off between the cost and the utility provided are selected in first and the less "efficient" attributes are chosen afterwards. The maximum available index is identical for the two firms: $A_1 = A_2 \to +\infty$.

The existence of a perfect equilibrium is studied when firms select indices and then prices. In the first stage, attribute choices are summarized in this proposition:

Proposition 3 *When unit costs are endogenous, there exists two perfect Nash equilibria verifying $p_i \geq p_j$ and differing only by the identity of firms. Firm i chooses the highest possible index such that $u'(a_i) = 0$. Firm j chooses the index equating the marginal utility of the consumer and the marginal cost of production: $u'(a_j) = c'(a_j)$.*

Proof: this proof is presented in Appendix 7.1.

Suppose that firm 2 sells the less appreciated product: its decision is affected when unit costs become endogenous and a lower attribute index is chosen. On the contrary, introduction of an attributes-dependent unit cost does not modify the choice of firm 1, the highest attribute index being selected. This result is a consequence of the weak nature of the Nash price equilibrium in the model. Indeed, firm 1 being a "reference" on the market, any increase in its unit costs also induces a raise of its rival's price and the position of 1 in the market is not weakened. As its profit is invariant with c_1, this reference firm selects the highest index, whatever the cost beard. Obviously, the relevance of this result does not lie in the exact value of attribute index computed but rather in the underlying strategic interactions highlighted.[10]

More precisely, this result can be linked with firms' practices and in particular with the strategy of "product supremacy" as carried out by Venetian tradesman.[11] Such a strategy was also followed by the

[10] In particular, as there is no outside option in the model, the attribute index is not bounded upside.

[11] As stated by Rapp (1975, p. 507-508), "Venice had considerable monopoly power in the Mediterranean emporium and this permitted her to maximize the quality of her industrial wares, giving little attention to cost, while clearing the market

Caterpillar company until 1982, the search of quality turning to obsession.[12] Consequently, Caterpillar acquires a *reference status*, leading it to set very high prices[13].

At the equilibrium, product differentiation is both horizontal for a level a_2^* (firms offering similar attribute indices) and vertical for a level $a_1^* - a_2^*$ (firm 1 providing additional attributes). However, when unit costs are strongly convex, the horizontal dimension become insignificant on the market. An other important aspect is the study of profit hierarchy. When exogenous unit costs are identical, the firm with the highest index always makes the highest profit but this property is not necessarily verified when costs are attributes-dependent. In this framework, condition (2.7) becomes:

$$\Pi_1^* \leq \Pi_2^* \Leftrightarrow c(a_1^*) - u(a_1^*) \geq c(a_2^*) - u(a_2^*) \tag{4.1}$$

As the utility is concave and the unit costs convex, the following property is verified:

$$\frac{d(c(a) - u(a))}{da} > 0 \; \forall a \geq \overline{a} \;\text{ with }\; \overline{a} > 0 \;\text{ such that }\; c'(\overline{a}) = u'(\overline{a})$$

The index a_2^* verifying $a_2^* = \overline{q} < a_1^*$, the inequality (4.1) is always true: *the firm with the lowest attribute index always makes the highest profit.* Indeed, choosing a low attribute index reduces the unit cost while allowing to set a relatively high price because of the horizontal dimension of differentiation on the market. When the market is covered, the converse property holds in the vertical differentiation model with quadratic endogenous unit cost: the high-quality firm makes the highest profit at the perfect Nash equilibrium (Motta, 1993, p 124).[14] The absence of horizontal differentiation increases competition and thus the disadvantage of offering a low quality.

at high prices. (...) Its reputation for high quality was a strong selling point."

[12]"Caterpillar had devoted itself long and single-mindedly to building a better, more efficient crawler tractor than anybody else in the world" (Miller, 1990, p 22). Caterpillar developed new attributes increasing vertical dimension of differentiation, like an exclusive after-sales service: "Caterpillar offered its customers forty-eight-hour guaranteed parts-delivery service anywhere in the world, from a construction site in Nebraska to a village in Zaire. If it couldn't fulfill that promise, the customer got the part free" (*op. cit.* p 23).

[13]Thomas Peters, who ordered equipment for the Navy in the 1970's, writes: "We would go to almost any ends, stretching the procurement regulations to the limit, to specify the always more expensive Cat equipment. We had to, for we knew our field commanders would string us up if we didn't find a way to get them Cat."(Peters and Waterman, 1982, p 171)

[14]Kuhn (2007) showed that the low-quality firm is able obtain the largest profit when the market is not covered. Moreover, in an asymmetric oligopoly with horizontal differentiation and a variety-dependent unit cost (Waterson, 1990), the firm choosing the lowest price receives the highest profit.

The consequences of neglecting costs for a benchmark producer can be illustrated by the difficulties of Venetian tradesmen: the imitation of their products by Northern Europe low-cost rivals triggered their decline in the Mediterranean area. Caterpillar mets a similar problem at the beginning of the 1980's, the company being confronted to a profit erosion and to the growing success of its main competitor, Komatsu.[15] These observations are thus consistent with the finding that the low-index firm makes the highest profit.

To conclude, when unit costs are attributes-dependent, index choices in the DBA model provide a good description of a maximum quality strategy. However, note that a high-quality firm is not expected to choose the maximum index if it is aware of an imitation threat: in this case, the market outcome may be closer to the traditional quality setting observed in vertical differentiation models.

4.3 Attributes choices with endogenous fixed cost

The assumption of an endogenous fixed costs embraces the upstream point of view of an applied research service. The index of specific attributes proposed by this service depends on the research investment, which generates a fixed cost. This section keeps the assumption $A_1 = A_2 = +\infty$ and the properties of the function $u_i = u(a_i)$ previously exposed. It is now assumed that each firm bears a fixed cost $F : a \to \mathbb{R}^+$, with $F(0) = 0$, whereas unit costs are exogenous $c_1 = c_2 = c$. The function F increases in a convex way with the attribute index: $F'(a_i) > 0$ and $F''(a_i) \geq 0$. This assumption means that an increase in the number of specific attributes makes them more and more costly to discover.

However, the proof of equilibrium existence cannot be realized with functions of a general form. That is why the following setting is used:

S1: *The utility has a linear form $u_i = a_i$ and the fixed costs are quadratic $F_i = a_i^2$.*

These functions are both meaningful and quite standard in the literature: for instance, a linear utility is used by Anderson, de Palma and Thisse (1992) and a quadratic fixed cost function is assumed by Ronnen (1991), Motta (1993) or Pepall and Richards (1994).

As previously, the existence of a perfect equilibrium is studied in the two-stage game. When $p_i \geq p_j$

[15] "Cat's obsession with quality had boosted expenses to the point where it could no longer compete. Its production methods had become too inefficient to enable it to match Komatsu's prices" (Miller, 1990, p 226)

in the second stage, equilibrium prices are given by equations (2.5) and (2.6) and profits become:

$$\Pi_i(a_i) = Nu(a_i) - F(a_i) \ \ ; \ \ \Pi_j(a_j) = \frac{N(u(a_i) + u(a_j))(\sqrt{\Delta} - u(a_i))}{u(a_i) + \sqrt{\Delta}} - F(a_j) \tag{4.2}$$

with $\Delta = u(a_i)^2 + 4u(a_i)(u(a_i) + u(a_j))$. The study of index choices leads to the following proposition:

Proposition 4 *When $a_i \geq a_j$ for $i, j \in \{1, 2\}$ and $i \neq j$, Π_i is quasi-concave and its maximum verifies $u'(a_i) = F'(a_i)/N$. The first derivative of Π_j is zero if:*

$$\frac{4u_i u'(a_j)(u_i + u(a_j))}{\sqrt{\Delta}(\sqrt{\Delta} + u_i)} = \frac{F'(a_j)}{N}$$

The setting S1 guarantees the quasi-concavity of Π_j and the existence of a global perfect Nash equilibrium.

Proof: presented in Appendix 7.2.

Thus, concavity of utility and convexity of fixed costs are necessary but not sufficient conditions to prove the existence of the equilibrium. Suppose again that firm 1 sells the most appreciated product. The attribute index chosen by firm 1 is such that marginal utility equals marginal fixed cost by consumer. The index selected by firm 2 is lower. Consequently, product differentiation is both horizontal and vertical. The two indices increase with N, the number of consumers, as in the logit model. The marginal investment in quality is more profitable the more consumers they are (Anderson *et al.* 1992, p 245).

Under S1 and when $N = 1$, equilibrium indices are $a_1 = 0.5$ and $a_2 = 0.317954$. Firm 2 realizes a higher profit than its rival[16]. Here again, such a property is not verified in pure vertical differentiation, the high-quality firm making the highest profit whatever the endogenous fixed cost function used (Lehman-Grube, 1997).

5 Conclusion

This paper supposes that consumers follow a random decision rule based on products' attributes. A two-stage game is studied in which firms select their attribute index and then compete in prices. Existence of a perfect Nash equilibrium is established with exogenous costs and with attributes-dependent unit and

[16] This property seems true for many settings although no general conclusion can be drawn with attributes-dependent fixed costs

fixed costs. When costs are exogenous with the attributes, firms choose the highest indices, as in the logit oligopoly. When unit or fixed costs are endogenous, firms select different attribute indices and product differentiation is both horizontal and vertical at the equilibrium. This important finding is not a consequence of the reference price effect, as this effect vanishes in the setting of attributes-dependent fixed costs considered here (unit costs are identical). This finding is rather linked with the demand system used.

Moreover, this conclusion differs of that obtained in multi-dimensional models in which firms differentiate their products according to a single dominant characteristic. If this structure of differentiation is relevant on some markets[17], horizontal and vertical forms of differentiation are simultaneously observed for many everyday life products. These different findings may be explained by the different frameworks of product specification used. In multi-dimensional models, firms can differentiate their product without bearing costs and only according to a fixed number of characteristics given *a priori*. However, attribute choices can affect endogenously the form of differentiation on the market and the level of costs, these relations being taken into account in the DBA model. For example, when endogenous unit costs are strongly convex, the horizontal dimension of differentiation become insignificant on the market.

At the equilibrium, the low-index firm makes the highest profit in the DBA model. Choosing a weak attribute index keeps costs at a low level and the horizontal differentiation on the market makes a high price acceptable for consumers. This result contrasts with the high quality advantage highlighted in vertical differentiation models.

These results are obtained under the (rather classical) assumption that firms can not imitate the competing products. If this assumption is not expected to affect the low-index advantage, the vertical dimension of differentiation may be reduced if firms behave strategically when a threat of imitation exists. Consequently, introducing a third stage of product imitation in the game could be a natural extension of the current work.

[17]Irmen and Thisse (1998, p 78) give the example of the US weekly magazine market.

6 References

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7 Appendix: proofs on the attributes choice

7.1 Equilibrium with endogenous unit costs

The proof of proposition 3 is presented here. Suppose that firm 1 sells the most appreciated product ($a_1 \geq a_2$). The local equilibrium is identified and it is shown afterwards that the equilibrium is also global.

Local maximum. When $a_1 \geq a_2$, equilibrium prices at the end of the second stage have the following expressions:

$$p_1^* = c(a_1) + \frac{u(a_1) + \sqrt{\Delta}}{2} \quad ; \quad p_2^* = c(a_1) + u(a_1) + u(a_2)$$

with $\Delta = u(a_1)^2 + 4u(a_1)x$ and $x = u(a_1) + u(a_2) + c(a_1) - c(a_2)$.

For *firm 1*, first and second order conditions are given by:

$$\frac{\partial \Pi_1}{\partial a_1} = Nu'(a_1) = 0 \;\; ; \;\; \left.\frac{\partial^2 \Pi_1}{\partial a_1^2}\right|_{\frac{\partial \Pi_1}{\partial a_1}=0} = Nu''(a_1) \leq 0$$

As the utility is concave, firm 1 selects the highest index a_1^c, verifying $u'(a_1^c) = 0$.

For *firm 2*, the following derivative is obtained after simplifications:

$$\frac{\partial \Pi_2}{\partial a_2} = \frac{4Nu_1 x(u'(a_2) - c'(a_2))}{\sqrt{\Delta}(\sqrt{\Delta} + u_1)} \tag{7.1}$$

It equates zero for a value a_2^c verifying $u'(a_2^c) = c'(a_2^c)$. At the equilibrium, the second order condition is:

$$\left.\frac{\partial^2 \Pi_2}{\partial a_2^2}\right|_{\frac{\partial \Pi_2}{\partial a_2}=0} = \frac{4Nu_1 x(u''(a_2) - c''(a_2))}{\sqrt{\Delta}(\sqrt{\Delta}+u_1)} \leq 0 \tag{7.2}$$

Thus, Π_2 is quasi-concave if $u''(a_2) \leq c''(a_2)$: this condition always holds under the assumptions of concave utility and convex unit cost.

Finally, attribute indices verify $a_1^c > a_2^c$ which implies $u(a_1) \geq u(a_2)$ and $c(a_1) \geq c(a_2)$. Consequently, conditions (2.3) and (2.4) are respected and a price equilibrium verifying $p_1 \geq p_2$ exists in the second stage.

Global maximum. It is proven here that firms have no interest in deviating from the local maximum previously computed, which is a perfect subgame Nash equilibrium. At first, define the "reservation utility" threshold of consumers, u_α, which is associated to the maximum attribute index $\lim_{a_i \to A_i} u(a_i) = u_\alpha$.

Consider now the case of *firm 1*: there is no attribute index a_1^{cc} such that $a_1^{cc} \leq a_2^c$ and verifying $\Pi_1(a_1^c) < \Pi_1(a_1^{cc})$. As $u'(a_1^c) = 0$, the reference profit is $\Pi_1^c(a_1^c) = Nu_\alpha$. If a_1^{cc} exists, Π_1^{cc} is similar to Π_2^c: $\Pi_1^{cc}(a_1^{cc}) = [Nx(\sqrt{\Delta} - u_2^c)]/[(u_2^c + \sqrt{\Delta})]$ with $\Delta = (u_2^c)^2 + 4u_2^c x$ and $x = u(a_1^{cc}) + u_2^c + c_2^c - c(a_1^{cc})$. The first derivative looks like the equation (7.1) and, consequently, the optimal index is given by $u'(a_1^{cc}) = c'(a_1^{cc})$ (symmetrically, the profit is quasi-concave, as in equation (7.2)). As utility and cost functions are identical between firms, attribute indices equal $a_1^{cc} = a_2^c$ and thus costs $c_1^{cc} = c_2^c$ and utilities $u_1^{cc} = u_2^c$. This utility level is now defined by u_β. When firm 1 deviates from the local maximum, it chooses the same attribute index as firm 2 and, for this value, $\Pi_1^{cc}(a_1^{cc}) = Nu_\beta$. But as $a_2^c \leq a_1^c$, we have $u_\beta \leq u_\alpha$ and thus $\Pi_1^c \geq \Pi_1^{cc}$. Firm 1 can never improve its profit by deviating from the local maximum.

Consider the case of *firm 2*. It is studied if there exists an index a_2^{cc} belonging to the interval $a_2^{cc} \geq a_1^c$ and verifying $\Pi_2(a_2^c) < \Pi_2(a_2^{cc})$. The reference profit of firm 2 is given by $\Pi_2^c(a_2^c) = [Nx(\sqrt{\Delta} - u(a_1^c))]/[(u(a_1^c) + \sqrt{\Delta})]$. If firm 2 deviates from the local maximum, the attribute index chosen can however not be strictly higher than that of firm 1 since $a_1^c = A_1 = A_2$: firm 2 necessarily selects $a_2^{cc} = a_1^c$. In this case, firm 2 realizes a profit $\Pi_2^{cc}(a_2^{cc}) = Nu(a_1^c)$. Comparison of profits leads to the condition:

$$\Pi_2^{cc} \leq \Pi_2^c \Leftrightarrow c(a_1^c) - u(a_1^c) \geq c(a_2^c) - u(a_2^c) \tag{7.3}$$

As utility is concave and unit cost convex, this inequality is always true.

Indeed, the following property is verified:

$$\frac{d(c(a) - u(a))}{da} > 0 \; \forall a \geq \overline{a} \text{ with } \overline{a} > 0 \text{ such that } c'(\overline{a}) = u'(\overline{a})$$

As a_2^c verifies $u'(a_2^c) = c'(a_2^c)$, we have $a_2 = \overline{a}$ and (7.3) is always true when $a_1^c > a_2^c$. Thus, firm 2 cannot increase its profit by deviating from the local maximum.

To conclude, deviations are unprofitable and a price equilibrium exists at the second stage for the attribute indices chosen at the first stage: the existence of a perfect Nash equilibrium is established.■

7.2 Equilibrium with endogenous fixed cost

It is supposed again that firm 1 selects the highest index ($a_1 \geq a_2$) and the proof of Proposition 4 is demonstrated in this Appendix. First, the local equilibrium is identified. Second, it is shown that a perfect Nash equilibrium exists when the setting S_1 is used.

At the last stage of the game, profits are given by equation (4.2). Attributes choices of *firm 1* are studied:

$$\frac{\partial \Pi_1}{\partial a_1} = Nu'(a_1) - F'(a_1)$$

At the equilibrium, the index chosen verifies $u'(a_1) = F'(a_1)/N$ and the second order condition is:

$$\left.\frac{\partial^2 \Pi_1}{\partial a_1^2}\right|_{\frac{\partial \Pi_1}{\partial a_1}=0} = Nu''(a_1) - F''(a_1) \leq 0$$

As utility is concave and fixed cost convex, the profit is quasi-concave.

Consider now the attributes choice of *firm 2*. After simplifications, the first derivative is given by:

$$\frac{\partial \Pi_2}{\partial a_2} = \frac{4Nu_1 x u'(a_2)}{\sqrt{\Delta}(\sqrt{\Delta} + u_1)} - F'(a_2) \tag{7.4}$$

with $\Delta = u(a_1)^2 + 4u(a_1)[u(a_1) + u(a_2)]$. At this extreme point, the index chosen by 2 verifies :

$$\frac{4u_1 u'(a_2) x}{\sqrt{\Delta}(\sqrt{\Delta} + u_1)} = \frac{F'(a_2)}{N}$$

The second order condition is:

$$\left.\frac{\partial^2\Pi_2}{\partial a_2^2}\right|_{\frac{\partial\Pi_2}{\partial a_2}=0} = \frac{4Nu_1[u_1u'^2(a_2)(\sqrt{\Delta}+u_1+2x)+xu''(a_2)(u_1+\sqrt{\Delta})(u_1+4x)]}{\sqrt{\Delta}(u_1+4x)(u_1+\sqrt{\Delta})^2} - F''(a_2) \leq 0$$

Assumptions of convex fixed costs and concave utilities are not sufficient to ensure that profit is quasi-concave in the general case: the fixed cost function must also be sufficiently convex (or the utility strongly concave).

When profits are quasi-concave, equilibrium indices verify $a_1 \geq a_2$. Indeed, cost and utility functions are identical between firms and the following inequality is respected:

$$\frac{4u_1x}{\sqrt{\Delta}(\sqrt{\Delta}+u_1)} = \frac{4u_1x}{u_1^2+4u_1x+u_1\sqrt{\Delta}} < 1$$

Consequently, condition (2.3) holds and a local perfect Nash equilibrium exists.

Under S1 and when $N = 1$, equilibrium attribute indices are $a_1^c = 0.5$ and $a_2^c = 0.317954$. The second order condition is always verified for firm 1 and the profit of firm 2 is strictly quasi-concave: $\left.\frac{\partial^2\Pi_2}{\partial a_2^2}\right|_{\frac{\partial\Pi_2}{\partial a_2}=0} \approx -1.8$. Profits are given by $\Pi_1(a_1^c) \approx 0.25$ and $\Pi_2(a_2^c) \approx 0.28$.

The local equilibrium is also a global one, as it is shown now. For the value of a_2^c computed, it can be easily proven by using equation (7.4) that the profit of *firm 1* strictly increases with a_1 for all $a_1^{cc} \in [0, a_2^c]$. Thus, the most profitable deviation is $a_1^{cc} = a_2^c \approx 0.3179$. But $a_1^{cc} = a_2^c$ can not be an equilibrium because firm 1 has not selected the value a_2^c during its choice of a_1^c inside the interval $[a_2^c; +\infty[$. Firm 1 prefers choosing $a_1^c = 0.5$ rather than deviating.

Finally, the most profitable deviation of *firm 2* from the equilibrium is given by $a_2^{cc} = a_1^c = 0.5$. The profit realized is $\Pi_2(a_2^{cc}) = \Pi_1(a_1^c) < \Pi_2(a_2^c)$ and thus the equilibrium value $a_2^c = 0.3179$ remains more profitable.

The shapes of utility and costs functions in S1 guarantees the existence of a perfect Nash equilibrium and neither firm 1 nor firm 2 is incited to modify its choice.■