

ARE PROGRESSIVE INCOME TAXES STABILIZING?*

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Abstract

We assess the stabilizing effect of progressive income taxes in a monetary economy with constant returns to scale. It is shown that tax progressivity reduces, in parameter space, the likelihood of local indeterminacy, sunspots and cycles. However, considering plausibly low levels of tax progressivity does not ensure saddle-point stability and preserves as robust the occurrence of sunspot equilibria and endogenous cycles. It turns out that increasing progressivity, through its impact on after-tax income, makes labor supply more inelastic. However, even when large, tax progressivity does not neutralize the effects of expected inflation on current labor supply which may lead to expectation-driven business fluctuations.

Keywords: progressive income taxes, business cycles, sunspots, stabilization.

Journal of Economic Literature Classification Numbers: D33, D58, E32, E62, H24, H30.

1 Introduction

Income-dependent taxes and transfers have been proposed as efficient automatic stabilizers since, at least, Musgrave and Miller (1948) (see also Vickrey 1947 and 1949, Slitor 1948, Friedman 1948). In recent years, the development of dynamic general equilibrium models has proved useful to study how progressive fiscal policy may stabilize the economy's aggregate variables. This strand of literature specifically allows to evaluate the level of social insurance provided by given fiscal schemes in the presence of various shocks. In particular, Christiano and Harrison (1999), Guo and Lansing (1998) have shown that income taxes, when sufficiently progressive, rule out local indeterminacy and restore saddle-path convergence (see also Guo 1999, Dromel and Pintus 2007). However, this literature assumes that the laissez-faire economy is subject to (large) increasing returns to scale, so that volatile paths may indeed improve welfare, in comparison with stationary allocations. In fact, Christiano and Harrison (1999, p. 20) give some examples such that, in their terminology, the bunching effect dominates the concavity effect.

In the present paper, we assess the impact of progressive income taxation in a monetary economy with *constant* returns to scale, by extending results due to Woodford (1986) and Grandmont *et al.* (1998). Absent the bunching effect, one suspects expectation-driven volatility to unambiguously lead to welfare losses (by the concavity, or risk-

aversion, effect) that call for stabilization.¹ As shown by Woodford (1986) and Grandmont *et al.* (1998), the presence of money as a dominated asset is critical to generate local indeterminacy in the laissez-faire economy without taxes. We show that local indeterminacy (hence sunspots and endogenous cycles) is *robust* with respect to the introduction of income tax progressivity when it is set at plausibly low values. More specifically, although progressive taxes on labor income are shown to reduce, in parameter space, the likelihood of local indeterminacy (see Propositions 2.2 and 2.3), considering plausibly low values of tax progressivity still leaves room for sunspots and (Hopf or flip) cycles. As a corollary, we also conclude that regressive income taxes would, on the contrary, enlarge the set of parameter values that are associated with local indeterminacy.

In consequence, one may view our results as casting some doubt on the idea that progressive income taxes are useful automatic stabilizers. This seems reminiscent of earlier debates about the practical importance of “built-in flexibility” (e.g. Musgrave and Miller 1948, Vickrey 1947, Slitor 1948).² Our focus on low progressivity is dictated by the available evidence (e.g. our own computations that we present on page 12, or Bénabou 2002, Cassou and Lansing 2004). Our analysis abstracts from progressive taxation of capital income. This is consistent with the US tax code, which sets tax rates that are less progressive on income from capital than on wage income (see Hall and Rabushka 1995). Finally, we follow Feldstein (1969), Kanbur (1979), Persson (1983) by considering tax schedules that exhibit constant residual income progression.

The mechanism at the heart of our main result is the following. Labor is elastically supplied by money holders and it depends on both the current real wage and the expected inflation rate. When workers have optimistic expectations (say, about falling inflation), they want to raise their consumption and, accordingly, they devote a higher fraction of their time endowment to work so as to increase their income, which originates an expansion. With increasingly progressive taxes on wage income, labor supply becomes less and less responsive to the real wage and to expected inflation. There is, however, a major difference with respect to how labor reacts to both variables: eventually, as progressivity tends to its maximal level, expected inflation keeps having a negative impact on labor, whereas the effect of (before-tax) real wage tends to zero. In other words, *tax progressivity does not neutralize the effects of expected inflation on current labor supply*, which leaves room for expectation-driven business cycles.

Aside from assuming constant returns and money holdings, the setting with taxes that we consider differs from those of Christiano and Harrison (1999), Guo and Lansing (1998) in that we abstract from capital income taxation. To ease

¹Here, we adopt the common view that fiscal policy is stabilizing when it leads to saddle-point stability, hence to determinacy.

²Our main conclusion is not inconsistent with some recent results obtained in two-sector models by Guo and Harrison (2001) and Sim (2005), in which the mechanisms leading to indeterminacy are somewhat different, as they also rely on increasing returns.

comparison with the latter papers, we introduce small increasing returns to scale and we check the robustness of our main findings obtained under constant returns. Finally, note that a recent paper by Seegmuller (2004, section 5.2.2) studies, in an example, the effects of nonlinear tax rates in the same model, but he restricts the analysis to regressive taxation. We should also stress at the outset that the property of progressivity we focus on is the following: the income tax schedule that agents face is such that the marginal tax rate is larger than the average tax rate (see Assumption 2.2).

The rest of the paper is organized as follows. Section 2 presents the monetary economy with constant returns and discusses how progressive taxes and transfers make expectation-driven fluctuations less likely. Finally, some concluding remarks and directions for future research are gathered in Section 3, while three appendices present proofs.

2 Progressive Income Taxes in a Monetary Economy with Constant Returns to Scale

In this section, we sketch the benchmark model, following the lines set out in Woodford (1986), Grandmont *et al.* (1998), to which we add progressive income taxes. The economy consists of two types of competitive agents (workers and capitalists) who consume and have perfect foresight during their infinite lifetime. Identical agents called *workers* consume and work during each period. They supply a variable quantity of labor hours and may save a fraction of their income by holding two assets: productive capital and nominal outside money. A financial constraint is imposed on workers: their expenditures must be financed out of their initial money balances or out of the returns earned on productive capital. On the other side, *capitalists* consume and save an income composed of money balances and returns on capital. Most importantly, it is assumed that capitalists discount future utility less than workers. Therefore, capitalists end up holding the whole capital stock and the resulting nonautarkic steady state is characterized by the modified golden rule, i.e. the stationary real rental rate on productive capital (net of capital depreciation) equals the capitalists' discount rate. Therefore, at the steady state (and nearby), the real return on capital is positive and larger than that of money balances, which is assumed to be zero, so that capitalists choose not to hold outside money.

To summarize, the steady state (and nearby) savings structure is the following: capitalists own the whole capital stock and workers hold the entire nominal money stock. Finally, the financial constraint faced by workers becomes a liquidity constraint which is obviously binding at the steady state. In that framework, Woodford (1986) showed that

although workers have an infinite lifetime, they behave like a two period living agent: they choose optimally their labor supply for today and consequently their next period consumption demand. Equivalently, workers know the current nominal wage and the next period price for the consumption good (along an intertemporal equilibrium with perfect foresight) and choose the (unique under usual assumptions) optimal bundle of labor today and consumption tomorrow on their offer curve. Therefore, the liquidity constraint allows one to interpret the length of the period as, say, a month and eventual endogenous fluctuations occur at business cycle frequency.

2.1 Fiscal Policy and Intertemporal Equilibria

A unique good is produced in the economy by combining labor $l_t \geq 0$ and the capital stock $k_{t-1} \geq 0$ resulting from the previous period. Production exhibits constant returns to scale, so that output is given by:

$$F(k, l) \equiv Alf(a), \quad (1)$$

where $A > 0$ is a scaling parameter and the latter equality defines the standard production function defined upon the capital labor ratio $a = k/l$. On technology, we shall assume the following.

Assumption 2.1.

The production function $f(a)$ is continuous for $a = k/l \geq 0$, C^r for $a > 0$ and r large enough, with $f'(a) > 0$ and $f''(a) < 0$.

Competitive firms take real rental prices of capital and labor as given and determine their input demands by equating the private marginal productivity of each input to its real price. Accordingly, the real competitive equilibrium wage is:

$$\omega = \omega(a) \equiv A[f(a) - af'(a)], \quad (2)$$

while the real competitive gross return on capital is:

$$R = \rho(a) + 1 - \delta \equiv Af'(a) + 1 - \delta, \quad (3)$$

where $1 \geq \delta \geq 0$ is the constant depreciation rate for capital.

Fiscal policy is supposed to map market income x into disposable income $\phi(x)$. Disposable income is obtained from market income by adding transfers and subtracting taxes, both of which are proportional to pre-tax income. For

simplicity, we assume that $x \geq \phi(x)$, so that taxes net of transfers (taxes from now on) are positive. In this formulation, there are two benchmark cases. When $\phi(x)$ is proportional to x , then ϕ has unitary elasticity and the tax rate is flat. Decreasing the elasticity of $\phi(x)$ from one (when the net tax rate is constant) to zero may be interpreted as *increasing fiscal progressivity*. More precisely, one can postulate the following (see Musgrave and Thin 1948 for an early definition, and, for example, Lambert 2001, chap. 7-8).

Assumption 2.2.

Disposable income $\phi(x)$ is a continuous, positive function of market income $x \geq 0$, with $x \geq \phi(x)$, $\phi'(x) > 0$ and $0 \geq \phi''(x)$, for $x > 0$. The income tax-and-transfer scheme exhibits weak progressivity, that is, $\phi(x)/x$ is non-increasing for $x > 0$ or, equivalently, $1 \geq \psi(x) \equiv x\phi'(x)/\phi(x)$.

Then $\pi(x) \equiv 1 - \psi(x)$ is a measure of income tax progressivity. In particular, the fiscal schedule is linear when $\pi(x) = 0$, or $\psi(x) = 1$, for $x > 0$, and the higher $\pi(x)$, the more progressive the fiscal schedule.

One can reinterpret the condition $1 \geq \psi(x)$ as the property that the *marginal* tax rate $\tau_m \equiv \partial(x - \phi(x))/\partial x$ is larger than the average tax rate $\tau \equiv (x - \phi(x))/x$: it is easily shown that $\tau_m - \tau = \phi(x)/x - \phi'(x)$ so that $\tau_m \geq \tau$ when $1 \geq \psi(x)$ or $\pi(x) \geq 0$ for all positive x . One can easily check that the latter condition also implies that the average tax rate is an increasing function of pre-tax income. Finally, note that fiscal progressivity is naturally measured, for some x , by $\pi \equiv 1 - \psi$ when one notes that $\pi = (\tau_m - \tau)/(1 - \tau)$. To put it differently, $\psi = 1 - \pi$ measures (local) residual income progression (as defined by Musgrave and Thin (? , p. 507)). In the next subsection, we simplify the analysis of the local dynamics by assuming that π is constant.

As in most papers in the literature (e.g. Guo and Lansing 1998), we assume that the proceeds of taxes, net of transfers, are used to produce a flow of public goods g , with $g = x - \phi(x)$. Therefore, the government budget is balanced.

To complete the description of the model, we now characterize the behavior of both classes of agents, following Woodford (1986) and Grandmont *et al.* (1998). A representative worker solves the following utility optimization problem, as derived in Appendix A:

$$\text{maximize } \{V_2(c_{t+1}^w/B) - V_1(l_t)\} \text{ such that } p_{t+1}c_{t+1}^w = p_t\phi(\omega_t l_t), c_{t+1}^w \geq 0, l_t \geq 0, \quad (4)$$

where $B > 0$ is a scaling factor, c_{t+1}^w is next period consumption, l_t is labor supply, $p_{t+1} > 0$ is next period price of output (assumed to be perfectly foreseen), $\omega_t > 0$ is real wage, and $\phi(\omega_t l_t)$ is disposable wage income, as described in

Assumption 2.2. To keep things simple, we assume in this section that progressive taxes and transfers are applied to labor income only. Capital income taxes are studied in Dromel and Pintus (2006), where we show that similar results hold. In particular, we show there that flat-rate taxation of capital income has no impact on parameter values that are compatible with local indeterminacy and bifurcations. We consider the case such that leisure and consumption are gross substitutes and assume therefore the following:

Assumption 2.3.

The utility functions $V_1(l)$ and $V_2(c)$ are continuous for $l^ \geq l \geq 0$ and $c \geq 0$, where $l^* > 0$ is the (maybe infinite) workers' endowment of labor. They are C^r for, respectively, $0 < l < l^*$ and $c > 0$, and r large enough, with $V_1'(l) > 0$, $V_1''(l) > 0$, $\lim_{l \rightarrow l^*} V_1'(l) = +\infty$, and $V_2'(c) > 0$, $V_2''(c) < 0$, $-cV_2''(c) < V_2'(c)$ (that is, consumption and leisure are gross substitutes).*

The first-order condition of the above program (4) gives the optimal labor supply $l_t > 0$ and the next period consumption $c_{t+1}^w > 0$, which can be stated as follows:

$$v_1(l_t) = \psi(\omega_t l_t) v_2(c_{t+1}^w) \text{ and } p_{t+1} c_{t+1}^w = p_t \phi(\omega_t l_t), \quad (5)$$

where $v_1(l) \equiv lV_1'(l)$ and $v_2(c) \equiv cV_2'(c/B)/B$. Assumption 2.3 implies that v_1 and v_2 are increasing while v_1 is onto \mathbf{R}_+ . Therefore, Assumption 2.3 allows one to define, from Eqs. (5), $\gamma \equiv v_2^{-1} \circ [v_1/\psi]$ (whose graph is the offer curve), which is a monotonous, increasing function only if the elasticity of ψ is either negative or not too large when positive.

Capitalists maximize the discounted sum of utilities derived from each period consumption. They consume $c_t^c \geq 0$ and save $k_t \geq 0$ from their income, which comes exclusively from real gross returns on capital $R_t k_{t-1}$ and is not affected by fiscal rules. We assume, following Woodford (1986), that capitalists' instantaneous utility function is logarithmic. As easily shown (for instance by applying dynamic programming techniques), their optimal choices are then given by a constant savings rate:

$$c_t^c = (1 - \beta)R_t k_{t-1}, \quad k_t = \beta R_t k_{t-1}, \quad (6)$$

where $0 < \beta < 1$ is the capitalists' discount factor and $R_t > 0$ is the real gross rate of return on capital. As usual, equilibrium on capital and labor markets is ensured through Eqs. (2) and (3). Since workers save their wage income in the form of money, the equilibrium money market condition is:

$$c_t^w = \phi(\omega(a_t)l_t) = M/p_t, \quad (7)$$

where $M \geq 0$ is money supply, assumed to be constant in the sequel, and p_t is current nominal price of output. Finally, Walras' law accounts for the equilibrium in the good market, that is, $c_t^w + c_t^c + k_t - (1 - \delta)k_{t-1} + g_t = F(k_{t-1}, l_t)$. From the equilibrium conditions in Eqs. (2), (3), (5), (6), (7), one easily deduces that the variables c_{t+1}^w , c_t^w , l_t , p_{t+1} , p_t , c_t^c and k_t are known once (a_t, k_{t-1}) are given. This implies that intertemporal equilibria may be summarized by the dynamic behavior of both a and k .

Definition 2.1.

An intertemporal perfectly competitive equilibrium with perfect foresight is a sequence (a_t, k_{t-1}) of \mathbf{R}_{++}^2 , $t = 1, 2, \dots$, such that, given some $k_0 \geq 0$,

$$\begin{cases} v_2(\phi(\omega(a_{t+1})k_t/a_{t+1})) &= v_1(k_{t-1}/a_t)/\psi(\omega(a_t)k_{t-1}/a_t), \\ k_t &= \beta R(a_t)k_{t-1}. \end{cases} \quad (8)$$

In view of Eqs. (8) and recalling that $a = k/l$, the nonautarkic steady states are the solutions (\bar{a}, \bar{l}) in \mathbf{R}_{++}^2 of $v_2(\phi(\omega(\bar{a})\bar{l})) = v_1(\bar{l})/\psi(\omega(\bar{a})\bar{l})$ and $\beta R(\bar{a}) = 1$. Equivalently, in view of Eq. (3), the steady states are given by:

$$\begin{cases} v_2(\phi(\omega(\bar{a})\bar{l})) &= v_1(\bar{l})/\psi(\omega(\bar{a})\bar{l}), \\ \rho(\bar{a}) + 1 - \delta &= 1/\beta. \end{cases} \quad (9)$$

We shall solve the existence issue by choosing appropriately the scaling parameters A and B , so as to ensure that one stationary solution coincides with, for instance, $(\bar{a}, \bar{l}) = (1, 1)$. For sake of brevity, both the result (Proposition B.1) and the proof are given in Appendix B.

2.2 Sunspots and Cycles under Progressive Income Taxes

We now study the dynamics of Eqs. (8) around (\bar{a}, \bar{k}) . These equations define locally a dynamical system of the form $(a_{t+1}, k_t) = G(a_t, k_{t-1})$ if the derivative of $\omega(a)/a$ with respect to a does not vanish at the steady state, or equivalently if $\varepsilon_\omega(\bar{a}) - 1 \neq 0$, where the notation ε_ω stands for the elasticity of $\omega(a)$ evaluated at the steady state under study. For simplicity, we assume that ϕ has a *constant elasticity* $\psi = 1 - \pi$ with $1 > \pi \geq 0$. Convenience aside, it appears that economic theory does not place strong restrictions on how the elasticity $\psi(x)$ of after-tax income varies with pre-tax income x (see e.g. Lambert 2001). Therefore, we choose to be parsimonious and introduce fiscal progressivity through a *single* constant parameter, that is, $\pi = 1 - \psi$. In other words, we assume constant residual income progression, as in

Feldstein (1969), Kanbur (1979), Persson (1983) (in static models), and Guo and Lansing (1998), Bénabou (2002) (in growth models). As it will soon appear, the following analysis could be easily adapted to account for a non-constant elasticity.

Straightforward computations yield the following proposition.

Proposition 2.1 (Linearized Dynamics around the Steady State).

Under the assumptions of Proposition B.1, suppose that ϕ has constant elasticity at the steady state (\bar{a}, \bar{k}) of the dynamical system in Eqs. (8), i.e. $\psi(\omega(\bar{a})\bar{l}) = 1 - \pi$, with $0 < \pi < 1$ is constant level of labor tax progressivity. Let $\varepsilon_R, \varepsilon_\omega, \varepsilon_\gamma$ be the elasticities of the functions $R(a), \omega(a), \gamma(l)$, respectively, evaluated at the steady state (\bar{a}, \bar{k}) and assume that $\varepsilon_\omega \neq 1$. The linearized dynamics for the deviations $da = a - \bar{a}, dk = k - \bar{k}$ are determined by the linear map:

$$\begin{cases} da_{t+1} &= -\frac{\varepsilon_\gamma/(1-\pi)+\varepsilon_R}{\varepsilon_\omega-1} da_t + \frac{\bar{a}}{\bar{k}} \frac{\varepsilon_\gamma/(1-\pi)-1}{\varepsilon_\omega-1} dk_{t-1}, \\ dk_t &= \frac{\bar{k}}{\bar{a}} \varepsilon_R da_t + dk_{t-1}. \end{cases} \quad (10)$$

The associated Jacobian matrix evaluated at the steady state under study has trace T and determinant D , where

$$T = T_1 - \frac{\varepsilon_\gamma - 1}{(1 - \pi)(\varepsilon_\omega - 1)}, \quad \text{with} \quad T_1 = 1 + \frac{|\varepsilon_R| - 1/(1 - \pi)}{\varepsilon_\omega - 1},$$

$$D = \varepsilon_\gamma D_1, \quad \text{with} \quad D_1 = \frac{|\varepsilon_R| - 1}{(1 - \pi)(\varepsilon_\omega - 1)}.$$

Moreover, one has $T_1 = 1 + D_1 + \Lambda$, where $\Lambda \equiv -\pi|\varepsilon_R|/[(1 - \pi)(\varepsilon_\omega - 1)]$.

Our main goal now is to show that sunspots and cycles are robust with respect to the introduction of tax progressivity, when the latter is set at plausibly low levels. Direct inspection of Eqs. (10) shows that the case of *flat-rate* taxes ($\pi = 0$ or $\psi = 1$) is equivalent to the case without government taxes, as studied by Grandmont *et al.* ?. In addition, when $1 > \pi > 0$, ε_γ is replaced, in the model with progressive taxes, by $\varepsilon_\gamma/(1 - \pi) > \varepsilon_\gamma$. Therefore, *one expects the picture that is obtained when π is not too large to be similar to the configuration occurring in the model without taxes* (or, for that matter, with flat-rate taxes), up to the change of parameter $\varepsilon_\gamma \rightarrow \varepsilon_\gamma/(1 - \pi)$. This is what we now show.

We assume, without loss of generality, that the steady state has been normalized at $(\bar{a}, \bar{k}) = (1, 1)$ (see Proposition B.1). Then we fix the technology (i.e. ε_R and ε_ω), at the steady state, and vary the parameter representing workers' preferences $\varepsilon_\gamma > 1$. In other words, we consider the parameterized curve $(T(\varepsilon_\gamma), D(\varepsilon_\gamma))$ when ε_γ describes $(1, +\infty)$. Direct inspection of the expressions of T and D in Proposition 2.1 shows that this locus is a half-line Δ that starts

close to (T_1, D_1) when ε_γ is close to 1, and whose slope is $1 - |\varepsilon_R|$, as shown in Fig. 1. The value of $\Lambda = T_1 - 1 - D_1$, on the other hand, represents the deviation of the generic point (T_1, D_1) from the line (AC) of equation $D = T - 1$, in the (T, D) plane.

Insert Figure 1 here.

The task we now face is locating the half-line Δ in the plane (T, D) , i.e. its origin (T_1, D_1) and its slope $1 - |\varepsilon_R|$, as a function of the parameters of the system. The parameters we shall focus on are the depreciation rate for the capital stock $1 \geq \delta \geq 0$, the capitalist's discount factor $0 < \beta < 1$, the share of capital in total income $0 < s = \bar{a}\rho(\bar{a})/f(\bar{a}) < 1$, the elasticity of input substitution $\sigma = \sigma(\bar{a}) > 0$, and fiscal progressivity $1 > \pi = \pi(\omega(\bar{a})\bar{l}) \geq 0$, all evaluated at the steady state (\bar{a}, \bar{k}) under study. In fact, it is not difficult to get the following expressions.

$$\begin{aligned} D_1(\sigma) &= (\theta(1-s) - \sigma)/[(1-\pi)(s-\sigma)], & \Lambda(\sigma) &= -\pi\theta(1-s)/[(1-\pi)(s-\sigma)], \\ T_1(\sigma) &= 1 + D_1(\sigma) + \Lambda(\sigma), & slope_\Delta &= 1 - \theta(1-s)/\sigma, \end{aligned} \tag{11}$$

where $\theta \equiv 1 - \beta(1 - \delta) > 0$ and all these expressions are evaluated at the steady state under study.

Our aim now is locating the half-line Δ , i.e. its origin $(T_1(\sigma), D_1(\sigma))$ and its slope in the (T, D) plane when the capitalists' discount rate β , as well as the technological parameters δ , s , and the level of fiscal progressivity π at the steady state are fixed, whereas the elasticity of factor substitution σ is made to vary. We get confirmation that the benchmark economy with constant tax rate $\pi = 0$ is equivalent to the no-tax case presented in Grandmont *et al.* (1998): the origin $(T_1(\sigma), D_1(\sigma))$ of Δ is located on the line (AC) , i.e. $\Lambda \equiv 0$ (see Fig. 1). The immediate implication of the resulting geometrical representation is that local indeterminacy and endogenous fluctuations emerge only for low values of σ (for σ lower than s , the share of capital in output) while, on the contrary, local determinacy is bound to prevail for larger values of σ . One corollary of this is *that flat tax rates on labor income do not affect the range of parameter values that are associated with local indeterminacy and bifurcations* (see also Guo and Harrison (2004) for a related discussion).

The key implication of increasing progressivity π from zero can be seen, starting with the benchmark case with linear taxes, by focusing on how the following intersection points vary with π (see Fig. 1). First, direct inspection of Eqs. (11) shows that $\Lambda(\sigma)$ (the deviation of $(T_1(\sigma), D_1(\sigma))$ from (AC)) is negative when σ is small enough (that is, $T_1(\sigma) < 1 + D_1(\sigma)$ when $\sigma < s$). In fact, the locus of $(T_1(\sigma), D_1(\sigma))$ generated when σ increases from zero describes a line Δ_1 which intersects (AC) at point I when $+\infty$ (i.e. $\Lambda(+\infty) = 0$). From Eqs. (11), one immediately sees that $D_1(+\infty)$ increases with π , so that point I goes north-east when π increases from zero. Second, Eqs. (11) imply that Δ_1

intersects the T -axis of equation $D = 0$ when $\sigma = \theta(1 - s)$ (that is, $D_1(\theta(1 - s)) = 0$), and that $\Lambda(\theta(1 - s))$ decreases, from zero, with π . An equivalent way of summarizing these two observations is that, when π increases from zero, point I (where Δ_1 intersects (AC)) goes north-east, along (AC) , whereas the *slope of Δ_1* decreases from one, so that several configurations occur in the (T, D) plane. We concentrate, in the next proposition, on the most plausible case such that progressivity is not too large. All the other cases, associated with larger progressivity, can be derived through the very same analysis and are presented in Dromel and Pintus (2006).

Proposition 2.2 (Local Stability and Bifurcations of the Steady State).

Consider the steady state that is assumed to be set at $(\bar{a}, \bar{k}) = (1, 1)$ through the procedure in Proposition B.1. If, moreover, $\theta(1 - s) < s$ and $0 < \pi < 1 - \theta(1 - s)/s$ (that is, fiscal progressivity is not too large), the following generically holds (see Fig. 1).³

1. $0 < \sigma < \sigma_F$: the steady state is a sink for $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$, where $\varepsilon_{\gamma H}$ is the value of ε_γ for which Δ crosses $[BC]$. Then the steady state undergoes a Hopf bifurcation (the complex characteristic roots cross the unit circle) at $\varepsilon_\gamma = \varepsilon_{\gamma H}$, and is a source when $\varepsilon_\gamma > \varepsilon_{\gamma H}$.
2. $\sigma_F < \sigma < \sigma_H$: the steady state is a sink when $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$. Then the steady state undergoes a Hopf bifurcation at $\varepsilon_\gamma = \varepsilon_{\gamma H}$ and is a source when $\varepsilon_{\gamma H} < \varepsilon_\gamma < \varepsilon_{\gamma F}$. A flip bifurcation occurs (one characteristic root goes through -1) at $\varepsilon_\gamma = \varepsilon_{\gamma F}$ and the steady state is a saddle when $\varepsilon_\gamma > \varepsilon_{\gamma F}$.
3. $\sigma_H < \sigma < \sigma_I$: the steady state is a sink when $1 < \varepsilon_\gamma < \varepsilon_{\gamma F}$. A flip bifurcation occurs at $\varepsilon_\gamma = \varepsilon_{\gamma F}$ and the steady state is a saddle if $\varepsilon_\gamma > \varepsilon_{\gamma F}$.
4. $\sigma_I < \sigma < s$ and $s < \sigma$: the steady state is a saddle when $\varepsilon_\gamma > 1$.

Proof: See Appendix C.

Proposition 2.2 reveals an important implication. The upper bound that is imposed on π appears to be large for plausible parameter values. In fact, $\theta = 1 - \beta(1 - \delta)$ is bound to be close to zero when the period is commensurate with business-cycle length, as $\beta \approx 1$ and $\delta \approx 0$ when the period is, say, a month. Therefore, the condition that $\pi < 1 - \theta(1 - s)/s$ is not restrictive. In other words, for sensible parameter values, tax progressivity does not rule out

³The expressions of σ_F , σ_H , σ_I , $\varepsilon_{\gamma H}$ and $\varepsilon_{\gamma F}$ are given in Proposition 2.3.

local indeterminacy by ensuring saddle-point convergence, even when it is quite large. The next statement shows how the critical values involved in Proposition 2.2 (see also Fig. 1) move with π .

Proposition 2.3 (Income Tax Progressivity and Local Indeterminacy).

The critical values involved in Proposition 2.2 (see Fig. 1) are such that $\sigma_F = \theta(1-s)/2$ and $\sigma_H = s[1 + \theta(1-s)/s - \sqrt{1 - \theta(1-s)/s}]/2$ are independent of π , while $\varepsilon_{\gamma F} = (1-\pi)(2s + \theta(1-s) - 2\sigma)/[2\sigma - \theta(1-s)]$, $\varepsilon_{\gamma H} = (1-\pi)(s - \sigma)/[\theta(1-s) - \sigma]$, and $\sigma_I = \theta(1-s)/2 + s(1-\pi)/(2-\pi)$ are decreasing functions of π .

In consequence, income tax progressivity reduces the set of parameter values that are associated with local indeterminacy.

Suppose now that we extend our results to consider negative values of π . In view of Assumption 2.2, $\pi < 0$ implies that income taxes are *regressive*.⁴ Then one corollary of Proposition 2.3 is that decreasing π from zero would enlarge the set of parameter values that are associated with local indeterminacy, as in Guo and Lansing (1998) or Seegmuller (2004).

Besides the qualitative statements contained in Proposition 2.3, it may be informative to assign numerical values to parameters and then ask how sensitive the critical values are with respect to tax progressivity. The first thing to notice is that the assumption of a finance constraint imposed on workers suggests to interpret the period as, say, a month. Then setting $\beta = 0.997$ and $\delta = 0.008$ is the monthly counterpart of the values adopted in the literature and based on annual data (that is, $\beta = 0.96$ and $\delta = 0.1$). With $s = 1/3$, one then has that $\theta \approx 0.01$, which implies that the upper bound appearing in Proposition 2.2 is $\pi < 1 - \theta(1-s)/s \approx 0.98$. This corroborates our previous conclusion that the case pictured in Fig. 1 is the most relevant. Moreover, based on US marginal tax rates that are provided by Stephenson (1998), our own computations deliver that income tax progressivity has ranged in [4%, 11%] over 1940-93, with an average around 6% (in accord with Bénabou 2002 or Cassou and Lansing 2004). In view of Proposition 2.3, one expects σ_F and σ_H to be close to zero when θ is close to zero, as is the case in our numerical example. Therefore, we focus on σ_I which is equal to $[\theta(1-s) + s]/2 \approx 0.17$ when $\pi = 0$. With tax progressivity, one gets that $\sigma_I = \theta(1-s)/2 + s(1-\pi)/(2-\pi) \approx 0.16$ when $\pi = 0.11$, that is, when tax progressivity is set at the modal value observed over the period 1940-93. Therefore, plausibly low values of income tax progressivity leaves the range $(0, \sigma_I)$ of parameter values associated with local indeterminacy virtually unchanged. In contrast, reducing σ_I by 50% requires $\pi \approx 0.67$, which seems improbable in view of the available evidence.

⁴In that case, one has to modify Assumption 2.2 so as to impose that the absolute value of π is not too large to ensure concavity of the workers' decision problem.

2.3 Explaining the Limits of Progressive Taxes as Automatic Stabilizers

Our last step is to provide some intuitive explanation of the mechanisms at work. In the related literature dealing with increasing returns in a non-monetary Ramsey economy, the main effect of progressive tax rates is to reverse the condition on the slopes of both labor demand and labor supply (Guo and Lansing 1998). In such models, the mechanisms leading to indeterminacy are different. We postulate instead constant returns so that, in particular, labor demand is (as a function of real wage) downward sloping. What we now illustrate intuitively is that expectation-driven business cycles occur because of an expected inflation effect that is absent from the related literature. Most importantly, we would like to understand why even *large* income tax progressivity fails to ensure saddle-point stability. As we now explain, key to the results is the fact that the more progressive taxes on labor income, the more stable disposable wage income and, therefore, the less responsive workers' labor supply. However, this effect is not strong enough to neutralize the impact of *expected inflation* on labor supply, which can lead to "animal spirits".

It is helpful to start with the benchmark case of a constant tax rate (which also covers the case with zero taxes and transfers) on labor income. In that case, workers' decisions are summarized by Eqs. (5) that may be written as follows, as ϕ reduces to the identity function and $\psi = 1$:

$$v_1(l_t) = v_2(p_t \omega_t l_t / p_{t+1}), \quad (12)$$

which defines implicitly labor supply $l(p_t \omega_t / p_{t+1})$. The latter first-order condition shows that when workers expect, in period t , that the price of goods p_{t+1} will go, say, down tomorrow, they wish to increase their consumption at $t + 1$ and, therefore, to work more today (remember that gross substitutability is assumed) so as to save more in the form of money balances to be consumed tomorrow. Moreover, the dynamical system in Eqs. (8) may be written as follows:

$$\begin{cases} v_2(\omega_{t+1} l_{t+1}) = v_1(l_t), \\ k_t = \beta R(k_{t-1} / l_t) k_{t-1}. \end{cases} \quad (13)$$

A higher labor supply l_t will lead to greater output, larger consumption and a smaller capital-labor ratio k_{t-1} / l_t and, therefore, to a higher return on capital R_t , so that, from Eqs. (13), capital demand k_t and investment will increase. Moreover, a larger capital stock k_t tomorrow will tend to increase both tomorrow's labor demand and tomorrow's real wage ω_{t+1} which will trigger an increase in tomorrow's labor supply. However, a higher capital stock will also tend to increase the ratio of capital/labor and, eventually, the effect on capitalists' savings will turn negative: a higher capital-labor ratio leads to a lower rate of return on capital and, therefore, to lower capital demand and investment. This will eventually lead to lower wage, lower labor supply, etc: the economy will experience a reversal of the cycle.

Note that this intuitive description relies on the presumption that both wage and interest rate are elastic enough to the capital-labor ratio: the elasticity of input substitution σ must be small enough.

Now, we would like to shed some light on the fact that although progressive income taxation makes the occurrence of self-fulfilling fluctuations less likely, it does not rule them out. Assume again, for simplicity, that ϕ has constant elasticity at steady state. In that case, Eqs. (5) reduce to:

$$v_1(l_t) = \psi v_2(p_t \phi(\omega_t l_t) / p_{t+1}). \quad (14)$$

When π increases from zero to one, the volatility of wage income decreases to zero: eventually, a highly progressive tax rate on labor income (that is, π close to one) leads to an almost constant after-tax wage bill, which in turn leads to a more stable consumption and, thereby, to a smaller reaction of labor supply in comparison to the case of flat-rate taxes. More specifically, Eq. (14) shows that a large progressivity π decreases the elasticity of labor supply. To see this, differentiate Eqs. (14) to get:

$$(\varepsilon_\gamma - 1 + \pi) \frac{dl}{l} = -\frac{dt^e}{t^e} + (1 - \pi) \frac{d\omega}{\omega}, \quad (15)$$

where $\gamma \equiv v_2^{-1} \circ [v_1/\psi]$, t^e denotes expected inflation, that is, $t_{t+1}^e \equiv p_{t+1}/p_t$. Eq. (15) clearly shows how the higher fiscal progressivity π , the less responsive labor supply to the real wage: this elasticity tends to zero when π tends to one. *However, large progressivity does not completely neutralize the impact of expected inflation on labor supply:* the corresponding elasticity does not vanish when $\pi = 1$. Consequently, optimistic expectations (say, a reduction in p_{t+1}) still lead to an increase of consumption and labor when fiscal policy is highly progressive so that expectation-driven business cycles occur.

2.4 Extending the Analysis: the Case of Small Externalities

The purpose of this section is to ask whether our results are robust with respect to the introduction of small increasing returns to scale. The presence of either externalities or internal increasing returns (as in Guo and Lansing 1998) has been shown to enlarge the range of capital-labor substitution elasticities compatible with local indeterminacy (Cazzavillan *et al.* 1998). More precisely, local indeterminacy and bifurcations occur when σ belongs to some interval, provided that ε_γ is small enough (Cazzavillan *et al.* 1998, Prop. 4.2). We now show that adding progressive labor taxes tends to restrict such an interval of values for σ . In other words, extending the analysis to introduce small externalities does not change our main conclusion: *increasing the level of tax progressivity π reduces the set of parameter values such that*

local indeterminacy and bifurcations occur.

So as to focus on the relevant cases, we assume that externalities come from labor only. In the notation of Cazzavillan *et al.* (1998, Ass. 2.1), we set $\varepsilon_\psi = 0$ and let $\nu > 0$ be the level of externalities. This is consistent with the existing literature, which has stressed that labor externalities are necessary for local indeterminacy to arise, but that capital externalities are not. In addition, we focus on small external effects by assuming that ν is smaller than a (large) threshold. This assumption accords with most empirical studies, that have shown how the assumption of constant returns to scale is consistent with the available data.

Exactly as in the previous section, we use the fact that, here again, adding progressive labor taxes in Cazzavillan *et al.* (1998) amounts to the parameter change $\varepsilon_\gamma \rightarrow \varepsilon_\gamma/(1-\pi)$. Then it is not difficult to derive the following statement, which is the analog of Proposition 2.1.

Proposition 2.4 (Linearized Dynamics around the Steady State with Small Externalities).

The linearized dynamics for the deviations $da = a - \bar{a}$, $dk = k - \bar{k}$ are determined by the linear map:

$$\begin{cases} da_{t+1} &= -\frac{\varepsilon_\gamma/(1-\pi)+\varepsilon_{R,a}(1+\varepsilon_{\Omega,k})}{\varepsilon_{\Omega,a}-1}da_t + \frac{\bar{a}}{k}\frac{\varepsilon_\gamma/(1-\pi)-(1+\varepsilon_{\Omega,k})(1+\varepsilon_{R,k})}{\varepsilon_{\Omega,a}-1}dk_{t-1}, \\ dk_t &= \frac{\bar{k}}{a}\varepsilon_{R,a}da_t + (1+\varepsilon_{R,k})dk_{t-1}. \end{cases} \quad (16)$$

The associated Jacobian matrix evaluated at the steady state under study has trace T and determinant D , where

$$T = T_1 - \frac{\varepsilon_\gamma - 1}{(1-\pi)(\varepsilon_{\Omega,a} - 1)}, \quad \text{with } T_1 = 1 + \frac{|\varepsilon_{R,a}| - 1/(1-\pi) - \varepsilon_{R,k} + \varepsilon_{R,k}\varepsilon_{\Omega,a} + \varepsilon_{\Omega,k}|\varepsilon_{R,a}|}{\varepsilon_{\Omega,a} - 1},$$

$$D = \varepsilon_\gamma D_1, \quad \text{with } D_1 = \frac{|\varepsilon_{R,a}| - 1 - \varepsilon_{R,k}}{(1-\pi)(\varepsilon_{\Omega,a} - 1)}.$$

Moreover, one has $T_1 = 1 + D_1 + \Lambda$, where $\Lambda \equiv \frac{(1-\pi)[\varepsilon_{R,k}\varepsilon_{\Omega,a} + \varepsilon_{\Omega,k}|\varepsilon_{R,a}|] + \pi[\varepsilon_{R,k} - |\varepsilon_{R,a}|]}{(1-\pi)(\varepsilon_{\Omega,a} - 1)}$.

Furthermore, we directly borrow from Cazzavillan *et al.* (1998, p. 81) the expressions of the elasticities of Ω and R that are related to technology, to get the following expressions.

$$D_1 = (\theta(1-s) - \sigma)/[(1-\pi)(s - \sigma(\nu + 1))], \quad \Lambda = \theta[\nu - \pi(1-s + \nu)]/[(1-\pi)(s - \sigma(\nu + 1))], \quad (17)$$

$$T_1 = 1 + D_1 + \Lambda, \quad \text{slope}_\Delta = 1 - \theta(1-s)/\sigma,$$

where $\theta \equiv 1 - \beta(1-\delta) > 0$ and all these expressions are evaluated at the steady state under study.

Assume that $\nu < (s - \theta(1-s))/(\theta(1-s))$, which is easily satisfied if we focus on small externalities and remember that θ is likely to be close to zero. Then two cases have to be considered. If $\pi < \nu/(1+\nu)$, the geometrical picture arising when $\pi > 0$ is similar to Cazzavillan *et al.* (1998, Fig. 7), so that local indeterminacy and bifurcations occur

when σ belongs to $(0, \sigma_{F2}) \cup (\sigma_{H2}, +\infty)$, provided that ε_γ is small enough. If, on the contrary, $\pi > \nu/(1 + \nu)$, then local indeterminacy occurs only if σ belongs to $(0, \sigma_{F2})$. In that configuration, increasing returns are so small that they are dominated by the opposite impact of tax progressivity. As a consequence, indeterminacy occurs only for low values of σ , as in the case without externalities. In summary, if $\pi > \nu/(1 + \nu)$ then income tax progressivity eliminates local indeterminacy when $\sigma > \sigma_{H2}$ but preserves it when $\sigma < \sigma_{F2}$. On the other hand, the change of parameter $\varepsilon_\gamma \rightarrow \varepsilon_\gamma/(1 - \pi)$ ensures that all the critical values of ε_γ , below which local indeterminacy prevails, are decreasing functions of π . This allows us to state the following.

Proposition 2.5 (Tax Progressivity and Local Indeterminacy under Small Externalities).

Assume that labor externalities are small enough, that is $\nu < (s - \theta(1 - s))/(\theta(1 - s))$. Then the range of values of σ that is compatible with local indeterminacy and bifurcations is either $(0, \sigma_{F2}) \cup (\sigma_{H2}, +\infty)$, when $\pi < \nu/(1 + \nu)$, or $(0, \sigma_{F2})$, when $\pi > \nu/(1 + \nu)$.

The critical values are such that $\sigma_{F2} = [(1 - \pi)(2s + \theta(1 - s + \nu)) + \theta(1 - s)]/[2 + 2(1 - \pi)(\nu + 1)]$ is a decreasing function of π , while $\sigma_{H2} = [(1 - \pi)s - \theta(1 - s)]/[(1 - \pi)(\nu + 1) - 1]$ is an increasing function of π .

Therefore, income tax progressivity reduces the set of parameter values that are associated with local indeterminacy.

3 Conclusion

We have shown, in a monetary economy model with constant returns, that sunspots and endogenous cycles are robust with respect to the introduction of tax progressivity, when it is set at realistic levels. Although fiscal progressivity reduces, in parameter space, the likelihood of sunspot equilibria and endogenous cycles, considering plausibly low values of tax progressivity still leaves room for sunspots and (Hopf or flip) cycles. In an expanded version, Dromel and Pintus (2006), we show that similar results hold with capital income taxes, or in an OLG economy with consumption in old age. In particular, it is shown there that adding progressive taxes on *capital* income in the Woodford model results in a negligible reduction of the set of parameter values that is associated with local indeterminacy. Moreover, extending the analysis to introduce increasing returns to scale does not change the main message of this paper.

Some directions for future research seem natural to us. It would be useful to generalize the analysis to the realistic case whereby progressivity varies with income. In addition, it seems relevant to introduce, in the OLG setting, consumption/savings choices in the first period of life. In that context, one expects that a smaller level of progressivity on

both capital and labor incomes could rule out local indeterminacy and bifurcations by stabilizing both young and old agents' consumptions. It remains to be seen if this is more in line with actual levels of fiscal progressivity. It is also expected that progressive taxes and transfers are inefficient to rule out endogenous fluctuations when consumption is financed, even partially, by the returns from financial assets that would remain untaxed. Moreover, although we have studied the stabilizing power of income taxation, similar results are expected in other frameworks with business taxes, e.g. in models with credit-constrained firms and collateral requirements related to cash-flows.

A Progressive Labor Income Taxes and Workers' Choices

In this section, we show how workers' decisions can be reduced to a two-period problem. Workers solve the following problem:

$$\max \sum_{t=1}^{+\infty} \beta_w^{t-1} V_2(c_t^w/B) - \beta_w^t V_1(l_t), \quad (18)$$

subject to

$$M_{t-1}^w + (r_t + (1 - \delta)p_t)k_{t-1}^w + p_t\phi(\omega_t l_t) \geq p_t c_t^w + p_t k_t^w + M_t^w, \quad (19)$$

$$M_{t-1}^w + (r_t + (1 - \delta)p_t)k_{t-1}^w \geq p_t c_t^w + p_t k_t^w, \quad (20)$$

where $B > 0$ is a scaling parameter, $0 < \beta_w < 1$ is the discount factor, $c_t^w \geq 0$ is consumption, $l_t \geq 0$ is labor supply. On the other hand, $M_{t-1}^w \geq 0$ and $k_{t-1}^w \geq 0$ are respectively money demand and capital holdings at the beginning of period t , $p_t > 0$ is the price of consumption goods, $w_t > 0$ is nominal wage, $r_t > 0$ is nominal return on capital and $1 \geq \delta \geq 0$ is capital depreciation, while $p_t\phi(\omega_t l_t)$ is disposable wage income (see Section 2) and $\omega = w/p$ defines real wage.

Define $\lambda_t \geq 0$ and $\epsilon_t \geq 0$ as the Lagrange multipliers associated, respectively, to (19) and (20) at date t . Necessary conditions are then the following.

$$\begin{aligned} 0 &\geq \beta_w^{t-1} V_2'(c_t^w/B)/B - (\lambda_t + \epsilon_t)p_t, & = 0 \text{ if } c_t^w > 0, \\ 0 &\geq -(\lambda_t + \epsilon_t)p_t + (\lambda_{t+1} + \epsilon_{t+1})(r_{t+1} + (1 - \delta)p_{t+1}), & = 0 \text{ if } k_t^w > 0, \\ 0 &\geq -\lambda_t + \lambda_{t+1} + \epsilon_{t+1}, & = 0 \text{ if } M_t^w > 0, \\ 0 &\geq -\beta_w^t V_1'(l_t) + \lambda_t p_t \omega_t \phi'(\omega_t l_t), & = 0 \text{ if } l_t > 0. \end{aligned} \quad (21)$$

Therefore, capital holdings are zero at all dates ($k_t^w = 0$) if the second inequality of (21) is not binding, that is, if:

$$V_2'(c_t^w/B) > \beta_w(r_{t+1}/p_{t+1} + 1 - \delta)V_2'(c_{t+1}^w/B), \quad (22)$$

if one assumes that $c_t^w > 0$ for all t (we will show that this is the case around the steady state). Condition (22) implies that workers choose not to hold capital, and it depends on workers' preferences because of the financial constraint (20).

Moreover, the financial constraint (20) is binding if $\epsilon_t > 0$, that is, if:

$$\omega_t \phi'(\omega_t l_t) V_2'(c_t^w/B)/B > \beta_w V_1'(l_t), \quad (23)$$

if one assumes that $l_t > 0$ (again, we will show that this is the case around the steady state). Condition (23) therefore implies that (20) is binding.

Under conditions (22) and (23), workers spend their money holdings, i.e. $p_t c_t^w = M_{t-1}^w$, and save their wage income in the form of money, i.e. $M_t^w = p_t \phi(\omega_t l_t)$, so as to consume it tomorrow, i.e. $p_{t+1} c_{t+1}^w = M_t^w$. Therefore, workers choose $l_t \geq 0$ and $c_{t+1}^w \geq 0$ as solutions to:

$$\max \{V_2(c_{t+1}^w/B) - V_1(l_t)\} \text{ s.t. } p_{t+1} c_{t+1}^w = p_t \phi(\omega_t l_t). \quad (24)$$

The solutions to (24) are unique under Assumption 2.2 and 2.3 and characterized by the following first-order condition, which is identical to (5) in the main text:

$$v_1(l_t) = \psi(\omega_t l_t) v_2(c_{t+1}^w), \quad p_{t+1} c_{t+1}^w = p_t \phi(\omega_t l_t), \quad (25)$$

where $v_2(c) \equiv c V_2'(c/B)/B$, $v_1(l) \equiv l V_1'(l)$.

Finally, it is straightforward to show that, under the assumptions that capitalists discount future less heavily than workers (that is, $\beta_w < \beta$) and that $\beta_w < 1$, conditions (22) and (23) are met at the steady state under study defined in Proposition B.1.

B Existence of Steady State: Proposition B.1

Proposition B.1 (Existence of a Normalized Steady State).

Under Assumptions 2.1, 2.2 and 2.3, $\lim_{c \rightarrow 0} c V_2'(c) < V_1'(1)/\psi(\omega(1)) < \lim_{c \rightarrow +\infty} c V_2'(c)$, $(\bar{a}, \bar{k}) = (1, 1)$ is a steady state of the dynamical system in Eqs. (8) if and only if $A = (1/\beta - 1 + \delta)/f'(1)$ and B is the unique solution of $\psi(\omega(1))\phi(\omega(1))V_2'(\phi(\omega(1))/B)/B = V_1'(1)$.

Proof: In view of Eqs. (8) and recalling that $a = k/l$, the nonautarkic steady states are the solutions (\bar{a}, \bar{l}) in \mathbf{R}_{++}^2 of $v_2(\phi(\omega(\bar{a})\bar{l})) = v_1(\bar{l})/\psi(\omega(\bar{a})\bar{l})$ and $\beta R(\bar{a}) = 1$. Equivalently, in view of Eq. (3), the steady states are given by:

$$\begin{cases} v_2(\phi(\omega(\bar{a})\bar{l})) &= v_1(\bar{l})/\psi(\omega(\bar{a})\bar{l}), \\ \rho(\bar{a}) + 1 - \delta &= 1/\beta. \end{cases} \quad (26)$$

We shall solve the existence issue by setting appropriately the scaling parameters A and B , so as to ensure that one stationary solution coincides with, for instance, $(\bar{a}, \bar{l}) = (1, 1)$. The second equality of Eqs. (26) is achieved by scaling the parameter A , while the first is achieved by scaling the parameter B . That is, we set $A = (1/\beta - 1 + \delta)/f'(1)$ to ensure that $\bar{a} = 1$. On the other hand, $\psi(\omega(\bar{a})\bar{l})v_2(\bar{c}) = v_1(\bar{l})$ is then equivalent to

$$\psi(\omega(1))\frac{\phi(\omega(1))}{B}V_2'\left(\frac{\phi(\omega(1))}{B}\right) = V_1'(1). \quad (27)$$

From Assumption 2.3, v_2 is decreasing in B so the latter condition is satisfied for some unique B if and only if:

$$\lim_{c \rightarrow 0} cV_2'(c) < V_1'(1)/\psi(\omega(1)) < \lim_{c \rightarrow +\infty} cV_2'(c). \quad (28)$$

C Proof of Proposition 2.2

To prove formally the proposition, our first task is to show that the point $(T_1(\sigma), D_1(\sigma))$, as a function of σ , indeed describes part of a line Δ_1 . From the fact that $T_1(\sigma) = 1 + D_1(\sigma) + \Lambda(\sigma)$ and $D_1(\sigma)$ are fractions of first degree polynomials in σ with the same denominator (see Eq. (11)), we conclude that the ratio of their derivatives $D_1'(\sigma)/T_1'(\sigma)$, or $D_1'(\sigma)/(D_1'(\sigma) + \Lambda'(\sigma))$, is independent of σ . Straightforward computations show that the slope of Δ_1 is:

$$\text{slope}_{\Delta_1} = \frac{D_1'(\sigma)}{T_1'(\sigma)} = \frac{s - \theta(1 - s)}{s - \theta(1 - s) + \pi\theta(1 - s)}. \quad (29)$$

From Eq. (11), we conclude that $\Lambda(\sigma)$ vanishes when σ goes to infinity. It follows that Δ_1 intersects the line (AC) at a point I of coordinates $(T_1(+\infty), D_1(+\infty))$, where $D_1(+\infty) = 1/(1 - \pi) > 0$ (see Fig. 1). We shall focus throughout on the configuration presented in Fig. 1, where $D_1(+\infty) \geq 1$ and the slope of Δ_1 is smaller than 1 (that is, $\pi \geq 0$). We shall ensure the latter condition by imposing, as in the case of linear (or of no) taxes, that $\theta(1 - s) < s$ (that is, the share of capital is large enough). This condition is not very restrictive when $\theta = 1 - \beta(1 - \delta)$ is small, which is bound to be the case when the period is short since β is then close to one and δ is close to zero. Note that the geometrical method can be applied as well when these conditions are not met.

Then it follows that both $\Lambda(\sigma)$ and $D_1(\sigma)$ are decreasing functions (see Eq. (11)), so that $T_1(\sigma)$ is also a decreasing

function, i.e. $T_1'(\sigma) = D_1'(\sigma) + \Lambda'(\sigma) < 0$. Accordingly, the slope of Δ_1 is smaller than one. From the above assumptions, one also gets all the necessary information to appraise the variations of $(T_1(\sigma), D_1(\sigma))$ as well as of the slope of Δ , when σ moves from 0 to $+\infty$. In particular, $T_1(0)$ and $D_1(0) = \theta(1-s)/[s(1-\pi)]$ are positive and the corresponding point is below I on the line Δ_1 when $\pi > 0$ (see Fig. 1). As σ increases from 0, $T_1(\sigma)$ and $D_1(\sigma)$ are decreasing and tend to $-\infty$ when σ tends to s from below. When $\sigma = s$, the function $\omega(a)/a$ has a critical point, i.e. its derivative with respect to a vanishes, and the dynamical system derived from Eqs. (8) is not defined. When σ increases from s to $+\infty$, $T_1(\sigma)$ and $D_1(\sigma)$ are still both decreasing, from $+\infty$ to $(T_1(+\infty), D_1(+\infty))$, which is represented by the point I in Fig. 1. In addition, the slope of Δ as a function of σ increases monotonically from $-\infty$ to 1 as σ moves from 0 to $+\infty$, and vanishes when $D_1(\sigma) = 0$. Moreover, the half-line Δ is above Δ_1 when $\sigma < s$, and below it when $\sigma > s$.

Therefore, several configurations arise when π increases from zero. When $\pi < 1 - \theta(1-s)/s$ (Proposition 2.2) then $D_1(0) < 1$: the geometric picture is as in Fig. 1 and it is not qualitatively different from the case of linear (or no) taxes. Second, $D_1(0) > 1$ when $\pi > 1 - \theta(1-s)/s$, and two cases arise depending on whether π is smaller or larger than $[s - \theta(1-s)]/[s - \theta(1-s)/2]$. The latter cases are presented in Dromel and Pintus ?.

We now derive all bifurcation values as functions of the structural parameters. We define $\theta \stackrel{\text{def}}{=} 1 - \beta(1 - \delta)$, and $s_\Delta(\sigma) \stackrel{\text{def}}{=} 1 - \theta(1-s)/\sigma$ as the slope of the half-line Δ .

An eigenvalue of -1 : the flip bifurcation: The equality $s_\Delta(\sigma) = -1$ allows one to derive $\sigma_F = \theta(1-s)/2$, so that $s_\Delta(\sigma) < -1$ when $\sigma < \sigma_F$.

Equation $1 + T(\varepsilon_\gamma) + D(\varepsilon_\gamma) = 0$ yields $\varepsilon_{\gamma F} = (1 - \pi)(2s + \theta(1-s) - 2\sigma)/[2\sigma - \theta(1-s)]$.

The condition that $1 + T_1(\sigma) + D_1(\sigma) = 0$ or, equivalently $\varepsilon_{\gamma F} = 1$, gives the last flip bifurcation value $\sigma_I = [\theta(1-s)(2 - \pi) + 2s(1 - \pi)]/[2(2 - \pi)]$ so that $\varepsilon_{\gamma F} > 1$ when $\sigma < \sigma_I$.

A pair of eigenvalues of modulus 1: the Hopf bifurcation: The condition that $T(\varepsilon_{\gamma H}) = -2$ when $D = 1$, i.e. when $\varepsilon_\gamma = \varepsilon_{\gamma H} = 1/D_1$, is rewritten as $Q_H(\sigma) \stackrel{\text{def}}{=} a\sigma^2 + b\sigma + c$, the roots of which contain the bifurcation value σ_H . The coefficients of $Q_H(\sigma)$ are:

$$\begin{aligned} a &= 4, \\ b &= -4[s + \theta(1-s)], \\ c &= \theta(1-s)[\theta(1-s) + 3s]. \end{aligned}$$

It is easily shown that there must exist two distinct real roots, and that $\sigma_H = s[1 + \theta(1-s)/s - \sqrt{1 - \theta(1-s)/s}]/2$ is the lowest. Finally, the bifurcation value $\varepsilon_{\gamma H} = (1 - \pi)(s - \sigma)/[\theta(1-s) - \sigma]$ follows from $D = \varepsilon_\gamma D_1 = 1$, i.e. $\varepsilon_{\gamma H} = 1/D_1$.

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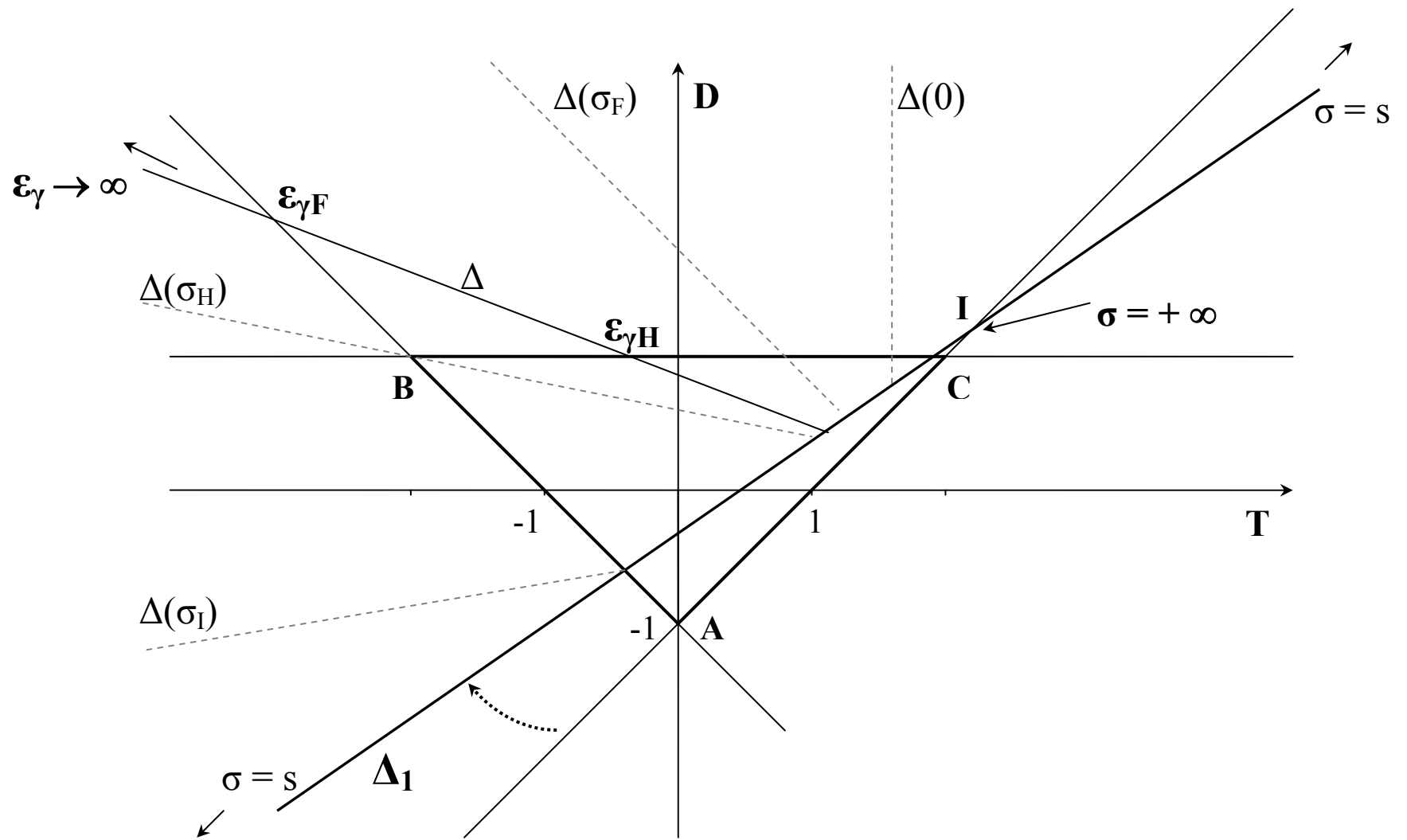


Figure 1
 Proposition 2.2