Rectangularization and the Rise in Limit-Longevity in a Simple OLG Model

Gregory Ponthiere[1]

F.R.S.-FNRS, CREPP, University of Liege

Correspondence: CREPP, University of Liege, Boulevard du Rectorat, 7, bat. B31, Sart-Tilman, 4000 Liege, Belgium. Tel: 0032-43663198. E-mail: G.Ponthiere@ulg.ac.be

Abstract:

Whereas OLG models with endogenous longevity do not distinguish between the rectangularization phenomenon and the rise in limit-longevity, these constitute two different demographic phenomena requiring a distinct modelling. This paper presents a two-period OLG model where the probability of survival from the first to the second period, as well as the maximum length of life, are endogenously determined and influenced by public policies. The issues of existence, uniqueness and stability of a steady-state are studied. It is shown that the transition towards the steady-state exhibits, under mild conditions, the observed succession of phases of rectangularization and derectangularization of survival curves.

Keywords: longevity, limit-longevity, OLG model, public policy, rectangularization.

JEL classification numbers: O41, E13, H51, I12.

[1] I gratefully acknowledge comments and suggestions made on a previous version of this paper by Luc Champarnaud, Alessandro Cigno, Gabrielle Demange, Laurence Lasselle, André Masson, Panu Poutvaara, Pierre Pestieau, Kevin Sheedy, Rick van der Ploeg, Stanley Winer, participants of the 2006 Séminaire Scientifique of the Caisse des Dépôts et Consignations in Bordeaux, participants of the 2006 CESifo Area Conference on Public Sector Economics in Munich, participants of the 2006 INFER Annual Meeting in Cork, and participants of the 2006 ASSET Annual Meeting in Lisbon. I am also grateful to two anonymous referees for their helpful comments. The usual disclaimer applies.

1 Introduction

Whereas these have been evolving relatively slowly during the longest part of history, economic activity and human longevity have grown more strongly during the last two centuries. As shown by Maddison (2001), while real GDP per capita was multiplied, in Western Europe, by a factor of 3 between years 1000 and 1820, it was multiplied by 16 between 1820 and 2000.[2] Moreover, while average life expectancy at birth in Western Europe was multiplied by 1.5 between years 1000 and 1820 (from about 24 to 36 years), it doubled over 1820-2000 (from 36 to 78 years).

The strong conjoint growth of economic activity and human longevity during the last 200 years (illustrated on Figure 1 for France) raises the twofold question of the relationship between those two phenomena: on the one hand, to what extent did the lengthening of life affect accumulation and growth; on the other hand, to what extent did economic expansion contribute to raise human longevity?

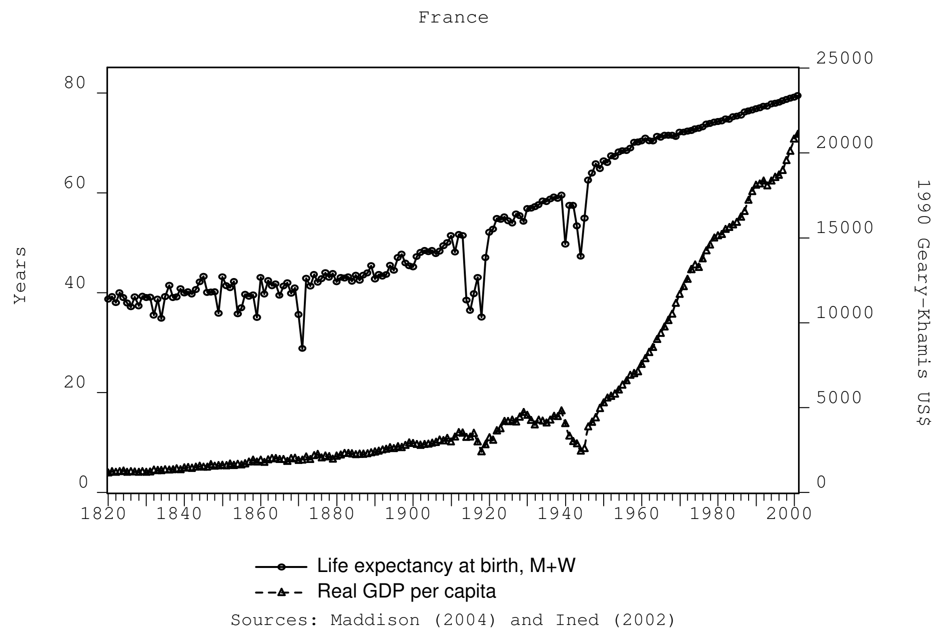


Figure 1: Period life expectancy at birth and real GDP per capita, France, 1820-2001.

[2]Indeed, real GDP per head has grown from about \$400 to \$1,204 (in international 1990 dollars) over 1000-1820, but from \$1,204 to \$19,002 over 1820-2000 (see Maddison, 2001).

The growth-longevity relationship has been largely studied in the economic growth literature, firstly in models analyzing the impact of longevity on accumulation mechanisms (e.g. savings, education) under an exogenous longevity, then, in models where prosperity influences longevity through various channels, such as private and/or public health expenditures.[3] Those models succeeded in showing not only how survival conditions, by determining the temporal horizon faced by agents, affect accumulation decisions, and, hence, economic development, but also, how the relationship between economic activity and longevity can lead to poverty traps or virtuous paths (longevity and production enhancing each others).

However, given that existing OLG models with endogenous longevity rely on a single-variable longevity (either a probability of survival from a life-period to another, or the length of a life-period), these tend to mix two phenomena, which are generally distinguished by demographers when analyzing the evolution of survival curves over time: on the one hand, the rectangularization phenomenon, that is, the rise in the proportion of people reaching high ages for a fixed maximum age at death; on the other hand, the rise in what demographers call the 'limit-longevity' (i.e. the maximum length of life that can be lived by a member of a cohort).

To illustrate that distinction, Figure 2 shows, for France, the evolution of survival curves over the last two centuries. As shown by the shift of the survival curve to the right, there has been a significant rise in the limit-longevity, in the sense that people can now reach ages that could not be reached in the past. However, there has also been a tendency of the survival curve to become more rectangular: a larger proportion of people can reach high ages for a fixed maximum age at death, so that the variance in the age at death is reduced.[4]

[3] On the first set of models, see Ehrlich and Lui (1991), de la Croix and Licandro (1999), Pecchenino and Utendorf (1999), Zhang *et al* (2001), and Boucekkine *et al* (2002). On the second kind of models, see Chakraborty (2004), Bhattacharya and Qiao (2005), and Pestieau *et al* (2006).

[4] One way to measure rectangularization proposed by Kannisto (2000) is to compute the coefficient ${}_{10}C_{50}$, which is the smallest age-interval covering 50 % of the deaths outside the first 10 years of life. On the measurement of rectangularization, see Vallin and Berlinguer (2002).

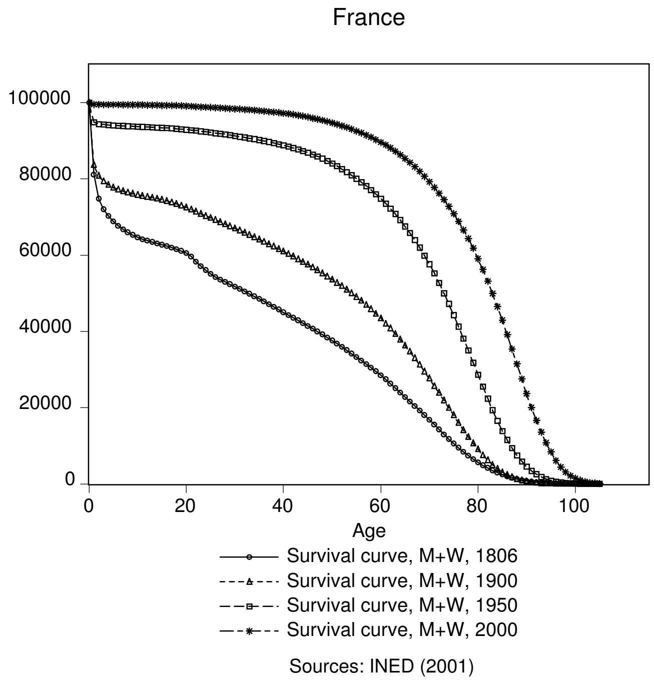


Figure 2: Period survival curves, men and women, France.

Although the evolution of survival curves over time is generally characterized by both a rectangularization and a rise in limit-longevity, demographers emphasized that those two phenomena did not take place to the same extent across countries and periods (see Yashin *et al,* 2001). This can be illustrated here with the example of France. As shown by Figure 2, the rectangularization process has dominated the rise in limit-longevity between 1806 and 1900: the right tail of the survival curve has been left unchanged over that period, while the rectangularization has been strong.[5] However, over 1950-2000, there has been a parallel shift of the survival curve to the right, leading to an increase in the tail of the survival distribution. Hence, over that period, the rise in limit-longevity has dominated the rectangularization process.[6]

The varying importance of rectangularization and the rise in limit-longevity

[5] For a detailed analysis of rectangularization in France, see Jeune and Kannisto (1997).

[6] See Yashin *et al* (2001) for a similar analysis for Japan, Sweden and the U.S. The evolution of survival curves in those countries - while having a country-specific timing - is characterized by a succession of periods where rectangularization dominates the rise in limit-longevity, and periods where, on the contrary, some derectangularization takes place.

raises the question of the factors affecting those two phenomena. That question can be addressed by examining the causes of death at different ages.[7] For instance, Nusselder and Mackenbach (2000) showed, in their analysis of the causes of variations in life expectancy at ages 60 and 85 in the Netherlands, that a substantial part of the rise of life expectancy at age 60 for men between years 1980-1984 and 1990-1994 is explained by a fall in the prevalence of lung cancers, which can thus be regarded as a leading cause of rectangularization for men. Nusselder and Mackenbach showed also that a significant part of the fall of life expectancy at age 85 for men and women over that period was caused by a rise in mental disorders, which suggests that mental disorders prevented a rise in limit-longevity over that period.

Such an analysis of the underlying causes of death is not without consequences for public policy, because the government can affect, through various channels, the different causes of death. According to the above example, preventive public policies against lung cancer would lead to a larger rectangularization, whereas medical research against ageing diseases (e.g. Alzheimer's disease) would, by reducing mental disorders, increase limit-longevity. Hence, the particular structure of public intervention can significantly affect each aspect of longevity.[8]

The goal of this paper is to study, in an OLG model, the phenomena of rectangularization and rise in limit-longevity, and, then, to discuss the form of optimal public intervention in such an economy. For that purpose, a two-period OLG model is developed, where the probability of survival from the first to the second period, as well as the length of the second period, can be affected by distinct public policies.

This paper is organized as follows. Section 2 presents the model. Section 3 analyzes the existence, uniqueness and stability of steady-states. Section 4 concen-

[7] On the difficulties raised by the decomposition of deaths by causes, see Meslé (2002).

[8] Note that the distinction between programs affecting rectangularization and programs affecting limit-longevity can be illustrated by various other examples. For instance, whereas expenditures promoting safety on roads contribute mainly to rectangularization, programs dealing with various aspects of ageing (not only mental, but, also, physical aspects) focus on limit-longevity.

trates on comparative statics. The short-run dynamics is examined numerically in Section 5. The optimal public policy is studied in Section 6. Section 7 concludes.

2 The model

2.1 Environment

Let us consider a two-period OLG model with identical households within each generation. Time is discrete. Each generation is denoted by its time of birth.

All individuals of a generation t live the first period for sure. During that period of unitary length, all individuals work, and save some money to fund their consumption during the second period, which is a period of retirement.

However, not all members of a cohort t will live the second period: only a proportion ϕ_{t+1} of generation t will do so ($0 \leq \phi_{t+1} \leq 1$).[9] The survivors of cohort t will enjoy a second period of length λ_{t+1} ($0 \leq \lambda_{t+1} \leq 1$). Hence, while the sum $1 + \phi_{t+1}\lambda_{t+1}$ is the life expectancy, $1+ \lambda_{t+1}$ constitutes the limit-longevity.

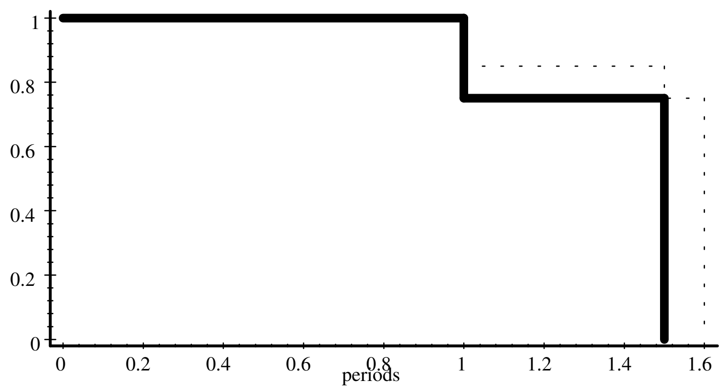


Figure 3: Survival curves

Although rudimentary, the survival curves in this model capture the two aspects of longevity under study (see Figure 3). The curve can become more rectangular

[9]Given that all members of a generation live the first period *entirely*, this amounts to assuming that the risk of death is fully concentrated at the beginning of the second period.

when the proportion of survivors of the first period increases, but there can also be a rise in limit-longevity, as shown by a shift of the survival curve to the right.

2.2 Consumption and savings

Consumption during the first period, denoted by c_t, is equal to:

$$c_t = (1 - \tau)w_t - s_t \tag{1}$$

where w_t is the wage, τ is the tax rate, while s_t is savings. It is assumed that, at the end of the first period, each individual deposits his savings at a mutual fund. The mutual fund invests these savings in capital, and guarantees a gross return equal to $\tilde{R}_{t+1}$ to the surviving old. The gross return $\tilde{R}_{t+1}$ is equal, under perfect competition, to R_{t+1}/ϕ_{t+1} (where R_{t+1} denotes the return on invested capital). For convenience, it is also supposed that a perfect annuities market exists.[10]

Total consumption during the second period is thus:

$$\lambda_{t+1} d_{t+1} = \tilde{R}_{t+1} s_t \tag{2}$$

where d_{t+1} denotes annuitized consumption when being old.

Let us now consider the savings decision. Each member of a cohort t must, at time t, choose a savings level s_t, but that decision must be made without a perfect knowledge of what the proportion of survivors in cohort t (i.e. ϕ_{t+1}) and the length of the second period (i.e. λ_{t+1}) will be. Given that no agent knows, at time t, the levels of ϕ_{t+1} and λ_{t+1}, the savings decision must be based on expectations for

[10] There must also exist some form of contract allowing that the production process takes place over a period of unitary length, while the consumption of the old (thanks to their savings income) lasts only over a period of length λ_{t+1}.

those variables, which shall be denoted by respectively ϕ^e_{t+1} and λ^e_{t+1}.

As it is well-known, various assumptions can be made on how agents form their expectations. One could, for instance, assume *fixed expectations*: agents of all epochs would expect the same levels of ϕ_{t+1} and λ_{t+1} [i.e. $\phi^e_{t+1} = \phi$ and $\lambda^e_{t+1} = \lambda$]. However, it is hard to see why agents would rely forever on such fixed beliefs. An alternative assumption consists of assuming *perfect foresight*: agents might perfectly anticipate the levels of ϕ_{t+1} and λ_{t+1} [i.e. $\phi^e_{t+1} = \phi_{t+1}$ and $\lambda^e_{t+1} = \lambda_{t+1}$]. Nevertheless, that assumption amounts to assuming not only that agents know the functional forms for ϕ_{t+1} and λ_{t+1}, but, also, that they have accurate estimates of the parameters involved in these. In the light of the difficulties faced by demographers in their attempts to quantify the influence of various factors affecting survival conditions, assuming perfect expectations in the present context is inadequate.[11]

Hence, we shall make here the intermediate postulate according to which agents have *adaptive expectations*, in the sense that agents of cohort t believe that demographic variables ϕ_{t+1} and λ_{t+1} will exhibit the same levels as the ones achieved by the previous cohort $t-1$ [i.e. $\phi^e_{t+1} = \phi_t$ and $\lambda^e_{t+1} = \lambda_t$].[12]

If one assumes adaptive expectations, as well as time-additive lifetime welfare, individual expected lifetime utility is, under logarithmic temporal welfare:

$$\begin{aligned} u_t &= \ln(c_t) + \beta\left[\phi_t\left(\lambda_t \ln\left(d_{t+1}\right)\right) + (1-\phi_t)0\right] \\ &= \ln\left(c_t\right) + \beta\phi_t\lambda_t \ln\left(d_{t+1}\right) \qquad (3) \end{aligned}$$

where β is a discount factor ($\beta > 0$), while the factor in bracket is the second period

[11] The estimation problems faced by demographers are partly due to the difficulty to identify the different causes of death (see Meslé, 2002).

[12] The assumption $\phi^e_{t+1} = \phi_t$ can be deduced from the usual adaptive expectations formula $\phi^e_{t+1} = \phi^e_t + \chi(\phi_t - \phi^e_{t-1})$, where the correction parameter χ equals 1 [note that fixed expectations (i.e. $\phi^e_{t+1} = \phi^e_t = \phi$) are obtained under χ equal to 0].

expected utility (under the assumption that the utility of being dead is zero).[13]

Substituting for the first and second period budget constraints in (3) yields:

$$\ln\left(w_t(1-\tau)-s_t\right)+\beta\phi_t\lambda_t\ln\left(\frac{R_{t+1}s_t}{\lambda_t\phi_t}\right) \tag{4}$$

Hence, the optimal savings level s_t^* is:

$$s_t^*=\frac{\beta\phi_t\lambda_t}{1+\beta\phi_t\lambda_t}(1-\tau)w_t \tag{5}$$

The expected probability of survival and length of the second period have a positive - but declining - influence on the propensity to save.

2.3 Production and capital accumulation

Production technology has the Cobb-Douglas form, with capital K_t and labour L_t:

$$Y_t=AK_t^{\alpha}L_t^{1-\alpha} \tag{6}$$

where Y_t is the output, A is a productivity parameter ($A>0$), α is a parameter ($0 < \alpha < 1$), while L_t grows at a constant rate n: $L_{t+1}=L_t(1+n)$.[14]

Profit maximization by firms implies:

[13] The function u_t exhibits what Bommier (2003) calls (net) risk-neutrality with respect to the length of life, which involves a simplification. Relaxing that assumption is left for future research.

[14] That assumption, although non trivial, is an analytically convenient simplification for the purpose at hand. Extending this model to connect fertility to longevity is left for future research.

$$w_t = (1-\alpha)Ak_t^{\alpha} \quad (7)$$

$$R_t = \alpha Ak_t^{\alpha-1} \quad (8)$$

where k_t denotes the capital per worker.

It is supposed that there is a full depreciation of capital after one period. Hence, the capital market equilibrium $K_{t+1} = L_t s_t$ implies:

$$k_{t+1} = \frac{(1-\alpha)(1-\tau)}{1+n} Ak_t^{\alpha} \frac{\beta\phi_t\lambda_t}{1+\beta\phi_t\lambda_t} \quad (9)$$

2.4 Government

The government implements two kinds of policies: on the one hand, public spending g_t affecting the proportion of survivors in a cohort (e.g. preventive programs against lung cancer); on the other hand, public spending r_t, which raise the length of life enjoyed by survivors (e.g. research against ageing diseases).[15]

The government's budget constraint is:

$$\tau w_t = g_t + r_t \quad (10)$$

For convenience, we shall introduce the parameter ψ_t $(0 \leq \psi_t \leq 1)$, which is the share of the budget dedicated to policies g_t affecting ϕ_{t+1}, while the proportion $(1-\psi_t)$ of the budget is dedicated to r_t. The composition of the budget is assumed to be constant over time ($\psi_t = \psi \; \forall t$), so that $g_t = \psi\tau w_t$ and $r_t = (1-\psi)\tau w_t$.

[15] Note that we shall, for simplicity, abstract here from public spending affecting directly *both* ϕ_{t+1} and λ_{t+1}.

2.5 Longevity

The proportion of survivors ϕ_{t+1}^{t} and the length of the second period λ_{t+1}^{t} are supposed to be influenced positively by economic prosperity, following the large empirical evidence showing that richer countries are generally characterized by better survival conditions than poorer countries.[16]

However, as Dreze and Sen (1989) and Anand and Ravallion (1993) emphasized, countries with equal development levels are not equally good at converting wealth in health, and public policies constitute a crucial channel by which the wealth of Nations affects the health of Nations. Thus, we shall distinguish here two influences of wealth on ϕ_{t+1} and λ_{t+1}: first, the influence of prosperity independently from public intervention, which is captured by the net wage $\tilde{w}_t$ (i.e. net of tax); second, the influence of the government, through public spending.[17] Public intervention takes here the form of two expenditures: g_t (affecting ϕ_{t+1}), and r_t (affecting λ_{t+1}).

Finally, we shall also assume that ϕ_{t+1} and λ_{t+1} depend not only on the level of economic prosperity and public policies, but, also, on past longevity achievements. Actually, demographers such as Vallin *et al* (2001) emphasized the crucial role played by habits and customs in mortality, so that it makes sense to suppose that the longevity achievements of a particular cohort are, to some extent, dependent on longevity achievements of previous cohorts. Such a dependency on the past may explain why two countries with the same level of development and the same public policy do not necessarily exhibit identical survival conditions.

Hence, ϕ_{t+1} and λ_{t+1} are assumed to be determined as follows:

[16] On this, see Pritchett and Summers (1996) for a panel data study.

[17] One should notice that the influence of economic development - independently of public intervention - on survival conditions is usually difficult to quantify. While prosperity seems to lead to lower mortality at the micro-level (see Valkonen, 2002), economic development at the macro-level may imply a deterioration of health (see Easterlin, 1999; Sartor, 2002).

$$\phi_{t+1} = \frac{B\phi_t + \gamma \tilde{w}_t + \delta g_t}{1 + \gamma \tilde{w}_t + \delta g_t} \quad (11)$$

$$\lambda_{t+1} = \frac{R\lambda_t + \varepsilon \tilde{w}_t + \upsilon r_t}{1 + \varepsilon \tilde{w}_t + \upsilon r_t} \quad (12)$$

where $\tilde{w}_t \equiv (1-\tau)w_t$ is the net wage. B and R are habits parameters ($0 < B < 1$, $0 < R < 1$), γ and δ reflect the impacts of $\tilde{w}_t$ and g_t on ϕ_{t+1}, and ε and υ reflect the effects of $\tilde{w}_t$ and r_t on λ_{t+1}.

Besides their analytical conveniency, the postulated production functions for ϕ_{t+1} and λ_{t+1} exhibit some plausible properties: provided δ and υ are positive, public expenditures g_t and r_t have a positive but declining effect on ϕ_{t+1} and λ_{t+1} respectively. Moreover, public expenditures and the net wage are here supposed to be substitutes (e.g. the positive effect of g_t on ϕ_{t+1} is decreasing in $\tilde{w}_t$), which is supported by empirical evidence suggesting that public policy can serve as a substitute for private wealth (see Dreze and Sen, 1989).[18]

In this paper, we shall suppose that capital per worker has a globally positive influence on ϕ_{t+1} and λ_{t+1}, which amounts to assuming that the inequalities $(1-\tau)\gamma + \tau\delta\psi > 0$ and $(1-\tau)\varepsilon + \tau\upsilon(1-\psi) > 0$ are satisfied. Note that this assumption seems to be, in the light of the empirical literature, a quite plausible postulate. That assumption is compatible with a negative effect of $\tilde{w}_t$ on ϕ_{t+1} and λ_{t+1} (i.e. a negative γ and ε), as long as public intervention is sufficiently large and productive.

[18] See Bhattacharya and Qiao (2005) for a model where public policy and private wealth are complementary in the production of longevity.

3 Steady-state equilibrium

3.1 Existence

In order to study the existence of a non-trivial steady-state equilibrium, we can first define the kk, $\phi\phi$ and $\lambda\lambda$ loci in the 3-dimensional (k, ϕ, λ) space, and formulate the question of the existence of a non-trivial steady-state as the issue of whether those three loci intersect somewhere in the (k, ϕ, λ) space.

Fixing $k_{t+1} = k_t = k$ in the capital equilibrium equation (9) allows us to derive the kk locus, which gives us, for each combination of ϕ and λ, the value of k such that k is constant:

$$k = \left(\frac{(1-\tau)(1-\alpha)A\beta\phi\lambda}{(1+n)(1+\beta\phi\lambda)} \right)^{\frac{1}{1-\alpha}} \tag{13}$$

Similarly, fixing $\phi_{t+1} = \phi_t = \phi$ in the survival production function (11) allows us to derive the $\phi\phi$ locus:

$$k = \left(\phi \frac{1-B}{A(1-\phi)(1-\alpha)(\gamma(1-\tau)+\delta\tau\psi)} \right)^{\frac{1}{\alpha}} \tag{14}$$

Finally, fixing $\lambda_{t+1} = \lambda_t = \lambda$ within the longevity production function (12) allows us to derive the $\lambda\lambda$ locus:

$$k = \left(\lambda \frac{1-R}{A(1-\lambda)(1-\alpha)(\varepsilon(1-\tau)+\upsilon\tau(1-\psi))} \right)^{\frac{1}{\alpha}} \tag{15}$$

When the loci kk, $\phi\phi$ and $\lambda\lambda$ intersect not only at the origin, but, also, at another point in the space (k, ϕ, λ), the latter point corresponds to a non-trivial steady-state, that is, a state of the economy at which both k, ϕ and λ are constant over time. Such an intersection of the three loci is illustrated on Figure 4.[19]

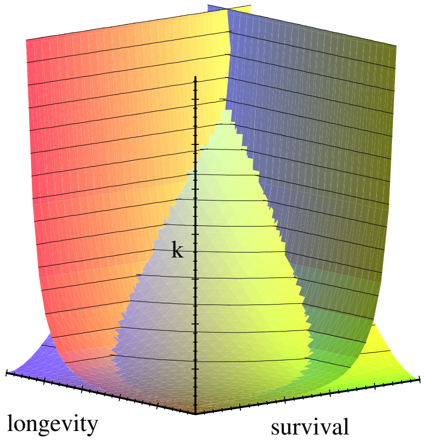


Figure 4: Existence

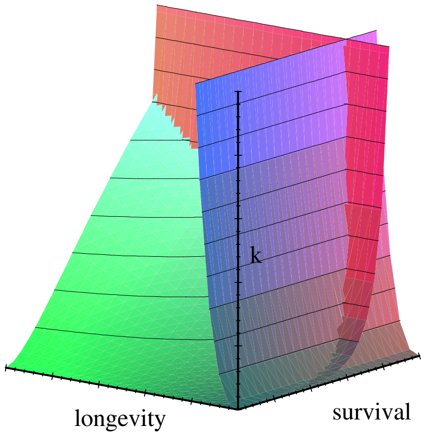


Figure 5: Non existence

Does such an intersection always exist? The answer to that question is negative. As this is shown on Figure 5, it might be the case that the three loci intersect only at the origin of axes, so that a non-trivial steady-state does not exist.

Let us now consider conditions guaranteeing the existence of a non-trivial steady-state equilibrium. As stated in Proposition 1, provided the total influence of capital per worker on ϕ and λ is positive [i.e. $(1-\tau)\gamma+\tau\delta\psi > 0$ and $(1-\tau)\varepsilon+\tau\upsilon(1-\psi) > 0$], the condition $\alpha < 1/3$ guarantees the existence of a non-trivial steady-state.[20]

Proposition 1 *Provided* $(1-\tau)\gamma + \tau\delta\psi > 0$ *and* $(1-\tau)\varepsilon + \tau\upsilon(1-\psi) > 0$*, a sufficient condition for the existence of a non-trivial steady-state is* $\alpha < 1/3$.

Given that the capital per worker k is likely to have a positive net effect on demographic variables ϕ and λ, the crucial part of Proposition 1 concerns the elasticity of output with respect to capital α. An elasticity lower than 1/3 implies that

[19] On Figures 4 and 5, parameters values are: $A = 20$, $\beta = 0.99$, $B = 0.5$, $\gamma = 0.1$, $\delta = 0.3$, $R = 0.5$, $\varepsilon = 0.1$, $\upsilon = 0.3$. For Figure 4: $\alpha = 0.5$, $\tau = 0.25$ and $\psi = 0.5$; For Figure 5: $\alpha = 0.7, \tau = 0.25$, and $\psi = 0.5$. Those values are purely illustrative.

[20] See Appendix A for the proof.

a non-trivial steady-state exists. On the contrary, if $\alpha \geq 1/3$, the existence of a non-trivial steady-state equilibrium is not guaranteed.

Note that $\alpha < 1/3$ is only a sufficient condition for the existence of a non-trivial steady-state, and not a necessary condition, so that economies with a higher α can also, under some circumstances, exhibit such a steady-state. That point is worth being underlined, as recent empirical estimates suggest that α is likely to be slightly higher than 1/3, and equal to 0.36 (see Ambler, 2000).

3.2 Uniqueness

Regarding the uniqueness of non-trivial steady-states, Proposition 2 states that the conditions guaranteeing the existence of a steady-state imply also its uniqueness.[21]

Proposition 2 *Provided* $(1-\tau)\gamma + \tau\delta\psi > 0$ *and* $(1-\tau)\varepsilon + \tau\upsilon(1-\psi) > 0$*, a sufficient condition for the uniqueness of a non-trivial steady-state is that* $\alpha < 1/3$.

Thus, under a globally positive effect of capital per worker on demographic variables ϕ and λ, the condition $\alpha < 1/3$ guarantees the existence and uniqueness of a non-trivial steady-state. On the contrary, if $\alpha \geq 1/3$, then neither the existence nor the uniqueness of the steady-state is guaranteed: a steady-state may not exist (as on Figure 5), or several non-trivial equilibria may exist (as on Figure 6).[22]

[21] See Appendix B for the proof.

[22] Parameters values are: $A = 20$, $\alpha = 0.4$, $\beta = 0.99$, $B = 0.5$, $\gamma = 0.1$, $\delta = 0.3$, $R = 0.5$, $\varepsilon = 0.1$, $\upsilon = 0.3.$, $\tau = 0.9$, and $\psi = 0.99$. Those values are purely illustrative.

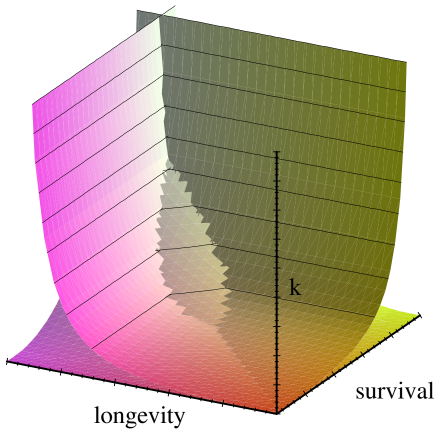


Figure 6: Multiple equilibria

The possible *multiplicity* of non-trivial steady-states is a desirable property of this model, as it is fully compatible with the empirical literature showing the large heterogeneity of historical evolutions in the (k, ϕ, λ) space (see Pritchett and Summers, 1996). Actually, panel data on production and survival conditions around the world show that, while a group of industrialized countries has, during the last decades, converged towards a steady-state exhibiting a high prosperity and favorable survival conditions, other countries have remained at low levels of production, and kept on exhibiting extremely bad survival conditions. Such a heterogeneity of historical evolutions cannot be explained by a model where the non-trivial steady-state is unique, because, in such a model, all economies must converge towards the same equilibrium. However, the present model, where multiple equilibria can, under $\alpha \geq 1/3$, occur, accounts for the heterogeneity in historical experiences.

3.3 Stability

Having discussed the existence and uniqueness of non-trivial steady-states, one should also notice that the non-trivial equilibrium, when it exists and is unique, is, under mild conditions, locally stable. Whereas the formal identification of the conditions under which the stability holds is left to the Appendix, one can have an idea of the stability properties of the steady-state by studying, on the phase diagram (Figure 4), the dynamics prevailing around the different loci.

While k must grow (resp. fall) if it lies below (resp. above) the kk locus, ϕ must grow (resp. fall) on the left (resp. right) of the $\phi\phi$ locus. Moreover, λ must increase (resp. decrease) if it is on the right (resp. left) of the $\lambda\lambda$ locus. Hence, the non-trivial equilibrium, if it exists and is unique, is likely to be locally stable. If, for instance, the economy is initially located at a point on the left of the $\phi\phi$ and kk loci, and on the right of the $\lambda\lambda$ locus, capital must fall, while both longevity and the probability of survival must grow, so that the economy will necessarily move downwards towards the kk locus, but, given that ϕ and λ are still growing in that area, the economy will move on the other side of the kk locus, and enter an area in which both k, ϕ and λ will grow, until an intersection of the three loci is reached, so that the non-trivial steady-state is locally stable.

When several equilibria exist, as on Figure 6, the intermediate equilibrium is unstable, because all dynamic forces around it point out towards either the other non-trivial steady-state, or, on the contrary, towards the origin of axes. This result is in conformity with the existence of *poverty traps*. The existence of such traps is hardly surprising: given that, under the assumptions made, longevity and capital accumulation tend to reinforce each others, it is crucial, if capital plays a substantial role in production, that initial survival conditions and capital are sufficiently high: otherwise, neither economic nor demographic take-off can take place.

4 Comparative statics

Let us now consider the influences of various parameters on the steady-state values of k, ϕ and λ. The signs of those effects are summarized on Table 1.

The parameters A, β and n, which affect the accumulation of capital directly, also influence the steady-state levels of ϕ and λ in the same directions. This indirect influence follows from the postulated endogeneity of ϕ and λ: capital accumulation has here a positive impact on ϕ and λ. However, while any factor favouring capital accumulation raises also ϕ^* and λ^*, whether the implied rise in limit-longevity exceeds the tendency towards rectangularization depends on whether k has a larger effect on ϕ or on λ. This is determined not only by the parameters in the production functions of ϕ and λ, but, also, by public policy parameters τ and ψ.

Moreover, the parameters entering the survival production function - i.e. γ, δ and B - do not only affect ϕ^* positively, but, also, *via* their positive effect on capital accumulation, λ^*. For instance, if preventive public expenditures against lung cancers become more productive in terms of ϕ (i.e. δ is raised), this will also raise k^* and λ^*. This indirect influence comes from the savings decision: more prevention against lung cancer increases ϕ, which, by increasing savings, leads to a higher output, and, *in fine,* to a higher longevity λ. The same kind of indirect effect holds for all parameters entering the production of λ. Hence, a major feature of this model is that the rise in limit-longevity and the rectangularization tend to reinforce each other indirectly, *via* their common relationship with capital accumulation.

Table 1: Comparative statics results

Parameter	Effect on k^*	Effect on ϕ^*	Effect on λ^*
A	$+$	$+$	$+$
β	$+$	$+$	$+$
n	$-$	$-$	$-$
B	$+$	$+$	$+$
γ	$+$	$+$	$+$
δ	$+$	$+$	$+$
R	$+$	$+$	$+$
ε	$+$	$+$	$+$
υ	$+$	$+$	$+$
τ	?	?	?
ψ	?	?	?

Finally, it should also be stressed that the influence of public policy parameters τ and ψ on the steady-state is ambiguous.

The unknown impact of τ follows from the fact that taxation has two opposite effects on capital accumulation: on the one hand, a higher τ, by shrinking the share of the wage available for savings, tends to slowdown capital accumulation; on the other hand, if one supposes that governmental intervention brings some extra-value as far as survival conditions are concerned, a higher τ leads to a rise in ϕ and λ, which promotes capital accumulation. Hence, those opposite effects imply that the impact of raising τ on steady-state capital is unknown. But, given that ϕ and λ are also influenced by the untaxed wage, the effect of τ on their levels is also unknown.

The impact of ψ on the steady-state is also ambiguous: in some cases, a higher proportion ψ of the government's budget allocated to survival expenditures g_t may affect the steady-state positively, whereas it may have, in other cases, a negative effect on not only $k^*$and λ^*, but, also, on ϕ^*. The reason why such a result may occur is that changing ψ amounts to moving the $\phi\phi$ and $\lambda\lambda$ loci in opposite directions (while the kk locus remains unchanged). Hence, given that the steady-state ϕ^* depends also on where the $\lambda\lambda$ locus lies, an optimal policy requires, as we shall see in Section 6, some kind of 'balance' between the two kinds of public programs, balance whose precise form depends on the parameters determining ϕ and λ.

5 The transition towards the steady-state

After having studied the long-run dynamics of the model, let us now explore its short-run dynamics. For that purpose, this Section examines numerically the capacity of the model to account for the observed succession of phases of rectangularization and derectangularization during the transition towards the steady-state.

As demographers emphasized (see Vallin and Berlinguer, 2001), the rectangularization and the rise in limit-longevity have not taken place to the same extents in all countries and all epochs. Recent studies of the evolution of survival curves over the last two centuries emphasized that the demographic history of a country can be generally decomposed into two successive *phases* (see Yashin *et al*, 2001). Firstly, a phase where the tendency towards rectangularization exceeds the rise in limit-longevity, so that the survival curve, although shifting to the right, becomes more rectangular. That phase is followed by a second one, where the rise in limit-longevity exceeds the rectangularization, so that the survival curve makes a quasi-parallel shift to the right. This constitutes the 'derectangularization' phase.

Demographers can explain the diversity of evolutions of survival curves across countries, by considering that all economies occupy different positions in that general dynamic framework, some countries being still in the first phase, whereas others have entered the second phase (e.g. France, United-States, Sweden).[23]

A virtue of our model is that it can replicate that two-phase transition, where

[23] See Yashin *et al* (2001).

a period of rectangularization is followed by a period of derectangularization.[24] Actually, two sets of assumptions allow us to reproduce numerically a transition exhibiting the succession of rectangularization and derectangularization phases.

One possibility is to assume that the global effect of prosperity on the proportion of survivors ϕ_{t+1} is much larger than its impact on limit-longevity λ_{t+1}. This amounts to assuming that the parameter γ in expression (11) exceeds significantly the parameter ε in (12), and that the parameter δ in (11) exceeds also the parameter υ in (12). Those assumptions are quite plausible: raising, by means of a public policy, the proportion of agents surviving the first period is an easier task than making people reach extremely high ages (i.e. $\delta > \upsilon$). Moreover, the 'natural' impact of prosperity on ϕ_{t+1} exceeds probably its impact on λ_{t+1} (i.e. $\gamma > \varepsilon$).

Figure 7 shows the evolution of ϕ_t, λ_t and life-expectancy $1 + \phi_t\lambda_t$ for the first 15 periods, starting from $k_0 = 1$, $\phi_0 = 0.4$ and $\lambda_0 = 0.5$ (implying, under a period of length 40 years, an initial life expectancy of 73 years). While the demographic parameters B and R are fixed to 0.8, it is assumed that γ, equal to 0.15, exceeds ε, equal to 0.01. Moreover, δ, equal to 0.25, exceeds significantly υ, equal to 0.02.[25]

During the first three periods, the substantial rise in the proportion of survivors ϕ_t exceeds the rise in the length of the second period λ_t. This corresponds to the first phase, during which rectangularization is dominant. Then, during the next 6 periods, the opposite occurs: the rise in λ_t exceeds the rise in ϕ_t: this is the derectangularization phase. Finally, after period 9, ϕ_t and λ_t are close to their steady-state levels, and there is neither a rectangularization, nor a derectangularization, as the survival curve remains constant over time. Hence, while the life expectancy $1 + \phi_t\lambda_t$ grows during periods 0 to 9, the forces at work vary over time:

[24] However, given that this model does not study infant mortality, this Section can only replicate the shifts of survival curves during the last century, but not before.

[25] Production parameters α and A are equal to 0.30 and 30, β is equal to 0.30 (equivalent to a subjective quarterly discount factor of 0.99). Fertility is supposed to be constant (i.e. $n = 0$). Policy parameters τ and ψ are fixed to 0.30 and 0.50.

during the first phase, the growth in life expectancy arises mainly thanks to the rise in ϕ_t, while the later growth is caused mainly by the rise in λ_t.[26]

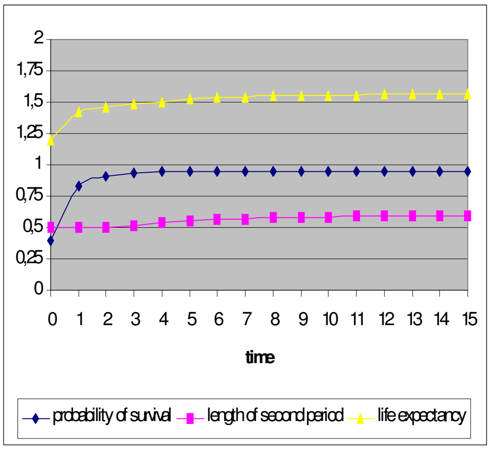


Fig. 7: Dynamics of ϕ, λ and $1 + \phi\lambda$

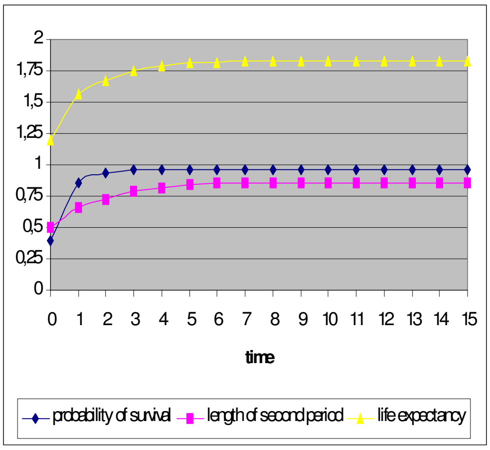


Fig. 8: Dynamics of ϕ, λ and $1 + \phi\lambda$

It is also possible to replicate the observed succession of phases of rectangularization and derectangularization under $\gamma \approx \varepsilon$ and $\delta \approx \upsilon$, by assuming a disequilibrium in the government's intervention in favour of policies promoting ϕ_t [i.e. $\psi >> 1/2$]. That disequilibrium is plausible, as public spending promoting survival at young ages exceed generally public spending promoting survival at very high ages.

This alternative way to replicate the succession of phases is illustrated on Figure 8, which shows the evolution of ϕ_t, λ_t and life-expectancy $1 + \phi_t\lambda_t$, starting from $k_0 = 1$, $\phi_0 = 0.4$ and $\lambda_0 = 0.5$. While γ and δ keep the same values as above, ε is fixed to 0.05, and υ is fixed to 0.10. Moreover, it is assumed that 95 % of the government's budget is dedicated to expenditures g_t (i.e. $\psi = 0.95$).[27]

[26] Note that the early gains of life expectancy were 'cheap', in the sense that these did not require a high level of capital. On the contrary, the larger difficulty to increase limit-longevity implies that it is only at high levels of k that λ can grow (unlike ϕ). Recent gains in life expectancy, caused mainly by a rise in λ, are thus more expensive than earlier gains, caused by rises in ϕ.

[27] Production parameters α and A are equal to 0.30 and 30, β is equal to 0.30. Fertility is supposed to be constant (i.e. $n = 0$). The tax rate τ is fixed to 0.30.

Figure 8 shows, during periods 0 and 1, a phase of dominant rectangularization (where the rise in ϕ_t exceeds the rise in λ_t), which is followed, during periods 2 to 6, by a phase of derectangularization (where the rise in λ_t exceeds the rise in ϕ_t).

6 Optimal public policy

Let us now examine the optimal public intervention. For that purpose, we shall assume that the fiscal capacity is fixed (i.e. the tax rate τ is given), and estimate the parameter ψ maximizing the steady-state utility of a representative agent.[28]

Although such a focus on the steady-state ignores all generations living during the transition, analyzing the optimal public policy from a long-run perspective is nonetheless relevant, because the economy will remain forever at that equilibrium once this is reached. Note that a focus on a representative agent is also a simplification: one may want, on ethical grounds, to depart from that average view.

Case A: neutral prosperity: $\gamma = \varepsilon$ Let us first consider the case where prosperity is 'neutral', in the sense that it would affect, in the absence of public intervention, the variables ϕ_{t+1} and λ_{t+1} in an *equal* manner. That assumption amounts to assuming that the net wage has the same effect on ϕ_{t+1} and λ_{t+1}.

Under that postulate, the optimal long-run ψ depends significantly on the extent to which the two public spending g_t and r_t affect respectively ϕ_{t+1} and λ_{t+1}. The sensitivity of ψ^* to the demographic parameters δ and υ is illustrated on Figure 9, which shows the avegare lifetime welfare at the steady-state as a function of ψ.[29]

Steady-state average lifetime welfare is a non-monotonic function of ψ, whose maximum depends on parameters δ and υ. If the two public policies are equally efficient at raising respectively ϕ_{t+1} and λ_{t+1} (i.e. $\delta = \upsilon$), the optimal ψ is 1/2.

[28] For that purpose, we shall fix $\alpha < 1/3$, to have a unique non-trivial steady-state.

[29] α and A are equal to 0.30 and 30, β is equal to 0.30. Fertility is constant (i.e. $n = 0$). The tax rate τ is fixed to 0.25. B and R are equal to 0.8, and $\gamma = \varepsilon = 0.05$.

On the contrary, if it is easier to raise the proportion of survivors rather than limit-longevity (i.e. $\delta > \upsilon$), ψ^* is higher than $1/2$, and growing with the δ/υ ratio. However, even if δ is three times larger than υ, the optimal ψ, equal to 0.86, remains smaller than unity, so that it is still desirable to dedicate a significant part of the budget to r_t. That result comes from the postulated utility function, which treats ϕ_{t+1} and λ_{t+1} in a *symmetric* way, so that some 'balance' is always required between the resources dedicated to ϕ_{t+1} and to λ_{t+1}.

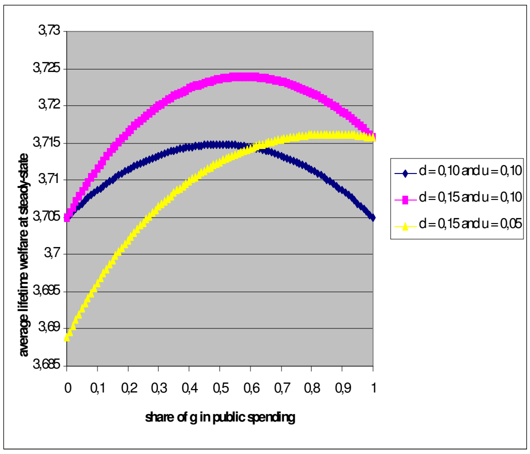


Fig. 9: Sensitivity of ψ^* to δ and υ

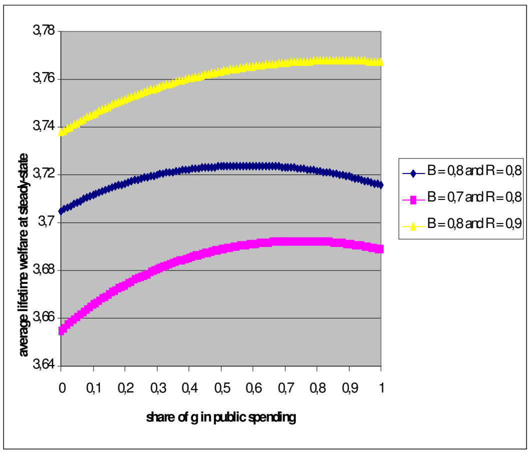


Fig. 10: Sensitivity of ψ^* to B and R

Whereas Figure 9 emphasized that differences in δ and υ affect ψ^* under an equal dependence of ϕ_{t+1} and λ_{t+1} on the past [i.e. $B = R$], that latter assumption is quite restrictive. Actually, the impact of the past is likely to be stronger for the determination of λ_{t+1} than for ϕ_{t+1}. For instance, some major causes of death for adults - lung cancers - can only be tackled by repeated preventive actions on the populations concerned, and, thus, can hardly be affected by past achievements, so that the influence of the past may, for ϕ_{t+1}, be lower than for λ_{t+1}.

As shown by Figure 10, once a differential in habits parameters is introduced in favour of λ_{t+1} [i.e. $B < R$], ψ^* becomes significantly higher: ψ^* equals 75 % under $B = 0.7$ and $R = 0.8$, against only 58 % under $B = R = 0.8$.[30] Moreover, under

[30] α and A are equal to 0.30 and 30, β is equal to 0.30. Fertility is constant (i.e. $n = 0$). The tax rate τ is fixed to 0.25. δ and υ are equal to respectively 0.15 and 0.10, and $\gamma = \varepsilon = 0.05$.

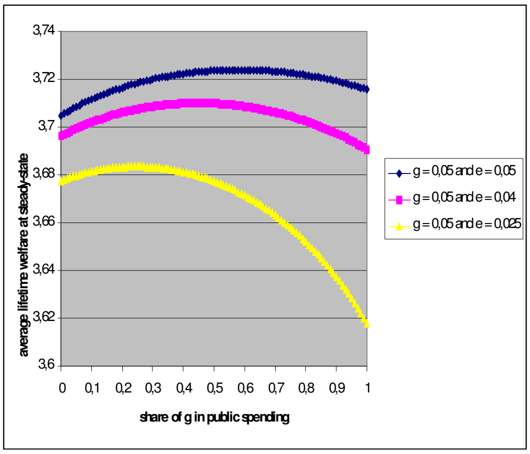
$B = 0.8$ and $R = 0.9$, the optimal long-run policy is to dedicate 87 % of the budget to g_t. Thus, the conjunction of a positive 'natural' influence of prosperity on λ_{t+1} with stronger customs makes public spending r_t less needed *ceteris paribus*.

Case B: non-neutral prosperity: $\gamma \neq \varepsilon$ While we assumed so far that prosperity would, without public intervention, affect ϕ_{t+1} and λ_{t+1} in a similar manner [i.e. $\gamma = \varepsilon$], such a neutrality lacks empirical support. Although it is hard to separate the contributions of each - private and public - factors on longevity, the 'natural' influence of prosperity is likely to be stronger on ϕ_{t+1} than on λ_{t+1}, as suggested by the historical fact that, at periods of limited public intervention, longevity progress began with a fall in mortality at low ages, before affecting mortality at older ages.[31]

As shown on Figure 11, reducing ε from 0.05 to 0.04 makes ψ^* fall from 58 to 45 %.[32] The tendency of ψ^* to fall when ε tends to 0 is not surprising: if the impact of the net wage on limit-longevity is very small, public intervention becomes the unique way to raise λ_{t+1}, so that the budget should be unbalanced in favour of r_t. Hence, ψ^* depends on the extent to which ϕ_{t+1} and λ_{t+1} can grow *without* governmental intervention. If one has good reasons to believe that λ_{t+1} can be less easily raised, without public intervention, than ϕ_{t+1}, this justifies a larger share of the budget to be spent on policies raising λ_{t+1}, even if the impact of r_t spending on λ_{t+1} is smaller than the impact of g_t on ϕ_{t+1} [i.e. $\delta > \upsilon$].

[31] See Vallin and Berlinguer (2002).

[32] α and A are equal to 0.30 and 30, β is equal to 0.30. Fertility is constant (i.e. $n = 0$). The tax rate τ is fixed to 0.25. δ and υ are equal to respectively 0.15 and 0.10, and $B = R = 0.8$.



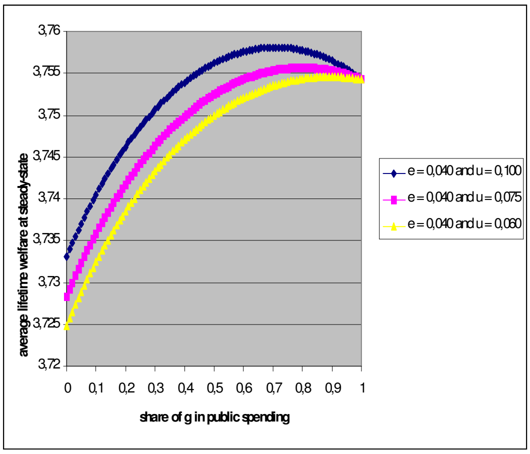


Fig. 11: Sensitivity of ψ^* to $\gamma$and ε Fig.12: Plausible values for ψ^*.

All in all, what is the optimal long-run public policy? Figure 12, which shows the average lifetime welfare at the steady-state for different combinations of parameters, aims at summarizing the above discussions.[33] All curves on Figure 12 assume that the net wage has, *ceteris paribus*, a slightly larger impact on ϕ_{t+1} than on λ_{t+1} [i.e. $\gamma = 0.05 > \varepsilon = 0.04$], and that habits have a larger influence on λ_{t+1} than on ϕ_{t+1} [i.e. $B = 0.8 < R = 0.9$]. However, these differ regarding the impact of public health spending: whereas δ is fixed to 0.15, υ equals 0.100, 0.075 and 0.060.

As shown on Figure 12, reducing the impact of spending r_t on λ_{t+1} increases significantly ψ^*, from 71 % under $\upsilon = 0.100$ to 89 % under $\upsilon = 0.060$. However, even when the extra-value brought by spending r_t in terms of λ_{t+1} is small, the optimal policy is still to dedicate a significant part of the public budget to expenditures promoting limit-longevity (11 %).

Hence, despite the plausible lower impact of expenditures r_t and the larger strength of habits in the determination of λ_{t+1} - which both support a high ψ^*- the mere fact that public health expenditures r_t can bring a significant extra-value in terms of λ_{t+1} in comparison with the *laissez-faire* suffices to rule out extremely unbalanced public policies favouring expenditures g_t over spending r_t.

[33] On Figure 12, α and A are equal to 0.30 and 30, β is equal to 0.30. Fertility is constant (i.e. $n = 0$). The tax rate τ is fixed to 0.25.

To sum up, this numerical exercise tends to question the optimality, from a long-run perspective, of extremely unbalanced public budgets favouring spending raising the proportion of survivors over spending enhancing limit-longevity.

7 Conclusions

Whereas existing OLG models with endogenous longevity do generally not distinguish between the rectangularization of survival curves and the rise of limit-longevity, this paper developed a two-period OLG model, where the probability of survival to the second period, and the length of that second period, are influenced - to various extents - by economic prosperity and by distinct public policies.

It was shown that, even if prosperity has a globally positive effect on each longevity dimension, the take-off of an economy towards a steady-state with a high prosperity and a high longevity is not always guaranteed: poverty traps may exist, depending on the importance of capital in the production process. This theoretical result is compatible with the empirical evidence of rising differentials between the economic and demographic achievements of rich and poor countries.

Another key feature of this model is that it is able to replicate the observed succession, over time, of two demographic phases: firstly, a phase where the tendency towards rectangularization exceeds the rise in limit-longevity, secondly, a phase where the rise in limit-longevity dominates the rectangularization process.

This paper showed also that the optimal long-run policy - i.e. maximizing steady-state average lifetime welfare - could hardly be extremely unbalanced between survival-enhancing policies and programs raising limit-longevity.

Finally, it should be stressed that this paper could be extended in several ways. First, one may assume an asymmetric treatment of ϕ and λ in agents' preferences, which would influence the optimal policy. One may also extend public intervention, by introducing pensions systems.[34] Hence, much work is to be done in the future.

8 References

Aisa, R. & Pueyo, F. (2004): "Endogenous longevity, health and economic growth: a slow growth for a longer life?", *Economics Bulletin*, 9 (3), pp. 1-10.

Ambler, S. (2000): "Optimal time consistent fiscal policy with overlapping generations", *CREFE Working Paper* 111, University of Montréal.

Anand, S. & Ravallion, M. (1993): "Human development in poor countries: on the role of private income and public services", *Journal of Economic Perspectives*, 7 (Winter), pp. 133-150.

Bhattacharya, J. & Qiao, X. (2005): "Public and private expenditures on health in a growth model", *Iowa State University Working Paper*, #05020.

Bommier, A. (2003): "Risk aversion, intertemporal elasticity of substitution and correlation aversion", *LEA Working Paper*, # 0307.

Boucekkine, R., de la Croix, D. & Licandro, O. (2002) : "Vintage human capital, demographic trends, and endogenous growth", *Journal of Economic Theory*, 104, pp. 340-375.

Chakraborty, S. (2004): "Endogenous lifetime and economic growth", *Journal of Economic Theory*, 116, pp. 119-137.

de la Croix, D. & Licandro, O. (1999) : 'Life expectancy and endogenous growth', *Economics Letters*, 65, pp. 255-263.

Dreze, J. & Sen, A.K. (1989): *Hunger and Public Action*, WIDER Studies in Development Economics. Oxford: Clarendon Press.

Easterlin, R. (1999): "How benevolent is the market? A look at the modern history of mortality", *European Review of Economic History*, 3, pp. 257-294.

Ehrlich, I. & Lui, F.T. (1991): "Intergenerational trade, longevity, and economic growth", *Journal of Political Economy*, 99, pp. 1029-1060.

[34] See Pestieau *et al* (2006) for a first step in that direction.

Jeune, B. & Kannisto, V. (1997): "Emergence of centenarians and super-centenarians", in Robine, J.-M., Vaupel, J.W., Jeune, B. & Allard, M. (Eds.): *Longevity: To the limits and Beyond*, Springer, Berlin, pp. 77-89.

Kannisto, V. (2000): "Measuring the compression of mortality", *Demographic Research*, 3, 6, (available online at http://www.demographic-research.org)

Lichtenberg, F.R. (2002): "Sources of U.S. longevity increase, 1960-1997", *NBER Working Paper* n°8755.

Maddison, A. (2001): *The World Economy. A Millenial Perspective.* Paris: OECD.

Medio, A. & Lines, M. (2001): *Nonlinear Dynamics. A Primer.* Cambridge: Cambridge University Press.

Meslé, F. (2002): "Les causes médicales de décès", in G. Caselli, J. Vallin & G. Wunsch (eds.) *Démographie : analyse et synthèse, volume III : Les déterminants de la mortalité.* Paris: Editions de l'Institut National d'Etudes Démographiques.

Nusselder, W.J. & Mackenbach, J.P. (2000): "Lack of improvement of life expectancy at advanced ages in the Netherlands", *International Journal of Epidemiology*, 29, pp. 140-148.

Pecchenino, R.A. & Utendorf, K.R. (1999): "Social security, social welfare and the aging population", *Journal of Population Economics*, 12, 4, pp. 607-623.

Pestieau, P., Ponthiere, G. & Sato, M. (2006) : "Longevity and pay-as-you-go pensions", *CORE Discussion Paper* 54.

Preston, S.H. (1975): "The changing relation between mortality and level of economic development", *Population Studies*, 29, 2, pp. 231-248.

Preston, S.H. (1980): "Causes and consequences of mortality declines in less developed countries during the twentieth century", in R.A. Easterlin (ed.): *Population and Economic Changes in Developing Countries.* Chicago: Chicago Press.

Pritchett, L. & Summers, L.H. (1996): "Wealthier is healthier", *Journal of Human Resources*, 31, 4, pp. 841-868.

Sartor, F. (2002) : "Les facteurs environnementaux de la mortalité", in G. Caselli, J. Vallin

& G. Wunsch (eds.): *Démographie : analyse et synthèse, volume III : Les déterminants de la mortalité*. Paris: Editions de l'Institut National d'Etudes Démographiques.

Soliani, L. & Lucchetti, E. (2002) : "Les facteurs de la mortalité", in G. Caselli, J. Vallin & G. Wunsch (eds.): *Démographie : analyse et synthèse, volume III : Les déterminants de la mortalité*. Paris: Editions de l'Institut National d'Etudes Démographiques.

Valkonen, T. (2002): "Les inégalités sociales devant la mort", in Caselli, G., Vallin, J. & Wunsch, G. (eds.): *Démographie : analyse et synthèse, volume III: Les déterminants de la mortalité*. Paris: Editions de l'Institut National d'Etudes Démographiques.

Vallin, J. & Berlinguer, G. (2002): "De la mortalité endogène aux limites de la vie humaine", in G. Caselli, J. Vallin & G. Wunsch (eds.): *Démographie : analyse et synthèse, volume III*. Paris: Editions de l'Institut National d'Etudes Démographiques.

Vallin, J., Caselli, G. & Surault, P. (2002) : "Comportements, styles de vie et facteurs socio-culturels de la mortalité", in G. Caselli, J. Vallin & G. Wunsch (eds.): *Démographie : analyse et synthèse, volume III*. Paris: Editions de l'Institut National d'Etudes Démographiques.

Wilmoth, J.R. & Horiuchi, S. (1999): "Rectangularization revisited: variability of age at death within human populations", *Demography*, 36, 4, pp. 475-495.

Wilmoth, J.R., Deegan, L., Lundstrom, H. & Horiuchi, S. (2000): "Increase in the maximum life-span in Sweden, 1861-1999", *Science*, 289, 29 (september), pp. 2366-2368.

Yashin, A.I., Begun, A.S., Boiko, S.I., Ukraintseva, S.V., & Oeppen, J. (2001): "The new trends in survival improvement require a revision of traditional gerontological concepts", *Experimental Gerontology*, 37, pp. 157-167.

Zhang, J., Zhang, J. & Lee, R. (2001): "Mortality decline and long-run economic growth", *Journal of Public Economics*, 80, pp. 485-507.

9 Appendix

9.1 Appendix A: Existence of steady-states

In order to derive the conditions guaranteeing the existence of a non-trivial steady-state, let us first rewrite the $\phi\phi$ locus as:

$$\phi = \frac{k^{\alpha}V}{k^{\alpha}V + 1 - B} \tag{16}$$

where $V \equiv A(1-\alpha)(\gamma(1-\tau)+\delta\psi\tau)$. Given the assumptions $A > 0$, $0 < \alpha < 1$ and $\gamma(1-\tau)+\delta\psi\tau > 0$, it follows that V must be strictly positive.

In a similar way, the $\lambda\lambda$ locus can be written as:

$$\lambda = \frac{k^{\alpha}W}{k^{\alpha}W + 1 - R} \tag{17}$$

where $W \equiv A(1-\alpha)(\varepsilon(1-\tau)+\upsilon(1-\psi)\tau)$. Here again, $W > 0$.

Then, substituting these two expressions in the kk locus yields:

$$k = \left(\frac{A\beta(1-\alpha)(1-\tau)\left(\frac{k^{\alpha}V}{k^{\alpha}V+1-B}\right)\left(\frac{k^{\alpha}W}{k^{\alpha}W+1-R}\right)}{(1+n)\left(1+\beta\left(\frac{k^{\alpha}V}{k^{\alpha}V+1-B}\right)\left(\frac{k^{\alpha}W}{k^{\alpha}W+1-R}\right)\right)} \right)^{\frac{1}{1-\alpha}} \tag{18}$$

If we denote the right-hand side as the function $H(k)$, the question of the existence of an equilibrium becomes the search for a fixed point of $H(k)$, that is, a value of k such that $H(k) = k$. It is clear that 0 is a fixed point. But we would like to derive conditions guaranteeing the existence of another fixed point.

For that purpose, we shall follow the method proposed in Chakraborty (2004) and show that the function $H(k)$ satisfies the properties (i)-(iv), that is:

(i) $H(0) = 0$

(ii) $H'(k) \geq 0 \; \forall k \in [0, \bar{k}]$

(iii) $\lim_{k \to \bar{k}} H(k)/k < 1$

(iv) $\lim_{k \to 0} H'(k) = +\infty$ if $\alpha < \alpha^*$.

(i)

$$H(0) = \left(\frac{A\beta(1-\alpha)(1-\tau)00}{(1+n)} \right)^{\frac{1}{1-\alpha}} = 0 \tag{19}$$

(ii) To show this, we shall assume that there exists an upper bound steady-state k, denoted by $\bar{k}$, i.e. a value of k that the steady-state capital never exceeds. Given that the postulated Cobb-Douglas production function satisfies the Inada conditions, there can be no doubt that such an upper bound exists. Moreover, that upper bound is independent from ϕ and λ. Throughout this Appendix, we shall take $\bar{k} = \left(\frac{A\beta(1-\alpha)(1-\tau)}{(1+n)(1+\beta)} \right)^{\frac{1}{1-\alpha}}$

In the interval $[0, \bar{k}]$, the first-order derivative of H with respect to k has, under mild conditions, the desired sign. Indeed, $H'(k)$ is positive if and only if:

$$\frac{1 + \beta \left(\frac{k^\alpha V}{k^\alpha V + 1 - B} \right) \left(\frac{k^\alpha W}{k^\alpha W + 1 - R} \right)}{\beta \left(\left(\frac{k^\alpha V}{k^\alpha V + 1 - B} \right) \left(\frac{k^\alpha W}{k^\alpha W + 1 - R} \right) \right)} > 1 \tag{20}$$

which is always true.

(iii) When k tends to $\bar{k}$, the $\lim_{k \to \bar{k}} H(k)/k$ is:

$$\lim_{k \to \bar{k}} H(k)/k = \frac{\left(\frac{A\beta(1-\alpha)(1-\tau)(\lim_{k \to \bar{k}} \phi)(\lim_{k \to \bar{k}} \lambda)}{(1+n)(1+\beta(\lim_{k \to \bar{k}} \phi)(\lim_{k \to \bar{k}} \lambda))} \right)^{\frac{1}{1-\alpha}}}{\left(\frac{A\beta(1-\alpha)(1-\tau)}{(1+n)(1+\beta)} \right)^{\frac{1}{1-\alpha}}} < 1 \tag{21}$$

given that ϕ and λ are bounded by 1 (i.e. their values when $k \to +\infty$), the above ratio remains inferior to 1 as required.

(iv) The $\lim_{k \to 0} H'(k)$ is:

$$\frac{\left(0^{\frac{3\alpha-1}{1-\alpha}}\right)\left[\frac{C^{\frac{1}{1-\alpha}}}{1-\alpha}\left(\frac{(\alpha VW)^{\frac{1}{1-\alpha}}((1-B)(0^{\alpha}W+1-R)+(1-R)V(0^{\alpha}V+1-B))}{(0^{\alpha}V+1-B)^{\frac{2-\alpha}{1-\alpha}}(0^{\alpha}W+1-R)^{\frac{2-\alpha}{1-\alpha}}}\right)\right]}{(1+n)^{\frac{1}{1-\alpha}}\left(1+\beta\frac{0^{2\alpha}VW}{(0^{\alpha}V+1-B)(0^{\alpha}W+1-R)}\right)^{\frac{2-\alpha}{1-\alpha}}} \tag{22}$$

It is straightforward to see that $H'(k)$ tends to $+\infty$ provided $\alpha < \alpha^* = 1/3$.

Hence, in the light of conditions (i)-(iv), it appears that, under $\alpha < 1/3$, a non-trivial equilibrium must exist: the function $H(k)$ must intersect the 45° line at least once on the interval $\left[0, \bar{k}\right]$. However, for $\alpha \geq 1/3$, one cannot be sure that a non-trivial equilibrium exists.

Regarding the uniqueness issue, one can see, from the above expression, that, under $\alpha < 1/3$, $H''(k)$ is strictly negative, so that H is concave, which implies that the non-trivial equilibrium must be, under that condition, unique.

9.2 Appendix B: Stability of steady-states

While the discussion of the stability of equilibria carried out in Section 3 in the light of phase diagrams suggested that non-trivial equilibria are likely to be stable (except the intermediate equilibrium), a more formal study of the stability is nonetheless required, because phase diagrams can sometimes simplify the picture significantly.

To discuss the stability of equilibria, let us first notice that the present system is non-linear, so that the conventional analysis of the Jacobian matrix (composed of the first-order derivatives of dynamic equations with respect to state variables) can only inform us on the stability of equilibria *provided* these are hyperbolic. Actually, if a fixed-point is hyperbolic, the Hartman-Grobman Theorem states that the stability of the linearized system (or its non-stability) implies the local stability of the non-linear system (or its non-stability) (see Medio and Lines, 2001). However, if the fixed-point is not hyperbolic, then the analysis of the linearized system does not allow us to draw any conclusion on the local stability of the non-linear system.

As stated in Medio and Lines (2001), fixed-points are, in discrete-time systems, hyperbolic if none of the eigenvalues of the Jacobian matrix, evaluated at the equilibrium, is equal to 1 in modulo.

To discuss the hyperbolicity of non-trivial equilibria, let us first compute the Jacobian matrix associated with the model of Section 2:

$$J \equiv \begin{pmatrix} \frac{\partial M(k_t,\phi_t,\lambda_t)}{\partial k_t} & \frac{\partial M(k_t,\phi_t,\lambda_t)}{\partial \phi_t} & \frac{\partial M(k_t,\phi_t,\lambda_t)}{\partial \lambda_t} \\ \frac{\partial N(k_t,\phi_t,\lambda_t)}{\partial k_t} & \frac{\partial N(k_t,\phi_t,\lambda_t)}{\partial \phi_t} & \frac{\partial N(k_t,\phi_t,\lambda_t)}{\partial \lambda_t} \\ \frac{\partial O(k_t,\phi_t,\lambda_t)}{\partial k_t} & \frac{\partial O(k_t,\phi_t,\lambda_t)}{\partial \phi_t} & \frac{\partial O(k_t,\phi_t,\lambda_t)}{\partial \lambda_t} \end{pmatrix} \tag{23}$$

where $M(k_t, \phi_t, \lambda_t)$, $N(k_t, \phi_t, \lambda_t)$ and $O(k_t, \phi_t, \lambda_t)$ denote:

$$k_{t+1} = M(k_t, \phi_t, \lambda_t) \equiv \frac{(1-\alpha)(1-\tau)}{1+n} A k_t^{\alpha} \frac{\beta\phi_t\lambda_t}{1+\beta\phi_t\lambda_t} \tag{24}$$

$$\phi_{t+1} = N(k_t, \phi_t, \lambda_t) \equiv \frac{B\phi_t + (1-\alpha)Ak_t^{\alpha}(\gamma(1-\tau)+\delta\psi\tau)}{1+(1-\alpha)Ak_t^{\alpha}(\gamma(1-\tau)+\delta\psi\tau)} \tag{25}$$

$$\lambda_{t+1} = O(k_t, \phi_t, \lambda_t) \equiv \frac{R\lambda_t + (1-\alpha)Ak_t^{\alpha}(\varepsilon(1-\tau)+\upsilon(1-\psi)\tau)}{1+(1-\alpha)Ak_t^{\alpha}(\varepsilon(1-\tau)+\upsilon(1-\psi)\tau)} \tag{26}$$

$M(k_t, \phi_t, \lambda_t)$, $N(k_t, \phi_t, \lambda_t)$ and $O(k_t, \phi_t, \lambda_t)$ are obtained, respectively, from substituting for the wage in expression (5), and from substituting for the net wage and for public health expenditures in expressions (11) and (12).

The relevant first-order derivatives are:

$$\frac{\partial M(k_t,\phi_t,\lambda_t)}{\partial k_t} = \frac{(1-\alpha)(1-\tau)}{(1+n)} A\alpha k_t^{\alpha-1} \frac{\beta\phi_t\lambda_t}{1+\beta\phi_t\lambda_t} > 0$$

$$\frac{\partial M(k_t,\phi_t,\lambda_t)}{\partial \phi_t} = \frac{(1-\alpha)(1-\tau)}{(1+n)} A k_t^{\alpha} \frac{\beta\lambda_t}{(1+\beta\phi_t\lambda_t)^2} > 0$$

$$\frac{\partial M(k_t,\phi_t,\lambda_t)}{\partial \lambda} = \frac{(1-\alpha)(1-\tau)}{(1+n)} A k_t^{\alpha} \frac{\beta\phi_t}{(1+\beta\phi_t\lambda_t)^2} > 0$$

$$\frac{\partial N(k_t,\phi_t,\lambda_t)}{\partial k} = \frac{(1-B\phi_t)(1-\alpha)A\alpha((1-\tau)\gamma+\delta\psi\tau)k_t^{\alpha-1}}{(1+Ak_t^{\alpha}((1-\tau)\gamma+\delta\psi\tau)(1-\alpha))^2} > 0$$

$$\frac{\partial N(k_t,\phi_t,\lambda_t)}{\partial \phi} = \frac{B}{1+Ak_t^{\alpha}((1-\tau)\gamma+\delta\psi\tau)(1-\alpha)} > 0$$

$$\frac{\partial N(k_t,\phi_t,\lambda_t)}{\partial \lambda} = 0$$

$$\frac{\partial O(k_t,\phi_t,\lambda_t)}{\partial k} = \frac{(1-R\lambda_t)(1-\alpha)A\alpha((1-\tau)\varepsilon+\upsilon(1-\psi)\tau)k_t^{\alpha-1}}{(1+Ak_t^{\alpha}((1-\tau)\varepsilon+\upsilon(1-\psi)\tau)(1-\alpha))^2} > 0$$

$$\frac{\partial O(k_t,\phi_t,\lambda_t)}{\partial \phi} = 0$$

$$\frac{\partial O(k_t,\phi_t,\lambda_t)}{\partial \lambda} = \frac{R}{1+Ak_t^{\alpha}((1-\tau)\varepsilon+\upsilon(1-\psi)\tau)(1-\alpha)} > 0$$

An equilibrium is hyperbolic if no eigenvalue of the Jacobian matrix, evaluated at the equilibrium, is equal to 1 in modulo. A sufficient condition for this hyperbolicity condition is that the trace of the Jacobian matrix (i.e. the sum of eigenvalues) is positive but strictly lower than 1, and that the determinant of the Jacobian matrix (i.e. the product of eigenvalues) is positive but strictly lower than one.

Actually, it is easy to see that those conditions are also sufficient conditions for the local stability of the equilibrium, which requires that all eigenvalues of the Jacobian matrix are strictly lower than 1 in modulo.

Those two conditions imply that, at the equilibrium (k^*, ϕ^*, λ^*):

$$0 < \alpha + \frac{B}{1+k^{*\alpha}V} + \frac{R}{1+k^{*\alpha}W} < 1 \text{ (*)}$$

and

$$0 < \alpha\frac{RB}{(1+k^{*\alpha}V)(1+k^{*\alpha}W)}\left[1 - \frac{1}{1+\beta\phi^*\lambda^*}\left(\frac{1-B}{B} + \frac{1-R}{R}\right)\right] < 1 \text{ (**)}$$

Regarding condition (*), the positivity of the sum in question is obvious. Hence (*) can be reformulated as:

$$\alpha < 1 - \frac{B}{1+k^{*\alpha}V} - \frac{R}{1+k^{*\alpha}W}$$

which is likely to be satisfied when α is low, and when the equilibrium is reached at a high level of capital.

Regarding (**), the positivity constraint can be rewritten as:

$$1 > \frac{1}{1+\beta\phi^*\lambda^*}\left[\frac{1-B}{B}+\frac{1-R}{R}\right]$$

which is, here again, a plausible condition if the equilibrium under study is characterized by sufficiently high steady-state ϕ^* and λ^*.

Regarding the second part of condition (**), it implies that:

$$\alpha\frac{RB}{(1+k^{*\alpha}V)(1+k^{*\alpha}W)}\left[1-\frac{1}{1+\beta\phi^*\lambda^*}\left(\frac{1-B}{B}+\frac{1-R}{R}\right)\right] < 1$$

Given that the first two factors are always smaller than 1, and that the third factor, if positive (see above), is also smaller than one (because it is equal to 1 minus something positive), the condition requires that the third factor is not too large. Here again, this condition is likely to prevail, especially when k^* is high, so that the steady-state is likely to be locally stable.

To conclude, the present discussion of the stability of the equilibrium suggests that this stability cannot be fully guaranteed in all cases. However, as this was shown, the equilibrium is likely to be stable when it corresponds to a sufficiently high combination of k^*, ϕ^* and λ^*. The reason for this is simple: being distant from the other equilibrium - i.e. the (0, 0, 0) point - insures stability, whereas an equilibrium that is close to the (0, 0, 0) is likely to be unstable.