Carry Trades and Speculative Dynamics*

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Abstract

When a currency trader borrows Japanese yen at 1 percent to fund the purchase of US dollar assets that yield 5 percent, the trader makes a profit unless the dollar depreciates. We examine how such "carry trades" can create speculative dynamics in foreign exchange markets. We develop a dynamic asset pricing model in which speculative dynamics can be ruled out in the absence of funding externalities. The additional assumption that carry traders create positive funding externalities for each other changes the nature of the price dynamics drastically. Not only does uncovered interest parity fail, but a currency with a high interest rate will exhibit the classic price pattern of "going up by the stairs, and coming down in the elevator".

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1 Introduction

A currency carry trade is constructed by selling a low interest rate currency to fund the purchase of a high interest rate currency - that is, by selling a currency forward that is at a significant forward premium. The "yen carry trade" in particular has been the focal point of debate over the last decade or more given the extended period of low interest rates in Japan. The failure of uncovered interest parity is clearly a precondition for the success of a carry trade. If the low interest rate currency were to appreciate relative to the high interest rate currency as predicted by uncovered interest parity, then any gain on the interest rate differential from the carry trade will be exactly offset by the capital loss resulting from the exchange rate movement, leaving the carry trader no better off.

A popular view among market commentators is that the failure of uncovered interest parity is not only a *pre-condition* for carry trades, but may also be a *consequence* of carry trades. A typical quote is the one below from the Economist magazine[1].

> "One obvious possibility is that the actions of carry traders are self-fulfilling; when they borrow the yen and buy the dollar, they drive the former down and the latter up."

Brunnermeier, Nagel, and Pedersen (2008) offer evidence which suggests that carry trades can play a significant role in exchange rate dynamics.[2] The idea that the profit from carry trades may be self-confirming was put forward by Froot and Thaler (1990), but recent experiences in the foreign exchange market have garnered many more adherents to such a view among

[1] *Carry on Speculating*, The Economist magazine, February 22nd 2007

[2] We discuss their findings in detail in Section 5.

market participants. The reasoning behind such a view starts with the observation that most central banks set official overnight interest rates mainly with domestic considerations in mind, rather than the external exchange rate environment. Inflation-targeting central banks, for instance, set their policy interest rate in response to the prospects for domestic inflation. When official interest rates are held fixed by central banks in this way, the wedge in interest rates that opens up across currencies allows a carry trader to book a profit even if the exchange rate were to remain constant over time by collecting on the flow payoff arising from the interest rate differential. Even if the carry trader calculates that the exchange rate will move against him, the expected length of time over which exchange rate will adjust is crucial in the trader's calculations. Even if the exchange rate will move against him *eventually*, if the adjustment is slow enough, then it may be profitable to enter a carry trade.

But then, this is a recipe for the exchange rate to deviate further from the predictions of uncovered interest parity. If a trader believes that the dollar will remain strong against the yen, the optimal response would be to buy the dollar and sell the yen. If such beliefs were widespread, the actions of traders acting on their beliefs will put upward pressure on the value of the dollar relative to the yen. In this way, a collective view that the dollar will remain strong against the yen will set in motion trades that will validate such a view. The belief in the success of carry trades can thus become self-fulfilling, and the failure of uncovered interest parity becomes the *consequence* of carry trades.

This paper develops a theoretical model in which the scenario painted above is the unique equilibrium outcome. We develop our theory on two premises. The first is that exchange rates are sensitive to the underlying flow of funds into or out of a currency, at least in the short run. Our second premise is that even though exchange rates may deviate from fundamentals in the short run, there is nonetheless some long term "fundamental anchor" that prevents exchange rates being completely decoupled from economic fundamentals. Both premises have empirical bases. For instance, Evans and Lyons (2002) find a significant impact of net order flows on currency returns. Froot and Ramadorai (2005) outline the evidence both for the short term dependence of prices on flows as well as how fundamentals exert themselves in the long-run.

Our benchmark result rests on these two premises alone. The result is a stark one. Based on these two premises alone, our benchmark result establishes the *opposite* of our main result. Anticipating that the fundamentals will exert themselves eventually, speculators hasten the return of exchange rates to the level implied by fundamentals, so that speculation serves to keep the exchange rate tightly tethered to the fundamentals. In this sense, speculation is stabilizing in the way that Friedman (1953) envisaged. In fact, under the benchmark case, stabilizing speculation is not only the unique equilibrium of the trading game, it is the only possible outcome that is consistent with common knowledge that speculators are rational.

The impossibility of speculative dynamics in our benchmark case serves the purpose of highlighting the role of funding externalities in our main result. To establish our main result on speculative dynamics, we posit that each trader's funding constraints are relaxed when many other speculators are already engaged in the carry trade. If carry traders create such positive funding externalities for each other, the stabilizing nature of speculation can be tipped over into a mutually reinforcing mode of speculation in which my incentive to engage in the carry trade is enhanced when others also engage in it. The action to engage in the carry trade become strategic complements across traders, rather than strategic substitutes.

Our final step is to refine the equilibrium in the trading game with funding externalities by introducing uncertainty in the evolution of fundamentals. We draw on the tools developed by Frankel and Pauzner (2000) and Burdzy, Frankel and Pauzner (2001) to solve dynamic coordination games. When combined with the strategic complementarity of actions, we derive price paths for the exchange rate as a unique, dominance solvable equilibrium of the trading game. In this equilibrium, extended periods of slow appreciations of the high rate currency are stochastically punctuated by endogenous crashes. Currency traders refer to such patterns as "going up by the stairs and coming down in the elevator" (see Breedon, 2001).

Related Literature

Our paper belongs to the strand of theoretical literature which shows that in the presence of positive externalities among speculators, rational speculation can be destabilizing. De Long et al. (1990) have shown that irrational "positive feedback trading" creates such externalities and can lead rational speculators to grow bubbles. More recently, Bernardo and Welch (2004) and Morris and Shin (2004) have examined market crashes in a static context, where traders may be induced to sell in anticipation of others selling. For Bernardo and Welch, such episodes are called "market runs" in analogy with bank runs. For Morris and Shin (2004), they borrow the term "liquidity black holes" used by practitioners. Closest to our approach is the contribution of Abreu and Brunnermeier (2003), who provide a formalization of the bursting of a bubble in a stock market in terms of the endogenous transition from one price path to another one.

Our main contribution to this literature is to offer a model in which purely static funding externalities generate entire speculative price paths. Namely, both the slow build-up of speculative carry trades and the subsequent sudden reversal caused by their unwindings are endogenous, and the equilibrium is unique. In other words, we offer a simple theoretical description of the slow rise and sudden burst of the "crowded trades" that seem to have played an important role in recent financial crises.

The plan for the rest of the paper is as follows. We begin in the next section by outlining our baseline model in which speculation can be ruled out in quite a strong sense. In section 3, we incorporate carry costs and funding externalities and show how the basic character of the model changes abruptly into one where speculative trading become strategic complements across players, rather than strategic substitutes. In section 4, we introduce our main model with stochastic fundamentals, which will be instrumental in refining the outcome in the game to a unique, dominance-solvable equilibrium. Our model generates new qualitative predictions on the relationship between the size of the carry trade, the path of the exchange rate, and the probability of a currency crash. Section 5 discusses the existing empirical evidence. Section 6 concludes.

2 Baseline Model

Time is continuous and is indexed by $t \in [0, +\infty)$. There are two assets. One asset is denominated in Japanese yen and serves as the *numéraire*. For the benchmark model, we may think of the numeraire asset as a Japanese yen deposit. The other asset is U.S. dollar denominated, and we may construe this second asset as a U.S. dollar deposit. The price (in terms of yen assets) of the dollar-denominated asset at date t is denoted p_t. There is a mass m of risk-neutral traders who faces the binary choice of holding either one dollar asset or p_t yen assets. We denote by x_t the mass of traders who hold dollars at date t.

Our baseline model rests on two key features. The first is that fund flows affect currency returns. In particular, we will examine the consequences of return dynamics of the form:

$$\frac{\dot{p}_t}{p_t} = k\left(\frac{\dot{x}_t}{m}\right) \tag{1}$$

where $k > 0$. In other words, the dollar appreciates in proportion to the rate at which traders move into the dollar asset and out of the yen asset. Accordingly, the price p_t is normalized to

$$p_t = e^{\frac{kx_t}{m}}.$$

We are not wedded to any particular microfoundations for (1), but one possible rationalization is in terms of the interactions between a brokerage sector that supply passive demand and supply curves which form the backdrop for the active trading decisions of the speculators. In Appendix A, we provide one possible simple microfoundation for (1) in such terms.

We now turn to the second key feature of our model. Although the short term price movements are dictated by the flows, we will suppose that there is a fundamental anchor to the asset price p_t. In the foreign exchange context, the fundamental anchor as given, say, by purchasing power parity, may be quite weak. But however weak the anchor is, the forward-looking behavior of the traders will serve to make its presence felt throughout the analysis. We model the fundamental anchor by assuming that there is a fundamental value v that is common knowledge among all traders, and that with a Poisson arrival rate ρ, the value of the dollar asset snaps back to v and remains there forever. The idea here is similar to the notion of a "day of reckoning" in Duffie, Gârleanu, and Pedersen (2002) on which an exogenous event reveals the value of future consumption generated by the dollar asset to all market participants. The assumption that the price remains at v forever once it has snapped back to v is offered as a simplification. Our focus is on how traders behave *in anticipation* of this anchor to the fundamental. Consumption takes place only at the day of reckoning.

Traders face a small friction in how often they can trade. A speculator can only trade at discrete designated trading dates that are generated by a Poisson process with intensity λ. The processes are independent across traders, so that a fraction λdt of the traders gets a chance to trade between t and $t + dt$. This small friction may be interpreted as the time it takes for a hedge fund to structure a large deal with prime brokers, or for a proprietary trader to clear internal risk controls before a large trade. In a very active market such as the FX market, we would expect the traders to have a free hand in trading, which corresponds to a large λ. Let x_t denote the mass of traders who are invested in dollars at date t. This mass has the following dynamics:

$$\begin{cases} \dot{x}_t = -\lambda x_t & \text{when traders sell the dollar} \\ \dot{x}_t = \lambda \left(m - x_t\right) & \text{when traders buy the dollar} \end{cases}$$

This departure from continuous trading strategies is the key feature of the model that warrants equilibrium uniqueness in Section 3.

At trading date t, a trader who holds the dollar asset faces a binary decision - to keep it or to sell it for p_t yen assets. For a trader who does not already hold the dollar asset, the binary decision is either to buy it at price p_t, or to maintain her yen holdings. At the time of making a decision, the trader can condition on the realized price path as well as the calendar date

t. Thus, the trading strategy of a trader is a mapping:

$$\left(t, (p_u)_{u \leq t}\right) \mapsto \{\text{dollar asset, yen asset}\} \tag{2}$$

that specifies whether a trader will hold dollars or yens for all pairs of dates and price histories.

Dominance Solvable Outcome

Our baseline model allows us to draw a very strong conclusion - starting from any price p_0, the price until the day of reckoning returns to the fundamental value v at the fastest possible rate. Any other outcome can be ruled out by the iterated deletion of strictly dominated strategies. The purpose of the benchmark analysis is to accentuate the contrast with our main result on the possibility of speculative dynamics.

Suppose that the price is p_t. The most pessimistic scenario for the holder of the dollar asset is that all future traders either switch out of it, or refrain from buying it so that the price path is declining over time. Under this most pessimistic scenario, the price path is given by $\{p_{t+u}\}_{u \geq 0}$, where

$$\frac{\dot{p}_{t+u}}{p_{t+u}} = -\frac{\lambda k x_{t+u}}{m}. \tag{3}$$

The price converges to 1, as each trader whose trading date arrives switches out of the dollar asset.

Even under this most pessimistic scenario, there is a price at which a trader is better off holding the dollar asset than the yen asset. Consider a speculator who has a chance to trade at date t. If the price path from date t onward is given by $\{p_{t+u}\}_{u \geq 0}$ then the expected excess rate of return on the dollar is:

$$\int_0^\infty \frac{\lambda\left(p_{t+u}-p_t\right)+\rho\left(v-p_t\right)}{p_t}e^{-(\lambda+\rho)u}du. \quad (4)$$

Thus, if the future price path is given by $\{p_{t+u}\}_{u\geq 0}$, the trader buys the dollar asset or holds on to it whenever (4) is greater than 0.

Note that the price dynamics (3) implies that

$$\begin{aligned}\frac{p_{t+u}-p_t}{p_t} &= \int_0^u \frac{\dot{p}_{t+s}}{p_t}ds \\ &= \int_0^u -\frac{\lambda k}{m}x_t e^{-\lambda s}.\frac{p_{t+s}}{p_t}ds \geq -\frac{kx_t}{m}\left(1-e^{-\lambda u}\right).\end{aligned}$$

Plugging this inequality into (4) shows that the expected excess return under this scenario is larger than

$$\frac{-\lambda^2 kx_t}{m\left(\lambda+\rho\right)\left(2\lambda+\rho\right)}+\frac{\rho}{\lambda+\rho}\frac{v-p_t}{p_t} \quad (5)$$

As $x_t \to 0$, the first term in (5) becomes small, while the second term tends to

$$\frac{\rho}{\lambda+\rho}\left(v-1\right)>0.$$

Thus there exists $\underline{x^0}\in(0,1)$ such that (5) is strictly positive for all $x_t \leq \underline{x^0}$. Thus the excess return on the dollar is positive in this pessimistic scenario if the price p_t is lower than $\underline{p}^0 = e^{\frac{k\underline{x}^0}{m}}$. But then, the most pessimistic price path given by (3) is *too pessimistic* in that it assumes that some future traders may choose dominated actions. By ruling out trading strategies that are dominated the most pessimistic price path now becomes:

$$\left\{\max\left(\underline{p}^0, e^{\frac{kx_te^{-\lambda u}}{m}}\right)\right\}_{u\geq 0} \quad (6)$$

Since (6) implies strictly higher prices than (3) beyond some date in the future, we can define a new threshold price given by $\underline{p}^1$ below which holding yen is dominated. Clearly, $\underline{p}^0 \leq \underline{p}^1$. If the price is below $\underline{p}^1$, the trader will not hold yen. Thus, any trading strategy in which a trader chooses the yen asset at a price below $\underline{p}^1$ is ruled out after *two* rounds of deletion of dominated strategies.

We can iterate this argument. After $n+1$ rounds of deletion of dominated strategies, the most pessimistic price path starting from p_t is given by:

$$\left\{ \max\left(\underline{p}^n, e^{\frac{kx_t e^{-\lambda u}}{m}} \right) \right\}_{u \geq 0}$$

This sets a new threshold $\underline{p}^{n+1}$ for the trading strategy, in which choosing yen for any price below $\underline{p}^{n+1}$ is ruled out by $n+2$ rounds of deletion of dominated strategies. We thus obtain the increasing sequence:

$$\underline{p}^0 \leq \underline{p}^1 \leq \underline{p}^2 \leq \cdots \leq \underline{p}^n \leq \cdots$$

Since price is bounded above, this sequence converges to some limit, denoted by $\underline{p}$. No trader will choose yen below $\underline{p}$ in any rationalizable outcome, since such an action is ruled out by iterated dominance. Thus, $\underline{p}$ constitutes a floor for the price of the dollar asset in any price path $\{p_{t+u}\}_{u \geq 0}$.

Analoguously, we can define a *decreasing* sequence of thresholds that corresponds to the most *optimistic* price paths that are consistent with n rounds of deletion of dominated strategies. If the price is sufficiently close to the upper bound e^k, then yen is strictly preferred since the price will never rise sufficiently to compensate for the risk that it could possibly fall to its fundamental value v. Let $\bar{p}^0$ be the price above which selling is dominant. Thus, the price path will never rise above this level. We can then iterate the argument to derive the decreasing sequence:

$$\bar{p}^0 \geq \bar{p}^1 \geq \bar{p}^2 \geq \cdots$$

Denote by $\bar{p}$ the limit of this sequence. This limit would constitute a ceiling for any price path. Clearly,

$$\underline{p} \leq \bar{p}. \tag{7}$$

We will now show that the reverse inequality must hold, too. Consider the floor price $\underline{p}$. We must have $\underline{p} \geq v$. To see this, suppose (for the sake of argument) that $\underline{p} < v$. Since no trader sells dollars below $\underline{p}$, the future path $\{p_{t+u}\}_{u \geq 0}$ lies on or above $\underline{p}$. Thus, conditional on a price $\underline{p}$, the expected return on the dollar asset is *strictly* greater than one since all possible future values of the asset are larger than $\underline{p}$ and $\underline{p} < v$. But this contradicts the fact that $\underline{p}$ is the upper limit of dominance thresholds. Buying the dollar would still be a strictly dominant strategy for some $p_t > \underline{p}$ in this case. Hence, we must have

$$\underline{p} \geq v. \tag{8}$$

From an exactly analoguous argument, we conclude that $v \geq \overline{p}$. Thus, we have

$$\underline{p} \geq v \geq \overline{p} \tag{9}$$

From (9) and (7), we conclude that $\underline{p} = \overline{p} = v$. We have thus proved the following.

Proposition 1 *In any subgame, the only trading strategy that survives the iterated deletion of dominated strategies is to hold the dollar asset when* $p_t \leq v$ *and hold the yen asset when* $p_t > v$.

Corollary 2 *In the unique equilibrium price path in the subgame that starts with price* p_t*, the price converges to the fundamental value at the maximum speed that trading constraints allow for.*

Our baseline model is in line with the hypothesis of the stabilizing role of speculation, as argued by Friedman (1953). No matter how loose the anchor is to the fundamentals (a loose anchor corresponds to a small ρ), the speculative behavior of traders pushes the price to coincide with the fundamentals.

The iterative argument presented above rests on the price space being bounded, which may be regarded as an unappealing feature. However, we stress that the main purpose of our paper is to show that speculative dynamics is possible even with forward-looking traders. Having a bounded price space makes our task that much harder. Hence, the bounded nature of the price space could be seen as a feature that adds to the appeal of our main result, which follows in section 4.

In contrast to our main result, the impossibility of speculation given by our result above can be understood as the resolution of two competing externalities generated by the predecessors of the date t trader. As the predecessors throw more "weight of money" into the dollar asset, there are two effects. First, the positive externality is that the future resale values $(p_{t+u})_{u\geq 0}$ will be high, other things being equal. But the negative externality is of course that the dollar asset is currently expensive. Because of the risk that the dollar asset reverts to its fundamental value, the negative externality ultimately wins out. Thus, a trader has no incentive to join in pushing the price away from its fundamental value. Instead, the trader will seek to trade against her predecessors to bring the price back into line with fundamentals. When λ is large and ρ small, fundamental risk is small compared to the risk that other speculators create an adverse price move. In this case, the competition between positive and negative externalities is more even, in the sense that additional positive externalities tip the balance toward conditions that are more fertile to the emergence of destabilizing speculation, as we see now.

3 Funding Externalities

We add to our baseline model two features that capture realistic aspects of yen carry trades. First, we introduce an interest rate differential between the dollar and the yen. For simplicity, assume that the yen asset yields zero in interest, and that the dollar asset pays interest at the rate δ. The interest is converted into yen at the exchange rate prevailing at the time it is earned.[3]

Second, we take into account that speculation requires capital. When she enters a carry trade at date t (i.e. borrows yen to obtain a dollar asset) an investor needs to tie up some capital. In particular, only a fraction of the dollar asset equal to

$$1 - h$$

where $h \in (0, 1)$ can be financed by the sale of yen assets. The remaining fraction h has to be financed by the trader's own capital.

This feature of our model captures the haircut that a prime broker would require as collateral from the speculator. Our key assumption is that *the capital requirement h decreases with respect to the dollar value p.* In other words, the banks that lend to carry traders have a procyclical risk management policy. Thus, leverage on the carry trade increases when prices are buoyant. This assumption follows Brunnermeier and Pedersen (2007), who describe a variety of practical situations in which the margin requirements of leveraged speculators have this procyclical feature. Adrian and Shin (2007) document evidence of procyclical lending by prime brokers, both from aggregate data and from individual broker balance sheet data. We revisit the

[3]Introducing an interest smaller than δ paid by the yen asset would be straightforward, but would not offer any additional insight.

empirical basis for this assumption in section 5. Formally, we denote $h(x_t)$ the capital requirement corresponding to a dollar value $p_t = e^{\frac{kx_t}{m}}$, where $h(.)$ is a continuously decreasing function.

A plausible explanation for this property of the capital requirement is that the lenders are less informed than the speculators, and thus do not know if an increase in p_t is due to a speculative flow or reflects some fundamental news. In the latter case, the collateral value of the dollar asset is enhanced in their eyes. If more cash in the market implies a possible higher collateral value of the trade in the eyes of the financiers, then speculators' leverage should increase with respect to p_t. An explicit modelling of such funding frictions is beyond the scope of this paper. Rather, we take this feature as given and study its impact on exchange rate dynamics.

Capital requirements are updated for each trader at each trading date. Thus, after entering a carry trade at date t, the trader subsequently incurs an opportunity cost of $\Delta h(x_t) \cdot p_t$ yen per unit of time until the next trading date (or the day of reckoning if it occurs before) because she ties up an amount $h(x_t) \cdot p_t$ of her own funds in the trade. In this new environment with financing constraints, the expected profit from a one dollar carry trade is the sum of two terms, as given below in (10).

$$\begin{aligned} &\overbrace{\int_0^{+\infty} (\lambda p_{t+u} + \rho v)\, e^{-(\lambda+\rho)u} du - p_t}^{\text{capital gain or loss due to exchange rate fluctuations}} \\ &+ \underbrace{\int_0^{+\infty} (\lambda+\rho)\, e^{-(\lambda+\rho)u} \left(\int_0^u (\delta p_{t+s} - \Delta h(x_t) \cdot p_t)\, ds \right) du}_{\text{cumulative carry minus cost of capital}} \end{aligned} \tag{10}$$

The first term in (10) is the expected capital gain or loss from movements in the exchange rate, where exchange rate movement can come either from the endogenous dynamics of the exchange rate or from the arrival of the day of reckoning. The second term is the expected accumulated flow payoff from the carry element minus the expected accumulated cost of tying up capital. Note that traders do not default on their yen debt, and absorb all capital losses even if they exceed the capital requirement. The alternative assumption that traders have an option to default with impunity, and without reputational costs would reinforce our results by adding a risk-shifting motive for the carry trade.

The fact that the capital requirement on a carry trade decreases with p_t implies that speculators create additional positive externalities for each other by entering carry trades. We now show that such positive externalities may dramatically change the underlying strategic incentives as compared to the baseline model.

Proposition 3

1. A price $p = p(x)$ solving

$$\delta = \Delta h(x) - \rho \left(\frac{v - p}{p} \right)$$

is a steady state.

2. Assume

$$\Delta h(m) < \delta < \Delta h(0)$$

If λ is sufficiently large and ρ sufficiently small, then there are multiple steady states starting from any price p_t. In particular, there is both a steady state in which all traders enter the carry trade after t, and also a steady state in which traders exit the carry trade after t.

Proof. See the appendix.

In the statement of proposition 3, "enter the carry trade" is a shorthand for the statement that a trader enters the carry trade if she is not already engaged in the carry trade, or maintains the carry trade if she is already engaged in the carry trade. Similarly, "exit the carry trade" means that the trader exits the carry trade if she is engaged in the carry trade, and does not enter if she is not already so engaged.

To gain an intuition for this result, we can decompose (10) into three terms. First, there is the capital gain or loss due to the endogenous movement in the exchange rate. Second, there is the capital gain or loss due to the arrival of the day of reckoning. Finally, there is the accumulated carry minus cost of capital. When λ is large and ρ small, traders engaging in the carry trade do not expect the day of reckoning to occur before they have a chance to unwind the trade. Thus they worry less about fundamental mispricing, unless possibly when the exchange rate p_t is very high relative to its fundamental value v. However, in this case, the flow payoff from the carry element is large also, since $h(.)$ is decreasing in x_t. Thus, it may be possible to sustain a path where entering the carry trade becomes mutually reinforcing, even though there is fundamental risk that the exchange rate reverts to its fundamental value. A similar reasoning yields the existence of self-justified unwindings of the carry trade.

Analyzing the situation in which λ tends to infinity is instructive. If $\lambda \rightarrow +\infty$, we get closer to a single-shot game with simultaneous moves between the traders. The two extreme steady states (all enter, all exit) resemble Nash equilibria of this one-shot game between the traders. The fact that the two extreme steady states resemble multiple equilibria in the single-shot game suggests that trading decisions are strategic complements - that is, the more other traders enter, the greater my incentive is to enter (and conversely, the greater the other traders exit, the more I want to exit). Thus, the strategic incentives become inverted, as compared to the benchmark case. We commented after our benchmark Proposition in the previous section that the reason why speculation is stabilizing comes from the fact that the *negative* externalities created by previous buyers outweigh the *positive* externalities. In Proposition 3, the roles are reversed. The positive externality of raising the price higher is larger than the negative externality. This is because for λ large and ρ small, fundamental risk over the horizon of a trade is so small that the positive funding externalities always offset the risk of holding an overvalued asset.

When funding constraints create such strategic complementarities, the price path itself will influence expected payoffs, and we cannot come to any firm conclusions regarding predictable outcomes without additional argument. In general, we can envisage very complicated dynamic strategies that try to balance the negative and positive externalities between traders, and we cannot say much more without additional structure on the problem. Rather than going further in investigating complex dynamics, we will now go in a different direction. We will now examine what happens when the carry itself is stochastic.

4 Stochastic Fundamentals

It turns out that the multiplicity of equilibria in the previous section is not robust to the addition of some variation in the carry δ. Adding (possibly arbitrarily small) shocks on δ, we obtain a unique dominance-solvable equilibrium. We draw on the work of Burdzy, Frankel and Pauzner (2001) and Frankel and Pauzner (2000), who showed that in binary action coordination games with strategic complementarities, the addition of small stochastic shocks to the fundamentals of the payoffs generates a unique, dominance solvable outcome. The arguments in these papers are similar to the global game arguments of Carlsson and van Damme (1993) and Morris and Shin (1998). We return to an interpretation of the results later in the paper.

Formally, we assume in this section that the carry obeys the process:

$$\delta_t = \delta + \sigma W_t,$$

where W_t is a standard Brownian motion, and $\sigma > 0$.[4]

In order to apply the mathematical framework developed by Burdzy, Frankel and Pauzner, we need to impose that the sensitivity of the haircut $h(.)$ to dollar value be sufficiently large. The particular requirement on the $h(\cdot)$ function can be stated in terms of the following inequality:

$$\min_x \{-h'(x)\} > \frac{ke^k}{2m}\left(h(0) + \frac{(2\lambda + k)}{2\Delta}e^k\right). \tag{11}$$

The exact role of this condition will appear shortly. The main result of the paper is the following:

Proposition 4

Suppose condition (11) holds. Then, if ρ is sufficiently small, there is a decreasing function $Z(.)$ such that in any subgame starting at date t with a carry δ_t and an exchange rate p_t, there is a unique, dominance solvable solution to the trading game. In this solution, a trader who trades at date t engages in the carry trade if and only if $\delta_t \geq Z(p_t)$.

It is interesting to contrast Proposition 4 with the results reported in Proposition 3. With a deterministic carry, Proposition 3 states that there are

[4] We could also add a stochastic interest on the yen asset and a random walk component to the log-return of the currency without modifying the results.

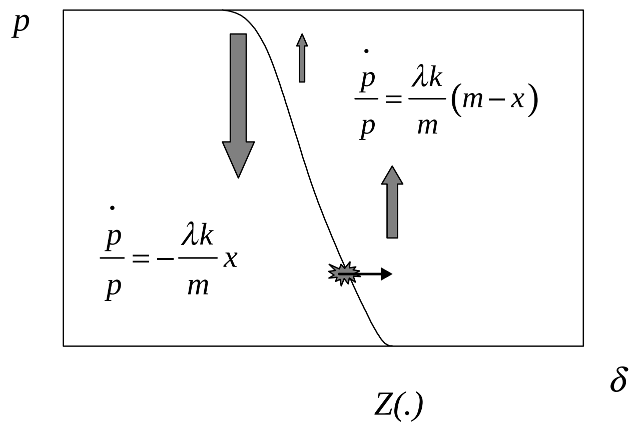

Figure 1: Unique equilibrium with stochastic δ_t

values of δ for which there are multiple equilibria starting from any price p_t. Adding Brownian shocks to δ, *even arbitrarily small*, changes this picture, since Proposition 4 implies that for ρ sufficiently small, the equilibrium is unique, and dominance-solvable.

Proposition 4 can be illustrated in figure 1. The curve $Z(p_t)$ divides the (δ, p)-space into two regions. Proposition 4 states that in the unique equilibrium, any trader decides to enter the carry trade to the right of the $Z(.)$ curve, and exit the carry trade to the left of the $Z(.)$ curve. Thus, p_t will tend to rise in the right hand region, and tend to fall in the left hand region, as indicated by the arrows in figure 1.

The dynamics of the flow of funds implied by the unique equilibrium is given by:

$$dx_t = 1_{\left\{\delta_t > Z\left(e^{\frac{kx_t}{m}}\right)\right\}} \lambda(m - x_t)dt - 1_{\left\{\delta_t < Z\left(e^{\frac{kx_t}{m}}\right)\right\}} \lambda x_t dt. \quad (12)$$

where $1_{\{.\}}$ denotes the indicator function that takes the value 1 when the condition inside the curly brackets is satisfied. These processes are known as *stochastic bifurcations,* and are studied in Bass and Burdzy (1999) and Burdzy et al. (1998). From Theorem 1 in Burdzy et al. (1998), for a given initial x_0, and for almost every sample path of δ, there exists a unique Lipschitz solution $(x_t)_{t\geq 0}$ to the differential equation (12) defining the price dynamics for Z Lipschitz decreasing.

Some suggestive features of the price dynamics can be seen from figure 1. When the dollar has appreciated for a while so that p_t is close to e^k, the rate of return if the currency appreciates is given by

$$\frac{\dot{p}}{p} = \frac{\lambda k}{m}(m - x) \simeq 0.$$

However, if the price crosses the Z boundary, the rate of depreciation is

$$\frac{\dot{p}}{p} \simeq -\lambda k$$

In other words, when p is high and the currency crosses the Z boundary from above, there is a sharp depreciation that was preceded by a slow appreciation. Such dynamics are suggestive of the price paths of high-yielding currencies in carry trades that "go up by the stairs and come down in the elevator".

To make our paper self-contained, we provide a proof of Proposition 4 that follows closely the argument given by Frankel and Pauzner (2000) for their discussion of binary coordination games. The difference between our setup and the game studied in Frankel and Pauzner (2000) is that viewed from date t, the future instantaneous profits at date $t + u$ depend on p_{t+u}, but also on p_t (see expression (10)). Their proof applies in a very similar fashion, however, provided ρ is sufficiently small.

Let $Z_0(.)$ denote the function such that if she believes that the other traders will exit the carry trade after t, a date-t trader enters the carry trade if and only if

$$\delta_t \geq Z_0(p_t).$$

We start with the following result.

Lemma 5

$Z_0(.)$ is Lipschitz and nonincreasing for ρ sufficiently small.

Proof. See the appendix.

Condition (11) is important in the proof of Lemma 5. It implies that the expected return on the carry trade increases as x_t increases, holding beliefs about future buy and sell orders constant. This is because condition (11) implies that as x rises, the gain of a reduced cost of carry overcomes the expected loss caused by a higher downside risk and a smaller upside risk.

Relaxing condition (11) would imply that the function $Z_0(.)$ can be increasing for some prices and decreasing for others. Addressing this case raises the mathematical question of the existence and uniqueness of Lipschitz price paths in this case, that we are unable to answer. Condition (11) warrants that $Z_0(.)$ is decreasing, which implies that the proof developed by Frankel and Pauzner (2000) applies to our setup.

Refer now to figure 2. Ruling out any strategy in which the trader holds yen to the right of Z_0, we can derive a boundary Z_1 for the second-round dominance region which indicates the region where it is dominant to hold dollar in the absence of any first-round dominated trading strategies. In other words, if she knows that other traders hold dollars at least when they are on the right of Z_0 at their trading dates, a trader will be willing to hold dollar at least when she is on the right of Z_1. We have

Lemma 6

If ρ is such that $Z_0(.)$ is Lipschitz and nonincreasing, then so is $Z_1(.)$. The Lipschitz constant of $Z_1(.)$ is smaller than the Lipschitz constant of $Z_0(.)$.

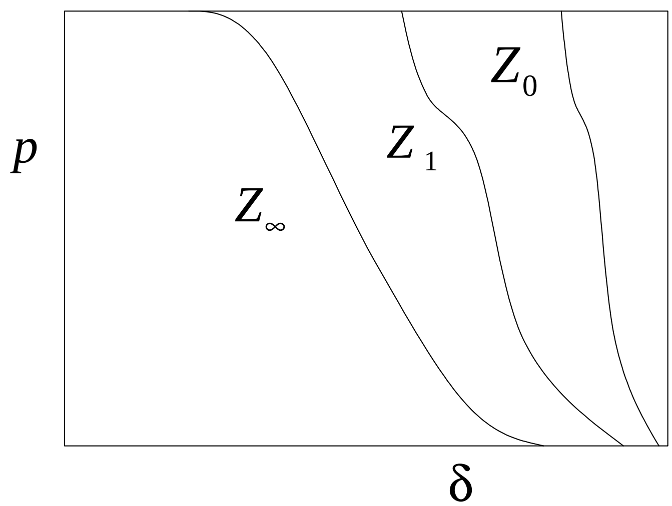

Figure 2: Iterative dominance from right. The curves Z_i are recursively defined as follows. Z_i is such that, if $\delta \geq Z_i(p)$, then a trader is willing to hold dollar if she believes that other traders hold dollar when they are on the right of Z_{i-1}.

Proof. See the appendix.

By iterating this process, we can obtain the boundary Z_∞ for the region where a trader holding yen can be eliminated by iterated dominance. Z_∞ is decreasing Lipschitz as a limit of decreasing Lipschitz functions with decreasing Lipschitz constants. The boundary Z_∞ defines an equilibrium strategy since, if all traders hold yen to the left and hold dollar to the right, the indifference point between dollar and yen for the trader also lies on Z_∞.

Consider now a translation to the left of Z_∞ so that the whole of the curve lies in a region where holding yen is dominant. Call this translation Z_0'. To the left of Z_0', holding yen is dominant. Then construct Z_1' as the *rightmost translation* of Z_0' such that a trader must choose yen to the left of Z_1' if she believes that other traders will play according to Z_0'. By iterating this process, we obtain a sequence of translations to the right of Z_0'. Denote by Z_∞' the limit of the sequence. Refer to figure 3. The boundary Z_∞' does not necessarily define an equilibrium strategy, since it was constructed as a translation of Z_0'. However, we know that if all others were to play according to the boundary Z_∞', then there is at least one point A on Z_∞' where the trader is indifferent between holding yen and holding dollar. If there were no such point as A, this suggests that Z_∞' is not the *rightmost* translation, as required in the definition.

We claim that Z_∞' and Z_∞ coincide exactly. The argument is by contradiction. Suppose that we have a gap between Z_∞' and Z_∞. Then, choose point B on Z_∞ such that A and B have the same height - i.e. have the same second component. But then, since the shape of the boundaries of Z_∞' and Z_∞ are identical, the stochastic bifurcation process starting from A must have the same distribution over payoffs as the process starting from B. Thus, the uncertainty governing the expected payoffs are identical at points A and

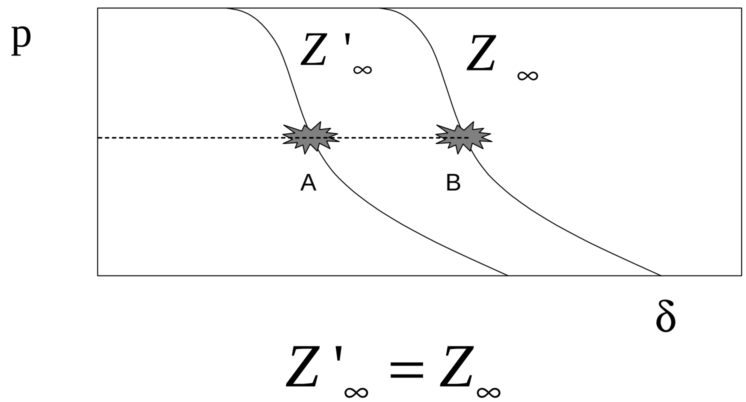

Figure 3: If a trader is in A and thinks that other traders enter the carry trade if and only if they are to the right of Z'_∞, then future price trajectories will just be horizontal translations of the trajectories realized when a trader is in B and thinks that other traders enter the carry trade if and only if they are to the right of Z_∞. Thus a trader can be indifferent between both situations only if A and B correspond to the same δ and thus $Z_\infty = Z'_\infty$.

B, except for the fact that B has a higher current value δ_t. This contradicts the hypothesis that a trader is indifferent between the two actions both at A and at B. If she were indifferent at A, she would strictly prefer to hold dollar at B, and if she is indifferent at B, she would strictly prefer to hold yen at A. But we constructed A and B so that traders are indifferent. Thus, there is only one way to make everything consistent, namely to conclude that $A = B$. Thus, there is no "gap", and we must have $Z'_\infty = Z_\infty$. In other words, we have the situation depicted in figure 1 as claimed.

Interpreting the Results

Proposition 4 demonstrates the impact of adding some uncertainty to the carry δ_t. The multiplicity of equilibria reported in the previous section resulted from the feature that, if the fundamentals were fixed and known, then one cannot rule out all other players trading in one direction, provided that the fundamentals were consistent with such a strategy. However, the introduction of shocks changes the picture radically. Trades are far less nimble than the shifts in the carry. Thus, choosing to enter the carry trade *versus* exiting the carry trade entails a substantial degree of commitment over time to fix one's trading strategy. Suppose that the (δ, p) pair is close to a dominance region, but just outside it. If δ is fixed, it may be possible to construct an equilibrium for both actions, but when δ moves around stochastically, it may wander into the dominance region between now and the next opportunity that the trader gets to trade. This gives the trader some reason to hedge her bets and take one course of action for sure. But then, this shifts out the dominance region, and a new round of reasoning takes place given the new boundary, and so on. Essentially, adding shocks to the carry enables us to extend to the two dimensional space of (δ, p) pairs the dominance argument we showed in our benchmark result without funding externalities in section 1.

That $Z(.)$ is nonincreasing implies that price paths exhibit hysteresis. If the dynamic system (δ_t, p_t) is in the area where buying is dominant $(\delta_t > Z(p_t))$, then the buy pressure takes the system away from $Z(.)$, making the continuation of a bullish market even more likely, all else equal. The reader may wonder whether Brownian excursions completely swamp this effect at the proximity of $Z\,(.)$, so that runs never develop and the system is "trapped" in the vicinity of $Z(.)$. The next proposition shows that it is not the case provided σ is sufficiently small.

Proposition 7

Assume that the system is in the state (p_t, δ_t) *such that*

$$\delta_t = Z\left(p_t\right),$$

and that ρ is such that Proposition 4 *applies. For any $\varepsilon > 0$, as $\sigma \to 0$, the last time at which the system hits $Z(.)$ before x_{t+u} becomes larger than $m - \varepsilon$ or smaller than ε tends to t in distribution. The probability that the price will go up tends to*

$$\frac{m - x_t}{m}.$$

Proof Note that for a given ρ sufficiently small, Proposition 4 guarantees the existence of the frontier $Z(\sigma, .)$ for any $\sigma > 0$, since all that is used in the proof if the fact that $\sigma \neq 0$. Thus, we can apply Theorem 2 in Frankel, and Pauzner (2000).■

The broad intuition for this result is that when σ is small, the price path around $Z(.)$ is mostly driven by changes in x_t: Fund flows are more important than changes in the carry δ_t. The speed at which the price goes up is $\frac{\lambda k}{m}(m - x_t)$, while it decreases with speed $-\frac{\lambda k}{m} x_t$. The price path does not revert to $Z\,(.)$ once it has headed off towards one direction, and the ratio of the probabilities to go up or down is the ratio of the speeds at which the price goes in each direction. If the system hits $Z\,(.)$ when p_t is very high (low), then it is most likely to bifurcate downwards (upwards).

In other words, if σ is sufficiently small, an econometrician who observes a sample of paths of (δ, p) generated by our model between dates 0 and T that start with a large (small) δ_0 will observe mostly upwards (downwards) bifurcations, and thus many small positive (negative) returns followed sometimes by a large negative (positive) return in the rare cases in which a reversal occurs before T.

5 Discussion

Evidence from Brunnermeier, Nagel, and Pedersen (2008)

Brunnermeier, Nagel and Pedersen (2008) present empirical evidence that is consistent with the qualitative properties of our model. They find that carry trade returns are subject to crash risk because currencies that are used to fund carry trades have positive skewness while high rate currencies have a negative skewness. Also, net long positions in currency futures increase with the interest rate differential, and with broad measures of risk appetite such as the VIX index. Moreover, the price of protection against a currency crash increases shortly after a crash despite the fact that a subsequent crash is less likely. This offers empirical support for our assumption that the funding of a carry trade is more difficult after a crash of the high rate currency. All these findings square with our description of a market in which speculative capital flows slowly, so that an interest rate hike will lead a currency to appreciate only gradually, with possible sudden reversals.

In a related study, Gagnon and Chaboud study the relative frequencies of large upward and downward movements of three currencies relative to the dollar. They find that these frequencies are similar for the euro, which carried an interest rate close to the U.S. rate over the sample period. Conversely, large appreciations are relatively more frequent than large depreciations for the yen, while large depreciations are relatively more frequent for the Australian dollar. Gyntelberg and Remolona (2007) find similar conditional skewness for another sample of currencies.

Carry trades and leverage

One of our key assumptions is that the haircut $h(x_t)$ is decreasing in x_t. We provide some further discussion of this assumption based on the evolution of the yen borrowing by foreign banks offices in Japan described in Hattori and Shin (2007). Although the carry trade is often portrayed purely as a bet on the foreign exchange markets, the significance of the carry trade extends more widely. For example, a hedge fund that wishes to take on a larger position in a security obtains funding from its prime broker (a Wall Street investment bank, say) by pledging assets in a repurchase agreement. The prime broker, for its part, funds the loan to the hedge fund by borrowing from another party.

If the Wall Street bank borrows in New York, it will pay a rate closely tied to the short term US Dollar interbank rate. However, if it were to borrow in Tokyo, and in yen, it can borrow at the much lower yen overnight rate. A bank with global reach can borrow yen through its Tokyo office. The Tokyo office of the Wall Street bank then has yen liabilities to Japanese banks, but has yen assets against its New York head office. The lending by the Japan office of the Wall Street bank to its head office is captured in its "interoffice" accounts, and reported to the Bank of Japan. By monitoring the waxing and waning of the interoffice accounts of foreign banks in Tokyo, it is possible to gain a window on yen liquidity that funds general increases in balance sheets outside Japan.

For instance, until recently, foreign banks maintained a net long position in Japanese assets through their interoffice accounts. However, in the period leading up to the credit crisis of 2007, yen liabilities of foreign banks surged, leading to an unprecedented net *short* position in Japanese assets. These net short positions were sharply unwound in August 2007, coinciding with the peak of the credit crisis of 2007.

In addition, as found in Adrian and Shin (2007) for the fluctuations in US primary dealer balance sheets, Hattori and Shin (2007) find that the fluctuations in the size of the net interoffice accounts is correlated with the VIX index of implied volatility on the broader US stock market. The periods when foreign banks have large yen liabilities are also those periods with low readings of the VIX index.

Hattori and Shin also find that the difference between the yen overnight rate and a summary measure of overnight rates in developed countries mirrors closely the overall size of the net interoffice accounts. Yen liabilities are high when foreign overnight rates are high relative to overnight rates in Japan. Conversely, when foreign overnight rates are close to Japanese rates, foreign banks have low yen liabilities. During the period of exceptionally low US interest rates in 2002 to 2004, foreign banks maintained low yen liabilities, suggesting that they could satisfy their funding needs by borrowing in US dollars without tapping the yen market. These facts seem to lend some support to the idea that the yen carry trade is associated with buoyant financial conditions when leveraged institutions lay on larger bets. Thus the fact that the haircut is falling with price is in keeping with the empirical fact that leverage is high when balance sheets are large, as shown by Adrian and Shin (2007).

6 Concluding Remarks

We have developed a dynamic asset pricing model in which speculators face a coordination problem because of procyclical capital requirements. Using recent methodological advances in game theory, we have obtained a unique equilibrium price that has appealing qualitative features. It implies a risk premium that is time-varying and countercyclical. The required return on dollar depends in a highly non-linear fashion on the dynamics of the interest rate differential. A natural route for future research is to investigate whether such stochastic bifurcations models can also generate interesting quantitative implications.

Appendix A

In this appendix we give simple possible microfoundations of equation (1) in terms of the interactions between the speculators and a passive brokerage sector.

Assume that when trading at date t, a trader meets the brokerage sector. The brokerage sector consists of a continuum of dealers with mass m who have heterogeneous valuations of the dollar asset with c.d.f. $F(.)$ until the day of reckoning. The density could be seen as arising from heterogeneity in their inventories, or from heterogeneous beliefs about the fundamentals v. Like the traders, each dealer can be long up to one dollar asset. At each trading date t, the trader submits a supply or demand schedule to the dealers, and the non-filled part is cancelled. As a result, p_t solves:

$$\frac{x_t}{m} = F(p_t) \tag{13}$$

where x_t is the mass of traders who hold one dollar asset at date t. Equation (13) formalizes that the date t trader buys the dollar asset from the dealer who owns it and values it the least at date t, or sells it to the dealer who does not own it and values it the most at date t. This corresponds to a trade with a dealer with a valuation of $F^{-1}(\frac{x_t}{m})$ in both cases. Our specification corresponds to the particular case in which

$$F^{-1}(z) = e^{kz}.$$

This particular specification implies that dollar returns are simply proportional to net dollar flows. All our results would hold with more general specifications of the function $F(.)$ as long as the function $\ln F^{-1}(.)$ is continuously differentiable with bounded derivatives.

Appendix B

In this appendix, we provide the proofs of results reported in the text.

Proof of Proposition 3

1. This result stems directly from solving for a zero expected profit (10) with a constant price path ($p_t = p$ for all t).

2. If she expects that all traders will exit the carry trade after date t, a trader who has a chance to enter the carry trade at date t expects a profit given by (10):

$$\int_0^{+\infty} \left[\rho \left(v - p_t \right) + \int_0^u \left[\lambda \dot{p}_{t+s} + \left(\lambda + \rho \right) \left(\delta p_{t+s} - \Delta h \left(x_t \right) p_t \right) \right] ds \right] e^{-(\lambda+\rho)u} du$$

Since

$$\dot{p}_{t+s} = -\frac{\lambda k}{m} . p_{t+s} . x_{t+s} \leq -\frac{\lambda k}{m} . x_t e^{-\lambda s},$$

and

$$p_{t+s} \leq p_t,$$

then this expected profit is smaller than

$$\frac{\rho}{\lambda + \rho} \left(v - p_t \right) - \frac{k \lambda^2}{m \left(\lambda + \rho \right) \left(2\lambda + \rho \right)} x_t + \frac{1}{\lambda + \rho} \left(\delta - \Delta h \left(x_t \right) \right) p_t.$$

For ρ sufficiently small, this expression gets arbitrarily close to

$$-\frac{k\lambda}{2m} x_t + \frac{1}{\lambda} \left(\delta - \Delta h \left(x_t \right) \right) p_t,$$

which is strictly negative for all x_t for λ sufficiently large since the second term is strictly negative for x_t sufficiently close to 0. Thus the trader is unwilling to enter the carry trade.

Similarly, if she expects that all traders will enter the carry trade after date t, then future price changes are given by

$$\dot{p}_{t+s} = \frac{\lambda k}{m} . p_{t+s} \left(m - x_{t+s} \right) \geq \frac{\lambda k}{m} \left(m - x_t \right) e^{-\lambda s},$$

and

$$p_{t+s} \geq p_t.$$

Thus the expected profit is larger than

$$\frac{\rho}{\lambda + \rho} \left(v - p_t \right) + \frac{k\lambda^2}{m \left(\lambda + \rho \right) \left(2\lambda + \rho \right)} \left(m - x_t \right) + \frac{1}{\lambda + \rho} \left(\delta - \Delta h \left(x_t \right) \right) p_t.$$

Again, for λ sufficiently large and ρ sufficiently small, this term is strictly positive for all values of x_t . Thus the trader enters the carry trade.■

Proof of Lemma 6

To establish Lemmas 6 and 7, we will use the following result.

Claim 8 *Consider two prices such that $p'_t > p_t$. Consider a pair of future price processes $\left(p'_{t+u} \right)_{u \geq 0}$ and $\left(p_{t+u} \right)_{u \geq 0}$ such that other traders enter or exit the carry trade at the same future dates for each process. Then, along almost each future path, and for all $s \geq 0$*

$$\frac{\dot{p}'_{t+s}}{p'_t} - \frac{\dot{p}_{t+s}}{p_t} > -\frac{\lambda k e^k}{m} \left(\left(1 - k \right) e^{-\lambda s} + k \right) \left(x'_t - x_t \right).$$

Proof. If they buy and sell at the same time, then along each future path and for all s

$$\dot{x}'_{t+s} - \dot{x}_{t+s} = -\lambda \left(x'_{t+s} - x_{t+s} \right),$$

and so

$$x'_{t+s} - x_{t+s} = \left(x'_t - x_t \right) e^{-\lambda s}.$$

Now,

$$\frac{\dot{p}'_{t+s}}{p'_t}-\frac{\dot{p}_{t+s}}{p_t}=\underbrace{\frac{p'_{t+s}}{p'_t}\left(\frac{\dot{p}'_{t+s}}{p'_{t+s}}-\frac{\dot{p}_{t+s}}{p_{t+s}}\right)}_{(A)}+\underbrace{\frac{\dot{p}_{t+s}}{p_{t+s}}\left(\frac{p'_{t+s}}{p'_t}-\frac{p_{t+s}}{p_t}\right)}_{(B)}.$$

We have

$$A=\frac{p'_{t+s}}{p'_t}.\left(\frac{-\lambda k}{m}\left(x'_t-x_t\right)e^{-\lambda s}\right)>\frac{-\lambda k e^k}{m}\left(x'_t-x_t\right)e^{-\lambda s},$$

and

$$\begin{aligned} B &= \frac{\dot{p}_{t+s}}{p_{t+s}}\left(e^{\frac{k\left(x'_{t+s}-x'_t\right)}{m}}-e^{\frac{k\left(x_{t+s}-x_t\right)}{m}}\right)\geq\frac{\lambda k^2 e^k}{m}\left(x'_{t+s}-x'_t-x_{t+s}+x_t\right) \\ &= \frac{-\lambda k^2 e^k}{m}\left(1-e^{-\lambda s}\right)\left(x'_t-x_t\right). \end{aligned}$$

■

We now prove lemma 6. Starting from a point (δ_t,p_t) on $Z_0(.)$, the expected return $r(\delta_t,p_t)$ is

$$\frac{\rho}{\lambda+\rho}\frac{v-p_t}{p_t}+\int_0^{+\infty}e^{-(\lambda+\rho)u}\int_0^u\left[\lambda\frac{\dot{p}_{t+s}}{p_t}+(\lambda+\rho)\left(\delta_t\frac{p_{t+s}}{p_t}-\Delta h\left(x_t\right)\right)\right]ds=0.$$

Compare now two points on $Z_0(.)$, (δ_t,p_t) and (δ'_t,p'_t), such that $p'_t>p_t$.

$$\begin{aligned} 0 &= r(\delta'_t,p'_t)-r(\delta_t,p_t)=\frac{\rho}{\lambda+\rho}\left(\frac{v-p'_t}{p'_t}-\frac{v-p_t}{p_t}\right) \\ &\quad+\int_0^{+\infty}e^{-(\lambda+\rho)u}\int_0^u\left[\begin{array}{c}\lambda\left(\frac{\dot{p}'_{t+s}}{p'_t}-\frac{\dot{p}_{t+s}}{p_t}\right)\\ +(\lambda+\rho)\left(\delta'_t\frac{p'_{t+s}}{p'_t}-\delta_t\frac{p_{t+s}}{p_t}-\Delta h\left(x'_t\right)+\Delta h\left(x_t\right)\right)\end{array}\right]ds. \end{aligned}$$

Note that Claim 8 applies to the pair of price paths $\left(p'_{t+u}\right)_{u\geq 0}$ and $(p_{t+u})_{u\geq 0}$ since other traders always sell. Applying Claim 8 and integrating yields:

$$\begin{aligned} 0 &= r(\delta'_t, p'_t) - r(\delta_t, p_t) > \frac{\rho}{\lambda+\rho}\left(\frac{v-p'_t}{p'_t} - \frac{v-p_t}{p_t}\right) \\ &\quad + \frac{1}{\lambda+\rho}\left[\Delta \min\{-h'\} - \frac{\lambda^2 k e^k (\lambda(1+k)+\rho)}{m(2\lambda+\rho)(\lambda+\rho)}\right](x'_t - x_t) \\ &\quad + \underbrace{\int_0^{+\infty} (\lambda+\rho) e^{-(\lambda+\rho)u} \int_0^u \left(\delta'_t \frac{p'_{t+s}}{p'_t} - \delta_t \frac{p_{t+s}}{p_t}\right) ds.}_{(C)} \end{aligned}$$

Now, we decompose

$$\delta'_t \frac{p'_{t+s}}{p'_t} - \delta_t \frac{p_{t+s}}{p_t} = \frac{p'_{t+s}}{p'_t}(\delta'_t - \delta_t) + \delta_t\left(\frac{p'_{t+s}}{p'_t} - \frac{p_{t+s}}{p_t}\right).$$

Noting that

$$\frac{p'_{t+s}}{p'_t} - \frac{p_{t+s}}{p_t} \geq \frac{-ke^k}{m}\left(1-e^{-\lambda s}\right)(x'_t - x_t),$$

and that on $Z_0(.)$, for ρ sufficiently small

$$0 < \delta_t < \Delta h(0) + \frac{ke^k}{2}$$

yields

$$\begin{aligned} (C) &> \left(\int_0^{+\infty} (\lambda+\rho) e^{-(\lambda+\rho)u}\left(\int_0^u \frac{p'_{t+s}}{p'_t} ds\right) du\right)(\delta'_t - \delta_t) \\ &\quad - \frac{\lambda k e^k}{m(2\lambda+\rho)(\lambda+\rho)}\left(\Delta h(0) + \frac{ke^k}{2}\right)(x'_t - x_t). \end{aligned}$$

Finally this yields that for ρ sufficiently small,

$$\begin{aligned} 0 &= r(\delta'_t, p'_t) - r(\delta_t, p_t) > \frac{1}{\lambda+\rho}\left[\begin{array}{c} \Delta \min\{-h'\} - \frac{\lambda}{2\lambda+\rho}\frac{ke^k}{m}\left(\Delta h(0) + \frac{ke^k}{2}\right) \\ - \frac{\lambda^2 k e^k(\lambda(1+k)+\rho)}{m(2\lambda+\rho)(\lambda+\rho)} \end{array}\right](x'_t - x_t) \\ &\quad + \left(\int_0^{+\infty} (\lambda+\rho) e^{-(\lambda+\rho)u}\left(\int_0^u \frac{p'_{t+s}}{p'_t} ds\right) du\right)(\delta'_t - \delta_t). \end{aligned}$$

Condition (11) implies that the first term on the right-hand side is strictly positive. Thus, it must be that $\delta'_t < \delta_t$. For ρ sufficiently small, $Z_0(.)$ is therefore strictly decreasing.

That $Z_0(.)$ is Lipschitz follows from the fact that

$$\begin{aligned} 0 &= r(\delta'_t, p'_t) - r(\delta_t, p_t) < \frac{\rho}{\lambda+\rho}\left(\frac{v-p'_t}{p'_t} - \frac{v-p_t}{p_t}\right) \\ &+ \frac{1}{(\lambda+\rho)}\left(\Delta \min\{-h'\} . (x'_t - x_t) - e^{-k}(\delta_t - \delta'_t)\right). \end{aligned}$$

Thus for ρ sufficiently small,

$$\frac{\delta_t - \delta'_t}{x'_t - x_t} < e^k \Delta \min\{-h'\}, \tag{14}$$

and $Z_0(.)$ is Lipschitz decreasing.■

Proof of Lemma 7

Consider two points (δ_t, p_t) and (δ_t, p'_t) on the left of $Z_0(.)$ such that $p'_t > p_t$. If $Z_0(.)$ was a vertical line in the plane (δ, p), then the conditions of Claim 8 would be satisfied. Using the same argument as in the proof of Lemma 6, we would obtain that the expected return starting from (δ_t, p'_t) is larger than the expected return starting from (δ_t, p_t) by comparing returns along pairs of paths with similar innovations in the carry. Thus $Z_1(.)$ would have to be decreasing.

But since in addition $Z_0(.)$ is decreasing, dollar will in fact be bought more often along the paths starting from (δ_t, p'_t) than along the path starting in (δ_t, p_t), thereby generating additional positive future returns starting with this higher price p'_t relative to the case in which dollar is bought and sold at the same time regardless of the starting price. This reinforces the fact that $Z_1(.)$ is decreasing.

Consider now two points (δ_t, p_t) and (δ'_t, p'_t) on $Z_1(.)$ such that $p'_t > p_t$. Assume that these points satisfy

$$\frac{\delta_t - \delta'_t}{x'_t - x_t} > M,$$

where M corresponds to the Lipschitz constant of $Z_0(e^{\frac{kx}{m}})$. Then it must be that any path starting from p_t will hit $Z_0(.)$ before the corresponding path starting from p'_t. Otherwise, this would contradict the fact that $Z_0(e^{\frac{kx}{m}})$ is Lipschitz with constant M. Thus, along pairs of paths with similar innovations in the carry, the difference $x'_{t+u} - x_{t+u}$ will decrease even faster than it would if traders were selling dollar all the time. But then, this implies that $Z_1(.)$ must be Lipschitz with at most the same constant as $Z_0(.)$, which contradicts

$$\frac{\delta_t - \delta'_t}{x'_t - x_t} > M.$$

■

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