

Bundling and Competition for Slots*

(Very Preliminary: Comments Welcome)

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April 11, 2008

Abstract

We consider competition among n upstream firms when each of them sells a portfolio of distinct products to a downstream firm having limited slots (or shelf space). We study how bundling affects competition for slots. When the downstream has k number of slots, efficiency requires that it purchase the best k products among all upstream firms' products. We find that without bundling, equilibrium often does not exist and the outcome is often inefficient. This is because each upstream firm faces both internal competition (i.e. competition among its own products) and the external competition (i.e. competition from other firms' products). On the contrary, bundling removes the internal competition and restores efficiency: an efficient equilibrium always exists. In particular, in the case of Digital good, the equilibrium outcome is unique if firms do not use slotting contracts. However, inefficient equilibria can exist if firms use slotting contracts. In addition, in the case of physical good, inefficient equilibria can exist due to full-line forcing as well.

Key words: Bundling, Competition among Portfolios, Slots (or Shelf Space), Full-Line Forcing, Slotting Contracts

JEL Code: D4, K21, L13, L41, L82

*We thank the seminar participants at University of Cergy-Pontoise. We also thank the comments from Jay Pil Choi, Denis Gromb and Régis Renault. Jeon gratefully acknowledges the financial support from the Spanish government under SEJ2006-09993/ECON and Ramon y Cajal grant. This research was partially funded by the Net Institute whose financial support is gratefully acknowledged.

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1 Introduction

In vertical relations, very often each upstream firm sells a portfolio of distinct products which compete for limited slots (or shelf space) of downstream firms. In this situation, upstream firms may employ bundling as a strategy to win over the competition for the limited slots. Even though bundling has been a major antitrust issue and a subject of intensive research, to the best of our knowledge, the literature seems to have paid little attention to competition among portfolios of distinct products and, in particular, no paper seems to have studied how bundling affects competition among portfolios for limited slots. In this paper, we attempt to provide a new perspective on bundling by addressing this question.

Examples of the situation we described above are abundant both among the digital products and among physical products. For instance, in the movie industry, each movie distributor has a portfolio of distinct movies and buyers (either movie theaters or TV stations) have limited slots. More precisely, the number of movies that can be projected in a season (or in a year) by a theater is constrained by time and the number of projection rooms. Likewise, the number of movies that a TV station can show at prime time during a year is also limited. Actually, allocation of slots in movie theaters has been one of the main issues of the last presidential election in France regarding the movie industry¹. Furthermore, bundling in the movie industry (known as block booking²) was declared illegal in two supreme court decisions in U.S.: *Paramount Pictures* (1948), where blocks of films were rented for theatrical exhibition, and *Loew's* (1962), where blocks of films were rented for television exhibition. In addition, recently in *MCA Television Ltd. v. Public Interest Corp.* (11th Circuit, April 1999), the court of appeals reaffirmed the *per se* illegal status of block booking.

Another situation we have in mind is that of manufacturers' competition for retailers' shelf space. Actually, slotting arrangements, the payment by manufacturers for retail shelf space, have become increasingly important and have been the subject of recent antitrust

¹Cahiers du Cinema (April, 2007) proposes to limit the number of copies per film since certain movies by saturating screens limit other films' access to screens and asks the presidential candidates' opinions about the policy proposal.

²Block booking refers to "the practice of licensing, or offering for license, one feature or group of features on the condition that the exhibitor will also license another feature or group of features released by distributors during a given period" (*United States v. Paramount Pictures, Inc.*, 334 U.S. 131, 156 (1948)).

litigation³ and the focus of Federal Trade Commission studies.⁴ In addition to slotting contracts, manufacturers having a large portfolio of products (for instance, think about all the products sold under the brand name of Nestle) may practice bundling (often called full-line forcing) to win the competition for slots and there has been antitrust cases related to this practice⁵. For instance, the French Competition Authority fined Société des Caves de Roquefort for using selectivity or exclusivity contracts with supermarket chains.⁶

Furthermore, since we build a general model in which each competing firm can bundle any number of products, our paper also contributes to the growing literature on bundling a large number of (information) good (for instance, Bakos and Brynjolfsson, 1999, 2000).

In our model, we assume away any asymmetric information or any uncertainty about values of products. This will allow us to depart from the existing literature on block booking or bundling (see the review of the related literature later on in this section) and to identify what seems to us a first-order effect of bundling associated with the downstream firm's slot constraint. Actually, in the case of movie industry, Kenney and Klein (1983) point out that price discrimination explanation is inconsistent with the facts of *Paramount* and *Loew's* since the prices of the blocks varied a great deal across markets. Furthermore, in the Digital era, the prices are more and more tailored to buyers' characteristics as in the case of pricing of academic journals.

We consider a simultaneous pricing game among n upstream firms who sell their products to a downstream firm having $k(> 0)$ number of slots. Each upstream firm i has a portfolio of n_i distinct products. In our setting, a product needs to occupy a slot to generate some value (i.e. a profit) to the downstream firm. Products are heterogenous (in terms of the value that each of them generates to the downstream firm) and independent⁷ (i.e. the value that a product generates does not depend on the set of the other products that occupy the slots). Therefore, in the absence of the slot constraint, there is no competition among the upstream firms. In this setup, social efficiency requires the downstream firm's

³See, for example, *R.J. Reynolds Tobacco Co. v. Philip Morris, Inc.*, 199 F. Supp. 2d 363 (M.D.N.C. 2002), *aff'd per curiam*, 67 Fed. Appx. 810 (4th Cir. 2003); *American Booksellers Ass'n, Inc. v. Barnes & Noble, Inc.*, 135 F. Supp. 2d 1031 (N.D. Cal. 2001); *Intimate Bookshop, Inc. v. Barnes & Noble, Inc.*, 88 F. Supp. 2d 133 (S.D.N.Y. 2000); *FTC v. H.J. Heinz Co.*, 116 F. Supp. 2d 190 (D.C.C. 2000), *rev'd*, 246 F.3d 708 (D.C. Cir. 2001).

⁴See FTC Report (2001) and FTC Study (2003).

⁵*Procter & Gamble / Gillette*, DG Competition case COMP/M.3732; Société des Caves de Roquefort, Conseil de la Concurrence, Decision 04-D-13, 8th April 2004.

⁶Société des Caves de Roquefort's market share in the Roquefort cheese market was 70% but, through the contract, could occupy eight among all nine brands that Carrefour, a supermarket chain, sold.

⁷We plan to extend our main results to the general case in which products can be substitutes or complements.

slots to be allocated to the best k products among all products. We study how bundling affects the set of the products that occupy the slots.

In addition, when we study bundling, we are interested in knowing how contractual arrangements such as full-line forcing and slotting contracts affect competition depending on whether products are digital good or physical good. For this purpose, we define bundling as a contract that specifies a set of products (included in the bundle), a fixed price for the right to buy products in the bundle, and one individual price for each product in the bundle that the downstream should pay in case it decides to buy the product. A contract without bundling is the special case in which the fixed fee is zero. Therefore, it is obvious that each upstream prefers bundling to no-bundling. A full-line forcing contract is a special bundling contract that forces the downstream to buy all products in the bundle if the latter buys any. Mathematically, a full-line forcing contract can be represented as a bundling contract that specifies a positive fixed fee but zero individual prices. Finally, when an upstream firm sells a bundle with a slotting contract, upon accepting the deal, the downstream firm should buy all products in the bundle and allocate one slot to each product.

As the main result, we find that without bundling, equilibrium (in pure strategy) often does not exist and hence the outcome is often inefficient while with bundling, there always exists an efficient equilibrium regardless of whether firms can use full-line forcing or slotting contracts. Under individual sale, the internal competition (i.e. competition among a firm's own products) coexists with the external competition (i.e. competition from other firms' products) and this can create equilibrium non-existence and inefficiency: we provide the intuition through a simple example in section 3.⁸ On the contrary, bundling removes the internal competition and the external competition among bundles makes it optimal for each upstream firm to sell only the products that belong to the best k : the outcome is always efficient in the absence of full-line forcing and slotting contracts.

To give the intuition, consider digital good and assume that firms do not use slotting contracts. Suppose that an upstream firm has a product that belongs to the k best among all the products in the industry. Then, since the downstream firm can strictly increase the profit by replacing an inferior product with this product, the upstream firm can always sell it at a strictly positive price. Therefore the outcome is always efficient and this result holds even if firms can use full-line forcing as long as the production cost per product is zero or small. However, if the cost is large, inefficient equilibria can exist due to full-line pricing.

⁸Since equilibrium often does not exist in a simultaneous pricing game, in our net institute working paper (Jeon-Menicucci 2007), we study a sequential pricing game without bundling and characterizes the equilibrium. We find that the coexistence of both kinds of competition induces each upstream firm to face a trade-off between quantity and rent extraction, which makes equilibrium often inefficient.

Then, if a firm induces the downstream firm to buy a bundle including a product that is slightly inferior to a product owned by a rival firm, the rival firm cannot sell its superior product even though it charges a price equal to the cost since the cost is higher than the increase in the surplus generated by replacing the inferior product with the superior one. Furthermore, if firms use slotting contracts, inefficient equilibria exist even for digital good. If a firm decides to occupy all slots with a slotting contract, a rival firm cannot sell a superior product unless this product can be combined with other products to constitute an alternative package that beats the package offered by the former. Nevertheless, we show that an efficient equilibrium always exist. In particular, there exists an efficient equilibrium in which all firms offer their products with a simple tariff (composed of a fixed fee and each individual price equal to the cost): no single firm has an incentive to deviate even though it can use full-line forcing or slotting contracts.

We think that this unambiguous welfare-enhancing effect of bundling is pretty novel. Our result has strong policy implications that go beyond the rule of reason supported by the existing literature analyzing bundling in a second-degree price discrimination framework.

There are only a few papers on block booking. According to the leverage theory, on which the Supreme Court's decisions were based, block booking allows a distributor to extend its monopoly power in a desirable movie to an undesirable one. This theory was criticized by Chicago School (see e.g. Bowman 1957, Posner 1976, Bork 1978) since the distributor is better off by selling only the desirable movie at a higher price. As an alternative, Stigler (1968) proposed a theory based on second-degree price discrimination. However, Kenney and Klein (1983) point out that simple price discrimination explanation is inconsistent with the facts of *Paramount* and *Loew's* and argue that block booking mainly prevents exhibitors from oversearching, (i.e. from rejecting films revealed ex post to be of below-average value).⁹

Most papers on bundling study bundling of two (physical) goods in the context of second-degree price discrimination and focus on either surplus extraction (Schmalensee, 1984, McAfee et al. 1989, Salinger 1995 and Armstrong 1996, 1999) in a monopoly setting or entry deterrence (Whinston 1990, Choi-Stefanadis 2001 and Nalebuff 2004) in a duopoly setting. Bakos and Brynjolfsson (1999, 2000)'s papers are an exception, in that they study bundling of a large number of information goods, but they maintain the second-degree price discrimination framework. Their first paper shows that bundling allows a monopolist to extract more surplus (since it reduces the variance of average valuations by the law of large

⁹Their hypothesis is empirically tested in a recent paper by Hanssen (2000) but the author finds little support for the hypothesis. But Kenny and Klein (2000) do not agree with Hanssen's analysis.

numbers) and thereby unambiguously increases social welfare;¹⁰ the second paper applies this insight to entry deterrence. Since we assume complete information and hence full surplus extraction is possible under the monopoly setting, the rent extraction issue does not arise in our framework and there is no use in applying the law of large number. It is also important to remind that we consider a static simultaneous pricing game and do not address the entry deterrence issue.

In Jeon-Menicucci (2006), we take a framework similar to the one in this paper to study bundling electronic academic journals. More precisely, publishers owning portfolios of distinct journals compete to sell them to a library who has a fixed budget to allocate between journals and books. We find that bundling is a profitable strategy both in terms of surplus extraction and entry deterrence. Conventional wisdom says that bundling has no effect in such a setting and this is true in the absence of the budget constraint. However, when the budget constraint binds, we find that each firm has a strict incentive to adopt bundling but bundling reduces social welfare by reducing the library's consumption of journals and books. In this paper, instead of focusing on the budget constraint of the buyer, we focus on the slot constraint. Another difference is that Jeon-Menicucci (2006) focus on products (journals) of homogeneous value while in this paper we consider products of heterogeneous value. In spite of similarities of the frameworks, the result we obtain here is completely opposite to the one in the previous paper since we find that the allocation under bundling is efficient while the allocation under individual sale is not necessarily efficient.

Shaffer (1991)¹¹ considers an upstream monopolist selling two substitutable products with variable quantity and finds that brand specific two-part tariffs alone do not allow the monopolist to capture the maximum rent from the downstream firm but full-line forcing (equivalent to bundling) does. We consider products of independent values and hence the rent extraction issue Shaffer considers does not arise.

Finally, to some extent, our efficiency result of bundling is related to Bernheim and Whinston (1985; 1998) and O'Brien and Shaffer (1997) who show that when two single-product firms simultaneously offer non-linear tariffs to a common retailer, the vertically-integrated outcome is obtained.¹² However, they do not consider the slot constraint: if there is no slot constraint in our model, all firms fully extract the downstream firm's surplus regardless of they practice bundling or not. In addition, we consider competition among n upstream firms when each firm can bundle any number of products and study how

¹⁰See also Armstrong (1999).

¹¹See also Vergé (2001) who performs the social welfare analysis in the setup of Shaffer (1991).

¹²O'Brien-Shaffer (2005) show that this result also holds under simultaneous Nash bargaining for the case of N single-product firms.

different contractual arrangements (bundling or not, full-line forcing, slotting contracts) affect competition depending on whether they sell digital good or physical good.

In what follows, section 2 reviews the Chicago School Criticism of leverage theory with a simple model and explains our contribution with respect to it. Section 3 illustrates the key results with a simple example. Section 4 presents the model. Section 5 presents the main results when firms do not use slotting contracts. Section 6 studies the situation when firms can use slotting contracts. Section 7 derives implications on horizontal mergers. Section 8 concludes.

2 Chicago School Criticism of Leverage Theory¹³

According to the leverage theory of tying (or bundling), a multiproduct firm with monopoly power in one market can monopolize a second market using the leverage provided by its monopoly power in the first market. The theory, however, was largely discredited as a result of criticisms originating in the Chicago School (see e.g. Bowman 1957, Posner 1976, Bork 1978). In this section, we review the Chicago School Criticism of leverage theory with a simple model and explains our framework and contribution with respect to it.

Consider two independent products 1 and 2 and two firms A and B. Firm A is the monopolist of product 1 and A and B compete in the market for product 2. There is a single buyer who has a unit demand for each product. The buyer's willingness to pay for product 1 is $u_A^1 > 0$: the buyer's willingness to pay for product 2 produced by A (or B) is $u_A^2 > 0$ ($u_B^2 > 0$). Assume that the cost of production is zero for all products. In addition, we assume $u_A^1 + u_A^2 > u_B^2$, which implies that by bundling, A can force the buyer to buy both products from A.

In the absence of bundling, firm $i (= A, B)$ simultaneously chooses a price for product $j (= 1, 2)$ $p_i^j \in \mathbf{R}_+$. In equilibrium, A always sells product 1 at $p_A^1 = u_A^1$ and sells product 2 at $p_A^2 = \max \{0, u_A^2 - u_B^2\}$ if and only if $u_A^2 \geq u_B^2$. Hence, A's profit without bundling is given by $p_A^1 + p_A^2 = u_A^1 + \max \{0, u_A^2 - u_B^2\}$. Note that under individual sale, the outcome is always socially efficient.

Suppose now that A bundles both products and charge P_A . Then, in equilibrium, A succeeds in selling both products at $P_A = u_A^1 + u_A^2 - u_B^2$. Note that under bundling, the outcome is socially inefficient if $u_A^2 < u_B^2$.

Comparing A's profit without bundling with its profit with bundling shows that bundling does not affect the profit if A is more efficient than B in product 2 (i.e. $u_A^2 \geq u_B^2$) and decreases it otherwise. This shows that A never has the incentive to practice bundling for

¹³This section more or less follows section 2 of Choi (2006).

the purpose of monopolizing the tied product market. Furthermore, a laissez-faire policy always achieves social efficiency since firm A's private incentive to practice bundling is aligned with social incentive.

However, we notice that Chicago School's criticism is a *weak argument* in the double sense: a social planner never has any strict incentive to favor bundling (since outcome is always socially efficient without bundling but it can be inefficient with bundling) and firms never have any strict incentive to practice bundling (since a firm can never strictly increase its profit with bundling).

In our paper, we consider a general model of competition among any number of firms when each of them sells a portfolio of any number of distinct products to a downstream firm (i.e. a buyer), called D . We assume that all products are independent: the value that D obtains from a product does not depend on the set of other products that D buys. Instead, we assume that D has a limited number of slots available and this creates competition among products. In this setting, we find a *strong argument* for laissez-faire regarding bundling. In particular, we show that the outcome of competition among portfolios is efficient under bundling but can be inefficient without bundling. In addition, we show that each firm weakly or strictly prefers bundling to no bundling since bundling allows a firm to avoid cannibalization due to the internal competition among its own products. In the next section, we illustrate these results through a simple example.

3 Illustration with a simple example

There are two upstream firms, called A and B. A has two products of value $(u_A^1, u_A^2) = (4, 3)$ and B has one product of value $u_B^1 = 2$: u_i^j means the value that the downstream firm D obtains from the j -th best product among firm i 's products. Note that now all three products are distinct and hence there is no direct competition among the products. However, D has only two slots, which generates competition among them. The production cost is zero for all products. We note that social efficiency requires that the two slots be occupied by only A's products.

3.1 Without bundling

3.1.1 Equilibrium non-existence under simultaneous pricing

Consider first a simultaneous game of pricing without bundling: firm $i(= A, B)$ simultaneously chooses a price for product $j(= 1, 2)$ $p_i^j \in \mathbf{R}_+$. We show below that this game has no equilibrium in pure strategy. We assume as a tie-breaking rule that if D is indifferent

among several products, D buys the products with highest (gross) values. Without loss of generality, we can assume that A chooses prices such that $4 - p_A^1 \geq \max\{0, 3 - p_A^2\}$: the net profit that D makes from buying A's best product is positive and larger than the one it makes from buying A's second best product.

First, there is no equilibrium in which A sells only its best product (i.e. there is no equilibrium with $p_A^2 > 1$). Suppose first that A charges $p_A^2 > 3$. Then, B's best response is $p_B^1 = 2$. A charges $p_A^1 = 4$ and hence achieves a profit equal to 4. This cannot be an equilibrium since A can deviate and charge for instance $p_A^{2'} = 3$ and $p_A^{1'} = 4$. Then A sells both products and realizes a profit equal to 7. Suppose now that A charges $p_A^2 \in (1, 3]$. Then, B can sell its product by charging $p_B^1 = p_A^2 - 1 - \varepsilon$ with $\varepsilon (> 0)$ small enough. $4 - p_A^1 \geq \max\{0, 3 - p_A^2\}$ implies that A charges $p_A^1 = 1 + p_A^2$ and hence A's profit is $1 + p_A^2$. Consider now A's deviation in which A charges $p_A^{2'} = p_A^2 - \varepsilon$ and $p_A^{1'} = p_A^1 - \varepsilon$. Then A sells both products and realizes a profit equal to $1 + 2(p_A^2 - \varepsilon)$, which is larger than $1 + p_A^2$.

Second, there is no equilibrium in which A sells both products (i.e. there is no equilibrium with $p_A^2 \leq 1$). Note first that $p_A^2 \leq 1$ together with $4 - p_A^1 \geq \max\{0, 3 - p_A^2\}$ implies that $p_A^1 \leq 1 + p_A^2$ and therefore A's profit cannot be larger than 3 ($\geq 2 + p_A^2$). However, A can realize a profit equal to 4 by choosing $p_A^1 = 4$ and $p_A^2 = 3$ regardless of B's strategy.

The above example illustrates well some dilemmas that A faces. On the one hand, there is a commitment issue: if A can commit not to sell its second best product, then A avoids competition from B and realizes a profit equal to 4 by extracting D's whole surplus from A's best product. However, A cannot commit to this policy: since B in turn responds by charging a monopoly price extracting D's whole surplus, A is tempted to sell its second best product as well by undercutting p_B^1 . On the other hand, starting from a situation in which A sells only the best product at $p_A^1 = 4$, if A wants to sell the second best product as well, A suffers from internal competition. In other words, matching $u_B^1 - p_B^1 (> 0)$ by charging a price $p_A^2 \leq u_A^2 - (u_B^1 - p_B^1)$ requires A also to reduce p_A^1 in order to maintain $u_A^1 - p_A^1 = u_A^2 - p_A^2$. Since any $p_A^2 > 1$ will be undercut by B, the Sup of the profit that A can achieve by selling both products is 3, which is lower than the profit A can achieve by selling only the best product (by charging for instance $p_A^1 = 4$ and $p_A^2 = 3$). Therefore, we have a circular argument and this is why the equilibrium does not exist. Furthermore, the argument shows that even if we consider a mixed-strategy equilibrium, it cannot be efficient.

3.1.2 Inefficiency in the sequential pricing

Consider now a sequential game of pricing in which A first chooses (p_A^1, p_A^2) and then B chooses p_B^1 after observing (p_A^1, p_A^2) . Under the sequential game, A can commit to its pricing

strategy and hence the equilibrium in pure strategy exists. However, we below show that the outcome is not socially efficient.

As we have seen before, if A wants to sell only its best product, it can charge for instance $p_A^1 = 4$ and $p_A^2 = 3$ and realizes a profit equal to 4. By contrast, if A wants to sell both products, because of the internal competition, it must charge $p_A^1 = 2$ and $p_A^2 = 1$. Then, its profit is 3. Therefore, without bundling, A prefers selling only its best product and the outcome is inefficient from social point of view.

3.2 Bundling

Consider now that A sells a bundle of both products and charges a price $P_A \in \mathbf{R}_+$. For notational consistency, let $P_B \in \mathbf{R}_+$ denote the price that B charges for its product. We can show that in the example that we consider, regardless of whether the pricing game is simultaneous or sequential, there is a unique equilibrium outcome and it is efficient. Furthermore, A's profit under bundling is higher than its profit without bundling.

For instance in the simultaneous game of pricing, the unique equilibrium is $P_A = 5$ and $P_B = 0$. In the equilibrium, D buys A's bundle and hence the outcome is socially efficient. It is easy to see why this is an equilibrium. A has no incentive to charge a higher price; then D prefers buying B's product instead of A's bundle. Given that B's profit is zero, $P_B = 0$ is one of B's best responses.

The intuition for why bundling restores efficiency is the following: A's bundling gets rid of the internal competition between A's own products and makes the external competition with respect to B's product efficient. To explain this, let us consider an imaginary situation in which A sells a bundle composed of only its best product and charges $P_A = 4$. Then, D will buy it for sure and also buy B's product. Suppose now that A includes the second best product into the bundle and charges $P'_A = 4 + \varepsilon$ with $\varepsilon (> 0)$ small enough: i.e. after adding the second best product, A increases the price of the bundle by $\varepsilon (> 0)$ small enough. Given that A's second-best product is superior to B's product by 1, D will buy A's bundle and replace B's product with A's second best product as long as $P'_A - P_A$ is smaller than $u_A^2 - u_B^1 = 1$. In a general model in which D has k number of slots, we show that a firm has a strict incentive to add any product that belongs to the k best among all products in the industry since adding such a product allows it to charge a strictly higher price. This is why the equilibrium under bundling is efficient. By contrast, without bundling, if A charges $p_A^1 = 4$ and $p_A^2 = \varepsilon$ with $\varepsilon (> 0)$ small enough, there will be cannibalization between A's two products and D will switch from A's first-best product to the second-best product.

Therefore, our simple example illustrates well our strong argument for laissez-faire with

respect to bundling. The analysis of a more general model later on shows that an efficient equilibrium always exists under bundling although the efficient outcome may not be unique depending on whether products are digital good or not and whether firms use full-line forcing or slotting contracts.

4 The Model

4.1 The setting

There are n upstream firms, denoted by $i = 1, \dots, n$, and a downstream firm, denoted by D; we use "he" for each upstream firm and "she" for D. Each firm i has a portfolio of n_i products, and all products are distinct. Firm D's distribution of a product requires one unit of slot, but D has a limited number of slots (or shelf space), $k(\geq 1)$, to distribute the upstream firms' products. Therefore, D can distribute at most k number of products. We assume that the unit cost of production is $c \geq 0$ for each upstream firm, and the cost of distributing each product is zero for D.¹⁴

In this setup, we consider products of heterogenous value and study how bundling affects the set of the products occupying D's limited slots. More precisely, we are interested in knowing when D distributes the best k number of products. We use ij to denote firm i 's j -th best product: for instance 12 represents firm 1's 2nd best product. Let u_i^j be the gross profit (or surplus) that D obtains from distributing the product ij ;¹⁵ thus $u_i^1 \geq u_i^2 \geq \dots \geq u_i^{n_i} > 0$ for $i = 1, \dots, n$. The net profit of D is given by the sum of the gross profits of the products she distributes, minus the money spent to buy these products.

In the case in which $n_i \geq k$, it is obvious that only the k best products of firm i matter in our setting. In the case of $n_i < k$, we define $u_i^{n_i+1} = \dots = u_i^k = 0$. In this way we can think, without loss of generality, that each firm's portfolio consists of k products. Let w^j denote D's gross profit from the j -th best product among all products offered by the n upstream firms, thus $w^1 \geq w^2 \geq \dots \geq w^{nk}$ and we assume $w^k > \max\{c, w^{k+1}\}$. We use w_{-i}^j to represent the value of the j -th best product among the products offered by all firms different from i , for $j = 1, \dots, k$.

We denote with FB the set of products with values w^1, \dots, w^k (the first best products), while FB_i represents the set of products in FB offered by firm i ; q_i^{fb} is the cardinality of FB_i and $U_i^{FB} \equiv u_i^1 + \dots + u_i^{q_i^{fb}}$ (obviously, $q_1^{fb} + \dots + q_n^{fb} = k$). In order to fix the ideas, we suppose without loss of generality that for some n^{fb} between 1 and n the following

¹⁴The latter assumption is without loss of generality: see footnote 15 below.

¹⁵If D bears cost $c_i^j \geq 0$ to distribute this product, then we can consider $u_i^j - c_i^j$ as D's gross profit.

hold: $q_i^{fb} \geq 1$ (that is, $FB_i \neq \emptyset$) for $i = 1, \dots, n^{fb}$, and $q_i^{fb} = 0$ (that is, $FB_i = \emptyset$) for $i = n^{fb} + 1, \dots, n$.

Although we do not explicitly model consumers, we assume that consumer surplus is maximized when all k slots are allocated to the products belonging to FB . Furthermore, it is obvious that the sum of all firms' profits is maximized when all k slots are allocated to the products belonging to FB . Therefore, we say that an equilibrium is (socially) efficient if in the equilibrium all k slots are allocated to the products belonging to FB .

4.2 Contracts and game

In what follows, we consider a simultaneous pricing game with bundling. By bundling, we mean that each upstream firm i proposes a contract composed of a fixed price $F_i \geq 0$ for i 's bundle and a price $p_{ij} \geq 0$ for the product with value ij , for $j = 1, \dots, k$. The interpretation of the prices is the following. If D wants to buy at least one product from firm i , then she must pay F_i to firm i for the right to buy. In addition, D pays the individual prices of the products she selects to buy. Note that the individual pricing without bundling is a special case with $F_i = 0$. Therefore, it is obvious that any firm has an incentive to practice bundling. Let $\sigma_i = (F_i, p_{i1}, \dots, p_{ik})$ denote the strategy followed by firm i . Since $p_{ij} = \infty$ can be chosen, we can restrict attention to the bundle of all products of i without loss of generality.

A particular bundling contract of interest is full-line forcing contract. More precisely, firm i might impose D to buy a subset of its products (or all its products) if D wants to buy any product. For instance, if i forces D to buy its $q_i (\leq k)$ best products, then upon accepting the deal, D should pay $F_i + p_{i1} + \dots + p_{iq_i}$. Therefore, a full-line forcing contract can be represented by a contract that includes $p_{ij} = 0$ for each product ij that is forced. For instance, the previous full-line forcing contract can be represented by $(F'_i, p'_{i1}, \dots, p'_{ik})$ where $F'_i = F_i + p_{i1} + \dots + p_{iq_i}$, $p'_{ij} = 0$ for any $j \leq q_i$ and $p'_{ij} = p_{ij}$ for any $j > q_i$. Furthermore, when $c > 0$, charging $p_{ij} = \varepsilon (> 0)$ where ε is close to zero and much smaller than c is not different from charging $p_{ij} = 0$. Therefore, we call any bundling contract that includes $p_{ij} < c$ a *de facto* full-line forcing contract.

In what follows, we will distinguish two cases depending on whether slotting contracts are used or not. If firm i does not use any slotting contract, D has full degree of freedom in deciding how D allocates its slots to the products bought from i . By contrast, if firm i sells a bundle of q_i number of products with a slotting contract, D has to allocate q_i number of slots to the products of the bundle if D accepts the contract. In section 5, we study competition among bundles without slotting contracts and in section 6, we study competition among

bundles with slotting contracts. In section 6, to get equilibrium existence, we have to expand the contract space such that each upstream firm i proposes a menu bundles. In section 6.1, we define a menu of bundles and explain why we need it.

In section 5, we consider a two-stage game in which

- At stage one, each upstream firm i simultaneously chooses $\sigma_i = (F_i, p_{i1}, \dots, p_{ik})$.
- At stage two, D observes the upstream firms' strategies $(\sigma_1, \dots, \sigma_n)$ and buys the combination of products which maximizes her profit.

At stage two, as a tie-breaking rule, we assume that in case D is indifferent between different combination of products, she chooses the one that maximizes her gross profits.

5 Bundling without slotting contracts

In this section, we study competition among bundles without slotting contracts. In section 5.1, we give the key intuition through the case of Digital good (i.e. $c = 0$). In section 5.2, we describe an efficient NE for the general case with any $c \geq 0$. Section 5.3 shows that any NE is efficient if c is small. Section 5.4 shows that if firms cannot use any full-line forcing (including de facto full-line forcing), any NE is efficient as well regardless of the value of c .

5.1 Digital good

Since the propositions in later subsections include $c = 0$ as a particular case, we here consider the case of digital good (i.e. $c = 0$) only to highlight the key intuition. Consider competition among pure bundles: each firm i offers only a single bundle of all its products and charges a price $F_i \geq 0$ and $p_{i1} = \dots = p_{ik} = 0$. So D either buys all the products of i at price F_i or buys nothing from i . Assume free disposal.

Let $U^{FB} \equiv U_1^{FB} + \dots + U_{n^{fb}}^{FB} = u^1 + \dots + u^k$ and $U_{-i}^{FB} \equiv u_{-i}^1 + \dots + u_{-i}^k$. Consider the following price

$$F_i^* = U^{FB} - U_{-i}^{FB}.$$

When firm i chooses its price, it assumes that all the other bundles are bought and charges the price equal to the extra value that D can get from buying its bundle. Note that $F_i^* > 0$ if firm i has at least a product which belongs to the k best products among all products (i.e. $q_i^{fb} > 0$): otherwise, $F_i^* = 0$. It is easy to see that this $\{F_i^*\}$ is an equilibrium. Obviously, firm i has no interest in reducing the price. Firm i has no incentive to increase its price either: since D is indifferent between buying its bundle and not buying it, any increase in

price above F_i^* induces D to stop buying it. Furthermore, this equilibrium is efficient since D can choose among all nk products.

Even though the equilibrium in this pure bundling is pretty simple, the intuition for which this generates efficiency is very important. In this case of free disposal, starting from a bundle of firm i with price F_i , adding an additional product into the bundle never makes the bundle less attractive and therefore i can command at least the same price. In other words, firm i does not need to worry about internal competition among its own products under bundling. Therefore, selling a bundle that includes all firm i 's products weakly dominates selling a bundle that does not include all i 's products. In equilibrium, all firms end up selling their bundles and therefore D can choose the best k products to occupy the slots, which makes the equilibrium efficient. Note that free disposal assumption is not necessary in the above argument as long as D can choose the products she wants to buy in each firm i 's bundle after paying F_i . In the next subsection, we show that this result extends to any $c > 0$.

5.2 An efficient equilibrium

We now consider the general case with $c \geq 0$. We first introduce further notation. This notation is useful to describe the products which D buys in case she does not buy any product of firm i . Precisely, for each firm $i = 1, \dots, n^{fb}$, let s_i be the number in $\{0, \dots, k - q_i^{fb}\}$ defined as follows: (i) $s_i = 0$ if $c > u_{-i}^{k - q_i^{fb} + 1}$; (ii) $s_i = q_i^{fb}$ if $u_{-i}^k \geq c$; (iii) if $u_{-i}^{k - q_i^{fb} + 1} \geq c > u_{-i}^k$, then s_i is the number in $\{1, \dots, q_i^{fb} - 1\}$ such that $u_{-i}^{k - q_i^{fb} + s_i} \geq c > u_{-i}^{k - q_i^{fb} + s_i + 1}$. The interpretation of s_i is as follows. Imagine that D buys only products offered by firms different from i . Suppose that D has purchased all the products in $\cup_{h \neq i} FB_h$ and that the price of each product which is not included in FB is equal to c and $F_i = 0$ for $i = n^{fb} + 1, \dots, n$. Then, in addition to buying $k - q_i^{fb}$ products belonging to FB , D buys also a set of products that we denote by SB_{-i} and has cardinality equal to s_i ; let $U_{-i}^{SB} \equiv u_{-i}^{k - q_i^{fb} + 1} + \dots + u_{-i}^{k - q_i^{fb} + s_i}$.

We now prove that an efficient NE exists. Consider the following strategy for firm i , which we denote with $\hat{\sigma}_i$:

$$\begin{aligned} \hat{F}_i &= \sum_{t=1}^{q_i^{fb}} (u_i^t - c - \max\{u_{-i}^{k - q_i^{fb} + t} - c, 0\}) \quad \text{for } i = 1, \dots, n^{fb} \\ &= U_i^{FB} - cq_i^{fb} - (U_{-i}^{SB} - cs_i) \\ \hat{F}_i &= 0 \quad \text{for } i = n^{fb} + 1, \dots, n \\ \hat{p}_{ij} &= c \quad \text{for } j = 1, \dots, k, \quad \text{for } i = 1, \dots, n \end{aligned} \tag{1}$$

Note that when $c = 0$, we have $\hat{F}_i = U^{FB} - U_{-i}^{FB}$ and $\hat{p}_{ij} = 0$, which is the equilibrium that

we described in the previous subsection. The next proposition establishes that the profile $(\hat{\sigma}_1, \dots, \hat{\sigma}_n)$ is a NE and the outcome is such that D distributes all the products in FB. This generates a social surplus equal to $\sum_{i=1}^{n^{fb}} (U_i^{FB} - cq_i^{fb})$, which is split as follows among D and upstream firms: firm i 's profit is \hat{F}_i for $i = 1, \dots, n$, while D's profit is $\sum_{i=1}^{n^{fb}} (U_{-i}^{SB} - cs_i)$.

Proposition 1 *The strategy profile $(\hat{\sigma}_1, \dots, \hat{\sigma}_n)$ is a NE for any $c \geq 0$. In the equilibrium, firm i 's profit is \hat{F}_i for $i = 1, \dots, n$, while D's profit is $\sum_{i=1}^{n^{fb}} (U_{-i}^{SB} - cs_i)$.*

Proof. We split the proof into two steps.

Step 1 When the upstream firms plays $(\hat{\sigma}_1, \dots, \hat{\sigma}_n)$, D buys the set of products FB and thus each firm i 's profit is \hat{F}_i and D's profit is $\sum_{i=1}^{n^{fb}} (U_{-i}^{SB} - cs_i)$.

We start by proving by contradiction that D buys at least some products offered by firm 1. First, suppose that D buys nothing from firm 1, and that he buys \bar{q} products from the other firms, with values $v_{-1}^1, \dots, v_{-1}^{\bar{q}}$ such that $v_{-1}^1 \geq \dots \geq v_{-1}^{\bar{q}}$. Obviously, $\bar{q} \leq k$ since buying more than k products implies that some of them would not be used and then D may reduce his outlay without reducing his gross payoff. Actually, however, $\bar{q} \leq k - q_1^{fb} + s_1$ as $p_i^j = c$ for $j = 1, \dots, k$ and any $i \neq 1$, and there are only $k - q_1^{fb} + s_1$ products offered by firms different from 1 with values not lower than c . But then we can prove that D can increase his payoff by buying from 1 the products in FB_1 , and refusing to buy the products of other firms with values $v_{-1}^{k-q_1^{fb}+1}, \dots, v_{-1}^{\bar{q}}$.¹⁶ In this way the profit of D varies by

$$\begin{aligned} \Delta\pi_D &= -\hat{F}_1 - cq_1^{fb} + \sum_{t=1}^{q_1^{fb}} u_1^t - \sum_{t=1}^{\bar{q}-k+q_1^{fb}} (v_{-1}^{k-q_1^{fb}+t} - c) \\ &= u_{-1}^{k-q_1^{fb}+1} + \dots + u_{-1}^{k-q_1^{fb}+s_1} - s_1c - [v_{-1}^{k-q_1^{fb}+1} + \dots + v_{-1}^{\bar{q}} - (\bar{q} - k + q_1^{fb})c] \end{aligned}$$

and notice that the set $A \equiv \{v_{-1}^{k-q_1^{fb}+1}, \dots, v_{-1}^{\bar{q}}\}$ is a subset of $B \equiv \{u_{-1}^{k-q_1^{fb}+1}, \dots, u_{-1}^{k-q_1^{fb}+s_1}\}$, and the set $B \setminus A$ has cardinality $s_1 - (\bar{q} - k + q_1^{fb})$. In case that $s_1 > \bar{q} - k + q_1^{fb}$, $\Delta\pi_D = \sum_{u \in B \setminus A} (u - c)$ is positive as $u > c$ for any $u \in B$. If instead $s_1 = \bar{q} - k + q_1^{fb}$, then $B \setminus A = \emptyset$ and $\Delta\pi_D = 0$. But then it is still in the interest of D to buy the products in FB_1 in order to increase his gross profit.

In this way we have proved that D buys at least one product of firm 1, and then pays the fixed fee \hat{F}_i . This reveals that D buys at least all the products in FB_1 , since all products of all firms have the same price c . Therefore, for $i = 1, \dots, n^{fb}$, D buys at least the products

¹⁶This applies in the case that $\bar{q} \geq k + 1 - q_1^{fb}$, as in the case that $\bar{q} \leq k - q_1^{fb}$, D can increase his payoff simply by buying the products in FB_1 and continuing to buy the products of the other firms with values $v_{-1}^1, \dots, v_{-1}^{\bar{q}}$

in FB_i . But since D will not buy more than k products, it must be the case that she buys the products in $FB_1 \cup \dots \cup FB_{n^{fb}}$ and buys nothing from firm $i = n^{fb} + 1, \dots, n$.

Step 2 When all firms different from i play $\hat{\sigma}_{-i}$, there exists no strategy of firm i which yields i a profit larger than \hat{F}_i .

In order to fix the ideas, we will prove this claim for firm 1. We start by noticing that when each firm plays (1), D buys the products in FB by Claim 1 and obtains a profit equal to

$$\begin{aligned}\hat{\pi}_D &\equiv \sum_{i=2}^{n^{fb}} \left[\sum_{t=1}^{q_i^{fb}} u_i^t - (\hat{F}_i + cq_i^{fb}) \right] + \sum_{t=1}^{q_1^{fb}} u_1^t - \left[\sum_{t=1}^{q_1^{fb}} u_1^t - \sum_{t=1}^{s_1} (u_{-1}^{k-q_1^{fb}+t} - c) \right] \\ &= \sum_{i=1}^{n^{fb}} (U_{-i}^{SB} - cs_i).\end{aligned}$$

If instead firm 1 deviates from $\hat{\sigma}_i$, then D can still make profit $\hat{\pi}_D$ by buying suitably from the other firms. Precisely, let D buy the products in $FB_2 \cup \dots \cup FB_{n^{fb}} \cup SB_{-1}$. D's payoff is then $\sum_{i=2}^{n^{fb}} [\sum_{t=1}^{q_i^{fb}} u_i^t - (\hat{F}_i + cq_i^{fb})] + \sum_{t=1}^{s_1} (u_{-1}^{k-q_1^{fb}+t} - c)$, which is equal to $\hat{\pi}_D$. Now suppose that there exists a strategy σ_1 of firm 1, different from $\hat{\sigma}_1$, such that given $(\sigma_1, \hat{\sigma}_{-1})$, D buys the products of firms $1, \dots, n$ in $S_1 \cup \dots \cup S_n$ (some of this sets may be empty), and that D pays to firm 1 more than $\hat{F}_1 + c \cdot (\#S_1)$ to buy the products in S_1 – this means that 1 makes a profit higher than \hat{F}_1 . The profit to D from buying $S_1 \cup \dots \cup S_n$ needs to be at least equal to $\hat{\pi}_D$, as she can obtain profit $\hat{\pi}_D$ by buying $FB_2 \cup \dots \cup FB_{n^{fb}} \cup SB_{-1}$. But this contradicts Step 1, since without the deviation of 1, D would have obtained from buying $S_1 \cup \dots \cup S_n$ a profit higher than $\hat{\pi}_D$ given that she would pay only $\hat{F}_1 + c \cdot (\#S_1)$ (and not more) in order to buy the products in S_1 . ■

A consequence of the proposition is that when each firm i offers $\hat{\sigma}_i$, no single firm has a strict incentive to deviate by using full-line forcing. In order to obtain some intuition about the NE, we present a useful property of the game we consider. In order to state this property, we gather in the set Z_i the strategies of firm i such that $p_{ij} = c$ for $j = 1, \dots, k$. Thus, two strategies σ_i and σ'_i in Z_i differ only because of the fixed fees, as the prices of the individual products are all equal to c for both σ_i and σ'_i . Then we can prove that no firm i loses anything by restricting to the strategies in Z_{-i} .

Lemma 1 *For any profile (σ_i, σ_{-i}) , let π_i denote the profit of firm i given (σ_i, σ_{-i}) . Then, firm i can make profit π_i also by playing σ'_i instead of σ_i , with $\sigma'_i \in Z_i$ and such that $F'_i = \pi_i$.*

Proof. Consider an arbitrary profile of strategies $(\sigma_1^*, \dots, \sigma_n^*)$ and let $\pi_1 > 0$ be the profit of firm 1 given $(\sigma_1^*, \dots, \sigma_n^*)$. We show that 1 can achieve the same profit π_1 by playing $\sigma_1 \in Z_1$

such that $F_1 = \pi_1$. In order to prove this result, it suffices to show that D buys at least one product from 1. We find that, (i) given $(\sigma_1, \sigma_{-1}^*)$, D can make the same profit that she makes from $(\sigma_1^*, \dots, \sigma_n^*)$ since she can buy the same products, at the same price; (ii) with $(\sigma_1, \sigma_{-1}^*)$, D cannot make a higher profit than with $(\sigma_1^*, \sigma_{-1}^*)$ without buying at least one product of firm 1 because otherwise D would not buy anything from 1 given $(\sigma_1^*, \sigma_{-1}^*)$, which contradicts $\pi_1 > 0$. ■

This means that, given σ_{-i} , in order to maximize his profit, firm i can restrict his attention to simple strategies such that $p_{ij} = c$ for any j – and use only the fixed fee as a rent extraction instrument. In a sense, this allows to think of the strategy space for each firm i as given by the possible values of F_i , that is $[0, +\infty)$.

Obviously, $\hat{\sigma}_i$ belongs to Z_i for any i and then it is simple to interpret the values of $\hat{F}_1, \dots, \hat{F}_n$. Indeed, suppose that D has purchased the products in $FB_2 \cup \dots \cup FB_{n^{fb}}$, and that she considers the profitability of buying also the products in FB_1 . By doing so, her profit increases by $U_1^{FB} - cq_1^{fb} - \hat{F}_1 = U_{-1}^{SB} - cs_1$. On the other hand, the best D can do by not buying FB_1 is to buy the products in SB_{-1} , which increases her payoff by $U_{-1}^{SB} - cs_1$ given $\hat{\sigma}_{-1}$. Therefore, \hat{F}_1 is the highest value of F_1 that induces D to buy the products in FB_1 , rather than buying SB_{-1} . It is clear then that the profit of firm 1 is smaller (and the profit of D is larger) the more valuable are the products in SB_{-1} , as the competition from these products exerts a downward pressure on \hat{F}_1 . In the limit case in which $SB_{-1} = \emptyset$, 1's profit is $U_1^{FB} - cq_1^{fb}$, which means that 1 captures the full surplus of D from the products in FB_1 . In particular, therefore, the profit of D given $(\hat{\sigma}_1, \dots, \hat{\sigma}_n)$ is positive if and only if $u_{-i}^{k-q_i^{fb}+1} > c$ for at least one i .

Example 1 (Illustration of an efficient equilibrium) Consider the case in which $k = 2$, $n = 2$, $c = 3$ and

$$(u_1^1, u_1^2) = (10, 8), \quad (u_2^1, u_2^2) = (9, 4)$$

Then $\hat{p}_{ij} = 3$, $\hat{F}_1 = (10 - 3) - (4 - 3) = 6$ and $\hat{F}_2 = (9 - 3) - (8 - 3) = 1$. Notice that firm 1 makes a profit of 6 while firm 2 makes a profit of just 1, even though product 21 has a value only slightly smaller than product 11. The reason is the competition exerted by product 12, which keeps small the profit of firm 2. On the other hand, product 22 has a small value and reduces the profit of firm 1 only by $4 - 3 = 1$ with respect to the net surplus generated by 11, which is $10 - 3 = 7$. Notice that if u_2^2 were smaller than 3, then \hat{F}_1 would be equal to 7.

5.3 Efficiency for a small c : Uniqueness

The game we are considering has many NE different from $(\hat{\sigma}_1, \dots, \hat{\sigma}_n)$ (see Example 2 below), but we can prove that if $c < u^k - u^{k+1}$ then in any NE the set of products distributed by D is given by FB. This makes the issue of multiplicity not very serious from the point of view of social welfare.

Proposition 2 *In any NE, D buys the set FB if $c < u^k - u^{k+1}$.*

Proof. Suppose that a NE $(\sigma_1, \dots, \sigma_n)$ exists such that D buys the products in $S_1 \cup \dots \cup S_n$ and S_1 does not include all the products in FB_1 ; that is, $FB_1 \setminus S_1 \neq \emptyset$. Let π_1 denote the profit of firm 1 in this NE, while π_D is the profit of D. Consider the strategy $\sigma_1 \in Z_1$ of firm 1 in which $F_1 = \pi_1 + \varepsilon$ for $\varepsilon > 0$ and small. We prove that D buys at least one product from firm 1, and therefore 1's profit increases to $\pi_1 + \varepsilon$. In order to do this, we notice that if D does not buy any product from 1, she cannot make a profit higher than π_D [otherwise she would not buy $S_1 \cup \dots \cup S_n$ given $(\sigma_1, \dots, \sigma_n)$]. Thus it suffices to show that D can earn more than π_D by selecting suitable products, which include some products offered by firm 1. Precisely, let D buy the products in $S_1 \cup \dots \cup S_n \cup \{1j\} \setminus \{hj'\}$, where $1j$ is in $FB_1 \setminus S_1$ and hj' is the lowest valued product in $S_1 \cup \dots \cup S_n$. If $h = 1$, then the change in profit for D is $u_1^j - \varepsilon - u_h^{j'} > 0$. If instead $h \neq 1$, the change in the profit of D is $u_1^j - c - \varepsilon - u_h^{j'} + p_h^{j'} \geq u_1^j - c - \varepsilon - u_h^{j'}$. Obviously, $u_1^j \geq u_1^{q_1^{fb}}$ since $1j \in FB$ and $u_{-1}^{k-q_1^{fb}+1} \geq u_h^{j'}$ since $hj' \notin FB$. Thus $u_1^j - c - \varepsilon - u_h^{j'} > 0$ if $u_1^{q_1^{fb}} - u_{-1}^{k-q_1^{fb}+1} > c$, and $u^k - u^{k+1} > c$ is a sufficient condition for this inequality to hold. ■

Proposition 2 holds because of an obvious principle. If a profile of strategies $(\sigma_1, \dots, \sigma_n)$ induces D to buy the products in $S_1 \cup \dots \cup S_n$ and some product $1j$ belongs to FB_1 but not to S_1 , then firm 1, by choosing $p_{1j} = c$, can make any product $1j$ in FB_1 more competitive than any product hj' outside FB since the difference in values between the two products is at least c by assumption, and thus $u_1^j - c > u_h^{j'} - p_h^{j'}$. Then 1 can expropriate a part of this increase in D's profit by raising slightly F_1 .

We remark however that, although efficiency is a property of any NE, the upstream firms' profits (and D's profits) may vary across NE, as the following example shows.

Example 2 (Indeterminacy of profits) Consider the case in which $k = 2$, $n = 2$, $c = 1$ and

$$(u_1^1, u_1^2) = (7, 0), \quad (u_2^1, u_2^2) = (10, 5)$$

Since $c < u^2 - u^3$ holds, Proposition 2 applies and therefore in any NE D distributes the two products in $FB = \{11, 21\}$. However, we exhibit below infinitely many NE, which

differ because of the profit of firm 1 and the profit of D. :

$$\begin{aligned}\sigma_1 & : F_1 = 1 + x, & p_{11} = 1, & p_{12} = 0 \\ \sigma_2 & : F_2 = 9, & p_{21} = 1, & p_{22} = x\end{aligned}$$

For any $x \in [0, 1]$, (σ_1, σ_2) is a NE in which D buys $FB = \{11, 21\}$ and $\pi_1 = F_1 = 1 + x$, $\pi_2 = F_2 = 9$, $\pi_D = 17 - 1 - x - 9 = 7 - x$.

In this example no profitable deviation for 1 exists because if 1 increases $F_1 + p_1^1$ above $2 + x$, then D's profit from buying product 11 is lower than $5 - x$, and it is more convenient for her to buy product 22 rather than 11. Notice that 2 is indifferent with respect to the value of $x \in [0, 1]$ (as product 22 is not going to be purchased by D in equilibrium) but the value of x determines the competition faced by product 11 and affects the price this product can command.¹⁷ We conjecture that the equilibrium profits can be uniquely pinned down as those in Proposition 1 if a firm cannot charge an individual price lower than c for products that are not sold in equilibrium.

5.4 Efficiency and Inefficiency for a large c

When we consider the case in which $c \geq u^k - u^{k+1}$, Proposition 2 does not hold (indeed, our argument following the proposition does not apply). When $c \geq u^k - u^{k+1}$, there exist inefficient NEs: the following example shows that an inefficient NE can exist due to full-line forcing.

Example 1' (full-line forcing and inefficiency) Consider the setting of Example 1: $k = 2$, $n = 2$, $c = 3$ and

$$(u_1^1, u_1^2) = (10, 8), \quad (u_2^1, u_2^2) = (9, 4)$$

Then $c > u^2 - u^3$ holds, and the NE below is such that D buys $\{11, 12\}$ rather than FB .

$$\begin{aligned}\sigma_1 & : F_1 = 11, & p_{11} = p_{12} = 0 \\ \sigma_2 & : F_2 = 6, & p_{21} = p_{22} = 0\end{aligned}$$

Given (σ_1, σ_2) , D buys the two products of firm 1, earning a profit of 7. There is no profitable deviation for firm 2 because (for instance), in order to sell product 21, he needs to receive at least 3 and this yields D a profit of 6, but this is not enough to induce D to drop product 12, which yields a profit of 8.

¹⁷Notice that x cannot be larger than 1, as then firm 2 would have an incentive to reduce p_{22} slightly below x in order to make a profit of about $x - 1$ by inducing D to buy product 22 rather than 11; when instead $x \in [0, 1]$, D has no such an incentive as $x - 1 \leq 0$.

Under full-line forcing, a firm can force D to buy a product that does not belong to FB . This in turn makes a rival firm unable to sell a product that belongs to FB even at the price equal to c as long as c is larger than the difference between both products' surpluses.

We remark, however, that the efficient NE $(\hat{\sigma}_1, \dots, \hat{\sigma}_n)$ described by Proposition 1 exists also for large values of c . Furthermore, we can prove an interesting result if firms are restricted to satisfy the condition

$$p_{ij} \geq c \quad \text{for } j = 1, \dots, k, \quad \text{for } i = 1, \dots, n. \quad (2)$$

Precisely, under (2) we can show that (regardless of the value of c) all the NE are outcome equivalent: across all of the NE, the products distributed by D are given by the set FB and the profits of the upstream firms, and of D, are like in the NE of Proposition 1. Furthermore, lemma 1 shows that firm i can always find a best response that satisfies (2).

Proposition 3 *Suppose that each firm must satisfy (2). Then, in any NE, D buys the products in FB and pays $\hat{F}_i + cq_i^{fb}$ to firm $i = 1, \dots, n^{fb}$, and earns $\sum_{i=1}^{n^{fb}} (U_{-i}^{SB} - cs_i)$.*

Step 1 In any NE, D buys the products in FB .

Suppose that a NE exists such that D buys the products in $S_1 \cup \dots \cup S_n$ and S_1 does not include all the products FB_1 (that is, $FB_1 \setminus S_1 \neq \emptyset$). Let π_1 denote the profit of firm 1 in this NE, while π_D is the profit of D. Consider the strategy of firm 1 in which $F_1 = \pi_1 + \varepsilon$ (for $\varepsilon > 0$ and small) and $p_{1q_1} = c$ for $q_1 = 1, \dots, k$. In order to prove that this is a profitable deviation, it suffices to show that D buys at least one product from firm 1, as this yields firm 1 a profit of $\pi_1 + \varepsilon$. In the case in which D does not buy any product from 1, she can make at most a profit equal to π_D . However, D can make a higher profit by buying from 1 the set S_1 and a product in $FB_1 \setminus S_1$ (say with value u_1), from the other firms the set $S_2 \cup \dots \cup S_n$ except the lowest valued product in $S_2 \cup \dots \cup S_n$ (say with value u_{-1} and price \bar{p}). The change in payoff for D is $u_1 - c - \varepsilon - u_{-1} + \bar{p}$, which is at least as large as $u_1 - \varepsilon - u_{-1}$ since $\bar{p} \geq c$, and $u_1 - \varepsilon - u_{-1} > 0$. Thus, after 1's deviation, D will buy at least one product of 1 and this makes 1's profit equal to $\pi_1 + \varepsilon$.

Step 2 In any NE, firm i 's profit is at least \hat{F}_i .

This claim is obvious for $i = n^{fb} + 1, \dots, n$, as $\hat{F}_i = 0$ for these firms. About firm $i = 1, \dots, n^{fb}$, we show that if firm i plays (1), then (regardless of the strategies followed by the other firms), D buys all products in FB_i . Hence, firm i gains \hat{F}_i and this establishes that his equilibrium profit is not lower than \hat{F}_i . The proof is by contradiction and is worked out for firm 1 to fix the ideas. Suppose that D buys nothing from firm 1, and that D buys \bar{q} products from the other firms, with values $v_{-1}^1, \dots, v_{-1}^{\bar{q}}$ such that $v_{-1}^1 \geq \dots \geq v_{-1}^{\bar{q}}$. Obviously, $\bar{q} \leq k - q_1^{fb} + s_1$ as $p_{iq_i} \geq c$ for any q_i and any $i \neq 1$, and there are only $k - q_1^{fb} + s_1$ products

offered by firms different from 1 with values not lower than c . But then we can prove that D can increase her payoff by buying from 1 the products in FB_1 , and stopping buying the products of other firms with values $v_{-1}^{k+1-q_1^{fb}}, \dots, v_{-1}^{\bar{q}}$.¹⁸ In this way the profit of D varies by

$$\Delta\pi_D = -\hat{F}_1 - cq_1^{fb} + \sum_{t=1}^{q_1^{fb}} u_1^t - \sum_{t=1}^{\bar{q}-k+q_1^{fb}} [v_{-1}^{k-q_1^{fb}+t} - p_{-1}(k - q_1^{fb} + t)]$$

where $p_{-1}(k - q_1^{fb} + t)$ represents the individual price charged for the product of value $v_{-1}^{k-q_1^{fb}+t}$. $\Delta\pi_D$ is at least as large as $-\hat{F}_1 - cq_1^{fb} + \sum_{t=1}^{q_1^{fb}} u_1^t - \sum_{t=1}^{\bar{q}-k+q_1^{fb}} (v_{-1}^{k-q_1^{fb}+t} - c)$ since $p_{-1}(k - q_1^{fb} + t) \geq c$. Then we can argue like in the proof of Step 1 in the proof of Proposition 1 to prove that the latter term is non negative.

Step 3 In any NE, firm i 's profit is at most \hat{F}_i .

Suppose that there exists a NE such that D buys the products in FB and pays to firm 1 an amount $\hat{F}_1 + cq_1^{fb} + \delta_1$ with $\delta_1 > 0$ (this means that firm 1 makes a profit by δ_1 higher than \hat{F}_1). Then we show that for some firm $i \neq 1$ a profitable deviation exists. First we investigate some properties of δ_1 , and for this purpose we denote with β the payoff of D from buying $FB_2 \cup \dots \cup FB_{n^{fb}}$, while $U_{-1}^{SB} \equiv u_{-1}^{k-q_1^{fb}+1} + \dots + u_{-1}^{k+s_1-q_1^{fb}}$ is the aggregate value of the products in SB_{-1} . Then we notice that after buying $FB_2 \cup \dots \cup FB_{n^{fb}}$, D's increase in profit from buying FB_1 (and paying $\hat{F}_1 + cq_1^{fb} + \delta_1$) is $U_{-1}^{SB} - s_1c - \delta_1$. Hence it is necessary that D is unable to make a profit higher than the equilibrium profit by buying only from firms different from 1. For this reason we introduce the set M of products with the interpretation that if D buys only products offered by firms different from 1, then she maximizes her profit by buying $FB_2 \cup \dots \cup FB_{n^{fb}} \cup M$ [and $(FB_2 \cup \dots \cup FB_{n^{fb}}) \cap M = \emptyset$], and let $\mu \geq 0$ be such that D's resulting payoff is $\beta + \mu$ (notice that $M \subseteq SB_{-1}$). Thus, in order for D to buy FB_1 it is necessary that $U_{-1}^{SB} - s_1c - \delta_1 \geq \mu$, that is $U_{-1}^{SB} - s_1c - \mu \geq \delta_1$. But then, if $\delta_1 < U_{-1}^{SB} - s_1c - \mu$ it is possible for 1 to increase his profit (by increasing F_1) until $\hat{F}_1 + cq_1^{fb} + U_{-1}^{SB} - s_1c - \mu$ because D cannot do better by buying only from firms different from 1; thus, $\delta_1 = U_{-1}^{SB} - s_1c - \mu$. Since $\delta_1 > 0$ by assumption, this equality reveals that $SB_{-1} \neq \emptyset$ and, in particular, $\mu < U_{-1}^{SB} - s_1c$. This means that if D buys the products in SB_{-1} , she must pay a price strictly larger than c at least for some of these products.

Now we find a profitable deviation for a firm different from 1. Consider a firm $i \neq 1$ with a product in M which is priced at $\bar{p} > c$, and let π_i denote its NE profit.¹⁹ Then a

¹⁸This applies in the case that $\bar{q} \geq k + 1 - q_1^{fb}$, as in the case that $\bar{q} \leq k - q_1^{fb}$, D can increase his payoff simply by buying the products in FB_1 and continuing to buy the products of the other firms with values $v_{-1}^1, \dots, v_{-1}^{\bar{q}}$

¹⁹If $M = \emptyset$ (and thus $\mu = 0$), then consider a firm i with a product in SB_{-1} which is priced $\bar{p} > c$ (notice that $SB_{-1} \neq \emptyset$, otherwise $\delta_1 = 0$).

profitable deviation for firm i is as follows: $F_i = \pi_i + \varepsilon$ (with $\varepsilon > 0$ and small), $p_i(q_i) = c$ for $q_i = 1, \dots, k$. Then D's profit from buying $FB_2 \cup \dots \cup FB_{n,fb} \cup M$ is $\beta + \mu + \bar{p} - c - \varepsilon$, which is larger than $\beta + \mu$.²⁰ On the other hand, D can make at most $\beta + \mu$ from buying only the products offered by firms different from i . Thus D will buy some products from i , which allows D to make a profit of $\pi_i + \varepsilon (> \pi_i)$. ■

From this proposition we derive a simple corollary for the case of $c = 0$ since the assumption (2) is automatically satisfied.

Corollary 1 *In the case of digital good, both the equilibrium allocation and the equilibrium profits are uniquely determined: the equilibrium is always efficient and each upstream firm i 's profit is equal to \hat{F}_i .*

6 Bundling with slotting contracts

In this section we study a setting in which each firm i can use slotting contracts. We first introduce menu of bundles and then study the setting in which all firms use slotting contracts. Last, we study the case in which each firm can decide whether to use a slotting contract or not.

6.1 Menu of bundles

Suppose that all firms use slotting contracts. Then, we have to expand the contract space to menus of bundles: more precisely, each firm i offers D a menu of bundles $\{B_i^j, P_i^j\}_{j=1}^k$, in which $B_i^j = \{i1, \dots, ij\}$ is the bundle of the j best products of firm i , while P_i^j is the price for B_i^j . Note that since a slotting contract includes full-line forcing, we do not need to specify individual products prices. Furthermore, if D purchases B_i^j under a slotting contract, then D should allocate j number of slots to the products of the bundle.

In the previous section, each upstream firm uses a strategy consisting of offering only one bundle that includes all its products. This restriction in strategy is without loss of generality since even if we allow the firm to use a menu of bundles, we can find his best response within the restricted strategy. However, if we consider firms using slotting contracts, we have a serious problem of equilibrium non-existence when we restrict each firm to propose only one bundle as the following result illustrates:

²⁰If $M = \emptyset$, then consider D's profit from buying $FB_2 \cup \dots \cup FB_{n,fb}$ and the product in SB_{-1} which is mentioned in footnote 19.

Lemma 2 *Suppose that all n upstream firms use slotting contracts. If each upstream firm offers only one bundle, there exists no equilibrium in which all n firms occupy some slots.*

Proof. Consider the case of duopoly (firm 1 and firm 2): the proof can be easily adapted to $n(> 2)$ number of firms. Suppose that there exists an equilibrium in which firm i ($= 1, 2$) occupies $q_i(> 0)$ number of slots with $q_1 + q_2 = k$. This implies that firm i proposes a bundle $B_i^{q_i}$ composed of its q_i best products. Given that 2 occupies only q_2 number of slots, 1 has monopoly power over the remaining slots and hence will choose

$$P_1^{q_1} = \sum_{j=1}^{q_1} u_1^j.$$

For the same reason, 2 will also extract the whole surplus from its best q_2 products.

$$P_2^{q_2} = \sum_{j=1}^{q_2} u_2^j.$$

However $(B_i^{q_i}, P_i^{q_i})$ cannot be an equilibrium. Since firm 1, for instance, has an incentive to sell a bundle that includes its k best products and charge a price

$$P_1^k = \sum_{j=1}^k u_1^j - \varepsilon$$

where $\varepsilon > 0$ is small enough. Therefore, we find a contradiction. ■

Therefore, we assume that each firm proposes a menu of bundles.

6.2 When all firms use slotting contracts

Suppose that all firms use slotting contracts. Since now buying a bundle requires D to distribute all products in the bundle, it may be profitable for a firm i to induce D to buy a bundle bigger than $B_i^{q_i^{fb}}$ to crowd out his opponents. In extreme cases, i may try to occupy all of D's slots with his own products by setting P_i^1, \dots, P_i^{k-1} very high and choosing P_i^k to induce D to buy B_i^k , as it occurs in the following example.

Example 3 (slotting contracts and inefficiency) Suppose that $n = 3, k = 3, c = 0$ and

$$(u_1^1, u_1^2, u_1^3) = (10, 8, 6); \quad (u_2^1, u_2^2, u_2^3) = (9, 7, 1); \quad (u_3^1, u_3^2, u_3^3) = (9, 7, 1)$$

Here $FB = \{11, 21, 31\}$, so that each firm occupies only one slot. However, an inefficient NE exists, in which P_i^j are high for $i = 1, 2, 3$ and $P_1^3 = 7, P_2^3 = P_3^3 = 0$. In words, each

firm offers only the big bundle and Bertrand competition among B_1^3, B_2^3, B_3^3 determines the above prices. In this NE, firm 1 occupies the three slots and realizes a profit larger than the profit in the efficient equilibrium. The reason for having this inefficient equilibrium is that firm 2 and firm 3 fail to coordinate by not offering menu of bundles. For instance, if 2 offers a bundle of the two best products at price equal to 0.4 and 3 offers the best product at price equal to 0.4, D will reject 1's offer even if 1 charges zero price.²¹

It is worthwhile to compare Example 3 with Proposition 2, which shows that efficiency is always attained for small values of c , under no slotting contracts. The reason for why such a result does not hold with slotting contracts is that for a firm with a product $ij \in FB$ which goes unsold it may be impossible to induce D to buy that product (ie, to buy a bundle which includes ij). Indeed, in Example 3, firm 1 does not allow D to buy B_1^1 and B_1^2 by setting P_1^1 and P_1^2 very high and then buying product 21 would require D to buy no product at all from 1; that implies a loss for D which is not compensated by the profit she can make on product 21, or on other products of firm 2.

Notwithstanding Example 3, we can show that an efficient NE always exists, also with slotting contracts, and it is outcome equivalent to the NE of Proposition 1 for the setting without slotting contracts. In order to describe this NE, we define

$$\hat{p}_i^j = \max\{c, u_i^j - \max\{u_{-i}^{k-j+1} - c, 0\}\} \quad \text{for } j = 1, \dots, k, \quad \text{for } i = 1, \dots, n$$

These prices are used to find the prices of the bundles B_1^1, \dots, B_1^k :

$$\hat{P}_i^j = \hat{p}_i^1 + \dots + \hat{p}_i^j \quad \text{for } j = 1, \dots, k, \quad \text{for } i = 1, \dots, n \quad (3)$$

We prove below that a NE exists such that each firm i chooses prices according to (3), but we first study a few properties of this price schedule. In particular, we prove that $\hat{p}_i^1 \geq \dots \geq \hat{p}_i^{q_i^{fb}} > c$ and $\hat{p}_i^{q_i^{fb}+1} = \dots = \hat{p}_i^k = c$. As a consequence, \hat{P}_i^j is increasing and "concave" for j in $\{1, \dots, q_i^{fb}\}$, and is "linear" with slope c for j in $\{q_i^{fb} + 1, \dots, k\}$.

First, we notice that $\hat{p}_i^j \geq \hat{p}_i^{j+1}$ and, in particular if $u_{-i}^{k-j+1} \leq c$, then u_{-i}^{k-j+1} does not matter and $\hat{p}_i^j = \max\{c, u_i^j\}$. Furthermore, we find that $\hat{p}_i^{q_i^{fb}} > c$, since either $u_{-i}^{k-q_i^{fb}+1} > c$ and then $\hat{p}_i^{q_i^{fb}} = u_i^{q_i^{fb}} - u_{-i}^{k-q_i^{fb}+1} + c > c$, or $u_{-i}^{k-q_i^{fb}+1} \leq c$ and then $\hat{p}_i^{q_i^{fb}} = u_i^{q_i^{fb}} > c$. This property, together with that \hat{p}_i^j decreases with j , implies that $\hat{p}_i^j > c$ for $j = 1, \dots, q_i^{fb}$. If $j > q_i^{fb}$, then either $u_i^j < c$ or $\hat{p}_i^j < \hat{p}_i^{k-j+1}$ (ie, $u_i^j < \max\{c, u_{-i}^{k-j+1}\}$); in either case, $\hat{p}_i^j = c$.

²¹We can also provide an example of an inefficient equilibrium with duopoly. Suppose $n = 2, k = 3$ and $c = 0$ and let the values for firms 1 and 2 be $(10, 8, 6)$ and $(9, 7, 1)$ as in example 3. Consider the profile of strategies in which both firms choose high prices for their bundles, except for the big bundles, priced at $P_1^3 = 7, P_2^3 = 0$. Then this is a NE.

Proposition 4 (*menu of bundles and slotting contracts*) *In the simultaneous pricing game with slotting contracts in which each upstream firm offers a menu of bundles, there exists a NE in which each firm i chooses the schedule (3), and D buys $B_1^{q_1^{fb}} \& \dots \& B_{n^{fb}}^{q_{n^{fb}}^{fb}}$, paying $\hat{P}_i^{q_i^{fb}} = \hat{F}_i + cq_i^{fb}$ to firm $i = 1, \dots, n^{fb}$.*

Step 1 When firms play $\{\hat{P}_1^j\}_{j=1}^k, \dots, \{\hat{P}_n^j\}_{j=1}^k$, D buys $B_1^{q_1^{fb}} \& \dots \& B_{n^{fb}}^{q_{n^{fb}}^{fb}}$.

First, it is obvious that if D buys $B_1^{q_1} \& \dots \& B_n^{q_n}$, then $q_1 + \dots + q_n = k$ because $q_1 + \dots + q_n < k$ implies $q_1 < q_1^{fb}$ (for instance) and D can increase his profit by buying $B_1^{q_1+1}$ as $\hat{p}_1^{q_1+1} \leq u_1^{q_1}$. Second, we can prove that $q_i = q_i^{fb}$ for $i = 1, \dots, n^{fb}$. If $q_1 < q_1^{fb}$ (for instance) then let $v_{-1}^1, \dots, v_{-1}^{\bar{q}}$ be defined as in the proof of Step 1 in the proof of Proposition 1, and notice that $\bar{q} = k - q_1$; in order to fix the ideas, suppose that the product with value $v_{-1}^{k-q_1}$ is j -th best product of firm 2, with $j > q_2^{fb}$. Then D increases her payoff by buying $B_1^{q_1+1} \& B_2^{q_2-1} \& \dots \& B_n^{q_n}$ rather than $B_1^{q_1} \& B_2^{q_2} \& \dots \& B_n^{q_n}$, because the difference in D 's profit is $u_1^{q_1+1} - \hat{p}_1^{q_1+1} - (v_{-1}^{k-q_1} - p_2^j) = u_1^{q_1+1} - (u_1^{q_1+1} - u_{-1}^{k-q_1} + c) - (v_{-1}^{k-q_1} - c) = u_{-1}^{k-q_1} - v_{-1}^{k-q_1}$ and $u_{-1}^{k-q_1} \geq v_{-1}^{k-q_1}$ by definition of $u_{-1}^{k-q_1}$: the best $(k - q_1)$ -th product of firms different from 1, while $v_{-1}^{k-q_1}$ is the best $(k - q_1)$ -th best product among the products D buys from firms different from 1. Also notice that, for $i = 1, \dots, n^{fb}$, $\hat{P}_i^{q_i^{fb}} = U_i^{FB} - (U_{-1}^{SB} - cs_1) = \hat{F}_i + cq_i^{fb}$, so that each firm makes the same profit as in the NE of Proposition 1.

Step 2 When the firms different from i play according to (3), there exists no strategy of firm i which yields i a profit larger than \hat{F}_i .

Let $\tilde{q}_2, \dots, \tilde{q}_n$ be such that $B_2^{\tilde{q}_2} \cup \dots \cup B_n^{\tilde{q}_n} = FB_2 \cup \dots \cup FB_{n^{fb}} \cup SB_{-1}$. Then it is possible to verify that D 's profit from buying $B_2^{\tilde{q}_2} \& \dots \& B_n^{\tilde{q}_n}$ is $\alpha \equiv \sum_{i=2}^n \sum_{t=1}^{\tilde{q}_i} u_i^t - \hat{P}_2^{\tilde{q}_2} - \dots - \hat{P}_n^{\tilde{q}_n} = \sum_{i=2}^{n^{fb}} \sum_{t=1}^{q_i^{fb}} u_i^t + \sum_{t=1}^{s_1} (u_{-1}^{k-q_1^{fb}+t} - c) - \hat{P}_2^{q_2^{fb}} - \dots - \hat{P}_{n^{fb}}^{q_{n^{fb}}^{fb}}$, where the equality holds because $p_i^j = c$ for any $j > q_i^{fb}$. This is the same payoff D earns if she buys $B_1^{q_1^{fb}} \& \dots \& B_{n^{fb}}^{q_{n^{fb}}^{fb}}$: $\sum_{i=2}^{n^{fb}} \sum_{t=1}^{q_i^{fb}} u_i^t - \hat{P}_2^{q_2^{fb}} - \dots - \hat{P}_{n^{fb}}^{q_{n^{fb}}^{fb}} + \sum_{t=1}^{q_1^{fb}} u_1^t - (\sum_{t=1}^{q_1^{fb}} u_1^t - \sum_{t=1}^{s_1} (u_{-1}^{k-q_1^{fb}+t} - c))$. Then we prove that there is no deviation of 1 such that D buys $B_1^{\tilde{q}_1} \& \dots \& B_n^{\tilde{q}_n}$ with $P_1^{\tilde{q}_1} > \hat{P}_1^{q_1^{fb}} + c(q_1 - q_1^{fb})$. Indeed, if this were the case then the resulting payoff for D would be at least α , but since $\hat{P}_1^{\tilde{q}_1} \leq \hat{P}_1^{q_1^{fb}} + c(\tilde{q}_1 - q_1^{fb})$,²² we infer that without deviation D would obtain a payoff higher than α by buying $B_1^{\tilde{q}_1} \& \dots \& B_n^{\tilde{q}_n}$. ■

We have already mentioned that there is a strict connection between this NE and the NE of Proposition 1 in the game without slotting contracts, and the two NE are outcome equivalent. This connection is due to the fact that, in both NE, D can buy the products which are outside FB at the constant price c , and this property determines the profit each

²²This is obvious if $\tilde{q}_1 > q_1^{fb}$, while for $\tilde{q}_1 < q_1^{fb}$ we have $p_1^{\tilde{q}_1} = u_1^{\tilde{q}_1} - \max\{u_{-1}^{k-q_1^{fb}+1} - c, 0\} > c$ and thus $\hat{P}_1^{\tilde{q}_1} < \hat{P}_1^{q_1^{fb}} + c(q_1 - q_1^{fb})$ for $q_1 < q_1^{fb}$.

firm is able to earn. In order to make clearer the link, readers can verify that in the setting of Section 5, the profile such that $F_i = 0$ and $p_{ij} = \hat{p}_i^j$ is a NE.

In addition, in the case of digital good, we can show that all NE exist if we restrict each firm to offer a price schedule $\{\hat{P}_i^j\}$ that is weakly increasing in j . This restriction makes sense since if D buys less, D weakly pays less.

Proposition 5 *Suppose that each upstream firm offers a menu of bundles and uses slotting contracts, In the case of Digital good, if each firm is constrained to offer a price schedule $\{\hat{P}_i^j\}$ that is weakly increasing in j , all NE are efficient.*

Proof. To be written. ■

6.3 When each firm can decide whether to use slotting contracts or not

In this subsection, we consider the game in which each upstream firm simultaneously decides (i) whether to use slotting contracts or not and (ii) accordingly offers a bundle or a menu of bundle. Note that both decisions ((i) and (ii)) occur simultaneously. We show that the strategy profile $(\hat{\sigma}_1, \dots, \hat{\sigma}_n)$ that we consider in section 5 is a NE for any $c \geq 0$ even if a firm can use slotting contracts.

Proposition 6 *The strategy profile $(\hat{\sigma}_1, \dots, \hat{\sigma}_n)$ is a NE for any $c \geq 0$ even if each firm can use slotting contracts.*

Proof. The proof of proposition 1 applies to this proposition as well even if a firm can deviate by using slotting contracts. ■

Proposition 6 shows that if each upstream firm i proposes a single bundle in which it charges \hat{F}_i for the right to buy and $\hat{p}_{ij} = c$ for each product ij , no single firm has an incentive to use a slotting contract to force D to buy more than the efficient number. Given that proposing such a single bundle is much simpler than proposing a menu of bundles with prices described by (3), if small firms are concerned about a big firm's abuse of slotting contracts, it might be better for them not to use any slotting contracts.

7 Bundling and horizontal merger

To be written

8 Conclusion

To be written

9 References

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