

Essential interest-bearing money

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Abstract

In the quasi-linear model of Lagos and Wright [A unified framework for monetary theory and policy analysis, *J. Polit. Econ.* 113 (2005) 463–484], money is essential and—if lump-sum taxation is feasible—the Friedman rule implements the first-best allocation. In this paper, I impose the additional restriction of voluntary trade; so that lump-sum taxation is infeasible. Despite this added restriction, I show that the first-best remains implementable under an incentive-feasible monetary policy. Under an incentive-feasible policy, at least some agents are willing to contribute a lump-sum payment that resembles a voluntary lump-sum tax. The set of incentive-feasible policies necessarily entails a non-negative inflation rate and a strictly positive nominal rate of interest.

Key words: Money; Interest; Friedman rule; Voluntary trade; Incentive-feasible policies; Efficient implementation

1 Introduction

In this paper, I examine a dynamic quasi-linear environment in which money is essential; see Lagos and Wright [4]. Absent policy intervention, the monetary equilibrium is inefficient. That is, the real rate of return on money is too low; so that individuals are motivated to economize excessively (from a social perspective) on their real money balances.

If lump-sum taxation is feasible, then this distortion can be corrected by the Friedman rule policy of equating the real return on money with the rate of time-preference. In principle, this might be accomplished with zero inflation and a strictly positive nominal interest rate; with the government's interest obligation financed by way of lump-sum taxation. But it can also be accomplished in the standard manner; i.e., with a zero nominal interest and a defla-

tion generated by way of lumps-sum contractions of the money supply. That is, interest-bearing money is not essential.

In environments where all trade is restricted be voluntary, lump-sum taxation is an infeasible policy instrument. I am motivated to ask what this additional restriction implies in terms of efficient implementation. A reasonable conjecture is that the constrained-efficient policy entails zero intervention (at least, this was my own prior). But I demonstrate below that this in fact not the case; i.e., there exist policies that can strictly improve on the allocation associated with zero intervention. Indeed, I demonstrate that the first-best remains implementable under a suitably designed policy.

The restriction to voluntary trade imposes restrictions on the nature of an optimal policy. In particular, policies are now constrained to be incentive-feasible (so that lump-sum payments, if they are to be made, must be individually-rational). I demonstrate that the standard Friedman rule prescription of deflating at the rate of time-preference is no longer incentive-feasible. In particular, the class of incentive-feasible policies necessarily entails a non-negative inflation rate and a strictly positive nominal interest rate. It is in this sense then that interest-bearing money (and inflation) is essential.

2 The Environment

The economy is populated by a continuum of *ex ante* identical agents, distributed uniformly on the unit interval and indexed by $i \in [0, 1]$. Each period $t = 0, 1, 2, \dots, \infty$ is divided into two subperiods; which for convenience are labeled *day* and *night*. Agents meet at a central location in both subperiods.¹

All agents have common preferences and abilities during the day. Let $x_t(i) \in \mathbb{R}$ denote the consumption (interpreted as production, if negative) of output in the day by agent i at date t . The key simplifying assumption is that preferences are linear in this term (transferable utility). As output produced in the day is nonstorable, an aggregate resource constraint implies:

$$\int x_t(i) di \leq 0; \tag{1}$$

for all $t \geq 0$.

At night, agents realize a shock that determines their type for the night. In particular, agents either have either a desire to consume or an ability to produce; assume that this occurs with equal probability. Moreover, assume

¹ Many papers in this literature assume instead that trade is decentralized (random pairwise meetings) in one of the subperiods. My main conclusions below are not sensitive to which of these structures is adopted.

that this stochastic process is *i.i.d.* across agents and across time. Refer to these types as *consumers*, and *producers*, respectively.

A consumer has utility $u(c)$ and a producer has utility $-g(y)$; where $c \in \mathbb{R}_+$ and $y \in \mathbb{R}_+$ denote consumption and production of the night good, respectively. Assume that $u'' < 0 < u'$, $\lim_{c \rightarrow 0} u'(c) = \infty$ and $g', g'' > 0$ with $\lim_{y \rightarrow 0} g'(y) = 0$. As the night good is also nonstorable, there is another aggregate resource constraint given by:

$$\int c_t(i) di \leq \int y_t(i) di; \quad (2)$$

for all $t \geq 0$.

As agents are *ex ante* identical, their preferences are represented by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{x_t(i) + 0.5 [u(c_t(i)) - g(y_t(i))]\}. \quad (3)$$

where $0 < \beta < 1$. Note that there is no discounting across subperiods.

Weighting all agents equally, a planner maximizes (3) subject to the resource constraints (1) and (2). As utility is linear in $x_t(i)$, agents are indifferent across any lottery over $\{x_t(i) : t \geq 0\}$ that delivers a given expected value. Without loss of generality, a planner may set $x_t(i) = 0$ for all i and all $t \geq 0$. Since g is strictly convex, all producers will be required to produce a common level of output $y \geq 0$. Given the strict concavity of u , all consumers will be allocated a common level of consumption $c \geq 0$. As the population is divided equally among producers and consumers at night, the resource constraint (2) implies $c = y$. Hence, conditional on a given level of y (and invoking the fact that $E_t[x_t(i)] = 0$), *ex ante* welfare is represented by:

$$W(y) = 0.5(1 - \beta)^{-1} [u(y) - g(y)]. \quad (4)$$

Clearly, there is a unique maximizer $0 < y^* < \infty$ satisfying:

$$u'(y^*) = g'(y^*). \quad (5)$$

In what follows, I refer to y^* as the *first-best* allocation. Associated with this allocation is any lottery over $x_t(i)$ that generates $E_t[x_t(i)] = 0$.

I impose the following additional restrictions on the environment. First, I assume that agents lack commitment. Second, I assume that agents are anonymous in the sense that their individual trading (and reporting) histories can be costlessly falsified (alternatively, there is no record-keeping technology). The second restriction rules out reputational equilibria; and together with the first restriction renders any form of private debt worthless. Third, I assume that society (the government) can create durable, divisible, and non-counterfeitable tokens. Fourth, I assume that all trade must be voluntary. And finally, I restrict trade among individuals to occur in competitive spot markets (though

exchange between individuals and the government is not restricted in this manner).

3 Market Structure, Timing, and Policy

As agents are anonymous and lack commitment, no form of valued private debt can exist in equilibrium. Hence credit transactions must take the form of *quid-pro-quo* swaps of fiat money (a form of social credit) for output. Agents begin time endowed with some money; this initial distribution can be arbitrary. Let (v_1, v_2) denote the (non-zero and finite) value of money in the day and night markets, respectively.

Government policy will be described in detail below, but it will be useful to explain here the aspects of policy that are relevant for individual decision-making. The government's policy rule operates at the beginning of the day, prior to day-market trading. In particular, an agent who enters the day with money balances m has the *option* of transforming these balances into $Rm - T$ units of money. Here, R is the gross nominal interest rate paid on money balances and T is a *voluntary* lump-sum tax. In other words, money is like an interest-bearing bond subject to a redemption fee. If an agent declines the redemption option, he simply enters the day-market with m units of money.

Following day-market activity, agents carry money into the night and realize their types (whether producer or consumer). Subsequent to night-market activity, agents carry any remaining money balances forward to the next day, where they are once again presented with the option of redeeming their money for interest.

4 Individual Decision-Making

4.1 The Day-Market

Let $m_1 \geq 0$ denote an agent's money balances at the beginning of a day; and let $m_2 \geq 0$ denote the money balances carried forward into the night. Let $\omega \in [0, 1]$ denote the probability of exercising the available redemption option. Subsequent to this choice, the agent is free to purchase or sell utility x at the going market price v_1 ; the day budget constraint is given by,

$$x = v_1 [\omega(Rm_1 - T) + (1 - \omega)m_1 - m_2]. \quad (6)$$

It will be convenient to transform variables from nominal to real terms. To this end, define $a \equiv v_1 m_1$ and $q \equiv v_2 m_2$. As well, let $\tau \equiv v_1 T$ and define $\phi \equiv v_1/v_2$. The day budget constraint can now be expressed as,

$$x = [\omega(Ra - \tau) + (1 - \omega)a - \phi q]. \quad (7)$$

I seek a recursive representation of an agent's choice problem. Let $D(a)$ represent the agent's maximum value function at the beginning of the day with real money balances $a \geq 0$; and let $N(q)$ represent the agent's maximum value function at the beginning of the night (prior to realizing his type) with real money balances $0 \leq q \leq \bar{q}$, where \bar{q} is an arbitrarily large, but finite, upper bound. These two value functions must satisfy the following recursion,

$$D(a) \equiv \max_{\omega, q} \{[\omega(Ra - \tau) + (1 - \omega)a - \phi q] + N(q)\}. \quad (8)$$

I make the following assumption,

[A1] *The function $N : \mathbb{R}_+ \rightarrow \mathbb{R}$ is twice continuously differentiable and satisfies $N'' < 0 < N'$; with $\phi < N'(0)$.*

As N and ϕ are equilibrium objects, I will have to verify later on that the properties assumed in **[A1]** are in fact valid.

The quasi-linear structure here simplifies matters considerably. In particular, the redemption choice ω and money demand q can be characterized independently of each other. The optimal redemption choice satisfies,

$$\hat{\omega} = \begin{cases} 1 & \text{if } (R - 1)a > \tau; \\ \omega \in [0, 1] & \text{if } (R - 1)a = \tau; \\ 0 & \text{if } (R - 1)a < \tau. \end{cases} \quad (9)$$

That is, assuming that $R > 1$ and $\tau > 0$, only agents with sufficiently large money balances a will find it individually-rational to pay the lump-sum "tax" τ .

[A1] implies that the demand for real money balances $0 < \hat{q} < \infty$ is determined uniquely by,

$$\phi = N'(\hat{q}). \quad (10)$$

As emphasized by Lagos and Wright [4], the demand for money at this stage is independent of initial money holdings a , so that all agents enter the night with identical money balances. Unlike Lagos and Wright [4], the value function D is not (in general) linear in a . In particular,

$$D'(a) = \begin{cases} R & \text{if } (R - 1)a > \tau; \\ 1 & \text{if } (R - 1)a < \tau. \end{cases} \quad (11)$$

That is, by the Theorem of the Maximum, D is continuous in a . But if $R > 1$ and $\tau > 0$, D will be piece-wise linear (and convex) in a ; and non-differentiable at the point $a = (R - 1)^{-1}\tau$.

4.2 The Night-Market

4.2.1 Consumers

Let $C(\hat{q})$ denote the value associated with being a consumer, entering the night-market with real money balances \hat{q} . This money is used to make purchases of output $0 \leq y_c \leq \bar{y} < \infty$ at the prevailing price-level v_2^{-1} . Hence, future money balances must obey $m_1^+ = m_2 - v_2^{-1}y_c$, with the further restriction that $m_1^+ \geq 0$. Expressed in real terms, this constraint is given by $a_c^+ = (v_1^+/v_1) \phi(\hat{q} - y_c) \geq 0$. The choice problem can be stated as,

$$C(\hat{q}) \equiv \max_{y_c, a_c^+} \left\{ u(y_c) + \beta D(a_c^+) : a_c^+ = (v_1^+/v_1) \phi(\hat{q} - y_c) \geq 0 \right\}. \quad (12)$$

Let (\hat{y}_c, \hat{a}_c^+) denote the solution to this problem. The solution here is at the very least an upper hemi-continuous correspondence; but in what follows, I anticipate that—in equilibrium—the relevant range for \hat{a}_c^+ will fall below the critical value $(R - 1)^{-1}\tau$. In other words, a consumer will not find it desirable to exercise his future redemption option. As D is differentiable below this range, the solution will in this case be a pair of functions. In fact, I will go even further here in assuming that the solution to (12) is characterized by a binding cash-constraint; i.e.,

$$[\mathbf{A2}] \quad \hat{y}_c = \hat{q} \text{ and } \hat{a}_c^+ = 0.$$

If the conjecture **[A2]** is valid, then it implies that consumers returning to the day-market will find it individually-rational not to pay taxes. By the envelope theorem, we have

$$C'(\hat{q}) = u'(\hat{y}_c). \quad (13)$$

4.2.2 Producers

Let $P(\hat{q})$ denote the value associated with being a producer, entering the night-market with real money balances \hat{q} . If a producer makes sales of output $0 \leq y_p \leq \bar{y} < \infty$ at the prevailing price-level v_2^{-1} , his future money balances are given by $m_1^+ = m_2 + v_2^{-1}y_p$. In real terms, this constraint is given by $a_p^+ = (v_1^+/v_1) \phi(\hat{q} + y_p)$. Clearly, the constraint $a_p^+ \geq 0$ will not bind in this case; and the choice problem may be formulated as,

$$P(\hat{q}) \equiv \max_{y_p, a_p^+} \left\{ -g(y_p) + \beta D(a_p^+) : a_p^+ = (v_1^+/v_1) \phi(\hat{q} + y_p) \right\}. \quad (14)$$

Let (\hat{y}_p, \hat{a}_p^+) denote the solution to this problem. In this case, I anticipate that **[A3]** $\hat{a}_p^+ > (R - 1)^{-1}\tau$ whenever $R > 1$.

If the conjecture **[A3]** is valid, then it implies that producers returning to the

day-market will find it individually-rational to pay taxes.

Note that by (11), $D'(\hat{a}_p^+) = R$. Hence, the supply of output at night $0 < \hat{y}_p < \bar{y}$ is characterized by,

$$g'(\hat{y}_p) = \phi \left(v_1^+ / v_1 \right) R\beta. \quad (15)$$

Moreover, by the envelope theorem,

$$P'(\hat{q}) = \phi \left(v_1^+ / v_1 \right) R\beta. \quad (16)$$

4.2.3 Gathering Restrictions

The *ex ante* value function associated with entering the night-market with money balances \hat{q} is given by $N(\hat{q}) \equiv 0.5 [C(\hat{q}) + P(\hat{q})]$. Employing the envelope results (13) and (16), we have,

$$N'(\hat{q}) = 0.5 \left[u'(\hat{y}_c) + \phi \left(v_1^+ / v_1 \right) R\beta \right]. \quad (17)$$

Note that if condition **[A2]** holds, then the function N essentially inherits the properties of u ; in other words,

Lemma 1 *Condition [A2] implies [A1].*

Now, combining (9) with (17) and employing (15),

$$\phi = 0.5 [u'(\hat{y}_c) + g'(\hat{y}_p)]. \quad (18)$$

Multiply both sides of (18) by $(v_1^+ / v_1) R\beta$, so that

$$\phi \left(v_1^+ / v_1 \right) R\beta = \left(v_1^+ / v_1 \right) R\beta 0.5 [u'(\hat{y}_c) + g'(\hat{y}_p)];$$

or, by again employing (15),

$$g'(\hat{y}_p) = \left(v_1^+ / v_1 \right) R\beta 0.5 [u'(\hat{y}_c) + g'(\hat{y}_p)].$$

Finally, rearrange the expression above in the following manner,

$$\left(v_1^+ / v_1 \right) R\beta u'(\hat{y}_c) = \left[2 - \left(v_1^+ / v_1 \right) R\beta \right] g'(\hat{y}_p). \quad (19)$$

5 Government Policy

Recall that as a matter of operating procedure, the government intervenes at the beginning of each day, prior to day-market trading. The policy rule is to pay a nominal interest rate R on money balances presented for redemption to agents willing to pay a lump-sum redemption fee T .

Let M denote the supply of money during any given period; with M^- denoting “previous” period’s money supply. Condition **[A2]** implies that at the

beginning of any day, producers hold the entire previous-period money supply M^- . Condition **[A3]** implies that producers will find it individually rational to pay the lump-sum tax T . Hence, the government faces an aggregate interest obligation $(R - 1)M^-$ and an aggregate revenue source $0.5T$.

The government may also print new money $M - M^-$. Hence, any feasible government policy will have to satisfy the government budget constraint, $(R - 1)M^- = M - M^- + 0.5T$; or

$$RM^- = M + 0.5T;$$

Let μ denote the (gross) rate of money supply expansion, so that $M = \mu M^-$. Hence, the government budget constraint can now be expressed as $T = [R/\mu - 1] 2M$. Multiplying both sides of this latter expression by v_1 and by noting that $v_1 M \equiv \phi q$, the government budget constraint can alternatively be expressed in real terms by,

$$\tau = [R/\mu - 1] 2\phi q. \quad (20)$$

In what follows, I refer to (R, μ) as a government policy. An incentive-feasible policy is a government policy that satisfies (20), together with conditions **[A2]** and **[A3]**.

6 Equilibrium

In this section, I characterize the steady-state monetary equilibrium conditional on a given incentive-feasible government policy.

To begin, market-clearing at night implies,

$$\hat{y}_c = \hat{y}_p = \hat{y}. \quad (21)$$

Combining (21) with (18) implies,

$$\hat{\phi} = 0.5 [u'(\hat{y}) + g'(\hat{y})]. \quad (22)$$

By condition **[A2]**, consumers exhaust their money balances at night; i.e., $\hat{q} = \hat{y}$. Hence, the equilibrium value of money at night must satisfy,

$$\hat{v}_2 = \hat{y}/M. \quad (23)$$

As $\hat{v}_1 \equiv \hat{\phi}\hat{v}_2$, and as stationarity implies $\hat{y} = \hat{y}^+$, it follows that,

$$\left(\hat{v}_1^+/\hat{v}_1\right) = \left(\hat{v}_2^+/\hat{v}_2\right) = (1/\mu). \quad (24)$$

Combining (21) and (24) with (19) yields,

$$\left(\frac{R\beta}{\mu}\right) u'(\hat{y}) = \left[2 - \left(\frac{R\beta}{\mu}\right)\right] g'(\hat{y}). \quad (25)$$

Condition (25) characterizes the equilibrium level of night-market output \hat{y} conditional on an incentive-feasible policy. With \hat{y} so determined, condition (22) characterizes the equilibrium relative price $\hat{\phi}$. Since $\hat{q} = \hat{y}$, condition (20) then delivers an expression for the equilibrium tax,

$$\hat{\tau} = [R/\mu - 1] \hat{\phi} 2\hat{y}. \quad (26)$$

Finally, consider the equilibrium distribution of real money balances at the beginning of the day. By condition **[A2]**, $\hat{a}_c^+ = 0$. Producers augment their money balances by their sales, so that $\hat{a}_p^+ = (\hat{\phi}/\mu)2\hat{y}$. Hence, the steady-state distribution for a is given by,

$$F(a) = \begin{cases} 0.5 & \text{for } 0 \leq a < (\hat{\phi}/\mu)2\hat{y}; \\ 1 & \text{for } (\hat{\phi}/\mu)2\hat{y} \leq a < \infty. \end{cases} \quad (27)$$

6.1 Existence

The key policy variable in this environment is $\delta \equiv R/\mu$; i.e., the real rate of return on government money. The effect of this policy parameter on the equilibrium level of output at night is described by (25); which I reproduce here for convenience,

$$\delta\beta u'(\hat{y}) = [2 - \delta\beta] g'(\hat{y}). \quad (28)$$

This restriction implicitly defines a function $\hat{y}(\delta)$ on the domain $(0, \beta^{-1})$.

It is trivial to show that a monetary equilibrium exists under a policy of zero intervention; i.e., $R = \mu = 1$. Under zero intervention, the equilibrium level of output produced at night is given by $\hat{y}(1) \equiv y_0$, where $0 < y_0 < y^*$ and the equilibrium tax is $\hat{\tau} = 0$. The cash-constraint for consumers is known to bind in this case; so that **[A2]** holds. Condition **[A3]** is irrelevant here, as $R = 1$.

In fact, it should be clear that there exists a class of incentive-feasible policies (R, μ) satisfying $\delta = R/\mu = 1$ and $R > 1$ that implement the zero intervention allocation y_0 as an equilibrium. That is, as the real rate of return on money remains equal to one, the cash-constraint for consumers binds as before; so that **[A2]** holds. The equilibrium tax in this case is also $\hat{\tau} = 0$; that is, the government's interest obligation in this case is financed entirely by new money creation. As producers carry strictly positive money balances into the day, condition **[A3]** must necessarily hold when $\hat{\tau} = 0$.

The interesting question, of course, is whether it might be possible to implement an allocation that improves upon y_0 . As *ex ante* welfare $W(y)$ is strictly increasing in y over the range $[y_0, y^*)$, and as (28) implies that $\hat{y}(\delta)$ is strictly

increasing in δ , in what follows I restrict attention to policies that satisfy $\delta > 1$. In addition, I restrict attention to policies that satisfy $\delta\beta < 1$; since otherwise, a monetary equilibrium will fail to exist.² Together then, the relevant range of policies is given by,

$$1 < \delta < \beta^{-1}. \quad (29)$$

7 Welfare-Improving Incentive-Feasible Policies

The equilibrium allocation \hat{y} characterized by (28) is predicated on the validity of **[A2]** and **[A3]**. I now turn to checking the validity of these assumptions for any given policy (R, μ) satisfying (29). For any such policy, condition (26) implies that $\hat{\tau} > 0$.

Consider first **[A3]**, which asserts that producers at night will find it optimal to exercise the redemption option the next day. In the proposed equilibrium, producers enter the day-market with real money balances $\hat{a}_p^+ = (\hat{\phi}/\mu)2\hat{y}$. By (9), exercising the redemption option is individually-rational if and only if $(R - 1)(\hat{\phi}/\mu)2\hat{y} \geq \hat{\tau}$; or, by appealing to (26),

$$(R - 1)(\hat{\phi}/\mu)2\hat{y} \geq (\delta - 1)\hat{\phi}2\hat{y}.$$

Simplifying, we have $(\delta - \mu^{-1}) \geq (\delta - 1)$; or $\mu \geq 1$. Producers will *strictly* prefer to exercise their redemption option if,

$$\mu > 1. \quad (30)$$

The restriction to policies that ensure strict rather than weak preference has the added technical benefit of allowing me to avoid the non-differentiability in the value function D that occurs at the point $(R - 1)^{-1}\hat{\tau}$; i.e., see (11).

In any case, if lump-sum taxation is to be individually-rational, deflationary policy is not an option. In particular, the “standard” Friedman rule prescription of setting $(R, \mu) = (1, \beta)$ is not incentive-feasible. Under the policy of fixing the real return on money $\delta = R/\mu$, inflation is superneutral (in the payoff relevant variables); in particular, the equilibrium real tax $\hat{\tau}$ is invariant to μ . And while a high inflation lowers the real value of money brought into the day, the correspondingly higher nominal return is more than enough to offset this effect.

Consider next **[A2]**, which asserts that consumers at night will be cash-constrained (so that exercising their future redemption option is necessarily suboptimal). Imagine, by way of contradiction, that consumers are not cash-constrained; i.e., so that $\hat{q} > \hat{y}$. In this case, we must consider the possibility that consumers carry a sufficiently large quantity of real money balances forward to make exercising the redemption option optimal. Hence, one of two

² That is, except in the limit as $\delta \nearrow \beta^{-1}$.

things must be true; i.e.,

$$\text{Case 1: } \hat{a}_c^+ < (R - 1)^{-1}\hat{\tau};$$

$$\text{Case 2: } \hat{a}_c^+ > (R - 1)^{-1}\hat{\tau};$$

where $\hat{a}_c^+ = (\hat{\phi}/\mu)(\hat{q} - \hat{y})$. (I ignore the razor's edge case).

As consumers are not cash-constrained, their demand for output at night determined by (12) must satisfy $u'(\hat{y}_c) = (v_1^+/v_1)\phi\beta D'(\hat{a}_c^+)$; or, by employing equilibrium prices, $u'(\hat{y}_c) = (\hat{\phi}/\mu)\beta D'(\hat{a}_c^+)$. Referring to (11), note that $D'(\hat{a}_c) = 1$ in Case 1 and $D'(\hat{a}_c) = R$ in Case 2. Moreover, note that by (17), the value function N is linear in \hat{q} whenever consumers are not cash-constrained.

Let me consider Case 1 first. In this case, $N'(\hat{q}) = 0.5 [(\hat{\phi}/\mu)\beta + \hat{\phi}\delta\beta]$. Existence of a monetary equilibrium requires that $0 < \hat{q} < \infty$ satisfy condition (10), so that

$$\hat{\phi} = 0.5 [(\hat{\phi}/\mu)\beta + \hat{\phi}\delta\beta].$$

But this can only be true if $\delta = 1/\mu$; a condition that is violated by the fact that $\delta > 1$ and $\mu > 1$.

Let me now consider Case 2. In this case, $N'(\hat{q}) = \hat{\phi}\delta\beta$. Once again, existence of a monetary equilibrium requires $0 < \hat{q} < \infty$ satisfying condition (10), so that

$$\hat{\phi} = \hat{\phi}\delta\beta.$$

But this condition is violated by the fact that $\delta\beta < 1$; i.e., see (30). Thus, for the range of policy parameters considered here, [A2] must hold.

Hence, it appears that policy parameters constrained to satisfy (29) and (30), the conjectured properties [A2] and [A3] are in fact properties of the (stationary) monetary equilibrium characterized above. The main conclusion then is summarized by the following proposition.

Proposition 1 *Under the range of incentive-feasible policies described by (29) and (30), there exists a stationary monetary equilibrium with a night allocation $y_0 < \hat{y}(\delta) < y^*$ characterized by (28).*

Note that as $\hat{y}(\delta) \nearrow y^*$ as $\delta \nearrow \beta^{-1}$, it follows as a corollary that under an appropriately designed policy, the equilibrium allocation \hat{y} can be made arbitrarily close to the first-best.

8 Discussion

I have analyzed above a quasi-linear environment in which money is essential. Absent intervention, the monetary equilibrium is inefficient relative to the first-best. It is well-known that if lump-sum taxation is feasible, then this

inefficiency can be corrected by the standard Friedman rule prescription of deflating at the rate of time-preference. In its more general form, the Friedman rule asserts that efficiency is restored by any policy that equates the real return on money to the rate of time-preference. This is obviously consistent with paying interest on money; as long as the government finances at least some of its interest obligations by way of lump-sum taxation. However, interest-bearing money is not essential.

If the environment is restricted so that all trade must be voluntary, then lump-sum taxation is infeasible. Financing nominal interest payments entirely by money creation cannot work either; as a “Fisher effect” leaves the real return on money unchanged. A reasonable conjecture is that type-contingent transfers of money at night (in particular, directed toward the cash-constrained consumers) might serve to improve welfare. Berentsen, Camera, and Waller [2] and Andolfatto [1], however, demonstrate that this cannot be the case; at least, not for the environment considered here.

In light of these considerations, it seems natural to conclude that when trade is restricted to be voluntary, the monetary equilibrium associated with zero intervention is constrained-efficient. Such a conclusion, however, appears not to be warranted; as it ignores the possibility that policy may be designed in a manner to render lump-sum tax payments individually-rational. When this is so, not all of the payment of interest on money needs to be monetized. This implies that inflation does not increase one-for-one with an increase in the nominal interest rate on money, allowing the government to implement a higher than unity real return on money holdings.

The main conclusion then is that an incentive-feasible monetary policy (that improves welfare beyond what is achievable with zero intervention) must entail a strictly positive nominal interest/inflation rate. It is in this sense that interest-bearing money (and inflation) is essential.

The conclusion here is not to be confused with that of Kocherlakota [3]; who demonstrates the essentiality of an interest-bearing (illiquid) nominal bond. In Kocherlakota’s environment, a policy of type-contingent money transfers can improve welfare. If types are private information, such transfers are infeasible. Kocherlakota [3] then demonstrates how the introduction of an illiquid bond can replicate what might otherwise have been achieved by way of type-contingent transfers. However, as I have alluded above, type-contingent transfers are “neutral” in the environment considered here. It follows as a corollary that an illiquid bond can play no welfare-enhancing role here; again, see Andolfatto [1].

There is, however, a relationship between my paper and that of Berentsen, Camera, and Waller [2]. These authors, who examine an environment similar to the one considered here, also make the case for essential interest-bearing money. In their model, this is accomplished by introducing a “bank” in the day-market that pays interest on deposits of cash from producers and redirects

these funds to consumers in the form of interest-bearing loans. For this solution to work, the bank must be endowed with at least a limited record-keeping technology. While modifying the environment in this manner seems entirely reasonable, my Proposition 1 suggests that it is not essential to do so if policy is designed correctly.

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