Electronic intermediation and two-sided markets: what happens when side-switching is possible?*

Pierre Gazé and Anne-Gaël Vaubourg†

Laboratoire d'Économie d'Orléans
Faculté de Droit, Economie, Gestion
Rue de Blois, BP 6739
45 067 Orléans cedex 02
France

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†Corresponding author, e-mail: Anne-Gael.Vaubourg@univ-orleans.fr

Abstract

We define electronic platforms as two-sided markets in which agents can switch from one side to the other side of the market. We consider two platforms in a duopoly through which sellers and buyers interact during two successive sessions. Between both sessions, some sellers become buyers and vice-versa. Equilibrium participation fees and profit are interpreted in terms of rewards and penalties, relatively to the equilibrium without side-switching. We also show that equilibrium fees and profit increase with relative mobility provided this mobility is large enough. We finally demonstrate that the equilibrium profit also rises with global mobility.

JEL Codes: L11, L13, L86

Key words: Electronic intermediation, externalities, side-switching, two-sided markets

1 Introduction

In two-sided markets (TSM), at least two groups of agents interact through an intermediary called a platform. As each group gives value to the participation of the other group, two-sided markets are characterized by a specific class of network externalities, called cross externalities. Rochet & Tirole (2003) and Evans (2003) review many industries that exhibit such a feature. Videogame platforms, credit card payment systems, newspapers and dating agencies provide well-known examples of TSM. As pointed out by Armstrong (2006) in a duopoly model, the participation of agents from one group crucially depends on how the TSM is attractive for agents belonging to the other group. This implies that participation (or registration) fees, i.e. fees charged to agents when they access the platform, are lower than they would be without any externality effects. Armstrong (2006) also shows that participation fee charged to agents from one group decreases with the valuation of their participation by agents from the opposite group[1].

For many years, the Internet network has given an opportunity for several existing TSM to be developed or reorganized. It is the case for auctions, traveling services or media industries. Some activities even owe their existence to the Internet network. Massively Multiplayer Online Role-Playing Games (MMORPG), the virtual purse or some types of electronic money belong to this category. In both cases (creating or developing preexisting activities), Internet is a new way of making transactions. On-line markets are clearly attractive to consumers in terms of selection, availability and prices, compared to their physical counterparts, thereby explaining their rapid growth. On-line retail sales in the United-States reached 114 billions in 2006, up from 93 billions in 2005 (an increase of 22.7%). In the US, e-commerce sales in the first quarter of 2007 accounted for 3.2 percent of total retail sales (source: Monthly retail Trade Survey). This trend is similar in Europe with 42 billions euros being spent in 2006 in the United Kingdom, 22 billions euros in Germany and 12 billions euros in France (source: Forrester Research, Inc.).

The specificities of Internet TSM have been examined by Caillaud & Jullien (2001, 2003). They argue that electronic intermediaries are able to monitor transactions, which allows them to charge not only participation fees but also transaction fees, i.e. fees that are paid by consumers each time a transaction occurs. The authors also argue that due to Internet platforms, agents get the opportunity to make multihoming, i.e. to connect to several platforms. Caillaud & Jullien (2001) show that in a duopoly with registration fees and single-homing, there exists an asymmetric equilibrium in which a dominant firm captures the whole market and earns positif profit. When transaction fees are added to participation fees, the profit

[1]The literature also proposes theoretical analysis of specific markets. Credit card payments have been analysed by Rochet & Tirole (2002), Guthrie & Wright (2003) and Chakravorti & Roson (2006) while Gabszewicz, Laussel & Sonnac (2001, 2004), Anderson & Gabszewicz (2006), Ferrando, Gabszewicz, Laussel & Sonnac (2008) and Kaiser & Wright (2006) devote a special attention to the media industry. For complete surveys, see Roson (2005) and Rochet & Tirole (2006).

becomes zero. This findings also holds in a symmetric equilibrium (Caillaud & Jullien (2003)). When the single-homing assumption is alleviated, the dominant firm's profit becomes zero and both sides of the market are offered a free access to the platform. In the symmetric equilibrium exhibited by Caillaud & Jullien (2003), the result is quite different: both platforms earn positive profit and only one group of agents benefits from free participation.

In this paper, we propose another specificity of electronic TSM. The main idea of our work is that electronic intermediaries are platforms in which buyers can easily become sellers and vice-versa[2]. For example, in the Internet-based virtual world Second Life, each gamer can buy or sell items (virtual or real commodities or services) to improve his satisfaction such that the group of sellers and the group of buyers are in perpetual reformation. Some identical characteristics prevail in the famous auction website eBay and in other electronic marketplaces such as Price Minister or Webidz. On these platforms, agents can buy or sell items according to their needs and desires without any (or with light) institutional constraints. Switching is easy, reversible and costless. The most well-known electronic payment PayPal also has this peculiar design. PayPal is a person-to-person on-line payment instrument designed for any type of monetary transfer such as auctions (eBay), gifts, etc. between platform members. PayPal users can receive or send funds indifferently, without beeing identified as a merchant or as a customer once and for all. What is very different from traditional on-line payment systems is that agents do not need any complex hardware or software device to receive funds: electronic mail is enough. We conceive a model to analyze these specific classes of TSM. Extending the Armstrong (2006)'s framework, we examine what happens when agents can migrate from one group to another and we investigate how mobility affects platforms' equilibrium prices and profits.

The remainder of the paper is organized as follows. In Section 2, we present the model. The equilibrium is characterized in Section 3. Section 4 deals with the comparative statics analysis of the equilibrium. Section 5 concludes the article.

2 The model

2.1 Assumptions

Following Armstrong (2006), we consider two platforms i and j in a duopoly as well as two groups of agents, sellers and buyers, denoted respectively s and b.

Agents are uniformly located on a unit segment while platforms are located at each of its extremities. Agents incur a unit transport cost, denoted t. This cost is supposed to be the same for sellers and buyers, which is consistent with our idea

[2]It is true that side-switching can exist in physical TSM. For example, in flea markets, agents can switch from the buyer side to the seller side several times within a day. Nevertheless, this is not a crucial characteristic of physical TSM since examples are very rare and the extent of switching probably modest.

that electronic TSM are particularly flexible: whether an agent is seller or buyer does not affect the cost he pays to connect to platforms. For platforms, the cost of providing the matching service to sellers and to buyers is f_s and f_b respectively.

The agents' utility increases with the number of agents from the other group: a seller's valuation for the participation of one buyer is given by α_s while a buyer's valuation for the participation of one seller is α_b[3].

We now introduce the crucial assumption of our model. It aims at accounting for the specificity of virtual platforms, in which sellers can easily become buyers and *vice versa*. For example, an agent who initially bought a camera on eBay may then buy a CD player or sell a piece of furniture on the same platform. This feature characterizes electronic second hand markets, in which private individuals and collectors participate without any definitive seller's or buyer's status. In our model the period during which sellers offer their items on-line while buyers browse advertisements is called a matching session, whether a transaction finally takes place or not.

We assume that agents choose one of the two platforms to participate in *two* successive and independent sessions. These two matching sessions are denoted M_1 and M_2 respectively. Between M_1 and M_2, a proportion β of initial sellers become buyers and a proportion λ of initial buyers become sellers[4]. These mobility rates are exogeneous. In the remainder of the paper, we will call initial sellers (resp. initial buyers) agents who are sellers (resp. buyers) during M_1, whatever their type during M_2.

Let $p_{s,i}$ (resp. $p_{s,j}$) be the price charged to sellers and $p_{b,i}$ (resp. $p_{b,j}$) the price charged to buyers by the platform i (resp. j). These prices, also called participation fees, are fixed and independent of the outcome of the matching and the amount of the potential transaction.

We finally denote $n^1_{s,i}$ (resp. $n^1_{s,j}$) the number of sellers and $n^1_{b,i}$ (resp. $n^1_{b,j}$) the number of buyers on the platform i (resp. j) during M_1 with $n^1_{s,i}+n^1_{s,j} = n^1_{b,i}+n^1_{b,j} = 1$. Similarly, $n^2_{s,i}$ (resp. $n^2_{s,j}$) is the number of sellers and $n^2_{b,i}$ (resp. $n^2_{b,j}$) the number of buyers on the platform i (resp. j) during M_2.

Turning to agents' utilities, we call $U^1_{s,i}$ (resp. $U^1_{s,j}$) the utility obtained by a seller and $U^1_{b,i}$ (resp. $U^1_{b,j}$) the utility obtained by a buyer on the platform i (resp. j) during M_1. We also refer to $U^2_{s,i}$ (resp. $U^2_{s,j}$) as the expected utility get by an initial seller on the platform i (resp. j) during M_2 and to $U^2_{b,i}$ (resp. $U^2_{b,j}$) as the expected utility get by an initial buyer on the platform i (resp. j) during M_2.

2.2 Timing of actions

The timing of actions is as follows:

[3]We do not account here for negative intra-group externalities among agents belonging to the same group. For an analysis of this phenomenon, see Belleflamme & Toulemonde (2007).

[4]Buyers and sellers cannot enter or exit the market between the two sessions. Although this restriction is not very realistic, modeling a net flow of agents into the market between M_1 and M_2 would introduce useless complexity and could mask the effects of side-switching on the equilibrium.

(i) Each agent knows if he will participate in M_1 as a seller or as a buyer. No agent knows if he will remain in his initial group or if he will switch between M_1 and M_2[5]. We suppose that all agents and both platforms know the mobility rate of each group.

(ii) Platforms then set the prices that sellers and buyers will pay respectively for their participation in M_1 and M_2. These fees are set once and for all and do not depend on whether the session is M_1 or M_2[6].

(iii) Agents then choose the platform through which they will participate in *both* sessions[7].

(iv) Each agent identifies himself as a seller or as a buyer for session M_1 and pays his participation fee to his platform. Session M_1 takes place.

(v) A proportion β of initial sellers become buyers and a proportion λ of initial buyers become sellers.

(vi) Each agent identifies himself as a seller or as a buyer for session M_2 and pays his fee. Session M_2 takes place.

We consider a two-stage game. In the first stage, the two platforms compete in prices. In the second stage, each agent chooses its platform for both sessions.

3 Equilibrium

3.1 The second-stage subgame: agents choose between the two platforms

We first examine the agents' choice between the two platforms. During M_1, agents' utilities are as follows:

$$U_{s,i}^1 = (\alpha_s n_{b,i}^1 - p_{s,i}),$$
$$U_{s,j}^1 = (\alpha_s n_{b,j}^1 - p_{s,j}),$$
$$U_{b,i}^1 = (\alpha_b n_{s,i}^1 - p_{b,i}),$$
$$U_{b,j}^1 = (\alpha_b n_{s,j}^1 - p_{b,j}).$$

The agents' expected utility during M_2 depends on whether they switch or not. For example, if an initial seller does not switch, he enjoys the participation of buyers;

[5] This assumption illustrates current situations in which one discovers an electronic marketplace from the buyer or the seller side without knowing if one will be a buyer or a seller for the next session.

[6] In accordance with reality, electronic marketplaces set prices for a given period and do not continuously adjust them according to agents' side switching.

[7] The case in which agents choose a platform before each session can be easily solved assuming that agents are involved in a game *à la* Armstrong (2006) two consecutive times.

if he switches, he values the participation of sellers. We thus have:

$$\begin{aligned}
E(U^2_{s,i}) &= (1-\beta)(\alpha_s n^2_{b,i} - p_{s,i}) + \beta(\alpha_b n^2_{s,i} - p_{b,i}),\\
E(U^2_{s,j}) &= (1-\beta)(\alpha_s n^2_{b,j} - p_{s,j}) + \beta(\alpha_b n^2_{s,j} - p_{b,j}),\\
E(U^2_{b,i}) &= (1-\lambda)(\alpha_b n^2_{s,i} - p_{b,i}) + \lambda(\alpha_s n^2_{b,i} - p_{s,i}),\\
E(U^2_{b,j}) &= (1-\lambda)(\alpha_b n^2_{s,j} - p_{b,j}) + \lambda(\alpha_s n^2_{b,j} - p_{s,j}).
\end{aligned}$$

Since agents are supposed to choose their platform for the two sessions, their choice depends on total expected utility, defined as follows:

$$\begin{aligned}
E(U_{s,i}) &= (\alpha_s n^1_{b,i} - p_{s,i}) + (1-\beta)(\alpha_s n^2_{b,i} - p_{s,i}) + \beta(\alpha_b n^2_{s,i} - p_{b,i}), && (1)\\
E(U_{s,j}) &= (\alpha_s n^1_{b,j} - p_{s,j}) + (1-\beta)(\alpha_s n^2_{b,j} - p_{s,j}) + \beta(\alpha_b n^2_{s,j} - p_{b,j}), && (2)\\
E(U_{b,i}) &= (\alpha_b n^1_{s,i} - p_{b,i}) + (1-\lambda)(\alpha_b n^2_{s,i} - p_{b,i}) + \lambda(\alpha_s n^2_{b,i} - p_{s,i}), && (3)\\
E(U_{b,j}) &= (\alpha_b n^1_{s,j} - p_{b,j}) + (1-\lambda)(\alpha_b n^2_{s,j} - p_{b,j}) + \lambda(\alpha_s n^2_{b,j} - p_{s,j}). && (4)
\end{aligned}$$

Since they choose their platform before M_1 once and for all, agents incur the transport cost according to their initial type, whatever their type during M_2[8]. As in Armstrong (2006), we use the well-known Hotelling specification to determine agents' participations. We obtain

$$\begin{aligned}
n^1_{s,i} &= \frac{1}{2} + \frac{E(U_{s,i}) - E(U_{s,j})}{2t}, && (5)\\
n^1_{s,j} &= \frac{1}{2} + \frac{E(U_{s,j}) - E(U_{s,i})}{2t}, && (6)\\
n^1_{b,i} &= \frac{1}{2} + \frac{E(U_{b,i}) - E(U_{b,j})}{2t}, && (7)\\
n^1_{b,j} &= \frac{1}{2} + \frac{E(U_{b,j}) - E(U_{b,i})}{2t}. && (8)
\end{aligned}$$

Contrary to a situation with no mobility, the population of sellers and of buyers evolves between M_1 and M_2. For example, the population of sellers during M_2 results from the switching of some initial buyers and from the immobility of some initial sellers. We thus have

$$\begin{aligned}
n^2_{s,i} &= (1-\beta)n^1_{s,i} + \lambda n^1_{b,i}, && (9)\\
n^2_{s,j} &= (1-\beta)n^1_{s,j} + \lambda n^1_{b,j}, && (10)\\
n^2_{b,i} &= (1-\lambda)n^1_{b,i} + \beta n^1_{s,i}, && (11)\\
n^2_{b,j} &= (1-\lambda)n^1_{b,j} + \beta n^1_{s,j}. && (12)
\end{aligned}$$

[8]Alternatively, we could assume that transport costs are not paid before M_1 but before each session. This would imply to determine, for each initial type of agents, an average transport cost weighted by mobility rates. But this does not qualitatively affect our results.

3.2 The first-stage subgame: platforms compete in prices

Platforms set equilibrium prices $p_{s,i}*$, $p_{b,i}*$, $p_{s,j}*$ and $p_{b,j}*$ according to the following program:

$$\begin{cases} \{p^*_{s,i}, p^*_{b,i}\} = \text{ArgMax}\Pi_i = \text{ArgMax}(p_{s,i} - f_s)(n^1_{s,i} + n^2_{s,i}) + (p_{b,i} - f_b)(n^1_{b,i} + n^2_{b,i}), \\ \{p^*_{s,j}, p^*_{b,j}\} = \text{ArgMax}\Pi_j = \text{ArgMax}(p_{s,j} - f_s)(n^1_{s,j} + n^2_{s,j}) + (p_{b,j} - f_b)(n^1_{b,j} + n^2_{b,j}). \end{cases}$$

where Π_i and Π_j denote plateforms' expected profits over the two sessions.

Owing to the complexity of the maximization program, we use the computation software Maple to solve it analytically[9]. Ensuring that the second-order condition is satisfied, we find that the determinant of the Hessian matrix is positive for sufficiently large values of t relatively to α_s and α_b. We obtain the following proposition:

Proposition 1. *For β and λ simultaneously different from 1 and for t high enough, there exists an equilibrium, which is unique and symmetric. Equilibrium prices are given by*

$$p^*_{s,i} = p^*_{s,j} = p^*_s = \hat{p}^*_s + \Delta p^*_s, p^*_{b,i} = p^*_{b,j} = p^*_b = \hat{p}^*_b + \Delta p^*_b,$$

$$\textit{with } \hat{p}^*_s = f_s + \frac{1}{2}t - \alpha_b,\ \hat{p}^*_b = f_b + \frac{1}{2}t - \alpha_s,$$

$$\Delta p^*_s = \frac{-\beta\frac{1}{2}(\lambda-\beta)\alpha_s - 2(1-\beta)\frac{1}{2}(\beta-\lambda)\alpha_b + \lambda\frac{1}{2}(\beta-\lambda)\alpha_b}{(2-\beta-\lambda)},$$

$$\Delta p^*_b = \frac{-\lambda\frac{1}{2}(\beta-\lambda)\alpha_b - 2(1-\lambda)\frac{1}{2}(\lambda-\beta)\alpha_s + \beta\frac{1}{2}(\lambda-\beta)\alpha_s}{(2-\beta-\lambda)}.$$

Since our equilibrium is symmetric, we restrict our discussion to p^*_s. $\hat{p}^*_s$ is the equilibrium price without side-switching ($\lambda = \beta = 0$). As pointed out by Armstrong (2006), it is decreasing with the externality coefficient of buyers[10].

Δp^*_s captures the effect of the mobility assumption on the equilibrium[11] . The crucial idea behind Δp^*_s is that the access to platforms is priced according to a game of rewards and penalties, *relatively to the equilibrium without side-switching.* According to the respective size of rewards and penalties, p^*_s can be lower or larger than $\hat{p}^*_s$. Under the mobility assumption, the populations of sellers and buyers during M_2 will be different from what they are in the equilibrium without side-switching: one of the two populations will be larger while the other one will be smaller. Agents are rewarded for the externalities they will exert on what we will

[9]The Maple file is available upon request to the authors.

[10]Note also that in $\hat{p}^*_s$ the transport cost is divided by 2 since it is paid once for both sessions. Under the assumption that transport costs are paid twice (before each session), we obtain Armstrong (2006)'s equilibrium price: $\hat{p}^*_s = f_s + t - \alpha_b$.

[11]When β and λ simultaneously tend toward 1, Δp^*_s and Δp^*_b tend to 0.

call "the larger population" and penalized for the externalities they will exert on "the smaller population". The identification of the larger and the smaller population depends on the sign of $\frac{1}{2}(\lambda-\beta)$ and $\frac{1}{2}(\beta-\lambda)$, which measure respectively the number of sellers and buyers during M_2, relatively to the equilibrium without mobility[12]. If $\lambda-\beta>0$, the larger population will be the sellers' one and the smaller population will be the buyers' one; if $\lambda-\beta<0$, it is the reverse.

Keeping in mind these preliminary remarks, we can now analyze Δp_s^* in details. The first term of the numerator is the reward or the penalty charged to an initial seller for its possible side-switching: if he becomes a buyer (with probability λ), his participation will be enjoyed during M_2 by sellers, for a total value of $\frac{1}{2}(\lambda-\beta)\alpha_s$ (the unit sellers' externality coefficient multiplied by the number of sellers relatively to the equilibrium without mobility). If $\lambda-\beta>0$ for example, the first term of the numerator is negative. It thus represents a reward for initial sellers, which is consistent with the fact that their participation as buyers during M_2 will be valued by the larger population (sellers).

The second term refers to the reward or the penalty charged to an initial seller for its potential immobility: if he remains a seller, its participation will be enjoyed during M_2 by buyers. If $\lambda-\beta>0$, this term is negative. It thus represents a penalty for initial sellers, which is consistent with the fact that their participation as sellers during M_2 will be valued by the smaller population (buyers). Since this term is the only one that accounts for the possibility to be a seller *during two sessions*, it is weighted twice as much as other terms.

The third term is the reward or the penalty charged to an initial buyer for its potential side-switching. If he becomes a seller, its participation will be enjoyed by buyers. If $\lambda-\beta>0$, this term is positive in the expression of Δp_b^*: initial buyers are penalized, which is consistent with the fact that their participation as sellers during M_2 will be valued by the smaller population (buyers). This penalty is passed on Δp_s^* under the shape of a reward (since it appears with the opposite sign). This means that an initial buyer is first penalized during M_1 for his potential side-switching (through the charging of Δp_b^*) before being compensated during M_2 by a reward (through the charging of Δp_s^*) if he actually switches.

Finally, as $\lambda+\beta$ measures global mobility, the division by $2-(\lambda+\beta)$ can be interpreted as a way to adjust the three terms described above by the distance between our equilibrium and the one withoutside-switching.

Substituting equilibrium prices in profit expressions allows us to derive the following proposition:

[12]Substituting equilibrium prices in expressions (1) to (8), we obtain: $n_{s,i}^{1*}=n_{s,j}^{1*}=n_{b,i}^{1*}=n_{b,j}^{1*}=\frac{1}{2}$. It comes from (9), (10), (11) and (12) that with mobility we have: $n_{s,i}^{2*}=n_{s,j}^{2*}=\frac{1}{2}(1-\beta+\lambda)$ and $n_{b,i}^{2*}=n_{b,j}^{2*}=\frac{1}{2}(1-\lambda+\beta)$ while without side-switching, we obtain: $\hat{n}_{b,i}^{2*}=\hat{n}_{b,j}^{2*}=\frac{1}{2}$. Hence, $\frac{1}{2}(\lambda-\beta)$ (resp. $\frac{1}{2}(\beta-\lambda)$) is the (positive or negative) equilibrium number of sellers (resp. buyers) on each platform during M_2, relatively to the equilibrium without mobility.

Proposition 2. *Platforms' equilibrium profit is given by*

$$\Pi_i^* = \Pi_j^* = \Pi^* = 2\hat{\Pi}^* + 2\Delta\Pi^*,$$

$$\textit{with } \hat{\Pi}^* = \frac{t - \alpha_s - \alpha_b}{2}, \;\; \Delta\Pi^* = \frac{-(1-\lambda)\frac{1}{2}(\lambda-\beta)\alpha_s + (1-\beta)\frac{1}{2}(\lambda-\beta)\alpha_b}{(2-\beta-\lambda)}.$$

Proposition 2 states that as equilibrium prices, the platforms' equilibrium profit can be written as the sum of two terms. $\hat{\Pi}^*$ is the equilibrium profit without side-switching[13]. As noticed by Armstrong (2006), it decreases with agents' externality coefficients.

$\Delta\Pi^*$ captures the effect of the side-switching assumption on the equilibrium profit. The first term of the numerator measures the platforms' loss or gain ensuing from the buyers' *immobility*. When $\lambda - \beta > 0$ for example, for example, initial buyers are rewarded since, if they remain buyers, they will exert externalities on the sellers' population, which is the larger one. This represents a loss for platforms, which is consistent with the negative sign of this term. The second term measures the platforms' loss or gain ensuing from the sellers' potential immobility. When $\lambda - \beta > 0$, initial sellers are penalized since, if they remain sellers, they will exert externalities on the buyers' population, which is the smaller one. This implies a gain for platforms, which is consistent with the positive sign of this term.

According to the relative size of the reward and the penalty, Π^* may be higher or lower than $2\hat{\Pi}^*$. It is straightforward that $\Delta\Pi^* > 0$ if $\lambda + \beta > \xi$ with $\xi \equiv \frac{(\lambda-\beta)(\alpha_s+\alpha_b)}{\alpha_b-\alpha_s} + 2$. This implies that for sufficiently high (resp. low) values of global mobility, the equilibrium profit is higher (resp. lower) than without side-switching[14].

4 Comparative statics analysis

We now turn to the comparative statics analysis of the equilibrium. We first investigate the impact of a change in relative mobility. We then study the impact of a shift in global mobility.

4.1 The effect of relative mobility

Concerning the effect of relative mobility, measured as $|\lambda - \beta|$, we obtain the following proposition:

Proposition 3.

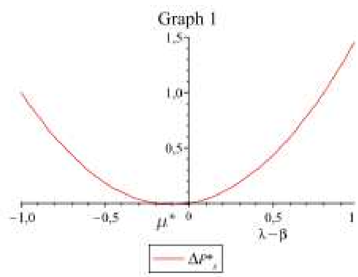
[13] As in Proposition 1, under the assumption that transport costs are paid before each session, we obtain the Armstrong (2006)'s equilibrium profit: $\hat{\Pi}^* = \frac{2t-\alpha_s-\alpha_b}{2}$.

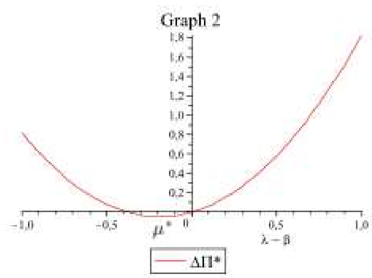
[14] As $\lambda + \beta \leq 2$, this condition is never satistied when $\xi > 2$ i.e. when $\frac{(\lambda-\beta)(\alpha_s+\alpha_b)}{\alpha_b-\alpha_s} > 0$. Π^* is thus always lower than $2\hat{\Pi}^*$ when the group having the highest externality parameter also has the lowest mobility rate ($\lambda - \beta > 0$ and $\alpha_s - \alpha_b > 0$ or $\lambda - \beta > 0$ and $\alpha_s - \alpha_b > 0$).

(a) There exists a threshold μ^* such that
if $-1 < \lambda - \beta < \mu^*$, equilibrium prices decrease with $\lambda - \beta$,
if $\mu^* < \lambda - \beta < 1$, equilibrium prices increase with $\lambda - \beta$.
(b) *There exists a threshold μ^{**} such that*
if $-1 < \lambda - \beta < \mu^{**}$, the platforms' profit decreases with $\lambda - \beta$,
if $\mu^{**} < \lambda - \beta < 1$, the platforms' profit increases with $\lambda - \beta$.

Proof. See Appendix.

Proposition 3 is illustrated in Graph 1 and Graph 2, in which the set of parameters is $\lambda + \beta = 1$, $\alpha_s = 1.5$ and $\alpha_b = 6$.





It establishes that platforms' prices and profit are increasing with relative mobility provided this mobility is large enough. To see that in details, let us examine Graph 2 (the same comment holds for Graph 3). Between -1 and μ^*, both $|\lambda - \beta|$ and Δp_s^* decrease. Hence Δp_s^* is an increasing function of relative mobility. Between μ^* and 0, $|\lambda - \beta|$ decreases while Δp_s^* increases. Δp_s^* is thus a decreasing function of relative mobility. Between 0 and 1, $|\lambda - \beta|$ and Δp_s^* increase such that Δp_s^* is an increasing function of relative mobility.

In the comments below, we will assume that $\lambda - \beta > 0$. We know from Proposition 1 and Proposition 2 that in this case, initial sellers are rewarded for their possible side-switching and penalized for their possible immobility while initial buyers are rewarded for their possible immobility and penalized for their possible side-switching. Note also that if $\lambda - \beta > 0$, an increase in relative mobility, *keeping global mobility unchanged*, corresponds to an increase in λ and a decrease in β.

The intuition behind Part (a) of Proposition 3 is as follows. As for Proposition 1, we limit our comment to p_s^*. Since $\hat{p}_s^*$ does not depend on relative mobility, we focus on Δp_s^*, considering each terms of its numerator successively. First, the fact that initial sellers are less likely to side-switch tends to reduce the reward they are charged for the externalities they will exert as buyers on the larger population (sellers) during M_2. But in the same time, these externalities will be more valued since the number of sellers during M_2 increases with $\lambda - \beta$. Under the combination of these two opposite effects, their reward may either decrease or increase. Second, the fact that initial sellers are less likely to side-switch tends to increase the penalty they are charged for the externalities they will exert as sellers on the smaller population (buyers). This effect is reinforced by the decreasing valuation of these externalities since the number of buyers during M_2 decreases with $\lambda - \beta$. Their penalty thus non ambiguously increases. Third, the fact that initial buyers are more likely to side-switch tends to increase the penalty they are charged for the externalities they will exert as sellers on the smaller population (buyers) during M_2. This effect is reinforced by the decreasing valuation of these externalities since the number of buyers during M_2 decreases. This leads to a rise in their penalty, which is passed on Δp_s^* under the shape of an increasing reward. The impact of relative mobility on Δp_s^* finally depends on the combination of all these effects. Part (a) of Proposition 3 states that for low levels of $\lambda - \beta$, a rise in relative mobility implies a decrease in Δp_s^* while for high levels of $\lambda - \beta$, it involves a rise in Δp_s^*.

To undertstand Part (b) of Proposition 3, remind that the platforms' equilibrium profit stems from agents' *immobility*. On the one hand, the fact that initial buyers are less likely to be immobile contributes to reduce the reward they are charged for the externalities they will exert as buyers on the larger population (sellers) during M_2. But as this effect is undermined by the increasing valuation of their participation during M_2, their reward may finally either decrease or increase. On the other hand, as explained above, the penalty charged to buyers for their possible immobility unambiguously rises with relative mobility. The impact of relative mobility on the equilibrium profit finally depends on the relative size of these two opposite effects. Part (b) of Proposition 3 states that below a certain threshold, a rise in relative mobility implies a decrease in the platforms' profit while above this threshold, it is the reverse.

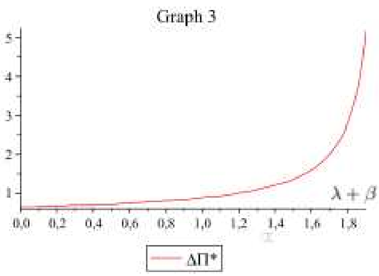
4.2 The effect of global mobility

Turning to the effect of global mobility, measured as $\lambda + \beta$, we have:

Proposition 4. The platforms' equilibrium profit increases with $\lambda + \beta$.

Proof. See Appendix.

Proposition 4 reports that both platforms make additional profit when global mobility rises. This is illustrated in Graph 3, in which the set of parameters is $\lambda - \beta = 0.4$, $\alpha_s = 1.5$ and $\alpha_b = 6$.



Of course, in a transaction fee regime, a change in global mobility stimulates business and profit earning. Proposition 4 reveals a more subtle effect that appears even in a participation fee regime. This phenomenon actually results from two effects. On the one hand, an increase in global mobility, *keeping relative mobility unchanged*, corresponds to an increase in λ and in β. As initial buyers are more likely to be immobile, they are less rewarded for the externalities they will exert as buyers on the larger population (sellers) during M_2. Similarly, as initial sellers are less likely to be immobile, they are less penalized for the externalities they will exert as sellers on the smaller population (buyers) during M_2[15]. The effect of global mobility on the numerator of $\Delta\Pi^*$ is thus undetermined. On the other hand, as global mobility rises, one moves away from the equilibrium without side-switching such that the denominator decreases. Proposition 4 states that the combination of all these effects finally results in an increase in the platforms' profit consecutively to a rise in global mobility.

5 Conclusion

In this paper, we define Internet platforms as two-sided markets in which agents can easily switch from one side to the other side of the market. To account for this specificity, we consider two platforms in a duopoly through which sellers and buyers match during two successive sessions. The crucial assumption of our paper is that each group is characterized by an exogenous mobility rate such that between the two sessions, some sellers become buyers and *vice-versa*.

We show that plateforms' equilibrium prices and profit are based on a game of rewards and penalties, relatively to the equilibrium without side-switching. The relative size of these rewards and penalties finally determines whether equilibrium prices and profit are higher or lower than without side-switching. We also identify global and relative mobility as sources of platforms' profit.

Side-switching in electronic TSM is a fruitful topic which deserves further investigation. First, our analysis could be refined by considering endogenous mobility

[15] Contrary to relative mobility, global mobility does not affect the size of populations during M_2. Hence the valuation of initial buyers' and initial sellers' participation during M_2 remains unchanged.

rates. Side-switching could notably depend on the skill acquired on one side of the market during previous sessions. For example, participating in a platform as a buyer can be a good way to learn how to better design a selling on-line advertisement (importance of pictures, choice of key-words etc). Symmetrically, a buyer can more easily estimate the seller's reservation price if he has been himself a seller during a previous session. It could be also interesting to account for the idea that side-switching possibilities enhance the attractiveness of platforms. For example, a consumer can be enticed to buy children's clothes or toys on a platform if he has in mind the opportunity to easily sell them later. This specificity could be integrated in the model as an increase in the quality of services delivered by platforms.

Appendix

Proof of Proposition 3.

Part (a). As the equilibrium is symmetric, we limit our attention to p_s^*. Since $\hat{p}_s^*$ does not depend on β and λ, we focus on Δp_s^*. To study it, we use an invertible change of variable. Setting $\theta = \lambda + \beta$ and $\mu = \lambda - \beta$ with $0 \leq \theta < 2$ and $-1 \leq \mu \leq 1$, we have $\beta = \frac{\theta+\mu}{2}$ and $\lambda = \frac{\theta-\mu}{2}$. It follows that

$$\Delta p_s^* = \Psi(\theta, \mu) = \frac{\mu(\alpha_s\theta - \alpha_s\mu + 3\alpha_b\theta - \alpha_b\mu - 4\alpha_b)}{4(\theta - 2)}.$$

Deriving $\Psi(\theta, \mu)$ with respect to μ yields

$$\frac{\delta\Psi(\theta, \mu)}{\delta\mu} = \frac{\alpha_s\theta - 2\alpha_s\mu + 3\alpha_b\theta - 2\alpha_b\mu - 4\alpha_b}{4(\theta - 2)}.$$

It is straighforward that

$$\frac{\delta\Psi(\theta, \mu)}{\delta\mu} = 0 \text{ for } \mu = \mu^* \equiv \frac{1}{2}\frac{(\alpha_b - \alpha_s)(\theta - 2)}{\alpha_b + \alpha_s}.$$

Therefore if $-1 < \mu < \mu^*$, then $\frac{\delta\Psi(\theta, \mu)}{\delta\mu} < 0$ and if $\mu^* < \mu < 1$, then $\frac{\delta\Psi(\theta, \mu)}{\delta\mu} > 0$.

Part (b). Resorting to the same invertible change of variable as above, we obtain

$$\Delta\Pi^* = \Phi(\theta, \mu) = \frac{\mu(-2\alpha_b - \alpha_b\mu - \alpha_s\theta + 2\alpha_s - \alpha_s\mu + \alpha_b\theta)}{2(\theta - 2)}.$$

It follows that

$$\frac{\delta\Phi(\theta, \mu)}{\delta\mu} = \frac{-2\alpha_b - 2\alpha_b\theta - 2\alpha_s\mu + 2\alpha_s - \alpha_s\theta + \alpha_b\theta}{2(\theta - 2)}$$

$$\text{and } \frac{\delta\Phi(\theta, \mu)}{\delta\mu} = 0 \text{ for } \mu = \mu^{**} \equiv \frac{\mu(\theta - 2)}{2\theta}.$$

Hence if $-1 < \mu < \mu^{**}$, then $\frac{\delta\Phi(\theta,\mu)}{\delta\mu} < 0$ and if $\mu^{**} < \mu < 1$, then $\frac{\delta\Phi(\theta,\mu)}{\delta\mu} > 0$.

Proof of Proposition 4. We start from $\Phi(\theta,\mu)$ defined in the proof of Proposition 3. Deriving $\Phi(\theta,\mu)$ with respect to θ yields

$$\frac{\delta\Phi(\theta,\mu)}{\delta\theta} = \frac{\mu^2(\alpha_s+\alpha_b)}{2(\theta-2)^2}.$$

It is straighforward that

$$\frac{\delta\Phi(\theta,\mu)}{\delta\theta} > 0.$$

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