Product Liability versus Reputation

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Abstract

Product Liability Law and the Tort process are socially costly tools to provide firms with incentives for safety. It has been argued that market reputation is, to a significant extent, a cheaper alternative to Product Liability. Our paper points out the sometimes overlooked fact that market forces inducing safety through reputation are not for free, but require to implement “market sanctioning” mechanisms that are costly for consumers and manufacturers. We show that Product Liability (the Law or formal legal mechanisms, more generally) positively affects the functioning of market reputation by reducing its costs. Thus, to an important extent, reputation and Product Liability are not substitutes but complements. We also specifically show the effects of different relevant legal policies, and namely that negligence reduces reputational costs more intensely than strict liability, and that legal and Court errors in determining liability interfere with the reputational cost reduction function of the Law. We complicate the basic analysis with endogenous prices and observability by consumers of the outcome of Court’s decisions. The analysis extends beyond safety in consumer markets and affects the overall interaction of reputation with formal (legal) instruments to influence behavior.

Keywords: Reputation, Relational Contracts, Product Liability and Consumer Protection.


*We are grateful to Kristoffel Grechenig, Luis Cabral, Jennifer Arlem, Cathy Sharkey, Jennifer Reinganum, Andrew Daughety, Lewis Kornhauser, Oren Bar-Gill, Mark Geistfeld, John Ferejohn, Martin Peitz and participants at seminars at Universitat Pompeu Fabra, NYU School of Law, Jornadas de Economia Industrial, Simposio de Análisis Económico, the American Association of Law and Economics, the Canadian Association of Law and Economics, and the Spanish Association of Law and Economics. The usual disclaimers apply.

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1 Introduction

This paper analyzes the interaction between market reputation (a form of implicit relational contract between the manufacturer of a product and consumers) and the law as tools to adequately address product hazards affecting consumers. The asymmetry of information between consumers and manufacturers with respect to the quality and safety of products generates incentive problems that may be solved or at least alleviated with either market reputation or the law. Thus, market reputation and the Law for product hazards -product liability- would appear to be alternative instruments for improving safety and quality in consumer markets.

Recently, Polinsky and Shavell (2010 a) have invigorated an important debate over the convenience of rethinking product liability, under the claim that it is a costly instrument and that part, even a large part, of its benefits in terms of incentives for safety may be achieved in a more efficient way by reputational forces influencing firms, and by public regulation. Others, in turn, have responded to this claim on different grounds (Goldberg and Zipursky 2010), and others have advanced further arguments, not focused on reputational incentives and public regulation (Rubin 2011; Viscusi 2011; Priest 2011). Our argument, however, has a broader scope of application, since reputation and law are tools to induce cooperation in a wide variety of settings. Klein and Laffer (1981) already argued that the informal, lawless instrument of market reputation is a less costly alternative to formal incentive schemes to induce cooperation in asymmetric information settings. Our analysis emphasizes that market reputation is not for free in social terms, and that organized legal instruments (tort and contract law, or regulation) may be very useful both to reduce the cost of relying on reputation to enhance desirable trade, but also to make cooperation

\footnote{Polinsky and Shavell. present in their original paper and in a reply to Goldberg and Zipursky (2010) [Polinsky and Shavell (2010 b)] other arguments concerning the overall cost-benefit assessment of product liability, and based on the compensation benefits of product liability, the incentives for safety flowing from ex ante public regulation, and legal and other costs stemming from product liability. We do not deal with any of these factors and thus we do not cast a vote in the “uneasy case for product liability” debate. Specially, we do not deal with ex ante regulation, since it does not affect the interaction between legal liability and reputation. Moreover, ex-ante regulation can be considered as providing the framework for the interplay between product liability and reputation, the tools that would deal with the incentive problems left yet unsolved by public ex ante regulation.}
sustainable in settings in which reputation on its own could not perform the trick of inducing socially advantageous trade. Specially when the informational asymmetry is severe, the firm’s surplus from future trade is not large, and the time horizon of many market participants is not long, the role of the legal system in encouraging cooperation becomes more relevant, perhaps essential.

Our ideas challenge the general validity of a claim that reputational incentives significantly weaken the justification of product liability, or legal liability more generally. First, that market reputation is not costless. Cooperation between consumers and firms in asymmetric information environments necessarily requires to implement punishment mechanisms that are costly for both consumers and firms. Second, that product liability reduces the "private" cost of market reputation. Technically, product liability allows to relax the incentive compatibility constraint for the functioning of market reputation. Thus, we agree with Polinsky and Shavell (2010) and others in a similar vein who claim, that the design of product liability law should take into account the existence and effectiveness of private instruments such as market reputation. However, this paper shows that such interaction does not necessarily imply that the level of legal liability should be reduced when market reputation is available.

Our approach is simple and, as indicated above, can be applied to other scenarios in which law and reputation are present. The core of the argument is the fact that consumers' knowledge that manufacturers may face potential legal liabilities for misbehavior\(^2\) facilitates cooperation between firms and consumers, since it reduces the need to rely on private "punishments" by consumers to deter manufacturers from "cheating" in safety or quality. When the existence of legal rules that may result in adverse consequences for the firm are common knowledge, the optimal reputational punishment goes down, and in equilibrium there will be trade for a larger range of parameter values.

\(^2\)This knowledge does not involve that of actual liabilities being imposed, simply the existence and features of the legal regime from which liabilities may ensue. When legal outcomes can be observed by consumers the effect is strengthened.
This complementarity between market reputation and legal rules, as far as we know, has not been fully recognized and analyzed before. There is a large body of literature on relational contracts and on the link between reputation and legal contract enforcement summarized in MacLeod (2007). However, most of this literature emphasizes substitution effects between both. Two papers show related but different complementary effects to the ones we want to analyze in this paper: Sobel (2006) compares partnerships supported through relational contracting and partnerships supported through formal legal institutions. The paper shows complementary effects in the form of opportunity costs of early cheating in relational contracts resulting from formal contract enforcement, thus increasing the number of such relationships. Greif (1994) provides historical evidence of a similar type of effect, showing how Genoese traders used formal contract enforcement to encourage new relationships, instead of using information sharing on past behaviour and ostracism to sanction opportunistic behaviour. The closest paper is Dhillon and Rigolini (2011), that also studies the interactions between formal and informal institution. Their focus, however, is not legal policy nor minimizing the cost of reputational sanctions. In particular, Dhillon and Rigolini (2011) analyzes, in a development context, an informal sanctioning mechanism, which may be reinforced by consumers' investment in being connected to other consumers, interacting with a formal enforcement mechanism which, in turn, may be made less effective by firms, through bribing activities. In their context, better informal enforcement reduces the incentives for bribing and indirectly, improves legal enforcement. This complementary effect link their project with ours.

A further contribution of our paper is to show how different legal policies affect the incentive compatibility constraints of firms, allowing us to compare them in terms of their effectiveness in reducing the private cost of reputation. In particular, we show that negligence is more effective for this purpose than strict liability. Our results are robust to two natural extensions of our framework, namely endogenous prices and observation by consumers of the outcome of product liability cases before they impose the market sanction.
Our modeling strategy is to choose a model of market reputation with imperfect information and analyze the effect of different product liability regimes. As we want to illustrate our main idea in the simplest possible way, we have adapted as a model of market reputation a simplified version of the collusion model of Green and Porter (1984) as presented in Tirole (1988) and Cabral (2005).\(^3\)

The model is as follows: Consumers cannot observe ex-ante the quality or safety of the product that determine the probability of accident. As increasing quality (reducing the probability of accident) is costly for the firm, in a static setting there is no trade, since only low quality would be produced in equilibrium. In a dynamic setting trade can be restored. The cooperative equilibrium in which firms produce high quality can be sustained by the consumers’ threat of refraining from buying the product in case that low quality or safety is ex-post detected. In case of perfect information (an accident perfectly reveals that the product is of low quality), cooperation can be implemented at zero cost if both players are patient enough. However, in the more realistic case in which producing high quality products does not completely eliminate the probability of malfunctioning and accidents, inducing cooperation requires incurring a disciplining cost to produce market reputation. This reputational cost is measured by the welfare loss in the number of periods in which there will be no trade after an accident happens in order to ”discipline” the manufacturer. This disciplining behavior by consumers is necessary to provide incentives to produce high quality, and they are also what we may call the “reputational costs” for inducing cooperation.

In this setting, we show that these ”reputational costs” are lower when the firm, in case an accident materializes, also faces legal liability towards the consumers who have been harmed by the product. This is because under the threat of product liability firms have more incentives to produce high quality, and this in turn reduces the need for consumers to rely on market disciplining.

\(^3\)In particular, Cabral (2005) presents a model of product safety and cooperation between firms and consumers very similar to our baseline model. This paper also points out that models based on repeated interaction should be denoted as trust models, while models based on bayesian updating should be called reputation models. We do not take a stance in this debate, but have decided to keep the term “reputation” since it seems to be more widely used.
measures. Moreover, we show that negligence rules are more effective in reducing the number of periods that are required for adequately punishing manufacturers. In the two extensions dealing with endogenous product prices and with information provided to the market by tort cases, the results also hold.

As explained before, these are just two theoretical - albeit important, in our view - dimensions to take into account when dealing jointly with reputation in the market and product liability. We do not attempt to measure the costs of actually imposing legal liability, or of running the legal system for this purpose, nor the actual costs of reputational mechanisms, so we do not intend to directly transplant our conclusions at the policy level. But we believe they require, together with other factors already discussed, appropriate consideration in order to understand the interaction between the legal system and market reputation.

The paper is organized as follows. In section 2 we present the basic model of reputation in our setting. In section 3, we characterize the equilibrium of the model. In section 4, we turn to the optimal equilibrium introducing product liability and establish the main results. In section 5, we show that, to an important extent, reputation and Product Liability are not substitutes but complements. Section 6 presents an extension of the basic model to endogenous prices. Section 7 considers the interaction between market reputation and legal liability when consumers observe the outcome of the tort process at the time of determining the market sanction. Section 8 contains a brief discussion of the implications, and concludes. All the proofs are relegated to a technical appendix.

2 The Model

We use a standard unilateral accident model with imperfect information. A competitive firm produces a good and chooses care (effort) in order to reduce the probability of accident when consumers use the product. In particular, we assume that the firm decides between two possible levels of care, \( e \in \{e, \bar{e}\} \). The choice of the firm (level of care) is private information (not observable...
by consumers). Exerting care is costly, $c_c < c_r$, and determines the probability of accident, $p_c > p_r$.

For simplicity and without loss of generality, we take $c_c = 0, c_r = c, p_c = 1$ and $p_r = \pi$. In case of accident, the representative consumer suffers a loss of $D$.

The firm may sell the product to a representative consumer with a willingness to pay for the good, $V$. In order to keep the model simple, we initially take as exogenous the price of the good, $P$. This is definitely a simplifying assumption, but it allows us to emphasize the basic link between market discipline and legal consequences. In section 4 we endogenize $P$, and show that beside the fact that prices will be influenced by the expectations about product failure, and by the legal regime, our main results hold.

Given this price, we assume that the representative consumer would buy the good if care is high, but not otherwise:

$$V - P - \pi D > 0 > V - P - D \iff D > V - P > \pi D.$$  

As supply has to be also profitable for the firm, the price also satisfies, $P \geq c$.\(^4\)

In a static framework, in which first the firm decides the level of care, and afterwards the consumer decides whether or not to buy the good without knowing the level of care, there would be no trade.

There are several ways to solve this market failure. We concentrate on two. On the one hand, the legal system through ex post regulation, tort liability or by enforcing explicit contracts, may provide sufficient incentives for the firm to exert high care, and trade will arise. On the other hand, without any intervention by the legal system, market reputation (a relational contract between the firm and its customers) may do the job. We now focus on this reputational mechanism by placing the interaction between the firm and the representative consumer in a dynamic framework.

\(^4\)In other words, we assume that the price is such that the participation constraint of the firm is satisfied. Thus, in the section 4 in which we consider a liability regime, this assumption will imply that the price also compensates the expected liability cost.
3 Market Reputation

Now we consider an infinite horizon framework with an infinitely lived firm and an infinitely lived representative consumer\(^5\), in which the basic game above is repeated over and over again. We start by assuming there is no legal liability, and contracts cannot be verified by a third party who could enforce a warranty provision. Then, only market reputation incentives are in place.

This repeated game has multiple equilibria, including the repetition of the solution of the static game. We will focus on equilibria supporting cooperation between the firm and the representative consumer. In particular, we consider the following grim strategy subgame perfect equilibrium inspired by Green and Porter (1984):

- Consumer starts trusting the firm in period 0, and buying the good at price \( P \).
- There is trade until consumer suffers an accident, i.e., firm chooses high effort and consumer trusts the firm by buying its products.
- When consumer observes an accident, she reacts by discontinuing to buy the product for \( T \) periods. After expiration of the \( T \) periods, the consumer is willing to buy the good from the firm again.

We will denote the missing trade surplus in \( T \) periods as the “cost of reputation” (below we formally justify this label). Both agents could be better off if they would not stop trading during the punishment phase. However, punishment is necessary to preserve incentives.

We are in a setting of ex-post imperfect information: the fact that an accident has occurred is an imperfect signal of the firm’s level of care. If the signal were perfect, then \( T \) could be infinite and the cost of reputation would be 0, since punishment is never imposed in equilibrium. In our setting, the imperfect information leads agents to incur a cost of reputation. We are going to

\(^5\) We could alternatively assume that there is an infinity sequence of one-period consumers who can observe the history of the game, under the additional assumption that consumers are able to coordinate in their punishment strategies.
concentrate on the “optimal” relational contract, the one that maximizes the number of periods in which trade occurs, or, equivalently, minimizes the number of periods in which the market sanction is imposed.

We assume that both agents face the same discount factor, \( \delta \in (0, 1) \). When consumer and firm play the strategy described above, let \( V^+ \) and \( V^- \) be the present discounted value of the firm profits depending on its level of care, high and low, respectively. We have:

\[
V^+ = P - c + (1 - \pi) \delta V^+ + \pi \delta V^-,
\]
\[
V^- = \delta^T V^+.
\]

Solving the equation system we obtain both present values in terms of the parameters of our model

\[
V^+ = \frac{P - c}{1 - (1 - \pi) \delta - \pi \delta^{T+1}}, \quad (1)
\]
\[
V^- = \delta^T V^+ = \frac{\delta^T (P - c)}{1 - (1 - \pi) \delta - \pi \delta^{T+1}}. \quad (2)
\]

Finally, to achieve this equilibrium we must add an incentive compatibility constraint. The following inequality captures the lack of incentives of the firm to choose low effort:

\[
V^+ \geq P + \delta V^-
\]

Using the definition of \( V^+ = P - c + (1 - \pi) \delta V^+ + \pi \delta V^- \), the incentive compatibility constraint can also be written as:

\[
(1 - \pi) \delta (V^+ - V^-) \geq c. \quad (3)
\]

We are interested in another equivalent expression for the inequality above, which can be found using the solution to the equation system \( V^+ \) and \( V^- \) (we plug equations (1) and (2) into (3)):

\[
(1 - \pi) \delta \frac{(1 - \delta^T)(P - c)}{1 - (1 - \pi) \delta - \pi \delta^{T+1}} \geq c.
\]
Let \( \Phi(T) \) be the left side of the incentive compatibility constraint above. For our purposes, this function has a useful property:

**Lemma 1** \( \Phi(T) \) is increasing in \( T \).

Hence, to solve optimally the infinitely repeated game, we want to choose \( T \) in order to maximize \( V^+ \).

\[
\max_T V^+ = \max_T \frac{P - c}{1 - (1 - \pi) \delta - \pi \delta^{T+1}}
\]
subject to the following constraint:

\[
\Phi(T) \geq c.
\]

Given that our function satisfies \( \frac{\partial V^+}{\partial T} < 0 \), then the optimal \( T^* \) for our problem will be the minimum \( T \) that satisfies the identity \( \Phi(T^*) = c \). But this equation has a unique solution, by Lemma ??.

The optimal punishment \( T^* \) refers to the welfare cost of reputation under imperfect information. Let \( W_{PI} = \frac{V - c - \pi D}{1 - \delta} \) be the welfare achieved under perfect information. Given \( T^* \), the present discounted value of welfare under imperfect information will be given by the following expression

\[
W_{IP} = V - c + (1 - \pi) \delta W_{IP} + \pi (\delta^{T^*+1} W_{IP} - D).
\]

Then,

\[
W_{IP} = \frac{V - c - \pi D}{1 - (1 - \pi) \delta - \pi \delta^{T^*+1}}.
\]

As the difference between welfare with perfect and with imperfect information, \( W_{PI} - W_{IP} \), is increasing in the optimal punishment \( T^* \), we denote \( T^* \) as the cost of reputation. Notice that this holds for a given \( \pi < 1 \). When \( \pi \) goes to 0, the imperfect information vanishes (accidents are perfect signals of low effort) and \( W_{PI} - W_{IP} \) goes to 0.

The optimal punishment \( T^* \) has been characterized for a given value of the discount factor \( \delta \), probability of accident under high effort \( \pi \), and marginal profit \( P - c \). Next Lemma establishes how the optimal punishment \( T^* \) depends on this set of parameters.
Lemma 2 \( T^* \) is increasing in \( \pi \) and decreasing in \( P - c \) and \( \delta \).

The intuition of Lemma 2 is as follows. The cost of reputation increases with \( \pi \) since it is a measure of the level of imperfect information, and decreases with \( P - c \) and \( \delta \), since they increase the cost of the missing trade.

4 Market Reputation with Product Liability

Now we introduce product liability law in our dynamic framework. The approach taken from now on assumes the existence of a Tort system, in which a court may verify, after an accident happens, whether the firm adopted one or another level of care and impose some degree of legal liability. We consider a rule such that, in case of an accident, the firm should pay a monetary amount consisting of a certain percentage of the harm caused to consumers. This percentage is not fixed but depends on the private information of the firm, i.e. the firm must pay:

- \( \alpha D \) if there is an accident and firm exerted care, where \( \alpha \in [0,1] \).
- \( \beta D \) if there is an accident and firm did not exert care, where \( \beta \geq \alpha \), and \( \beta \in [0,1] \).

Thus, any possible liability rule is a specification \((\alpha, \beta)\). Notice that this parametrization allows us to consider as particular cases the most relevant rules used in tort law and considered by the Law and Economic literature:

i. Strict Liability: Firm must pay the entire damage, no matter if it exerted high care or not, i.e: \( \beta = \alpha = 1 \)

ii. Negligence rule: Firm is liable for the entire damage if and only if it exerted low care, i.e: \( \beta = 1 \) and \( \alpha = 0 \).

iii. Negligence rule with errors in determining liability: This negligence rule includes the realistic complication that a jury or a court could incur in two possible errors when determining liability based on the true level of effort exerted by the firm. These errors are known as Type I
and Type II errors: Type I error means convicting an innocent (careful) firm, that is \( \alpha > 0 \), and Type II error takes place when acquitting a guilty (exerting low care) firm, i.e. \( \beta < 1 \).

iv. No liability: \( \beta = \alpha = 0 \).

To solve the new infinite horizon game, we follow a similar procedure to the one in the previous section.

Let \( V^+_R \) and \( V^-_R \) be the Present Discounted Value of the firms’ profits incorporating the expected monetary sanction imposed by product liability law. By definition, we have:

\[
V^+_R = P - c + (1 - \pi) \delta V^+_R + \pi \delta [V^-_R - \alpha D],
\]
\[
V^-_R = \delta^T V^+_R.
\]

Notice that we are assuming that the liability payment takes place in the next period. Otherwise, the incentive compatibility constraint trivially holds, and liability rules allow trade in the static game. This gap is necessary to implement the legal mechanism of payment, since agents must wait for the court resolution. Similarly to the previous section, we solve the equation system:

\[
V^+_R = \frac{P - c - \pi \delta \alpha D}{1 - (1 - \pi) \delta - \pi \delta^T + 1}, \tag{4}
\]
\[
V^-_R = \delta^T V^+_R = \frac{\delta^T (P - c - \pi \delta \alpha D)}{1 - (1 - \pi) \delta - \pi \delta^T + 1}, \tag{5}
\]

Liability rules also affect the incentive compatibility constraint, so in order to express that the firm has no incentive to exert low effort, now we have:

\[
V^+_R \geq P + \delta [V^-_R - \beta D]
\]

We use the definition of \( V^+_R = P - c + (1 - \pi) \delta V^+_R + \pi \delta [V^-_R - \alpha D] \), to rewrite the incentive compatibility constraint as:

\[
\delta (1 - \pi) [V^+_R - V^-_R] + \delta [\beta - \pi \alpha] D \geq c \tag{6}
\]

Following similar computations as in the previous section (we plug equations (??) and (??) into (??)), we obtain the incentive compatibility constraint under product liability as the inequality...
given by:

$$
\Psi(T, \alpha, \beta) \geq c
$$

where this new function is:

$$
\Psi(T, \alpha, \beta) = (1 - \pi)\delta \frac{(1 - \delta^T)(P - c - \delta \pi \alpha D)}{1 - (1 - \pi) \delta - \pi \delta^T + 1} + \delta D(\beta - \pi \alpha)
$$

(7)

Notice that, by construction, $$\Phi(T) = \Psi(T, 0, 0)$$. In words, that the incentive compatibility constraint when $$\alpha$$ and $$\beta$$ are equal to zero is the same as the one when only reputation is in place.

We are interested in comparing both incentive compatibility constraints, the one achieved through market forces alone, versus the one including this generic effect of legal liability. More specifically, we want to focus on the optimal number of periods during which consumers stop buying the product when $$(\alpha, \beta) = (0, 0)$$ versus any other values of the parameters $$\alpha, \beta$$.

We start by analyzing how the incentive compatibility constraint (the function $$\Psi(T, \alpha, \beta)$$) depends on the punishment and on the specification of the liability rule $$(\alpha, \beta)$$.

**Lemma 3** $$\Psi(T, \alpha, \beta)$$ is increasing in $$T$$, increasing in $$\beta$$ and decreasing in $$\alpha$$.

The intuition of Lemma ?? is related to the incentive compatibility constraint as follows: The larger the punishment is, the easier it is that the incentive compatibility constraint is satisfied. This is because exerting effort reduces the probability of punishment, $$\alpha \leq \beta$$. In the same line, increasing $$\beta$$ (decreasing $$\alpha$$) makes exerting effort more attractive, and this makes more likely that the incentive compatibility constraint is satisfied.

We define $$T^*_R(\alpha, \beta)$$ as the optimal number of periods in which there is no trade under the liability rule $$(\alpha, \beta)$$, $$T^*_R(\alpha, \beta)$$ is the solution to the following problem

$$
\max_T \ V^+_R = \frac{P - c - \pi \delta \alpha D}{1 - (1 - \pi) \delta - \pi \delta^{T+1}}
$$

subject to the incentive compatibility constraint:

$$
\Psi(T, \alpha, \beta) \geq c.
$$
As in the previous case, given that \( \frac{\partial V^*_R}{\partial T} < 0 \), and \( \Psi(T, \alpha, \beta) \) is increasing in \( T \), \( T^*_R(\alpha, \beta) \) is the unique solution to the equation \( \Psi(T^*_R, \alpha, \beta) = c \). The next proposition uses the implicit characterization of \( \Psi(T^*_R, \alpha, \beta) = c \) and Lemma ?? to analyze how the optimal punishment under legal liability depends on the liability parameters \((\alpha, \beta)\).

**Proposition 1** The optimal punishment (reputational cost) under liability, \( T^*_R(\alpha, \beta) \), is decreasing in \( \beta \) and increasing in \( \alpha \).

Proposition ?? is illustrated by figure 1 where \( \alpha' > \alpha \) and \( \beta' > \beta \).

![FIGURE 1 Around here.]

The next corollary of Proposition ?? ranks the two major liability rules in terms of their effect on reputational costs.

**Corollary 1** The optimal reputational punishment is higher under strict liability than under negligence, i.e. \( T^*_R(1,1) > T^*_R(0,1) \).

Moreover, negligence rule is the best policy from the welfare point of view among all possible legal rules \((\alpha, \beta)\), since \( T^*_R(0,1) \) is the minimum among all possible \( T^*_R(\alpha, \beta) \).

Also, the previous results have implications for the effect of judicial errors (under negligence) on reputational costs.

**Corollary 2** The optimal reputational punishment is increasing in the probability of judicial errors.

Another direct application of Proposition ?? is that negligence is superior to the benchmark case (without legal liability), \( T^*_R(0,0) > T^*_R(0,1) \). However, if we want to compare strict liability with the benchmark case, we cannot invoke Proposition ??, since the effect of both parameters \( \alpha \) and \( \beta \) on the optimal length of the market punishment is ambiguous. The next result provides a general comparison between liability rules and the benchmark case.
Proposition 2 The optimal reputational punishment under any liability rule \((\alpha, \beta)\) is lower than without legal liability.

In summary, any liability rule improves the functioning of market reputation by reducing its costs.

5 Product Liability and Reputation: Complements or Substitutes?

As product liability and reputation may achieve in isolation the same outcomes in terms of incentives, it is clear, and in fact it seems to be widely shared idea, that they are substitutes as instruments to induce adequate behavior. In this line, in the previous section we have also showed that legal liability reduces the optimal reputational sanctions, which emphasizes the substitution effects between both instruments. In this section, we want to point out the complementarity effects between product liability and reputation: product liability reduces the cost of reputational sanctions. This becomes particularly obvious when one considers the range of parameters for which trade between firms and consumers can be sustained. Product liability makes it possible that market reputation allows cooperation to happen for a larger set of parameter values than market reputation alone would be able to induce to equilibrium. In other words legal, liability makes reputation more successful in ensuring trade in markets.

Proposition 3 i) In the absence of legal liability, the trade equilibrium may arise only if \(\delta \geq \delta_{\text{min}}^{*} = \frac{c}{r(1-\pi)}\). ii) In presence of legal liability, the trade equilibrium may arise for a larger set of discount rates.

The minimum discount rate \(\delta_{\text{min}}^{*}\) is characterized using: i) \(\delta_{\text{min}}^{*}\) is associated to the maximum penalty \(T^{*} = \infty\). ii) For \(\delta_{\text{min}}^{*}\) the incentive compatibility constraint is binding \(\Psi(\infty, 0, 0) = c\). The second part of Proposition ?? follows from the fact that in the presence of the legal liability the incentive compatibility constraint is not longer binding for \(\delta_{\text{min}}^{*}\) and \(T^{*} = \infty\).
A natural extension to our basic analysis would endogenize product prices. We allow for this complication in our framework in the simplest possible way. Consider that there are several identical firms that may sell the product in the market, and heterogeneous consumers (with different willingness to pay). Consumers with the highest willingness to pay are matched with the firms that have entered in the market, and the pairs will engage in the long term relationship we have described in the previous section. Then, the more firms participate in the market, the lower the price will be due to the lower willingness to pay of the marginal consumer. We assume that the equilibrium price is determined by a free entry condition such that the expected long term profits should be equal to an entry cost, \( k \) (for example, advertisement expenditures). This condition is independent of the legal regime in place. In other words, the free entry condition implies that \( V^*_R = k \) for all values of \((\alpha, \beta)\) including \((0,0)\) (no legal liability).

Given the previous analysis and the free entry condition in the industry, the equilibrium price \( P^*_E \) and the optimal punishment \( T^*_E \) are going to be the solution to the following system of equations.

\[
V^*_R(P, T, \alpha) = k, \\
\Psi(P, T, \alpha, \beta) = c. 
\]  

Notice that although we have introduced the dependence on \((P; T)\), functions \( V^*_R \) and \( \Psi \) are the ones characterized in the previous section by equations (??) and (??).

In the following, we assume that the system above has an unique solution. This assumption is not innocuous. It is clear, for example, that if \( k \) is close to 0 (we are close to perfect competition), we have the well know non-equilibrium existence result of implicit contracts, since some rents are

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\( ^6 \) An alternative and simple way to endogenize prices would be to give all bargaining power to the firm that sets the price in such a way that consumers are indifferent between participating in the market or not, \( V - \pi(1 - \alpha)D \). The advantage of this more abstract and flexible way to characterize the equilibrium price, is the possibility of making comparative static exercises regarding the degree of competition.
necessary for satisfying the incentive compatibility constraint. But for large enough $k$, the system of equations should have a unique solution. $V^+_R(P;T)$ is increasing in $P$ and decreasing in $T$, implying that for every $k$, there is an increasing function $T_k(P)$ such that $V^+_R(P,T_k(P),\alpha) = k$ for all $P$. Similarly, $\Psi(P,T,\alpha,\beta) = c$ is increasing in both $P$ and $T$, which implies that for every $c$, there is a decreasing function $T_c(P)$ such that $\Psi(P,T_c(P),\alpha,\beta) = c$ for all $P$. Then for a pair $(k,c)$, we may obtain an equilibrium price and an optimal punishment $(P^*_E,T^*_E)$ such that $T^*_E = T_k(P^*_E) = T_c(P^*_E)$. This “supply” and “demand” equilibrium is nicely illustrated by figure 2.

[FIGURE 2 Around here.]

Intuitive comparative statics can be derived using these $T_k(P)$ and $T_c(P)$ inverse functions. Consider that the entry cost increases to $k' > k$, implying a reduction in competition, which shifts upwards the profit inverse function, $T_{k'}(P) < T_k(P)$. If we keep the price fixed, higher profits involve lower punishment, and $T_c(P)$ does not change. Figure 3 shows that the new equilibrium is characterized by higher prices and lower punishment. Intuitively, less competition increases firms’ rents and the cost of punishment (the opportunity cost of no trade) increases, which in turn leads to lower punishment in equilibrium.

[FIGURE 3 Around here.]

In the same line, consider now that the cost of taking care increases to $c' > c$. This moves upwards the incentive inverse function, $T_{c'}(P) > T_c(P)$. If we keep the price fixed, higher costs of taking care involve higher punishment, and $T_k(P)$ does not change. Figure 4 shows that the new equilibrium is characterized by higher prices and higher punishment. Intuitively, higher costs make more difficult to satisfy the incentive compatibility constraint, then higher punishment is required. Finally, higher punishment must be compensated with higher equilibrium price for keeping constant the equilibrium price. Figure 4 illustrates these comparative statics exercises.

[FIGURE 4 Around here.]
The formal characterization of the equilibrium can be obtained with the following procedure. We plug the expression of \( \Psi \) characterized by equation (??) into the incentives equation (??). Then we identify \( V_+^R(P; T) \) and replacing it by \( k \).

\[
\begin{align*}
\Psi(P, T, \alpha, \beta) &= c \\
(1 - \pi)\delta(1 - \delta^T)(P - c - \delta \pi \alpha D) + \delta D(\beta - \pi \alpha) &= c \\
(1 - \pi)\delta(1 - \delta^T)V_+^R(P; T) + \delta D(\beta - \pi \alpha) &= c \\
(1 - \pi)\delta(1 - \delta^T)k &= c - \delta D(\beta - \pi \alpha)
\end{align*}
\]

Thus, the optimal punishment \( T_E^*(\alpha, \beta) \) is defined by the equality

\[
\delta T_E^*(\alpha, \beta) = \delta - \frac{c - \delta D(\beta - \pi \alpha)}{k(1 - \pi)}
\]

Using that the left side of the equality is decreasing in \( T_E^* \) and the right side is increasing in \( \beta \) and decreasing in \( \alpha \), we can characterize how the optimal punishment depends on \( \beta \) and \( \alpha \).

**Proposition 4** (i) The optimal punishment with endogenous prices, \( T_E^*(\alpha, \beta) \), is decreasing in \( \beta \) and increasing in \( \alpha \). (ii) The optimal punishment under any liability rule \( (\alpha, \beta) \) is lower than without product liability law, i.e. \( T_E^*(\alpha, \beta) < T_E^*(0, 0) \).

Therefore, Proposition ?? shows that our main results are robust to the introduction of endogenous prices. In particular, (i) states that the cost of reputation is decreasing in the probability of judicial errors, and (ii) that the cost of reputation is lower when a tort system exists. We can also formalize the qualitative analysis undertaken above. \( T_E^* \) is increasing in \( \pi \) and \( c \), and decreasing in \( D \) and \( k \).

The next step is to characterize the equilibrium price by plugging the equilibrium punishment, \( T_E^* \), into the profits constraint equation.

\[
V_+^R(P^*(\alpha, \beta), \alpha) = k
\]
Proposition 5  (i) The equilibrium prices are increasing in $\alpha$ and decreasing in $\beta$, in other words, equilibrium prices are increasing in judicial errors.  
(ii) For every constellation of parameters, there is a cut-off $0 < \alpha^* \leq 1$, such that if $\alpha < \alpha^*$ the equilibrium price is lower with legal liability than without it.

Part i) of Proposition ?? follows from the profit function $V_R^+$ being decreasing in $\alpha$ and $T_E^*(\alpha, \beta)$, and the optimal punishment $T_E^*(\alpha, \beta)$ being increasing in $\alpha$ and decreasing in $\beta$. As profits remain constant, if the profit function increases (decreases), equilibrium price should decrease (increase). Part ii) follows from the fact that the equilibrium price with $\alpha = 0$ is lower than the equilibrium price in the benchmark case without legal liability. This is because the firm does not incur any liability costs (in case of exerting care) and $T_E^*(0, \beta) < T_E^*(0, 0)$, which increases the profit function and leads to a lower equilibrium price. Finally, the equilibrium price increases in $\alpha$ and may or not be higher than the equilibrium price without liability depending on the level of harm, $D$.

7 The Tort System Provides Information to the Market.

In previous sections we have disregarded the possibility that consumers may observe the outcome of the tort process following an accident, and thus, are able to make the reputational sanction -the number of periods in which trade with the firm is discontinued- contingent on the Court’s decision implementing product liability law. Now, we introduce feedback from the Tort system to the market, so that market reputation and market sanctions may depend upon the liability findings of courts.

There are various factors that may affect the plausibility of assuming this information provision to the market, stemming not from the existence of product liability law, but from the specific legal outcomes of the cases involving the products of a given firm. The immediacy of lawsuits in the aftermath of a product-related accident, the length of legal proceedings, the availability of
settlement between the firm and the plaintiffs in the suit and the confidentiality of the terms of settlement, the clarity of court’s decisions concerning the allocation of liability, and the level of consumers’ knowledge over the final decisions in the cases, and their content, all seem to influence the way in which consumers would be able to base their future purchase-or non-purchase-decisions upon the outcome of particular court cases.

In this section, thus, we are agnostic towards the degree of practical relevance of the information provision to the market arising from specific decisions in the legal process. But we believe this function may be of real-world significance under certain circumstances. In any case, it is theoretically important to explore the possibility of feedback to consumers from particular court outcomes, and here we present an extension of the previous model with product liability allowing for consumers’ sanctions to depend on the liability judgement of the Court if an accident has taken place and a lawsuit has been filed.

Formally, we define a new infinite horizon game in which the representative consumer makes the market sanctions depend on Court outcomes. After an accident, the consumer observes whether or not the firm has been declared liable and then sets the market sanctions accordingly. We proceed as in the previous cases by computing the Present Discounted Value of the firms’ profits, including the expected monetary sanctions imposed by product liability law and also the punishment by consumers, now based on the imposition of actual legal liabilities. Notice that given that there are Type I and Type II errors, consumers may find optimal to punish the firm in case of accident even if the firm is found not liable by the Court. Thus, in case of an accident, a punishment phase starts but the length of this punishment depends on the Court’s decision. In particular, we have:

\[
V_F^+ = P - c + (1 - \pi)\delta V_F^+ + \pi\delta \alpha[V_F^- - D] + \pi(1 - \alpha)V_F^N, \\
V_F^- = \delta T_F V_F^+, \\
V_F^N = \delta T_N V_F^+. 
\]
Solving the equation system, we obtain:

\[
V_F^+ = \frac{P - c - \pi\delta\alpha D}{1 - (1 - \pi)\delta - \pi(\alpha\delta T_L + (1 - \alpha)\delta T_{NL})},
\]

Liability rules also affect the incentive compatibility constraint, so in order to express that the firm has no incentive to exert low effort, now we have:

\[
V^+ \geq P + \delta[\beta V_{FL}^- - D] + (1 - \beta) V_{FNL}^-.
\]

Following similar computations than in the previous sections, we obtain the incentive compatibility constraint under product liability as the inequality given by:

\[
\Psi_F(T_L, T_{NL}, \alpha, \beta) \geq c
\]

where this new function is:

\[
\Psi_F(T_L, T_{NL}, \alpha, \beta) = \frac{\delta [(1 - \pi) + \pi(\alpha\delta T_L + (1 - \alpha)\delta T_{NL}) - (\beta\delta T_L + (1 - \beta)\delta T_{NL})] (P - c - \pi\delta\alpha D)}{1 - (1 - \pi)\delta - \pi\delta(\alpha\delta T_L + (1 - \alpha)\delta T_{NL})} + \delta D(\beta - \pi\alpha).
\]

Notice that if \(T_L = T_{NL} = T\) by construction, \(\Psi_F(T, T, \alpha, \beta) = \Psi(T, \alpha, \beta)\) and \(\Psi_F(T, 0, 1, 1) = \Psi(T, 1, 1)\).

We are interested in characterizing the optimal punishment with feedback from the tort process, which will be the solution to the following problem

\[
\max_{T_L, T_{NL}} V_F^+ = \frac{P - c - \pi\delta\alpha D}{1 - (1 - \pi)\delta - \pi\delta(\alpha\delta T_L + (1 - \alpha)\delta T_{NL})}
\]

subject to the incentive constraint:

\[
\Psi_F(T_L, T_{NL}, \alpha, \beta) \geq c.
\]

Then, we need to determine the optimal punishment when the firm is liable, \(T_L\), and when it is not, \(T_{NL}\). In order to compare the solution to this problem (with two punishment variables \((T_L, T_{NL})\)) with the optimal punishment in the previous framework with only one instrument \(T_R\), we focus on the impact of the punishment on the objective function. We say that \((T_L, T_{NL})\)
generates lower expected punishment costs than \( T_R \) if \( \alpha \delta^{T_L} + (1 - \alpha) \delta^{T_{NL}} > \delta^{T_R} \). In fact, the solution to the problem is the pair \((T^*_L, T^*_NL)\) that satisfies the incentive compatibility constraint and maximizes, \( \alpha \delta^{T_L} + (1 - \alpha) \delta^{T_{NL}} \), (minimizes the expected punishment costs).

**Proposition 6**

(i) The optimal reputational punishment with feedback, \((T^*_L, T^*_NL)\) targets the liable firm, minimizing the punishment when the firm is not liable, \((T^*_L, T^*_NL) \in \{(T, 0) \cup (\infty, T)\}\).

(ii) The optimal reputational punishment with feedback, \((T^*_L, T^*_NL)\), generates lower expected punishment costs than without it, \( T^*_R \), i.e. \( \alpha \delta^{T_L} + (1 - \alpha) \delta^{T_{NL}} > \delta^{T^*_R} \).

Figure 5 illustrates the intuition of part (i) of Proposition ??

[FIGURE 5 Around here.]

The optimal punishment \((T^*_L, T^*_NL)\) belongs to the set \( \{(T, 0) \cup (\infty, T)\} \) since among all the points in the iso-curve \( \alpha \delta^{T_L} + (1 - \alpha) \delta^{T_{NL}} = U^* \), the set \( \{(T, 0) \cup (\infty, T)\} \) minimizes the punishment when the firm is not liable and maximizes the expected punishment of the firm that does not exert effort, and this relaxes the incentive compatibility constraint and maximizes \( \alpha \delta^{T_L} + (1 - \alpha) \delta^{T_{NL}} \).

Part (ii) of Proposition ?? is a direct implication from the idea the non-feedback equilibrium \( T_L = T_{NL} = T^*_R \) is feasible but it is not the optimal solution.

Finally, the negative relationship between the performance of the liability system and judicial errors also holds when the market receives information from the tort process.

**Proposition 7** The optimal reputational punishment with feedback, \((T^*_L, T^*_NL)\) is increasing \((\alpha \delta^{T_L} + (1 - \alpha) \delta^{T_{NL}} \) is decreasing\) in judicial errors, decreasing in \( \beta \) and increasing in \( \alpha \).

8 Conclusions

The Law interferes in several ways with consumer markets, and in manufacturers/consumers interactions. One of the most important channels for this is the use of ex post legal sanctions to provide incentives to manufacturers in order to achieve desirable -at least in the eyes of the
lawmaker or regulator- levels of safety and quality in consumer products. We observe, thus, across legal systems, a wide range of tools to this effect: monetary fines for the violation of regulations on product safety and quality, product liability when goods cause bodily or other harm to consumers or even bystanders, and legal warranties that require the manufacturer, in case of lack of conformity of the product with the agreed or reasonably expected quality, to repair or replace the product, or to reduce price, allow for rescission, and/or pay damages to the consumer who bought the non-conforming product. All these legal instruments intend to improve, one way or another, manufacturers’ incentives for providing safety and quality.

These tools are not cheap to design and to implement. Especially, product liability has received severe criticism for its high costs, among other failures [Polinsky and Shavell (2010 a); Viscusi (2011)]. It is thus tempting to think of turning to the market for solutions to the possibility of insufficient safety and quality, and specially, to rely more heavily on market reputation to provide the necessary incentives: Consumers, after having observed that the product of a given manufacturer has caused accidents, or has stirred discontent or dissatisfaction in other consumers, would ”sanction” the manufacturer by ceasing to buy his products for some time. Under this threat, manufacturers will be subject to the right incentives to invest in safety and quality up to the desirable levels.

However, when there is imperfect information on the part of consumers, that is, accidents that result in harm, and defects or non-conformities in products happen also even if manufacturers have optimally invested in safety and quality, the provision of incentives implies in equilibrium the need to incur positive sanctioning and reputational costs. These costs constitute a social welfare loss incurred for the entire period in which consumers ”punish” the manufacturer who has caused the accident or the defect, as a result of refraining to buy his products. This is an unavoidable cost of market reputation under imperfect information. We have also shown that our findings are robust-or even reinforced-with endogenous prices and with feedback from actual legal cases after an accident occurs.
We have shown that the Law may improve matters by reducing these reputational costs. If consumers know that market forces are not alone in providing incentives for manufacturers, the size and duration of the ”market sanction” decreases in order to induce levels of effort and care by manufacturers that consumers desire. In this setting, the Law makes market forces cheaper to operate. Of course, some level of perception by consumers of the ex post ”sanctioning” effected by the Law is necessary for that positive impact on reputational costs. This is likely to be satisfied, especially in well-publicized or far-reaching cases, which are also those able to generate more costly market sanctions.

We have also shown that the performance of different legal regimes matters for this complementary effect of legal sanctions and market sanctions. Errors -both type I and type II- by Courts when imposing liability to manufacturers for their failure to live up to the required levels of effort in safety and quality diminish the positive effect of legal sanctions on market sanctions. The same happens with more indiscriminate or less tailored types of liability regimes, such as strict liability, that imposes liability regardless of the adequacy of behavior on the part of the manufacturer.\footnote{We consider only the incentive dimension of liability regimes, and not other possible and important properties, such as compensation. It is obvious that strict liability entails higher levels of expected compensation for the victims of product accidents or product defects, and thus consumers would react differently to strict liability and to negligence considering this dimension. Given that our interest lies only in the provision of incentives for manufacturers, we disregard this effect. It is as if we imagine strict liability decoupled into a rule for incentives to the potential injurer (here the manufacturer) and an insurance policy covering the victim’s harm. Our analysis refers only to the former.} Negligence, at least if the standards are properly determined, and the level of error in its functioning is limited, being more discriminating in the application of legal sanctions, and directing them only towards those manufacturers which fall below the legal standards of safety and quality is a superior regime. Again, the question of observability by consumers, who are the ones taking the decisions to impose market sanctions, becomes an important assumption, but we think that public perception of legal consequences is accurate enough to distinguish between the two basic legal regimes.

We have chosen to show this important property of product liability—and other related areas of the Law dealing with safety and quality of consumer products— in the simplest setting that we
were able to devise. Many complications are possible and would indeed be relevant. We will just mention two of them. The market structure may have an impact on the effects of legal remedies, as it has been shown in the context of some legal remedies - rescission or termination of the contract if there is a defect.\textsuperscript{8} Collective action problems among consumers and issues of litigation (from class actions to litigation fees and selection and compensation structures for lawyers\textsuperscript{9}) have also been entirely set aside in our analysis, despite their undeniable importance. Still, we believe that the effects we have identified and presented in this paper may have a bearing upon policy debates concerning the desirability of legal liability and its design, taking into account the market forces that operate in consumer markets.

Our results may also have important implications for the general relationship between informal (reputational) and formal (specially legal) instruments to induce desirable behavior. The latter complement the former reducing their cost of implementation and sustaining advantageous interactions that reputation alone could not make to work. The importance of these effects vary with the informational asymmetries, the stakes of the productive interaction, and the time horizon of the agents.

\section*{A Appendix}

\textbf{Proof of Lemma ??:} Let \( \varphi(x) = \frac{1-x}{1-(1-\pi)\delta - \pi x \delta} \). Now, we have \( \Phi(T) = (1-\pi)\delta \varphi(x(T)) \), for \( x(T) = \delta T \). As \( x(T) \) is decreasing, in order to show that \( \Phi \) is increasing in \( T \), we have to show that \( \varphi(x) \) is decreasing in \( x \).

\textbf{Proof of Lemma ??:} We write the binding incentive compatibility condition that characterizes the optimal punishments as follows, \( \Phi(T^*(a), a) - c = 0 \), where \( a \in \{\pi, \delta, P - C\} \). By the implicit function theorem we obtain \( T^*(a) = \frac{\partial \Phi(T^*(a))}{\partial P - c} \). Given that for Lemma ?? \( \frac{\partial \Phi(T^*(a))}{\partial P - c} > 0 \), the \( \text{sign}\{T^*(a)\} = -\text{sign}\{a\} \). Given that, i) \( \frac{\partial \Phi(T^*P - c)}{\partial P - c} = (1-\pi)\delta \frac{1-(1-\pi)\delta - \pi T + \pi}{1-(1-\pi)\delta - \pi \delta^2 + \pi} > 0 \) and \( \frac{\partial T^*}{\partial P - c} < 0 \).

\textsuperscript{8}Stremitzer (2012, forthcoming)
\textsuperscript{9}See, Spier (2007).
ii) \( \frac{\partial \Phi(T^*, \pi)}{\partial \pi} = (P - c)(1 - \delta T) \delta \left[ \frac{-(1-(1-\pi)\delta - \pi \delta T^+)}{(1-(1-\pi)\delta - \pi \delta T^+)^2} \right] < 0 \) and \( \frac{\partial T^*}{\partial \pi} > 0. \) Finally,

\[
\frac{\partial \Phi(T^*, \delta)}{\partial \delta} = (P - c)(1 - \pi) \left[ \frac{(1 - (T + 1)\delta T) (1 - (1 - \pi) \delta - \pi \delta T^+)}{(1 - (1 - \pi) \delta - \pi \delta T^+)^2} \right] \\
= (P - c)(1 - \pi) \left[ \frac{(1 - \delta T^+ - (T + 1)\delta T + (T + 1)\delta T^+)}{(1 - (1 - \pi) \delta - \pi \delta T^+)^2} \right] \\
= (P - c)(1 - \pi) \left[ \frac{(1 - (T + 1)\delta T + T \delta T^+)}{(1 - (1 - \pi) \delta - \pi \delta T^+)^2} \right] > 0
\]

Where the sign positive comes from the fact that \( 1 - (T + 1)\delta T + T \delta T^+ \) is strictly decreasing and 0, when \( \delta = 1, \) therefore for all \( \delta < 1, \) the expression is positive. Then \( \frac{\partial \Phi(T^*, \delta)}{\partial \delta} > 0 \) and \( \frac{\partial T^*}{\partial \delta} < 0. \) ■

**Proof of Lemma ??**: Calculating the partial derivatives of the function \( \Psi \) we obtain:

\[
\frac{\partial \Psi}{\partial \beta} = \delta D > 0 \\
\frac{\partial \Psi}{\partial \alpha} = (1 - \pi)\delta (1 - \delta T) (-\pi D) \frac{1 - (1 - \pi) \delta - \pi \delta T^+}{1 - (1 - \pi) \delta - \pi \delta T^+} - \delta D \pi < 0
\]

Where the last inequality is due to the negative numerator and positive denominator of the first term. Finally, to show \( \Psi \) is increasing in \( T, \) it suffices to remind the fact that \( \Phi(T) \) is increasing in \( T \) and both functions depend on \( T \) in the same way. ■

**Proof of Proposition ??**: By the implicit function theorem and the definition of \( T_R^*; \) \( \Psi(T_R^*, \alpha, \beta) = c, \) we obtain \( \frac{\partial T_R^*}{\partial \alpha} = -\frac{\partial \Psi}{\partial \alpha} = -\frac{\partial \Psi}{\partial \beta} = -\frac{\partial \Psi}{\partial \beta} > 0. \) Similarly, \( \frac{\partial T_R^*}{\partial \beta} = -\frac{\partial \Psi}{\partial \beta} = -\frac{\partial \Psi}{\partial \beta} < 0. \) ■

**Proof of Corollary ??**: The result follows directly from part i) of Proposition ??, since \( \frac{\partial T_R^*}{\partial \alpha} > 0. \) ■

**Proof of Corollary ??**: The result follows directly from Proposition ??.

**Proof of Proposition ??**: It suffices to compute \( \Psi \) for a particular pair of liability parameters \( (\alpha, \beta) \neq (0, 0) \) and no liability, and given the restrictions on the parameters of the model, it can be shown that:

\[ \Psi(T, \alpha, \beta) > \Psi(T, 0, 0) \]

This inequality follows from substituting the particular cases considered. First, incentive compatibility constraint under \( (\alpha, \beta) \) takes the form:

\[
(1 - \pi)\delta (1 - \delta T) (P - c - \delta \pi \alpha D) \frac{1 - (1 - \pi) \delta - \pi \delta T^+}{1 - (1 - \pi) \delta - \pi \delta T^+} + \delta D (\beta - \pi \alpha) \geq c
\]
On the other hand, incentive compatibility constraint according to the case of no regulation takes the form:

\[ \delta(1 - \pi) \frac{(P - c)(1 - \delta^T)}{1 - (1 - \pi)\delta - \pi\delta^{T+1}} \geq c \]

Thus, we need to know which of both left sides of the inequalities above is greater. Comparing both expressions we find:

\[ \Psi(T, \alpha, \beta) = \Psi(T, 0, 0) - \frac{\delta(1 - \pi)\delta\pi\alpha D(1 - \delta^T)}{1 - (1 - \pi) - \pi\delta^{T+1}} + \delta D(\beta - \pi\alpha) \]

But our objective is equivalent to show

\[ (1 - \pi)\delta - \frac{(1 - \delta^T)\delta\pi\alpha D}{1 - (1 - \pi)\delta - \pi\delta^{T+1}} < \delta D(\beta - \pi\alpha) \]

\[ (1 - \pi)\delta(1 - \delta^T)\delta\pi\alpha D < \delta D(\beta - \pi\alpha)(1 - (1 - \pi)\delta - \pi\delta^{T+1}) \]

\[ (\delta - \pi\delta - \delta^{T+1} + \pi\delta^{T+1} + 1 - (1 - \pi)\delta - \pi\delta^{T+1})\pi\alpha < \beta(1 - (1 - \pi)\delta - \pi\delta^{T+1}) \]

\[ (1 - \delta^{T+1})\pi\alpha < \beta(1 - (1 - \pi)\delta - \pi\delta^{T+1}) \]

\[ -\pi\delta^{T+1} + \pi < 1 - \delta + \pi\delta - \pi\delta^{T+1} \]

\[ (1 - \pi)\delta < 1 - \pi \]

Finally, from this result we conclude that:

\[ T^*_R(\alpha, \beta) < T^*_R(0, 0) \]

Proof of Proposition ??: i) Given Lemma 2, the minimum discount rate \( \delta^*_\text{min} \) is associated to the maximum penalty \( T^* = \infty \). Then, the incentive compatibility constraint, \( \Phi(T) = c \) (which should be binding for \( \delta^*_\text{min} \)) simplifies to

\[ (1 - \pi)\delta^*_\text{min} \frac{(P - c)}{1 - (1 - \pi)\delta^*_\text{min}} = \frac{c}{P(1 - \pi)} \]

For lower discount rates of \( \delta^*_\text{min} \) the incentive compatibility constraint cannot be satisfied. ii) The proof of Proposition ?? shows that for any pair of liability parameters \( (\alpha, \beta) \neq (0, 0) \), \( \Psi(T, \alpha, \beta) > \Psi(T, 0, 0) \). Then for all \( \delta \geq \delta^*_\text{min} \), the incentive compatibility constraint is satisfied for the maximum punishment and it is not binding, \( \Psi(\infty, \alpha, \beta) > c \). Therefore, trade may arise in equilibrium if \( \delta \geq \delta^*_\text{min} \) and for continuity of \( \Psi(\infty, \alpha, \beta) \) in \( \delta \), the incentive compatibility should be satisfied for values of \( \delta \) strictly lower than \( \delta^*_\text{min} \).
Proof of Proposition ??: The optimal punishment with endogenous prices $T_E^*(\alpha, \beta)$ is defined by the equality

$$\delta T_E^*(\alpha, \beta) = \delta - \frac{c - \delta D(\beta - \pi\alpha)}{k(1 - \pi)}$$

As the left side of the equality is decreasing on $T_E^*$ and the right side is increasing on $\beta$ and decreasing on $\alpha$, then $T_E^*(\alpha, \beta)$, is decreasing in $\beta$ and increasing in $\alpha$. Finally, as $\delta D(\beta - \pi\alpha) \geq 0$, for the same argument, $T_E^*(\alpha, \beta) < T_E^*(0, 0)$, since

$$\delta - \frac{c}{k(1 - \pi)} < \delta - \frac{c - \delta D(\beta - \pi\alpha)}{k(1 - \pi)}.$$

Proof of Proposition ??: i) We plug the optimal punishment $T_E^*(\alpha, \beta)$ into the profits equation.

$$V_R^+(P^*; T_E^*(\alpha, \beta), \alpha) = k$$

Then, increasing $\alpha$ increases the price, $\frac{\partial P_E^*}{\partial \alpha} = -\frac{\partial^2 V_R^+}{\partial \alpha^2} = -\frac{0}{0} > 0$,

because the profit function $V_R^+$ is decreasing in $\alpha$

$$\frac{dV_R^+}{d\alpha} = \frac{\partial^2 V_R^+}{\partial \alpha^2} + \frac{\partial V_R^+}{\partial T_E^*} \frac{\partial T_E^*}{\partial \alpha} < 0$$

since $\frac{\partial V_R^+}{\partial \alpha} < 0$, $\frac{\partial V_R^+}{\partial T_E^*} < 0$, and $\frac{\partial T_E^*}{\partial \alpha} > 0$.

For the same taken, we can show that increasing $\beta$ decreases the price, $\frac{\partial P_E^*}{\partial \beta} = -\frac{\partial^2 V_R^+}{\partial \beta^2} = -\frac{0}{0} < 0$,

because the profit function $V_R^+$ is increasing in $\beta$, $\frac{\partial V_R^+}{\partial \beta} = \frac{\partial V_R^+}{\partial T_E^*} \frac{\partial T_E^*}{\partial \beta} > 0$.ii) See the arguments provided in the main text.

Proof of Proposition ??:

(i) We rewrite the incentive compatibility constraint.

$$\delta \left[ (1 - \pi)(1 - (\alpha \delta T_L + (1 - \alpha) \delta T_{NL})) + (\beta - \alpha)(\delta T_{NL} - \delta T_L) \right] \left( P - c - \pi \delta \lambda D \right) + \delta D(\beta - \pi\alpha) \geq c$$

Consider the following change of variable $U = \alpha \delta T_L + (1 - \alpha) \delta T_{NL}$, which implies $\delta T_L = \frac{U}{\alpha} - \frac{(1 - \alpha)}{\alpha} \delta T_{NL}$, and then $\delta T_{NL} - \delta T_L = \frac{\delta T_{NL}}{\alpha} - \frac{U}{\alpha}$. 

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\[
\delta \left[ (1 - \pi)(1 - U) + (\beta - \alpha)\left(\frac{\delta T_{NL}}{\alpha} - \frac{U}{\alpha}\right) \right] \frac{(P - c - \pi \delta \alpha D)}{1 - (1 - \pi) \delta - \pi \delta U} + \delta D(\beta - \pi \alpha) \geq c
\]

As the left hand side of the equality is decreasing in \(U^*\), and increasing in \(\delta T_{NL}\), this implies that \(\frac{\partial U^*}{\partial \delta T_{NL}} > 0\). Then, the optimal policy requires to maximize \(\delta T_{NL}\) (minimize \(T_{NL}\)). This implies that in the optimal solution, \(T_L^* \neq \infty \rightarrow T_{NL}^* = 0\), or alternatively \(T_{NL}^* \neq 0 \rightarrow T_L^* = \infty\). Hence, \((T_{NL}^*, T_L^*) \in \{0, T\} \cup \{T, \infty\}\).

(ii) The proof that the optimal punishment with feedback, \((T_L^*, T_{NL}^*)\), generates lower expected punishment cost than without it, \(T_R^*\), i.e. \(U^* = \alpha \delta T_L + (1 - \alpha) \delta T_{NL} > \delta T_R\), it is just to notice that \(T_L = T_{NL} = T_R^*\) was feasible and it is not optimal. This is on the other hand easy to verify by comparing the two binding incentive compatibility constraints.

\[
\frac{(1 - \pi)(1 - U^*)}{1 - (1 - \pi) \delta - \pi \delta U^*} = \frac{\delta D(\beta - \pi \alpha)}{\delta (P - c - \pi \delta \alpha D)} - \frac{(\beta - \alpha)\left(\frac{\delta T_{NL}}{\alpha} - \frac{U^*}{\alpha}\right)}{1 - (1 - \pi) \delta - \pi \delta U^*}
\]

\[
\frac{(1 - \pi)(1 - \delta T_R^*)}{1 - (1 - \pi) \delta - \pi \delta T_R} = \frac{c - \delta D(\beta - \pi \alpha)}{\delta (P - c - \pi \delta \alpha D)}
\]
Notice that the left side of both equalities is the same and it is a decreasing function of $U^*$ and $\delta^{TR}$. The right hand side of the first equality (10) is lower (the second term is negative) than the right side of (13) and this implies that $U^* = \alpha \delta^{TL} + (1 - \alpha)\delta^{TNL} > \delta^{TR}$.

**Proof of Proposition 2:** Following the same arguments than in (ii) in Proposition ??, the proof follows from the fact that the right hand side of the equality (10) above is decreasing in $\beta$ and increasing in $\alpha$.

**References**


\[ \alpha \delta^{TL} + (1 - \alpha) \delta^{NL} = U^* \]

\( (\bar{\delta}^{T^*_L}, \bar{\delta}^{T^*_NL}) \)