Elite-Led and Majoritarian Institutional Reform

Avinash Dixit, Princeton University

Presentation at the workshop
The dynamic of institution
Paris, October 3 and 4, 2008
Effect of Size on Self-Enforcement

General point: unequal size makes it harder to sustain cooperation.

Prisoner’s Dilemma stage game

<table>
<thead>
<tr>
<th>Trader 1</th>
<th>Honest</th>
<th>Cheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honest</td>
<td>$k_1$, $k_2$</td>
<td>$-\ell k_1$, $k_2 + g k_1$</td>
</tr>
<tr>
<td>Cheat</td>
<td>$k_1 + g k_2$, $-\ell k_2$</td>
<td>0, 0</td>
</tr>
</tbody>
</table>
Indefinite repetition with effective discount rate $r$:

<table>
<thead>
<tr>
<th></th>
<th>Trader 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trigger $(T)$</td>
<td>$k_1 + \frac{k_1}{r}, \ k_2 + \frac{k_2}{r}$</td>
</tr>
<tr>
<td>Defect $(D)$</td>
<td>$k_1 + g k_2, \ -\ell k_2$</td>
</tr>
</tbody>
</table>

$(D, D)$ is always an equilibrium in this game. $(T, T)$ also equilibrium if and only if

$$\frac{k_1}{r} > g k_2 \quad \text{and} \quad \frac{k_2}{r} > g k_1,$$

(1) or

$$g r < \frac{k_1}{k_2} < \frac{1}{g r}.$$  

(2)
Stricter requirement: risk-dominance

\((T, T)\) is risk-dominant if

\[
\left( \frac{k_1}{r} - g k_2 \right) \left( \frac{k_2}{r} - g k_1 \right) > (\ell k_1)(\ell k_2).
\]

This becomes a quadratic in \(k = k_1/k_2\), and the condition is

\[
\phi(k) \equiv k^2 - \frac{r}{g} \left( \frac{1}{r^2} + g^2 - \ell^2 \right) k + 1 < 0.
\] (3)

The roots \(k_L, k_H\) have product 1, and satisfy

\[
r g < k_L < 1 < k_H < \frac{1}{r g},
\]

\((T, T)\) is risk dominant for \(k_L < k < k_H\), a narrower range than (2).
Weaker requirement: transfers possible

Let $k \equiv k_1/k_2 > 1$; player 1 makes transfer $t$ to player 2:

<table>
<thead>
<tr>
<th>Trader 1</th>
<th>Trader 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trigger ($T$)</td>
</tr>
<tr>
<td>Trigger ($T$)</td>
<td>$k - t + \frac{k - t}{r}$, $1 + t + \frac{1 + t}{r}$</td>
</tr>
<tr>
<td>Defect ($D$)</td>
<td>$k + g$, $-\ell$</td>
</tr>
</tbody>
</table>

Conditions for $(T, T)$ to be a (Pareto-preferred) equilibrium:

1's honesty: $k - t + \frac{k - t}{r} > k + g$, or $t < \frac{r}{1 + r} \left( \frac{k}{r} - g \right)$, \hspace{1cm} (4)

2's honesty: $\frac{1 + t}{r} > g k$, or $t > g r k - 1$. \hspace{1cm} (5)

Condition for simultaneous validity weaker than (2); vacuous if $r$ small enough.
General principle: Honesty is self-enforcing if the ratio of the sizes $k_1/k_2$ is within a range $[k_L, k_H]$ where $k_L < 1 < k_H$ and $k_L k_H = 1$.
Outside this range, smaller player has the greater incentive to cheat.

Question: Implications for the benefits of establishing, at cost $C'$, formal governance institution to ensure honest behavior.

Two Sizes, Pairwise Matching

Continuum population of mass 1. Matched randomly in pairs; mass of matches $= \frac{1}{2}$.
Fraction $(1 - \lambda)$ is size 1; fraction $\lambda < 1/2$ is size $k > k_H > 1$.
Once pair is formed, sizes of both partners are common knowledge.
Then each pair plays the repeated PD game above.
Honesty is self-enforcing in $\frac{1}{2} \left[ \lambda^2 + (1 - \lambda)^2 \right]$ matches; not in $\lambda (1 - \lambda)$, losing value $V = (1 + k)(1 + r)/r$ each.
Formal institution legal system that enforces honesty will add value $\lambda (1 - \lambda) V$.

Therefore formal institution is socially desirable if

$$\lambda (1 - \lambda) V > C.$$
Coase Theorem

If an ex ante Coasian meeting agrees to establish formal governance:

- each of $\lambda$ large players stands to gain $k(1 + r)/r$ with probability $(1 - \lambda)$
- each of $(1 - \lambda)$ small players stands to gain $(1 + r)/r$ with probability $\lambda$

Total expected gain

$$
\lambda (1 - \lambda) k \frac{1 + r}{r} + (1 - \lambda) \lambda \frac{1 + r}{r} = \lambda (1 - \lambda) (1 + k) \frac{1 + r}{r} = \lambda (1 - \lambda) V.
$$

This is the full social gain, so the outcome socially optimal.

Elite-Led Reform

Each of $\lambda$ elite stands to gain $(1 - \lambda) k (1 + r)/r$.

If they can agree and share costs, net benefit positive if

$$(1 - \lambda) k (1 + r)/r > C/\lambda,$$

or

$$\lambda (1 - \lambda) k (1 + r)/r > C.$$

Left hand side is $k/(1 + k)$ of social gain $\lambda (1 - \lambda) (1 + k) V$.

So elite provision will be suboptimal, but only slightly if $k$ large.
Majoritarian Reform

Each \((1 - \lambda)\) small traders stands to gain \(\lambda (1 + r)/r\).
They are majority; can impose the cost on the whole population.
But constitution prevents discriminatory taxes; each pays \(C\).
This is in the interest of this majority if
\[
\lambda (1 + r)/r > C.
\]

When \(k > 1\) and \(\lambda < 1/2\), \((1 - \lambda)(1 + k) > 1\). Therefore the condition implies
\[
\lambda (1 - \lambda)(1 + k)(1 + r)/r > C,
\]
or
\[
\lambda (1 - \lambda)V > C.
\]
So majoritarian reform also socially suboptimal.

Possible cases shown in Figures 1 and 2. Plot three functions of \(\lambda \in [0, 0.5]\).

Majoritarian provision: \(f_1(\lambda) = \lambda > C(1 + r)/r\)
Elite provision: \(f_2(\lambda) = \lambda (1 - \lambda)k > C(1 + r)/r\)
Social optimality: \(f_3(\lambda) = \lambda (1 - \lambda)(1 + k) > (1 + r)/r\)
Region where majoritarian reform is possible is a strict subset of region where elite-led reform is possible. As $C$ decreases, elite reform will occur first.
Comparison of regions depends on size of $\lambda$

For small $\lambda$, elite reform first; for larger $\lambda$, majoritarian.

Summary: Elite-led reform if big inequality of size, numbers.
Continuum of Sizes, Pairwise Matching

Size $s \in [1, \infty)$, CDF $F(s) = 1 - s^{-\alpha}$.

Payoff from honest trade proportional to size. Finite total payoff: $\alpha > 1$.

Let $x = \ln(s)$ and $h = \ln(k_H)$; parameters chosen to make $h = 1$.

Reform led and paid for by large players $[x, \infty)$ possible if $x \in [x_{\min}, x_{\max}]$.

So $1 - G(x_{\max})$ is smallest fraction of population that can lead reform.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$x_{\min}$</th>
<th>$x_{\max}$</th>
<th>$1 - G(x_{\max})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.42</td>
<td>2.00</td>
<td>2.00</td>
<td>0.063</td>
</tr>
<tr>
<td>1.40</td>
<td>1.75</td>
<td>2.38</td>
<td>0.036</td>
</tr>
<tr>
<td>1.35</td>
<td>1.60</td>
<td>2.93</td>
<td>0.020</td>
</tr>
<tr>
<td>1.30</td>
<td>1.51</td>
<td>3.55</td>
<td>0.010</td>
</tr>
</tbody>
</table>
**Regulating a Monopolist**

One large player deals with several small players simultaneously, e.g. insurance company pools risks among many policyholders.
Both may cheat: moral hazard for policyholders, unfair denial of claims by company.
The above model needs different interpretation in this case.
If company chooses to cheat, it will do so on massive scale,
so in this context it may be the smaller player with greater temptation.
Then regulatory institution may need to be established by large player
in this case, namely the population of policyholders; so majoritarian vote
or similar political process representing the numerous population.

**Future Work:**

Major omission above: [1] Details of how elite or majority take collective action.