

# Punishment strategies and the quality of goods: Judges' intervention

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## Abstract

This paper analyses the impact of a double adverse selection problem on the quality of products. We consider two markets: a goods market where the consumer is the principal of the firm (the supervisor) and a labor market where the supervisor is the principal of workers (the agents). The main goal of this paper is to define an optimal contract that shows how the judge incites the incompetent supervisor to not participate on the market. When consumer is scammed, he sues a claim to the court. Our result shows that the level of punishment to avoid incompetent supervisor increases with the level of quality of good. It is also shown that the presence of a legal punishment system improves the consumer's welfare and keeps only competent supervisors.<sup>1</sup>

JEL Classification: L22,D82,D86

**Key words:** Quality of Goods, quality of Labor, legal system, firm design, labeling goods.

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# 1 INTRODUCTION

Consumers' claims paradigm became a major implement for the advent of consumers' associations. Usually, when consumers buy a product, they discover the true quality of the purchased good *ex post*. These associations defend consumers by reporting the low quality of good that consumers buy instead of high quality of good that they require.

In an asymmetric information environment, relevant indexes that guarantee the quality of products are only the price and, if there is, a product certification. This may not be sufficient to protect the consumers' interests.

For example, even if drugs are certificated, scam is a frequent phenomenon according to the french public database on consumers' claims by sectors. Such as, "the MEDIATOR" drug in France (in the health sector) is considerate as a scandal given its harmful effects on the patients. In fact, despite of this major damage, not many caseload have been recorded. Moreover, another example should be noted. In 2015, in spite of the certification and the high price of cars of Volkswagen, about 500 000 seats of Volkswagen were declared damaged says "El Pais".<sup>2</sup>

These examples reveal that consumers can not be sure of the high quality of purchased goods and they can be scammed by the firms. This can be explained by internal problem within the firm when the supervisor is incompetent for example and labels wrongly the products on the goods market. Therefore, these questions remain unresolved: how can we limit agency problems inside organizations and how can we protect consumers' welfare when he is scammed? This is the purpose of this paper. We consider an asymmetric environment information with adverse selection on the goods market and on the labor market. On the goods market, consumers are the principals of the firm: they delegate to a 2-product monopoly the production of a good which has two different levels of quality. There are two types of consumers according to their preferences to quality. After labeling products, the firm determines each price for each level of good. The control technology is imperfect so that a good quality of good can be labeled as a low quality and low quality of good can be labeled as a high quality. On the labor market, the supervisor is the principal of workers: there are two types of workers. Each type of worker is charged to produce each quality of good. The aversion to effort for each worker is not observable to the supervisor. When the consumer is scammed, he sues a claim to the judge.

The problem is resolved using the backward resolution technique. We start by resolve consumers' program then the supervisor's program.

The first important result is that the amount of the punishment increases with the level of quality of good: the higher the quality of the good is the higher the amount of the claim is. The second important result of this paper is that, the low type of workers' effort shut-down when there supervisor is incompetent. However, with a legal sanctions system, it is not the case anymore. In this case, it will be only the competent supervisor on the market. Consequently, the consumer will not be scammed.

In the theoretical literature, many authors analyze the optimal control problem within the firm. To effectively engage their workers and ensure good production, firms hire a super-

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<sup>2</sup>[http://www.lemonde.fr/economie/article\\_moteurs-truques-volkswagen-seat-dans-la-tourmente-en-espagne-enquete-du-mexique.html](http://www.lemonde.fr/economie/article_moteurs-truques-volkswagen-seat-dans-la-tourmente-en-espagne-enquete-du-mexique.html)

visor in order to incite the agents (the workers) and to transmit information, see Antle 1982. This approach appears an effective solution, however, Tirole (1986) emphasizes that there is always a risk of collusion between the supervisor and the worker. Results show that this collusion harms adversely the organization. Empirical studies argue that this strategy of monitoring presents a form of opportunism, see Dalton (1959), Crozier (1967) and Mintzberg (1983).

This structure of agency entails not only the possibility of a collusion between the supervisor and the worker but also the possibility of a collusion between the supervisor and the principal, see Vafai (2002) and Vafaï (2010).

Following this line of thought, Laffont and Tirole (1992) propose another issue of optimal control: the firm delegates control to an external audit company where an auditor is charged to report workers' effort to the principal. The authors show that this method of monitoring is more efficient than the previous one because they consider that the external auditor is more objective than the intern one. Nagarajan, Bainman and Bam (1991) and Kofman and Lawarree (1993) analyze this structure of control and underline that this later exposes the firm to unofficial activities between the supervisor and the audit company. They suggest also that there is a distortion of information by practicing this strategy.

This collusion does not only affect the labor efficiency but also the quality of products proposed to the consumers. To guarantee the good quality, many firms certify its products. This method has been criticized by many authors, see Auriol et al. (2003), Das (2013), Mason (2011) and Tirole (1988). The authors evince that firms certify its products in order to differentiate compared to the other firms. Consequently, once again they show that there is a possibility of a collusion between firms and the certification companies.

The remainder of the paper is organized as follows. Section 2 is devoted to the presentation of the model. Section 3 analyses the behavior of each type of consumer. Section 4 studies the behavior of the supervisor on the good market and on the labor market. Section 5 presents the integrated solutions of each type of consumer and the optimal contract of the supervisor taking account of the optimal legal system. Section 6 concludes.

## 2 THE MODEL

Under an asymmetric information environment with double adverse selection problem, we consider a 2-product monopoly that produces two different levels of qualities of good: a high quality of good and a low quality of good. The monopoly is represented by the supervisor. We consider three players: a consumer, a supervisor and worker. The firm operates on two markets: the goods market and the labor market. On the goods market, the consumer delegates the production to the firm. Outface the demand of its consumers, the supervisor hires two types of workers to produce each level of quality of good on the labor market. This paper studies the case when the supervisor is either competent or incompetent inside an organization and analyses its consequences on the quality of good and on the consumers' welfare. A competent supervisor labeled correctly the workers' production on the goods market and an incompetent supervisor establishes labeling the reverse. This analysis is conducted by constituting a link between the goods market and the labor market. The sequence

of the game is presented as follows:

1. On the labor market, the supervisor hires two types of workers. With probability  $\nu$ , a worker is efficient as long as he holds a low production cost  $\underline{\varepsilon} \in R_+$  and makes a high level of effort  $\bar{e} \in R_+^2$  to produce a high quality of good  $q(\bar{e}) = \bar{q}$  where  $\bar{q} \in Q := [Q_0, Q_1]$ . With probability  $1 - \nu$  a worker is less efficient as long as he has a high production cost  $\bar{\varepsilon} \in R_+$  and makes a low level of effort  $\underline{e} \in R_+^2$  to produce low quality of good  $q(\underline{e}) = \underline{q}$  where  $\underline{q} \in \underline{q} := [\underline{q}_0, \underline{q}_1]$  where  $\underline{q}_1 < Q_0$ <sup>3</sup>.

The relation between the level of each effort and the level of each quality of good is represented as

$$q : \begin{cases} E & \rightarrow & Q \\ e & \mapsto & q(e) \end{cases} \quad (1)$$

The supervisor sets the level of quality depending on the demand. Once the consumer fixes his optimal level of quality of good, the supervisor defines exactly each level of quality to produce. In this model, it is important to note that the firm makes profit only on the high quality of good  $\bar{q}$ .

2. The monitoring technology is imperfect which implies that the supervisor can be of two types depending on his competence. With probability  $\alpha$  the supervisor is competent and has a  $\underline{\gamma} \in \Gamma := [\Gamma_0, \Gamma_1]$  supervision cost. In this case, he labels good according to their true quality. With probability  $1 - \alpha$  the supervisor is not competent and has a supervision cost  $\bar{\gamma} \in \bar{\gamma} := [\gamma_0, \gamma_1]$  where  $\gamma_1 < \Gamma_0$ . In this case, he does not label the good effectively.
3. There are two types of heterogeneous consumers according to their preferences of quality of good. A  $\underline{\theta}$ -consumer prefers a high quality of good  $\bar{q}$  where  $\underline{\theta} \in \bar{\Theta} := [\Theta_0, \Theta_1]$ . A  $\bar{\theta}$ -consumer prefers a low quality of good  $\underline{q}$  where  $\bar{\theta} \in \underline{\Theta} := [\theta_0, \theta_1]$  and  $\theta_1 < \Theta_0$ . We note that  $\bar{q}$  is selling at price  $\bar{p}$ , the  $\underline{q}$  is selling at price  $\underline{p}$  with  $\underline{p} < \bar{p}$ . Given that we are under asymmetric information environment, the consumer discovers the quality of the good that he buys *ex post*.

The timing of the game is thus the following:

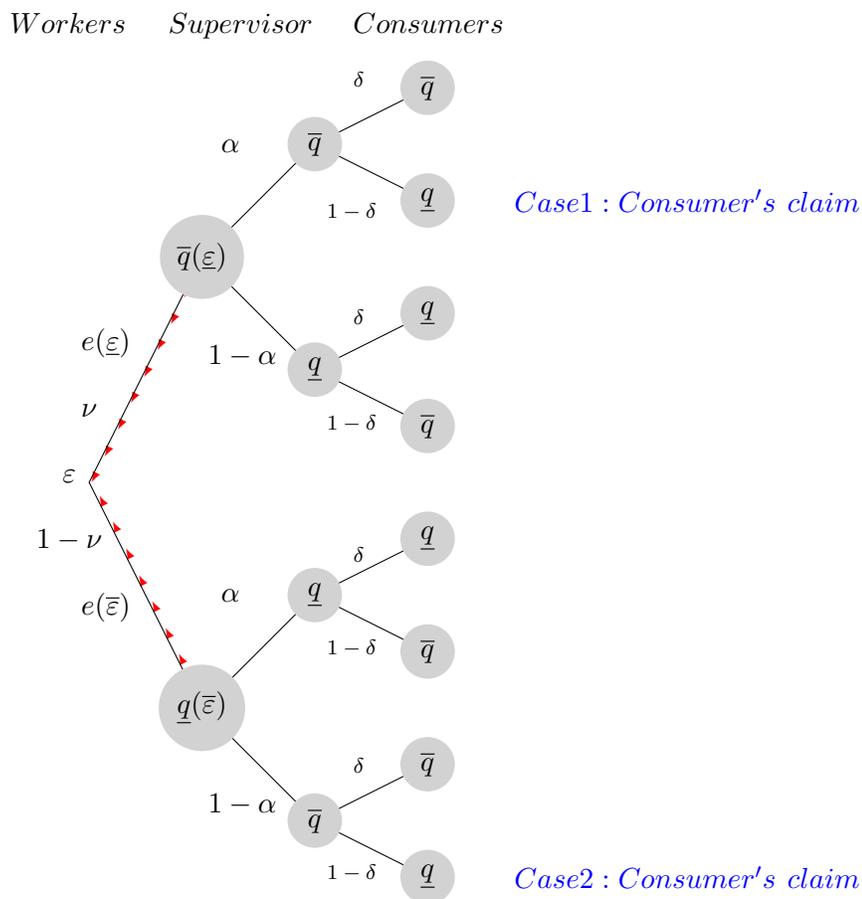
- The evolution of the game on the labor market is as follows: (1) The supervisor offers a contract to workers. (2) The workers accept or refuse the contract. (3) The worker makes  $\bar{e}(\underline{\varepsilon})$  with probability  $\nu$  or  $\underline{e}(\bar{\varepsilon})$  with probability  $1 - \nu$  and control takes place. (4) There exist two types of supervisors: with probability  $\alpha$  the supervisor is competent and with probability  $1 - \alpha$  he is non-competent. In this case, when the  $\bar{e}$ -worker produces a low quality of goods  $\underline{q}$ , the incompetent supervisor labeled it as a high quality of good  $\bar{q}$  on the goods market. (5) The supervisor advertises respectively prices  $\bar{p}$  or  $\underline{p}$  depending on the supervisor's competence. (6) The contract is executed.

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<sup>3</sup>The later condition is to avoid the situation of a pooling equilibrium.

- The evolution of the game on the goods market is as follows: (1) After the step (5) of the timing on the labor market, the consumers buy goods depending on their preferences to the quality  $\bar{q}$  or  $\underline{q}$ . Each product is characterized to satisfy the level of quality. (2) The consumers discover true quality of the purchased good *ex post*. With probability  $\delta$ , consumers are satisfied with their purchase and with probability  $1 - \delta$ , they are not satisfied. (3) The  $\bar{\theta}$ -consumer always sues a claim when he is not satisfied with probability  $1 - \delta$ . In this case, the punishment is always applied. The  $\underline{\theta}$ -consumer never sues a claim.

The 3-player game can be presented as a sequential tree of decision as follows



Note that it is a game in extensive form. In order to keep the time consistency, the resolution of the problem is by using the backward resolution technique. Consequently, the first step is solving the consumer's program program, then the supervisor's program and finally the workers' program. This schematic shows three different cases:

- The case 1 is the case where the  $\bar{\theta}$ -consumer buys an effective  $\bar{q}$  but he is not satisfied.

- The case 2 is the case where the  $\bar{\theta}$ -consumer thinks that he buys an  $\bar{q}$  whereas it is a  $\underline{q}$ : the worker produces an  $\underline{q}$  and the supervisor declares it on the goods market as an  $\bar{q}$  product. Indeed, in this case, the supervisor is incompetent.

When one of these consumers sues a claim to the court, the judge restores the information which means that there is no asymmetric information in this step. Consequently, this paper focuses only on the last agency problem case when the supervisor is not competent and labeled reverse products.

### 3 THE BEHAVIOR OF CONSUMERS ON THE GOODS MARKET

From the sequential tree of decision, we can identify different cases where each type of consumer  $\bar{\theta}$  or  $\underline{\theta}$  does not agree with the purchased quality. Hence, the consumer can bring the case to court. From the game tree, this claim is realized with probability  $\sigma$ . This is resumed in the first case and the second one in the game tree. In this paper we focus only on the second case: when the worker provides an effort  $\underline{e}$  at a cost  $\bar{\varepsilon}$  to produce  $\underline{q}$  quality of good and the supervisor labels it as  $\bar{q}$  instead of  $\underline{q}$ . In this case, the consumer is swindled by the firm. Let us define  $L$  the amount of the claim that the judge attributes to the consumer when he sues a claim. This amount  $L \in R^+$  is paid by the firm: it represents the punishment that costs to the firm when the supervisor is not competent. We suppose that the judge restores the information, therefore there is no asymmetric information at this level.

The expected utility function of the  $\bar{\theta}$ -consumer is defined as follows

$$EU(q, \bar{p}, \bar{\theta}, \alpha, \nu, \delta) = [\nu\alpha\delta + (1 - \nu)(1 - \alpha)\delta][\bar{\theta}(A - B\bar{q})\bar{q}] + [\nu\alpha(1 - \delta) + (1 - \nu)(1 - \alpha)(1 - \delta)][\bar{\theta}(A - B\underline{q})\underline{q} + \sigma\bar{q}L] - \bar{p}\bar{q}, \quad (2)$$

The expected utility function of the  $\underline{\theta}$ -consumer is as follows

$$EU(\underline{q}, \underline{p}, \underline{\theta}, \alpha, \nu, \delta) = [(1 - \nu)\alpha\delta + \nu(1 - \alpha)\delta][\underline{\theta}(A - B\underline{q})\underline{q}] + [\nu(1 - \alpha)(1 - \delta) + (1 - \nu)\alpha(1 - \delta)][\underline{\theta}(A - B\bar{q})\bar{q}] - \underline{p}\underline{q}. \quad (3)$$

It should be stressed here that  $\theta(A - Bq)q$  is the consumers' utility function. The outline of the choice of this utility function is that it provides a linear demand function.

In the existing literature, it is always the supervisor who incites his agent (the worker) not to deviate. Given that, in this paper, the consumer is the principal of the firm on the goods market, which implies that he controls it. This later must incite the supervisor to be competent. Consequently, like in the literature, the consumer sets the incentive constraints and the participation constraints in his program.

As mentioned above, it is only the  $\bar{\theta}$ -consumer can be scammed. In all cases, the  $\underline{\theta}$ -consumer never sues a claim. In fact, even though the supervisor labels  $\bar{q}$  as  $\underline{q}$ , the  $\underline{\theta}$ -consumer has never interest to complaint. This section analyses the behavior of each type of consumer.

### 3.1 THE BEHAVIOR OF $\bar{\theta}$ -CONSUMER :

This model is studied under asymmetric information environment with adverse selection: the consumer discovers the true quality of product *ex post*. Given that the consumer is the principal of the firm on the goods market, he wants to make sure that he will not be scammed. Therefore, he incites the supervisor to not deviate.

This subsection studies the behavior of the  $\bar{\theta}$ -consumer. This consumer is demanding with respect to the high quality that he wants to buy. The incentive constraints of the  $\bar{\theta}$ -consumer as follows

$$\bar{p}q - \frac{1}{2}\gamma\bar{q}^2 \geq \underline{p}q - \frac{1}{2}\gamma\underline{q}^2 - \sigma\bar{q}L, \quad (4)$$

$$\underline{p}q - \frac{1}{2}\gamma\underline{q}^2 \geq \bar{p}q - \frac{1}{2}\gamma\bar{q}^2. \quad (5)$$

The participation constraints of the  $\bar{\theta}$ -consumer are represented as follows:

$$\bar{p}q - \frac{1}{2}\gamma\bar{q}^2 \geq 0, \quad (6)$$

$$\underline{p}q - \frac{1}{2}\gamma\underline{q}^2 \geq 0. \quad (7)$$

The participation constraints mean that the profit of the firm must be positive to ensure its production.

The incentive constraints (4) and (5) and the participation constraints (6) and (7) allow us to draw the  $\bar{\theta}$ -consumer's program by using (2) as the objective function:

$$\begin{aligned} & \max_{\bar{q}} [\alpha\nu\delta + (1-\nu)(1-\alpha)\delta][\bar{\theta}(A - B\bar{q})\bar{q}] + \\ & [\nu\alpha(1-\delta) + (1-\nu)(1-\alpha)(1-\delta)][\bar{\theta}(A - B\underline{q})\underline{q} + \sigma\bar{q}L] - \bar{p}\bar{q}, \\ & \text{subject to } \left\{ \begin{array}{ll} \bar{p}q - \frac{1}{2}\gamma\bar{q}^2 \geq \underline{p}q - \frac{1}{2}\gamma\underline{q}^2 - \sigma\bar{q}L & (8) \\ \underline{p}q - \frac{1}{2}\gamma\underline{q}^2 \geq \bar{p}q - \frac{1}{2}\gamma\bar{q}^2 & (9) \\ \bar{p}q - \frac{1}{2}\gamma\bar{q}^2 \geq 0 & (10) \\ \underline{p}q - \frac{1}{2}\gamma\underline{q}^2 \geq 0 & (11) \\ \bar{p}q - \frac{1}{2}\gamma\bar{q}^2 \geq \underline{p}q - \frac{1}{2}\gamma\underline{q}^2 & (12) \end{array} \right. \end{aligned}$$

The equation (12) the constraint of monotonicity of the firm: The profit generated of production of high quality of good is more important than the profit of low quality of good. This constraint is checked *expost*.

$$\bar{p}q - \frac{1}{2}\gamma\bar{q}^2 < 0.$$

An incentive compatible contract of a  $\bar{\theta}$ -consumer is defined by the solution of the following program  $P$ :

$$\begin{aligned} \max_{\bar{q}} \quad & [\nu\alpha\delta + (1-\nu)(1-\alpha)\delta][\bar{\theta}(A - B\bar{q})\bar{q}] + [\nu\alpha(1-\delta) + (1-\nu)(1-\alpha)(1-\delta)][\bar{\theta}(A - B\underline{q})\underline{q} + \sigma\bar{q}L]\bar{p}\bar{q} \\ \text{subject to} \quad & \left\{ \begin{aligned} \bar{p}q - \frac{1}{2}\gamma\bar{q}^2 &= \underline{p}q - \frac{1}{2}\gamma\underline{q}^2 - \sigma\bar{q}L & (13) \\ \underline{p}q - \frac{1}{2}\gamma\underline{q}^2 &= 0 & (14) \end{aligned} \right. \end{aligned}$$

*Proof is given in Appendix.*

From the equality (13) :

$$\bar{p} = \frac{1}{2}\gamma + \frac{\underline{p}q - \frac{1}{2}\gamma\underline{q}^2 - \sigma L\bar{q}}{\bar{q}}. \quad (15)$$

This equality is checked *expost*.

### 3.2 THE BEHAVIOR OF $\underline{\theta}$ -CONSUMER :

As previously explained, the  $\underline{\theta}$ -consumer is always satisfied even if the supervisor does not labeled correctly the products on the goods market. Consequently, no particular incentive constraints for this type of consumer. Even though he is the principal of the consumer.

Using (3) as an objective function, a rational  $\underline{\theta}$ -consumer solves the following optimal menu of contract:

$$\begin{aligned} \max_{\underline{q}} \quad & [\nu(1-\alpha)\delta + (1-\nu)\alpha\delta][\underline{\theta}(A - B\underline{q})\underline{q}] + \\ & [\nu(1-\alpha)(1-\delta) + (1-\nu)\alpha(1-\delta)][\underline{\theta}(A - B\bar{q})\bar{q}] - \underline{p}q \\ \text{subject to} \quad & \end{aligned}$$

$$\underline{p}q - \frac{1}{2}\gamma\underline{q}^2 = 0 \quad (16)$$

As mentioned above, the firm does not make profit on the low quality of good so that (17) = 0.

### 3.3 THE BEHAVIOR OF $\underline{\theta}$ -CONSUMER :

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Using (3) as an objective function, a rational  $\underline{\theta}$ -consumer solves the following optimal menu of contract:

$$\max_{\underline{q}} [\nu(1 - \alpha)\delta + (1 - \nu)\alpha\delta][\underline{\theta}(A - B\underline{q})\underline{q}] +$$

$$[\nu(1 - \alpha)(1 - \delta) + (1 - \nu)\alpha(1 - \delta)][\underline{\theta}(A - B\bar{q})\bar{q}] - \underline{p}\underline{q}$$

subject to

$$\underline{p}\underline{q} - \frac{1}{2}\bar{\gamma}\underline{q}^2 = 0 \quad (17)$$

As mentioned above, the firm does not make profit on the low quality of good so that (17) = 0.

## 4 THE BEHAVIOR OF THE SUPERVISOR

In the existing literature, the supervisor is always the principal of the employees. The originality of our model is that the supervisor plays the role of the agent on the goods market. Whereas, he remains the principal on the labor market. Therefore the supervisor makes decision only on the labor market. On the labor market, he labels each good produced by the workers depending on his own type  $\underline{\gamma}$  or  $\bar{\gamma}$ .

### 4.1 THE BEHAVIOR OF THE SUPERVISOR ON THE LABOR MARKET

On the labor market, the supervisor labels every good produced by the workers and this is depending on his competence type  $\underline{\gamma}$  or  $\bar{\gamma}$ . Consequently, the supervisor takes into account the incentive constraints and the participation constraints of each type of worker.

The incentive constraints are represented as follows

$$\bar{w} - \underline{\varepsilon}\bar{e} \geq \underline{w} - \underline{\varepsilon}e, \quad (18)$$

$$\underline{w} - \bar{\varepsilon}e \geq \bar{w} - \bar{\varepsilon}\bar{e}. \quad (19)$$

The first constraint (18) means that the supervisor must ensures that the surplus of the  $\underline{\varepsilon}$ -worker is better than the surplus of the  $\bar{\varepsilon}$ -worker who pretends being a  $\underline{\varepsilon}$  one. The second

constraint (19) means that the surplus of the  $\bar{\varepsilon}$ -worker must be better than the surplus of the  $\underline{\varepsilon}$ -worker who makes in reality a  $\bar{\varepsilon}$ .

The participation constraints are given as follows

$$\bar{w} - \underline{\varepsilon}\bar{e} \geq 0, \quad (20)$$

$$\underline{w} - \bar{\varepsilon}\underline{e} \geq 0. \quad (21)$$

The participation constraints mean that each type of worker accepts to be hired in the firm. For simplicity we suppose that there is no opportunity cost.

Since the probability  $\alpha$  is the probability that the supervisor labels wrongly the products. That means that when the  $\bar{\varepsilon}$ -worker produces an  $\underline{q}$ , it is interesting for the supervisor to declare it as an  $\bar{q}$ . By applying this strategy, the firm attracts the  $\bar{\theta}$ -consumers and consequently he wins the gain generated by the  $\bar{q}$  with less cost  $\bar{\gamma}$ .

A rational supervisor maximizes the following program:

$$\max_{\bar{e}, \underline{e}} [\nu\alpha + (1 - \nu)(1 - \alpha)][\bar{q}(\bar{e}) - \bar{w}(\bar{e})] + [(1 - \nu)\alpha + \nu(1 - \alpha)][\underline{q}(\underline{e}) - \underline{w}(\underline{e})]$$

$$\text{subject to } \left\{ \begin{array}{ll} \bar{w} - \underline{\varepsilon}\bar{e} \geq \underline{w} - \underline{\varepsilon}\underline{e} & (22) \\ \underline{w} - \bar{\varepsilon}\underline{e} \geq \bar{w} - \bar{\varepsilon}\bar{e} & (23) \\ \bar{w} - \underline{\varepsilon}\bar{e} \geq 0 & (24) \\ \underline{w} - \bar{\varepsilon}\underline{e} \geq 0 & (25) \end{array} \right.$$

Let us define that  $\bar{q}(\bar{e}) = (\bar{A} - \bar{B}\bar{e})\bar{e}$  and  $\underline{q}(\underline{e}) = (\underline{A} - \underline{B}\underline{e})\underline{e}$ . Once again, the choice of these functions is explained by the fact that it provides a linear demand functions.

The program of a rational supervisor on the labor market becomes as follows

$$\max_{\bar{e}, \underline{e}} [\nu\alpha + (1 - \nu)(1 - \alpha)][(\bar{A} - \bar{B}\bar{e})\bar{e} - \bar{w}(\bar{e})] + [(1 - \nu)\alpha + \nu(1 - \alpha)][(\underline{A} - \underline{B}\underline{e})\underline{e} - \underline{w}(\underline{e})]$$

$$\text{subject to } \left\{ \begin{array}{ll} \bar{w} - \underline{\varepsilon}\bar{e} \geq \underline{w} - \underline{\varepsilon}\underline{e}, & (26) \\ \underline{w} - \bar{\varepsilon}\underline{e} \geq \bar{w} - \bar{\varepsilon}\bar{e}, & (27) \\ \bar{w} - \underline{\varepsilon}\bar{e} \geq 0, & (28) \\ \underline{w} - \bar{\varepsilon}\underline{e} \geq 0, & (29) \end{array} \right.$$

Therefore, an incentive compatible contract of the supervisor's program is solution of the following problem:

$$\max_{\bar{e}, \underline{e}} [\nu\alpha + (1 - \nu)(1 - \alpha)][(\bar{A} - \bar{B}\bar{e})\bar{e} - \bar{w}(\bar{e})] + [(1 - \nu)\alpha + \nu(1 - \alpha)][(\underline{A} - \underline{B}\underline{e})\underline{e} - \underline{w}(\underline{e})]$$

$$\text{subject to } \begin{cases} \bar{w} - \underline{\varepsilon}\bar{e} = \underline{w} - \underline{\varepsilon}\underline{e} & (30) \\ \underline{w} - \bar{\varepsilon}\underline{e} = 0 & (31) \end{cases}$$

*Proof is given in Appendix.*

## 5 THE INTEGRATED SOLUTIONS WITH ASYMMETRIC INFORMATION

In this section, since consumers are the principals of the firm on the goods market, they require the level of quality corresponding to their preferences. Therefore, by using the backward induction resolution technique, we start by solving the consumer's program.

### 5.1 THE OPTIMAL LEVEL OF QUALITY OF GOODS AND THE AMOUNT OF THE CLAIM

Under asymmetric information environment with adverse selection, the firm establishes a portfolio of menu of contract. This portfolio is composed by the menu of contract on the goods market and the one associated to the labor market. The main of this subsection is then to prove Proposition 1. *PROPOSITION 1: Under asymmetric information, the optimal portfolio of contracts entails:*

1. *On the goods market the price corresponding to each optimal level of quality of good is incentive compatible*
2. *On the labor market the wage of each type of worker corresponding respectively to their optimal level of effort are compatible*
- *On the goods market, the optimal menu of contract is characterized as follows*

$$\exists : \bar{\theta} \in \bar{\Theta}, \bar{\theta} = \frac{\gamma(\bar{A}^2 - \underline{\varepsilon}^2) - 4\bar{B}\sigma L(1 - \delta)(Z + 1)}{4\delta Z \bar{A}\bar{B} - 2ZB\delta(\bar{A}^2 - \underline{\varepsilon}^2)}, \text{ and } \exists \bar{q}^* \in Q, \text{ such that} \quad (32)$$

$$\bar{q}^* = \frac{\sigma L(1 - \delta)[\alpha\nu + (1 - \alpha)(1 - \nu) + 1] + \delta[\alpha\nu + (1 - \alpha)(1 - \nu)]\bar{\theta}A}{2B\bar{\theta}\delta[\alpha\nu + (1 - \alpha)(1 - \nu)] + \gamma}, \quad (33)$$

$$\bar{p}^* = \frac{1}{2}\gamma \left[ \frac{\sigma L(1 - \delta)(Z + 1) + \delta Z \bar{\theta}A}{2B\bar{\theta}\delta Z + \gamma} \right] - \sigma L + \frac{1}{2}(\bar{\gamma} - \gamma) \frac{2B\bar{\theta}\delta(Z + \gamma)(A\bar{\theta}\delta Z)^2}{[2B\bar{\theta}\delta Z + \bar{\gamma}]^2[\sigma L(1 - \delta)(Z + 1) + \delta Z \bar{\theta}A]}. \quad (34)$$

$$\exists : \underline{\theta} \in \underline{\Theta}, \underline{\theta} = \frac{[Y(\underline{A} - \bar{\varepsilon}) - (1 - Y)(\bar{\varepsilon} - \underline{\varepsilon})][4Y^2\underline{B}^2A - B[Y(\underline{A} - \bar{\varepsilon}) - (1 - Y)(\bar{\varepsilon} - \underline{\varepsilon})]]}{4A\delta XY^2\underline{B}^2}, \quad (35)$$

$$\underline{q}^* = \frac{A\theta\delta[\nu(1-\alpha) + \alpha(1-\nu)]}{2B\delta[\nu(1-\alpha) + \alpha(1-\nu)] + \bar{\gamma}}, \quad (36)$$

$$\underline{p}^* = \frac{\bar{\gamma}}{2} \left[ \frac{A\theta\delta[\nu(1-\alpha) + \alpha(1-\nu)]}{2B\delta[\nu(1-\alpha) + \alpha(1-\nu)] + \bar{\gamma}} \right]. \quad (37)$$

Where  $Z = \nu(1-\alpha) + \alpha(1-\nu)$ ,  $X = \nu(1-\alpha) + \alpha(1-\nu)$ ,  $Y = \alpha(1-\nu) + (1-\alpha)\nu$  and  $1-Y = \alpha\nu + (1-\alpha)(1-\nu)$ .

- The optimal menu of contracts on the the goods market is compatible with the optimal menu of contract on the labor market. On the labor market, the optimal menu of contract is characterized by

$$\bar{w}^* = \frac{\underline{\varepsilon}(\bar{A} - \underline{\varepsilon})}{2\bar{B}} + (\bar{\varepsilon} - \underline{\varepsilon}) \left[ \frac{\bar{A} - \underline{\varepsilon} - \bar{\varepsilon}}{2\bar{B}} - \frac{[\alpha\nu + (1-\alpha)(1-\nu)](\bar{\varepsilon} - \underline{\varepsilon})}{[\alpha(1-\nu) + \nu(1-\alpha)]2\bar{B}} \right], \quad \bar{e}^* = \frac{\bar{A} - \underline{\varepsilon}}{2\bar{B}} \quad (38)$$

$$\underline{w}^* = \bar{\varepsilon} \left[ \frac{A - \bar{\varepsilon}}{2\underline{B}} - \frac{[\alpha\nu + (1-\alpha)(1-\nu)](\bar{\varepsilon} - \underline{\varepsilon})}{[\alpha(1-\nu) + \nu(1-\alpha)]2\underline{B}} \right], \quad (39)$$

$$\underline{e}^* = \frac{A - \bar{\varepsilon}}{2\underline{B}} - \frac{[\alpha\nu + (1-\alpha)(1-\nu)](\bar{\varepsilon} - \underline{\varepsilon})}{[\alpha(1-\nu) + \nu(1-\alpha)]2\underline{B}}. \quad (40)$$

**Proof.** As pointed it above, the proof is conducted by using the backward resolution technique. Consequently, we start by solving each type of consumer's program then the supervisor's program. Let start by solving the  $\bar{\theta}$ -consumer's program. From (13) we obtain the price of the high quality of good  $\bar{q}$ :

$$\bar{p} = \frac{1}{2}\gamma\bar{q} - \sigma L + \frac{pq - (1/2)\gamma q^2}{\bar{q}}$$

Which means that

$$\bar{p}\bar{q} = \frac{1}{2}\gamma\bar{q} + pq - \frac{1}{2}\gamma q^2 - \sigma\bar{q}L$$

Replace this expression (2) into the  $\bar{\theta}$ -consumer's function we obtain the following first order solution

$$\delta[\alpha\nu + (1-\alpha)(1-\nu)]\bar{\theta}A - 2\delta[\alpha\nu + (1-\alpha)(1-\nu)]B\bar{q}^*\bar{\theta} + \sigma L(1-\delta)[\alpha\nu + (1-\alpha)(1-\nu)]\gamma\bar{q}^* + \sigma L = 0$$

Isolating  $\bar{q}^*$  we define the optimal level of high quality of good

$$\bar{q}^* = \frac{\sigma L(1-\delta)[\alpha\nu + (1-\alpha)(1-\nu) + 1] + \delta[\alpha\nu + (1-\alpha)(1-\nu)]\bar{\theta}A}{2B\bar{\theta}\delta[\alpha\nu + (1-\alpha)(1-\nu)] + \underline{\gamma}}$$

Using (17) we obtain the price of the low quality of good  $\underline{q}$

$$\underline{p} = \frac{1}{2}\bar{\gamma}\underline{q}$$

Replacing this expression into the objective function of  $\theta$ -consumer to obtain the first order condition

$$\delta[\nu(1 - \alpha) + \alpha(1 - \nu)]A\theta - 2B\underline{q}^*\delta[\nu(1 - \alpha) + \alpha(1 - \nu)] - \bar{\gamma}\underline{q}^* = 0$$

Isolating  $\underline{q}^*$  we obtain optimal level of low quality of good

$$\underline{q}^* = \frac{A\theta\delta[\nu(1 - \alpha) + \alpha(1 - \nu)]}{2B\delta[\nu(1 - \alpha) + \alpha(1 - \nu)] + \bar{\gamma}}, \quad (41)$$

Replacing each optimal level of quality of good into the optimal prices  $\bar{p}$  and  $\underline{p}$  we obtain the following solutions

- $\exists \underline{\gamma} \in \Gamma := [\Gamma_0, \Gamma_1]$  such that the optimal level of price of high quality of good if as follows

$$\bar{p}^* = \frac{1}{2}\underline{\gamma} \left[ \frac{\sigma L(1 - \delta)[(1 - Z) + 1] + \delta(1 - Z)\bar{\theta}A}{2B\bar{\theta}\delta(1 - Z) + \underline{\gamma}} \right] - \sigma L \quad (42)$$

$$+ \frac{1}{2} \frac{(\bar{\gamma} - \underline{\gamma})[2\delta(1 - Z)\bar{\theta}B + \underline{\gamma}](\delta ZA\theta)^2}{[2\delta(1 - Z)\bar{\theta}B + \bar{\gamma}]^2[\sigma L(1 - \delta)(1 - Z) + 1] + \delta(1 - Z)\bar{\theta}A}, \quad (43)$$

- $\exists \bar{\gamma} \in \gamma := [\gamma_0, \gamma_1]$  where  $\gamma_1 < \Gamma_0$  such that the optimal level of price of low quality of good if as follows

$$\underline{p}^* = \frac{\bar{\gamma}}{2} \left[ \frac{A\theta\delta[\nu(1 - \alpha) + \alpha(1 - \nu)]}{2B\delta[\nu(1 - \alpha) + \alpha(1 - \nu)] + \bar{\gamma}} \right]. \quad (44)$$

We now prove the optimal menu of contract on the labor market. From the supervisor's program we determine the optimal wages of each type of worker  $\underline{\varepsilon}$  and  $\bar{\varepsilon}$  as well as their optimal level of effort.

$$\bar{w}^* = \underline{\varepsilon}\bar{e}^* + (\bar{\varepsilon} - \underline{\varepsilon})e^*, \quad (45)$$

$$\underline{w}^* = \bar{\varepsilon}e^*. \quad (46)$$

From (31) we obtain  $\underline{w} = \bar{\varepsilon}e$ . Replacing the later expression into the objective function to obtain the optimal menu of contract.

The solution is given by determining the First Order Condition:

$$\bar{e}^* = \frac{\bar{A} - \underline{\varepsilon}}{2\bar{B}}, \quad (47)$$

$$\underline{e}^* = \frac{\underline{A} - \bar{\varepsilon}}{2\underline{B}} - \frac{[\nu\alpha + (1 - \nu)(1 - \alpha)](\bar{\varepsilon} - \underline{\varepsilon})}{[\alpha(1 - \nu) + (1 - \alpha)\nu]2\underline{B}}. \quad (48)$$

Finally, optimal wages corresponding to each type of worker are obtained from (45) and (46)

$$\bar{w}^* = \frac{\underline{\varepsilon}(\bar{A} - \underline{\varepsilon})}{2\bar{B}} + (\bar{\varepsilon} - \underline{\varepsilon}) \left[ \frac{\bar{A} - \bar{\varepsilon}}{2\bar{B}} - \frac{[\nu\alpha + (1 - \nu)(1 - \alpha)](\bar{\varepsilon} - \underline{\varepsilon})}{[(1 - \nu)\alpha + \nu(1 - \alpha)]2\bar{B}} \right], \quad (49)$$

$$\underline{w}^* = \bar{\varepsilon} \left[ \frac{\underline{A} - \bar{\varepsilon}}{2\underline{B}} - \frac{[\nu\alpha + (1 - \nu)(1 - \alpha)](\bar{\varepsilon} - \underline{\varepsilon})}{[\alpha(1 - \nu) + (1 - \alpha)\nu]2\underline{B}} \right]. \quad (50)$$

## 5.2 THE OPTIMAL LEVEL OF PUNISHMENT STRATEGIES

As pointed above, when  $\bar{\theta}$ -consumer is scammed by the firm, he sues a claim to the court. In this model, we suppose that there is no asymmetric information in front of the judge: he restores the information. The legislator's optimal strategy consists to determine a level of punishment such that it prevents from the incompetent supervisor.

**Proposition 2:** The level that the legislator fixes when there is a claim is defined as follows

$$L = \frac{2\bar{\theta}W(\delta B\bar{A}^2 - B\delta\underline{\varepsilon}^2 - \delta A\bar{B}) + \underline{\gamma}(\bar{A}^2 - \underline{\varepsilon}^2)}{4\sigma\bar{B}(1 - \delta)(W + 1)}. \quad (51)$$

Where  $W = \alpha\nu + (1 - \alpha)(1 - \nu)$ .

**Proof.** On the goods market, the optimal level of high quality of good is:

$$\bar{q}^* = \frac{\sigma L(1 - \delta)[\alpha\nu + (1 - \alpha)(1 - \nu) + 1] + \delta[\alpha\nu + (1 - \alpha)(1 - \nu)]\bar{\theta}A}{2B\bar{\theta}\delta[\alpha\nu + (1 - \alpha)(1 - \nu)] + \underline{\gamma}}$$

On the labor market, we have  $\bar{q}(\bar{e}) = (\bar{A} - \bar{B}\bar{e})\bar{e}$ . The optimal level of  $\underline{\varepsilon}$ -worker's effort is:

$$\bar{e}^* = \frac{\bar{A} - \underline{\varepsilon}}{2\bar{B}}$$

Replacing  $\bar{e}^*$  in the expression of  $\bar{q}(\bar{e})$  we obtain:

$$\bar{q}(\bar{e}^*) = \left[ \bar{A} - \bar{B} \left( \frac{\bar{A} - \underline{\varepsilon}}{2\bar{B}} \right) \right] \left( \frac{\bar{A} - \underline{\varepsilon}}{2\bar{B}} \right)$$

In the equilibrium,  $\bar{q}^* = \bar{q}(\bar{e}^*)$ . From what we isolate the amount of the sanction.

As the result shows, more the quality of good is high more the level of punishment is important. This is depends also of the consumer's preferences for the product.

## 6 CONCLUSION

The existing literature analyzes optimal control inside the organization to prevent collusion and/or abuse of authority by introducing a principal-supervisor-agent hierarchies, where the supervisor can be intern or external to the firm. The main shortcoming of this agency structure is that it implies for the organization a high exposure to a risk of individual opportunism behavior.

This paper proposes a new method of control. The purpose is to show under an asymmetry information environment with adverse selection, how can we eliminate an incompetent supervisor from the market? In this paper, we analyze how an incompetent supervisor affects the quality of production and makes consumers scammed.

Our approach suggests that the consumer is the principal of the firm on the goods market given that he is the residual claimant of the quality of goods.

When the incompetent supervisor labels goods in reverse, high type of consumer is scammed by the firm. Consequently, he sues a claim to the justice. We show that the amount of the claim increases with the level of quality of good: the higher is the quality of good, the more the amount of the claim is important. Moreover, on the labor market when the supervisor is the principal of the worker, this paper shows that the low type of worker has always interest to deviate with the incompetent supervisor. For this reason, when the high type of consumer sues a claim to the courts of justice, this type of supervisor and the worker are no longer incited to deviate given the amount of punishment. The presence of system of legal punishments restricts not only the individual opportunism behavior within the firm generated by the incompetent supervisor but also eliminates his presence on the market.

## Appendix 1.A: Proof of high consumer's program

If (4.11) holds as an equality then:  $\underline{pq} = \frac{1}{2}\bar{\gamma}\underline{q}^2$ .

Replacing  $\underline{pq} = \frac{1}{2}\bar{\gamma}\underline{q}^2$  into (4.8) we obtain:

$$\bar{p}\bar{q} - \frac{1}{2}\underline{\gamma}\bar{q}^2 \geq \frac{1}{2}\bar{\gamma}\underline{q}^2 - \frac{1}{2}\underline{\gamma}\underline{q}^2 - \sigma L\bar{q}$$

$$\bar{p}\bar{q} - \frac{1}{2}\underline{\gamma}\bar{q}^2 \geq \frac{1}{2}\underline{q}^2(\bar{\gamma} - \underline{\gamma}) - \sigma L\bar{q}$$

$\frac{1}{2}\underline{q}^2 > 0$  and  $(\bar{\gamma} - \underline{\gamma}) > 0$  so that the participation constraint (4.10) is true if  $\frac{1}{2}\underline{q}^2(\bar{\gamma} - \underline{\gamma}) > \sigma L\bar{q}$ .

If the incentive constraint (4.8) is an equality then:

$$\bar{p}\bar{q} - \frac{1}{2}\underline{\gamma}\bar{q}^2 = \frac{1}{2}\bar{\gamma}\underline{q}^2 - \frac{1}{2}\underline{\gamma}\underline{q}^2 - \sigma L\bar{q}$$

$$\bar{p}\bar{q} - \frac{1}{2}\underline{\gamma}\bar{q}^2 = \frac{1}{2}\underline{q}^2(\bar{\gamma} - \underline{\gamma}) - \sigma L\bar{q}$$

$$\bar{p}\bar{q} - \frac{1}{2}\bar{\gamma}\bar{q}^2 - \frac{1}{2}\underline{\gamma}\bar{q}^2 + \frac{1}{2}\underline{\gamma}\bar{q}^2 = \frac{1}{2}\underline{q}^2(\bar{\gamma} - \underline{\gamma}) - \frac{1}{2}\bar{\gamma}\bar{q}^2 + \frac{1}{2}\underline{\gamma}\bar{q}^2 - \sigma L\bar{q}$$

After simplification we obtain:

$$\bar{p}\bar{q} - \frac{1}{2}\bar{\gamma}\bar{q}^2 = \frac{1}{2}(\bar{\gamma} - \underline{\gamma})(\underline{q}^2 - \bar{q}^2) - \sigma L\bar{q}$$

The later equality means that  $\bar{p}\bar{q} - \frac{1}{2}\bar{\gamma}\bar{q}^2 < 0$  which means that (4.9) is always true.

## Appendix 2.B: Proof of the supervisor's program

If (25) holds as an equality then  $w = \bar{e}e$ .

Replacing  $w = \bar{e}e$  in the incentive constraint (22) we obtain:

$$\bar{w} - \underline{\varepsilon}\bar{e} \geq \bar{e}e - \underline{\varepsilon}e$$

$$\bar{w} - \underline{\varepsilon}\bar{e} \geq e(\bar{e} - \underline{\varepsilon})$$

$\bar{e} - \underline{\varepsilon} > 0$  so that the participation constraint (eq24) is always true.

If (eq22) is an equality then:

$$\bar{w} - \underline{\varepsilon}\bar{e} = e(\bar{e} - \underline{\varepsilon})$$

$$\bar{w} - \bar{e}e - \underline{\varepsilon}\bar{e} + \underline{\varepsilon}\bar{e} = e(\bar{e} - \underline{\varepsilon}) - \bar{e}e + \underline{\varepsilon}\bar{e}$$

After simplification we obtain:

$$\bar{w} - \bar{e}e = (\bar{e} - \underline{\varepsilon})(\bar{e} - e)$$

$(\bar{e} - e) > 0$  and  $(\bar{e} - \underline{\varepsilon}) < 0$  so that the incentive constraint (eq23) is always true.

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