Free-Trade Integration and Destabilizing Tax Policies*

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Abstract: *This paper studies free-trade integration as a stabilizing channel of expectation-driven fluctuations due to an income tax policy that is destabilizing in autarky. We examine a two-country, two-factor, two-good Ramsey model with endogenous labor supply. Countries differ by their technologies and their tax policies. We show first that a countercyclical income tax policy is destabilizing in autarky. After a free-trade reform, characterized by perfect mobility of labor and capital, whe show that the integrated economy is saddle-path stable under elastic enough labor supply. Hence, we conclude that free-trade integration prevents from the transmission of expectation-driven fluctuations between countries and stabilizes a country that is originally indeterminate in autarky.*

Keywords: *Indeterminacy, expectation-driven business cycles, fiscal policy, balanced-budget rule, two-sector model, free-trade.*

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1 Introduction

While the benefits and drawbacks of international trade in terms of welfare are one of the most well-established conclusions, wether free-trade integration decreases macroeconomic volatility or not is less clear. For instance, empirical evidence exhibits contrasted results: Kose *et al.* [11] suggest that a stronger international integration is associated with less macroeconomic volatility in developped countries while di Giovanni and Levchenko [6] argue that increase in trade amplifies business fluctuations, due to sectoral effects. The theoretical ground also reveals a similar ambiguity. For instance, a direct argument is that entering international markets leads to a bigger exposure to industry-specific shocks, given a greater specialization, or by the international spillover induced by more trade integration. Such effect might be offset by an improvement in countries' risk-sharing, thereby reducing uncertainty and macroeconomic fluctuations. Yet, the debates about the source of aggregate fluctuations in a world economy often focus on shocks on the fundamentals. In contrast, more recent investigations also suggest nonfudamental causes to macroeconomic volatility and argue that globalization and free-trade integration may be the source of expectation-driven and endogenous fluctuations.

These investigations may be separated in two strands. On the one hand, contributions considering Heckscher-Ohlin-Samuelson (HOS thereafter) model claim that expectation-driven fluctuations still occur when two symmetric and destabilized countries open to trade (see Nishimura and Shimomura [16]). On the other hand, the second strand argues that in non-HOS environment, expectation-driven fluctuations and endogenous cycles are emergent properties when countries open to trade. As shown by the works of Nishimura, Venditti and Yano ([17], [18]), a country characterized by aggregate instability when closed may spread sunspot-based fluctuations toward a locally determinate country when engaging into free-trade integration. In addition, Nishimura, Venditti and Yano [19] shows that endogenous cycles may emerge in the integrated economy while the two countries exhibit monotone convergence when closed.[1] The present investigation takes a differ-

[1] See also the study by Le Riche [12] in OLG economies engaging into free-trade in an Heckscher-Ohlin model and Ghiglino [8] that studies different level of integration in goods and factor markets in an infinite horizon model with heterogeneous agents.

ent view and emphasize that engaging a free-trade reform restore uniqueness of equilibrium trajectory, in the special case where capital and labor are perfectly mobile between countries. In particular, we focus on the case where a country opens to international trade while it faces a destabilizing tax policy under balanced-budget rule in autarky.

An extensive literature in macroeconomics highlights the destabilizing property of fiscal policies with balanced-budgets. Since the seminal contribution of Schmitt-Grohé and Uribe [23], extensive contributions study this issue considering the effect of heterogeneous agents, multi-sectors with variable non-decreasing returns to scale or finance-constrained economies.[2] These contributions share the characteristic to focus on a closed-economy framework, although modern economies are embedded in a globalized world. To the best of our knowledge, only a few number of studies, so far, consider the destabilizing effect of tax policy and balanced-budget in open economies model as in Velasco and Meng [14] and Naito [15]. However, these contributions assume a small-open economy structure and therefore do not consider trade issues. The present paper aims therefore to fill the gap and proposes to investigate the stability properties of a balanced-budget tax policy under free-trade integration.

We consider a two-sector infinite horizon economy where governments levy an income tax. There are two countries that produce two goods, consumption and investment under constant return to scale technologies that differ between countries. Countries' firms use two factors, capital and labor, which are perfectly mobile between countries.[3] In each country, the government follows a balanced-budget rule. A wasteful amount of government spending is financed through a tax on the income created inside his territory (i.e. source-based taxation). This assumption is relevant as it is the most common tax regime among developed countries according to em-

[2]See respectively, for instance, Bosi and Seegmuller [3], Guo and Harrison [10], Lloyd-Braga *et al.* [13].

[3]Although the latter assumption seems extreme, especially concerning a perfectly mobile labor market, this is a useful assumption to characterize the free-trade equilibrium path. We restrict therefore our investigation to countries with a high mobility of labor. In particular, large countries in the European Union that share some borders and economically close such as Germany, France and United Kingdom or the Scandinavian countries. See the study of Recchi *et al.* [21] on the labor market mobility inside the European Union. Our study may also encompass U.S. States and Canada.

pirical evidence. Furthermore, the income tax rate is endogenous so that it adjusts with the public spending over GDP ratio. Finally, households get utility from consumption and leisure and have identical preferences across countries.

Our study provides a two-step analysis according to the degree of trade integration of countries. First, we investigate the destabilizing income tax policy in a two-sector closed-economy. We derive that a countercyclical tax that is intermediate, but high enough, leads to expectation-driven fluctuations when labor is sufficiently elastic. This is a direct extension of the seminal contribution of Schmitt-Grohé and Uribe [23] with factor-intensity differences. In the second step, we consider the free-trade economy. We show that when the two countries engage in a free-trade integration with perfecly mobile capital and labor, the economy is saddle-path stable for a high enough elasticity of labor supply. Hence, and this is our main result, an economy that is indeterminate under autarky stabilizes against expectation-driven fluctuations when it opens to trade. Similarly, if a country is determinate under autarky and begins to trade with an indeterminate closed-economy, free-trade integration prevents from the transmission of such expectation-driven fluctuations in the determinate country. Furthermore, our result holds independently of countries' capital-intensity difference and pattern of trade. To the best of our knowledge, this is the first paper that consider the issue of the stabilizing effect of free-trade when closed-economies experience local indeterminacy due to a destabilizing tax policy.

Our conclusions can be related to the contribution of Sim and Ho [22] who claims that an indeterminate closed-economy stabilizes when it opens to trade with a determinate country. However, they consider a HOS model with productive externalities with immobile capital and labor. An opposite conclusion is provided by Nishimura *et al.* [17], [18] and [19]. These contributions consider closed-economies that are determinate and open to trade. They highlight the destabilizing effect of international trade since such determinate closed-economies experience local indeterminacy and or persistent endogenous cycles under free-trade. Furthermore, they also outline the possibility of transmission of expectation-driven fluctuations. Yet, a major difference with our contribution is that they consider decreasing returns, capital mobility only and inelastic labor supply.

The paper is organized as follows: in Section 2, we introduce the closed-economy model and provides the conditions to get local indeterminacy. Section 3 solves the economy under free-trade integration and highlights the stabilizing effect of free-trade integration while Section 4 gives some concluding comments. The Appendix is given in Section 5.

2 The Closed-Economy Model

We consider an infinite horizon dynamic general equilibrium model in which a government levies an income tax to finance a variable public spending. The production side is characterized by two factors, capital and labor, and two goods, consumption and investment.

2.1 Government Policy

We assume that the government runs a balanced-budget. The government sets the level of a wasteful amount of public spending $G(t)$ financed through an income tax, $\tau(t)$. The balanced-budget rule is therefore given by:

$$G(t) = \tau(t)[r(t)k^h(t) + w(t)l(t)] \tag{1}$$

with $r(t)k(t) + w(t)l(t)$ the national income, $G(t)$ the government spending, $\tau(t)$ the tax rate on income.

The public spending varies with respect to the national income according to:

$$G(r(t)k(t) + w(t)l(t)) = [r(t)k(t) + w(t)n(t))]^{\varepsilon_g} \tag{2}$$

Since the elasticity of the tax rate with respect to the tax base is given by $\varepsilon_g - 1$, the tax rate is countercyclical if $\varepsilon_g < 1$, constant if $\varepsilon_g = 1$ and procyclical if $\varepsilon_g > 1$.

2.2 Production Structure

The productive structure of the economy is described by two sectors. The first one produces a consumption good whose price is normalized to unity while the second supplies an investment good with a relative price denoted $p(t)$. Both sectors use capital and labor to produce their respective output. The sectoral technology is Cobb-Douglas such that $y_j(t) = \bar{\xi}_j k_j(t)^{\alpha_j} n_j(t)^{1-\alpha_j}$ with $j = c, i,$. The inputs k_j and n_j are respectively the stock of capital and the amount of labor respectively used in sector j to produce output y_j, $\alpha_j \in (0,1)$ is the capital intensity in sector j and $\bar{\xi}_j = \alpha_j^{-1}(1-\alpha_j)^{-1}\xi_j^{-1}$ is a scaling constant. Thereafter, subscripts c and i respectively denote the consumption good and the investment good sector. Both factors are mobile between sectors. A firm of the sector j chooses the optimal amount of factors through the following cost-minimization program :

$$\begin{array}{c}\phi(r(t), w(t), y_j(t)) = Min_{\{k_j(t), n_j(t)\}} \quad r(t)k_j(t) + w(t)n_j(t) \\ s.t. \quad y_j(t) \geq \bar{\xi}_j k_j(t)^{\alpha_j} n_j(t)^{1-\alpha_j}\end{array} \tag{3}$$

where $r(t)$ and $w(t)$ respectively denote the interest and the wage rate. From the first order conditions of program (3), we derive the total cost function:

$$\phi_j(r(t), w(t), y_j(t)) = \xi_j^{-1} r(t)^{\alpha_j} w(t)^{1-\alpha_j} y_j(t) \tag{4}$$

Perfect competition implies that the marginal cost is equal to the marginal benefit such that:

$$1 = \xi_c^{-1} r(t)^{\alpha_c} w(t)^{1-\alpha_c} \tag{5}$$

$$p(t) = \xi_i^{-1} r(t)^{\alpha_i} w(t)^{1-\alpha_i} \tag{6}$$

with $p(t)$ the relative price of the investment good in terms of the consumption good. We derive the competitive factor prices as function of the relative price of output by solving (5) and (6) with respect to $w(t)$ and $r(t)$ such that:

$$w(p(t)) = \left[\frac{\left(\xi_i p(t)\right)^{\alpha_c}}{\left(\xi_c\right)^{\alpha_i}}\right]^{\frac{1}{\alpha_c-\alpha_i}} \tag{7}$$

$$r(p(t)) = \left[\frac{\left(\xi_i p(t)\right)^{-(1-\alpha_c)}}{\left(\xi_c^{-(1-\alpha_i)}\right)}\right]^{\frac{1}{\alpha_c-\alpha_i}} \tag{8}$$

These two expressions characterize the factor price-commodity price frontier which is associated to the well-known Stolper-Samuelson property that an increase in the relative price of a good increases the price of the factor in which that good is relatively abundant.

2.3 Households behavior

We consider an economy with a continuum of measure one of identical and infinitely-lived agents. Without loss of generality, we assume that population is constant and time is continuous. An agent gets his utility from consumption $c(t)$ and disutility from labor $l(t)$. Agents' income is obtained from their labor supply and the net accumulation of capital. One agent maximizes its intertemporal utility according to:

$$\begin{aligned} \max_{c(t),k(t),l(t)} \quad & \int_{t=0}^{+\infty} e^{-\rho t} U\left(c(t), l(t)\right) dt \\ s.t. \quad & c(t) + p(t)I(t) = (1-\tau(t))[r(t)k(t) + w(t)l(t)] \\ & \dot{k}(t) = I(t) - \delta k(t) \\ & k(0) > 0 \text{ given} \end{aligned} \tag{9}$$

where $I(t)$ denotes the investment good, $\rho > 0$ the discount factor and $\delta > 0$ the depreciation rate of capital. We assume the households take the income tax $\tau(t)$ as given. The instantaneous utility function is separable in consumption and labor such that:

$$U(c(t), l(t)) = ln(c(t)) - \Gamma \frac{l(t)^{1+\chi}}{(1+\chi)}$$

with χ the inverse of the elasticity of labor supply and $\Gamma > 0$ a scaling constant.

Denote $\lambda(t)$ the shadow price of capital $k(t)$. The first-order conditions are:

$$\frac{p(t)}{c(t)} = \lambda(t) \tag{10}$$

$$\Gamma l(t)^{\chi} = \frac{(1-\tau(t))w(t)}{c(t)} \tag{11}$$

$$\dot{\lambda}(t) = \lambda(t)\left[(1-\tau(t))\frac{r(t)}{p(t)} - \rho - \delta\right] \tag{12}$$

$$\dot{k}(t) = (1-\tau(t))\frac{(r(t)k(t) + w(t)l(t))}{p(t)} - \delta k(t) - \frac{c(t)}{p(t)} \tag{13}$$

and the transversality condition:

$$\lim_{t\to+\infty} e^{-\rho t}\lambda(t)k(t) = 0 \tag{14}$$

2.4 Equilibrium and Steady State

Consider first the factor markets given by:

$$l(t) = n_c(t) + n_i(t) \tag{15}$$

$$k(t) = k_c(t) + k_i(t) \tag{16}$$

with $n_j(t)$ and $k_j(t)$ the sectoral demand for labor and capital given by the first-order conditions of (3), $k(t)$ the total capital stock and $l(t)$ the households' labor supply. As shown in Appendix 5.1, using the factor markets clearing-conditions (15), we define the sectoral supply functions $y_c(t) \equiv y_c(k(t), l(t), p(t))$ and $y_i(t) \equiv y_i(k(t), l(t), p(t))$:

$$\begin{aligned} y_c(t) &= \frac{(1-\alpha_i)r(p(t))k(t)-\alpha_i w(p(t))l(t)}{\alpha_c-\alpha_i} \\ y_i(t) &= \frac{\alpha_c w(p(t))l(t)-(1-\alpha_c)r(p(t))k(t)}{p(t)(\alpha_c-\alpha_i)} \end{aligned} \tag{17}$$

For the commodity market clearing, we assume the government uses a share of public spending in order to purchase each goods. In order to focus on the effect of the income tax policy, we shall choose a particular share that avoid any compositional effect of the government purchases on clearing of commodity markets. We assume that a fraction $\gamma(t) = \frac{y_c(t)}{y_c(t)+p(t)y_i(t)}$ goes to the consumption good sector while a fraction $1-\gamma(t) = \frac{p(t)y_i(t)}{y_c(t)+p(t)y_i(t)}$ goes to the investment good sector.[4] The commodity markets clearing-conditions are:

$$c(t) + \gamma(t)G(t) = y_c(t), \quad I(t) + (1-\gamma(t))\frac{G(t)}{p(t)} = y_i(t) \tag{18}$$

Using the balanced-budget, these rewrite:

$$c(t) = (1-\tau(t))y_c(t), \quad I(t) = (1-\tau(t))y_i(t)$$

Summing equations (18) and using the households' budget constraint we get the aggregate resource constraint:

$$c(t) + p(t)I(t) + G(t) = y_c(t) + p(t)y_i(t) \tag{19}$$

Let us characterize now the intertemporal equilibrium. Using (7)-(8), we solve the first-order conditions (10)-(11) to get $l(c(t), p(t), \tau(t))$ and

[4] Several studies focus on the effect of the composition of public spending used to purchased the goods. See for instance Raurich [20] in the case of an endogenous growth model and Chang *et al.* [4] in a modified Benhabib-Farmer model. In the latter, the fraction of purchase of public spending is constant since it is financed by a lump-sum tax.

$\lambda(c(t), p(t), \tau(t))$ while we derive $p(k(t), l(t), y_i(t))$ and $y_c(k(t), l(t), y_i(t) \equiv y_c(k(t), l(t), p(k(t), l(t), y_i(t))$ from the sectoral supply functions (17). We use the clearing market condition of the consumption good in the labor supply and solve to obtain $l(k(t), y_i(t), \tau(t))$. From equation (2), we derive $\tau(k(t), y_i(t))$ and solve backward to obtain $l(k(t), y_i(t))$, $c(k(t), y_i(t))$, $y_c((k(t), y_i(t))$, $p((k(t), y_i(t))$, $w(k(t), y_i(t))$, $r(k(t), y_i(t))$, and $\lambda(k(t), y_i(t))$.

Differentiating the first-order condition (10) with respect to time and solving in equation (12), the Euler equation writes:

$$\dot{y}_i = \frac{\left\{\frac{(1-\tau(k(t),y_i(t))r(k(t),y_i(t)))}{p(k(t),y_i(t))} - \rho - \delta - \left(\frac{\varepsilon_{c,k}}{k(t)} - \frac{\varepsilon_{p,k}}{k(t)}\right)[(1-\tau(k(t),y_i(t)))y_i(t) - \delta k(t)]\right\}}{\left(\frac{\varepsilon_{c,y_i}}{y_i(t)} - \frac{\varepsilon_{p,y_i}}{y_i(t)}\right)} \tag{20}$$

where:

$$\varepsilon_{cx} = \frac{\partial c(k(t),y_i(t))}{\partial x(t)} \frac{x(t)}{c(k(t),y_i(t))}, \quad \varepsilon_{px} = \frac{\partial p(k(t),y_i(t))}{\partial x(t)} \frac{x(t)}{p(k(t),y_i(t))}, \quad x(t) = k(t), y_i(t)$$

Substitute the clearing market condition of the investment good sector into the capital accumulation equation to get:

$$\dot{k} = (1 - \tau(k(t), y_i(t)))y_i(t) - \delta k(t) \tag{21}$$

An intertemporal equilibrium is a path $\{k(t), y_i(t)\}_{t \geq 0}$, given the initial condition $k(0) > 0$, satisfying equations (20)-(21) and the transversality condition (14).

A steady-state equilibrium of the closed-economy is a solution (k, y_i, τ) satisfying the following system of equations:

$$\begin{array}{c} (1-\tau)y_i = \delta k \\ (1-\tau)\frac{r(k,y_i)}{p(k,y_i)} = (\rho + \delta) \\ \tau = [r(k, y_i)k + w(k, y_i)l(k, y_i)]^{\varepsilon_g - 1} \end{array} \tag{22}$$

When solving the two first equations we obtain $\bar{k} \equiv k(\tau)$ and $\bar{y}_i \equiv y_i(\tau)$. Substitute these in the third equation to get $\tau = \Omega(k(\tau), y_i(\tau))$. The latter equation potentially yields several tax rates as solutions since it may be represented by a Laffer curve.

Proposition 1. *Let $(\bar{k}, \bar{y}_i)$ be the steady-state solution of (20)-(21). There exist τ^*, $\tau^{**} \in (0, 1)$ such that, if $\varepsilon_g < 1$, there exist two steady-state tax rates (τ^*, τ^{**}) while if $\varepsilon_g > 1$, there is a unique steady-state tax rate τ^*, satisfying the balanced-budget rule when $(k, y_i) = (\bar{k}, \bar{y}_i)$.*

Proof: see Appendix 5.2.

In the next section, we study the destabilizing effect of an income tax policy in the closed-economy.

2.5 Expectation-driven fluctuations in the closed-economy

We linearize the dynamical system (20)-(21) in the neighbourhood of the steady state. The characteristic polynomial is given by $P(z) = z^2 - \mathcal{T}z + \mathcal{D}$ with $\mathcal{T}$ and $\mathcal{D}$, the trace and the determinant of the Jacobian matrix of (20)-(21), given by equation (67) in Appendix 5.3. Since the system (20) possesses one predetermined and one forward variable, the local dynamics are indeterminate if and only if both roots of the characteristic polynomial are negative and determinante otherwise. We get local indeterminacy if $\mathcal{T} < 0$ and $\mathcal{D} > 0$ and saddle-path stability if $\mathcal{D} < 0$. Let us denote:

$$\bar{\tau} = \frac{(1-\alpha_i)}{1+\alpha_c-\alpha_i-\alpha_c\varepsilon_g}$$

$$\underline{\tau} = \frac{\left[\frac{\alpha_i^2}{\alpha_c}+\chi\left[1+\frac{(\alpha_c-\alpha_i)[\rho(\alpha_c-\alpha_i)+\delta(1-\alpha_i)(1-\alpha_c)]}{\alpha_c[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}\right]\right]}{(1-\varepsilon_g)\left[\frac{(\rho+\delta)}{[\rho+\delta(1+\alpha_c-\alpha_i)]}-\alpha_i+\frac{\chi\delta(\alpha_c-\alpha_i)^2[\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)][\rho+\delta(1+\alpha_c-\alpha_i)]}\right]+\left[\frac{\alpha_i^2}{\alpha_c}+\chi\left[1+\frac{(\alpha_c-\alpha_i)[\rho(\alpha_c-\alpha_i)+\delta(1-\alpha_i)(1-\alpha_c)]}{\alpha_c[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}\right]\right]}$$

The next proposition summarizes the conditions to get local indeterminacy.

Proposition 2. *There exist $\bar{\rho} \in (0, +\infty)$, $\bar{\alpha_i} \in (0, 0.5)$, $\bar{\chi} \in (0, +\infty)$, $\underline{\tau} \in (0, 1)$ and $\bar{\tau} \in (\underline{\tau}, 1)$ such that for any $\rho \in (0, \bar{\rho})$, $\alpha_i \in (0, \bar{\alpha_i})$, $\chi \in (0, \bar{\chi})$ and $\varepsilon_g < 1$, the steady state is locally indeterminate if and only if and $\tau \in (\underline{\tau}, \bar{\tau})$.*

Proof: see Appendix 5.3.

This result extends the conclusion of Schmitt-Grohé and Uribe [23] in a two-sector model with differences in factor-intensity ranking between sectors. More precisely, given a large enough elasticity of labor supply, a countercyclical tax rate that is high enough leads to expectation-driven fluctuations. Furthermore, one easily shows that the lower bound on the tax rate is increasing with respect to ε_g. Hence, the more countercyclical the tax rate is, the more likely local indeterminacy occurs.[5] The economic intuition behind this result goes as follows. Departing from the steady-state situation, suppose that households expect the future tax rate to increase such that the expected after-tax return of capital decreases. It makes capital income less attractive leading to a fall in the demand of the investment good which results in a drop in its relative price. The decreases in demand for the investment good and its relative price have two consequences. On the one hand, households report the fall in demand of the investment good to increase their demand in the consumption good. On the other hand, from the Stolper-Samuelson theorem, we know that a decrease in the relative price of the investment good leads to a decrease in the factor price in which it is relatively abundant. Assume first that the investment good is more labor intensive than the consumption good ($\alpha_c > \alpha_i$) such that the drop in the relative price leads to a decrease in the wage rate. This drop in wage together with the increase in consumption demand implies from the consumption-leisure trade-off (11) that households reduce their labor supply. Since the capital stock is predetermined, households have less income resulting in a lower tax base. It follows that under a countercyclical tax policy, the tax rate increases. Expectations are self-fulfilled. Assume now that the investment good is more capital-intensive than the consumption good sector ($\alpha_c < \alpha_i$). In such case, the interest rate decreases while in contrast the wage rate increases. Given the increase in consumption, the effect on the labor supply in (11) is ambiguous. However, if the tax rate is high enough, the increase in wage is dominated by the the increase in consumption and results in a decrease in the labor supply and therefore in households' income. Since the tax base decreases and the tax rate is countercyclical, the tax rate increases and expectations are self-fulfilling. This explains the requirement of a higher tax rate if the investment good is capital intensive.

In the next section, we examine the effect of opening this economy to free-trade with perfect mobility of labor and capital between countries.

[5]For the effects of a change in factor-intensities α_c and α_i on $\underline{\tau}$ and $\bar{\tau}$, see Abad [1]. The latter discusses in addition the plausibility of such destabilizing tax policies and argues that a two-sector economy is more likely to be destabilized than a the one-sector.

3 The Model under Free-Trade

In this section, we present and compare the local dynamics of the model described in Section 2 when two economies operates a free-trade integration.

3.1 The Decentralized Model

We assume that two economies similar to the one described in section 2 opens to free-trade. In particular, the two countries produce and exchange both goods. The technology and the tax policy differ between countries while the households' preferences are identical between countries. We assume that capital and labor are perfectly mobile between the countries and therefore that the countries' after-tax factor prices are equalized at each point in time. For the description of the model in the current Section, each time the two countries will be simultaneously considered, all the symbols will be affected by a superscript A or B. Otherwise, the superscript h with $h = A, B$ is used.

The households of country h maximize their intertemporal utility according to the following program:

$$\begin{array}{ll} \max\limits_{c^h(t),k^h(t),l^h(t),\theta_k^h,\theta_l^h} & \int_{t=0}^{+\infty} e^{-\rho t} U\left(c(t)^h, l^h(t)\right) dt \\ s.t. & c^h(t) + p^h(t) I^h(t) = (1-\tau^h(t))[\theta_k^h(t) r^h(t) k^h(t) + \theta_l^h(t) w^h(t) l^h(t)] \\ & +(1-\tau^*(t))[(1-\theta_k^h(t)) r^*(t) k^h(t) + (1-\theta_l^h(t)) w^*(t) l^h(t)] \\ & \dot{k^h}(t) = I^h(t) - \delta k^h(t) \\ & k^h(0) > 0 \text{ given} \end{array} \tag{23}$$

with $\theta_k^h(t)$ and $\theta_l^h(t)$ the share of capital and labor of households of country h supplied domestically, $\tau^*(t)$, $r^*(t)$ and $w^*(t)$ respectively the foreign country tax rate, interest rate and wage rate.

As in the previous section, we derive the expression of country's h wage rate and interest rate:

$$w^h(p^h(t)) \quad = \quad \left[\frac{\left(\xi_i{}^h p^h(t)\right)^{\alpha_c{}^h}}{\left(\xi_c{}^h\right)^{\alpha_i{}^h}} \right]^{\frac{1}{\alpha_c{}^h - \alpha_i{}^h}} \tag{24}$$

$$r^h(p^h(t)) \quad = \quad \left[\frac{\left(\xi_i{}^h p^h(t)\right)^{-(1-\alpha_c{}^h)}}{\left(\xi_c\right)^{-(1-\alpha_i{}^h)}} \right]^{\frac{1}{\alpha_c{}^h - \alpha_i{}^h}} \tag{25}$$

Government of country h levies a source-based income tax to finance a wasteful public spending. In particular, the balanced-budget of country's h government is:

$$G^h(t) = \tau^h(t)[r^h(t)k^h(t) + w^h(t)n^h(t)] \tag{26}$$

Note that the tax base is not the aggregate income of the citizens' of country h but the income acquired in country h by the labor supplied and the capital owned of households of both countries. The government's spending follows a rule similar to (2):

$$G^h(r^h(t)k^h(t) + w^h(t)n^h(t)) = \frac{\eta^h}{Z^h}[r^h(t)k^h(t) + w^h(t)n^h(t)]^{\varepsilon_g^h} \tag{27}$$

where η^h is a parameter related to the steady state tax rate and Z^h is a scaling constant. As a result, the tax rate in country h adjusts so that the budget is balanced:

$$\tau^h(t) = \frac{\eta^h}{Z^h}[r^h(t)k^h(t) + w^h(t)n^h(t)]^{\varepsilon_g^h - 1} \tag{28}$$

Let us characterize the market equilibrium. Domestic factor markets satisfies:

$$\begin{aligned} n^h(t) &= n_c^h(t) + n_i^h(t) \\ k^h(t) &= k_c^h(t) + k_i^h(t) \end{aligned} \tag{29}$$

where $k^h(t)$ and $n^h(t)$ are the firms' demand of capital and labor in country $h = A, B$. As in the closed economy, we use these to derive the sectoral supply functions:

$$\begin{aligned} y_c^h(t) \equiv y_c^h(k(t)^h, n(t)^h, p(t)^h) &= \frac{(1-\alpha_i^h)r^h(p(t)^h)k(t)^h - \alpha_i^h w^h(p(t)^h)n(t)^h}{\alpha_c^h - \alpha_i^h} \\ y_i^h(t) \equiv y_i^h(k(t)^h, n(t)^h, p(t)^h) &= \frac{\alpha_c^h w^h(p(t)^h)n(t)^h - (1-\alpha_c^h)r^h(p(t)^h)k(t)^h}{p(t)^h(\alpha_c^h - \alpha_i^h)} \end{aligned} \tag{30}$$

Note that the following identity holds:

$$y_c^h(t) + p^h(t)y_i^h(t) = r^h(t)k^h(t) + w^h(t)n^h(t) \tag{31}$$

Because of capital and labor perfect mobility, the factor markets of the integrated economies satisfy:

$$l(t)^A + l(t)^B \equiv l(t) = n^A(t) + n^B(t)$$
$$k(t) = k^A(t) + k^B(t) \tag{32}$$

with $l(t)$ and $k(t)$ the aggregate labor supply and capital stock.

Similarly to the closed-economy model, we assume that the goverment of each country uses a share $\gamma^h(t) = \frac{y_c^h(t)}{y_c^h(t)+p^h(t)y_i^h(t)}$ of the public spending to purchase the consumption good. Under a free-trade equilibrium, the clearing-market conditions for the goods are given by:

$$c^A(t) + c^B(t) = (1-\tau^A(t))y_c^A(t) + (1-\tau^B(t))y_c^B(t)$$
$$I^A(t) + I^B(t) = (1-\tau^A(t))y_i^A(t) + (1-\tau^B(t))y_i^B(t)$$

Hence, according to these conditions, the countries' net exports in the consumption good and in the investment good cancel each others:

$$\mathcal{N}\mathcal{X}_c^A + \mathcal{N}\mathcal{X}_c^B = 0$$
$$\mathcal{N}\mathcal{X}_i^A + \mathcal{N}\mathcal{X}_i^B = 0 \tag{33}$$

with $\mathcal{N}\mathcal{X}_c^h = (1-\tau^h(t))y_c^h(t) - c^h(t)$, and $\mathcal{N}\mathcal{X}_i^h = (1-\tau^h(t))y_i^h(t) - I^h(t)$, respectively the net export of the consumption good and the investment good in country h.

Although the decentralized model has the advantage to fully characterize the time-path of each variable together with their distribution between countries , it is technically demanding and suffers from a curse of dimensionality since the intertemporal equilibrium may not be reduced to a system of two-differential equation. However, using some well-known techniques borrowed from the literature in international trade and heterogeneous agents, we are able to consider a tractable version of this model that allows to study the local dynamics.

3.2 The Pseudo-Planner Problem

It is well known from the literature on the producer theory and the literature on international trade that we may use the dual equivalence of the firms' problem to study the consumption-investment trade-off by focusing on the Production Possibility Frontier (PPF). [6] In particular, define from (30) the

[6] See Drugeon [7] for the equivalence between the sectoral supply functions and the PPF approach.

relative price for a given level of capital stock, labor demand and supply of the investment good such that $p^h(k^h(t), n^h(t), y_i^h(t))$ and substitute it in the sectoral supply function of the consumption good to get:

$$y_c^h(k^h(t), n^h(t), p(k^h(t), n^h(t), y_i^h(t))) = T^h(k^h(t), n^h(t), y_i^h(t))$$

The latter expression is useful since its first derivatives are related to the factor prices and the relative price of the investment good such that:

$$\begin{array}{r}T_1^h(k^h(t), n^h(t), y_i^h(t)) = r^h(k^h(t), n^h(t), y_i^h(t)),\\ T_2^h(k^h(t), n^h(t), y_i^h(t)) = w^h(k^h(t), n^h(t), y_i^h(t)),\\ T_3^h(k^h(t), n^h(t), y_i^h(t)) = -p^h(k^h(t), n^h(t), y_i^h(t))\end{array} \tag{34}$$

Let us now consider the households' side by considering the social welfare function faced by a pseudo-planner. The latter have to maximize the utility of consumers of the integrated economy subject to market-clearing for a given state of technology and countries' tax policies. Assume a free-trade equilibrium exists in this economy. We denote by λ^h, $h = A, B$, the country h's marginal utility of wealth associated with the free-trade equilibrium. Given $\lambda = (\lambda^A, \lambda^B)$, we define the social welfare function:

$$\mathcal{W}(k(t), l(t), y_i(t); \lambda) = \max_{\substack{c^h, l^h, k^h, n^h, y_i^h \\ h=A,B}} \frac{ln(c(t)^A)}{\lambda^A} + \frac{ln(c(t)^B)}{\lambda^B} - \frac{\Gamma l^A(t)^{1+\chi}}{(1+\chi)\lambda^A} - \frac{\Gamma l^B(t)^{1+\chi}}{(1+\chi)\lambda^B}$$

$$\begin{array}{l} s.t.\ c^A(t) + c^B(t) \leq (1-\tau^A(t))T^A(k^A(t), n^A(t), y_i^A(t)) + (1-\tau^B(t))T^B(k^B(t), n^B(t), y_i^B(t)) \\ \quad y_i(t) \geq (1-\tau^A(t))y_i^A(t) + (1-\tau^B(t))y_i^B(t) \\ \quad k(t) \geq k^A(t) + k^B(t) \\ \quad l^A(t) + l^B(t) \geq n^A(t) + n^B(t) \end{array} \tag{35}$$

Where the taxes $\tau^A(t)$ and $\tau^B(t)$ are taken as given. More precisely, the country h's income tax is determined in equilibrium according to the country's balanced-budget rules. Using the relationship between production and income (31), we define:

$$\begin{array}{c}\tau^h(t) \equiv \tau^h(k^h(t), n^h(t), y_i^h(t)) \\ = \frac{\eta^h}{Z^h}[T^h(k^h(t), n^h(t), y_i^h(t)) - T_3^h(k^h(t), n^h(t), y_i^h(t))y_i^h(t)]^{\varepsilon_g^h - 1}\end{array} \tag{36}$$

We can deal with the program above in two-step. First, the Negishi approach states that in an economy with heterogeneous agents, the set of Pareto allocations can be obtained by solving a pseudo-social planner's problem (i.e. taking the tax rates as given) with a utility function as given by (35). The competitive equilibrium is, then, the Pareto optimal allocation obtained using the "right" set of weights. However, under the assumption of identical utility functions across countries, we can show that the free-trade allocation does not depend on the particular values of the weights (since marginal rates of substitution need to be equalized across countries). The second step considers the production side. It defines the aggregate social production function as the sum of the after-tax national social production function subject to the integrated factor markets and market clearing of the investment good. As a result, we get countries' factors $(k^A(t), k^B(t), n^A(t), n^B(t))$ as function of aggregate variable $(k(t), l(t), y_i(t))$ and countries' tax rates $(\tau^A(t), \tau^B(t))$. We define thereafter the world social production function as $T(k(t), l(t), y_i(t), \tau^A(t), \tau^B(t))$.

Proposition 3. *Under free-trade, an equilibrium path of aggregate consumption, $c(t)$, labor, $l(t)$, and capital stock, $k(t)$, is determined in such a way that it solves the following maximization problem:*

$$\begin{aligned} &\max_{c(t),k(t),l(t),y_i(t)} \int_{t=0}^{+\infty} e^{-\rho t} ln(c(t)) - \frac{l(t)^{1+\chi}}{1+\chi} \\ &s.t.\ \ c(t) \leq T(k(t), l(t), y_i(t), \tau^A(t), \tau^B(t)) \\ &\dot{k}(t) = y_i(t) - \delta k(t) \end{aligned} \tag{37}$$

Once the aggregate consumption and labor path, $c(t)$ and $l(t)$ are determined, the two countries' consumption paths, $c^A(t)$ and $c^B(t)$, and labor supply paths, $l^A(t)$ and $l^B(t)$, are given by:

$$c^A(t) = \frac{(1/\lambda^A)c(t)}{(1/\lambda^A)+(1/\lambda^B)}, \qquad c^B(t) = \frac{(1/\lambda^B)c(t)}{(1/\lambda^A)^{\sigma}+(1/\lambda^B)}$$
$$l^A(t) = \frac{(1/\lambda^A)^{\frac{1}{\chi}} l(t)}{(1/\lambda^A)^{\frac{1}{\chi}}+(1/\lambda^B)^{\frac{1}{\chi}}}, \quad l^B(t) = \frac{(1/\lambda^B)^{\frac{1}{\chi}} l(t)}{(1/\lambda^A)^{\frac{1}{\chi}}+(1/\lambda^B)^{\frac{1}{\chi}}}$$

Proof: See Appendix 5.4

From this Proposition, we conclude that we can focus on the equilibrium aggregate paths of consumption and labor since it is independent of the Negishi weights, λ^A and λ^B. The distributions between countries of consumption and labor supply are determined by countries h's share of the marginal utility of wealth in the total marginal utility of wealth $\frac{(1/\lambda^h)}{\Sigma_h(1/\lambda^h)}$. On the production side, it is useful to define the aggregate social production function $T(k(t), l(t), y_i(t), \tau^A(t), \tau^B(t))$ in order to characterize the equilibrium according to the aggregate quantities, the world relative price and the world factor prices.

The current-value Hamiltonian associated with (37) is then:

$$\mathcal{H} = ln(T(k(t), l(t), y(t), \tau^A(t), \tau^B(t)) - \frac{\Gamma l(t)^{1+\chi}}{1+\chi} + \lambda(t)(y_i(t) - \delta k(t)) \quad (38)$$

We derive the following first-order conditions:[7]

$$\begin{aligned} \frac{p(k(t),l(t),y_i(t),\tau^A(t),\tau^B(t))}{c(k(t),l(t),y_i(t),\tau^A(t),\tau^B(t))} &= \lambda(t) \\ \Gamma l^\chi(t) &= \frac{w(k(t),l(t),y_i(t),\tau^A(t),\tau^B(t))}{c(k(t),l(t),y_i(t),\tau^A(t),\tau^B(t))} \\ -\dot{\lambda}(t) &= \lambda(t)\left(\frac{r(k(t),l(t),y_i(t),\tau^A(t),\tau^B(t))}{p(k(t),l(t),y_i(t),\tau^A(t),\tau^B(t))} - \rho - \delta\right) \end{aligned} \quad (39)$$

and the transversality condition:

$$\lim_{t\to+\infty} e^{-\rho t}\lambda(t)K(t) = 0 \quad (40)$$

Similarly to the closed-economy, we solve the two first equations in (39) to obtain $l(k(t), y_i(t), \tau^A(t), \tau^B(t))$ and $\lambda(k(t), y_i(t), \tau^A(t), \tau^B(t))$. Besides, the countries h's income taxes are determined by the national quantities of capital $k^h(t)$, labor $n^h(t)$ and investment output $y_i^h(t)$ which are themselves determined by the aggregate quantites $k(t)$, $l(t)$ and $y_i(t)$, we derive that $(\tau^A(t), \tau^B(t)) = (\tau^A(k(t), y_i(t)), \tau^B(k(t), y_i(t)))$. Substitute back these expressions to derive $l(k(t), y_i(t))$. It follows that the aggregate social production function writes $c(k(t), l(k(t), y_i(t)), \tau^A(k(t), y_i(t)), \tau^B(k(t), y_i(t))) \equiv c(k(t), y_i(t))$.

Thus, differentiating the first equation of the system (39) with respect to time and the Euler equation for $\dot{y}_i(t)$, we get:

[7] Note that from the supply-side problem, we get equalization of the relative price such that $p(k(t), l(t), y_i(t), \tau^A(t), \tau^B(t)) = p^h(k(t), l(t), y_i(t), \tau^A(t), \tau^B(t))$. Similarly, we get $r(k(t), l(t), y_i(t), \tau^A(t), \tau^B(t)) = (1 - \tau^h(t))r^h(k(t), l(t), y_i(t), \tau^A(t), \tau^B(t))$ and $w(k(t), l(t), y_i(t), \tau^A(t), \tau^B(t)) = (1 - \tau^h(t))w^h(k(t), l(t), y_i(t), \tau^A(t), \tau^B(t))$.

$$\dot{y}_i(t) = \frac{\left[\frac{r(k(t),l(t),y_i(t),\tau^A(t),\tau^B(t))}{p(k(t),l(t),y_i(t),\tau^A(t),\tau^B(t))} - \rho - \delta - \left(\frac{\varepsilon_{c,k}}{k(t)} - \frac{\varepsilon_{p,k}}{k(t)}\right)(y_i(t) - \delta k(t))\right]}{\left(\frac{\varepsilon_{c,y_i}}{y_i(t)} - \frac{\varepsilon_{p,y_i}}{y_i(t)}\right)} \tag{41}$$

The accumulation of the aggregate stock of capital is then given by:

$$\dot{k}(t) = y_i(t) - \delta k(t) \tag{42}$$

An intertemporal equilibrium is a path $(k(t), y_i(t))$ satisfying (41)-(42), given the initial condition $k(0) > 0$, and the transversality condition (40).

3.3 Stationary allocation under free-trade

In this section, we characterize the stationary allocation of the model under free-trade. It is well known, at least in the Heckscher-Ohlin model, that the distribution of capital between countries is indeterminate (see Chen [5], Nishimura and Shimomura [16]) and depends on the countries' initial conditions on capital stock $k^h(0)$ and marginal utility $\lambda^h(0)$. However, we may choose a particular point of that distribution and solve using some normalization constant.

Let us first focus on the stationary allocation of capital $k = k^A + k^B$ under free-trade. Consider that country h imports capital such that $k^{h*} < (1 - \tau^h) y_i^{h*}$ while the other country exports capital. Once we have characterized the distribution of exports and imports of capital, we are able then to derive the associated level of consumption for each country and therefore the export and import of the consumption good for countries A and B. Consider a particular solution θ satisfying a symmetric property such that $\delta k^A = \theta(1 - \tau^A) y_i^A$ and $\theta \delta k^B = (1 - \tau^B)) y_i^B$. In the rest of the paper, we will consider economies with parameter values for which such an equilibrium exists. Let us denote $\underline{\theta} = \frac{\delta \alpha_c^A}{(\rho+\delta)} < 1$ and $\bar{\theta} = \frac{(\rho+\delta)}{\delta \alpha_c^B} > 1$.

The following proposition provides conditions for the existence of the free-trade allocation:

Proposition 4. *Let $\xi_c^B = \xi_i^B = 1$ and consider a constant $\theta \in (\underline{\theta}, \bar{\theta})$. The* *free-trade allocation* $k = k^{A*} + k^{B*}$ *with:*

$$\begin{aligned} k^{A*} &= \frac{\theta\alpha_c^A\alpha_c^B\mathcal{C}^{\frac{-\chi}{1+\chi}}[xi_i^{A*}(1-\tau^A)(\rho+\delta)^{-\alpha_i^A}]^{\frac{1}{1-\alpha_i^A}}}{\alpha_c^B[(\rho+\delta)-\theta\delta\alpha_i^A]+\alpha_c^A[(\rho+\delta)-\delta\theta\alpha_i^B]} = \frac{\theta(1-\tau^A)y_i^{A*}}{\delta} \\ k^{B*} &= \frac{\alpha_c^A\alpha_c^B\mathcal{C}^{\frac{-\chi}{1+\chi}}[(1-\tau^B)(\rho+\delta)^{-\alpha_i^B}]^{\frac{1}{1-\alpha_i^B}}}{\alpha_c^B[(\rho+\delta)-\theta\delta\alpha_i^A]+\alpha_c^A[(\rho+\delta)-\delta\theta\alpha_i^B]} = \frac{(1-\tau^B)y_i^{B*}}{\theta\delta} \end{aligned} \tag{43}$$

is a solution of equation (41)-(42) if and only if $\xi_c = \xi_c^{A*}$ *and* $\xi_i = \xi_i^{A*}$ *with:*

$$\begin{aligned} \xi_c^{A*} &= \xi_i^{A*\frac{(1-\alpha_c^A)}{(1-\alpha_i^A)}}\left(\frac{\rho+\delta}{1-\tau^A}\right)^{\frac{(\alpha_c^A-\alpha_i^A)}{(1-\alpha_i^A)}}\left(\frac{\rho+\delta}{1-\tau^B}\right)^{\frac{-(\alpha_c^B-\alpha_i^B)}{(1-\alpha_i^B)}} \\ \xi_i^{A*} &= (1-\tau^B)^{\frac{(1-\alpha_i^A)}{(1-\alpha_i^B}}(1-\tau^A)^{-1}(\rho+\delta)^{\frac{\alpha_i^A-\alpha_i^B}{1-\alpha_i^B}} \end{aligned} \tag{44}$$

The associated free-trade allocation of consumption is such that $c^{A*} = (1-\tau^B)T^{B*}$ *and* $c^B = (1-\tau^A)T^{A*}$. *Moreover, country A is a net importer of capital for any* $\theta \in (1, \bar{\theta})$ *while country B is a net importer of capital for any* $\theta \in (\underline{\theta}, 1)$.

Proof: see Appendix 5.5.

Note that using appropriate values of ξ_c^{A*} and ξ_i^{A*}, we may characterize the autarkic distribution $\theta = 1$ in which countries trade along the transition path but do not export or import at steady state such that $(1-\tau^h)y_i^h = \delta k^h$. However, the allocation still depends on the tax rates of country A and B. As in the closed-economy configuration, we may have multiple stationary taxes satisfying (36) for a given stationary allocation. We use therefore the scaling constants Z^A and Z^B in order to consider constant steady state taxes $\tau^A = \eta^A \in (0,1)$ and $\tau^B = \eta^B \in (0,1)$ and characterize the Normalized Steady State (NSS thereafter).

Proposition 5. *Let* $\theta \in (\underline{\theta}, \bar{\theta})$, $\xi_c^A = \xi_c^{A*}$ *and* $\xi_i^A = \xi^{A*}$. *There exists unique value* $Z^{A*} > 0$ *and* $Z^{B*} > 0$ *such that when* $Z^A = Z^{A*}$ *and* $Z^B = Z^{B*}$, *the stationary allocation given in Proposition 4 is a NSS with* $(\tau^A, \tau^B) = (\eta^A, \eta^B)$.

Proof: see Appendix 5.5.

In the next section, we investigate the local dynamics of an integrated economy with tax policies.

3.4 The Stabilizing Effect of Free-Trade

Before beginning the investigation of the local dynamics, a word of caution is required. It is well known that under constant returns to scale private technology, the aggregate social production function $T(k(t), l(t), y_i(t))$ is not well defined since its second-order derivatives are nil. However, as shown in Appendix 5.6, the presence of an income tax policy allows to get a non-degenerate aggregate social production function. In particular, only $T_{m3}(k(t), l(t), y_i(t)) = T_{3m}(k(t), l(t), y_i(t)) = 0$ with $m = 1, 2, 3$. The reason for the latter is that income taxes have only an indirect effect on the relative price of the goods through the after-tax factor prices. But since these are equalized in equilibrium and are symmetric, their respective impacts on the relative price cancel each other. Furthermore, to conserve a non-degenerate social production function in the integrated economy, we shall rule out the case where one country sets a constant tax $\varepsilon_g^h = 1$. Similarly, the aggregate social production function is not well defined in the Heckscher-Ohlin-Samuelson configuration where countries have access to identical technology such that $\alpha_c^A = \alpha_c^B$ and $\alpha_i^A = \alpha_i^B$. Hence, we strictly assume a non-HOS model such that $\alpha_c^A \neq \alpha_c^B$ and $\alpha_i^A \neq \alpha_i^B$.

After having linearized the system (41)-(42), we derive the characteristic polynomial $P(z) = z^2 - \mathcal{T}z + \mathcal{D}$ with $\mathcal{T}$ and $\mathcal{D}$, the trace and the determinant of the Jacobian matrix, given in Appendix 5.6

Proposition 6. *Let $\xi_c^A = \xi_c^{A*}$, $\xi_i^A = \xi_i^{A*}$, $\xi_c^B = \xi_i^B = 1$, $Z^A = Z^{A*}$, $Z^B = Z^{B*}$ and $\theta \in (\underline{\theta}, \bar{\theta})$. Under a free-trade equilibrium, the NSS is always locally determinate. Furthermore, there exists $\bar{\chi}^{FT} > 0$ such that for any $\chi \in (0, \bar{\chi}^{FT})$, the NSS is locally saddle-path stable while for any $\chi \in (\bar{\chi}^{FT}, +\infty)$, the NSS is either locally a source or locally a saddle-path.*

Proof: see Appendix 5.6

According to Proposition 6, free-trade openness with perfect mobility of capital and labor rules out local indeterminacy. Furthermore, if labor is sufficiently elastic, the NSS is locally a saddle-path while the NSS may be a source or a saddle-path if labor is inelastic enough. Since we focus on a situation where the economy is indeterminate when closed, we shall assume that we are in the former case with elastic enough labor supply. From Proposition 2, we define $(\underline{\tau}^h, \bar{\tau}^h)$ as the interval of destabilizing tax rates in country h when closed, $\bar{\chi}^h$, $\bar{\rho}^h$ and $\bar{\alpha}_i^h$ respectively the upper-bounds on the elasticity of labor supply, on the discount factor in country and on the capital intensity in the investment good in country h. Let us assume:

Assumption 1. *$\rho \in (0, min(\bar{\rho}^A, \bar{\rho}^B))$, ε_g^A, $\varepsilon_g^B < 1$, $\alpha_i^A < \bar{\alpha}_i^A$, $\alpha_i^B < \bar{\alpha}_i^B$ and $\chi \in (0, min(\bar{\chi}^A, \bar{\chi}^B))$.*

In order to compare the effect of trade integration, we have to set the technological parameters ξ_c^h and ξ_i^h in the closed-economy at the same value than the free-trade equilibrium, given by (44). We conclude therefore:

Corollary 1. *Let $\theta \in (\underline{\theta}, \bar{\theta})$, $Z^A = Z^{A*}$, $Z^B = Z^{B*}$ and Assumption 1 hold. There exist $\underline{\alpha}_i^A \in (0, \bar{\alpha}_i^A)$ and $\underline{\alpha}_i^B \in (0, \bar{\alpha}_i^B)$ such that the free-trade equilibrium is locally saddle-path stable, while when closed:*

1. *country A is locally indeterminate if $\eta^A \in (\underline{\tau}^A, \bar{\tau}^A)$ and $\alpha_i^A \in (\underline{\alpha}_i^A, \bar{\alpha}_i^A)$, and country B is locally saddle-path stable if $\eta^B \notin (\underline{\tau}^B, \bar{\tau}^B)$. Free-trade prevents from the transmission of expectation-driven fluctuations in country B and stabilizes country A.*

2. *both countries are locally indeterminate if $\eta^A \in (\underline{\tau}^A, \bar{\tau}^A)$, $\alpha_i^A \in (\underline{\alpha}_i^A, \bar{\alpha}_i^A)$, $\eta^B \in (\underline{\tau}^B, \bar{\tau}^B)$ and $\alpha_i^B \in (\underline{\alpha}_i^B, \bar{\alpha}_i^B)$. Free-trade stabilizes both countries.*

Proof: see Appendix 5.7.

According to Corollary 1, free trade integration, with perfect factor mobility, is stabilizing economies that were locally indeterminate before trade integration. Note that this holds if one or both countries are locally indeterminate when closed. Hence, not only free-trade is stabilizing but it also prevents from expectation-driven fluctuations. Besides, these conclusions are independent of the pattern of trade and of the factor-intensity ranking of the countries.

A similar conclusion is provided by Sim and Ho [22] in HOS model with country-specific productive externalities and immobile factors. In particular, they show that if one of the trading country is locally determinate in autarky, the world economy is also determinate. Opposite results characterize the contributions of Nishimura *et al.* [17], [18] and [19]. These papers find that, in the non-Heckcher-Ohlin case, opening to free-trade a determinate economy may lead to endogenous cycles and/or local indeterminacy. Similarly, Nishimura and Shimomura [16] claims that opening to free-trade maintains the local indeterminacy property of a closed economy in the dynamic HOS model with productive externalities. However, a major difference with these papers is that we consider perfect mobility on both labor and capital while they assume immobile factor or perfect mobility of capital only.

Our result relies on the perfect mobility of factors associated with the integration of factor markets. Because factors are mobile across countries, an equilibrium requires the after-tax wage and after tax interest rate to equalize between countries. Because of that, any deviation from an equilibrium path generates some arbitrage which results in flows of both capital and labor from one country to another. Hence, a change in expectations may be not self-fulfilling since a country now may have access, from import, to capital stock. Suppose that country A imports capital and that its technology for the consumption good is more capital intensive than the investment good. Starting from the steady state, suppose that households expects an increase in the future income tax of country A which leads to a decrease in the expected after-tax rate of returns of capital in this country. As a consequence, households' demand of the investment good in country A decreases leading to a decline in the relative price of the investment good in that country. From Stolper-Samuelson theorem, this leads to a decline in the wage rate and an increase in the interest rate. In a closed-economy, households would reduce their labor supply and, given the capital stock is predetermined, have less revenues. However, under free-trade with perfectly mobile factors, an arbitrage in after-tax interest rates and after-tax wages results in flows of capital and labor between countries. In such case, households move labor from country A to B while country A imports more capital from B. But since country A uses more capital stock, households in country A also increases their labor supply. It follows that households' income, and therefore the tax base, increases. Under a countercyclical tax policy, the tax rate decreases. The initial expectations is therefore self-destroying.

4 Concluding comments

In this paper, we study the stabilizing effect of free-trade integration with perfectly mobile capital and labor. In particular, we show that trade openness prevents sunspot equilibria in a country that experiences expectation-driven fluctuations, due to a high enough countercyclical income tax, before trade integration. As a consequence, a policy that entails a full integration of both good markets and factor markets seems a powerful tool to stabilize one or several countries, for a given income tax policy. To the best of author's knowledge, this is the first paper to adress the issue of the stabilizing effect of trade integration under a destabilizing tax policy.

Our conclusion relies on the perfect mobility of all factors and constant returns to scale techology. Hence, further research may concentrate on studies focusing on the particular effect of partial mobility of factors. The assumption of perfect mobility of capital and even more of labor is, of course, extreme since some frictions exist between countries, especially in labor markets due to regional and cultural particularities. In a similar way, considering constant-return to scale technologies is also a crucial assumption since this implies a degenerate social production function. Assuming small decreasing returns, in at least one sector, seems therefore a promising path for research. It may also clarify the effect of perfect mobility of labor in comparison to the previous contribution of Nishimura *et al.* [17], [18] and [19]. Finally, the present paper has focused on an income tax policy that is specific to countries. A common policy, such as a full fiscal integration, over a unique tax base may also be an interesting research avenue.

5 Appendix

5.1 The social production function $T(k, l, y_i)$

Consider the firm's program:

$$\begin{array}{c} \phi_j(t) = Min_{\{k_j(t), n_j(t)\}} \quad \bar{\xi}_j r(t) k_j(t) + w(t) n_j(t) \\ s.t. \quad y_j(t) \geq \bar{\xi}_j k_j(t)^{\alpha_j} n_j(t)^{1-\alpha_j} \end{array} \tag{45}$$

Solving the system given by the first-order conditions allows to get factor demands for capital k_j, labor n_j and the Lagrange multiplier $\mu_j(t)$ as function of the rental rate of capital $r(t)$, the wage rate $w(t)$ and sectoral output y_j :

$$n_j(t) = (1-\alpha_j)\xi_j^{-1}(\frac{w(t)}{r(t)})^{-\alpha_j} y_j(t) \tag{46}$$

$$k_j(t) = \alpha_c \xi_j^{-1}(\frac{w(t)}{r(t)})^{1-\alpha_j} \tag{47}$$

$$\mu_j(t) = \xi_j^{-1} r(t)^{\alpha_j} w(t)^{1-\alpha_j} \tag{48}$$

where we have used $\bar{\xi}_j = \alpha_j^{-\alpha_j}(1-\alpha_j)^{-(1-\alpha_j)}\xi$. We derive the total cost function $\phi(t)$ by substituting equations (46)-(47) in (45) and obtain:

$$\phi_j(r(t), w(t), y_j(t)) = \xi_j^{-1} r(t)^{\alpha_j} w(t)^{1-\alpha_j} y_j(t) \tag{49}$$

Labor l and capital k must clear on the factor market such that: [8]

$$l = n_c + n_i \tag{50}$$

$$k = k_c + k_i \tag{51}$$

Using Shephard's Lemma, the sectoral input are given by $n_j = \phi_j^w(r, w, y_j)$ and $k_j = \phi_j^r(r, w, y_j)$:

Considering these together with the sectoral profit maximizations, we get :

$$\begin{array}{l} \phi_j^r(r, w, y_j) = \alpha_j \xi_j^{-1} r^{\alpha_j - 1} w^{\alpha_j} y_j = \frac{\alpha_j p_j y_j}{r} \\ \phi_j^w(r, w, y_j) = (1-\alpha_j)\xi_j^{-1} r^{\alpha_j} w^{-\alpha_j} y_j = \frac{\alpha_j p_j y_j}{w} \end{array} \tag{52}$$

where $p_c = 1$ and $p_i = p$.

Factor markets are then given by :

$$\begin{array}{rcl} l &=& \frac{\beta_c y_c}{w} + \frac{\beta_i p y_i}{w} \\ k &=& \frac{\alpha_c y_c}{r} + \frac{\alpha_i p y_i}{r} \end{array} \tag{53}$$

Solving these two expressions for in y_c and y_i, we get the sectoral supply functions :

$$\begin{array}{rcl} y_c(k, n, p) &=& \frac{(1-\alpha_i) r(p) k - \alpha_i w(p) n}{\alpha_c - \alpha_i} \\ y_i(k, n, p) &=& \frac{\alpha_c w(p) n - (1-\alpha_c) r(p) k}{p(\alpha_c - \alpha_i)} \end{array} \tag{54}$$

We derive the following relationship between the sectoral supply functions, the capital stock, the labor supply and the relative price.

[8] From now on, we omit the argument (t).

$$\frac{dy_c}{dk} = \frac{(1-\alpha_i)r}{(\alpha_c-\alpha_i)}, \qquad \frac{dy_i}{dk} = \frac{-(1-\alpha_c)r}{(\alpha_c-\alpha_i)p}$$

$$\frac{dy_c}{dn} = \frac{-\alpha_i w}{(\alpha_c-\alpha_i)}, \qquad \frac{dy_i}{dn} = \frac{\alpha_c w}{(\alpha_c-\alpha_i)p} \tag{55}$$

$$\frac{dy_c}{dp} = \frac{-[(1-\alpha_c)(1-\alpha_i)rk+\alpha_c\alpha_i wl]}{p(\alpha_c-\alpha_i)^2}, \quad \frac{dy_i}{dp} = \frac{[(1-\alpha_c)(1-\alpha_i)rk+\alpha_c\alpha_i wl]}{p^2(\alpha_c-\alpha_i)^2}$$

Note that $T(k,l,y_i) - T_3(k,l,y_i)y_i = rk + wl$.

From (54), define $p(k,l,y_i)$ and substitute this expression in $y_c(k,l,p)$. This yields $yc(k,l,p(k,l,y_i)) \equiv y_c(k,l,y_i) = T(k,l,y_i)$. The latter function represents the Production Possibility Frontier. Using (55) and implicitly differenciating, we find :

$$\begin{aligned} T_1(k,l,y_i) &= \frac{dy_c(k,l,p)}{dk} + \frac{dy_c(k,l,p)}{dp}\frac{dp}{dy_i(k,l,p)}\frac{dy_i(k,l,p)}{dk} = r \\ T_2(k,l,y_i &\frac{dy_c(k,l,p)}{dl} + \frac{dy_c(k,l,p)}{dp}\frac{dp}{dy_i(k,l,p)}\frac{dy_i(k,l,p)}{dl} = w \\ T_3(k,l,y_i) &= \frac{dy_c(k,l,p)}{dp}\frac{dp}{dy_i(k,l,p)} = -p \end{aligned}$$

It follows that the identity $y_c + py_i = T(k(t),l(t),y_i(t)) - T_3(k(t),l(t),y_i(t))y_i(t)$ is satisfied.

We also derive the second-order derivatives of the $T(k,l,y_i)$ function:

$$\begin{aligned} T_{11} &\equiv \frac{\partial r}{\partial p}\frac{\partial p}{\partial k} = \frac{-(1-\alpha_c)^2T_1^2}{[(1-\alpha_c)(1-\alpha_i)rk+\alpha_c\alpha_i wl]} & T_{12} &\equiv \frac{\partial r}{\partial p}\frac{\partial p}{\partial l} = \frac{(1-\alpha_c)\alpha_c T_1T_2}{[(1-\alpha_c)(1-\alpha_i)rk+\alpha_c\alpha_i wl]} \\ T_{13} &\equiv \frac{\partial r}{\partial p}\frac{\partial p}{\partial y_i} = \frac{(1-\alpha_c)(\alpha_c-\alpha_i)T_1T_3}{[(1-\alpha_c)(1-\alpha_i)rk+\alpha_c\alpha_i wl]} & T_{22} &\equiv \frac{\partial w}{\partial p}\frac{\partial p}{\partial l} = \frac{-\alpha_c^2T_2^2}{[(1-\alpha_c)(1-\alpha_i)rk+\alpha_c\alpha_i wl]} \\ T_{23} &\equiv \frac{\partial w}{\partial p}\frac{\partial p}{\partial y_i} = \frac{-\alpha_c(\alpha_c-\alpha_i)T_2T_3}{[(1-\alpha_c)(1-\alpha_i)rk+\alpha_c\alpha_i wl]} & T_{33} &\equiv -\frac{\partial p}{\partial y_i} = \frac{-(\alpha_c-\alpha_i)^2T_3^2}{[(1-\alpha_c)(1-\alpha_i)rk+\alpha_c\alpha_i wl]} \end{aligned} \tag{56}$$

5.2 Proof of Proposition 1

The steady state is given by the solution of :

$$\begin{aligned}
(1-\tau)y_i &= \delta k \\
(1-\tau)\tfrac{r}{p} &= (\rho+\delta) \\
y_c &= \tfrac{(1-\alpha_i)rk-\alpha_i wl}{(\alpha_c-\alpha_i)} \\
y_i &= \tfrac{\alpha_c wl-(1-\alpha_c)rk}{p(\alpha_c-\alpha_i)} \\
\Gamma l^{1+\chi} &= \tfrac{w}{y_c} \\
w &= \left[\frac{\left[(\xi_i p)\right]^{\alpha_c}}{\left[\xi_c\right]^{\alpha_i}}\right]^{\frac{1}{\alpha_c-\alpha_i}} \\
r(p) &= \left[\frac{\left[(\xi_i p)\right]^{-(1-\alpha_c)}}{\xi_c^{-(1-\alpha_i}}\right]^{\frac{1}{\alpha_c-\alpha_i}} \\
\bar{g}(y_c+py_i)^{\varepsilon_g} &= \tau(t)(r(t)k(t)+w(t))
\end{aligned}$$

We use the two first equations in the sectoral supply functions to derive :

$$\begin{aligned}
k &= \tfrac{(1-\tau)\alpha_c w(p)l}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)]p} \\
y_i &= \tfrac{\delta\alpha_c w(p)l}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)]p} \\
y_c &= \tfrac{((\rho+\delta(1-\alpha_i))w(p)l}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}
\end{aligned} \tag{57}$$

Solving the steady-state Euler equation for p gives:

$$p(\tau) = \xi_c\left\{\xi_i^{-(1-\alpha_c)}\left[\frac{(1-\tau)}{(\rho+\delta)}\right]^{\alpha_c-\alpha_i}\right\}^{\frac{1}{1-\alpha_i}} \tag{58}$$

And the expressions for $w(\tau)$ and $r(\tau)$:

$$\begin{aligned}
r(\tau) &= \xi_c\left\{\xi_i^{-(1-\alpha_c)}\left[\tfrac{(1-\tau)}{(\rho+\delta)}\right]^{-(1-\alpha_c)}\right\}^{\frac{1}{1-\alpha_i}} \\
w(\tau) &= \xi_c\left\{\xi_i^{\alpha_c}\left[\tfrac{(1-\tau)}{(\rho+\delta)}\right]^{\alpha_c}\right\}^{\frac{1}{1-\alpha_i}}
\end{aligned} \tag{59}$$

Using the consumption-labor trade-off with $\Gamma = 1$ and the expression of y_c, we drive:

$$l = \left[\frac{[\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}\right]^{\frac{-1}{1+\chi}} \tag{60}$$

The existence and the uniquess or multiplicity of steady state are then determined by τ in the balanced budget rules given by :

$$G(r(\tau)k(\tau)+w(\tau)) = \tau[r(\tau)k(\tau)+w(\tau)] \tag{61}$$

One easily derives that the right-hand side of the rule describes a Laffer curve for any $\tau \in (0,1)$. The left-hand side determines therefore the number of tax rates such that the rules is satisfied according to the value of ε_g. We have therefore to solve:

$$\Omega(\tau) \equiv \tau \left[\frac{[\rho+\delta(1+\alpha_c-\alpha_i)]l}{[(1-\alpha_c)\rho+(1-\alpha_i)]}(1-\tau)^{\frac{\alpha_c}{1-\alpha_i}}\right]^{(\varepsilon_g-1)} = 1 \tag{62}$$

If $\varepsilon_g > 1$, the right-hand side is stricly increasing for any $\tau \in (0,1)$ and goes from zero to infinity. As a result, there exists a unique steady state. However, if $\varepsilon_g < 1$, the steady state may not exist or there may be multiplicity of steady state since $f(0) = f(1) = 0$.

We derive that:

$$\Omega'(\tau) = \frac{\alpha_c}{(1-\alpha_i)}\tau\left[\frac{[\rho+\delta(1+\alpha_c-\alpha_i)]l}{[(1-\alpha_c)\rho+(1-\alpha_i)]}(1-\tau)^{\frac{\alpha_c}{1-\alpha_i}}\right]^{-(\varepsilon_g-1)}\left[\frac{(1-\alpha_i)}{\alpha_c}+\frac{\tau(\varepsilon_g-1)}{1-\tau}\right] \tag{63}$$

Hence, $\Omega'(\tau) > (<)0$ if and only if $\tau < (>)\frac{1-\alpha_i}{1-\alpha_i+\alpha_c(1-\varepsilon_g)}$.

It follows that there exists $\bar{\tau} \in (0,1)$ such that $f(\tau^*) = maxf(\tau)$ and there are two steady-state tax rate that satisfy $G(t) = \tau(rk+wl)$.

□

5.3 Proof of Proposition 2

Let us first derive $\frac{dc}{dk}, \frac{dc}{dy_i}, \frac{dp}{dk}, \frac{dp}{dy_i}$.

Rewrite first the first order conditions (10) and (11) using the market clearing condition for the consumption good, $c = (1-\tau(k,y_i))T(k,l,y_i)$, such that:

$$\begin{gathered}\frac{T_3(k,l,y_i)}{(1-\tau(k,y_i))T(k,l,y_i)} = -\lambda \\ \Gamma l^{\chi} = \frac{T_2(k,l,y_i)}{T(k,l,y_i)}\end{gathered} \tag{64}$$

Solve the system to obtain $\lambda(k,y_i)$ and $l(k,y_i)$ and differentiate this system with respect to k, y_i, l and λ and solve in order to derive:

$$\begin{aligned} \frac{dl}{dk} &= \frac{\frac{T_{21}}{T_2} - \frac{T_1}{T}}{\frac{\chi}{l} + \frac{T_2}{T} - \frac{T_{22}}{T_2}} \\ \frac{dl}{dy_i} &= \frac{\frac{T_{23}}{T_2} - \frac{T_3}{T}}{\frac{\chi}{l} + \frac{T_2}{T} - \frac{T_{22}}{T_2}} \end{aligned}$$

If we totally differentiate $c(k, l(k, y_i), y_i) = (1 - \tau(k, y_i))T(k, l(k, y_i, y_i)$, we get:

$$\begin{aligned} \frac{dc}{dk} &= (1-\tau)T_1 - \frac{d\tau}{dk}T + (1-\tau)T_2\frac{dl}{dk} \\ \frac{dc}{dy_i} &= (1-\tau)T_3 - \frac{d\tau}{dy_i}T + (1-\tau)T_2\frac{dl}{dy_i} \end{aligned}$$

Similarly, we get from the definition of $p(k, l(k, y_i), y_i)$:

$$\begin{aligned} \frac{dp}{dk} &= T_{31} + T_{32}\frac{dl}{dk} \\ \frac{dp}{dy_i} &= T_{33} + T_{32}\frac{dl}{dy_i} \end{aligned}$$

Using these expressions, we derive the Euler equation (20) such that:

$$\dot{y}_i = \frac{\left\{\frac{(1-\tau)T_1}{T_3} + \rho + \delta - \left[\frac{T_{31}}{T_3} - \frac{T_1}{T} + \frac{d\tau}{dk}\frac{1}{1-\tau} + \frac{dl}{dk}\left(\frac{T_{32}}{T3} - \frac{T_2}{T}\right)\right][(1-\tau)y_i - \delta k]\right\}}{\left[\frac{T_{33}}{T_3} - \frac{T_3}{T} + \frac{d\tau}{dy_i}\frac{1}{1-\tau} + \frac{dl}{dy_i}\left(\frac{T_{32}}{T3} - \frac{T_2}{T}\right)\right]} \tag{65}$$

Finally, we use the identity $rk + wl = y_c + py_i \equiv T(k, y_i) - T_3(k, y_i)y_i$ to write the tax rule (2) such that $\tau(k, y_i) = (T(k, y_i) - T_3(k, y_i)y_i)^{\varepsilon_g - 1}$. We get then:

$$\begin{aligned} \frac{d\tau(k(t), y_i(t))}{dk(t)} &= \frac{\tau(\varepsilon_g - 1)}{[T - T_3 y_i]}[T_1 - T_{31}y_i + (T_2 - T_{32}y_i)\frac{dl}{dk}] \\ \frac{d\tau(k(t), y_i(t))}{dy_i(t)} &= \frac{\tau(\varepsilon_g - 1)}{[T - T_3 y_i]}[-T_{33}y_i + (T_2 - T_{32}y_i)\frac{dl}{dy_i}] \end{aligned}$$

We totally differentiate the equations (21) and (65) in the neighborhood of the NSS. we obtain

$$\begin{aligned} \frac{d\dot{k}}{dk} &= -\left(\frac{d\tau}{dk}y_i + \delta\right) \\ \frac{d\dot{k}}{dy_i} &= 1 - \tau - \frac{d\tau}{dy_i} \\ \frac{d\dot{y}_i}{dk} &= \frac{(\rho+\delta)[\frac{T_{31}}{T_3} - \frac{T_{11}}{T_1} + \frac{dl}{dk}(\frac{T_{32}}{T_3} - \frac{T_{12}}{T_1}] + (\frac{d\tau}{dk}y_i + \delta)(\frac{T_1}{T} - \frac{\tau_k}{(1-\tau)}) + \frac{dl}{dk}(\frac{T_{32}}{T3} - \frac{T_2}{T})]}{\left[\frac{T_{33}}{T_3} - (\frac{T_3}{T} - \frac{\tau y_i}{(1-\tau)}) + \frac{dl}{dy_i}(\frac{T_{32}}{T3} - \frac{T_2}{T})\right]} \\ \frac{d\dot{y}_i}{dy_i} &= \frac{(\rho+\delta)[\frac{T_{33}}{T_3} - \frac{T_{13}}{T_1} + \frac{dl}{dy_i}(\frac{T_{32}}{T_3} - \frac{T_{12}}{T_1}] - (1 - \tau - \frac{d\tau}{dy_i})(\frac{T_1}{T} - \frac{\tau_k}{(1-\tau)}) + \frac{dl}{dk}(\frac{T_{32}}{T3} - \frac{T_2}{T})]}{\left[\frac{T_{33}}{T_3} - (\frac{T_3}{T} - \frac{\tau y_i}{(1-\tau)}) + \frac{dl}{dy_i}(\frac{T_{32}}{T3} - \frac{T_2}{T})\right]} \end{aligned}$$

where the T_{mn} evaluated at the NSS are given by:

$$\begin{array}{ll} T_{11} = \frac{-(1-\alpha_c)^2 T_1^2[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)(1-\alpha_c+\alpha_i)]\alpha_c T_2 l} & T_{12} = \frac{(1-\alpha_c)\alpha_c T_1 T_2[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)(1-\alpha_c+\alpha_i)]\alpha_c T_2 l} \\ T_{13} = \frac{(1-\alpha_c)(\alpha_c-\alpha_i)T_1 T_3[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)(1-\alpha_c+\alpha_i)]\alpha_c T_2 l} & T_{22} = \frac{-\alpha_c^2 T_2^2[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)(1-\alpha_c+\alpha_i)]\alpha_c T_2 l} \\ T_{23} = \frac{-\alpha_c(\alpha_c-\alpha_i)T_2 T_3[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)(1-\alpha_c+\alpha_i)]\alpha_c T_2 l} & T_{33} = \frac{-(\alpha_c-\alpha_i)^2 T_3^2[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)(1-\alpha_c+\alpha_i)]\alpha_c T_2 l} \end{array} \tag{66}$$

The trace and the determinant are respectively given by:

$$\begin{array}{c} \mathcal{T} = \frac{d\dot{k}}{dk} + \frac{d\dot{y}_i}{dy_i} \\ \mathcal{D} = \frac{d\dot{k}}{dk}\frac{d\dot{y}_i}{dy_i} - \frac{d\dot{k}}{dy_i}\frac{d\dot{y}_i}{dk} \end{array}$$

After some computations, we derive the expressions of the trace and the determinant:

$$\begin{array}{l} \mathcal{D} = \frac{\frac{(\rho+\delta)}{\alpha_c}(1+\chi)[(1-\alpha_i)(1-\tau)-\tau\alpha_c(1-\varepsilon_g)]}{\left[\frac{\alpha_i^2}{\alpha_c}+\chi\left[1+\frac{(\alpha_c-\alpha_i)[\rho(\alpha_c-\alpha_i)+\delta(1-\alpha_i)(1-\alpha_c)]}{\alpha_c[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}\right]\right](1-\tau)-\tau(1-\varepsilon_g)\left[\frac{(\rho+\delta)}{[\rho+\delta(1+\alpha_c-\alpha_i)]}-\alpha_i+\frac{\chi\delta(\alpha_c-\alpha_i)^2[\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)][\rho+\delta(1+\alpha_c-\alpha_i)]}\right]} \\ \mathcal{T} = \rho + \frac{\tau(1-\varepsilon_g)(1+\chi)(\rho+\delta)}{\left[\frac{\alpha_i^2}{\alpha_c}+\chi\left[1+\frac{(\alpha_c-\alpha_i)[\rho(\alpha_c-\alpha_i)+\delta(1-\alpha_i)(1-\alpha_c)]}{\alpha_c[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}\right]\right](1-\tau)-\tau(1-\varepsilon_g)\left[\frac{(\rho+\delta)}{[\rho+\delta(1+\alpha_c-\alpha_i)]}-\alpha_i+\frac{\chi\delta(\alpha_c-\alpha_i)^2[\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)][\rho+\delta(1+\alpha_c-\alpha_i)]}\right]} \end{array} \tag{67}$$

Denote:

$$\underline{\tau} = \frac{\left[\frac{\alpha_i^2}{\alpha_c}+\chi\left[1+\frac{(\alpha_c-\alpha_i)[\rho(\alpha_c-\alpha_i)+\delta(1-\alpha_i)(1-\alpha_c)]}{\alpha_c[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}\right]\right]}{(1-\varepsilon_g)\left[\frac{(\rho+\delta)}{[\rho+\delta(1+\alpha_c-\alpha_i)]}-\alpha_i+\frac{\chi\delta(\alpha_c-\alpha_i)^2[\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)][\rho+\delta(1+\alpha_c-\alpha_i)]}\right]+\left[\frac{\alpha_i^2}{\alpha_c}+\chi\left[1+\frac{(\alpha_c-\alpha_i)[\rho(\alpha_c-\alpha_i)+\delta(1-\alpha_i)(1-\alpha_c)]}{\alpha_c[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}\right]\right]}$$

$$\bar{\tau} = \frac{(1-\alpha_i)}{1-\alpha_i+\alpha_c(1-\varepsilon_g)}$$

Note first that $\underline{\tau}$ and $\bar{\tau}$ are higher than unity if $\varepsilon_g > 1$.

Assume now that $\varepsilon_g < 1$. A necessary condition to obtain local indeterminacy is that the latter is negative. It requires therefore that $\underline{\tau} < \tau$. Given this condition, a positive determinant is obtained if $\tau < \bar{\tau}$. We require then that $\underline{\tau} < \bar{\tau}$. This requires to set $\chi < \bar{\chi}$ with

$$\bar{\chi} = \frac{(1-\alpha_i)\frac{(\rho+\delta)}{[\rho+\delta(1+\alpha_c-\alpha_i)]} - \alpha_i}{\alpha_c + \frac{(\alpha_c-\alpha_i)[[\rho+\delta(1-\alpha_i)][\rho(\alpha_c-\alpha_i)+\delta(1-\alpha_i)^2]+\delta\alpha_c[\rho(\alpha_c-\alpha_i)+\delta(1-\alpha_i)(1-\alpha_c)]]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)][\rho+\delta(1+\alpha_c-\alpha_i)}} \tag{68}$$

From the denominator of $\bar{\chi}$, we derive that $\bar{\chi} > 0$ if α_i is low enough and negative otherwise. Furthermore, if $\alpha_i = 0.5$, the numerator is negative. It follows that there exists $\bar{\alpha}_i$ such that $\bar{\chi} \in (0, 0.5)$ for any $\alpha_i \in (0, \bar{\alpha}_i)$.

A necessary condition to obtain local indeterminacy is then $\tau \in (\underline{\tau}, \bar{\tau})$ and $\varepsilon_g < 1$. We need finally to ensure that the trace is negative. This requires the following to hold:

$$\frac{\rho\left[(1-\varepsilon_g)\left[\frac{(\rho+\delta)}{[\rho+\delta(1+\alpha_c-\alpha_i)]}-\alpha_i+\frac{\chi\delta(\alpha_c-\alpha_i)^2[\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)][\rho+\delta(1+\alpha_c-\alpha_i)]}\right]+\left[\frac{\alpha_i^2}{\alpha_c}+\chi\left[1+\frac{(\alpha_c-\alpha_i)[\rho(\alpha_c-\alpha_i)+\delta(1-\alpha_i)(1-\alpha_c)]}{\alpha_c[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}\right]\right]\right](\underline{\tau}-\tau)+(\rho+\delta)\tau(1-\varepsilon_g)}{\left[(1-\varepsilon_g)\left[\frac{(\rho+\delta)}{[\rho+\delta(1+\alpha_c-\alpha_i)]}-\alpha_i+\frac{\chi\delta(\alpha_c-\alpha_i)^2[\rho+\delta(1-\alpha_i)]}{[(1-\alpha_c)\rho+\delta(1-\alpha_i)][\rho+\delta(1+\alpha_c-\alpha_i)]}\right]+\left[\frac{\alpha_i^2}{\alpha_c}+\chi\left[1+\frac{(\alpha_c-\alpha_i)[\rho(\alpha_c-\alpha_i)+\delta(1-\alpha_i)(1-\alpha_c)]}{\alpha_c[(1-\alpha_c)\rho+\delta(1-\alpha_i)]}\right]\right]\right](\underline{\tau}-\tau)} \leq 0$$

Given that the denominator is negative, a negative trace requires a positive numerator. The first term of the denominator is negative while the second is positive. The numerator is positive for $\rho = 0$ but not necessarily for $\rho = +\infty$. Hence, by a continuity argument, there exists $\bar{\rho} \in (0, +\infty)$ such that given $\tau > \underline{\tau}$, $\rho < \bar{\rho}$ satisfies the previous inequality.

□

5.4 Proof of Proposition 3

Let us write the static lagrangian from the consumer side :

$$\begin{aligned} L = & \frac{ln(c(t)^A)}{\lambda^A} + \frac{ln(c(t)^B)}{\lambda^B} - \frac{\Gamma l^{A\,1+\chi}}{(1+\chi)\lambda^A} - \frac{\Gamma l^{B\,1+\chi}}{(1+\chi)\lambda^B} \\ & + \mu_c^c(c - c^A - c^B) + \mu_l[l - l^A - l^B] \end{aligned} \tag{69}$$

We derive the first-order conditions with respect to c^A, l^A, c^B,l^B,c:

$$\begin{aligned} (c(t)^A)^{-1} &= \mu_c\lambda^A \\ (c(t)^B){-1} &= \mu_c\lambda^B \\ \frac{\Gamma l^{A\chi}}{\lambda^A} &= \mu_l \\ \frac{\Gamma l^{B\chi}}{\lambda^B} &= \mu_l \\ c^A + c^B &= c \\ l^A + l^B &= l \end{aligned} \tag{70}$$

Solving this system of equations yields:

$$
\begin{array}{ll}
{\mu_c}^{-1} = \frac{c}{(1/\lambda^A + 1/\lambda^B)} & {\mu_l}^{-1} = \frac{l}{(1/\lambda^{A\frac{1}{\chi}} + 1/\lambda^{B\frac{1}{\chi}})} \\
c^A = \frac{1/\lambda^A}{(1/\lambda^A + (1/\lambda^B)} c & c^B = \frac{1/\lambda^B}{(1/\lambda^A + 1/\lambda^B)} c \\
l^A = \frac{(1/\lambda^A)^{\frac{1}{\chi}}}{(1/\lambda^A)^{\frac{1}{\chi}} + (1/\lambda^B)^{\frac{1}{\chi}}} l & l^B = \frac{(1/\lambda^A)^{\frac{1}{\chi}}}{(1/\lambda^A)^{\frac{1}{\chi}} + (1/\lambda^B)^{\frac{1}{\chi}}} l
\end{array} \tag{71}
$$

We substitute expressions (71) in the countries' utility function weighted by the Negishi weights to get:

$$
ln(c(t)) \left[\frac{ln\left(\frac{1/\lambda^A}{1/\lambda^A + 1/\lambda^B}\right)}{\lambda^A} + \frac{ln\left(\frac{1/\lambda^B}{1/\lambda^A + 1/\lambda^B}\right)}{\lambda^B} \right] - \frac{\Gamma l(t)^{1+\chi}}{1+\chi} \left[\frac{\left(\frac{(1/\lambda^A)^{\frac{1}{\chi}}}{(1/\lambda^A)^{\frac{1}{\chi}} + (1/\lambda^B)^{\frac{1}{\chi}}}\right)^{1+\chi}}{\lambda^A} + \frac{\left(\frac{(1/\lambda^A)^{\frac{1}{\chi}}}{(1/\lambda^A)^{\frac{1}{\chi}} + (1/\lambda^B)^{\frac{1}{\chi}}}\right)^{1+\chi}}{\lambda^B} \right]
$$

Because the utility function is additively-separable , the terms in bracket are only scaling the marginal utility of c and l and can therefore be dropped. We get then:

$$
U(c, l) = ln(c(t)) - \frac{\Gamma l(t)^{1+\chi}}{1+\chi}
$$

and countries' consumption and labor paths are determined according to (71).

Let us now consider the production side. The aggregate social production function is the solution of the following:

$$
\begin{gathered}
T(k, l, y_i) = \max_{\substack{k^h, n^h, y_i^h \\ h=A,B}} (1-\tau^A) T^A(k^A, n^A, y_i^A) + (1-\tau^B) T^B(k^B, n^B, y_i^B) \\
s.t. \quad k \geq k^A + k^B \\
y_i \leq (1-\tau^A) y_i^A + (1-\tau^B) y_i^B \\
l \geq n^A + n^B
\end{gathered} \tag{72}
$$

The first-order conditions are:

$$
(1-\tau^A) T_1^A(k^A, n^A, y_i^A) = \lambda_1 \tag{73}
$$

$$
(1-\tau^B) T_1^B(k^B, n^B, y_i^B) = \lambda_1 \tag{74}
$$

$$
(1-\tau^A) T_2^A(k^A, n^A, y_i^A) = \lambda_2 \tag{75}
$$

$$
(1-\tau^B) T_2^B(k^B, n^B y_i^B) = \lambda_2 \tag{76}
$$

$$T_3^A(k^A, n^A, y_i^A) = -\lambda_3 \tag{77}$$

$$T_3^B(k^B, n^B, y_i^B) = -\lambda_3 \tag{78}$$

$$n^A + n^B = l \tag{79}$$

$$k^A + k^B = k \tag{80}$$

$$(1-\tau^A)y_i^A + (1-\tau^B)y_i^B = y_i \tag{81}$$

Solving equations (75) and (77) and using (79) gives $n^h(k^A, y_i^A, k^B, y_i^B, l, \tau^A, \tau^B)$ with $h = A, B$. We substitute in equations (73), (74), (78), (76) and solve with (81) and (80) to obtain $y_i^h(k, y_i, l, \tau^A, \tau^B)$. Finally, we substitute the tax rate of each country $\tau^h(k^h, n^h, y_i^h)$ to obtain $k^h(k, l, y_i)$, $n^h(k, l, y_i)$ and $y_i^h(k, l, y_i)$. As a result, the aggregate social production function is given by $T(k, l, y_i)$ and by the Enveloppe Theorem, we have $\lambda_1 = T_1(k, l, y_i) \equiv r$, $\lambda_2 = T_2(k, l, y_i) \equiv w$ and $-\lambda_3 = T_3(k, l, y_i) \equiv -p$.

5.5 Proof of Propositions 4 and Proposition 5

A steady-state satisfies :

$$(1-\tau^A)r^A = (1-\tau^B)r^B = T_1 \tag{82}$$

$$(1-\tau^A)w^A = (1-\tau^B)w^B = T_2 \tag{83}$$

$$p^A = p^B = -T_3 \tag{84}$$

$$\frac{(1-\tau^A)r^A}{p^A} = (\rho + \delta) = \frac{(1-\tau^B)r^B}{p^B} \tag{85}$$

$$n^A + n^B = l \tag{86}$$

$$k^A + k^B = k \tag{87}$$

$$(1-\tau^A)y_i^A + (1-\tau^B)y_i^B = y_i \tag{88}$$

$$T = (1-\tau^A)y_c^A + (1-\tau^B)y_c^B \tag{89}$$

$$y_i = \delta k \tag{90}$$

$$T_2 = Tl^{\chi} \tag{91}$$

together with the expression of the sectoral supply functions in each country. Suppose that there is a constant θ such that: $\theta(1-\tau^A)y_i^A = \delta k^A$ and $(1-\tau^B)y_i^B = \theta\delta k^B$ hold.

We substitute the sectoral supply functions of each country in these relationships together with equation (85) and solve to get:

$$\begin{array}{ll} k^A = \frac{(1-\tau^A)\theta\alpha_c^A w^A n^A}{p^A[\theta(1-\alpha_c^A)(\rho+\delta)+\delta(\alpha_c^A-\alpha_i^A)]}, & k^B = \frac{(1-\tau^B)\alpha_c^B w^B n^B}{p^B[(1-\alpha_c^B)(\rho+\delta)+\theta\delta(\alpha_c^B-\alpha_i^B)]} \\ y_i^A = \frac{\alpha_c^A \delta w^A n^A}{p^A[\theta(1-\alpha_c^A)(\rho+\delta)+\delta(\alpha_c^A-\alpha_i^A)]}, & y_i^B = \frac{\theta\alpha_c^B \delta w^B n^B}{p^B[(1-\alpha_c^B)(\rho+\delta)+\theta\delta(\alpha_c^B-\alpha_i^B)]} \\ y_c^A = \frac{[(\rho+\delta)\theta-\delta\alpha_i^A]w^A n^A}{[\theta(1-\alpha_c^A)(\rho+\delta)+\delta(\alpha_c^A-\alpha_i^A)]}, & y_c^B = \frac{[(\rho+\delta)-\theta\delta\alpha_i^B]\alpha_c^B \delta w^B n^B}{[(1-\alpha_c^B)(\rho+\delta)+\theta\delta(\alpha_c^B-\alpha_i^B)]} \end{array} \tag{92}$$

Incomplete specialization for both countries requires $\theta \in (\underline{\theta}, \bar{\theta})$ with $\underline{\theta} = \frac{\delta^A}{(\rho+\delta)} < 1$ and $\bar{\theta} = \frac{(\rho+\delta)}{\delta\alpha_i^B} > 1$.

Note that $\mathcal{NX}_i^A = (1-\tau^A)y_i^A(\theta-1)$ and $\mathcal{NX}_i^B = (1-\tau^B)y_i^B\frac{\theta-1}{\theta}$. From the clearing market condition for the investment good, we derive:

$$\begin{array}{rcl} \theta(1-\tau^A)y_i^A & = & (1-\tau^B)y_i^B \\ k^A & = & \theta k^B \end{array} \tag{93}$$

Hence, to be an equilibrium, the following must hold:

$$n^A = \frac{\alpha_c^B[\theta(1-\alpha_c^A)(\rho+\delta)+\delta(\alpha_c^A-\alpha_i^A)]}{\alpha_c^A[(1-\alpha_c^B)(\rho+\delta)+\theta\delta(\alpha_c^B-\alpha_i^B)]}n^B \tag{94}$$

Where we used the equalities $p^A = p^B$ and $(1-\tau^A)w^A = (1-\tau^B)w^B$.

Consider now the consumption good clearing market and suppose that $c^A = \frac{(1-\tau^A)y_c^A}{\eta}$ and $c^B = \eta(1-\tau^B)y_c^B$. Since $\mathcal{NX}_c^A + \mathcal{NX}_c^B = 0$, this leads to:

$$(1-\tau^A)y_c^A = \eta(1-\tau^B)y_c^B \tag{95}$$

and therefore, we derive:

$$\eta = \frac{[(\rho+\delta)\theta-\delta\alpha_i^A]}{[\rho+\delta-\theta\delta\alpha_i^B]} \tag{96}$$

Consider now the consumption-leisure trade-off (91) with $T = (1-\tau^A)y_c^A + (1-\tau^B)y_c^B$. We express n^B as a function of the aggregate labor-supply such that :

$$n^B = \frac{\alpha_c^A[(1-\alpha_c^B)(\rho+\delta)+\theta\delta(\alpha_c^B-\alpha_i^B)]}{[\alpha_c^B[(\rho+\delta)\theta-\delta\alpha_i^A]+\alpha_c^A[(\rho+\delta)-\delta\theta\alpha_i^B]]l^{\chi}} \tag{97}$$

Hence, from the aggregate labor market $l = n^A + n^B$, we obtain:

$$l = \left[\frac{\alpha_c^A[(1-\alpha_c^B)(\rho+\delta)+\theta\delta(\alpha_c^B-\alpha_i^B)]+\alpha_c^B[(1-\alpha_c^A)\theta(\rho+\delta)+\delta(\alpha_c^A-\alpha_i^A)]}{\alpha_c^B[(\rho+\delta)\theta-\delta\alpha_i^A]+\alpha_c^A[(\rho+\delta)-\theta\delta\alpha_i^B]}\right]^{\frac{1}{1+\chi}} \equiv \mathcal{C}^{\frac{1}{1+\chi}}$$

We have now to ensure equalization of after-tax factor prices and equalization of the relative price through equations (82)-(84). Using the stationary euler equation (85), we obtain p^A and p^B given by:

$$\begin{aligned} p^A &= \xi_c^A \xi_i^{A\frac{-(1-\alpha_c^A)}{(1-\alpha_i^A)}} \left(\frac{\rho+\delta}{1-\tau^A}\right)^{\frac{-(\alpha_c^A-\alpha_i^A)}{(1-\alpha_i^A)}} \\ p^B &= \xi_c^B \xi_i^{B\frac{-(1-\alpha_c^B)}{(1-\alpha_i^B)}} \left(\frac{\rho+\delta}{1-\tau^B}\right)^{\frac{-(\alpha_c^B-\alpha_i^B)}{(1-\alpha_i^B)}} \end{aligned} \tag{98}$$

We normalize $\xi_c^B = \xi_i^B = 1$ and use the scaling constant $\xi_c^A = \xi_c^{A*}$ to ensure $p^A = p^B$ with:

$$\xi_c^{A*} = \xi_i^{A\frac{(1-\alpha_c^A)}{(1-\alpha_i^A)}} \left(\frac{\rho+\delta}{1-\tau^A}\right)^{\frac{(\alpha_c^A-\alpha_i^A)}{(1-\alpha_i^A)}} \left(\frac{\rho+\delta}{1-\tau^B}\right)^{\frac{-(\alpha_c^B-\alpha_i^B)}{(1-\alpha_i^B)}} \tag{99}$$

This ensures therefore that $(1-\tau^A)r^A = (1-\tau^B)r^B$. Similarly, we show that $(1-\tau^A)w^A = (1-\tau^B)w^B$ holds if and only if $\xi_i^A = \xi_i^{*A}$ with:

$$\xi_i^{A*} = (1-\tau^B)^{\frac{(1-\alpha_i^A)}{(1-\alpha_i^B)}}(1-\tau^A)^{-1}(\rho+\delta)^{\frac{\alpha_i^A-\alpha_i^B}{1-\alpha_i^B}} \tag{100}$$

Finally, we have to ensure that the budget is balanced in each country holds. We use the expression above to obtain $y_c^A + p^A y_i^A$ and $y_c^B + p^B y_i^B$ and derive:

$$\begin{aligned} \tau^A &= \frac{\eta^A}{Z^A}\left[\left(\frac{\rho+\delta)}{1-\tau^A}\right)^{\frac{1+\alpha_c{}^A-\alpha_i{}^A}{1-\alpha_i{}^A}} \left(\frac{\rho+\delta)}{1-\tau^B}\right)^{\frac{-(1+\alpha_c{}^B-\alpha_i{}^B)}{1-\alpha_i{}^B}} \frac{\alpha_c^B[(\rho+\delta)\theta+\delta(\alpha_c^A-\alpha_i^A)]\mathcal{C}^{\frac{-\chi}{1+\chi}}}{\alpha_c^B[(\rho+\delta)-\theta\delta\alpha_i^A]+\alpha_c^A[(\rho+\delta)-\delta\alpha_i^B}\right]^{\varepsilon_g^A-1} \\ \tau^B &= \frac{\eta^B}{Z^B}\left[\alpha_c^A\left(\frac{\rho+\delta)}{1-\tau^B}\right)^{\frac{-\alpha_c^B}{1-\alpha_i^B}} \frac{[(\rho+\delta)+\delta\theta(\alpha_c^B-\alpha_i^B)]\mathcal{C}^{\frac{-\chi}{1+\chi}}}{\alpha_c^B[(\rho+\delta)-\theta\delta\alpha_i^A]+\alpha_c^A[(\rho+\delta)-\delta\alpha_i^B}\right]^{\varepsilon_g^B-1} \end{aligned} \tag{101}$$

We use therefore the scaling constants Z^A and Z^B such that $(\tau^A, \tau^B) = (\eta^A, \eta^B)$ when $Z^A = Z^{A*}$ and $Z^B = Z^{B*}$ with:

$$Z^{A*} = \left[\left(\frac{\rho+\delta)}{1-\tau^A} \right)^{\frac{1+\alpha_c{}^A-\alpha_i{}^A}{1-\alpha_i{}^A}} \left(\frac{\rho+\delta)}{1-\tau^B} \right)^{\frac{-(1+\alpha_c{}^B-\alpha_i{}^B)}{1-\alpha_i{}^B}} \frac{\alpha_c^B[(\rho+\delta)\theta+\delta(\alpha_c^A-\alpha_i^A)]\mathcal{C}^{\frac{-\chi}{1+\chi}}}{\alpha_c^B[(\rho+\delta)-\theta\delta\alpha_i^A]+\alpha_c^A[(\rho+\delta)-\delta\alpha_i^B} \right]^{\varepsilon_g^A-1}$$

$$Z^{B*} = \left[\alpha_c^A (\frac{\rho+\delta)}{1-\tau^B})^{\frac{-\alpha_c^B}{1-\alpha_i^B}} \frac{[(\rho+\delta)+\delta\theta(\alpha_c^B-\alpha_i^B)]\mathcal{C}^{\frac{-\chi}{1+\chi}}}{\alpha_c^B[(\rho+\delta)-\theta\delta\alpha_i^A]+\alpha_c^A[(\rho+\delta)-\delta\alpha_i^B} \right]^{\varepsilon_g^B-1} \tag{102}$$

□

5.6 Proof of Proposition 6

We need first to derive the second-order derivatives T_{mn} with $m, n = 1, 2, 3$.

Lemma 1. *The aggregate social production function is characterized by:*

$$T_{11}=\frac{-T_1{}^4T_2{}^2T_3^2\tau^A(\varepsilon_g^A-1)\tau^B(\varepsilon_g^B-1)[(\alpha_c^B-\alpha_i^B)-(\alpha_c^A-\alpha_i^A)]^2}{(1-\tau^A)^A(1-\tau^B)^2\Phi} \quad T_{12}=\frac{-T_1^3T_2^3T_3^2\tau^A(\varepsilon_g^A-1)\tau^B(\varepsilon_g^B-1)[(\alpha_c^B-\alpha_i^B)-(\alpha_c^A-\alpha_i^A)]^2}{(1-\tau^A)^A(1-\tau^B)^2\Phi}$$
$$T_{21}=\frac{-T_1^3T_2^3T_3^2\tau^A(\varepsilon_g^A-1)\tau^B(\varepsilon_g^B-1)[(\alpha_c^B-\alpha_i^B)-(\alpha_c^A-\alpha_i^A)]^2}{(1-\tau^A)^A(1-\tau^B)^2\Phi} \quad T_{22}=\frac{-T_1^2T_2^4T_3^2\tau^A(\varepsilon_g^A-1)\tau^B(\varepsilon_g^B-1)[(\alpha_c^B-\alpha_i^B)-(\alpha_c^A-\alpha_i^A)]^2}{(1-\tau^A)^A(1-\tau^B)^2\Phi} \tag{103}$$

and $T_{13}=T_{23}=T_{31}=T_{32}=T_{33}=0$ *with:*

$$\Phi=|W|[T^A-T_3^Ay_i^A][(1-\alpha_c^A)(1-\alpha_i^A)T_1^Ak^A+\alpha_c^A\alpha_i^AT_2^An^A][T^B-T_3^By_i^B][(1-\alpha_c^B)(1-\alpha_i^B)T_1^Bk^B+\alpha_c^B\alpha_i^BT_2^Bn^B]$$

Let us characterize the second-order derivatives of the world social production function, namely, $T_{mn}(k, l, y_i)$ with $m, n = 1, 2, 3$. Totally differentiate the system (73)-(81). We get:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ dk \\ dl \\ dy_i \end{pmatrix} = W \begin{pmatrix} dk^A \\ dn^A \\ dy_i^A \\ dk^B \\ dn^B \\ dy_i^B \\ d\lambda_1 \\ d\lambda_2 \\ d\lambda_3 \end{pmatrix} \tag{104}$$

with W given by:

$$\left[\begin{array}{ccccccccc} & & & 0 & 0 & 0 & -1 & 0 & 0 \\ & W^A & & 0 & 0 & 0 & 0 & -1 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & & & & -1 & 0 & 0 \\ 0 & 0 & 0 & & W^B & & 0 & -1 & 0 \\ 0 & 0 & 0 & & & & 0 & 0 & 1 \\ & & & & & & 0 & 0 & 0 \\ & C^A & & & C^B & & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \end{array}\right] \tag{105}$$

with:

$$W^A = \begin{pmatrix} \lambda_1 \left[\frac{T_{11}^A}{T_1^A} - \pi_{k^A}^A\right] & \lambda_1 \left[\frac{T_{12}^A}{T_1^A} - \pi_{n^A}^A\right] & \lambda_1 \left[\frac{T_{13}^A}{T_1^A} - \pi_{y_i^A}^A\right] \\ \lambda_2 \left[\frac{T_{21}^A}{T_2^A} - \pi_{k^A}^A\right] & \lambda_2 \left[\frac{T_{22}^A}{T_2^A} - \pi_{n^A}^A\right] & \lambda_2 \left[\frac{T_{23}^A}{T_2^A} - \pi_{y_i^A}^A\right] \\ -\lambda_3 \frac{T_{31}^A}{T_3^A} & -\lambda_3 \frac{T_{32}^A}{T_3^A} & -\lambda_3 \frac{T_{33}^A}{T_3^A} \end{pmatrix} \quad W^B = \begin{pmatrix} \lambda_1 \left[\frac{T_{11}^B}{T_1^B} - \pi_{k^B}^B\right] & \lambda_1 \left[\frac{T_{12}^B}{T_1^B} - \pi_{n^B}^B\right] & \lambda_1 \left[\frac{T_{13}^B}{T_1^B} - \pi_{y_i^B}^B\right] \\ \lambda_2 \left[\frac{T_{21}^B}{T_2^B} - \pi_{k^B}^B\right] & \lambda_2 \left[\frac{T_{22}^B}{T_2^B} - \pi_{n^B}^B\right] & \lambda_2 \left[\frac{T_{23}^B}{T_2^B} - \pi_{y_i^B}^B\right] \\ -\lambda_3 \frac{T_{31}^B}{T_3^B} & -\lambda_3 \frac{T_{32}^B}{T_3^B} & -\lambda_3 \frac{T_{33}^B}{T_3^B} \end{pmatrix}$$

$$C^A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{-\tau^A}{dk^A} y_i^A & \frac{-d\tau^A}{dn^A} y_i^A & [(1-\tau^A) - \frac{-d\tau^A}{dy_i^A} y_i^A] \end{pmatrix} \quad C^B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{-\tau^B}{dk^B} y_i^B & \frac{d\tau^B}{dn^B} y_i^B & [(1-\tau^B) - \frac{d\tau^B}{dy_i^B} y_i^B] \end{pmatrix}$$

and where $\pi_{x^h}^h$ is given by $\frac{d\tau^h}{dx^h} \frac{1}{(1-\tau^h)}$ with $x^h = k^h, n^h, y_i^h$ and $h = A, B$ with:

$$\begin{aligned} \frac{d\tau^h}{dk^h} &= \frac{\tau^h(\varepsilon_g^h - 1)(T_1^h - T_{31}^h y_i^h)}{[T^h - T_3^h y_i^h]} \\ \frac{d\tau^h}{dn^h} &= \frac{\tau^h(\varepsilon_g^h - 1)(T_2^h - T_{32}^h y_i^h)}{[T^h - T_3^h y_i^h]} \\ \frac{d\tau^h}{dy_i^h} &= \frac{-\tau^h(\varepsilon_g^h - 1)T_{33}^h y_i^h}{[T^h - T_3^h y_i^h]} \end{aligned} \tag{106}$$

By definition, the following hold:

$$\begin{array}{lll} \frac{d\lambda_1}{dk} \equiv T_{11} = W_{7,7}^{-1} & \frac{d\lambda_1}{dl} \equiv T_{12} = W_{7,8}^{-1} & \frac{d\lambda_1}{dy_i} \equiv T_{13} = W_{7,9}^{-1} \\ \frac{d\lambda_2}{dk} \equiv T_{21} = W_{8,7}^{-1} & \frac{d\lambda_2}{d2} \equiv T_{22} = W_{8,8}^{-1} & \frac{d\lambda_2}{dy_i} \equiv T_{23} = W_{8,9}^{-1} \\ \frac{d\lambda_3}{dk} \equiv T_{31} = W_{9,7}^{-1} & \frac{d\lambda_3}{dl} \equiv T_{32} = W_{9,8}^{-1} & \frac{d\lambda_3}{dy_i} \equiv T_{33} = W_{9,9}^{-1} \end{array} \tag{107}$$

After cumbersome computations (available on demand), and since $\lambda_1 = T_1$, $\lambda_2 = T_2$ and $\lambda_3 = -T_3$, we derive the expressions above. Besides, note that $T_{11}T_{22} - T_{12}T_{21} = 0$.

$\square$

We compute now the elasticity ε_{ck}, ε_{cy_i}, ε_{pk} and ε_{py_i} using the market-clearing equation for the consumption good and the identity $p(k, l(k, y_i)y_i) = -T_3(k, l(k, y_i)y_i)$.

$$\begin{aligned} \varepsilon_{ck} &= \frac{T_1}{T} + \frac{T_2}{T}\frac{dl}{dk} \\ \varepsilon_{cy_i} &= \frac{T_3}{T} + \frac{T_2}{T}\frac{dl}{dy_i} \\ \varepsilon_{pk} &= \frac{T_{31}}{T_3} + \frac{T_{32}}{T_3}\frac{dl}{dk} \\ \varepsilon_{py_i} &= \frac{T_{33}}{T_3} + \frac{T_{32}}{T_3}\frac{dl}{dy_i} \end{aligned}$$

From the first order-conditions (39), we derive $\frac{dl}{dk}$ and $\frac{dl}{dy_i}$:

$$\frac{dl}{dk} = \frac{\frac{T_{21}}{T_2} - \frac{T_1}{T}}{\frac{\chi}{l} + \frac{T_2}{T} - \frac{T_{22}}{T_2}} \quad \frac{dl}{dy_i} = \frac{\frac{T_{23}}{T_2} - \frac{T_3}{T}}{\frac{\chi}{l} + \frac{T_2}{T} - \frac{T_{22}}{T_2}} \tag{108}$$

We are now ready to compute the trace and the determinant. Totally differentiating the dynamical system of the free-trade economy, we get:

$$\begin{pmatrix} d\dot{k} \\ d\dot{y}_i \end{pmatrix} = \begin{pmatrix} -\delta & 1 \\ \frac{(\rho+\delta)(\frac{T_{11}}{T_1} + \delta\frac{T_{12}}{T_1}\frac{dl}{dk}) - \delta(\frac{T_1}{T} + \frac{T_2}{T}\frac{dl}{dk})}{\frac{T_3}{T} + \frac{T_2}{T}\frac{dl}{dy_i}} & \frac{(\rho+\delta)\frac{T_{12}}{T_1}\frac{dl}{dy_i} + (\frac{T_1}{T} + \frac{T_2}{T}\frac{dl}{dk})}{\frac{T_3}{T} + \frac{T_2}{T}\frac{dl}{dy_i}} \end{pmatrix} \begin{pmatrix} dk \\ dyi \end{pmatrix}$$

Where we used the fact that $T_{13}(k, l, y_i) = T_{23}(k, l, y_i) = T_{31}(k, l, y_i) = T_{32}(k, l, y_i) = T_{33}(k, l, y_i) = 0$. After some simple computations, the trace and the determinant are then:

$$\mathcal{T} = \rho$$

$$\mathcal{D} = \frac{-(\rho+\delta)^3\left[\delta + \frac{\chi(\rho+\delta)}{\mathcal{C}}\right] T_3^4 \eta^A(\varepsilon_g^A - 1)\eta^B(\varepsilon_g^B - 1)[(\alpha_c^B - \alpha_i^B) - (\alpha_c^A - \alpha_i^A)]^2}{T_2\Phi\left[\frac{(\rho+\delta)^2 T_3^4 \eta^A(\varepsilon_g^A - 1)\eta^B(\varepsilon_g^B - 1)[(\alpha_c^B - \alpha_i^B) - (\alpha_c^A - \alpha_i^A)]^2}{T_2\Phi} + \frac{\chi}{\mathcal{C}^{\frac{1}{1+\chi}}}\right]} \tag{109}$$

with:

$$\Phi = |W| \frac{(\alpha_c^A\alpha_c^B)^4[(\rho+\delta)\theta + \delta(\alpha_c^A - \alpha_i^A)][(1-\alpha_c^A)(\rho+\delta)\theta + \delta(\alpha_c^A - \alpha_i^A)\alpha_i^A][\rho+\delta+\delta(\alpha_c^B - \alpha_i^B)\theta][(1-\alpha_c^B)(\rho+\delta) + \delta\theta(\alpha_c^B - \alpha_i^B)\alpha_i^B]}{\mathcal{C}^{\frac{\chi}{1+\chi}}[\alpha_c^B[(\rho+\delta)\theta - \delta\alpha_i^A] + \alpha_c^A[(\rho+\delta) - \delta\theta\alpha_i^B]}$$

$$\mathcal{C} = \left[\frac{\alpha_c^A[(1-\alpha_c^B)(\rho+\delta) + \theta\delta(\alpha_c^B - \alpha_i^B)] + \alpha_c^B[(1-\alpha_c^A)\theta(\rho+\delta) + \delta(\alpha_c^A - \alpha_i^A)]}{\alpha_c^B[(\rho+\delta)\theta - \delta\alpha_i^A] + \alpha_c^A[(\rho+\delta) - \theta\delta\alpha_i^B]}\right]$$

Since the $\mathcal{T}$ is positive, the NSS is always locally determinate. However, the sign of the determinant can not be directly determined. However, note that $\mathcal{D} = -\delta(\rho+\delta) < 0$ if $\chi = 0$ and the NSS is therefore locally saddle-path stable. In contrast, if labor is unelastic, we get:

$$\frac{(\rho+\delta)^4 T_3^4 \eta^A(\varepsilon_g^A - 1)\eta^B(\varepsilon_g^B - 1)[(\alpha_c^B - \alpha_i^B) - (\alpha_c^A - \alpha_i^A)]^2}{T_2 \Phi} \tag{110}$$

The sign of the determinant depends then of the cyclicality of the policy followed in country A and B and of $|W|$ the determinant of the matrix W. As a result, for a given sign of $|W|$, coordinating the ε_g^h may lead to saddle-path stability. For instance, if $|W| > 0$, $\varepsilon_g^A < 1 < \varepsilon_g^B$, or if $|W| < 0$ and $\varepsilon_g^A < 1$, $\varepsilon_g^B < 1$, the NSS is saddle-path stable .

It follows that there exists $\bar{\chi}^{FT}$ such that for any $\chi \in (0, \bar{\chi}^{FT})$, the NSS is locally a saddle-path stable and for any $\chi \in (\bar{\chi}^{FT}, +\infty)$, the NSS is either locally a source or locally saddle-path stable. □

5.7 Proof of Corollary 1

Let us first note that $\bar{\chi}^A$ and $\bar{\chi}^B$ can be made arbitrarily low when α_i^A tends to $\bar{\alpha}_i^A$ and α_i^B tends to $\bar{\alpha}_i^B$. Hence, there exists $\underline{\alpha}_i^A \in (0, \bar{\alpha}_i^A)$ and $\underline{\alpha}_i^B \in (0, \bar{\alpha}_i^B)$ such that the inequality $\bar{\chi}^A, \bar{\chi}^B < \bar{\chi}^{FT}$ is satisfied for any $\bar{\chi}^{FT} \in (0, +\infty)$.

Under Assumption 1 and $Z^h = Z^{h*}$, if $\eta_h \in (\underline{\tau}^h, \bar{\tau}^h)$, $\chi^h \in (0, \bar{\chi}^h)$ and $\alpha_i^h \in (\underline{\alpha}_i^h, \bar{\alpha}_i^h)$, country h satisfies the conditions given in Proposition 2 but also satisfies the condition to obtain saddle-path stability under free-trade integration.

□

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