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Energy Transition Under Irreversibility: A Two-Sector Approach

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Abstract This paper analyses the optimal energy transition of a two-sector economy (energy and final goods) under irreversible environmental catastrophe. First, it proposes a general appraisal of optimal switching problems related to energy transition showing: (1) the possibility of a catastrophe due to accumulation of pollution; and (2) technological regimes with the adoption of renewable energy. Second, it numerically shows that for given baseline parameter values, the most profitable energy transition path may correspond to the one in which the economy starts using both resources, crosses the pollution threshold by losing a part of its capital, and never adopts only clean energy. Third, it extends the model to allow for additional investment in energy saving technologies. We then find that this additional investment favours full transition to the sole use of renewable energy. It is then profitable to take advantage of these synergies by jointly promoting deployment of clean energy and providing incentives for investment in energy saving technologies.

Keywords Energy · Irreversibility · Pollution · Switch

JEL Classification Q30 · Q53 · C61

1 Introduction

In order to reduce global CO_2 emissions by 50 per cent from 2005 to 2050, some energy policies employ scenarios which focus on adoption of renewable energy (RE) sources and investment in energy saving technologies (EST). Despite growing investment in the produc-

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tion of RE [63–244 billion USD from 2006 to 2012 (GEA 2012)], fossil fuels, i.e. dirty energy sources are still mainly used (78.2 %) throughout the world. Therefore it is crucial not only to drastically change the way energy is produced, but also to find energy saving strategies. According to the Global Energy Assessment (GEA), about one-third of overall investment in the energy sector is efficiency-related, following the efficiency pathways (GEA 2012). This paper focuses on energy transition that involves decisions about both RE adoption and investment in EST. We analyse the optimal energy transition of a two-sector economy (energy and final goods) with exhaustible oil reserves, a renewable source of energy and a pollution threat.

Energy transition involves decisions about both RE adoption and investment in EST. The former concerns adopting clean energy sources as an alternative to polluting sources of energy, while the latter could help reduce overall energy consumption. In the early literature on natural resource economics, many authors adopted a different focus on the long run depletion of oil reserves and on the polluting feature of oil. Dasgupta and Heal (1974, 1979), Dasgupta and Stiglitz (1981) and Krautkraemer (1986) analyse the long run depletion of oil reserves, while Nordhaus (1994) and Tahvonen (1996, 1997) focus on the polluting aspects of oil. In this regard, one solution could be to adopt a backstop technology (a renewable resource for example) as a clean energy. More recently, several works (Acemoglu et al. 2014; Amigues et al. 2015; Tsur and Zemel 2003) focus on climate change as one of the important reasons for the transition to clean energy or to clean technologies. As the use of polluting energy resources generates pollution that accumulates over time, an ecological catastrophe may occur at some point in time. The catastrophic event will generate some irreversible damage[1] (Forster 1975; Tahvonen and Withagen 1996; Ulph and Ulph 1997; Pindyck 2002; Pommeret and Prieur 2009; Ayong Le Kama et al. 2014). Damage can also be partly reversible (Tsur and Zemel 1996; Nævdal 2006), or fully reversible (Kollenbach 2013).

There is no consensus in the literature about how to model environmental damage resulting from pollution. Some authors consider the damage as income loss (Karp and Tsur 2011; Tsur and Withagen 2012) or social welfare loss (Van der Ploeg and Withagen 2012; Prieur et al. 2013). Other authors focus on productive sectors: capital loss (Horii and Ikefuji 2012); or destruction capacity (Golosov et al. 2014). The present paper assumes that the economy experiences a catastrophic event when the level of pollution is above a certain critical threshold. Therefore, the economy loses part of its productive stock of capital. Moreover, to support the simultaneous use of both resources, many authors assume a convexity of the production cost of renewable energy (Chakravorty et al. 1997; Amigues et al. 2015) or an increasing extraction cost of fossil fuels (Tsur and Zemel 2005; Kollenbach 2013). For example, Amigues et al. (2015) studies energy transition in a deterministic framework and consider adjustment costs over production capacity of renewable energy. They identify three energy regimes in a partial equilibrium setting with an intermediate regime of simultaneous use of both resources. In addition, several studies assume imperfect or perfect substitution between inputs. Alternatively, we consider the case of an economy with rigidities such that oil and RE sources are complementary, as in Pelli (2012). Moreover, we also assume that capital use and energy are complementary, as in Pindyck and Rotemberg (1983), Boucekkine and Pommeret (2004) or Díaz and Puch (2013).

In a deterministic framework, Boucekkine et al. (2013) provide first order optimality conditions in an optimal regime switching problem with threshold effects. These optimality

[1] There are various types of irreversibility. It could be exhaustion of the natural regeneration capacity (Tsur and Withagen 2013), an irreversibility in the decision process (Pommeret and Prieur 2009; Ayong Le Kama et al. 2014) or a ceiling on the pollution stock (Lafforgue et al. 2008; Chakravorty et al. 2012).

conditions are the continuity of appropriate co-states and states variables, and that of the Hamiltonian. The present paper is mainly related to the application in that paper as it involves both the switching decision to cleaner energy sources and the pollution threshold effect as the main drivers of energy transition. However, the contribution of our paper is threefold. First, we use a two-sector approach in which the economy requires capital to produce energy that can be used as inputs to produce a final good. We do not allow a natural regeneration capacity, instead we consider the irreversibility of pollution for a loss of capital. In the same vein, we do not account for direct pollution damage, but only the loss of productive capital due to the occurrence of a catastrophe. In contrast to this paper, we allow a simultaneous use of both resources (dirty and clean energy sources). More precisely, we assume that there is a complementarity between both resource use and capital in the production of final goods.

Given the baseline parameter values, we numerically show that the most profitable energy transition path may correspond to the one in which the economy starts using both resources, crosses the pollution threshold by losing a part of its capital and never adopts clean energy only. This result is in line with arguments supporting the idea that a complete transition to a low carbon economy is likely to be very slow. Without innovations in the energy sector such as investment in energy efficiency, and because fossil fuels are needed to produce clean energy, it is more profitable to progressively reduce dependency on fossil fuels which is costless (except for the catastrophe, which occurs once), than to switch to sole use of a costly clean energy. Sensitivity analysis shows that high productivity of capital and energy services in the final goods sector and of capital in producing clean energy postpones the occurrence of an environmental catastrophe. Therefore, public policy should promote innovation that helps increase the productivity of capital and energy services in final goods and energy sectors.

Third, we extend our model to the adoption of energy saving technologies, which very few works deal with (Charlier et al. 2011; De Groot et al. 2001 and Acemoglu et al., 2012). In order to fill the gap in the literature about the importance of EST in energy transition, we extend our model to allow for investment decisions in EST. More precisely, the economy may decide to invest in energy saving appliances or in energy efficient systems to reduce overall energy consumption. This investment is additional to that made in clean energy to help reach energy transition targets. Numerical results mainly show that this additional investment favours full transition to the sole use of renewable energy in the sense that it postpones environmental catastrophe, is welfare-improving and allows a complete energy transition. It is then profitable to take advantage of these synergies by jointly promoting deployment of clean energy and providing incentives for investment in energy saving technologies. This is particularly important for developing countries which mostly rely on polluting energy resources and are the most vulnerable to climate change. The remainder of this paper is structured as follows: The model is presented in Sect. 2. We analyse the optimal energy transition path in Sect. 3. Section 4 extends the model to allow investment in EST. Conclusions are presented in Sect. 5.

2 Model

In this section, we present a model for transitory regimes (first and second regimes) in which both types of energy (dirty and clean) are used simultaneously, while production of dirty energy is cut out in the third and final regime (see Sect. 3.1 for the definition of regimes). We consider a closed economy that produces energy and final goods in a general equilibrium setting. The economy uses a dirty source (exhaustible oil reserves) and a clean source (for example solar panels) to produce energy. Part of the energy is used as energy services by a representative consumer through a separable utility function. The other part is used as input in a Leontief production function to produce final goods. The use of dirty energy by both final goods sector and households has a negative impact on the environment. Above a certain pollution threshold, the economy experiences a catastrophic event and loses a part of its stock of capital. In the following sections, we describe the energy sector, the final goods sector, household utility and pollution threats, respectively.

2.1 Energy Sector

Energy is an intermediate good that is produced using E_d, a non-renewable and dirty source, and E_c, a renewable and clean source. A representative consumer uses part E_C of the energy as energy services, while the other part E_Y is used as an input to produce final goods. Let us denote respectively E_{Cd}, E_{Cc}, E_{Yd} and E_{Yc} the parts of the dirty and the clean energy that households use and that the final goods sector uses. We assume that production of the dirty energy is costless. The stock S_t of the dirty energy source at each time t is generated by the following dynamics:

$$\dot{S}_t = -E_{dt} \tag{1}$$

where E_{dt} is the rate of extraction of the dirty energy source.

The production of clean energy requires the use of capital. For example, to produce solar (or wind) energy, one needs to install solar panels (or wind turbines) in order to transform sunlight (wind) into electricity. Hence we assume a "η-to-one" transformation of K_{Et}, a part ϕ of capital K_t as follows:

$$E_{ct} = \varphi K_{Et} = \eta\phi K_t \tag{2}$$

where η is the productivity of capital in the clean energy sector. Due to the high-efficiency energy conversion of clean technology such as solar panels, we assume that productivity is high and greater than one ($\eta > 1$).

In our model, pollution only comes from the use of dirty energy. The following energy market clearing conditions holds:

The dirty energy that the economy produces is fully consumed by households and as an input to produce final goods:

$$E_{dt} = E_{Ydt} + E_{Cdt}. \tag{3}$$

Total production of the clean energy is split between the final goods sector and household energy consumption:

$$E_{ct} = E_{Yct} + E_{Cct}. \tag{4}$$

Finally, the total energy used in the economy is from the dirty and clean energy sources:

$$E_{Yt} + E_{Ct} = E_{ct} + E_{dt}. \tag{5}$$

2.2 Pollution Threat

The use of dirty energy source either by households or as an input to produce final goods generates greenhouse gas emissions. Pollution accumulates in the environment (atmosphere) according to the following process:

$$\dot{Z}_t = E_{dt}. \tag{6}$$

We do not account for the natural regeneration capacity of the environment as in Van der Ploeg and Withagen (2012, 2014). Admittedly, our pollution dynamics are quite restrictive.

However, our assumption can be seen as the most pessimistic way to deal with the threat of pollution to justify the necessity of an energy transition. Moreover, though there are no formal statistics for natural assimilation, climate change experts usually report that half of the CO_2 emitted is currently removed from the atmosphere within a century (IPCC 2007). It may therefore have a very small effect on the path of fossil fuels. Relaxing this assumption would possibly delay a catastrophe, but will not affect an optimal transition that contains a regime after the catastrophe.

Ultimately the economy experiences a catastrophic event. When the level of pollution Z_t is above a certain critical threshold $\overline{Z}$, the economy loses once and for all a part θ of its capital stock when the catastrophe occurs. The stock of capital is then suddenly destroyed.

2.3 Final Goods Sector

In order to produce a final good Y_t, a part E_{Yt} of energy and a part $(1-\phi)$ of capital (K_{Yt}) serve as inputs in a Leontief production function. The interpretation runs as follows: There exist operating costs where the amount depends on the energy requirements of the capital, such that for any capital use there is a corresponding energy requirement. Such complementarity is assumed in order to be consistent with several studies arguing that capital and energy are complements (see for example Berndt and Wood 1975; Pindyck and Rotemberg 1983; and more recently Díaz and Puch 2013). The production function is defined as:

$$Y_t = min\{\alpha_2 K_{Yt}, \beta_2 E_{Yt}\}, \tag{7}$$

with

$$K_{Yt} = (1-\phi)K_t. \tag{8}$$

For analytical convenience we also assume that use of both the dirty and clean resources is complementary. The clean and the dirty sources may not be complementary, and in reality two types of explanations can be provided. First, using an econometric approach, Pelli (2012) proves that there exists some complementarity between dirty sources of energy (oil, coal, gases) and clean ones (hydroelectric, biomass-wood and waste, geothermal, solar/photovoltaic, wind and nuclear). The implication is that production of energy using a clean source, for example solar panels, requires oil to build the solar panels. Second, the presence of rigidities in a macroeconomic view may also explain the complementarity between dirty and clean sources: for example, it is not easy to substitute between oil and the electricity provided by solar panels. Several studies assume imperfect substitution (Michielsen 2014) or perfect substitution between energy sources (Van der Ploeg and Withagen 2012, 2014). While the latter assumption is unrealistic and is an extreme case, the complementarity assumption is also an extreme case of imperfect substitution. Therefore, reality lies between these two extreme cases (perfect substitution and complementarity). Moreover, this assumption allows us to highlight the implication of complementarity between the two types of energy sources in the energy transition. Relaxing this assumption would introduce energy transition paths where fossil fuels are solely used in the first regime (see for example Amigues et al. 2015).

We define E_{Yt} as:

$$E_{Yt} = min\left\{\frac{1}{\xi}E_{Ydt}, E_{Yct}\right\} \tag{9}$$

where ξ is the coefficient of the combination between the clean and the dirty sources of energy.

2.4 Households

We consider a representative household using energy services E_{Ct} and consuming a non-energy good C_t, excluding durable goods. We assume that utility can then be expressed over all goods as separable on energy services $u_t(E_{Ct})$ and on non-energy goods $u_t(C_t)$.[2,3] Therefore, the gross utility U_t represents consumer preferences that are expressed by the discounted sum of instantaneous separable utility flows:

$$U_t = \int_{T_0}^{\infty} \left[\frac{C_t^{1-\delta}}{1-\delta} + \frac{E_{Ct}^{1-\delta}}{1-\delta} \right] e^{-\rho t} dt, \tag{10}$$

where ρ is the discount rate, δ is a positive coefficient of utility that is different from 1 and T_0 is time 0 with $T_0 = 0$.

Both the clean and the dirty energy sources are complementary for the same reasons as in the final goods sector[4]

$$E_{Ct} = min \left\{ \frac{1}{\xi} E_{Cdt}, E_{Cct} \right\} \tag{11}$$

where ξ is the part of the dirty energy used in the energy mix as defined in Eq. (9).

Households own firms in both the energy and final goods sectors. They consume a part of the final goods production and invest the rest to produce clean energy and final goods:[5]

$$Y_t = C_t + \dot{K}_t, \tag{12}$$

with

$$K_t = K_{Et} + K_{Yt}. \tag{13}$$

In the following sections, we first analyse the optimal energy transition path. In Sect. 4, we provide the numerical results. Finally, we extend the model to the adoption of energy saving technologies in Sect. 5.

3 Optimal Energy Transition Path

In this section, we analyse energy transition paths that include a catastrophic event and/or sole use of clean energy. Three regimes can occur. In the first one, energy is produced by both dirty and clean resources that are complementary, and the level of pollution is below the threshold. In the second regime, the catastrophe has occurred and both energy sources

[2] A non-separable utility function (Cobb–Douglas or Constant Elasticity of Substitution) would capture crossing effects arising from a strong relationship between the use of dirty energy, clean energy and the level of consumption. To avoid such effects, we focus on non-durable goods.

[3] An alternative model in which households consume final goods combined with home-based services produced from capital and energy consumption would lead to similar results in the sense that our definition of energy services (mainly clean energy) implicitly incorporates capital. However, we only consider investments in productive sectors (i.e. the final goods sector and the energy sector) for simplicity. As stated in footnote 2, such an alternative model would capture crossing effects arising from strong Leontief relationships.

[4] This is a strong assumption but it is consistent with the complementarity assumption considered in the final goods sector (see Sect. 2.3). If there were an available technology in the final goods sector that allows for substitution between fossil fuel and renewable energy, this technology could be used by households as well.

[5] For simplicity and analytical tractability, we consider the particular case of no capital depreciation without loss of generality. The absence of capital depreciation, will simply induce a lower optimal level of investment.

are used again, but pollution is above the threshold. The third regime is characterized by the sole use of the clean energy. We assume that time starts at $T_0 = 0$, T_1 is the date at which the second regime starts, while the third regime starts at date T_2. T_1 and T_2 can take zero, strictly positive and infinite values. Crossing T_1 and T_2 defines nine energy transition paths. We first focus on the energy transition path that corresponds to strictly positive values for T_1 and T_2 ($T_1 > 0$, $T_2 > 0$), which we denote as the 'central' energy transition path because it is a succession of the three regimes. The second part of this section focuses on the remaining eight paths that we denote as 'corner' energy transition paths because there are specific cases. To solve for the corner energy transition paths, we simply need to set appropriate values (zero, infinite values for T_1 and T_2).

3.1 Central Energy Transition Path

In this section, we analyse the central energy transition path that is a succession of regimes for which the regime switch corresponds to a change of model as follows: The economy starts by using both sources of energy (dirty and clean) and therefore pollutes. The economy accumulates pollution up to the threshold $\overline{Z}$ (Sect. 3.1.3). Once pollution exceeds this critical level $\overline{Z}$, the economy experiences a catastrophic event. Then, a part θ of capital is then suddenly destroyed, (Sect. 3.1.2) but the economy still uses both sources of energy. Once the economy switches to sole use of the clean energy, the production of dirty energy is cut out (Sect. 3.1.1). We backward solve for the optimal general path starting from the third regime (sole use of clean energy) followed by the second regime and lastly by the first regime. We use the boundary conditions as in Boucekkine et al. (2013) to find the optimal time at which the economy crosses the critical pollution threshold and turns to the sole use of clean energy. As it is not possible to obtain an analytical solution, we solve it numerically.

3.1.1 Third Regime: Sole Use of Clean Energy

During the third regime, the economy solely uses clean energy. Therefore, constraints (1) and (6) both become irrelevant and Eqs. (9) and (11) drop and are replaced by $E_{Yt} = E_{Yct}$ and $E_{Ct} = E_{Cct}$, respectively. By combining Eq. (13) together with the Leontief conditions (LC, hereafter) applied to Eq. (7), Eq. (12) can be rewritten as (see the proof in Appendix A_1):

$$\dot{K}_t = \alpha_2 \left(K_t - \frac{1}{\eta} E_{Yt} - \frac{1}{\eta} E_{Ct} \right) - C_t. \tag{14}$$

The social planner solves the following programme:

$$V_3 = Max \int_{T_2}^{\infty} \left(\frac{C_t^{1-\delta}}{1-\delta} + \frac{E_{Ct}^{1-\delta}}{1-\delta} \right) e^{-\rho(t-T_2)} dt$$
$$\text{st Eq. (14)},$$

where T_2 is the switching time to the third regime.

The corresponding Hamiltonian is defined as:

$$H_3 = \frac{C_t^{1-\delta}}{1-\delta} + \frac{E_{Ct}^{1-\delta}}{1-\delta} + \lambda_t \left[\alpha_2 (K_t - \frac{1}{\eta} E_{Yt} - \frac{1}{\eta} E_{Ct}) - C_t \right],$$

with λ_t the co-state variable related to capital K.

The first order conditions (FOCs) with respect to C_t, E_{Ct} and K_t respectively give:

$$C_t^{-\delta} = \lambda_t \tag{15}$$

$$E_{Ct}^{-\delta} = \frac{\alpha_2}{\eta}\lambda_t \tag{16}$$

and

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - \alpha_2, \tag{17}$$

where λ_t is the co-state variable associated with capital.

One can easily identify the consumption versus savings arbitrage condition in Eqs. (15) and (16). It states that the marginal value of capital has to equal the marginal utility of consumption on the one hand, and the marginal utility of energy services on the other. Moreover, condition (17) implies a constant instantaneous return over capital.

Solving Eq. (14) using Eqs. (15)–(17), LC applied to Eq. (7) and the transversality condition (see the proof in Appendix A_2), we obtain:

$$K_t = -\frac{\Theta\delta}{\alpha_2 - \rho - \delta\Lambda}\lambda_{T_2}^{-\frac{1}{\delta}} e^{(\frac{\alpha_2-\rho}{\delta})(t-T_2)},$$

where $\Lambda = \frac{\alpha_2\beta_2\eta}{\alpha_2+\beta_2\eta}$, $\Theta = \frac{\alpha_2\beta_2}{\alpha_2+\beta_2\eta}(\frac{\alpha_2}{\eta})^{-\frac{1}{\delta}} + 1$ and λ_{T_2}, the marginal value of the capital at the switching time T_2 will be determined in Sect. 3.1.4 using boundary conditions.

We can easily deduce the value function V_3 during the third regime:

$$V_3 = -\frac{\delta[1 + (\frac{\alpha_2}{\eta})^{-\frac{1-\delta}{\delta}}]\lambda_{T_2}^{-\frac{1-\delta}{\delta}}}{(1-\delta)[\alpha_2(1-\delta)-\rho]}.$$

3.1.2 Second Regime: Simultaneous Use of Dirty and Clean Energy, Exhaustibility of the Dirty Source of Energy

In the second regime, both the clean and dirty energy sources are still used after the catastrophe. Therefore, the economy faces an exhaustibility problem Eq. (1) while Eq. (6) is irrelevant. Applying LC to Eq. (7) and used together with Eq. (8), Eq. (12) can be rewritten as:

$$\dot{K}_t = \alpha_2(1-\phi)K_t - C_t. \tag{18}$$

Using the LC from Eqs. (9) and (11), and summing up the two, Eq. (1) becomes:

$$\dot{S}_t = -E_{dt} = -\xi(E_{Yt} + E_{Ct}). \tag{19}$$

The social planner solves the following programme:

$$V_2 = Max \int_{T_1}^{T_2} \left(\frac{C_t^{1-\delta}}{1-\delta} + \frac{E_{Ct}^{1-\delta}}{1-\delta}\right) e^{-\rho(t-T_1)} + V_3 * e^{-\rho T_2}$$
$$\text{st Eqs. (18) and (19)},$$

where T_1 is the switching time to the second regime.

The corresponding Hamiltonian can be written as:

$$H_2 = \frac{C_t^{1-\delta}}{1-\delta} + \frac{E_{Ct}^{1-\delta}}{1-\delta} + \mu_t[\alpha_2(1-\phi)K_t - C_t] - \nu_t\xi(E_{Yt} + E_{Ct}),$$

with μ and ν the co-state variables associated with the capital K and the stock of the dirty source of energy S_t, respectively.

Resolution of the capital accumulation equation Eq. (18) using the FOCs of the above programme gives (see proof in Appendix A_3):

$$K_t = -(\overline{K_2} - K_{T_1}) * e^{(\frac{\alpha_2(1-\phi)-\rho}{\delta})(t-T_1)} + \overline{K_2},$$

where $\overline{K_2}$ is a constant and will be determined using boundary conditions in Sect. 3.1.4. Finally, using the fact that the dirty energy source is exhaustible and the fact that we have crossed the pollution threshold after a period of time T_1, we get (see proof in Appendix A_4):

$$\nu_{T_1} = f(\nabla, \overline{K_1}, \overline{K_2}, T_1, T_2),$$

where ∇ is the set of parameters, $\overline{K_1}$ a constant and ν_{T_1} and K_{T_1}, the marginal value of the stock of the dirty source of energy and the level of capital at the switching time T_1 respectively, that will be determined by the boundary conditions in Sect. 3.1.4.

3.1.3 First Regime: Simultaneous Use of Dirty and Clean Energy, Pollution Problem

At the beginning of the programme, the economy starts using both energy sources and faces a pollution problem Eq. (6). A catastrophic event may occur once the level of pollution reaches the critical threshold that results in loss of capital. We assume that dirty energy is abundant ($S_0 > \overline{Z}$) so that Eq. (1) is irrelevant. Therefore, the economy crosses the pollution threshold before complete depletion of the dirty energy source. From Eq. (19), Eq. (6) becomes:

$$\dot{Z}_t = -\dot{S}_t = \xi(E_{Yt} + E_{Ct}). \tag{20}$$

The social planner then solves:

$$V_1 = Max \int_0^{T_1} \left[\left(\frac{C_t^{1-\delta}}{1-\delta} + \frac{E_{Ct}^{1-\delta}}{1-\delta} \right) e^{-\rho t} \right] dt + V_2^* e^{-\rho T_1}$$
$$\text{st Eqs. (18) and (20),}$$

which looks like the programme solved in the second regime (Sect. 3.1.2) except for the sign of Eq. (20). We present the results of the first regime in Appendix A_5.

3.1.4 Boundary Conditions

We use three types of boundary conditions: (1) continuity of the co-state variable related to the capital μ_t ; (2) continuity of K_t; and (3) the equality of the Hamiltonian at the switching time. The co-state variable ν_t associated with the pollution stock Z is not continuous at the switching time T_1 because Z_t is fixed to $\overline{Z}$. At the switching time T_2, Z_t can be freely chosen and becomes continuous but it no longer exists during the third regime because clean energy is not polluting. The continuity of μ_t together with that of K_t helps to determine $\overline{K}_1$, $\overline{K}_2$, K_{T_1}, K_{T_2}, λ_{T_2}, ν_{T_0} and ν_{T_1}, respectively (The expressions and proof are available in the online appendix 9.2). We then simultaneously and numerically solve the equality of Hamiltonians at the switching time T_1 and T_2 to get T_1 and T_2. Now, let us consider the corner energy transition paths before providing the numerical value function.

3.2 Corner Energy Transition Paths

We exclude four corner energy transition paths among a total of eight because they are infeasible. The corner energy transition path of $T_1 = 0$ combined with $T_2 > 0$, $T_2 = 0$ or $T_2 = \infty$ cannot occur because the economy cannot start above the pollution threshold without consuming the polluting energy. If the economy starts with the clean energy source, it will never cross the pollution threshold as it is not polluting. Thus, the corner energy transition path that corresponds to the case $T_2 = 0$ and $T_1 > 0$ is not possible. Finally, we consider the following corner energy transition paths: (1) one switch to the sole use of clean energy (Sect. 3.2.1), (2) one switch above the pollution threshold (Sect. 3.2.2), (3) no switch (Sect. 3.2.3) and (4) starting with the clean energy (Sect. 3.2.4). In this section we present only the four relevant energy transition paths.[6]

3.2.1 One Switch to the Sole Use of Clean Energy ($T_1 = \infty$)

This case is a corner energy transition path in which the economy never exceeds the critical pollution threshold and therefore only switches to sole adoption of the clean energy. The economy starts using both the dirty and clean resources that are complementary, and pollution is below the critical level. After some time T, it switches to sole use of the clean source of energy before the level of pollution crosses the pollution threshold. Therefore, the economy escapes the catastrophe forever. To obtain the switching time T, it is sufficient to set $T_1 = \infty$ and $T_2 = T$.

3.2.2 One Switch Above the Pollution Threshold ($T_2 = \infty$)

This case corresponds to the transition from the first regime to the second regime without the switch to the third regime. Again, the economy starts using both the dirty and clean sources of energy with a level of pollution that is below the threshold level. Then, the economy switches to the regime in which both energy sources are still used but, the level of pollution is now above its critical threshold and the economy never makes a full transition to the sole use of clean energy. To obtain the switching time T and the dynamics of variables, one needs to set $T_2 = \infty$ and $T_1 = T$.

3.2.3 No Switch ($T_1 = \infty$ *and* $T_2 = \infty$)

On the no-switch energy transition path, the economy always uses both the dirty and clean sources of energy. Moreover, it does not solely use clean energy and the level of pollution remains below its critical threshold forever. This energy transition path corresponds to the first regime and one does not need to use boundary conditions to obtain the switching time. It is sufficient to set $T_1 = \infty$ and use the transversality conditions that give $\overline{K}_1 = 0$.

[6] Note that by assumption, the initial stock of the dirty source of energy S_0 is used in the first regime ($\overline{Z}$) and the remaining is used in the second regime ($S_0 - \overline{Z}$). Thus, any energy transition path that includes only the first regime or its combination with the third regime is characterized by S_{T_2} that goes to $S_0 - \overline{Z}$, while it goes to 0 for any energy transition path that includes both first and second regimes. When the energy transition path does not include either of the first two regimes, $S_{T_2} = S_0$.

3.2.4 Starting with Clean Energy ($T_1 = \infty$ *and* $T_2 = 0$)

On this energy transition path, the economy never uses the dirty source of energy and therefore does not pollute. The pollution threshold then becomes irrelevant. It corresponds to the third regime without any pollution threat. In this case, we need to set $T_1 = \infty$ and $T_2 = 0$.

4 Numerical Results and Sensitivity Analysis

In this section, we numerically solve for the switching times T_1 and T_2, and calculate the value functions of the central energy transition path and that of each of the corner energy transition paths. We present the parameter values that are used to obtain the numerical results. We also provide the numerical value functions and the sensitivity analysis.

4.1 Numerical Results

Due to lack of information about some parameters in our model, we can only provide numerical illustrations of our results. Therefore, we do not attempt to fully calibrate the model. As a consequence, the results hold only for the baseline values of the parameters. We also perform sensitivity analysis on parameters that are relevant to policy to assess the validity of the results. Parameter values have been chosen as follows: As in Van der Ploeg and Withagen (2014) we set the discount rate ρ at 0.014 and the inverse of the elasticity of intertemporal substitution δ at 2. Moreover, we consider the pollution threshold $\overline{Z} = 1200$ gigatonne of carbon (GtC) as in Prieur et al. (2013), which relies on a calibration exercise developed by Karp and Zhang (2012). Other parameters are arbitrarily chosen in order to provide a numerical illustration (see online appendix 9.1 for more details).

With the baseline parameters, we first numerically solve for the optimal levels of T_1 and T_2. Then, we derive the value functions of the central energy transition path and that of each of the corner energy transition paths, which we compare and select the one that gives the highest value function. The numerical results are summarised in Table 1. We also provide graphical comparisons of the value functions of energy transition paths in Fig. 1.

The most profitable energy transition path is the one that gives the highest value function to the social planner. Given our baseline parameters, it corresponds to the corner energy transition path in which $T_2 = \infty$. In such a case, the most profitable energy transition path can be described as follows: The economy starts using both sources of energy. Then, it crosses the pollution threshold and loses a part of its capital. Finally, the economy keeps using the dirty and clean energy and never switches to the sole adoption of clean energy in the long

Table 1 The value functions of the energy transition paths

Energy transition path	Value function
Central case ($T_1 = 28$; $T_2 = 100$)	−89.93
$T_1 = \infty$	−143.58
$T_2 = \infty$	**−43.6568**
$T_1 = \infty$; $T_2 = \infty$	*−86.59*
$T_1 = \infty$; $T_2 = 0$	−122.23

Bold indicates the most profitable energy transition path
Italic indicates the energy transition path that is close to the most profitable energy transition path

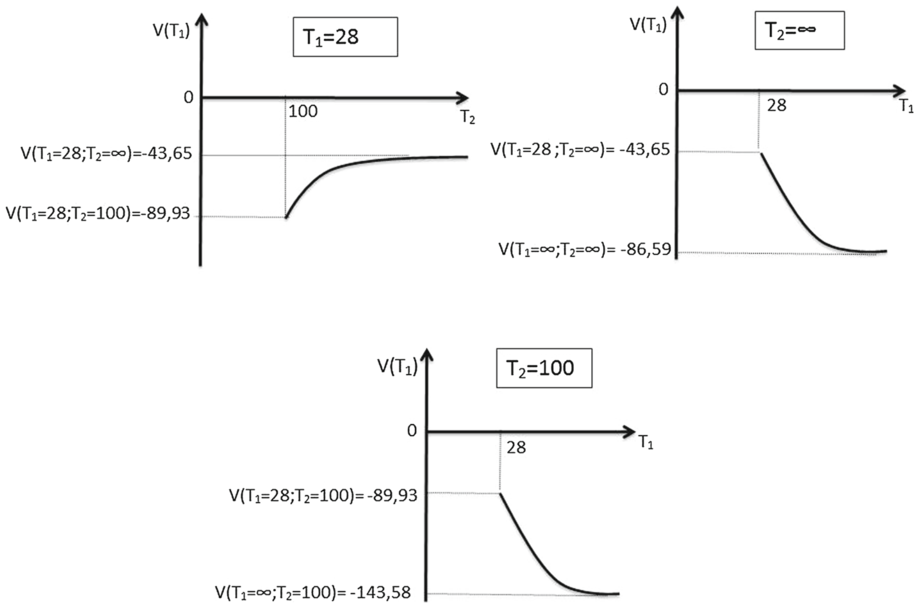

Fig. 1 Comparison between value functions of the energy transition paths

term. One could observe that the corner energy transition path that corresponds to $T_1 = \infty$ and $T_2 = \infty$ gives a value function that is higher than the one with cases $T_1 = \infty$; $T_1 = \infty$ and $T_2 = 0$ and the central case. This may be justified by the fact that the economy does not lose or gain enough by refraining from polluting more in order to never cross the pollution threshold. One can also observe that the central energy transition path is far from being the most profitable one. The numerical results with respect to the baseline parameters then show that there exist parameter values for which it is less profitable for the economy to switch to the sole use of the clean source of energy.

4.2 Sensitivity Analysis

As the baseline parameters rely mostly on values that are arbitrarily chosen, we devote this section to sensitivity analysis of parameters relevant to policy to assess the validity of the results. For each parameter, we chose two new values (one lower and one higher than the baseline value) which we compared with the result of the baseline value in order to isolate sensitivity to the value of the parameter. Specifically, for each new parameter value we calculated the switching times T_1 and T_2 and the value functions of the central and corner energy transition paths. Given these new values, we found that the corner energy transition path in which $T_1 > 0$ and $T_2 = \infty$ still has the highest value function and so is the most profitable one (see Table 5 in the online appendix 9.3). We then decided to focus only on the most profitable energy transition path and we present the sensitivity analysis with respect to the time T_1 at which the catastrophe occurs. The result of the sensitivity analysis is summarised in Table 2.

The discount rate ρ negatively affects the time at which the catastrophe occurs. The implication of this result is that more impatient people (i.e. with a higher discount rate) extract more fossil fuel and will then cross the critical pollution threshold more quickly. This result is in line with the intergenerational equity issue that refers to fair intertemporal

Table 2 The sensitivity analysis on the occurrence of the catastrophe

ρ	1.3 %	1.4 %	7 %
T_1	33	28	5
θ	0.01	0.05	0.1
T_1	25	28	35
$\overline{Z}$	600	1200	2000
T_1	10	28	68
η	1.05	1.5	2
T_1	26	28	34
α_2	0.00005	0.0001	0.0005
T_1	10	28	42
β_2	0.01	0.02	0.1
T_1	22	28	45
S_0	27000	28000	2800000
T_1	27.99	28	28.08

distribution of the endowment with natural assets such as fossil fuels. More impatient people do not care much about the future, over-exploit the dirty sources of energy today and then leave damage for future generations.

The occurrence of the catastrophe is positively affected by the size of the catastrophe (θ). In fact, if people know that the catastrophe will destroy a huge part of their stock of capital, they will fear the negative consequences of their dirty energy use more and will then reduce it. As a consequence, they could remain longer in the first regime before crossing the pollution threshold. This suggests that the economy that fears the negative consequences of climate change and the risk of ecological catastrophe is more favourable to the energy transition. In terms of policy implications, particular attention should be paid to innovations that help to reduce the use of energy, such as energy efficiency investment at the household level. Also, there is a need to promote curtailment actions such as through behaviour change. This can be done through raising public awareness of the potential consequences of the use of fossil fuels such as climate change. As expected, a high pollution threshold ($\overline{Z}$) increases the time of the occurrence of the catastrophe. If the critical pollution threshold that could provoke the catastrophic event is high, the pollution problem will become less rigid. The economy will have more freedom to use the dirty source of energy and will therefore stay longer in the pre-event regime.

The productivity of capital (α_2) and energy services (β_2) in the final goods sector and that of capital in producing renewable energy (η) positively affect the occurrence of the catastrophe. High productivity in the final goods sector would require a low quantity of fossil fuels to produce the final goods that will be used for more investment and more consumption. Likewise, high productivity in the energy sector requires less capital to produce more renewable energy. As a consequence, the economy puts less pressure on the dirty source of energy, pollutes less and postpones the occurrence of the environmental catastrophe. Therefore, public policy should promote innovation that helps to increase the productivity of capital and energy services in productive sectors.

Finally, the initial stock of the dirty source of energy has only a slight positive effect on the occurrence of the catastrophe. In fact, the environmental catastrophe that occurs at the end of the first regime is a consequence of pollution that accumulates over time in the environment. The stock of the dirty source of energy does not matter much in the first regime as the economy will reach the pollution threshold level before the dirty source of energy is completely exhausted. Therefore, the pollution problem is dominant in the first regime while the exhaustibility problem of the dirty source of energy arrives later on during the second regime after the catastrophe has occurred.

5 Introducing Investment in Energy Saving Technologies (EST)

Let us recall that E_{Yt} and E_{Ct} are energy services in the final goods sector and for households respectively. The final goods sector uses E_{Ydt} of the dirty source of energy and E_{Yct} of the clean source of energy, while households use E_{Cdt} of the dirty source of energy and E_{Cct} of the clean source of energy. At each period of time, in addition to consumption and investments in energy sector and final goods sector, the economy now invests a part of the final goods production q_t in energy saving technologies. We assume that q_t in energy saving technologies does not accumulate so that Eq. (12) becomes:

$$Y_t = C_t + \dot{K}_t + q_t$$

We assume that the investment q_t serves to reduce by $\varepsilon(q_t)$ units the resources that the economy needs in order to get the same energy services. Implicitly, it means that we do not account for a scale effect.[7] The idea behind the no scale assumption is as follows: Suppose that $\varepsilon(q_t)$ is the maximum amount of energy that can be saved due to investment q_t in EST. Given this maximum level, investment in EST will be optimally undertaken with respect to energy use in order to avoid any waste. Let us assume that $\varepsilon(q_t)$ is an increasing function ($\varepsilon^{'}(q_t) > 0$) and exhibits decreasing marginal returns ($\varepsilon^{''}(q_t) < 0$) in the abatement investment. $\varepsilon(q_t)$ is increasing in the sense that the more the economy invests in EST, the more it reduces use of the energy resource to get a given energy service. Moreover, as $\varepsilon(q_t)$ is increasing, we assume that $\varepsilon(q_t)$ is concave in order to have a maximum for q_t. Also, we avoid a complete elimination of the use of energy resources so that it will require an infinite amount of investment to do so.

5.1 Main Analytical Results

Due to the investment q_t in energy saving technologies, the dynamics of capital, the dirty source of energy and that of pollution are modified, while the household utility remains the same (see the proof in appendix B_1). Note that those dynamics do not change in terms of the extraction of energy resources, but only in terms of energy services. The same amount of energy resource provides more energy services when energy saving technologies are used. In comparison with the previous model, the social planner has to consider one additional control variable (investment q_t) to solve for the optimal energy transition.

The main change in the results is the fact that the level of capital at each period of time during the three regimes has an additional negative component. We therefore have chosen to present only the third regime (see the other regimes in Appendix B_2). For the third regime, all the previous FOCs remain the same. The main change comes from the FOC with respect to investment q_t:

[7] One should also consider that the investment q_t induces a scale effect. The scale effect is characterized by an energy saving which is proportional to the amount of energy use. This would make the present model very complex and unsolvable because of the interaction that may appear between q_t and all the preceding control variables such as energy services.

$$\varepsilon_Y^{'}(q_t) + \varepsilon_C^{'}(q_t) = \frac{\eta}{\alpha_2}. \tag{21}$$

Equation (21) highlights the arbitrage condition between the reduction of resources as a gain from the energy saving technologies and the constant marginal cost of investment. The solution of Eq. (21) gives the optimal investment in energy saving technologies. Now, let us specify the energy saving $\varepsilon_i(q_t)$ as a class of power function $cq_t^{\sigma_i}$ where $i \in \{Y, C\}$ and c, σ_i are the parameters. Moreover, we set $c = 1$ and $\sigma_i \in [0, 1]$ in order to meet the required properties defined before.

Without loss of generality, let us assume that investment in EST yields the same productivity either at the household level or at the industry level such that $\sigma_1 = \sigma_2 = \sigma$. Thus, we get:

$$q^* = \left[\frac{\eta}{2\sigma\alpha_2}\right]^{\frac{1}{\sigma-1}}. \tag{22}$$

By replacing the optimal value of investment in EST Eq. (22) into the equation of capital accumulation, we can solve the model as before to get the following expression of capital:

$$K_t = -\frac{\Theta\delta}{\alpha_2 - \rho - \delta\Lambda}\lambda_{T_2}^{-\frac{1}{\delta}} e^{(\frac{\alpha_2-\rho}{\delta})(t-T_2)} - \frac{\varpi}{\Lambda},$$

where $\varpi = \frac{2\Lambda}{\eta}(\frac{\eta}{2\sigma\alpha_2})^{\frac{\sigma}{\sigma-1}} - (\frac{\eta}{2\sigma\alpha_2})^{\frac{1}{\sigma-1}}$ and the remaining parameters are the same as the ones defined above.

In this new expression of capital, we have an additional component $-\frac{\varpi}{\Lambda}$ due to the investment in EST. This additional component is negative in the sense that it negatively affects the level of capital. In fact, the economy additionally uses a part of its income to invest in EST. This part could have been invested in a productive sector (final goods and energy) or consumed by households. Hence the share of income that goes to investment is reduced.

5.2 Empirical Results and Policy Implications

As before, here we discuss corner energy transition paths which we compare to the central energy transition path to isolate the most profitable one. In order to make our numerical results comparable, we used the same set of baseline parameter values as before. Additionally, we set the productivity of investment in EST both at the household level and at industry level σ to 0.5. We numerically solved for the switching times T_1 and T_2 and calculated the value functions of the central energy transition path and that of each of the corner energy transition paths. We compared the value functions among them and identified the most profitable one which gives the highest value function. The numerical results are presented in the Table 3.

The numerical results are threefold. First, investments in energy saving technologies increase the time at which the economy may experience the catastrophe and that of the sole adoption of renewable energy. In fact, investments in energy saving technologies help to reduce the consumption of energy for the same quality of energy services and therefore

Table 3 The values functions with EST

Central case ($T_1 = 80$; $T_2 = 150$)	$T_1 = \infty$	$T_2 = \infty$	$T_1 = \infty; T_2 = \infty$	$T_1 = \infty; T_2 = 0$
−23.3194	−30.4469	−30.4528	−30.5553	−30.4329

help to reduce pressure on the stock of fossil fuels. As a result, pollution is reduced and the economy can remain longer in the first regime before the level of pollution crosses its critical threshold level. Second, investment in EST increases the welfare of the society. Although investment in EST reduces the share of the income that goes to investment in both the final goods sector and the clean energy production sector, it increases the welfare of the society. The gain from investment in EST overcomes its forgone utility.

Last but not least, investments in EST change the most profitable energy transition path which becomes the central energy transition path where there is a full transition to the sole use of clean energy. In that sense, it favours full energy transition. The implication is that saving energy reduces energy expenses and decreases the use of fossil fuel for energy services. There is then less of a need for fossil fuels during the first two regimes and the economy can remain in the regime longer. The economy becomes less energy intensive and the gain from energy expenses can be reallocated to increase investment in renewable energy and investment in EST. Then, switching to the sole use of clean sources of energy becomes more attractive. After the complete exhaustion of fossil fuels, the economy is energy efficient and can fully rely on renewable energy.

Note that the four corner energy transition paths give very similar welfare. This can be explained by the potential synergies that may exist between energy saving technologies and clean energy. As corner energy transition paths do no include the three regimes, they may not benefit much from these synergies. Additionally, we perform a sensitivity analysis using the same boundaries for the parameters as in Sect. 4.2 and find that this result is robust. This result is in line with the results of the scenario 'Combined high renewables and efficiency' from a 2006 European Commission Energy and Transport report (Directorate General for Energy and Transport).[8] The combination of renewable energy and energy efficiency policies results in lower energy requirements, allows for more growth of the renewable energy share for primary energy needs and also leads to a strong decline in CO_2 emissions. Regarding policy implications, introducing EST policies will help to reduce pressure on fossil fuels and therefore postpone environmental catastrophe. This can be done through economic incentives for home renovation systems or energy efficient appliances for example. Interestingly, EST policy will also contribute to boost the full transition to clean energy. As investment in EST is welfare improving, it is then profitable for the economy to combine both adoption of clean energy and investments in energy saving technologies. Then, the economy will take advantage of the synergies that may arise from jointly promoting deployment of clean energy and providing incentives for investment in energy saving technologies.

6 Conclusion

This paper makes two main contributions. First, it analyses the optimal energy transition as optimal switching problems correspondent to the occurrence of environmental catastrophe and to the adoption of clean source of energy. We characterize two types of energy transition paths: (1) central energy transition path and (2) corner energy transition paths. The boundary conditions serve to isolate the optimal energy transition path. We find that for given baseline parameter values and in the absence of any possibility to invest in energy saving technologies, the most profitable energy transition path may correspond to the one in which the economy starts using both resources, crosses the pollution threshold by losing a part of its capital, and never adopts only clean energy.

8 For more details see https://ec.europa.eu/energy/sites/ener/files/documents/ee_and_res_scenarios.

This result is in line with some arguments supporting the idea that a complete transition to a low carbon economy is likely to be very slow (Fouquet 2010; Solomon and Krishna 2011). Three explanations can be provided. First, electric power from other sources of energy is still used in all of the manufacturing processes for producing renewable energy. For example, producing solar panels has some indirect downstream energy requirements (Ayres 2007). As the economy still needs fossil fuels to produce clean energy, it is more profitable to progressively reduce this costless dependence on fossil fuels (except for the catastrophe that occurs once) than to switch to the sole use of a costly clean energy. Second, without innovations in the energy sector such as energy efficiency investment, the global demand of energy is expected to increase and the economy will become more energy intensive. In this sense, it may be less profitable to fully rely on a costly renewable energy. Third, some recent studies show that the potential of global wind power (De Castro et al. 2011) and that of global solar electric (De Castro et al. 2013) might be even lower than the current final consumption of energy by means of fossil fuels. Therefore, an immediate and complete transition to an economy that only relies on renewable sources of energy may not be profitable.

The second contribution of this paper is the extension of this model to the adoption of energy saving technologies. We mainly find that investment in energy saving technologies favours full energy transition. In this sense, it postpones environmental catastrophe, it is welfare improving and it allows a complete transition to sole use of clean energy. In terms of policy implications, we can say that without additional investment in energy saving technologies and due to the need for fossil fuels in the production of clean energy, it is more profitable to progressively reduce dependence on fossil fuels which are costless, than to switch to the sole use of a costly clean energy. Public policy should also promote innovation that helps increase the productivity of capital and energy services in productive sectors and saves both money and energy. As investment in energy saving technologies can encourage the energy transition, it is therefore profitable to take advantage of the synergies that may arise from jointly promoting clean energy and providing incentives for investment in energy saving technologies.

In this paper, we can give a general view of energy transition by considering optimal switching problems, but this has required other stringent assumptions such as the complementarity assumed between dirty and clean sources of energy in both intermediate and final consumption. This assumption does not allow us to focus on energy transition as a process of gradually substituting clean to dirty energy. An alternative would consist of incorporating intermediary phases of a gradual substitution between energy sources after the phase of complementarity between clean and dirty energy. This may change the optimal energy transition path and therefore deserves further research. This paper can be extended to investigate factors that jointly favour the adoption of renewable energy and investments in energy saving technologies.

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Appendix

Appendix A_0

See Table 4.

Table 4 Variables and parameters

E_{ij}, with $i = Y, C$ and $j = d, c$	The quantity of the energy type "j" that is used by i
Subscript Y refers to	Final goods sector
Subscript C refers to	Household
Subscript d refers to	Dirty energy
Subscript c refers to	Clean energy
Subscript t refers to	Time
S	Stock of the dirty source of energy
K_E	Capital that is used to produce the clean energy
K_Y	Capital that is used to produce final goods
K	Total level of capital
q	Investment in energy saving technologies
Z	Stock of pollution
C	Level of consumption of non-energy goods by households
$T_0 = 0$	Beginning time of the first regime
T_1	Switching time to the second regime
T_2	Switching time to the third regime
Subscript T_0 refers to	Evaluation of the variable at $t = T_0$
Subscript T_1 refers to	Evaluation of the variable at $t = T_1$
Subscript T_2 refers to	Evaluation of the variable at $t = T_2$
η	Productivity of capital in the clean energy sector
ϕ	Part of capital that is used to produce clean energy
α_2	Productivity of capital in the final goods sector
β_2	Productivity of energy in the final goods sector
ξ	Part of dirty energy that is used in the energy mix
ρ	Discount rate
δ	Positive coefficient of the utility function
$\overline{Z}$	Pollution threshold above which the catastrophe occurs
θ	Part of capital that is loss due to the catastrophe
σ	Productivity of investment in energy saving technologies
S_0	The initial stock of the dirty source of energy
K_0	The initial stock of capital

Appendix A_1

Let us recall that the equation of capital accumulation is:

$$\dot{K}_t = Y_t - C_t. \tag{23}$$

We also know that: $Y_t = min\{\alpha_2 K_{Yt}, \beta_2 E_{Yt}\}$, where, $K_t = K_{Et} + K_{Yt}$ and $E_{ct} = \eta K_{Et}$. $E_{ct} = \eta K_{Et}$ implies that $K_{Et} = \frac{E_{ct}}{\eta}$. Then,

$$K_{Yt} = K_t - K_{Et} = K_t - \frac{E_{ct}}{\eta}. \tag{24}$$

From Leontief conditions in the final goods sector, we have:

$$Y_t = \alpha_2 K_{Yt} = \beta_2 E_{Yt}. \tag{25}$$

During the third regime, only the clean source of energy is used so that we have the following equalities: $E_{Yct} = E_{Yt}$, and $E_{Cct} = E_{Ct}$. By summing up the above two expressions and plugging this into successive Eqs. (24), (25) and into Eq. (23) gives $\dot{K}_t = \alpha_2 K_t - \alpha_2 \frac{E_{Yt}+E_{Ct}}{\eta} - C_t$.

Appendix A_2

To determine the expression of capital in the third regime, we need to solve the following equation of capital accumulation for the capital K_t: $\dot{K}_t = \Lambda K_t - \Theta \lambda_{T_2}^{-\frac{1}{\delta}} e^{(\frac{\alpha_2-\rho}{\delta})(t-T_2)}$, where $\Lambda = \frac{\alpha_2 \beta_2 \eta}{\alpha_2+\beta_2\eta}$ and $\Theta = \frac{\alpha_2\beta_2}{\alpha_2+\beta_2\eta}(\frac{\alpha_2}{\eta})^{-\frac{1}{\delta}}+1$. By making a change of variables $x_t = K_t e^{-\Lambda(t-T_2)}$ and using the following transversality conditions $\lim_{t\to\infty} \lambda_t K_t e^{-\rho(t-T_2)} = 0$, we get $K_t = -\frac{\Theta\delta}{\alpha_2-\rho-\delta\Lambda}\lambda_{T_2}^{-\frac{1}{\delta}} e^{(\frac{\alpha_2-\rho}{\delta})(t-T_2)}$, for $\alpha_2(1-\delta) < \rho$.

Finally, we need to impose the non-negativity condition on E_{Yt} so that:

$$E_{Yt} = \frac{\alpha_2}{\alpha_2+\eta\beta_2}(\eta K_t - E_{Ct}) > 0 \Leftrightarrow -\frac{\Theta\delta\eta}{\alpha_2-\rho-\delta\Lambda} - \left(\frac{\alpha_2}{\eta}\right)^{-\frac{1}{\delta}} > 0.$$

Appendix A_3

The expression of capital in the second regime is determined from FOCs as follows:

FOCs lead to: $\mu_t = \mu_{T_1} e^{(\rho-\alpha_2(1-\phi))(t-T_1)}$, $\nu_t = \nu_{T_1} e^{\rho(t-T_1)}$, $C_t = \mu_{T_1}^{-\frac{1}{\delta}} e^{(\frac{\alpha_2(1-\phi)-\rho}{\delta})(t-T_1)}$ and $E_{Ct} = (\xi\nu_{T_1})^{-\frac{1}{\delta}} e^{-\frac{\rho}{\delta}(t-T_1)}$. Using the above expression of C, the equation of capital accumulation becomes: $\dot{K} - \alpha_2(1-\phi)K = -\mu_{T_1}^{-\frac{1}{\delta}} e^{(\frac{\alpha_2(1-\phi)-\rho}{\delta})(t-T_1)}$. Using the same variable change as in Appendix B and taking K_t at t = T_1, gives:

$K_t = -(\overline{K_2} - K_{T_1}) * e^{(\frac{\alpha_2(1-\phi)-\rho}{\delta})(t-T_1)} + \overline{K_2}$, where $\overline{K_2}$ is unknown and will be determined using boundary conditions in Sect. 3.1.4.

Appendix A_4

We assume that the dirty source of energy is exhaustible and that we have crossed the second regime after a period of time T_1. Then, the initial stock of the dirty source of energy S_0 is equal to the sum of the part of the dirty source of energy that is used during the first regime which corresponds to the total amount of pollution $\overline{Z}$ and the part of the dirty source of energy that the economy uses during the second regime. We have: $S_0 = \underbrace{\int_0^{T_1} \xi(E_{Yt}+E_{Ct})dt}_{\overline{Z}} + \int_{T_1}^{T_2} \xi(E_{Yt} + E_{Ct})dt$. This implies that: $S_0 - \overline{Z} = \int_{T_1}^{T_2} \xi(E_{Yt}+E_{Ct})dt = \frac{\xi\alpha_2}{\beta_2}\int_{T_1}^{T_2} K_t dt + \xi \int_{T_1}^{T_2} E_{Ct} dt$, with $S_0 > \overline{Z}$.

The above equation gives:

$$\frac{1}{\xi}(S_0-\overline{Z}) = -(\xi\nu_{T_1})^{-\frac{1}{\delta}} * \frac{\delta}{\rho}\left[e^{-\frac{\rho}{\delta}(T_2-T_1)}-1\right] + \frac{\alpha_2(1-\phi)}{\beta_2}\overline{K_2}\,[(T_2-T_1)] - \frac{\delta\alpha_2}{\beta_2(\alpha_2(1-\phi)-\rho)} * (\overline{K_2}-K_{T_1}) * \left[e^{(\frac{\alpha_2(1-\phi)-\rho}{\delta})(T_2-T_1)}-1\right].$$

Appendix A_5

The level of capital at each time during the first regime is determined as follows:

FOCs give: $\mu_t = \mu_{T_0}e^{(\rho-\alpha_2(1-\phi))t}$, $\nu_t = \nu_{T_0}e^{\rho t}$, $C_t = \mu_{T_0}^{-\frac{1}{\delta}}e^{(\frac{\alpha_2(1-\phi)-\rho}{\delta})t}$ and $E_{Ct} = (-\nu_{T_0}\xi)^{-\frac{1}{\delta}}e^{-\frac{\rho}{\delta}t}$. As before, we also replace the expression of C_t in the equation of capital accumulation to get: $\dot{K}_t - \alpha_2(1-\phi)K_t = -\mu_{T_0}^{-\frac{1}{\delta}}e^{(\frac{\alpha_2(1-\phi)-\rho}{\delta})t}$. Solving the above equation and taking K_t at $t=0$ give $K_t = -(\overline{K_1}-K_0)e^{(\frac{\alpha_2(1-\phi)-\rho}{\delta})t} + \overline{K_1}$. Finally, at the end of the first regime, we cross the pollution threshold so that $\overline{Z} = \int_0^{T_1}\xi(E_{Yt}+E_{Ct})dt$. This equation then implies that:

$$\nu_{T_0} = -\frac{1}{\xi}\left[\left(-\frac{\overline{Z}}{\xi} + \frac{\alpha_2(1-\phi)}{\beta_2}\overline{K_1} * T_1 - \frac{\alpha_2(1-\phi)\delta}{\beta_2(\alpha_2(1-\phi)-\rho)}(\overline{K_1}-K_0) \left[e^{\left(\frac{\alpha_2(1-\phi)-\rho}{\delta}\right)*T_1}-1\right]\right) * \frac{\rho}{\delta\left[e^{-\frac{\rho}{\delta}T_1}-1\right]}\right]^{-\delta}$$

where ν_{T_0} and $\overline{K_1}$ are unknown and will be determined in Sect. 3.1.4 using boundary conditions.

Appendix B_1

Equations (12), (9) and (11) become respectively:

$$\dot{K}_t = Y_t - C_t - q_t, \qquad (26)$$

$$\begin{cases} E_{Yt} = min\{\frac{1}{\xi}E_{Ydt}, E_{Yct}\} + \varepsilon_Y(q_t), & t < T_2 \\ E_{Yt} = E_{Yct} + \varepsilon_Y(q_t), & t \geq T_2 \end{cases}$$

and

$$\begin{cases} E_{Ct} = min\{\frac{1}{\xi}E_{Cdt}, E_{Cct}\} + \varepsilon_C(q_t), & t < T_2 \\ E_{Ct} = E_{Cct} + \varepsilon_C(q_t), & t \geq T_2. \end{cases}$$

where $t < T_2$ corresponds to the first two regimes, while $t \geq T_2$ denotes the third regime.

Also, Eqs. (14), (19) and (20) become respectively:

$$\dot{K}_t = \alpha_2 K_t - \alpha_2\frac{(E_{Yt}+E_{Ct})-(\varepsilon_Y(q_t)+\varepsilon_C(q_t))}{\eta} - C_t - q_t.$$

$$\dot{S}_t = -E_{dt} = -\xi(E_{Yt}+E_{Ct}) + \xi(\varepsilon_Y(q_t)+\varepsilon_C(q_t))$$

and

$$\dot{Z}_t = E_{dt} = \xi(E_{Yt}+E_{Ct}) - \xi(\varepsilon_Y(q_t)+\varepsilon_C(q_t)).$$

Appendix B_2

Second regime

As before, the only change is the FOC with respect to q_t:

$$\varepsilon_Y'(q_t) + \varepsilon_C'(q_t) = \frac{\mu_t}{\xi \nu_t}. \tag{27}$$

Using the same specifications as before, the solution of Eq. (27) is :

$$q^* = \left[\frac{\mu_t}{2\sigma\xi\nu_t}\right]^{\frac{1}{\sigma-1}}. \tag{28}$$

Equation (28) helps to solve the model during the second regime as before. The expression of the capital during the second regime becomes:

$$K_t - \overline{K_2} = -\mu_{T_1}^{-\frac{1}{\delta}} * \frac{\delta}{\alpha_2(1-\phi)(1-\delta)-\rho} e^{\frac{\alpha_2(1-\phi)-\rho}{\delta}(t-T_1)}$$
$$+ \left[\frac{\mu_{T_1}}{2\sigma\xi\nu_{T_1}}\right]^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\alpha_2(1-\phi)\sigma} e^{-\frac{\alpha_2(1-\phi)}{\sigma-1}(t-T_1)}.$$

We should also observe here that the level of capital at each period of time during the second regime has a second negative component. As the share of the income that goes to investment is reduced by investment in energy saving technologies, one should expect a decrease in capital.

As in the case without any investment in EST, all the dirty sources of energy are extracted during the first and the second regimes such that:

$$S_0 - \overline{Z} = \int_{T_1}^{T_2} \xi(E_{Yt} + E_{Ct} - \varepsilon_Y - \varepsilon_C)dt.$$

Solving the above equation, we get:

$$S_0 - \overline{Z} = \xi\frac{\alpha_2}{\beta_2}\overline{K_2}[T_2 - T_1] + H_0\left[e^{-\frac{\rho}{\delta}(T_2-T_1)} - 1\right]\nu_{T_1}^{-\frac{1}{\delta}} + H_1\left[e^{\frac{\alpha_2(1-\phi)-\rho}{\delta}(T_2-T_1)} - 1\right]\mu_{T_1}^{-\frac{1}{\delta}}$$
$$+ H_2\left[e^{\frac{-\alpha_2(1-\phi)}{\sigma-1}(T_2-T_1)} - 1\right][\frac{\mu_{T_1}}{\nu_{T_1}}]^{\frac{1}{\sigma-1}} + H_3\left[e^{\frac{-\alpha_2\sigma(1-\phi)}{\sigma-1}(T_2-T_1)} - 1\right]\left[\frac{\mu_{T_1}}{\nu_{T_1}}\right]^{\frac{\sigma}{\sigma-1}} \quad \textbf{(Eq. A)}$$

where $H_0 = -\xi^{\frac{\delta-1}{\delta}}\frac{\delta}{\rho}$, $H_1 = -\frac{\xi\delta^2\alpha_2}{\beta_2[\alpha_2(1-\phi)(1-\delta)-\rho][\alpha_2(1-\phi)-\rho]}$, $H_2 = -\frac{\xi(\sigma-1)^2\alpha_2}{\beta_2(2\sigma\xi)^{\frac{1}{\sigma-1}}\sigma\alpha_2^2(1-\phi)^2}$ and $H_3 = \frac{2\xi(\sigma-1)}{\alpha_2(1-\phi)\sigma(2\sigma\xi)^{\frac{\sigma}{\sigma-1}}}$.

First Regime

As in the second regime the optimal investment in EST is:

$$q^* = \left[-\frac{\mu_t}{2\sigma\xi\nu_t}\right]^{\frac{1}{\sigma-1}}. \tag{29}$$

We then solve the equation of capital accumulation to get the following expression of capital during the first regime:

$$K_t - \overline{K_1} = -\mu_{T_0}^{-\frac{1}{\delta}} \frac{\delta}{\alpha_2(1-\phi)(1-\delta)-\rho} e^{\frac{\alpha_2(1-\phi)-\rho}{\delta}t}$$
$$+ \left[-\frac{\mu_{T_0}}{2\sigma\xi\nu_{T_0}}\right]^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma\alpha_2(1-\phi)} e^{\frac{-\alpha_2(1-\phi)}{\sigma-1}t}.$$

We still have an additional negative component of the capital due to investment in energy saving technologies.

At the end of the first regime, we cross the pollution threshold so that:

$$\overline{Z} = \int_0^{T_1} \xi(E_{Yt} + E_{Ct} - 2\varepsilon_t^*)dt.$$

By solving the above equation as before, we get the following expression:

$$\overline{Z} = \xi\frac{\alpha_2}{\beta_2}\overline{K_1}T_1 + H_0\left[e^{-\frac{\rho}{\delta}T_1} - 1\right](-\nu_{T_0})^{-\frac{1}{\delta}} + H_1\left[e^{\frac{\alpha_2(1-\phi)-\rho}{\delta}T_1} - 1\right]\mu_{T_0}^{-\frac{1}{\delta}}$$
$$+ H_4\left[e^{\frac{-\alpha_2(1-\phi)}{\sigma-1}T_1} - 1\right]\mu_{T_0}^{\frac{1}{-\delta}} + H_3\left[e^{\frac{-\alpha_2\sigma(1-\phi)}{\sigma-1}T_1} - 1\right]\left[\frac{\mu_{T_0}}{\nu_{T_0}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad \textbf{(Eq. B)}$$

where H_0, H_1 and H_3 are the same as defined before and $H_4 = \frac{\xi\delta(\sigma-1)\alpha_2}{\beta_2\alpha_2((1-\phi)[\alpha_2(1-\phi)(1-\delta)-\rho]}$.

Boundary Conditions

As in the case without any investments in EST, we apply some boundary conditions. Continuity of μ_t and continuity of K_t at the switching times T_1 and T_2 gives the following equation:

$$H_5\mu_{T_0}^{-\frac{1}{\delta}} - \frac{\varpi}{\Lambda} = H_6\mu_{T_0}^{-\frac{1}{\delta}} + H_7\left[\frac{\mu_{T_0}}{2\sigma\xi\nu_{T_1}}\right]^{\frac{1}{\sigma-1}} + H_8\left[\frac{-\mu_{T_0}}{2\sigma\xi\nu_{T_0}}\right]^{\frac{1}{\sigma-1}} + (1-\theta)K_0 \quad \textbf{(Eq. C)}.$$

where

$$H_5 = -\frac{\Theta\delta}{\alpha_2-\rho-\delta\Lambda}e^{\frac{\alpha_2(1-\phi)-\rho}{\delta}T_2},$$
$$H_6 = \frac{\delta}{\alpha_2(1-\phi)(1-\delta)-\rho}\left[\theta e^{\frac{\alpha_2(1-\phi)-\rho}{\delta}T_1} + (1-\phi) - e^{\frac{\alpha_2(1-\phi)-\rho}{\delta}T_2}\right],$$
$$H_7 = \frac{\sigma-1}{\alpha_2\sigma(1-\phi)}\left[-e^{\frac{\rho-\alpha_2(1-\phi)}{\sigma-1}T_1} + e^{[\frac{\rho}{\sigma-1}T_1 - \frac{\alpha_2(1-\phi)}{\sigma-1}T_2]}\right] \text{ and}$$
$$H_8 = \frac{(\sigma-1)(1-\theta)}{\alpha_2\sigma(1-\phi)}\left[-1 + e^{\frac{-\alpha_2(1-\phi)}{\sigma-1}T_1}\right].$$

Equations A, B and C express three different relationships between μ_{T_0}, ν_{T_0} and ν_{T_1} that we can simultaneously solve. Additionally, we simultaneously and numerically solve the equality of Hamiltonians at the switching time T_1 and T_2 to get T_1 and T_2.

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