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Primitive accumulation, growth and the genesis of social classes

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Primitive Accumulation, Growth and the Genesis of Social Classes

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Abstract:

Unlike Stiglitz, we show that an inegalitarian long run equilibrium can emerge in a Solow growth model framework, assuming a linear consumption function. We then interpret this result in line with Marxian economics, showing that this dynamic framework is consistent with Roemer's idea of endogenous class stratification. We extend this calculation by incorporating some features of the Pasinetti-Samuelson-Modigliani model, and provide an example of possible microfoundations.

Contrairement à Stiglitz, nous montrons qu'un équilibre inégalitaire de long terme peut apparaître dans le cadre du modèle de Solow, en supposant une fonction de consommation linéaire. Nous interprétons ce résultat en relation avec l'économie marxiste, en montrant que ce cadre dynamique est cohérent avec l'idée de Roemer d'une stratification endogène des classes sociales. Nous élargissons ensuite l'analyse en intégrant certains aspects du modèle de Pasinetti, Samuelson et Modigliani et fournissons un exemple de possibles fondements microéconomiques.

JEL : B50, E25, O40

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Primitive Accumulation, Growth and the Genesis of Social Classes

Le maître d'école : Dis-moi donc d'où la fortune de ton père lui est venue ?
L'enfant : Du grand-père.
Le maître d'école : Et à celui-ci ?
L'enfant : Du bisaïeul.
Le maître d'école : Et à ce dernier ?
L'enfant : *Il l'a prise.*

J.W. Goethe, quoted by Marx in the French edition of *Capital*.

Economic theory tries to explain inequality among agents in the long run, but is nevertheless divided between different traditions and visions of society. Within a classical and Marxian framework, capitalist society is divided into social classes defined by the ownership (or lack of ownership) of means of production, which gives power over production decisions. According to Marx, beyond the legal equality between parties to this exchange lies a real asymmetry between those who make production decisions and have control over "the hidden abode of production"[2], and those who are deprived of it. Besides, the classical and Marxian tradition accepts competitive conditions, at least as a first approximation: it considers that social inequality is consistent with a perfect working of the markets; it is not a result of market imperfections. By contrast, the marginalist revolution has been associated with the advent of consumer prominence position, in a world where firms are transparent. In the absence of idiosyncratic features (such as various psychological discount

[2] *Capital*, Part I, Chapter VI.

rates for example), the question of which conditions allow long run inequality to be compatible with a competitive environment remains partly open. The aim of this paper is to show that a simple neoclassical approach, such as Solow's canonical growth model, for all its oversimplified hypotheses[3], can nevertheless be accommodated to grasp some features and raise some questions relevant to the Marxist tradition, and so enrich the comparison between different economic approaches to social inequality.

This aim is rather paradoxical, since Solow's model cannot deal with asymptotic wealth inequality. The simplified intuition of this result is plain. In this model, wages as well as profit rate depend on the overall capital stock in the economy and so, with a given average (and marginal) propensity to save, the growth rate of an individual capital stock is inversely correlated with its level. Any inequality among agents vanishes asymptotically[4]. This result of convergence cross-agent, which parallels and slightly differs from the famous analogous one across countries, makes the relevance of the model questionable.

Stiglitz (1969) showed that, with a possible exception, this same result of an asymptotic and even distribution of income and wealth is valid if we suppose a linear savings function, such that the marginal propensity to save is constant but the average propensity increases in line with the income and is endogenously determined in equilibrium. In contrast, Schlicht (1975) showed that an asymptotic inequality is possible with a strictly convex savings function. In Bourguignon (1981), the same idea repeats a very classical argument to the benefit of social inequality: with a strictly convex saving function the saving capacity and growth possibilities of an economy are boosted by the inequality of income distribution. Even if all agents are otherwise the same, in the long run everybody could benefit from a social inequality between them. As the title of his article says, Bourguignon demonstates "Pareto superiority of unegalitarian equilibria".

Stiglitz' exception is very different indeed. What he has in mind is "the possible exception, in the case of negative savings at zero income, of a group with zero wealth" (1969, p.382). In that case we'll have no Pareto ranking possibility between egalitarian and inegalitarian equilibria, one group being better off and the other worse off in each situation compared to the other. We are interested in this case, because it gives a kind of indivisibility, a solution of continuity in the behavior of agents with zero wealth. Nowadays inequality models do generally assume some kind of indivisibility as well[5], as fixed cost to investment for example. Besides, assuming asymmetric information on the credit market, poor agents who are rationed cannot overcome the indivisible cost; so the ensuing inequality endures. But in contrast with this modern treatment of the question, Stiglitz' exception may generate everlasting inequality without assuming any market imperfection: in a closed economy, with no asymmetric information whatsoever, it is indeed rational to refuse to provide loans to agents with zero wealth who need these loans for consumption purposes. In our opinion, this raises the question of the endogenous emergence of a class deprived of any means of production in a competitive environment. As far as we know, Roemer (1982 a) and b)) is the first

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[3] There is only one kind of good, which may be consumed or used in production, only one kind of labour, perfect technical substitution, factor payments are effected at marginal productivity, etc.

[4] See Jacques et Rebeyrol (2001), which shows that the same result prevails if one admits that, as in Kaldor (1955-56), the propensity to save on profits is higher than it is on wages.

[5] See for example Galor and Zeira (1993), Matsuyama (2006), Moav (2002), Aghion and Bolton (1997) and Piketty (1997).

author to have discussed the logical endogenous determination of social class stratification, possibly of a Marxian type, and the associated emergence of new institutions. We think that the Stiglitz exception can provide a simple dynamic structure in which to discuss such ideas[6].

We first examine Stiglitz' model to show that his exception has a much larger scope that he thought himself, by studying his model inside a configuration of parameters he ruled out from the start (1). We then comment this configuration in line with Marx' ideas of a necessary primitive accumulation in the growing process of the capitalist society and the logical emergence of typical capitalistic social classes and institutions such as the labour market (2). We extend this result by considering, as in Pasinetti (1962), a model that deals with pure capitalist agents (that is, agents who earn profits but no wages) (3). In the last section, we show that the type of results we have found, particulary the existence of a Marxian type equilibrium, can easily be derived from simple microfoundations that rule out any difference between agents' preferences (4)[7].

1. Stiglitz' Model Revisited

The Stiglitz (1969) model is a Solow-like model. There is only one good, which can be used as capital or consumption, and is yielded by a "well-behaved" production function. The main difference is that individual consumption is based on a linear function, where $C_h = (1-s)Y_h + \beta$, with $\beta > 0$ (and $s \in]0,1[$) which is the same for all agents *h*: the marginal propensity to consume is constant but the average propensity is assumed to decrease with income. Individual income Y_h is composed of the wage rate w (everybody works in this economy) and of the profit earned on individual capital K_h. In a footnote however (footnote 9 on p.387), Stiglitz considers that "if there is a lower bound on the amount of capital that one can hold (an upper bound on indebtedness)" and if "the lower bound is zero", consumption will be limited by income when capital owned is nil. In that case we must compare wage income with the β/s threshold, because agents' savings will be positive as soon as $w \geq \beta/s$, even if $K_h = 0$. Thus, we can write:

$$C_h = \begin{cases} (1-s)Y_h + \beta & \text{if } K_h > 0 \ or \ Y_h \geq \beta/s \\ Y_h & \text{if } K_h = 0 \ and \ Y_h \leq \beta/s \end{cases} \qquad (1)$$

$\beta > 0,\ s \in]0,1[,\ K_h \geq 0.$

It should be noted that with this consumption function, agents who constantly "eat up their capital" by consuming more than they earn, will experience a sudden drop in their consumption level when their capital is completely depleted. In the last section of this article, we will provide a justification for this drop in a model in which agents live for one period only and, in utility terms, value their consumption more than that of their heirs.

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[6] We think that our framework is similar to that of Galor and Moav (2006). Instead of focusing on the genesis of capitalist social classes, they introduce human capital to discuss quite the reverse: "the demise of the class structure".

[7] Two methodological warnings before we proceed:
- unlike Roemer (1982), we don't aim at giving an interpretation of a marxist system of thought and nothing is said about the concept of exploitation.
- but like Roemer, our "construction is a logical one, and is not intended as historical" (1982 a), p.170).

As in Solow's model, a "well-behaved" aggregate production function is assumed, which is homogeneous of degree one, twice differentiable and satisfies Inada conditions. Average per capita income $f(k)$ is divided between gross profits $kf'(k)$ and wages $w(k)$. Capital depreciates at rate δ and demographic growth occurs at rate n. In Stiglitz'model, agents are gathered in homogeneous groups with equal division of wealth among heirs. Group i is a (constant) proportion a_i of the total population[8]. Let L be the total labour force, K_i the capital held by group i, k_i the ratio ${}^{K_i}/_{L}$ and k the overall capital/labour ratio (of course, $k = \sum_i k_i$). Assuming (1), group i savings is equal to $a_i L[sw(k) - \beta] + sK_i f'(k)$. Thus, the dynamics of k_i is governed by the following equation:

$$\dot{k}_i = \begin{cases} a_i[sw(k) - \beta] + [sf'(k) - (n+\delta)]k_i & \text{if } k_i > 0 \ or \ w(k) \geq \beta/s \\ 0 & \text{if } k_i = 0 \ and \ w(k) \leq \beta/s \end{cases} \qquad (2)$$

As long as everybody owns a positive amount of capital, the aggregate capital/labour ratio moves according to:

$$\dot{k} = sw(k) - \beta + [sf'(k) - (n+\delta)k] = sf(k) - (n+\delta)k - \beta \qquad (3)$$

Stiglitz assumed that equation (3) has two stationary solutions. This is depicted on Fig 1 (b) (in contrast with case (a) of Solow's model where $\beta = 0$, and with case (c), see below). In case (b), Stiglitz shows that the two long run positive equilibria that appear are both egalitarian. The lower equilibrium is unstable, whereas the upper one is stable, both with respect to aggregate income and to its distribution, which tends to become even. Thus a trap appears and the economy must disappear or converge towards the upper egalitarian equilibrium, depending on the initial capital/labour ratio. The "possible exception" he considers in his footnote (note 9 on p.387) is that inegalitarian equilibria appear near the lower root, in which case there is a "poor group" with zero wealth. In this footnote, this possibility relies on the sign of the third derivative of $f(k)$; it doesn't exist with a Cobb-Douglas production function.

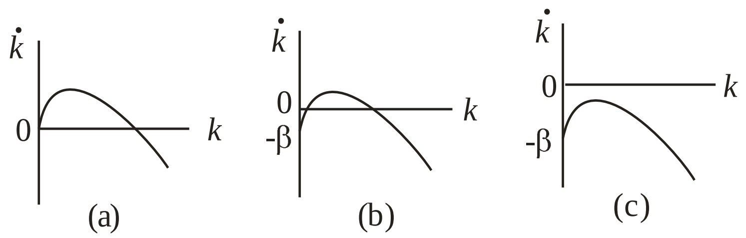

Figure 1

Consider now the configuration Stiglitz ruled out: suppose that parameter β is great enough (compared to $n + \delta$) to prevent the existence of any positive egalitarian equilibrium, that is, suppose the equation $\dot{k} = 0$ in (3) has no real roots, as shown in Fig 1 (c). Under such a dramatic hypothesis, as long as any K_i has not fallen to zero, the overall capital/labour ratio will continually decrease, threatening the very existence of the economy. In this case, can social inequality improve the situation ? The answer turns out to be positive. We give a possible phase diagram in the (k, k_i) plane in Fig. 2, in the case of two groups only.

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[8] From now on, subscript h refers to an agent or a household, whereas subscript i refers to a homogeneous group of agents.

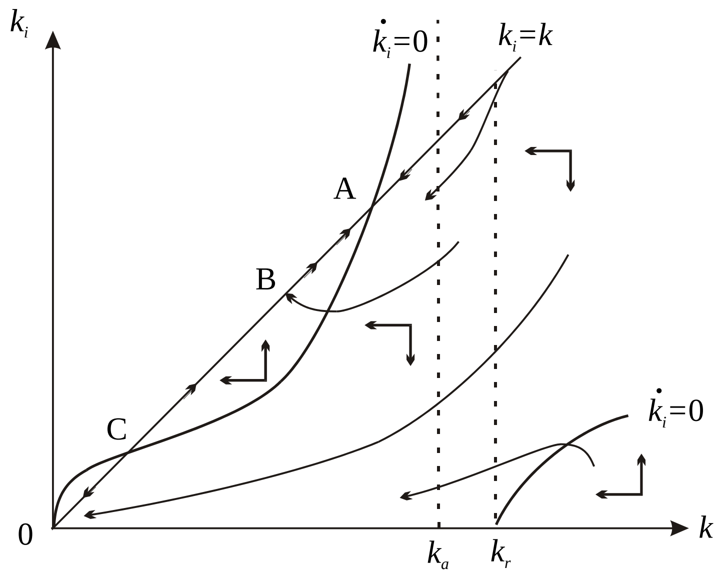

Figure 2

The equation for locus $\dot{k_i} = 0$ is $k_i = a_i(\frac{sw(k)-\beta}{n+\delta-sf'(k)})$. The left branch of its graph, before its asymptote at $k_a = f'^{-1}(\frac{n+\delta}{s})$, may lie entirely over the first bisector[9], but mathematically we can always choose a_i small enough to ensure that it crosses this bisector twice, as shown on Fig. 2. We define $k_r = w^{-1}(\frac{\beta}{s})$. Note that[10] $k_r > k_a$. Thus, on the left hand side of the diagram, below point $k = k_r$, labour marginal productivity (which increases with k) is too small to ensure positive savings on the part of agents deprived of all stock of means of production. The opposite is true on the right hand side, beyond k_r.

On the first bisector, as $k_i = k$, the dynamics of k is no longer described by equation (3) but by equation (2). k stays on the bisector whenever $k_i = k < k_r$. The phase diagram shows three equilibria, 0 and A being locally stable whereas C is unstable. No positive egalitarian equilibrium exists, but A is nevertheless a long run inegalitarian stable equilibrium.

This diagram illustrates the problem of primitive economies whose productivity is small enough to threaten their very existence. Their economy can indeed disappear along trajectories that converge towards the origin. But a given group, if it initially holds enough of the total stock of means of production, may see its part of the total wealth growing to unity. If a (small) group succeeds in holding enough capital, no matter how, it can find itself in a position to acquire the entire stock. If this happens, as it does in Fig. 2 at point B, at the close of a trajectory where the other group happens to be deprived of any capital, the dynamics of the overall k ratio switches from equation (3) to equation (2). Because at B the wage rate is less than β/s, consumption in the "poor" group will drop in a discontinuous manner. This will enhance overall growth: the drop in consumption of the poor is sufficient to put an end to the fall-off in capital and impede the vanishing of the economy.

[9] If this is the case for every group i, then the economy will definitely vanish.

[10] Assuming that the equation (3) has no real roots implies that whatever k, $\dot{k} < 0$. Hence at k_a, as $sf'(k_a) - (n+\delta) = 0$ by definition, equation (3) yields $\dot{k}_a = sw(k_a) - \beta < 0$. Because $sw(k_r) - \beta = 0$, we can conclude that $k_r > k_a$.

Afterwards, the economy will converge towards long run inegalitarian point A, where the wage rate stays below β/s.

We thus arrive at a final conclusion that is the opposite of Stiglitz': a Solow-like model with a linear consumption function can exhibit a long run inegalitarian equilibrium which, moreover, is locally stable. After point B has been reached, the poor are constrained to consume no more than they earn. This fact doesn't result from any market imperfection or asymmetric information or *arbitrary* "upper bound on indebtedness". In a closed economy, the poor cannot consume more than their real wage without diminishing the overall capital/labour ratio. With the assumption of Fig. 1 (c) that equation (3) has no real root, this level of consumption under duress is the condition for maintaining the capital/labour ratio. In the next section, we shall consider whether this idea can be used to shed light on Marx' primitive accumulation concept, which is logically anterior to capitalist social relationships.

2. Marx and Primitive Accumulation

In Karl Marx' view, "capital" is not defined as a stock of goods but as a social relationship which is historically determined:

> *"[Capital] can spring into life, only when the owner of the means of production and subsistence meets in the market with the free labourer selling his labour-power. And this one historical condition comprises a world's history."* (1867, chap.6)

Marx thought that a capitalist society calls for *primitive accumulation*, that is "an accumulation not the result of the capitalistic mode of production, but its starting point" (1867, chap.26), which has taken place through violence:

> *"In actual history it is notorious that conquest, enslavement, robbery, murder, briefly force, play the great part. (...) The methods of primitive accumulation are anything but idyllic."* (1867, chap.26)

> *"Force is the midwife of every old society pregnant with a new one. It is itself an economic power."* (1867, chap.31)

The historical process which results in a capitalist society is not only an economical but also a political one:

> *"The immediate producer, the labourer, could only dispose of his own person after he had ceased to be attached to the soil and ceased to be the slaver, serf, or bondsman of another. (...) Hence, the historical movement which changes the producers into wage-workers, appears, on the one hand, as their emancipation from serfdom and from the fetters of the guilds, and this side alone exists for our bourgeois historians. But, on the other hand, these new freedmen became sellers of themselves only after they had been robbed of all their own means of production, and of all the guarantees of existence afforded by the old feudal arrangements. And the history of this, their expropriation, is written in the annals of mankind in letters of blood and fire."* (1867, chap.26)

Of course, the economic modelling used in the first section of this article cannot describe "the old feudal arrangements" and the political process of their disintegration and collapse[11]. In our opinion, it can nevertheless shed light, more modestly, on a logical economic structure which can result in the workers' expropriation of their means of production. In a logical manner, *but definitely not a historical one*, assume a society composed of independent craftsmen in a world where constant returns to scale prevail, and lacking technical progress in a first approximation. Assume macroeconomists' usual simplifications that labour is homogenous and there is only one good to be used as a productive factor as well as a consumption good. Suppose craftsmen consume as described in equation (1) and that we have the very case of Fig.1 (c) (that is, whatever k, $sf(k) - (n+\delta)k - \beta < 0$. If all craftsmen are given the same wealth, the economy must definitely collapse. Now suppose that, as a result of whatever random or violent and unfair process, the population of craftsmen is divided into two different homogeneous groups, "poor" and "rich", according to their per capita stock of means of production. Equation (2) is no longer valid in this context, because each group gets idiosyncratic marginal productivity for its factors. If we keep the same conventions as in section 1 ($a_i = {L_i}/{L}$ and $k_i = {K_i}/{L}$), we get:

$$\dot{k}_i = a_i\left(sf\left(\frac{k_i}{a_i}\right) - \beta\right) - (n+\delta)k_i$$

The stock of means of production of the poor group will become nil at a time where the stock of the rich group is still positive. At that time poor agents will no longer be able to survive by themselves, because they will be "short of everything necessary for the realization of their labour-power". Rich agents will follow in the footsteps of the poor, except if they succeed in raising their income sufficiently by founding new institutions. Establishing a labour market and employing the poor will indeed improve the marginal productivity of their means of production. If this is the case, afterwards the economy will be such as described by the model in section 1. With a wage rate equal to the marginal productivity of labour, the per capita stock will evolve according to equation (2), once again coming to the fore with $k_i = k$. Thus, if a_1 refers to the proportion of the rich, the overall capital/labour ratio will now increase for all $k < k_a$ if and only if:

$$a_1 < \frac{[sf'(k) - (n+\delta)]k}{\beta - sw(k)}$$

If this condition is fulfilled, then the dynamic of *k* will be described by the trajectory from B to A in Fig. 2. After the establishment of the labour market, the stock of means of production will become capital in Marx' sense, because expropriated workers will have been forced to sell their labour-power. On the trajectory from B to A and in the long run inegalitarian equilibrium the value of the labour-power allows for the reproduction of a social class of workers deprived of means of production: the wage rate, being less than β/s, prevents this class from accumulating wealth. Hence, if wealth happens to be sufficiently concentrated in a few hands in the craftsmen's society, new institutions and social classes can emerge endogenously and impede the vanishing of the economy. This *primitive violence* has not been arbitrary, because in this framework the issue of the extent of

[11] Likewise, according to historical materialism, capitalism emerges as a result of a succession of modes of production, ancient then feudal, in a process that involves technical progress and the development of productive forces. This is obviously beyond the scope of our paper.

previous accumulation[12], understood as the level of accumulation necessary to avoid the collapse of the economy, cannot be separated from that of the distribution of that wealth[13].

As Roemer (1982 a and b) nevertheless point out, a labour market is not the single market structure that can arise in such a situation. More precisely, there is an isomorphism between an economy with a labour market and another with a credit market instead, and "the two markets coexisting in the same economy are redundant" (1982 a p.184). In our framework, this result stems directly from Euler's theorem. With constant returns to scale, the establishment of a labour market isn't necessary because poor agents can borrow the capital they need for their personal production from rich agents: whatever the market structure (labour or credit) and if markets are purely competitive the final allocation will be the same. As Roemer suggests, the fact that "labor markets have been historically more significant than credit markets in capitalism" may perhaps be explained "at a more concrete level of abstraction than that of this paper" if supervision costs are higher on credit operations than in mills (1982 a p.185), that is if the production relationships within its "hidden abode" are easier to control than the financial ones. At this point the conclusion is then that the concrete explanation for the emergence of the labour market requires the introduction of elements that are outside the scope of a basic purely competitive model, such as increasing returns to scale or moral hazard problems. Compared to Roemer's static analysis, our model allows for a dynamic understanding of the endogenous emergence of a group of workers that are short of everything.

3. The Model with Pure Capitalists

We shall now turn to two extensions of our representation. First of all, our description seems to be limited to poor and primitive economies, because we have ruled out (by assumption) Stiglitz' hypothesis of the existence of a high egalitarian equilibrium. Secondly, both in Marx' analysis as in Roemer's class decomposition there are pure capitalist agents who only earn profits whereas, in previous sections everybody was assumed to be working.

The idea of pure capitalist agents has been addressed by Pasinetti (1962) and Samuelson & Modigliani (1966). Pasinetti focuses not so much on "functional distributions" between wages and profits, but on "social distributions" between workers and (pure) capitalists. In the "Pasinetti case", a long run inegalitarian equilibrium prevails. But the mechanism at work is very different from that studied in part 1. In Pasinetti's model, capitalists and workers have different (average and marginal) exogenous propensities to save, and the possibility of an inegalitarian long run equilibrium is generated by a high enough ratio of their respective savings propensities[14]. Thus, enduring inequalities rest on behavioural differences. Moreover, Pasinetti's inegalitarian long run equilibrium is not of a "Marxian type", because in his model, workers own a positive amount of capital.

We now propose to introduce Pasinetti's pure capitalists agents into the framework of the previous sections where all agents, regardless of class, consume according to equation (1). Consider

[12] "The accumulation of stock must, in the nature of things, be previous to the division of labour." (Smith, 1876, p.292).
[13] This would not have been the case with the Stiglitz' hypothesis in Fig.1 (b), because there the trap given by the lower and unstable equilibrium is only defined by the level of the aggregate k.
[14] For a presentation of this model with a phase diagram, see Jacques & Rebeyrol (2001).

two groups of homogeneous agents, workers and capitalists. There are L workers (referred to by subscript ℓ) and M capitalists (referred to by subscript c). Each of these numbers grows at rate n over time and maintains proportion $\theta = M/L$ constant. With $k_\ell = {K_\ell}/{L}$, $k_c = {K_c}/{L}$ and $k = \frac{K_\ell + K_c}{L}$, the dynamics of these ratios is given by the following equations :

$$\dot{k}_\ell = \begin{cases} sw(k) - \beta + [sf'(k) - (n+\delta)]k_\ell & \text{if } k_\ell > 0 \ or \ w(k) \geq \beta/s \\ 0 & \text{if } k_\ell = 0 \ and \ w(k) \leq \beta/s \end{cases}$$

$$\dot{k}_c = \begin{cases} sk_c f'(k) - (n+\delta)k_c - \theta\beta & \text{if } k_c > 0 \\ 0 & \text{if } k_c = 0 \end{cases}$$

$$\dot{k} = \begin{cases} sf(k) - (n+\delta)k - (1+\theta)\beta & \text{if } k_c > 0 \ and \ k_\ell > 0 \ (or \ w(k) \geq \beta/s) \\ \dot{k}_c & \text{if } k_\ell = 0 \ and \ w(k) \leq \beta/s \\ \dot{k}_\ell & \text{if } k_c = 0 \end{cases}$$

We now assume that equation $sf(k) - (1+\theta)\beta - (n+\delta)k = 0$ has two distinct real roots, warranting the existence of a stable upper equilibrium (in the same manner as Stiglitz does in his model). A possible phase diagram, in the (k, k_c) plane, is drawn on Fig. 3 (a numerical example is given in footnote 20 below).

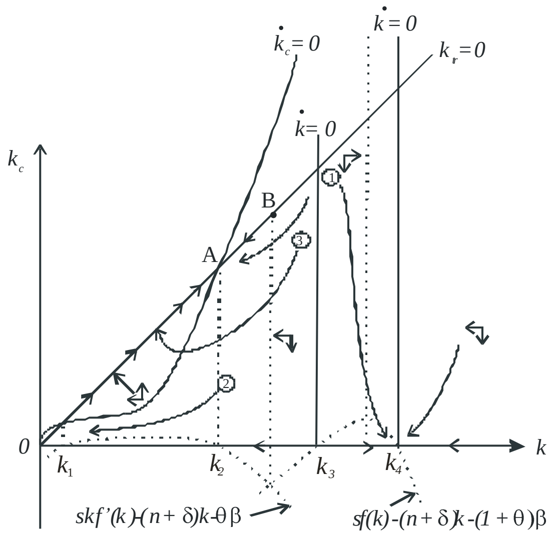

Figure 3

If the economy begins with an overall capital/labour ratio lower than k_1, it must disappear completely. At the other end of the spectrum, if the economy begins with $k > k_3$, it will converge towards an upper equilibrium at k_4: workers acquire capital and pure capitalist agents disappear asymptotically, along trajectories like that described by (1) in Figure 3[15]. The range between k_1 and k_3 raises the question of primitive accumulation. Over this range the analysis is similar to that already made in part 1: some trajectories, such as (2) on Fig. 3, make the overall capital/labour ratio shrink to zero. But if pure capitalists agents own a greater part of the total capital, as on trajectory (3), workers will finally be completely deprived of all capital. At this point, workers will earn less than β/s and will accordingly have to reduce their consumption discontinuously (indeed point B, on the

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[15] This illustrates the Walrasian dream of the extinction of social classes by all workers' accession to capital property.

first bisector, shows a coordinate on the k-axis which is level k_r such that[16] $w(k_r) = \beta/s)$. The dynamics of the overall capital/labor ratio is now given by $\dot{k}_c$ with $k_c = k$, that is by equation $\dot{k} = skf'(k) - (n+\delta)k - \theta\beta$ (the dotted line on the left of Fig. 3). Hence afterwards, global accumulation is revived and the economy converges to a long-run equilibrium of a "Marxian type" at point A.

To sum up, the introduction of pure capitalist agents allows us to consider the coexistence of two locally stable positive long-run equilibria. The first is of the Marxian type, contrary to the upper one, which is egalitarian. Of course, if the economy is stuck at the lower inegalitarian equilibrium, one is free to believe that future technical progress might shift the $\dot{k}_c = 0$ curve to the right, shifting point A towards B in such a manner that workers will begin to save and thus induce virtuous dynamics towards the upper egalitarian long run equilibrium, after the vanishing of the Marxian type equilibrium. One should however remember that the position of the $\dot{k}_c = 0$ curve also rests on the psychological parameter β that may well increase with historical progress: from a Marxian point of view, the value of labour-power is not physiologically, but historically determined.

4. Inequality and Individual Behaviour

Are there inequalities that are not the outcome of idiosyncratic features ? This is not a matter of fact. Idiosyncrasies do exist, but authors like Marx and Walras clearly think that individual differences are not relevant to the greater social questions[17]. If this idea is to be taken seriously, if the question of interest is the normative question of whether a society, by itself, can generate inequality among its members, then it is important to work out schemes where all agents are completely identical, apart from the inheritance they receive in a determined social framework. This is why it seems more satisfactory to assign the same demographic and savings behavior to everybody, as we do in this paper (it is not only a formal convenience). Nevertheless, in the part 3 model, we do not explain why pure capitalist agents do not work. Moreover, from the beginning, we have not explained the drop in consumption encountered in function (1).

So let's turn to a neoclassical modelling where preferences, including the propensity to opt for leisure, are made explicit and completely identical across agents, so that the effects of unequal social distribution of wealth can be analyzed per se. We use simple "microeconomic foundations", and provide a numerical example simply in order to explain individual behaviors and to show that "Marxian type" equilibria can easily arise.

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[16] This can be seen because the two dotted curves cross at this level, which implies that $skf'(k) = sf(k) - \beta$, or equivalently that $sw(k) = \beta$.

[17] "Let's get to the point. Is there, in our society, a form of misery that isn't the logical result of sloth, stupidity or ill fortune? ". ("Allons au fait. Y a-t-il, dans notre société, d'autre misère que celle qui résulte logiquement de la paresse, de l'inintelligence ou des revers de la fortune ?"), Walras (1860, p.iii).

"In times long gone-by there were two sorts of people; one, the diligent, intelligent, and, above all, frugal elite; the other, lazy rascals, spending their substance, and more, in riotous living. (...) Thus it came to pass that the former sort accumulated wealth, and the latter sort had at last nothing to sell except their own skins. And from this original sin dates the poverty of the great majority that, despite all its labor, has up to now nothing to sell but itself, and the wealth of the few that increases constantly although they have long ceased to work. Such insipid childishness is every day preached to us in the defence of property." Marx (1867, chap.26).

Let us assume a model in which, in discrete time, agents live for one period only. They maximize a static utility which is a function of consumption *C*, labor *L* and inheritance K_{+1}, which they will bequeath to their children[18]. The utility function is assumed to have the following form (where a_1, a_2, a_3, $\bar{C}$ and $\overline{K}$ are all positive parameters):

$$U(C_h, K_{h,+1}, L_h) = a_1 \ln\left(C_h - \overline{C}\right) + a_2 ln\left(K_{h,+1} + \overline{K}\right) + a_3 \ln(2 - L_h) \qquad (4)$$

First, let us consider that individual income Y_h is exogenous. Maximizing (4) subject to $C_h + K_{h,+1} \leq Y_h$, $C_h > \bar{C}$ and $K_{h,+1} \geq 0$ thus leads to the following solution, if any, in C_h and $K_{h,+1}$:

$$C_h = \begin{cases} (1-s)Y_h + \beta & (\text{and } K_{h,+1} = sY_h - \beta) \quad \text{if } Y_h \geq \beta/s \\ Y_h & (\text{and } K_{h,+1} = 0) \quad \text{if } \bar{C} < Y_h \leq \beta/s \end{cases} \qquad (5)$$

where $s = \frac{a_2}{a_1+a_2}$ lies between 0 and 1 and $\beta = \frac{a_1\overline{K}+a_2\bar{C}}{a_1+a_2}$ is positive. Note that $\frac{\beta}{s} = \bar{C} + \frac{a_1}{a_2}\overline{K} > \bar{C}$. When $Y_h \geq \beta/s$, agents want positive savings but when $\bar{C} < Y_h \leq \beta/s$, constraint $K_{h,+1} \geq 0$ is binding. In that case, we have $K_{h,+1} = 0$ and $C_h = Y_h$. The problem has no solution if $Y_h < \bar{C}$ (the parameter $\bar{C}$ can be viewed as a physiological subsistence level of consumption, so that it lies on the boundary of the consumption set). This solution (5) is clearly a discrete analogue of function (1).

Agent income is now the sum of wages (if employed) and the return from capital K_h, inherited from the previous generation. This capital is used in production where it depreciates at rate $\delta = 1$. Its net return is then $r = f'(k) - 1$. We thus have :

$$Y_h = wL_h + (1+r)K_h \qquad (6)$$

We assume work hours are indivisible: they can only have two values, 0 and 1. To solve the program agents, using (5) and (6), compare their utility when working (with $L_h = 1$) and when enjoying leisure ($L_h = 0$).

This simple model can generate various dynamics. We are interested in situations involving two homogeneous classes, workers deprived of any capital and who do not save on the one hand, capitalists who only earn profits, on the other. If there are, in such a situation, L workers and M capitalists, each of these numbers growing at rate *n* (with $\theta = M/L$), the dynamics of the overall capital/labour ratio will only reflect capitalists' behavior. It is given by[19]:

$$(1+n)k_{+1} = skf'(k) - \theta\beta$$

Let us take a simple numerical example. Let us assume[20] the production function to be $y = \sqrt{k}$. Take the values $\bar{C} = 0.01$, $\overline{K} = 0.135$, $a_1 = 0.8$, $a_2 = 1.2$ (consequently $s = 0.6$ and $\beta = 0.06$), $a_3 = 3$, $n = 0$, $\delta = 1$. Now suppose there are M agents, each of them owning capital of 0.10177,

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[18] No dynamic optimization is involved because we admit that inheritance is directly useful to the parents, without working out the offspring's utilities. We follow here Walras' (1874) practice that has become routine in growth theory and inequality litterature, cf. for example Galor and Moav (2006).

[19] In continuous time, letting the life time of the agents approach zero, this equation will be the same as the one used in the part 3 phase diagram: $\dot{k} = skf^{'}(k) - (n+\delta)k - \theta\beta$.

[20] The following numerical example ($y = \sqrt{k}$, $s = 0.6$, $\beta = 0.06$, $n = 0$, $\delta = 1$) can be used to exemplify the configuration of parameters behind part 2 Fig. 2.

and L agents who get no capital at all, no other commodity for sale than their labour-power, with $\theta = 0.35$. Then $K/L = 0.035619$, which is the highest, locally stable, of the two stationary solutions of the preceding difference equation (which will be refered to by k^*). We still have to check that in this situation, workers are indeed willing to work and willing to have zero savings, whereas capitalists choose leisure, the preferences of each being given by the utility function (4).

With $w(k^*) = 0.094365$ (which is the wage rate if capitalists indeed do not work), workers deprived of any other source of income cannot save (because $w(k^*) < \beta/s = 0.1$). If capitalists decided to work, the wage rate would be even lower (because the overall capital/labour would shrink), and the constraint more binding still. The L agents of course choose to work, because if they don't their income would be nil and would not ensure survival level $\bar{C}$.

As for the capitalists, they each get an individual capital of $k^*/\theta = 0.10177$ and, with $f'(k^*) = 2.6493$ earn 0.26962 in individual profits. Without working, they will consume 0.16785 and save 0.10177 (which will just ensure the reproduction of capital, no more, no less), reaching a level of utility of -1.1263. What happens if they work ? They may all act as competitive price takers, taking w and r as given. In that case, they will think that working gives them a global income of 0.36399, which will allow them to consume 0.2056 and save 0.15839, reaching a level of utility of -2.7768 only, because of the disutility of working[21]. They will thus choose not to work. If, on the contrary, these agents act strategically, they will consider that other capitalists could work too, which will consequently affect prices. In that case, the incentive to work will be still less, because the wage rate will drop and the profit rate will rise[22]. In any case, we can conclude that the situation is a long-run inegalitarian equilibrium of a "Marxian type", between absolutely identical agents, if you discount their capital endowment.

In this economy, what happens if the existing capital is equally redistributed among the $L + M$ agents? Every one now gets an individual capital of 0.026384. Consider only symmetrical solutions. If nobody works, there is no production and everybody will consume their entire capital of 0.026384,

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[21] In this example, the capitalist will choose leisure as soon as its weight in the utility function, a_3, is greater than 0.61877.

[22] More generally, a capitalist h whose profits are greater than β/s will have an incentive to work if and only if $w > (2^{\frac{a_3}{a_1+a_2}} - 1)[(1+r)K_h + \bar{K} - \bar{C}]$. Higher r and smaller w both discourage working.

To prove it, denote by Δv the difference between utility when working and when enjoying leisure. Such a capitalist will compute:

$$\Delta v = a_1 \, ln \frac{(1-s)(w+(1+r)K_h)+\beta-\bar{C}}{(1-s)(1+r)K_h+\beta-\bar{C}} + a_2 \, ln \frac{s(w+(1+r)K_h)-\beta+\bar{K})}{s(1+r)K_h-\beta+\bar{K}} - a_3 \, ln2$$

Notice that $\beta - \bar{C} = (1-s)(\bar{K} - \bar{C})$ and $\bar{K} - \beta = s(\bar{K} - \bar{C})$, consequently:

$$\Delta v = (a_1 + a_2) \, ln \frac{w + (1+r)K_h + \bar{K} - \bar{C}}{(1+r)K_h + \bar{K} - \bar{C}} - a_3 \, ln2$$

The agent will work if $\Delta v > 0$, that is if $\frac{w+(1+r)K_h+\bar{K}-\bar{C}}{(1+r)K_h+\bar{K}-\bar{C}} > 2^{\frac{a_3}{a_1+a_2}}$, or equivalently if $w > (2^{\frac{a_3}{a_1+a_2}} - 1)[(1+r)K_h + K-C$.

which is between C and β/s, enjoying a level of utility equal to -3.6127. Afterwards the economy will disappear (deprived of capital, children cannot secure their subsistence). On the other side, if everybody works, individual income will stand at 0.16243. Everybody will consume 0.12497 and save 0.037458, reaching a utility level of -3.8396 only, which is less than in the case of leisure. Hence the example illustrates the primitive accumulation problem: an egalitarian economy disappears while the concentration of capital among M capitalists allows the stationary path analyzed above. Now suppose the preference for leisure to be less strong, say, with $a_3 = 1$ instead of 3. None of the preceding results will be affected, except that the agents in the egalitarian economy will now choose to work (reaching a utility level of -3.8396 instead of -4.9989 if they choose leisure), saving more (0.037458) than they have received (0.026384). The economy will converge towards a higher level of capital[23]. This time the example illustrates the case of a growth path enhanced, beyond the "Marxian type" equilibrium, by an egalitarian distribution of wealth.

Thus the main result of this article, the existence of a Marxian type equilibrium, rests upon a threshold in the consumption behaviour of the agents that can easily be formalized in a standard neoclassical framework. The variations on the basic model, including the existence of pure capitalist agents and the type of dynamics generated, can be obtained through simple modifications of the parameters of the production and common utility functions.

In contrast with recent tradition which puts emphasis on the interaction between indivisibilities and imperfections in the credit market to explain social inequality, we show, by elaborating on Stiglitz' model (1969), that workers can be maintained in a poverty trap where they neither save nor consume more than their wage income, without market imperfections. This can be connected to the classical and Marxian tradition of a wage rate that allows the workers to survive but not accumulate capital, in a competitive environment. This modelling, however basic, includes the idea of a process of primitive accumulation, through which new institutions, social classes and inequality can emerge endogenously, without supposing any difference at all in individual characteristics.

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