Tradable deficit permits:
a way to ensure sub-national fiscal discipline?

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Abstract

This paper proposes a system of tradable deficit permits for implementing budgetary austerity at the local level. We evaluate the efficiency of the fiscal retrenchment allocation in a dynamic setting with a commitment problem. The way rights are allocated and traded on the market turns out to be decisive for the cost-effectiveness of the system. Indeed, the inability of the State to commit dynamically to a sharing rule of deficit rights generates perverse incentives which affect the local market. The market turns out to be inefficient - with heterogeneous jurisdictions - unless the State allows local decision-makers to trade permits through time.

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1 Introduction

The criteria for fiscal convergence implemented by the Maastricht Treaty, and strengthened by the Stability and Growth Pact (SGP) signed in Amsterdam (1997) compel each EMU Member State to a national budgetary discipline: the public deficit must remain below 3% of GNP and the public debt must not exceed 60% of GNP. To enforce budgetary rules, the SGP is based on both preventive and repressive mechanisms which may lead to financial sanctions imposed on the non-complying Member State’s central government.

A national internal incentives problem stems from the fact that deficit and debt ratios apply to the consolidated budget of all public administration bodies - including local jurisdictions - whereas only central authorities are held responsible for violations. Without a mechanism for explicitly sharing the efforts made to curb deficits, local jurisdictions would conduct tax and spending policies without taking their impact on the consolidated deficit into account. The central government can therefore be penalized, despite the fact that its budget is balanced, if local jurisdictions run excessive deficits. To abide by the terms of European commitments, the central government should therefore design and implement coordination mechanisms, that have the power to internalize budgetary externalities and to ensure the overall compatibility of local policies with national objectives\footnote{The need to secure fiscal discipline is not a major concern only in EMU Member States. Each decentralised country aims at ensuring the sustainability of public finances while exploiting the efficiency gains of decentralisation.}. In an "austerity" context, a crucial question is how the central government will share deficit-reduction efforts with local authorities. In other words, how can we solve this "tragedy of the commons"?

Most of the EMU Member States have adopted or strengthened fiscal rules at the local level - either enforced by the central government (Finland, Italy, Portugal) or set up by cooperative institutions (Belgium, Denmark, Germany and Spain) - in order to improve fiscal coordination. Among the wide variety of fiscal rules, the most widespread are budget balance requirements, restrictions on borrowing and, to a lesser extent, limits on the ability to increase spending or the tax burden\footnote{See Joumard and Kongsrud (2003) and Sutherland, Price and Joumard (2005).}. These traditional instruments for deficit regulation are certainly simple and easily understandable, but they often lack flexibility - especially in
bad times - and do not take the heterogeneity of the costs of enforcing the standard into account. Because local jurisdictions have different administrative rigidities, different electoral motivations, and different costs for public good provision, a uniform quantitative constraint imposes widely diverging costs which turns out to be inefficient. With a large number of local jurisdictions, it becomes particularly costly, not to say impossible, to extract all the necessary information about the marginal cost of fiscal retrenchment. When marginal costs vary according to time period, such a cost could not even be inferred from the response to the instrument. The implementation of Pigovian taxes thus also becomes inefficient. An alternative mechanism would be the implementation of a local market for tradable deficit permits. In Austria\(^3\), an internal stability pact introduced in 1999 - and revised in 2001 - implicitly includes the possibility of establishing a market for deficit permits which are transferable between Land governments and municipalities within the Land\(^4\).

A system of tradable deficit permits for implementing the fiscal constraints of the Maastricht treaty has already been studied by Casella (1999), but at the EMU level. Casella has based her approach on the environmental economic literature\(^5\) by considering deficits as a form of pollution. According to this literature, independently of the initial distribution, the trading of permits on a competitive market ensures an efficient decentralised allocation of pollution. Any polluter whose the marginal abatement cost is lower than the price at which permits are trading will have an incentive to abate more by selling permits, and \textit{vice versa}\(^6\). At the market equilibrium, marginal abatement costs are equalized, and the target is achieved at minimum cost. The parallel to the regulation of the deficit behaviour - at the EMU level or the national level - is immediate and... tempting!

\(^3\)This way has also been considered by Italia (Commissione Tecnica per la Spesa Pubblica, 1998).

\(^4\)In practice, it has not been implemented because the incentives for trading permits on a market have been blunted by a soft budget balance objective.

\(^5\)The theoretical foundation is based on the notion of property rights developed by Coase (1960). Dales (1968) pointed out its applicability for water and Crocker (1966) for air. Montgomery (1972) formally solved the problem of proving the existence of cost-effective permit market equilibrium. See Cropper and Oates (1992) for a survey.

\(^6\)A famous application of a market based system of tradable pollution permits follows the 1990 Clean Air Act Amendments.
In a static setting, Casella (1999) shows that the implementation of a tradable deficit permits market as a mechanism for sharing budgetary discipline among the EMU Member States is efficient. The optimal allocation equalizes the marginal abatement cost of fiscal retrenchment across all countries. However, fiscal coordination is mainly a dynamic issue. The deficit is a form of stock pollutant: each unit of current deficit adds permanently to the stock of the debt and to the level of the damage. In a dynamic setting, a soft budget constraint\textsuperscript{7} problem may arise if the behaviour of local jurisdictions in the market is influenced by the expectation of receiving additional deficit rights from the State. Our paper aims at evaluating the efficiency of a market for tradable deficit permits at the local level, in a dynamic setting with a commitment problem\textsuperscript{8}. Is the market-based mechanism for allocating budgetary austerity at the local level still efficient under a soft budget constraint policy? How does the ability of local jurisdictions to manipulate the distribution of rights affect the market?

The originality of our paper comes from modelling an intertemporal budgetary game between the regulator - the State - and the regulated agents - the local jurisdictions - for the distribution of deficit rights. The existence of the soft budget constraint is based on the local jurisdictions’ interest in behaving strategically in order to extract extra rights from the State, and on the State’s incentives to deviate \textit{ex post} from the fiscal discipline policy stated \textit{ex ante}. In this sequential game, local jurisdictions act as Nash competitors with each other, but they are first-movers in a Stackelberg game with the State. The total amount of rights\textsuperscript{9} to be shared between the local and central levels is assumed to be exogenous in each period. The State - which only intervenes in the market via the distribution of rights - splits this initial stock between itself and the local jurisdictions so that it maximizes the welfare

\textsuperscript{7}Following Rodden, Eskeland and Litvack (2003), a soft budget constraint can be defined as "the situation when an entity can manipulate its access to funds in undesirable ways". The inability of the rescuer to generate expectations of no bailout entails a soft budget constraint. See Kornai, Maskin and Roland (2003) for a survey.

\textsuperscript{8}The environmental economic literature has paid scant attention to the temporal inconsistency problem. See Batabyal (1996), Gersbach and Glazer (1999), Marsiliani and Renström (2000), Petrakis and Xepapadeas (2003).

\textsuperscript{9}This amount implicitly ensures the long-term solvency of public administrations.
minus its own cost of being constrained. A local jurisdiction may buy permits from another jurisdiction with a rights surplus or it may sell permits but must own enough of them to cover its current deficit in each period. The market price thus reflects the current value of the budgetary austerity.

We subsequently allow deficit permits to be traded over time. In such a setting, local jurisdictions may directly reduce their deficit as well as buy, sell, bank and borrow deficit permits in order to meet the applicable standards or to take advantage of any speculative opportunities. The intertemporal deficit permits trading thus lowers the cost of compliance with the deficit standards imposed by the State by allowing local jurisdictions to administer their budgets more flexibly over the two periods.

Our results can be summarized as follows. The sharing rule of deficit rights designed by the State turns out to be easily manipulable by local decision makers. Due to its aim of equalizing the marginal welfare across all local jurisdictions, the State always finds it optimal \textit{ex post} to reallocate rights in the second period when a local jurisdiction increases its first-period deficit. We show that the soft budget constraint phenomenon results in inefficiency of the local market for tradable deficit permits, unless local preferences are identical. The opportunistic behaviour of local decision makers consists in reducing first-period budgetary efforts, and this exerts upwards pressure on the price. However, the intertemporal trading of deficit permits restores the efficiency, \textit{i.e.} the marginal abatement costs equalization condition, due to the fact that local jurisdictions aim at smoothing their cost of being constrained over the two periods.

In the following, section 2 describes the model, the cost of being constrained and the functioning of the local market. Section 3 characterizes the rights allocation rule set by the State. Section 4 analyzes the impact of the soft budget constraint on the efficiency of the tradable deficit permits market. Section 5 introduces intertemporal flexibility in the permits trading and studies how this modifies the behaviour of local jurisdictions. Concluding remarks are gathered in section 6.
2 Analytical framework

The framework is a simple two-period intertemporal model. The economy consists of \( n \) local jurisdictions with the same level in the hierarchy of public authorities and a State. An internal budgetary austerity policy - laid down by an international treaty or self-imposed - commits the State to keep the consolidated public deficit, \( i.e. \) the sum of local jurisdictions’ deficit \( \sum_{i=1}^{n} d_{it} \) and its own deficit\(^{10} \) \( d_{ct} \), below an exogenous amount\(^{11} \overline{d}_t \):

\[
\sum_{i=1}^{n} d_{it} + d_{ct} \leq \overline{d}_t,
\]

in each period \( t = 1, 2 \).

The budgetary austerity is costly both for the local jurisdictions and for the State. Let \( C_{it}(\delta_{it}) \) measure the cost to local jurisdiction \( i \) of reducing its primary deficit to a level \( \delta_{it} \) in period \( t \), that is, the difference between the unconstrained utility, \( i.e. \) the local jurisdiction \( i \) runs a primary deficit \( \delta_{it}^* \), and the utility constrained by a budgetary rule, \( i.e. \) the local jurisdiction \( i \) runs \( \delta_{it} < \delta_{it}^* \). This cost of being constrained, which differs among local jurisdictions and through time\(^{12} \), may be interpreted as the value of foregone local public good provision, the value of the resources involved in improving efficiency and (or) can be viewed as an electoral cost. Note that the cost does not depend on the deficit \( d_{it} \) but on the primary deficit \( \delta_{it} \), \( i.e. \) the real amount of new resources devoted to the objectives of the local jurisdiction. Similarly, we define \( C_{ct}(\delta_{ct}) \) as the cost to the State of being constrained to a primary deficit \( \delta_{ct} \) in period \( t \). These functions are assumed to be twice continuously differentiable, decreasing \( (C'_{ct}(\delta_{ct}) \leq 0, C''_{ct}(\delta_{ct}) \leq 0 \; \forall i) \) and convex \( (C'''_{ct}(\delta_{ct}) \geq 0, C''_{ct}(\delta_{ct}) \geq 0 \; \forall i) \) in the amount of primary deficit, with a unique minimum at zero \( (-C''_{ct}(\delta_{ct}) = -C''_{it}(\delta_{it}^*) = 0 \; \forall i) \) at the unconstrained levels \( \delta_{ct}^* \) and \( \delta_{it}^* \). Hence additional costs from reducing one more unit of primary deficit, \( i.e. \) \( -C'_{ct}(\delta_{ct}) \) and \( -C'_{it}(\delta_{it}) \), also called marginal abatement costs, are increasing. By simplification, \( C'''_{ct}(\delta_{ct}) = 0 \) and \( C'''_{it}(\delta_{it}) = 0 \) \( \forall i \).

\(^{10}\)For a negative value of \( d_{it} \) (resp. \( d_{ct} \)), the jurisdiction \( i \) (resp. the State) runs a surplus.

\(^{11}\)\( \overline{d}_t \) could correspond to 3% of GDP of a EMU Member State.

\(^{12}\)In a political economy model, this cost would be higher the year before the election.
Each local jurisdiction \( i \) provides a local public good in quantity \( G_{it} \) financed by its primary deficit \( \delta_{it} \). Similarly\(^{13}\), the State provides a national public good in quantity \( G_{ct} \) financed by \( \delta_{ct} \). The citizens’ utility generated from the public goods provision is represented by the welfare function \( W_t(G_{1t}, \ldots, G_{it}, \ldots, G_{nt}, G_{ct}) \), which is twice continuously differentiable, increasing and concave in each argument, with \( W_t''' = 0 \) by simplification. We rule out spillover effects and substitutability between public goods, i.e. \( \frac{\partial^2 W_t}{\partial G_{it} \partial G_{jt}} = 0 \), \( \frac{\partial^2 W_t}{\partial G_{it} \partial G_{ct}} = 0 \), \( \frac{\partial^2 W_t}{\partial G_{ct} \partial G_{ct}} = 0 \), \( \frac{\partial^2 W_t}{\partial G_{it} \partial G_{ct-1}} = 0 \) and \( \frac{\partial^2 W_t}{\partial G_{it-1} \partial G_{it-1}} = 0 \) \( \forall i, j \).

The State perfectly observes\(^{14}\) the deficit \( d_{it} \) and debt \( D_{it} \) levels but has imperfect information about the marginal abatement cost \( -C_{it}'(\delta_{it}) \) of each local jurisdiction \( i \). In order to tackle this asymmetric information cost, it organizes a local market for tradable deficit permits which is supposed to ensure optimal sharing of the budgetary austerity among the local jurisdictions. In each period \( t \), the State allocates itself \( \alpha_t \overline{d}_t = \overline{d}_{ct} \) and splits \((1 - \alpha_t)\overline{d}_t = \sum_{i=1}^{n} \overline{d}_{it} \) between local jurisdictions, where \( \alpha_t \in [0, 1] \). The local entity \( i \) may either reduce its deficit to the initial stock of rights \( \overline{d}_{it} \) or be involved in the market, but it must have enough permits to cover the total deficit \( d_{it} \) at the end of the period \( t \). Let \( p_t \) denote the endogenous price of a permit in \( t \).

The primary deficit \( \delta_{it} \) of a local jurisdiction \( i \) in the period \( t \) is then defined as the total deficit \( d_{it} \) less the debt interest payment \( r D_{it} \), to which either the receipts from selling permits \( p_t (\overline{d}_{it} - d_{it}) > 0 \) are added or the expenses from buying permits \( p_t (\overline{d}_{it} - d_{it}) < 0 \) are deducted:

\[
\delta_{it} = d_{it} + p_t (\overline{d}_{it} - d_{it}) - r D_{it},
\]

where \( r \) is the debt interest, \( D_{i1} \) the exogenous level of the debt at the beginning of period 1 and \( D_{i2} = D_{i1} + d_{i1} \) the level of the debt at the beginning of period 2\(^{15}\). The State, being

\(^{13}\)There is no explicit taxation in our model but lump-sum taxes would not change the behaviour of the players.

\(^{14}\)In many countries, the State has many pieces of information relative to local jurisdictions’ accounts, insofar as local budgetary documents are transmitted to central administrative services, or publicly disclosed, and because the Public Treasury acts as a tax collector on behalf of local jurisdictions.

\(^{15}\)Assuming that the debt lives infinitely, the debt reimbursement tends to zero.
not involved in the market, uses all the rights it owns \((d_{ct} = \overline{d}_{ct} \ \forall t)\) under the assumption \(\delta_{ct} < \delta^*_c\) so that its primary deficit in the period \(t\) is

\[
\delta_{ct} = d_{ct} - rD_{ct},
\]

where \(D_{c1}\) is the exogenous level of the first-period debt and \(D_{c2} = D_{c1} + d_{c1}\) is the second-period debt.

Contrary to the environmental economic literature, it can be noted that the marginal abatement cost does not depend only on the use of the common-pool resource, \(i.e.\) the deficit in our setting. The endowment of rights, the debt level, and the market price also affect the cost of being constrained. We first point out an inverse relation between the endowment of rights and the marginal abatement cost \(\left(\frac{\partial(-C'_{it})}{\partial d_{it}} = -p_tC''_{it} < 0\right)\). An increase in the initial stock of rights induces a downwards translation of the marginal abatement cost, and conversely, as shown below by the Figure 1.

\[
\begin{align*}
\text{Figure 1: Effects of the initial stock of rights} \\
\text{A variation in the debt interest payment, either induced by the interest rate } r \text{ or the burden of the debt, leads to the same effects. An additional unit of deficit in } t \text{ generates a current gain because of the reduction in the current marginal abatement cost } \left(\frac{\partial(-C'_{it})}{\partial d_{it}} = -(1 - p_t)C''_{it} < 0\right) \text{ but it creates a loss for the future period } \left(\frac{\partial(-C'_{it+1})}{\partial d_{it}} = rC''_{it+1} > 0\right) \text{ due to the increase in the debt interest payment.}
\end{align*}
\]
In addition, the negative slope \( \frac{\partial(-C_{it}')}{\partial d_{it}} = -(1 - p_t) C_{it}'' < 0 \) of the marginal abatement cost increases with respect to the market price \( \frac{\partial^2(-C_{it}')}{\partial d_{it} \partial p_t} = C_{it}'' - (1 - p_t) C_{it}'''(\overline{d}_{it} - d_{it}) > 0 \) under the assumption \( C_{it}''' = 0 \)). As shown by the Figure 2, the straight line of the marginal abatement cost pivots around the point \( \overline{d}_{it} = d_{it} \) following a rise in the market price \( \frac{\partial(-C_{it}')}{\partial p_t} = -C_{it}''(\overline{d}_{it} - d_{it}) \).

![Figure 2: Effects of an increase in the market price](image)

It may thus result in exacerbating the inequalities among the local entities. The aggregated local deficit is unchanged but the variance may increase leading to the coexistence of "hot spots" jurisdictions with a high concentration of deficits and "cool spots" jurisdictions that run high surpluses.

The sequence of decisions is as follows. Before the game, the State is assumed to decide on the first-period level of rights for each local jurisdiction \( i \) and for itself. This initial decision is exogenous to the game to be played. In each period \( t \), local jurisdictions play as Nash competitors when choosing their level of deficit \( (d_{it}) \) and trading permits on the local market. Conversely, they play as Stackelberg leaders towards the State, \( i.e. \) they expect the way it will share the rights in period 2 and take into account its reaction function in their choice of first-period local deficit. We solve the model by backward induction.
The sharing rule of deficit rights

The first-period sharing of deficit rights is exogenous so we focus on the second-period problem. The State assumes that the local jurisdiction $i$ will provide the public good in quantity $\tilde{G}_{i2} = \bar{d}_{i2} - rD_{i2}$, with $D_{i2} = D_{i1} + d_{i1}$. Due to the asymmetric information - the local abatement cost being private information - the State is indeed unable to predict the market-based allocation of permits, the market price and, consequently, the local public good provision which justifies the use of $\tilde{G}_{i2}$ $\forall i$. The State designs the sharing rule so that it maximizes the welfare function minus its cost of being constrained\(^{16}\):

$$\begin{align*}
\max_{\bar{d}_{i2}, \ldots, \bar{d}_{i2}, \ldots, \bar{d}_{n2}, \bar{d}_{c2}} & \quad W_2(\tilde{G}_{12}, \ldots, \tilde{G}_{i2}, \ldots, \tilde{G}_{n2}, G_{c2}) - C_{c2}(\delta_{c2}) \\
\text{s.t.} & \quad \sum_{i=1}^{n} \bar{d}_{i2} + \bar{d}_{c2} = \bar{d}_2 \\
\text{with} & \quad \tilde{G}_{i2} = \bar{d}_{i2} - rD_{i2} \quad \forall i \\
& \quad G_{c2} = \delta_{c2} = \bar{d}_{c2} - rD_{c2}
\end{align*}$$

which yields the following first-order conditions:

$$\frac{\delta W_2}{\delta \tilde{G}_{c2}} - C'_{c2} = \frac{\delta W_2}{\delta \tilde{G}_{i2}} \quad \forall i$$

(1)

and

$$\sum_{i=1}^{n} \bar{d}_{i2} + \bar{d}_{c2} = \bar{d}_2.$$  

(2)

The internal sharing of deficit rights in the second period is such that the value from distributing one additional right to the State, in terms of welfare and abatement cost, is equal\(^{16}\) under the assumption $\frac{\partial^2 W_t}{\partial G_{i1} \partial G_{c2-1}} = \frac{\partial^2 W_t}{\partial G_{i2} \partial G_{c1-1}} = 0$, the first-period welfare function does not influence the second-period sharing rule.
to the welfare from distributing one extra right to the jurisdiction $i$, and that for all $i$. As a consequence, the sharing rule equalizes the marginal welfare across all local jurisdictions.

The design of the sharing rule is crucial because it conditions the behaviour of local jurisdictions. A key issue consists in determining the reaction function of the State, i.e. how it will modify the second-period sharing of rights following an increase in the first-period deficit of a local jurisdiction $i$. If the State feels pressure to increase the jurisdiction $i$’s deficit rights in period 2, it will create a soft budget constraint. Conversely, a hard budget constraint policy will be implemented if it resists temptation to reallocate rights *ex post*. From the differentiation of the first-order conditions (1) and (2) with respect to $d_{i1}$, $d_{i2}$ $\forall i$ and $d_{c2}$, we derive the State’s budgetary response$^{17}$:

**Proposition 1:** The State cannot credibly commit on a sharing rule of deficit rights: it always finds it optimal to deviate *ex post* from the originally stated second-period distribution of rights following an increase in jurisdiction $i$’s first-period deficit. Formally,

$$
\frac{d(d_{i2})}{d(d_{i1})} = \frac{(\frac{1}{\gamma})^{n-2} \left( \frac{1}{\gamma} - \sum_{k \neq i} X_k \right)}{\text{det } A} > 0,
$$

$$
\frac{d(d_{c2})}{d(d_{i1})} = \frac{(\frac{1}{\gamma})^{n-2} \left( -\frac{1}{\gamma} \right)}{\text{det } A} < 0,
$$

$$
\frac{d(d_{k2})}{d(d_{i1})} = \frac{(\frac{1}{\gamma})^{n-2} X_k}{\text{det } A} < 0,
$$

with

$$
X_i = C_{i2} - \frac{\delta^2 W_2}{\delta C_{i2}^2} < 0
$$

$^{17}$Note that

$$
\sum_{k=1}^{n} \left[ \sum_{i=1}^{n} \frac{d(d_{i2})}{d(d_{k1})} + \frac{d(d_{c2})}{d(d_{k1})} \right] = 0.
$$
\[
\det A = \left( \frac{1}{r} \right)^{n-1} \left( \frac{1}{r} - \sum_{i=1}^{n} X_i \right) > 0.
\]

Proof: Appendix 1

The State is then unable to commit dynamically. While it finds it optimal \textit{ex ante} to deny additional rights in order to impose budgetary discipline, it always finds it optimal to reallocate rights \textit{ex post} when a local jurisdiction increases its deficit in the previous period. Additional deficit rights distributed to local jurisdiction \( i \) are shifted onto the other jurisdictions and the State, constituting a negative externality. In the following section, we analyse the impact of the soft budget constraint on the local market for tradable deficit permits.

4 The local market for tradable deficit permits

4.1 Local jurisdictions’ behaviour

The market is supposed to be perfectly competitive. Each local jurisdiction behaves as a price taker. The Stackelberg leader’s position gives local decision makers a strategic advantage over the State: they choose their first-period deficit expecting the second-period reallocation of rights. Each local jurisdiction \( i \) aims at minimizing its cost of being constrained and the expenditure in permits over the two periods\(^{18}\), discounted at the rate \( e \):

\[
\begin{align*}
\min_{d_{i1}, d_{i2}} & \quad \sum_{t=1}^{2} \frac{1}{(1 + e)^{t-1}} \left[ C_i(t) (\delta_{it}) + p_t (d_{it} - \overline{d}_{it}) \right] \\
\text{s.t.} & \\
\delta_{i1} &= (1 - p_1) d_{i1} + p_1 \overline{d}_{i1} - r D_{i1} \\
\delta_{i2} &= (1 - p_2) d_{i2} + p_2 \overline{d}_{i2} - r (D_{i1} + d_{i1})
\end{align*}
\]

\(^{18}\)The local decision-maker does not take the consequences of its budgetary decisions for the future periods \((t > 2)\) into account. A variant would consist in adding a final-value term which captures the value of damages for all time periods after \( t = 2 \).
The solution \( d_i(\hat{p}_1, \hat{p}_2, \hat{d}_{i1}, \frac{d(\hat{d}_{i2})}{d(\hat{d}_{i1})}) \) is implicitly defined by the first-order condition:

\[
- (1 - p_1) C'_{i1} (\delta_{i1}) + \frac{r}{1 + e} C'_{i2} (\delta_{i2}) + \frac{p_2}{1 + e} (-C'_{i2} (\delta_{i2}) + 1) \frac{d(\hat{d}_{i2})}{d(\hat{d}_{i1})} = p_1. \tag{3}
\]

Compared with the following condition derived by Casella (1999) in a static case

\[
- (1 - p_1) C'_{i1} (\delta_{i1}) = p_1,
\]

two additional effects are at work. The first \( \left( \frac{r}{1 + e} C'_{i2} (\delta_{i2}) \right) \) is due to the dynamic of the model. The increase in the first-period deficit weighs the debt interest payment down which, in turn, raises the second period marginal abatement cost. The second effect \( \left( \frac{p_2}{1 + e} (-C'_{i2} (\delta_{i2}) + 1) \right) \) comes from the inability of the State to commit dynamically to an internal sharing of deficit rights. The expectation of additional rights in the second period - which lower both the cost of being constrained and the expenditure in permits - encourages the local jurisdiction to run more deficits. A direct implication of the soft budget constraint is that too much deficit is undertaken relative to the efficient level.

On the contrary, the condition which implicitly defines the second-period deficit \( d_{i2}(\hat{p}_2, \hat{d}_{i2}, \hat{d}_{i1}) \):

\[
- (1 - p_2) C'_{i2} (\delta_{i2}) = p_2, \tag{4}
\]

turns out to be the one derived in the static case with a hard budget constraint. This result is not surprising given the temporal horizon. In the second period, the decision maker does not take into account either the budgetary position he leaves its successor or the future sharing of rights. The second-period deficit allocation thus equalizes the marginal cost of fiscal retrenchment across all jurisdictions:

\[
-C'_{i2} (\delta_{i2}) = \frac{p_2}{1 - p_2} = -C'_{j2} (\delta_{j2}) \quad \forall i, j, \tag{5}
\]
which is a standard result in the environmental economic literature.

On the other hand, this condition is altered in the first period by the soft budget constraint, as stated by Proposition 2 and its corollary.

**Proposition 2:** The sharing of fiscal retrenchment efforts among the local jurisdictions in the first period is conditioned by the expectation of the second-period rights reallocation. The standard condition of marginal abatement costs equalization changes in a non-standard condition

\[-C'_{i1} + \frac{p_2}{(1+e)(1-p_1)(1-p_2)} \frac{d(\overline{d}_{i2})}{d(d_{i1})} = -C'_{j1} + \frac{p_2}{(1+e)(1-p_1)(1-p_2)} \frac{d(\overline{d}_{j2})}{d(d_{j1})} \quad \forall i, j. \quad (6)\]

Proof: Appendix 2

**Corollary of Proposition 2:** Under a hard budget constraint policy, the condition of marginal abatement costs equalization is satisfied again. Under a soft budget constraint policy, marginal abatement costs coincide for all local jurisdictions if, and only if, preferences for the local public good are identical

\[\frac{\delta^2 W_2}{\delta G_{i2}^2} = \frac{\delta^2 W_2}{\delta G_{j2}^2} \quad \forall i, j\]

which results in

\[\frac{d(\overline{d}_{i2})}{d(d_{i1})} = \frac{d(\overline{d}_{j2})}{d(d_{j1})} \quad \forall i, j.\]

To sum up, equilibrium market conditions are affected by the inability of the State to commit dynamically. By differentiating (3) and (4), we calculate respectively the effects of changes in $p_1$, $p_2$, $\overline{d}_{i1}$ and $\frac{d(\overline{d}_{i2})}{d(d_{i1})}$ on the local jurisdiction $i$’s first-period cost-minimizing level of deficit $d_{i1}(p_1, p_2, \overline{d}_{i1}, \frac{d(\overline{d}_{i2})}{d(d_{i1})})$, as well as the impact of $p_2$, $\overline{d}_{i2}$ and $d_{i1}$ on $d_{i2}(p_2, \overline{d}_{i2}, d_{i1})$. The main results are stated below.

**Result 1:** A rise in the market price always induces a deficit reduction for a net seller, i.e. a local jurisdiction with $\overline{d}_{it} > d_{it}$, but may increase the deficit of a net buyer, i.e. a local jurisdiction with $\overline{d}_{it} < d_{it}$, in the same period. Formally,
\[
\frac{d (d_{it})}{dp_t} = \frac{C'_{it} - (1 - p_t) C''_{it}(\bar{d}_{it} - d_{it}) - 1}{(1 - p_t) C''_{it}}.
\]

Proof: Given \( C'_{it} \leq 0 \) and \( C''_{it} \geq 0 \), \( \frac{d(d_{it})}{dp_t} < 0 \) for \( \bar{d}_{it} > d_{it} \). On the contrary, \( \frac{d(d_{it})}{dp_t} > 0 \) for \\
\(- (1 - p_t) C''_{it}(\bar{d}_{it} - d_{it}) > -C''_{it} + 1.\)

Contrary to the standard result which states that the use of the common-pool resource decreases with the market price, the growing cost of buying permits may increase the deficit of badly endowed jurisdictions. For the following, let us assume that the impact of the market price on the aggregated deficit demand is negative:

**Assumption 1:**

\[
\sum_{i=1}^{n} \frac{d (d_{it})}{dp_t} = \frac{\sum_{i=1}^{n} (C'_{it} - 1) - (1 - p_t) \sum_{i=1}^{n} C''_{it}(\bar{d}_{it} - d_{it})}{(1 - p_t) \sum_{i=1}^{n} C''_{it}} < 0
\]

**Result 2:** The first-period fiscal retrenchment decreases with the softness of the budget constraint. Formally,

\[
\frac{d (d_{i1})}{dp_2} = \frac{1}{(1 + e) (1 - p_2)^2 C'_{i1}} > 0.
\]

The level of the first-period deficit depends on the credibility of the second-period sharing rule. The expectation of additional rights results in an opportunistic behaviour which leads to lower budgetary efforts. These perverse incentives increase with the \( p_2 \) value of one additional right but decrease with the \( p_1 \) cost of one extra unit of deficit in the first period.

**Result 3:** Under a hard budget constraint policy, an increase in \( p_2 \) always deters the local jurisdiction from running deficit in the first period. On the contrary, the soft budget constraint phenomenon encourages the budgetary indiscipline of the local jurisdiction, following a rise in \( p_2 \), for \( \frac{d(\bar{d}_{i2})}{d(d_{i1})} > r \). Formally,

\[
\frac{d (d_{i1})}{dp_2} = \frac{\left[ \frac{d(\bar{d}_{i2})}{d(d_{i1})} - r \right]}{(1 + e) (1 - p_1)^2 (1 - p_2)^2 C'_{i1}}
\]
Proof: for \( \frac{d(d_{ij})}{d(d_{1i})} = 0 \), \( \frac{d(d_{1i})}{dp_2} = \frac{-r}{(1+e)(1-p_1)^2(1-p_2)^2C_{1i}} < 0 \) and for \( \frac{d(d_{ij})}{d(d_{1i})} > r \), \( \frac{d(d_{1i})}{dp_2} > 0 \).

For \( \frac{d(d_{ij})}{d(d_{1i})} > r \) (resp. \( \frac{d(d_{ij})}{d(d_{1i})} < r \)), the expected gain in terms of the valuation of additional rights is higher (resp. lower) than the increasing cost of the debt following a rise in \( p_2 \) which encourages (resp. discourages) the local policy-maker to borrow.

Finally, and obviously, the more rights the local jurisdiction owns, the lower its deficit

\[ \frac{d(d_{it})}{d(d_{it})} = \frac{-p_t}{1-p_t} < 0. \]

With a plethora of rights, the market price is equal to zero so that the distribution of rights does not affect the budgetary behaviour.

### 4.2 The market price at the equilibrium

For a given initial sharing of rights, the first-period equilibrium price \( p_1^m \) on the tradable deficit permits market is implicitly defined by the following market clearing condition:

\[ \sum_{i=1}^{n} d_{i1} \left( p_1^m, p_2, \bar{d}_{i1}, \frac{d(d_{i2})}{d(d_{i1})} \right) = \sum_{i=1}^{n} \bar{d}_{i1} \quad (7) \]

and similarly, the second-period equilibrium price \( p_2^m \) is implicitly defined by:

\[ \sum_{i=1}^{n} d_{i2} \left( p_2^m, \bar{d}_{i2}, d_{i1} \right) = \sum_{i=1}^{n} \bar{d}_{i2}, \quad (8) \]

under the assumption that the unconstrained demand is higher than the total rights:

\[ \sum_{i=1}^{n} d_{1i}^*> \sum_{i=1}^{n} \bar{d}_{i1} \quad \text{and} \quad \sum_{i=1}^{n} d_{2i}^*> \sum_{i=1}^{n} \bar{d}_{i2}. \]

The case where the equilibrium price is zero is ignored as it is neither realistic nor interesting.

From the first-order conditions ((3) and (4)) and the market clearing conditions ((7) and (8)), we derive

\[ p_1^m = p_1^m \left( p_2, \sum_{i=1}^{n} \bar{d}_{i1}, \frac{d(d_{i2})}{d(d_{i1})} \right) \quad (9) \]
where \( \frac{d(d_2)}{d(d_1)} = \left( \frac{d(d_{12})}{d(d_{11})}, \ldots, \frac{d(d_{n2})}{d(d_{n1})} \right) \), and

\[
p_2^m = p_2^m \left( \sum_{i=1}^{n} \frac{\bar{d}_{i1}}{\bar{d}_{i2}}, \sum_{i=1}^{n} \bar{d}_{i2} \right).
\] (10)

**Result 4:** Under Assumption 1, the soft budget constraint exerts an upwards pressure on the first-period market price. Formally\(^{19}\),

\[
\frac{dp_1^m}{d\frac{d(d_2)}{d(d_1)}} = \frac{-\frac{\partial d_{1i}(.)}{\partial \frac{d(d_2)}{d(d_1)}}}{\sum_{i=1}^{n} \frac{\partial d_{1i}(.)}{\partial p_1}} > 0.
\]

Intuitively, the expectation of additional rights in the second period increases the first-period demand for deficit permits which, in turn, exerts pressure on the market resulting in a higher first-period price. However, the price increase will tend to counteract attempts to encourage deficit, at least for net seller jurisdictions from Result 1.

**Result 5:** As expected, under Assumption 1, the market price is a decreasing function of the same period local deficit rights ceiling\(^{20}\):

\[
\frac{dp_i^m}{d\sum_{i=1}^{n} \bar{d}_{it}} = \frac{-\frac{\partial \sum_{i=1}^{n} d_{it}(.)}{\partial \sum_{i=1}^{n} \bar{d}_{it}} + 1}{\sum_{i=1}^{n} \frac{\partial d_{it}(.)}{\partial p_t}} < 0
\]

**Result 6:** Given Assumption 1, the first-period market price always decreases with the rise of the second-period market price under a hard budget constraint policy\(^{21}\). The soft budget constraint reverses this negative intertemporal relation between the market prices for \( \frac{d(d_2)}{d(d_1)} \) high enough \( \forall i \). Formally,

\[
\frac{dp_1^m}{dp_2} = \frac{-\frac{\partial d_{1i}(.)}{\partial p_2}}{\sum_{i=1}^{n} \frac{\partial d_{1i}(.)}{\partial p_1}}.
\]

\(^{19}\)The sign of this expression follows from Assumption 1 and Result 2.

\(^{20}\)The sign of this expression follows from Assumption 1 and \( \frac{\partial d_{it}(.)}{\partial d_{it}} < 0 \).

\(^{21}\)The sign of this expression follows from Assumption 1 and Result 3.
Proof: for \( \frac{d\bar{d}_{i2}}{d(d_{i1})} = 0 \) \( \forall i \), \( \sum_{i=1}^{n} \frac{d(d_{i1})}{dp_{2}} = \sum_{i=1}^{n} \frac{(1+e)(1-p_{1})^{2}(1-p_{2})^{2}C_{i1}^{n}}{C_{i1}} < 0 \) which leads to \( \frac{dp_{m}^{i}}{dp_{2}} < 0 \) under Assumption 1. For \( \sum_{i=1}^{n} \frac{d(d_{i2})}{dp_{2}} > 0 \), which is always the case for \( \frac{d\bar{d}_{i2}}{d(d_{i1})} > r \) \( \forall i \), \( \frac{dp_{m}^{i}}{dp_{2}} > 0 \) under Assumption 1.

Under a hard budget constraint policy, the intertemporal relation between the market prices is always negative. The expected increase in the second-period cost of being constrained reduces the first-period demand for permits and thus the first-period market price. On the contrary, the increasing valuation of one extra right in the second period pushes the local jurisdiction to run more deficits in the first period under a soft budget constraint policy which induces a positive relation between the market prices for \( \frac{d\bar{d}_{i2}}{d(d_{i1})} \) high enough for all \( i \).

### 4.3 Impact of the soft budget constraint at the equilibrium

The impact of local jurisdiction \( i \)'s soft budget constraint on its cost minimizing level of unabated deficit \( d_{i1}^{m} \left( p_{1}^{m}, p_{2}^{m}, \bar{d}_{i1}, \frac{d(d_{i2})}{d(d_{i1})} \right) \) as well as on local jurisdiction \( j \)'s cost minimizing level of unabated deficit \( d_{j1}^{m} \left( p_{1}^{m}, p_{2}^{m}, \bar{d}_{j1}, \frac{d(d_{i2})}{d(d_{j1})} \right) \), when evaluated at the equilibrium prices, are obtained by differentiating these two expressions \( d_{i1}^{m}(.) \) and \( d_{j1}^{m}(.) \) with respect to \( \frac{d\bar{d}_{i2}}{d(d_{i1})} \):

\[
\frac{\partial d_{i1}^{m}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} = \frac{\partial d_{i1}(.)}{\partial \frac{d(d_{i2})}{d(d_{i1})}} \frac{\partial \bar{d}_{i2}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} + \frac{\partial d_{i1}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \frac{\partial \bar{d}_{i2}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} + \frac{\partial d_{i1}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \frac{\partial \bar{d}_{i2}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \frac{\partial \sum_{i=1}^{n} \bar{d}_{i2}}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} + \frac{\partial d_{i1}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \frac{\partial \bar{d}_{i2}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \frac{\partial \sum_{i=1}^{n} \bar{d}_{i2}}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}},
\]

\[
\frac{\partial d_{j1}^{m}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} = \frac{\partial d_{j1}(.)}{\partial \frac{d(d_{i2})}{d(d_{i1})}} \frac{\partial \bar{d}_{i2}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} + \frac{\partial d_{j1}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \frac{\partial \bar{d}_{i2}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} + \frac{\partial d_{j1}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \frac{\partial \bar{d}_{i2}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \frac{\partial \sum_{i=1}^{n} \bar{d}_{i2}}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} + \frac{\partial d_{j1}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \frac{\partial \bar{d}_{i2}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \frac{\partial \sum_{i=1}^{n} \bar{d}_{i2}}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}},
\]

A change in \( \frac{d\bar{d}_{i2}}{d(d_{i1})} \) directly influences \( d_{i1}^{m}(.) - \frac{\partial d_{i1}(.)}{\partial \frac{d\bar{d}_{i2}}{d(d_{i1})}} \) being always positive from Result 2 - while it affects both \( d_{i1}^{m}(.) \) and \( d_{j1}^{m}(.) \) \( \forall j \neq i \) in an indirect manner through changes
in equilibrium prices. The term \( \frac{\partial d_{k1}^m}{\partial p_{m1}^m} \) for \( k = 1, \ldots, n \) represents the indirect effect on jurisdiction \( k \)'s level of unabated deficit that occurs when \( p_{m1}^m \) adjusts to the variation in demand, which is always negative for a net seller, i.e. \( d_{k1} > d_{k1} \), but may be positive for a net buyer, i.e. \( d_{k1} < d_{k1} \), from Results 1 and 4. The two other effects go through the reduction in \( p_{m2}^m \) following a higher amount of rights allocated at local level. From Results 1, 3 and 6 it turns out that the impact of \( \frac{d(d_{m2})}{d(d_{m1})} \) on \( d_{m1} \) depends on the initial endowment as well as on the comparison between \( \frac{d(d_{m2})}{d(d_{m1})} \) and \( r \). We do not rule out the case where \( \frac{\partial d_{m1}^m}{\partial d_{m2}} < 0 \) and \( \frac{\partial d_{m1}^m}{\partial d_{m1}} > 0 \). However, the following condition obtained by differentiating (7) must be satisfied:

\[
\frac{\partial d_{m1}^m}{\partial d_{m2}} = -\sum_{k \neq i} \frac{\partial d_{k1}^m}{\partial d_{m1}}.
\]

If a rise in \( \frac{d(d_{m2})}{d(d_{m1})} \) causes an increase in local jurisdiction \( i \)'s equilibrium level of unabated deficit, the equilibrium levels of unabated deficit for all other jurisdictions will decrease in equal magnitude, and conversely.

### 4.4 The inefficiency of the tradable deficit permits market

Suppose that an omniscient and benevolent planner towards the local jurisdictions acts instead of each decision maker on the local market over the two periods. In this joint-cost problem, the local regulator’s problem is to minimize the total costs of being constrained of the \( n \) heterogeneous local jurisdictions, expecting the inability of the State to commit dynamically:

\[
\begin{align*}
\text{min} & \quad \sum_{t=1}^{2} \frac{1}{(1 + e)^{t-1}} \left[ \sum_{i=1}^{n} C_{it} (\delta_{it}) + p_t (\sum_{i=1}^{n} d_{it} - \sum_{i=1}^{n} \bar{d}_{it}) \right] \\
\text{s.t.} & \quad d_{11}, \ldots, d_{n1} \\
& \quad d_{12}, \ldots, d_{n2}
\end{align*}
\]
\[ \delta_{i1} = (1 - p_1) d_{i1} + p_1 \bar{d}_{i1} - r D_{i1} \quad \forall i \]
\[ \delta_{i2} = (1 - p_2) d_{i2} + p_2 \bar{d}_{i2} - r (D_{i1} + d_{i1}) \quad \forall i \]

\[ \frac{d(\bar{d}_{i2})}{d(d_{i1})}, \frac{d(\bar{d}_{k2})}{d(d_{i1})} \quad \forall k \neq i, \forall i. \]

The first-period and second-period budgetary efforts are implicitly defined by the following first-order conditions:

\[ -(1 - p_1) C'_{i1} + \frac{r}{1 + e} C'_{i2} + \frac{p_2}{1 + e} (-C'_{i2} + 1) \frac{d(\bar{d}_{i2})}{d(d_{i1})} + \sum_{k \neq i} \frac{p_2}{1 + e} (-C'_{k2} + 1) \frac{d(\bar{d}_{k2})}{d(d_{i1})} = p_1 \quad \forall i \]
\[ -(1 - p_2) C'_{i2} - p_2 = 0 \quad \forall i. \] (11)

By comparing (11) and (3), it turns out that the optimal level of the first-period deficit from the local planner’s point of view is lower than the level of deficit chosen by the self-interested local jurisdiction \( i \). Without any regulation on the market, the local jurisdiction \( i \) does not take into account the budgetary externality it generates for the other jurisdictions because the fact that the additional deficit rights allocated to one jurisdiction are partially borne by others. The first-period market allocation is thus inefficient from the local planner’s point of view. On the contrary, the second-period market allocation is efficient.

Suppose now that an omniscient and benevolent national planner, implementing a hard budget constraint policy, aims at minimizing the total costs of being constrained of the \( n \) heterogeneous local jurisdictions. It turns out that the optimal allocation equalizes the marginal abatement cost in each period.

\[ \text{Indeed, the additional term } \sum_{k \neq i} \frac{p_2}{1 + e} (-C'_{k2} + 1) \frac{d(\bar{d}_{k2})}{d(d_{i1})} \text{ is negative from Proposition 1.} \]
5 Intertemporal trading of deficit permits

In addition to deficit permits trading between local jurisdictions, we now allow deficit permits trading through time. In such a setting, local jurisdictions may directly reduce deficit as well as buy, sell, bank and borrow deficit permits in order to meet applicable standards or to take advantage of any speculative opportunities. For a deficit $d_{i1} < \bar{d}_{i1}$, the local jurisdiction $i$ generates a permits surplus which may be sold to other jurisdictions, or deposited in a bank account to be used by the local jurisdiction itself, or sold to other jurisdictions in period 2. On the contrary, for a deficit $d_{i1} > \bar{d}_{i1}$, the local jurisdiction $i$ may buy permits from other jurisdictions or borrow permits, but its budget must be balanced at the end of period 2. Let $x_{i1} > 0$ (resp. $< 0$) denote the quantity of deficit permits bought (resp. sold) and $B_{i1} > 0$ (resp. $< 0$) denote the stock (resp. the borrowing) of permits. The intertemporal deficit permits trading thus lowers the cost of compliance with the deficit standards imposed by the State by allowing local jurisdictions to administer their budget more flexibly over the two periods.

At first sight, the intertemporal trading of deficit permits seems to be an additional tool at local jurisdiction’s disposal for manipulating its first-period budgetary choice and obtaining more deficit rights in the second period. By banking deficit permits, it increases its first-period deficit and thus receives additional rights in the second period.

The local jurisdiction $i$ chooses the first-period deficit $d_{i1}$, the second-period deficit $d_{i2}$ as well as the stock of permits $B_{i1}$ minimizing its cost of being constrained and the expenditure in permits over the two periods:

$$\min_{d_{i1}, d_{i2}, B_{i1}} \sum_{t=1}^{2} \frac{1}{(1 + e)^{t-1}} [C_{it}(\delta_{it}) + p_t x_{it}]$$

s.t.
\[
\delta_{i1} = d_{i1} - p_1 x_{i1} - r D_{i1}
\]
\[
\delta_{i2} = d_{i2} - p_2 x_{i2} - r (D_{i1} + d_{i1})
\]
\[
x_{i1} = d_{i1} - \bar{d}_{i1} + B_{i1}
\]
\[
x_{i2} = d_{i2} - \bar{d}_{i2} - B_{i1}.
\]

\[
\frac{d(\bar{d}_{i2})}{d(d_{i1})} = \frac{d(\bar{d}_{k2})}{d(d_{i1})} \quad \forall k \neq i
\]

This problem with intertemporal flexibility yields the following necessary conditions:

\[
-(1 - p_1) C'_{i1} (\delta_{i1}) + \frac{r}{1 + e} C'_{i2} (\delta_{i2}) + \frac{p_2}{1 + e} (-C'_{i2} (\delta_{i2}) + 1) \frac{d(\bar{d}_{i2})}{d(d_{i1})} = p_1 \quad \forall i \quad (13)
\]
\[
-(1 - p_2) C'_{i2} (\delta_{i2}) = p_2 \quad \forall i \quad (14)
\]
\[
p_1 (-C'_{i1} + 1) = p_2 \left(\frac{-C'_{i2} + 1}{1 + e}\right) \quad \forall i. \quad (15)
\]

The local jurisdiction \(i\)'s stock of deficit permits \(B_{i1} (p_1, p_2, d_{i1}, d_{i2}, \bar{d}_{i1}, \bar{d}_{i2})\), given by the condition (15), equalizes the marginal costs of the two periods. To some extent, the local jurisdiction smoothes the cost of budgetary austerity over the two periods.

**Proposition 3**: The intertemporal trading of deficit permits restores the efficiency of the market for tradable deficit permits

Proof: directly from (14) and (15).

By allowing local jurisdictions to trade permits through time, the State thus restores the standard condition of marginal abatement costs equalization in the first period. The inefficiency related to the soft budget constraint phenomenon vanishes.
6 Conclusion

The way rights are allocated and traded on the market is decisive for the cost-effectiveness of the system. The inability of the State to commit dynamically to a sharing rule of deficit rights generates perverse incentives. Local jurisdictions are encouraged to run more deficits in the first period in order to receive more rights in the second period, which in turn exerts an upwards pressure on the first-period market price. The market turns out to be inefficient - with heterogeneous jurisdictions - unless the State allows local decision makers to trade permits through time. Indeed, the intertemporal trading of deficit permits restores the marginal abatement costs equalization condition due to the fact that local jurisdictions aim at smoothing their cost of being constrained over the two periods.
References


7 Appendix

7.A Proof of Proposition 1

The differentiation of the first-order conditions (1) and (2) with respect to \( d_{i1}, d_{i2} \ \forall i \), \( \bar{d}_{c2} \) leads to:

\[
d(d_{i1}) = \frac{1}{r} d(\bar{d}_{i2}) + \frac{C''_{i2} - \delta^2 W^2}{\delta G_{i2}^2} d(\bar{d}_{c2}) \quad \forall i
\]

and

\[
\sum_{i=1}^{n} d(\bar{d}_{i2}) + d(\bar{d}_{c2}) = d(\bar{d}_{2}) \quad \text{with} \quad d(\bar{d}_{2}) = 0.
\]

To simplify notation, in the following, we denote

\[
X_i = \frac{C''_{i2} - \delta^2 W^2}{\delta G_{i2}^2} < 0.
\]

From the matrix system

\[
\begin{pmatrix}
\frac{1}{r} & 0 & 0 & 0 & X_1 \\
0 & \frac{1}{r} & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & 0 & \frac{1}{r} & 0 & X_i \\
0 & 0 & 0 & \frac{1}{r} & X_n \\
1 & 1 & 1 & 1 & 1 \\
\end{pmatrix} \begin{pmatrix}
d(\bar{d}_{i1}) \\
d(\bar{d}_{i2}) \\
\vdots \\
d(\bar{d}_{n1}) \\
d(\bar{d}_{c2}) \\
\end{pmatrix} = \begin{pmatrix}
d(d_{i1}) \\
d(d_{i1}) \\
\vdots \\
d(d_{i1}) \\
0 \\
\end{pmatrix}
\]

we compute the reaction function with the Cramer’s rule. Q.E.D.

7.B Proof of Proposition 2

By substituting \(-C''_{i2}(\delta_{i2})\) by the expression given by the first-order condition (4) in the first-order condition (3), the \( n \) conditions (3) boil down to:

\[
\frac{p_1}{(1-p_1)} + \frac{r}{1+e(1-p_1)(1-p_2)} \frac{p_2}{(1-p_1)} \frac{d(\bar{d}_{i2})}{(1-p_1)} = -(1-p_1) C'_{i1}(\delta_{i1}) + \frac{1}{1+e(1-p_1)} \frac{p_2}{(1-p_1)(1-p_2)} \frac{d(\bar{d}_{j1})}{(1-p_1)} \forall i, j.
\]

Q.E.D.