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Public and private environmental spending. A political economy approach

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Public and private environmental spending. A political economy approach*

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Abstract

This paper studies the determination of public investment in environmental quality when there are private alternatives. Public investment is chosen by majority voting. When consumption and environmental quality are complementary one may observe a solution of the type ”ends against the middle.”

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1 Introduction

There are various ways of dealing with environmental quality and with policies, public or private, aimed at maintaining it. In this paper we look at a problem such as the quality of water that deteriorates at a constant rate and can be preserved collectively or privately. We adopt a dynamic model of successive generations. Bequest motive is the source of savings. At each generation, the quality of environment can be enhanced by a public investment which is particularly efficient because of the public good characteristic of the environment. This public investment is financed by a flat rate income tax. Individuals differ in their labor productivity and in their initial endowment. Hence, the public environmental policy can be viewed as progressive. Each individual can also invest privately in environmental quality; this investment does not have any externality. To pursue with the example of water quality, one can think of collective purification plant and of individual water filters. Given the progressivity of the environmental policy, well-to-do households would prefer individual over collective techniques even though the latter is much more efficient than the former. Henceforth, if the tax rates and thus the level of public investment are chosen by votes, one can expect that the most preferred tax rate decreases with income and indeed reaches zero for some above average income.

This conjecture is not necessarily verified for low incomes. We assume that an individual's utility depends on consumption, environmental quality and the utility of his children. If consumption and environment are strong complements, a poor individual can vote against a too high tax rate that would lead him to a too low consumption level. Indeed we show that in case of complementarity between consumption and environmental quality, we can have a coalition of low and high income individuals opposing middle income individuals in the determination of the tax rate. This is what Epple and Romano (1996) call "ends against the middle" and it is a case where the median voter theorem does not apply. To get this particular result, one needs complementarity between consumption and the publicly provided private good and the possibility of supplementing public provision by private purchases. We have thus two key features of our paper, the possibility of private contribution and the complementary between consumption and environment. There is little work on the issue of voting for environmental quality. Note a recent paper by Kempf and Rossignol (2003) who show that public spending tends to be larger in societies with less inequality.

There is also a paper by Aidt (1998) who analyzes environmental policy in a common agency model of politics. Competition between lobby groups keeps the economy away from the efficiency Pigouvian rule. Also McAusland (2003) looks at the issue of voting for pollution policy within an open economy setting. She shows that poorer voters may be the greener voters within the electorate for reasons close to those developed in our paper. Even though richer voters are in favor of higher provision of environmental quality they may be unwilling to pay proportionality more. Finally, Jouvet *et al.* (2000) discuss the issue of environmental quality in a dynamic setting with altruism but without any political economy feature.

The remainder of this paper is organized as follows. Next section presents the model. Section 3 focuses on the steady-state solution; it gives the optimality conditions and the *laissez-faire* solution. Section 4 gives the voting equilibrium. A final section concludes.

2 The Model

At each period of time N altruistic agents live and work for one period. There are I type of individuals. Each agent of type i is characterized by her/his labor productivity α_i where $i = 1, ..., I$. In the economy there is a proportion p_i of agents of type i and we assume:

$$\sum_{i=1}^{I} p_i \alpha_i = 1 \tag{1}$$

2.1 Consumers

In period t, each agent of type i can improve her/his own environmental quality in a private way by an environmental expense e_{it}. The environmental quality for an agent is given by :

$$q_{it} = e_{it} + \Psi_t \tag{2}$$

where Ψ_t represents the contribution of public investment to the individual's environmental quality. We assume additivity of e and Ψ for the sake of simplicity.

Agent i 's budget constraint is given by:

$$c_{it} = (1 - \tau_t)\alpha_i w_t + R_t x_{it-1} - e_{it} - x_{it} \tag{3}$$

where c_{it} is consumption, w_t is wage per unit of efficient labor, R_t is the return to capital investment. This agent receives a bequest x_{it-1} and gives x_{it} to her/his child (constant population implies that each parent has only on child). τ_t is the environmental tax. Bequest are constraint to be non negative, $x_{it} \geq 0$.

Any agent of type i born in period t derive utility from consumption c_{it} environmental quality q_{it} and his/her child's utility. Agent's preferences for c and q are supposed to be homothetic and represented by a utility function $U(c_{it}, q_{it})$. $U(.)$ is an increasing, strictly concave, twice differentiable function and verifies Inada conditions. Then, the marginal rate of substitution is defined by:

$$\frac{U_c(c,q)}{U_q(c,q)} = h(\frac{q}{c}) \tag{4}$$

where $h(.)$ is an increasing bijection on $\Re_{++}$.

At period t, the welfare of an agent of type i, V_{it}, is defined by:

$$V_{it} = U(c_{it}, q_{it}) + \gamma V_{it+1} \tag{5}$$

with γ the degree of altruism, $\gamma \in (0,1)$. Agent i solves the following problem:

$$\begin{cases} \max_{c_{it},q_{it},x_{it},e_{it}} & \sum_{t=0}^{\infty} \gamma^t U(c_{it}, q_{it}) \\ \text{s.c.} & c_{it} = (1-\tau_t)\alpha_i w_t + R_t x_{it-1} - e_{it} - x_{it}, \\ & q_{it} = e_{it} + \Psi_t, \\ & x_{it} \geq 0, e_{it} \geq 0. \end{cases}$$

2.2 Government

The public side of environmental quality Ψ_t depends on total public spending, E_t: $\Psi_t = \Psi(E_t)$. Function Ψ is a concave and twice differentiable: $\Psi' > 0$, $\Psi'' < 0$ with $\Psi(0) = 0$, $\Psi'(0) = +\infty$ and $\Psi'(+\infty) = 0$. The specification of Ψ is crucial. The return to private spending is constant and equal to one.

Whatever the number of agents, the marginal return of public spending decreases from $+\infty$ to 0. Low (or not to large) E is more efficient than private spending, but for enough large E, it is the opposite. These properties imply that on pure efficiency grounds (identical individuals) public investment should prevail up to the point where its marginal return (Ψ') equals unity (the return of the private technology).
Because of its public good nature public investment offers another advantage: it dominates private investment up to the point where its marginal return times N ($N\Psi'$) equals 1.
As an illustration take $\Psi(E) = AE^{\varepsilon}$ where $\varepsilon < 1$ and $A > 0$ is a scale factor. In the identical individuals case, environmental quality is given by

$$q = e + AE^{\varepsilon}$$

subject to $e + E/N$ being a constant. It is thus clear that $e = 0$ as long as

$$E \leq (\varepsilon NA)^{\frac{1}{1-\varepsilon}}$$

In the following we assume that in the first-best case wherein individuals are made identical the optimal E denoted E^* is below that upper bound. In the second best with heterogenous individuals this condition does not hold anymore.

The environmental tax, τ_t, is used to finance E, public environmental spending. The government's revenue constraint is given by

$$E_t = \sum_{i=1}^{I} p_i N \alpha_i \tau_t w_t = \tau_t N w_t \tag{6}$$

2.3 Firms behavior

At each period t, competitive firms produce an homogeneous good Y_t with capital K_t and labor L_t. We assume a well-behaved production function (increasing, concave and homogeneous of degree one),

$$Y_t = F(K_t, L_t)$$

Assuming a total depreciation of capital after one period[1], a representative firm, in period t, maximizes its profits π_t,

$$\pi_t = F(K_t, L_t) - w_t L_t - R_t K_t$$

With perfect competition, price w_t and R_t are given and factors prices are equal to there marginal productivities,

$$F_L(K_t, L_t) = w_t \tag{7}$$

$$F_K(K_t, L_t) = R_t \tag{8}$$

2.4 Equilibrium for a given policy

The equilibrium conditions for an agent i are given by the following first order conditions:

$$-U_c(c_{it}, q_{it}) + U_q(c_{it}, q_{it}) \leq 0; = 0 \quad if \quad e_{it} > 0 \tag{9}$$

[1]Or equivalently that $F(K, L)$ includes capital after depreciation

and

$$-U_c(c_{it}, q_{it}) + \gamma R_{t+1} U_c(c_{it+1}, q_{it+1}) \leq 0; = 0 \quad if \quad x_{it} > 0. \tag{10}$$

The intertemporal equilibrium is defined, for a given sequence of government decisions τ_t, by a sequence of prices w_t and R_t, and individual variables satisfying all the equilibrium conditions. The government decisions satisfies its budget constraint (6. Consumers decisions maximize their utility 5 which yields (9) and (10). Firms decision implies (7) and (8).

The capital stock is equal to the sum of bequests,

$$K_{t+1} = \sum_i p_i N x_{it} = N\overline{x}_t \tag{11}$$

where $\overline{x}_t$ is the mean of bequests. The return to bequest is defined by the marginal productivity of capital (8). The markets of labor and good are clear, respectively,

$$L_t = \sum_i p_i \alpha_i N = N \tag{12}$$

and

$$Y_t = N\overline{c}_t + N\overline{e}_t + E_t + N\overline{x}_t \tag{13}$$

where $\overline{c}_t = \sum_i p_i c_{it}$, $\overline{e}_t = \sum_i p_i e_{it}$.

The initial conditions $x_{i,-1} \geq 0$ for all $i = 1, ..., I$ are given with $x_{i,-1}$ such that $\sum_i p_i N x_{i,-1} = K_0 > 0$

3 Equilibrium and optimum in the steady-state

With a constant tax rate $\tau_t = \tau$, the steady state satisfies $E = \tau N w$, $L_t = N$, $K_{t+1} = K$, $k = K/N = \sum_i p_i x_i = \overline{x}$, $e_{it} = e_i$, $x_{it} = x_i$, $c_{it} = c_i = (1-\tau)\alpha_i w + (R-1)x_i - e_i$ and $q_{it} = q_i = e_i + \Psi(E)$. A positive stock of capital implies that at least one bequest x_i is strictly positive and therefore with relation (10), we obtain the modified golden rule,

$$F_K(k,1) = \frac{1}{\gamma} \tag{14}$$

with $k = \widehat{k}$ being the stationary stock of capital. Then, at the steady-state equilibrium $R = \widehat{R} = 1/\gamma$, and $w = \widehat{w} = F_L(\widehat{k}, 1)$ and type's i's individuals have a life-cycle income:

$$\omega_i = (1-\tau)\alpha_i \widehat{w} + (\widehat{R} - 1)x_i \tag{15}$$

In the long run wealth distribution depends on $x_{i,-1}$ and on the dynamics. When there is no constraint on bequest each altruistic agent has the same behavior as an infinite lived agent faced to the following intertemporal budget constraint,

$$\sum_{t=0}^{\infty} \rho_t (c_{it} + e_{it}) = x_{i,-1} + \sum_{t=0}^{\infty} \alpha_i \rho_t w_t (1-\tau) \equiv \Omega_i \tag{16}$$

where $\rho_t = \rho_{t-1}/R_t$, with $\rho_0 = 1$, are the discount factors. Then, the long run net wealth distribution depends on the distribution of Ω_i and on the $x_{i,-1}$. When the ranking of $x_{i,-1}$ is the same as α_i ranking, the distribution of net wealth is the same as the distribution of labor productivities α_i. In order to simplify our study, we assume that bequests, at the stationary equilibrium, proportional to the labor productivities, *i.e.* $x_i = \alpha_i \overline{x} = \alpha_i \widehat{k}$.

3.1 *Laissez-faire* ($E = 0$, $\tau = 0$)

Each agent i maximizes his/her utility subject to his/her budget constraint with $\tau = 0$, i.e. $maxU(c_i, e_i)$ subject to $c_i + e_i = \omega_i = \alpha_i \widehat{w} + (\widehat{R} - 1)\alpha_i \widehat{k}$. Then the relation (9) implies

$$U_c(c_i, e_i) = U_q(c_i, e_i) \tag{17}$$

which is equivalent to $e_i/c_i = h^{-1}(1) \equiv \mu$. Therefore, with the assumption of homothetic preferences we obtain that private environmental spending is proportional to consumption, $e_i = \mu c_i$ and to the net income, ω_i, (and also to α_i).

3.2 Social Optimum and the decentralization

In a centralized economy after redistribution all agent have the same consumption, $c_i = c$ and $e_i = e$. The environmental quality is defined by $q = e + \psi(E)$. The central planer maximizes $U(c, e + \psi(E))$ with respect to c, e and E subject to $c + e + E/N = f(\widehat{k}) - \widehat{k}$, $c \geq 0$, $e \geq 0$ and $E \geq 0$.

The solution with $e^* = 0$ satisfies

$$U_c(c^*, \psi(E^*)) = N\psi'(E^*)U_q(c^*, \psi(E^*)) \tag{18}$$

The condition for $e^* = 0$ is equivalent to $\psi'(E^*) \geq 1/N$, at steady state the productivity of public spending is large than one of private.

Contrasting (17) and (18) is interesting. In a *laissez-faire* setting, given our assumption on preferences, each individual contributes to the quality of his/her environment. In the first-best optimum, we expect that $E^* > 0$ and $e^* = 0$. This mainly depends on $\Psi(E)$ and on N. But this is just an assumption. Nothing precludes Ψ to be so inefficient and N to be so low that even in the first-best, private contribution would be the best device to maintain environmental quality.

We now move to a positive setting and try to see what would be the private contribution for a given value of τ and thus of $E = \tau N \widehat{w}$.

3.3 Private environmental contribution for a given public policy

- **Case** $e_i > 0$.

If $e_i > 0$ then the relation (9) implies

$$U_c(c_i, q_i) = U_q(c_i, q_i) \tag{19}$$

which is equivalent to $q_i/c_i = h^{-1}(1) \equiv \mu$. The budget constraint is

$$c_i + e_i = (1-\tau)\alpha_i \widehat{w} + (\widehat{R} - 1)\alpha_i \widehat{k} \equiv \omega_i \tag{20}$$

and with $\mu c_i = q_i = e_i + \Psi(E)$ we obtain

$$e_i(1 + \frac{1}{\mu}) = \omega_i - \frac{\Psi(E)}{\mu} \tag{21}$$

Thus $e_i > 0$ if and only if $\omega_i > \Psi(E)/\mu$. This condition is equivalent to $h(\Psi(E)/\omega_i) < h(\mu) = 1$ and thus $e_i > 0$ if and only if $U_c(\omega_i, \Psi(E)) <$

$U_q(\omega_i, \Psi(E))$. An agent chooses a positive environmental contribution, $e_i > 0$, if the consumption of her/his net income ω_i induce a lower consumption marginal utility than the environmental quality financed by the government.

- **Case** $e_i = 0$

If $e_i = 0$, then $U_c(c_i, q_i) \geq U_q(c_i, q_i)$ with $q_i = \Psi(E)$ and $c_i = \omega_i$. The condition for $e_i = 0$ is $U_c(\omega_i, \Psi(E)) \geq U_q(\omega_i, \Psi(E))$.

The two cases can be resumed in the following way,

$$e_i = \max\{0, \mu c_i - \Psi(E)\} \tag{22}$$

and the corresponding stationary welfare $V_i(E)$ satisfies

$$V_i(E) = \sum_0^{\infty} \gamma^t U(c_i, q_i) = \frac{1}{1-\gamma} U(c_i, q_i) \tag{23}$$

We now turn to the central section of this paper: the determination of τ or E through majority voting. We first have to characterize the individual's indirect utility with E or τ as argument.

4 Voting Equilibrium

We show that the welfare function of an agent of type i, $V_i(E)$ is single peaked and we study the variations of his/her preferred public spending level with respect to his/her productivity parameter α_i. After that we analyze the result of the vote.

4.1 Study of the welfare function

We introduce two life-cycle utility functions for the constrained case and for the unconstrained one.

Given by relation (15) and $\tau\widehat{w} = E/N$, we write the income net of bequest of agent of type i as the following function of E,

$$\omega_i = (1-\tau)\alpha_i\widehat{w} + (\widehat{R}-1)\alpha_i\widehat{k} = \alpha_i\widehat{\omega} - \alpha_i E/N \equiv \omega_i(E) \tag{24}$$

where $\widehat{\omega} = \widehat{w} + (\widehat{R}-1)\widehat{k}$. We denote

$$U_i^0(E) = U(\omega_i(E), \Psi(E)) \tag{25}$$

as the life-cycle utility when consumption is equal to $\omega_i(E)$ and thus his/her environmental quality is $\Psi(E)$.

We show in the appendix that the strictly concave function $U_i^0(E)$ reaches its maximum at some point E_i^0 in$(0, N\widehat{\omega})$.

The "unconstrained" life-cycle utility is defined by choosing e_i, positive or negative, which maximizes

$$U(\omega_i(E) - e_i, \Psi(E) + e_i) \tag{26}$$

This maximum is reached when the partial derivatives U_c' and U_q' are equal and this is equivalent to $q_i = \Psi(E) + e_i = \mu c_i = \mu(\omega_i(E) - e_i)$. Thus the maximum of (26) is

$$U_i^1(E) = U(c_i^1(E), \mu c_i^1(E)) \tag{27}$$

where $c_i^1(E) = \frac{1}{1+\mu}(\omega_i(E) + \Psi(E))$. The strictly concave function $U_i^1(E)$ reaches its maximum at E_i^1, the solution of $\Psi'(E_i^1) = \alpha_i/N$.

¿From relation (21), the constraint $e_i \geq 0$ is binding if and only if $E \geq \overline{E}_i$, where $\overline{E}_i$ is the solution of $\Psi(\overline{E}_i) - \mu\omega_i(\overline{E}_i) = 0$. Thus we have

$$V_i(E) = \begin{cases} \frac{1}{1-\gamma}U_i^0\,(E) & \text{if} \quad E_i \geq \overline{E}_i \\ \frac{1}{1-\gamma}U_i^1\,(E) & \text{if} \quad E_i \leq \overline{E}_i \end{cases}$$

We show in the appendix

Proposition 1 *The function $V_i(E)$ is single peaked and it reaches its maximum either at E_i^1 if $E_i^1 \leq \overline{E}_i$ or at E_i^0 if $E_i^1 > \overline{E}_i$. In the later case, E_i^0 belongs the the interval $(\overline{E}_i, E_i^1)$.*

It may help the intuition to represent the problem at hand graphically. Figure 1a and 1b present the indirect utilities U_i^0 and U_i^1 for the two cases. For $E_i < \bar{E}_i$, U_i^0 prevails and for $E_i > \bar{E}_i$, U_i^1 prevails. The relevant indirect utility is given by the thick single-peaked curve.

Insert Figure 1a and Figure 1b

¿From these two figures we obtained the most preferred value of E for an individual of type α_i.

4.2 Variations of the preferred public spending level

• For an agent of type i, the unconstrained preferred public spending $E_i^1 = E^1(\alpha_i)$ is the solution of $\Psi'(E_i^1) = \alpha_i/N$. This level is feasible (with $e_i \geq 0$) if and only if $E^1(\alpha_i) \leq \overline{E}_i$, where $\overline{E}_i$ is the solution of $\Psi(\overline{E}_i) - \mu\omega_i(\overline{E}_i) = 0$.

These conditions:

$$E^1(\alpha_i) \leq \overline{E}_i$$

and

$$\frac{\overline{E}_i}{N} + \frac{\Psi(\overline{E}_i)}{\mu\alpha_i} = \widehat{\omega}$$

are equivalent to

$$g(\alpha_i) = \frac{E^1(\alpha_i)}{N} + \frac{\Psi(E^1(\alpha_i))}{\mu\alpha_i} \leq \widehat{\omega} \tag{28}$$

Since $E^1(\alpha) = \Psi'^{-1}(\alpha/N)$ is decreasing function of α, $g(\alpha)$ is decreasing and the agents of type i reach their unconstrained preferred level $E^1(\alpha_i)$ if and only if $\alpha_i \geq \widehat{\alpha}$, where $\widehat{\alpha}$ is the solution of $g(\widehat{\alpha}) = \widehat{\omega}$. For these agents, $E^1(\alpha_i)$ is a decreasing function of the productivity parameter α_i.

• If an agent of type i is constrained, *i.e.* $\alpha_i < \widehat{\alpha}$, his or her preferred level of public spending is $E_i^0 = E^0(\alpha_i)$ which is the solution of equation (A2) in the appendix:

$$\phi(E_i^0, \alpha_i) \equiv \Psi'(E_i^0) - \frac{\alpha_i}{N} h(\frac{\Psi(E_i^0)}{\alpha_i(\widehat{\omega} - E_i^0/N)}) = 0$$

The function $\phi(E_i^0, \alpha_i)$ is decreasing with respect to E_i^0 (since Ψ' is decreasing and h is increasing) and its derivative with respect α_i is equal to

$$\frac{\partial\phi(E_i^0, \alpha_i)}{\partial\alpha_i} = \frac{1}{N}(z_i^0 h'(z_i^0) - h(z_i^0)) \tag{29}$$

where $z_i^0 = \frac{\Psi(E_i^0)}{\alpha_i(\widehat{\omega} - E_i^0/N)}$ is the ratio of environmental quality to consumption. We have shown the following:

Proposition 2 *If $\alpha_i \geq \widehat{\alpha}$, the preferred public spending level of the agents of type i is $E^1(\alpha_i)$ which is a decreasing function of the productivity parameter α_i. For $\alpha_i \leq \widehat{\alpha}$, the preferred public spending level of the agents of type i is $E^0(\alpha_i)$ and the derivative of $E^0(\alpha_i)$ has the same sign as the elasticity of h at z_i^0 minus 1, where $z_i^0 = \frac{\Psi(E^0(\alpha_i))}{\alpha_i(\widehat{\omega} - E^0(\alpha_i)/N)}$.*

4.3 The political equilibrium

Only if the preferred public spending level is a monotonic function of the productivity the median voter theorem applies. This is the case when the elasticity of the function h is smaller or equal to 1: the two functions $E^0(\alpha)$ and $E^1(\alpha)$ are non-increasing.

Proposition 3 *If the elasticity of h is smaller or equal to 1, then the preferred public spending level (and the corresponding tax rate) is non-increasing with respect to the productivity parameter. Thus the political equilibrium is the level preferred by the median voter (see figure 2a).*

When the elasticity of h is not smaller or equal to 1, the analysis of the voter is a more complex. In order to obtain explicit results we consider a constant elasticity of substitution (CES) utility function,

$$U(c, q) = \frac{1}{1 - 1/\sigma}(c^{1-1/\sigma} + \beta q^{1-1/\sigma}) \tag{30}$$

where σ is the elasticity of substitution and β the environmental preferences parameter, $\beta > 0$. For this function, we have

$$h(\frac{q}{c}) = \frac{c^{-1/\sigma}}{\beta q^{-1/\sigma}} = \frac{1}{\beta}(\frac{q}{c})^{1/\sigma} \tag{31}$$

The function h has the constant elasticity $1/\sigma$. Then, if $\sigma > 1$, the preferred public spending level is decreasing with α_i. But $E^0(\alpha_i)$ is increasing if $\sigma < 1$.

Therefore, if we now consider a vote on the environmental tax τ we distinguish the following possibilities:

- If $\sigma > 1$, the preferred public spending is a decreasing function of α_i. Then, given the single peakedness of preferences, the median voter theorem applies. The individual of type α_m (median productivity) is therefore decisive.

- If $\sigma = 1$, E_i^0 is constant, the median voter theorem applies as well and under the assumption $\alpha_m \leq \widehat{\alpha}$ there is a majority vote in favor of E_i^0.

- If $\sigma < 1$, E_i^0 is an increasing function of α_i. Then, the median voter theorem does not apply and we have to use the Epple-Romano approach. That is, the voting equilibrium involves the worker with middle productivity individuals voting against a coalition of the lower productivity and the higher productivity individuals.

We know that if $\alpha_i > \hat{\alpha}$, the desired level of environmental quality decreases with α_i. For $\alpha_i < \hat{\alpha}$, any profile can be observed. Using a CES utility we know that it decreases also with α_i if $\sigma > 1$ and then the Condorcet winner is the median productivity α_m worker. If $\sigma < 1$, we have the "ends against the middle" solution. On Figure 2b there is a coalition of workers with $\alpha_i < \alpha_0$ and $\alpha_i > \alpha_1$ against workers with $\alpha_0 < \alpha_i < \alpha_1$.

Insert Figure 2a and Figure 2b

The intuition is very close to that of Epple-Romano (1996) or Casamatta *et al.* (2000). With complementarity low ability workers are not going to vote for a too high tax as their consumption is closely related to the net of tax wage: $\alpha w (1-\tau)$. With substituability between c and q, they instead vote for high rate of taxation realizing that they get a lot $\Psi(E)$ for paying little $\tau\alpha w_i$.

In these figures for the sake of intuition, we assume that the median wage $w_m < \bar{w} = 1$ which is standard and that $\hat{\alpha} > 1$, which is less standard and implies that only the richer workers privately contribute to the quality of their environment. Comparing Figures 2a and 2b we also see that the level of public investment tends to be lower in the case when "the end meet the middle".

We now look at the amount of private contribution that result from majority voting. We know that $\bar{E}_i$, the value of public spending that makes workers of type i indifferent between contributing and not contributing, increases with α_i. Let us denote $\tilde{\alpha}$ the productivity for which $\bar{E}_i$ and the majority choice of E are equal. As Figures 3a and 3b indicate all individuals with productivity above $\tilde{\alpha}$ will contribute to environmental quality. We note that $\bar{E}$ intersect the curve with the most preferred E at $\tilde{\alpha}$. Not surprisingly there will be more private contribution where "ends meet the middle" than when the median voter is decisive.

Insert Figure 3a and Figure 3b

5 Conclusion

In this paper we have considered the case where environmental quality can be maintained by either public investment or private contribution. Public investment is financed by a flat rate tax which implies that workers with income below average benefit from it. There is another reason why one could prefer public investment, namely the technology. Given the public good nature of environmental quality, public investment is more efficient than private one. This question is dealt within a growth model of successive generations where the motive for saving is parental altruism toward children. As a consequence, the modified golden rule is achieved in the long run and this drives some of the results. We are interested by the political economy choice of public investment by workers of different productivity. We show that preferences are single peaked in that in some case one readily applies the median voter theorem and in other cases one has to use the so-called "the ends meet the middle" approach.

One of the avenues of further research is to introduce the idea of bequeathing not only financial or human capital, but also environmental quality.

We would introduce a lag in the way public investment affect the environmental quality whereas there would not be any lag for private protection. Taking the example of water, public infrastructure investment takes time whereas domestic purification devices have an instantaneous effect. These differential lags make the analytic more difficult. Note that we could also allow for a choice between private contribution to public investment and going through the political process. Suppose that parents have the choice of helping their children by contributing s_i to a public investment or to invest time and money in a political process such as the one described here. There would be an interesting arbitrage between the efficiency loss linked to the "tragedy of the commons" and the loss associated with a redistributive political process.

Another extension we are thinking of is to link environmental deterioration to production which would take us away from the very convenient modified golden rule.

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Appendix

- The maximum of $U_i^0(E) = U(\omega_i(E), \Psi(E))$

The function $U_i^0(E)$ is defined and strictly concave on the interval $(0, N\widehat{\omega})$, since $U(c,q)$ is increasing and strictly concave, and $\omega_i(E) = \alpha_i(\widehat{\omega} - \overline{E}/N)$ and $\Psi(E)$ are concave. Its derivative

$$\frac{dU_i^0(E)}{dE} = -\frac{\alpha_i}{N}U_c' + \Psi'(E)U_q' \qquad \text{(A1)}$$

tends to $+\infty$ (resp. $-\infty$) when E tends to 0 (resp. to $N\widehat{\omega}$). Thus $U_i^0(E)$ reaches its maximum at E_i^0 where its derivative is equal to 0. Using $U_c'/U_q' = h(q/c)$ we obtain

$$\Psi'(E_i^0) - \frac{\alpha_i}{N}h(\frac{\Psi(E_i^0)}{\alpha_i\widehat{\omega} - \alpha_i E_i^0/N}) = 0 \qquad \text{(A2)}$$

- Proof of Proposition 1

a) The function $U_i^0(E)$ is decreasing for E such that $E \geq \overline{E}_i$ and $E \geq E_i^1$. Consider $E \geq \overline{E}_i$. Then the constraint $e_i \geq 0$ is binding and at $c_i = \omega_i(E)$ and $q_i = \Psi(E)$, we have $U_c' \geq U_q'$ and

$$\frac{dU_i^0(E)}{dE} \leq (-\frac{\alpha_i}{N} + \Psi'(E))U_q' \qquad \text{(A3)}$$

The LHS is negative if $\Psi'(E) < \alpha_i/N$, i.e. if $E > E_i^1$. Thus $U_i^0(E)$ is decreasing for E such that $E \geq \overline{E}_i$ and $E \geq E_i^1$.

b) If $E_i^1 \leq \overline{E}_i$, the maximum of $U_i^1(E)$ is reach at E_i^1 with $e_i \geq 0$ and

$U_i^0(E)$ is decreasing for $E \geq \overline{E}_i$. Thus the maximum of $V_i(E)$ is reached at E_i^1 and $V_i(E)$ is single peaked.

c) If $E_i^1 > \overline{E}_i$, then $U_i^1(E)$ is increasing for $E \leq \overline{E}_i$, and for $E \geq E_i^1$ $U_i^0(E)$ is decreasing. Thus the maximum of $V_i(E)$ is reached at E_i^0 which belongs to the interval $(\overline{E}_i, E_i^1)$ and $V_i(E)$ is single peaked.

Figure 1a: The case $E_i^1 < \overline{E}_i$ ($e_i > 0$)

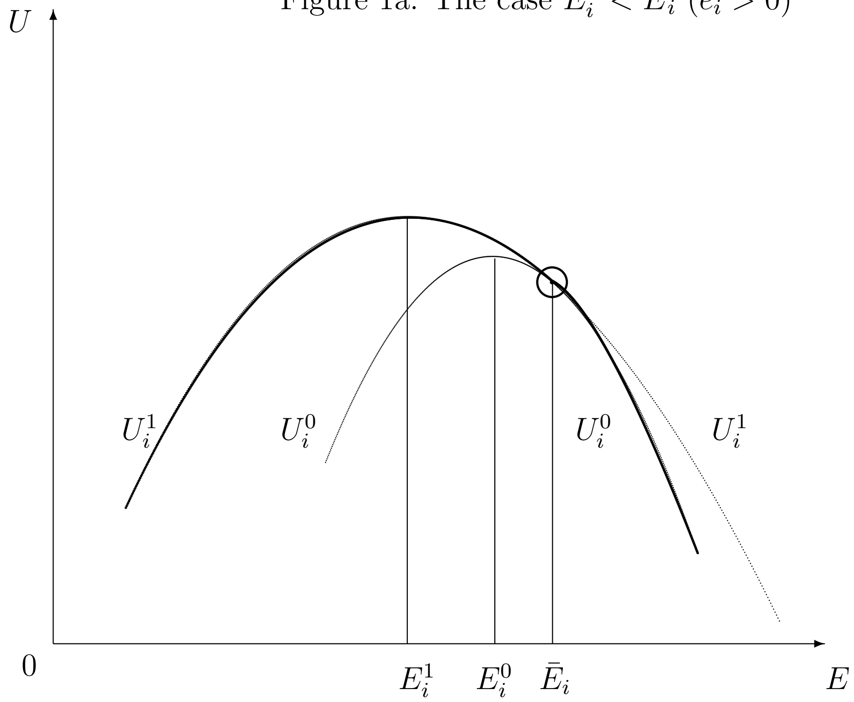

Figure 1b: The case $E_i^1 > \bar{E}_i$ ($e_i = 0$)

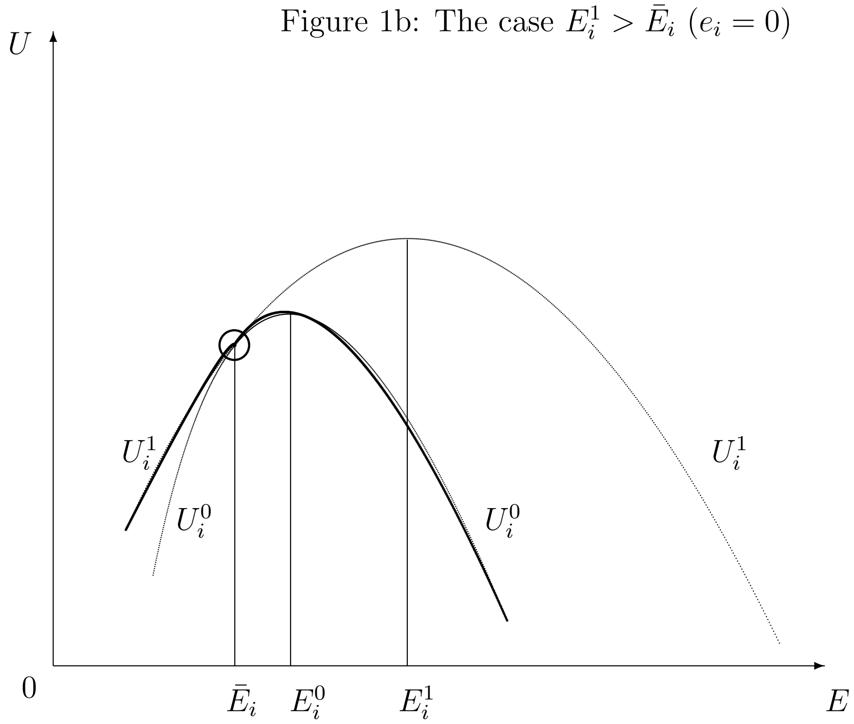

Figure 2a

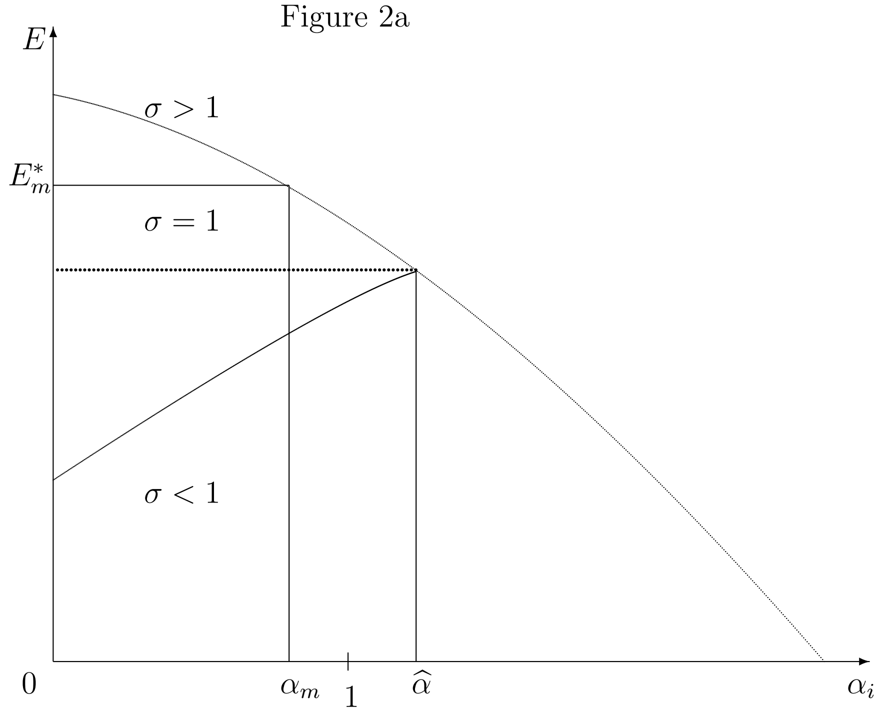

Figure 2b

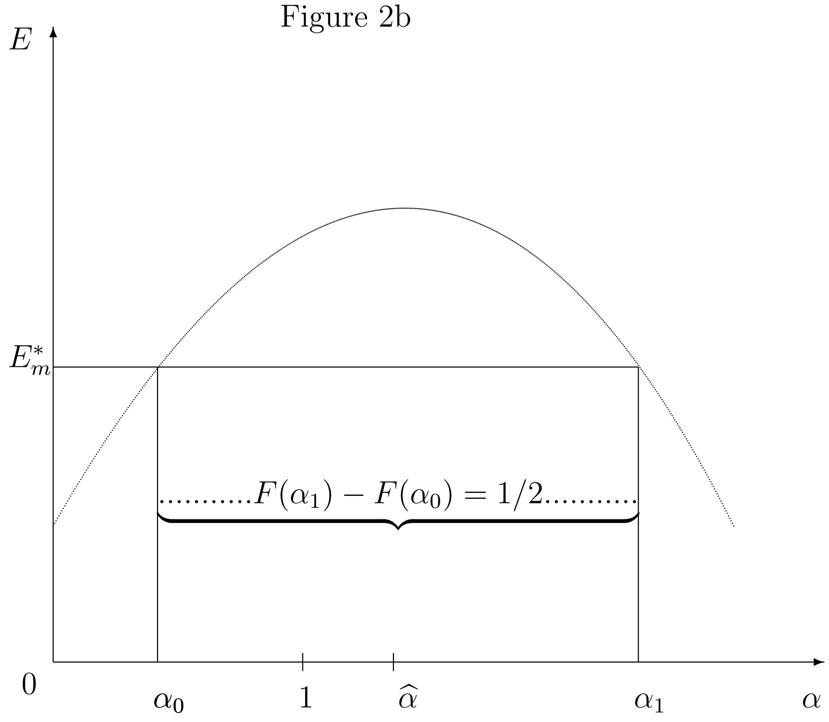

Figure 3a

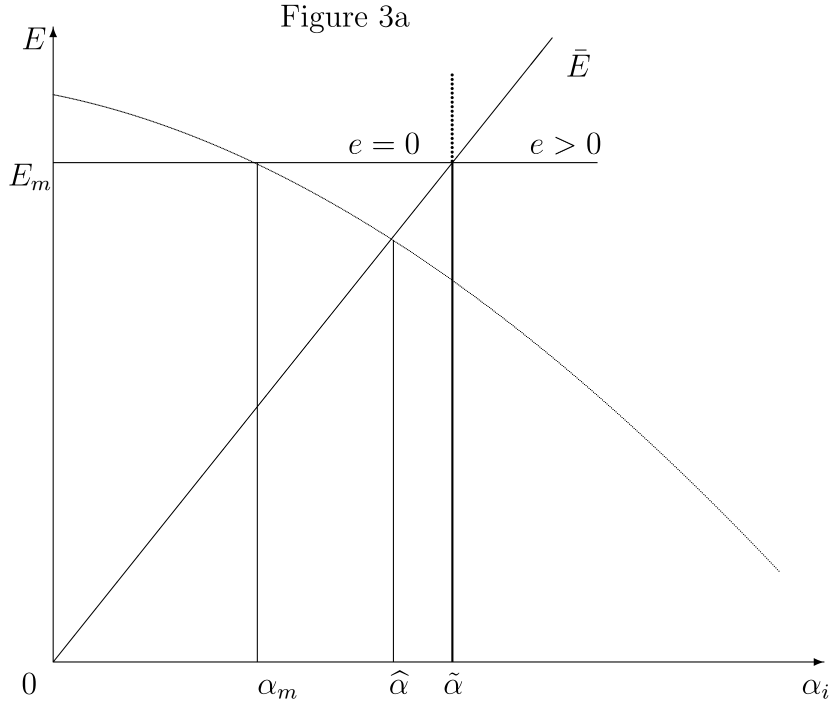

Figure 3b