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A Reappraisal of the Allocation Puzzle through the Portfolio Approach

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A Reappraisal of the Allocation Puzzle through the Portfolio Approach

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Abstract

The neoclassical growth model predicts that emerging countries with higher TFP growth should receive larger capital inflows. Gourinchas and Jeanne (2007) document that, in fact, countries that exhibited higher productivity catch-up received *less* capital inflows, even though they invested more in their domestic technology. This is the allocation puzzle. I show that introducing investment risk in the same neoclassical framework qualifies the predictions in terms of capital flows: countries with higher TFP growth invest more in their own production but they have to hold external bonds for precautionary savings motives. Contrary to the riskless approach, the portfolio approach predicts accurately the allocation of capital flows across developing countries.

Key Words: Growth accounting, Capital flows, Investment risk, Financial globalization, Portfolio choice.

JEL Class.: F21, F43, G11, O16.

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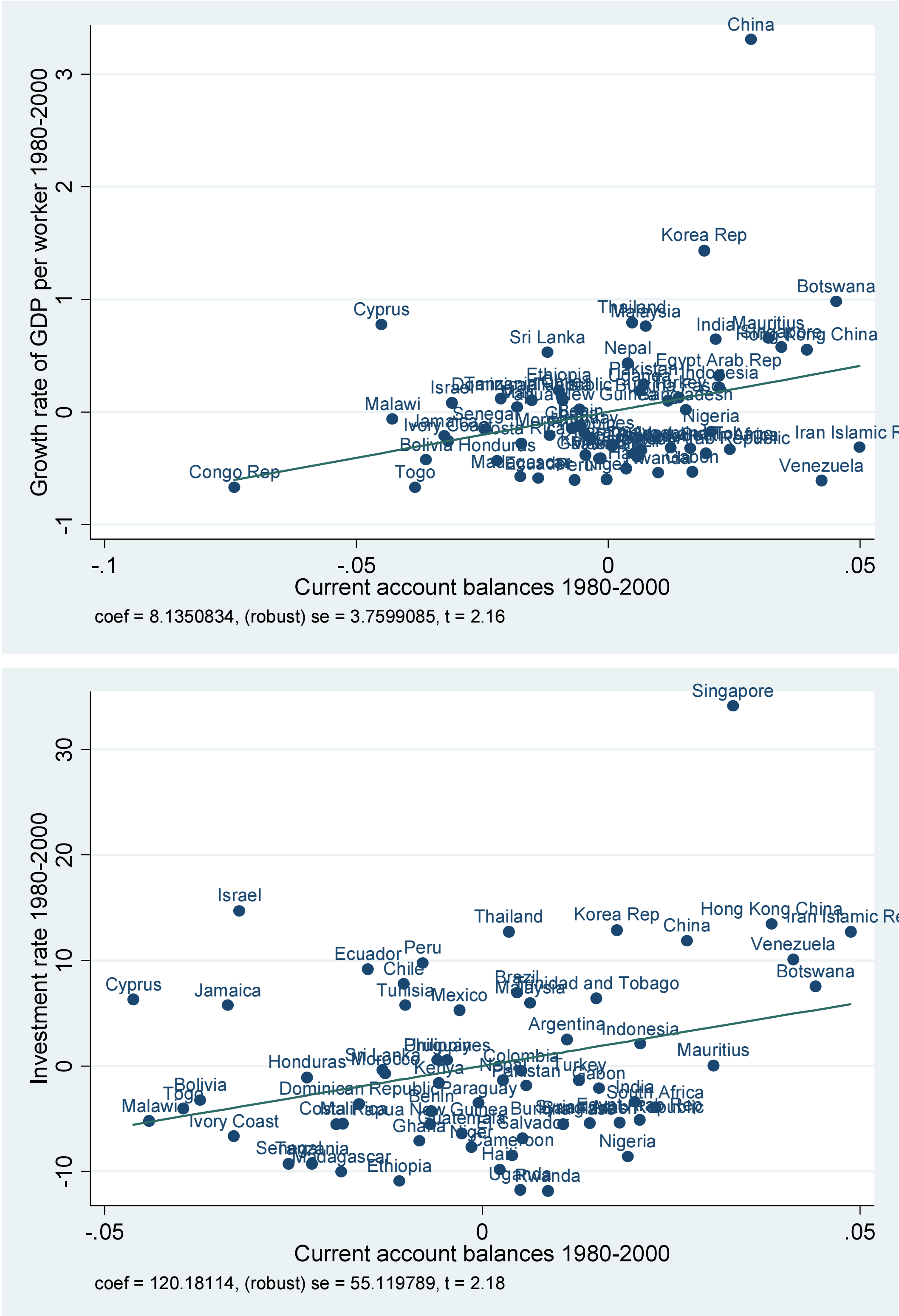
1 Introduction

The neoclassical growth model (Ramsey-Cass-Koopmans) predicts that a country whose marginal return on capital is above the world's interest rate and that opens to international bond markets increases its investment level through international borrowing. More precisely, when the return on domestic capital is higher than the cost of borrowing, it is optimal to borrow from the rest of the world to finance domestic investment. Under decreasing marginal returns, this takes place until the marginal return on capital equals the world's interest rate. The higher the initial discrepancy between both returns, the more the country invests and the more it has to borrow. This should generate a positive cross-country correlation between investment and capital inflows.

Two main elements can account for the difference between a country's marginal return on capital and the international interest rate: capital-scarcity and total factor productivity (TFP) gains. If the level of capital is low when financial markets open, then its marginal return is high relatively to the world's interest rate. Similarly, starting from equal domestic and foreign returns, an increase in TFP pushes the former above the latter. In both cases, both investment and foreign borrowing increase. Hall and Jones (1999) and Caselli (2004) show that TFP remains the main source of cross-country differences in income. Therefore, according to the textbook growth model, countries with higher productivity growth should attract more capital.

This prediction has recently been challenged by Gourinchas and Jeanne (2007). Using a calibrated neoclassical growth model in the spirit of the development accounting literature on a sample of 69 non-OECD economies between 1980 and 2000, they show that not only the model fails to predict the correct amount of capital inflows, but the predicted flows are negatively correlated with the actual ones. They call this paradox the "allocation puzzle". This puzzle comes from the fact that productivity growth is negatively correlated to capital inflows. Put differently, the more productive countries receive less capital from abroad. According to the model, countries with higher productivity growth should (i) invest more in their technology in order to keep up with productivity growth, and (ii) borrow from the rest of the world to finance their investment. Gourinchas and Jeanne (2007) show that these countries do invest more, but instead of borrowing more, they lend more. This puzzle is summarized by Figure 1, which presents the cross-country correlation between the growth rate of GDP per worker and the average current account balances during 1980-2000 on the one hand (upper graph), and between the average investment rate and the average current account balances during the same period on the other hand (lower graph). The figure shows that capital outflows are positively related to both growth

Figure 1: Growth of GDP per worker and investment rate against current account balances, 1980-2000



Source: Penn World Tables 6.2 (Heston et al., 2006), CHELEM database.

and investment[1]. Explaining the puzzle thus necessitates to account for this positive correlation between investment and capital outflows.

In this paper, I introduce investment risk in the neoclassical model used by Gourinchas and Jeanne (2007) to explain the positive correlation between investment and capital outflows. Two cases are then considered: the particular case where the level of risk is zero and the case where it is strictly greater than zero. The first case corresponds to the "riskless" approach and is similar to Gourinchas and Jeanne (2007), while the second case corresponds to a portfolio choice approach. In the riskless approach, private capital and bonds are perfect substitutes: if the marginal productivity of private capital is higher than the world interest rate on bonds, then it is optimal to borrow from the rest of the world in order to invest in private capital. In the portfolio approach, the composition of the portfolio matters. In particular, in the constant relative risk aversion (CRRA) case, bonds and private capital are constant shares of the portfolio. Intuitively, when the level of risk is high, one part of the portfolio (riskless bonds) is used to self-insure against the riskiness of the other part (risky capital). In this case, a more productive domestic capital makes a country more willing to invest in private capital, but in order to invest more it has to hold a higher amount of bonds. In the long term, this is possible because a higher productivity makes the country richer. It is therefore possible to exhibit a negative correlation between productivity growth and capital inflows (i.e. a positive correlation between productivity growth and capital outflows).

The two approaches are developed and calibrated on the same sample as Gourinchas and Jeanne (2007). When using the riskless approach, the same negative correlation between predicted and observed flows as in Gourinchas and Jeanne (2007) is found. As expected, the allocation puzzle is recovered. When relying on the portfolio approach, a positive correlation between predicted and observed flows is found. Therefore, the portfolio approach outperforms the riskless one in terms of capital flows allocation. Two main facts contribute to this result: (i) countries with higher TFP growth tend to experience smaller capital inflows; (ii) countries with larger capital shares in their portfolio at the beginning of period also experience smaller capital inflows. Fact (i) is at the core of the puzzle when using the riskless approach while it is consistent with the portfolio approach, according to the intuition developed above. Fact (ii) makes sense only within the portfolio approach and also contributes to solve the puzzle. This is because, contrasting with the riskless approach, the share of capital and safe assets in the

[1]The significant positive correlations are robust to the exclusion of China in the upper panel and Singapore in the lower panel. The resulting t-statistics are respectively 1.98 and 1.83.

portfolio at steady state is determined and unique across countries. Convergence towards the steady state then implies a smaller rise in the capital share in countries where capital is already a large part of the portfolio. Therefore, the level of bonds should decrease less in those countries. These two facts contribute strongly to the success of the portfolio approach in reproducing the right direction of flows. However, in assessing the magnitude of flows, the portfolio approach fares worse than the riskless one. Capital inflows to developing countries are over-estimated by several order of magnitude. Some extensions are developed in order to diminish this discrepancy.

The rest of the paper is structured as follows: Section 2 examines the robustness of the puzzle regarding the existing literature; Section 3 presents the model with the two approaches and their properties; Section 4 calibrates the model and confronts the predicted capital flows according to both approaches to the data; Section 5 provides some extensions.

2 Related literature

The allocation puzzle can be linked to the strand of literature that studied the positive link between growth and savings. This link has been documented at the country level by empirical studies such as Carroll and Weil (1994) and more recently Attanasio et al. (2000). Three main types of models can explain this link. It is legitimate to examine each of them in order to determine whether they could be extended to account for the growth-current account relationship. First, the neoclassical growth model itself: in closed economies, a higher saving rate can account for more growth through a higher level of capital. However, empirical studies conclude rather in favor of a growth-to-saving causality than the opposite. In the neoclassical model, a growth-to-saving channel exists, but it is counterfactual since growth affects savings negatively through the expectations of higher future income, i.e. the human wealth effect. Besides, this approach cannot be applied to the allocation puzzle since the impact of savings on growth does not survive the opening of financial markets.

Second, overlapping-generations growth models provide a potential theoretical channel for growth-to-saving causality. Modigliani's life-cycle hypothesis (Modigliani, 1986) implies that fast-growing countries have a higher aggregate saving ratio because young households, who are in the saving phase of their cycle, are richer than the old, which are in the dissaving phase. However, in Modigliani's approach, faster growth means only greater differences in income between young and old consumers, while it also implies higher expectations of future income at the individual level. Carroll and Summers (1991) show that the former effect, the aggregation effect, is outweighed by the latter, the human wealth effect: even if the life-cycle hypothesis is taken into account, the correlation between growth and savings should be negative.

Third, Carroll et al. (2000) stress the role of habit formation in the positive link between growth and savings. In the presence of habit, the elasticity of intertemporal substitution is higher, so the response of savings is more sensitive to a change in the interest rate. However, this can lead to a positive correlation between growth and savings only in a closed economy, where the interest rate depends on the productivity of capital. In an open economy, the interest channel, through which the growth rate affects savings, is muted, since the interest rate is fixed internationally. Therefore, explanations of the growth-saving relationship either have internal flaws, or rely on mechanisms that are specific to closed economies.

Some recent contributions on global imbalances have highlighted the role of financial development in shaping capital flows to developing countries. They are also potential candidates to account for the allocation puzzle. Dooley et al. (2005*b*), Mendoza et al. (2007), Matsuyama (2007) and Ju and Wei (2006, 2007) explain how low financial development in the South, through production risk, credit constraints or a poor financial intermediation system can lead to "uphill flows", that is, positive lending to the North. These approaches can be related to the Lucas puzzle. However, the allocation puzzle cannot be reduced to the Lucas paradox, since the latter is about the magnitude and not the direction of flows. The Lucas puzzle points to the fact that the capital flows that would enable the marginal productivity of capital to equalize across countries do not take place. It can be explained by the presence of an unobserved capital wedge, that depends on the country's institutions, and that would explain why the observed marginal productivity, measured by the capital to labor ratio, does not adjust: the ex-post (after-tax), unobserved private returns in fact do adjust. Even if this wedge is taken into account, which is the case in Gourinchas and Jeanne (2007), capital should still flow where the level of investment is higher, that is in countries where the wedge-adjusted productivity is higher. The contributions on global imbalances, which explain uphill flows by the presence of a wedge between the marginal return to capital and the rate of return, fail to explain the allocation puzzle: in order to be consistent with the latter, the wedge should be positively correlated with the growth rate, which is not the case in general.

Some other studies on global imbalances show a concern for growth. Caballero et al. (2008) build an intergenerational model where low financial development, that is the inability of the economy to store value, increases the demand for foreign assets. As a consequence, high growth economies can still export capital if they cannot generate enough assets. This model provides a convincing explanation for the joint phenomenon of the US deficit and Asian savings glut. But growth still has a negative impact on the long-term current account and external position because it increases the domestic supply of assets. In Aghion et al. (2006), domestic savings constitute a collateral and thus favors foreign investment, which has positive externalities on growth. But the consequences in terms of correlation between growth and the current account, that is savings minus investment, are unclear. Besides, they take the savings rate as exogenous, whereas empirical evidence suggests that they cause one another (Carroll and Weil, 1994; Attanasio et al., 2000).

Last but not least, the allocation puzzle can be related to the literature on export-led growth. Indeed, the current account balance can be viewed as exports minus imports. Rodrik (2006) stresses the role of export-oriented policies in promoting growth. Dooley et al. (2004), Dooley et al. (2005*a*) and Rodrik (2007) study how external policy, and in particular an undervalued exchange rate, can stimulate the manufacturing sector through trade. However, though very convincing, these studies do not tell us how growth feeds back on the trade balance. Except in Dooley et al. (2004) and Dooley et al. (2005*a*), where higher net exports in Asia are originated in higher savings, the trade balance is taken as exogenous[2]. Finally, any explanation of the link between growth and capital flows must be finance-based somehow since we must take into account the intertemporal behavior of agents.

3 Amending the neoclassical growth model

In this section, I build on the neoclassical growth model developed by Gourinchas and Jeanne (2007), in which capital flows are determined by their productivity path relative to the world technology frontier. The model features a small open economy and the rest of the world. The latter is unaffected by the small country's dynamics. What is examined specifically here is how investment risk modifies portfolio decisions and in particular capital flows.

Time is continuous, indexed by $t \in [0, \infty)$. There is a continuum of length 1 of infinitely-lived households, or families, indexed by i. Each household is composed of N_t members, and each member is endowed with 1 unit of labor. Labor is supplied inelastically in a competitive labor market. Each household owns a firm which employs labor in the competitive labor market. Households can invest only in the -risky- capital of their own firm or in a riskless bond. All

[2] In Rodrik (2006) and Rodrik (2007), it is not so much the trade balance itself than its composition in terms of exports and imports that matters

uncertainty is idiosyncratic, and hence all aggregates are deterministic.

3.1 Firms and technology

Denote household i's net capital income by dQ_t^i. It evolves according to:

$$dQ_t^i = (1-\tau)[F(K_t^i, A_t N_t^i) - w_t N_t^i]dt - \delta K_t^i dt + \sigma K_t^i dv_t^i \tag{1}$$

where K_t^i is the household's holdings in private capital, A_t the exogenous and deterministic level of productivity, N_t^i the amount of labor the firm hires in the labor market, w_t the wage rate, which is not firm-specific since the labor market is competitive. The parameter τ is a wedge on the gross capital return, that is, before subtracting capital depreciation. This is a deviation from Gourinchas and Jeanne (2007), where the wedge is on capital returns net of depreciation. This specification is chosen only for practical reasons[3] and does not change the results dramatically. As in Gourinchas and Jeanne (2007), this wedge can be interpreted as a tax on capital income, or as the result of other distorsions that would introduce a difference between social and private returns. We assume that this tax on capital return is distributed equally among households. The parameter δ is the depreciation rate. The production function F is assumed to follow a Cobb-Douglas specification: $F(K, AN) = K^\alpha (AN)^{1-\alpha}$, $\alpha \in (0,1)$. The technology is exactly identical to Gourinchas and Jeanne (2007), except that time is continuous and that investment risk is introduced through a standard Wiener process dv_t^i. This process is assumed to be iid across agents and time. It satisfies $E[dv_t^i = 0]$ and $E[(dv_t^i)^2] = 1$ for all i and $E[dv_t^i dv_t^j] = 0$ for all i, j, $i \neq j$. This risk can be interpreted as a production or a depreciation shock that affects the return on capital. It is assumed that this shock is averaged out across households, that is: $\int_0^1 dv_t^i = 0$. The parameter σ measures the amount of individual risk. Gourinchas and Jeanne (2007)'s specification is nested when $\sigma = 0$.

We show now that the capital income is linear in K_t^i. Denote $\tilde{k}_t^i = K_t^i/(A_t N_t^i)$ the capital per efficient unit of labor and $\tilde{y}_t^i = F(K_t^i, A_t N_t^i)/(A_t N_t^i) = f(\tilde{k}_t^i) = \tilde{k}_t^{i\alpha}$ the production per efficient unit of labor. Employment is chosen after the capital stock has been installed and the shock has been observed. Therefore, at each period t, the firm chooses employment in order to maximize $F(K_t^i, A_t N_t^i) - w_t N_t^i$, where w_t is the competitive wage per unit of labor. This yields $w_t = (1-\alpha) A_t (\tilde{k}_t^i)^\alpha$. Because the competitive wage is constant across firms $\tilde{k}_t^i$, the ratio of capital to efficient labor, is also constant across firms. Denote $\tilde{k}_t = \tilde{k}(\tilde{w}_t) = \tilde{w}_t^{1/\alpha}/(1-\alpha)$, where $\tilde{w} = w/A$ is the wage per efficient unit of labor. Using this result, we can write the capital

[3] The resulting amount can be expressed as a fraction of production.

income as follows:

$$dQ_t^i = r_t K_t^i dt + \sigma K_t^i dv_t^i$$

where $r_t = r(\tilde{w}_t, \tau) = (1-\tau)\alpha\tilde{k}(\tilde{w}_t)^{\alpha-1} - \delta$ is the private net return on capital. The net capital income is therefore linear in K_t^i, which makes the analysis tractable when $\sigma > 0$.

The country has an exogenous, deterministic productivity path $\{A_t\}_{t=0,\ldots,\infty}$, which is bounded by the world productivity frontier:

$$A_t \leq A_t^* = A_0^* e^{g^* t}$$

The world productivity frontier is assumed to grow at rate g^*. Following Gourinchas and Jeanne (2007), we define the difference between domestic productivity and the productivity conditional on no technological catch-up as follows:

$$e^{\pi_t} = \frac{A_t}{A_0 e^{g^* t}} \tag{2}$$

We assume that $\pi = lim_{t\to\infty}\pi_t$ is well defined. Therefore, the growth rate of domestic productivity converges to g^*.

3.2 The household's program

The household's preferences follow expected utility. Instantaneous utility is logarithmic. We assume, as Barro and Sala-i Martin (1995), that the representative member of the household is altruistic and maximizes the welfare of his descendants along with his own. He therefore maximizes the following family's welfare function:

$$U_t^i = E_t \int_t^\infty N_s \ln c_s^i e^{-\rho(s-t)} ds \tag{3}$$

where $\rho > 0$ is the discount rate and c_t^i is the individual consumption of the members of household i in period t. The growth rate of population is supposed to be exogenous and equal to n:

$$N_t = N_0 e^{nt}$$

For the utility function to be well defined, we must have $n < \rho$.

We now turn to the household's budget constraint.

Let B_t denote the household's holdings in riskless bond and H_t his human wealth defined as the present discounted value of future labor income and tax product: $H_t = \int_t^\infty e^{-(s-t)R^*}(N_s w_s + Z_s)ds$ where R^* is the international interest rate and $Z_t = \int_0^1 \tau[F(K_t^i, A_t N_t^i) - w_t N_t^i]di = \tau\alpha\tilde{k}(\tilde{w}_t)^{\alpha-1}K_t$ is the tax product, with $K_t = \int_0^1 K_t^i di$. We thus have:

$$dH_t = (R^* H_t - N_t w_t - Z_t)dt \tag{4}$$

Define effective wealth as the sum of financial wealth $B_t^i + K_t^i$, and human wealth:

$$\Omega_t^i = B_t^i + K_t^i + H_t$$

The evolution of the household's financial wealth obeys to:

$$d(B_t^i + K_t^i) = dQ_t^i + [R^* B_t^i + N_t w_t + Z_t - C_t^i]dt \tag{5}$$

It follows from (1), (4) and (5), that the evolution of effective wealth, in per capita terms, can be described by:

$$d\omega_t^i = [r_t k_t^i + R^*(b_t^i + h_t) - c_t^i - n\omega_t^i]dt + \sigma k_t^i dz_t^i \tag{6}$$

where lower case letters, except n, the population's growth rate, stand for per capita (i.e. per family member) values. For each variable X_t^i (X_t), x_t^i (x_t) stands for X_t^i/N_t (X_t/N_t).

A key element is that the return to capital is linear in K_t^i. This translates to the linearity of effective wealth. The household maximizes his utility with respect to (6). When we set $\sigma = 0$, the framework corresponds to that of Gourinchas and Jeanne (2007). Otherwise, the investment rules follow the classical portfolio choice rules with CRRA utility.

This framework is similar to Kraay and Ventura (2000) and Kraay et al. (2005), who, among others, apply the portfolio choice model to an open economy. But the portfolio approach has been applied only in AK contexts, which cannot account for such phenomena as decreasing returns and human wealth effects. Here, we use a transformation of the budget constraint introduced by Angeletos and Panousi (2007) in order to make it linear in wealth and apply the portfolio approach to the neoclassical growth model.

3.3 Household's behavior

The linearity of the evolution of the budget constraint along with the homotheticity of preferences ensures that the household's problem reduces to a homothetic problem à la Samuelson and Merton. It follows that the optimal policy rules are linear in wealth.

Lemma 1: *Define $\phi_t^i = k_t^i/\omega_t^i$, the fraction of effective wealth invested in private capital. For a given interest rate R^* and a given sequence of wages $\{W_t\}$, the policy responses of the household i are given by:*

$$c_t^i = (\rho - n)\omega_t^i \tag{7}$$

$$\phi_t^i = \phi_t = \frac{r_t - R^*}{\sigma^2} \tag{8}$$

Equation (7) shows the familiar result that consumption per capita equals the annualized value of wealth, taking into account population growth. It is a direct consequence of the log utility.

Equation (8) is the portfolio choice rule. It says that the risky share of the portfolio is increasing in the risk premium $r_t - R^*$ and decreasing in the amount of risk σ. When σ is large, the share of risky assets is low, while the share of safe assets is high. The share of safe assets can be viewed as a way for the household to self-insure against the potential bad shocks to the risky part of the portfolio. Even if the return on the safe assets is lower than the yield of private capital on average ($R^* < r_t$), they play the role of buffer-stock savings against uncertainty. Bonds are therefore held not only for their return but also for their insurance function.

Human wealth h_t and bond holdings b_t are both safe assets and are substitutes. Notice that Equation (8) implies that $b_t^i = (1-\phi_t)\omega_t^i - h_t$. When the household expects large labor and tax revenues in the future (h_t is large), he can borrow more (b_t^i decreases). This is the human wealth effect.

The share of capital in the portfolio ϕ_t is all the more reactive to the risk premium that σ is small. In the extreme case, when $\sigma = 0$, ϕ_t goes to infinity as long as the return on private capital is strictly higher than the return on bonds ($r_t > R^*$).

The individual rules (7) and (8) are linear in wealth and can therefore be written in aggregate terms: $c_t = (\rho - n)\omega_t$ and $k_t = \phi_t \omega_t$, where $y_t = \int_0^1 y_t^i di$ is the aggregate value for y_t^i. By dividing each side by A_t, they can also be written in terms of efficient units of labor. The following Lemma follows:

Lemma 2: *Let $\tilde{x}_t = \int_0^1 \tilde{x}_t^i di$ denote the aggregate value of $\tilde{x}_t^i$, where $\tilde{x}_t^i = X_t^i/(A_t N_t^i)$ is the value of X_t^i in efficient labor terms. For a given interest rate R^*, the aggregate dynamics of the economy is characterized by:*

$$\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = \bar{R}_t - \rho - (\dot{\pi}_t + g^*) \tag{9}$$

with $\bar{R}_t = \phi_t^i r_t + (1-\phi_t^i)R^*$, the average return on portfolio,

$$\dot{\tilde{k}}_t + \dot{\tilde{b}}_t = f(\tilde{k}_t) - \delta\tilde{k}_t + R^*\tilde{b}_t - \tilde{c}_t - (n + \dot{\pi}_t + g^*)(\tilde{k}_t + \tilde{b}_t) \tag{10}$$

and:

(i) *If $\sigma > 0$:*

$$\tilde{k}_t = \frac{\phi_t}{1-\phi_t}(\tilde{h}_t + \tilde{b}_t) \tag{11}$$

where ϕ_t satisfies (8) and:

$$\dot{\tilde{h}}_t = (1 - \alpha + \tau\alpha) f(\tilde{k}_t) - (n + \dot{\pi}_t + g^* - R^*)\tilde{h}_t \tag{12}$$

(ii) *If $\sigma = 0$:*

$$\tilde{k}_t = \tilde{k}^* = \left(\frac{\alpha(1-\tau)}{R^* + \delta}\right)^{\frac{1}{1-\alpha}} \tag{13}$$

Equation (9) is obtained by differentiating Equation (7) with respect to time and using the portfolio rule (8). It corresponds to the Euler equation of the economy. Consumption growth per efficient unit of labor increases with $\bar{R}_t$, the mean return to savings and decreases with $\dot{\pi}_t + g^*$, the growth of TFP.

Equation (10) is obtained from the aggregation of the individual budget constraints (6). Equation (11) is a rewriting of the portfolio choice rule (8).

Finally, Equation (13) derives from the no-arbitrage condition $r_t = R^*$ between bonds and domestic capital when $\sigma = 0$. This no-arbitrage condition is an equilibrium outcome that derives from the infinite elasticity of private capital demand to the return differential between capital and bonds. The concavity of the production function insures that this no-arbitrage condition is satisfied. This fixes the level of capital so that its private return equals the world interest rate. In this case, the average return on portfolio $\bar{R}_t$ is simply equal to the world interest rate on bonds R^*.

The labor market clears so the labor force is identical in all firms: $N_t^i = N_t$ for all i. To recover aggregate values, the per worker or per efficient units of labor values must therefore be multiplied by $A_t N_t$.

When $\sigma = 0$, Equations (9), (10) and (13), along with k_0, b_0 and the no-Ponzi conditions, characterize entirely the dynamics of the economy. When $\sigma > 0$, Equations (9), (10), (11) and (12) along with k_0, b_0 and the no-Ponzi conditions, characterize the dynamics. In that case, we must keep track of an additional variable, $\tilde{h}_t$, because households' wealth matters for investment.

3.4 Steady state

We define the steady state by $\dot{\tilde{k}}/\tilde{k} = 0$ and $\dot{\pi}_t = 0$. This condition implies different constraints on the world interest rate depending on σ.

Proposition 1:

(i) *If $\sigma > 0$, the open economy steady state exists if and only if $R^* - g^* < \rho$ and is defined by:*

$$(1-\tau) f'(\tilde{k}^*) - \delta - R^* = \sqrt{\sigma^2(\rho - R^* + g^*)} \tag{14}$$

$$\tilde{k}^* = \frac{\phi^*}{1-\phi^*}\left[\frac{(1-\alpha+\alpha\tau)f(\tilde{k}^*)}{R^*-g^*-n}+\tilde{b}^*\right] \tag{15}$$

with $\phi^* = \sqrt{\frac{\rho - R^* + g^*}{\sigma^2}}$.

(ii) *If $\sigma = 0$, the open economy steady state exists if and only if $R^* - g^* = \rho$ and is defined by:*

$$(1-\tau)f'(\tilde{k}^*) - \delta = R^* \tag{16}$$

$$\tilde{b}^* = -\tilde{k}^* - \frac{(1-\alpha+\alpha\tau)f(\tilde{k}^*) - \tilde{c}_0 e^{-\pi}}{\rho - n} \tag{17}$$

with $\tilde{c}_0 = (\rho - n)\left[(1-\alpha+\alpha\tau)f(\tilde{k}^*)\int_0^\infty e^{-(\rho-n)t+\pi_t}dt + \tilde{k}_0 + \tilde{b}_0\right]$.

Equation (14) derives from the stationarity of consumption in efficient labor terms and the Euler equation (9). It states that, in the steady state equilibrium, the risk premium (LHS) is constant and depends positively on the amount of risk σ and on the difference between the discount factor ρ and the world interest rate adjusted for the growth of the world productivity $\rho - (R^* - g^*)$. Equation (16) is another way to write the no-arbitrage condition $r_t = R^*$ when capital is not risky, but it can also be viewed as a particular case of Equation (14), where $\sigma = 0$.

Equation (15) is the steady-state version of the portfolio allocation rule (11), while Equation (17) derives from the long-term version of the budget constraint (10) and from the Euler equation (9). In the presence of risk, safe assets, including bond holdings, are a constant share of the portfolio which depends only on the parameters of the model. But in the absence of risk, the amount of bonds is determined only by initial wealth $\tilde{k}_0 + \tilde{b}_0$. Notice that in both cases, $\phi^* > 0$ since wealth and capital are necessarily positive, but we do not have necessarily $\phi^* < 1$. This is equivalent to $h^* + b^* > 0$, which is not necessarily the case in a small open economy, since b^* can be negative. When $\sigma = 0$, the steady-state share of capital in the portfolio ϕ^* depends on initial conditions and thus can take any value above zero. When $\sigma > 0$, it depends on the parameters. However, if σ is not too small (namely, if $\sigma > \rho - R^* + g^*$), then $\phi^* < 1$.

Equations (14) and (15) on the one hand, and (16) and (17) on the other, are sufficient to determine $\tilde{k}^*$ and $\tilde{b}^*$, the steady-state values for capital and bond holdings per efficient unit of labor. Equations (14) and (16) determine $\tilde{k}^*$ unambiguously and these values can be replaced respectively in Equations (15) and (17) to determine $\tilde{b}^*$.

3.5 Capital flows

Following the method of Gourinchas and Jeanne (2007), the model is confronted with the data observed over a finite period $[0, T]$. However, before deriving the level of bonds predicted by the model, some assumptions must be made. First, we abstract from unobserved future developments in productivity by assuming that all countries have the same productivity growth rate g^* after time T.

Assumption 1: $\pi_t = \pi$ for all $t \geq T$.

When $\sigma = 0$, $\tilde{k}_t = \tilde{k}^*$ for all t. The steady state is reached immediately. However, when $\sigma > 0$, $\tilde{k}_t$ is contingent on time, which makes it impossible to abstract on T from future $\tilde{k}_t$, except if $\tilde{k}_T$ is sufficiently close to the steady state. In particular, for T sufficiently high, $\tilde{k}_T$ is close to $\tilde{k}^*$, since $\tilde{k}$ converges to its steady state[4]. In the remainder of the analysis, it is therefore assumed that T is sufficiently large to be able to make the following approximation: $\tilde{k}_t = \tilde{k}^*$ for all $t \geq T$.

Denote by $\Delta B/Y_0 = (B_T - B_0)/Y_0$ the amount of capital flows between 0 and T. In order to distinguish the predicted capital flows according to the riskless and portfolio approaches, denote the former $\overline{\Delta B/Y_0}$ and the latter $\widehat{\Delta B/Y_0}$.

Proposition 2: *Under Assumption 1 and for T sufficiently large, the ratio of cumulated capital inflows to initial input is given by:*

(i) *If* $\sigma = 0$*:*

$$\overline{\frac{\Delta B}{Y_0}} = \frac{\tilde{k}_0 - \tilde{k}^*}{\tilde{k}_0^\alpha} e^{(n+g^*)T} + \left(e^{(n+g^*)T} - 1\right) \frac{\tilde{b}_0}{\tilde{k}_0^\alpha} - (e^\pi - 1) \frac{\tilde{k}^*}{\tilde{k}_0^\alpha} e^{(n+g^*)T}$$
$$-e^{\pi+(n+g^*)T} \frac{(1-\alpha+\alpha\tau)\tilde{k}^{*\alpha}}{\tilde{k}_0^\alpha} \int_0^T e^{-(\rho-n)t}(1 - e^{\pi_t - \pi})dt \tag{18}$$

(ii) *If* $\sigma > 0$*:*

$$\widehat{\frac{\Delta B}{Y_0}} = \frac{1-\phi^*}{\phi^*} \frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha} + \frac{1-\phi^*}{\phi^*} \frac{\tilde{k}^*}{\tilde{k}_0^\alpha} \left(e^{\pi+(n+g^*)T} - 1\right) + \frac{\tilde{k}_0}{\tilde{k}_0^\alpha} \left(\frac{1}{\phi^*} - \frac{1}{\phi_0}\right)$$
$$+e^\pi \frac{(1-\alpha+\alpha\tau)\tilde{k}^{*\alpha}}{\tilde{k}_0^\alpha} \int_0^T e^{-(R^*-(n+g^*))t} e^{\pi_t - \pi} \frac{\tilde{k}_t^\alpha}{\tilde{k}^{*\alpha}} dt \tag{19}$$

where $\phi_0 = \tilde{k}_0/(\tilde{k}_0 + \tilde{h}_0 + \tilde{b}_0)$ is the initial share of capital in portfolio, with $\tilde{h}_0 = (1 - \alpha + \tau\alpha) \left[\int_0^T e^{-[R^*-(n+g^*)]T+\pi_t} \tilde{k}_t^\alpha dt + e^{\pi+(n+g^*)T} \frac{\tilde{k}^{*\alpha}}{R^*-(n+g^*)}\right]$ the initial human wealth.

Equations (18) and (19) give the predicted capital outflows as a function of n, g^*, ρ, R^*, τ, the sequence of productivity catch-up $\{\pi_t\}_{t=1,..,T}$ and initial values $\tilde{b}_0$ and $\tilde{k}_0$. Note that $\tilde{k}^*$ is also

[4] See Angeletos (2007) and Angeletos and Panousi (2007) for the transitional dynamics of this kind of model.

a function of the parameters. In the risky environment, the sequence of capital per efficient unit of labor $\{\tilde{k}_t\}_{t=1,..,T}$[5] and the initial share of capital in wealth ϕ_0 depend also on these parameters.

Equation (18) is the continuous-time version of Gourinchas and Jeanne (2007). It can be decomposed into the same components. The same vocabulary and notations are therefore used here. Consider the first term:

$$\frac{\overline{\Delta B^c}}{Y_0} = \frac{\tilde{k}_0 - \tilde{k}^*}{\tilde{k}_0^\alpha} e^{(n+g^*)T}$$

The difference $\tilde{k}_0 - \tilde{k}^*$ is the amount immediately borrowed by the country to equalize its private return to capital to the world's interest rate. Following Gourinchas and Jeanne (2007), we call it the convergence term.

The second term,

$$\frac{\overline{\Delta B^t}}{Y_0} = \left(e^{(n+g^*)T} - 1\right) \frac{\tilde{b}_0}{\tilde{k}_0^\alpha}$$

represents the impact of the initial external position in the presence of trend growth ($n+g^* > 0$). It reflects the amount of capital outflows (or inflows) required to maintain the ratio of external position to output constant.

The third term,

$$\frac{\overline{\Delta B^i}}{Y_0} = -(e^\pi - 1) \frac{\tilde{k}^*}{\tilde{k}_0^\alpha} e^{(n+g^*)T}$$

reflects the impact of productivity catch-up on investment. Positive long-term productivity catch-up ($\pi > 0$) implies further needs in investment. It contribute negatively to the external position, because the country has to borrow from abroad.

Finally, the fourth term,

$$\frac{\overline{\Delta B^s}}{Y_0} = -e^{\pi+(n+g^*)T} \frac{(1-\alpha+\alpha\tau)\tilde{k}^{*\alpha}}{\tilde{k}_0^\alpha} \int_0^T e^{-(\rho-n)t}(1 - e^{\pi_t - \pi})dt$$

reflects the impact of savings on the external position. It represents the consumption smoothing behavior. Indeed, the households adjust their consumption according to their intertemporal wealth, which depends on their discounted flow of deterministic revenue $\tilde{w}_t + \tilde{z}_t$. The path of those revenues depends on $(1-\alpha+\alpha\tau)\tilde{k}^{*\alpha}$ and on the path of π_t. All these components correspond exactly to those analyzed by Gourinchas and Jeanne (2007).

Consider now Equation (19), which represents the predicted flows according to the portfolio approach, that is when $\sigma > 0$. The sign of $\phi^*/(1-\phi^*)$ is critical to determine the sign of the first and second terms. We have seen that the steady-state share of capital in wealth ϕ^* is strictly

[5]By solving the (9), (10), (11) and (12) system.

positive but that it is not necessarily below one in the general case. Namely, the parameter σ must be high enough, and more precisely follow the following assumption:

Assumption 2: $\sigma > \rho - R^* + g^*$

This assumption is maintained in the remaining analysis.[6] Some of the components of $\widehat{\frac{\Delta B}{Y_0}}$ can have the same interpretation as in the riskless approach. The first component,

$$\frac{\widehat{\Delta B^c}}{Y_0} = \frac{1-\phi^*}{\phi^*}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^{\alpha}}$$

represents the impact of convergence. If $\tilde{k}_0 < \tilde{k}^*$, the country increases its capital stock. But contrary to the riskless approach, this does not imply a decrease in the net external position. On the opposite, the increase in wealth following the accumulation of private capital induces a rise in foreign assets, which are a constant fraction of wealth.

The second term,

$$\frac{\widehat{\Delta B^i}}{Y_0} = \frac{1-\phi^*}{\phi^*}\frac{\tilde{k}^*}{\tilde{k}_0^{\alpha}}\left(e^{\pi+(n+g^*)T} - 1\right)$$

reflects long-term productivity catch-up. Again, the sign of the contribution of this term when $\pi > 0$ is opposite to the riskless approach. The intuition is the same as for the convergence term. The increase in investment induced by productivity growth increases wealth and makes the external position rise.

The third term,

$$\frac{\widehat{\Delta B^p}}{Y_0} = \frac{\tilde{k}_0}{\tilde{k}_0^{\alpha}}\left(\frac{1}{\phi^*} - \frac{1}{\phi_0}\right)$$

is the portfolio structure term. It reflects the impact of changes in the structure of portfolio on external bond holdings. If, for example, the share of capital increases ($\phi^* > \phi_0$), then, holding everything equal, external bond holdings should decrease.

[6] The constraint that $\phi^* < 1$ and equivalently that $\phi^*/(1-\phi^*) > 0$ can be rationalized by the following general equilibrium argument. Consider a world composed by a continuum of countries indexed by j, $j \in [0,1]$. Each country taken individually is small and is negligible regarding the others taken as a whole, which corresponds to our small open economy framework. Countries can differ with respect to τ and n, but have the same level of idiosyncratic risk σ (as we will assume in the calibration section). As a result, they have the same steady-state share of capital in the portfolio ϕ^*, according to Proposition 1. Summing (15) across countries, we obtain:

$$\int_0^1 \tilde{k}^{*j} dj = \frac{\phi^*}{1-\phi^*}\left[\int_0^1 \frac{(1-\alpha+\alpha\tau^j)f(\tilde{k}^{*j})}{R^* - g^* - n^j}dj + \int_0^1 \tilde{b}^{*j}dj\right]$$

Since the world bond market clears, we have $\int_0^1 \tilde{b}^{*j}dj = 0$. Therefore, $\phi^*/(1-\phi^*)$ is necessarily positive as long as $\sigma > 0$. This is made possible by the adjustment of the world interest rate R^* in order to clear the bond market. It is therefore consistent with the portfolio approach to assume that $\phi^*/(1-\phi^*) > 0$.

Finally, the fourth term,

$$\frac{\widehat{\Delta B^h}}{Y_0} = e^{\pi}\frac{(1-\alpha+\alpha\tau)\tilde{k}^{*\alpha}}{\tilde{k}_0^{\alpha}}\int_0^T e^{-(R^*-(n+g^*))t}e^{\pi_t-\pi}\frac{\tilde{k}_t^{\alpha}}{\tilde{k}^{*\alpha}}dt$$

is the human wealth term. It represents the impact of changes in human wealth between the beginning and the end of period. Holding the amount of safe assets constant, a decrease in human wealth must be compensated by an increase in bonds. This term can be related to the consumption smoothing term in the riskless approach, because it features the discounted sum of safe revenues. Notice that, contrary to the riskless approach, it does not only depend on the path of π_t, but also on the path of $\tilde{k}_t$. This is because, in the portfolio approach, the level of capital does not immediately adjust to its steady state value: it depends on the current level of wealth and not only on the world's interest rate. As a consequence, the path of deterministic revenues $\tilde{w}_t + \tilde{z}_t$ and therefore the consumption smoothing term are contingent on both the path of productivity catch-up π_t and the path of efficient capital $\tilde{k}_t$.

3.6 The role of productivity

Hall and Jones (1999) and Caselli (2004) show that TFP is a major source of the cross-country differences in income. Consistently, Gourinchas and Jeanne (2007) find that productivity growth is the main source of the allocation puzzle. It is therefore instructive to compare how it affects bond holdings in both approaches. It has been already noticed that π has opposite effects on the catch-up term in the two approaches, $\frac{\overline{\Delta B^i}}{Y_0}$ and $\frac{\widehat{\Delta B^i}}{Y_0}$. However, $\frac{\overline{\Delta B^s}}{Y_0}$ and $\frac{\widehat{\Delta B^h}}{Y_0}$ depend in a more complicated way on π and the path of π_t. In order to simplify the problem, the following assumption is made:

Assumption 3: $\pi_t = f(t)\pi$ *where* $f(.)$ *is common across countries and satisfies* $0 \leq f(t) \leq 1$ *and* $\lim_{t\to\infty} f(t) = 1$*.*

Under Assumption 3, we can rewrite $\frac{\overline{\Delta B^s}}{Y_0}$ as:

$$\frac{\overline{\Delta B^s}}{Y_0} = -e^{\pi+(n+g^*)T}\frac{(1-\alpha+\alpha\tau)\tilde{k}^{*\alpha}}{\tilde{k}_0^{\alpha}}\int_0^T e^{-(\rho-n)t}(1-e^{-\pi(1-f(t))})dt$$

This term is decreasing in the long-run productivity catch-up e^{π} as long as $\pi > -100\%$[7], which is a very weak condition[8]. Faster relative productivity growth implies higher future income,

[7]See the Appendix for a proof.

[8]In particular, it will be satisfied in the data.

leading to an increase in consumption and a decrease in savings. As a result, the external position deteriorates, including in the long run.

Therefore, in the absence of risk, in line with Gourinchas and Jeanne (2007), countries growing faster should borrow more, both through the investment and consumption channels. However, in the presence of risk, the picture is more nuanced.

To begin with, $\frac{\widehat{\Delta B^h}}{Y_0}$ can be rewritten as follows:

$$\frac{\widehat{\Delta B^h}}{Y_0} = \frac{(1-\alpha+\alpha\tau)\tilde{k}^{*\alpha}}{\tilde{k}_0^{\alpha}} \int_0^T e^{-(R^*-(n+g^*))t} e^{\pi f(t)} \frac{\tilde{k}_t^{\alpha}}{\tilde{k}^{*\alpha}} dt$$

which is increasing in e^{π}, as opposed to $\overline{\frac{\Delta B^s}{Y_0}}$. Faster relative productivity growth here increases the level of capital outflows. This is because higher expected revenues in the future encourage the households to borrow more both in $t = 0$ and in $t = T$, but the flow of revenues between 0 and T weighs only on borrowing in $t = 0$. Faster productivity growth between these dates will thus have a positive impact on borrowing at $t = 0$, thus increasing the level of capital outflows between 0 and T. A key element here is that in the portfolio approach, the beginning-of-period level of debt is wiped out at steady state by wealth effects. On the opposite, in the riskless approach, the long-run level of debt is contingent on the inherited one, so the amount of consumption-smoothing that took place in the beginning persists in the long run.

As a result, the presence of risk reverses the investment and consumption channels. According to these channels, countries growing faster should borrow less. These channels should therefore contribute to solving the puzzle. This should be nevertheless qualified by the portfolio adjustment term, which also depends on productivity growth.

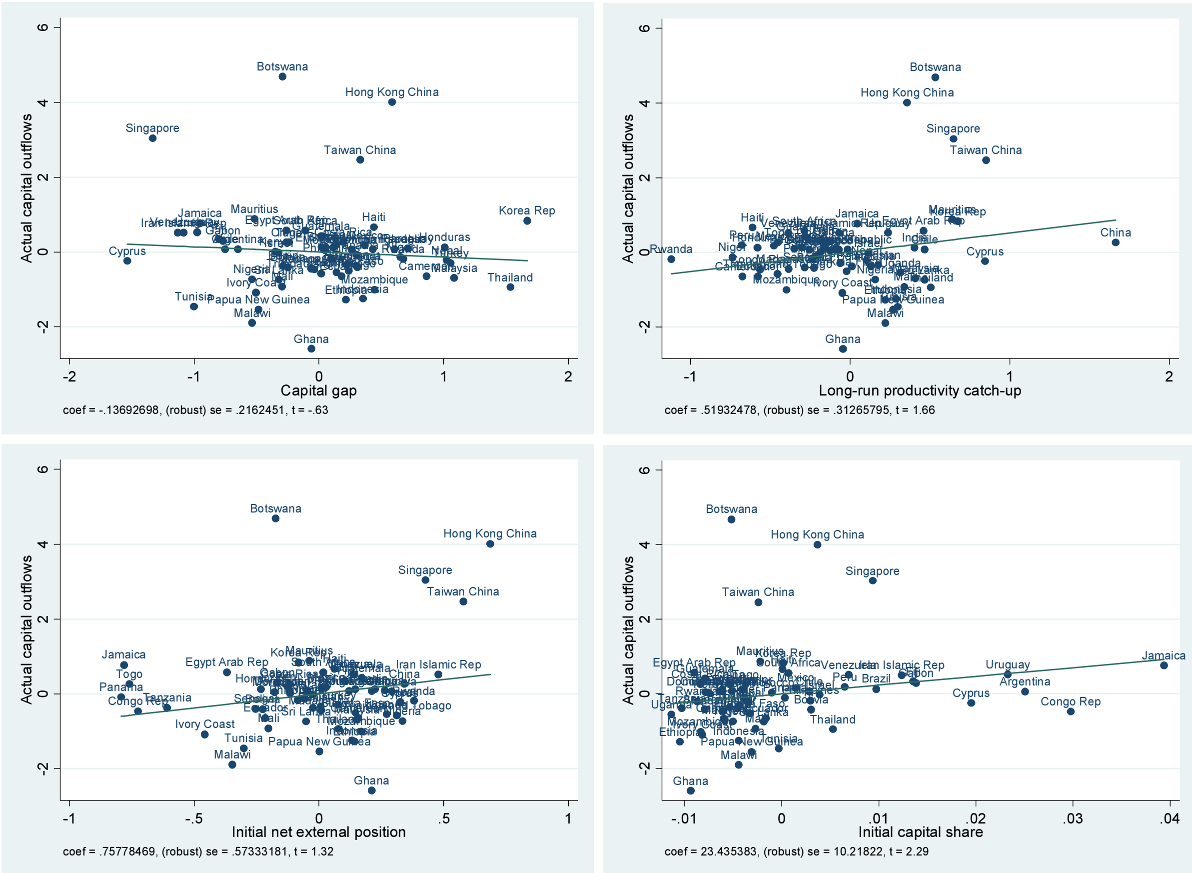
Indeed, the portfolio adjustment term depends on the share of capital in initial wealth, which in turn involves human wealth. Under Assumption 3, the initial human wealth can be written as follows:

$$\tilde{h}_0 = (1-\alpha+\tau\alpha)\left[\int_0^T e^{-[R^*-(n+g^*)]T+\pi f(t)} \tilde{k}_t^{\alpha} dt + e^{\pi+(n+g^*)T} \frac{\tilde{k}^{*\alpha}}{R^*-(n+g^*)}\right]$$

Since the productivity catch-up π affects positively the expected labor and tax income during the period, it also affects positively the initial human wealth. As a consequence, high π will have a negative contribution to capital outflows through the portfolio adjustment term. This is because when the initial share of capital in the portfolio is too low as compared to the long-run one, the external position has to decrease in order to adjust to the steady-state portfolio structure.

To summarize, the presence of risk qualifies the predictions of the neoclassical growth model in terms of capital flows. In the absence of risk, in line with Gourinchas and Jeanne (2007), countries growing faster should borrow *more*. In the presence of risk, the result is ambiguous. It depends whether the investment and consumption channels dominate the long-run portfolio adjustment.

Figure 2: Actual capital outflows between 1980 and 2000 (as a share of initial GDP), against their determinants: capital gap $((\tilde{k}^* - \tilde{k}_0)/\tilde{k}_0)$, long-run productivity catch-up (π), initial external position to GDP ratio $(\tilde{b}_0)/\tilde{y}_0)$ and initial share of capital in the portfolio (ϕ_0)



Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author's calculations.

In order to get an intuition of what approach is more likely to fit the reality, consider Figure 2, which plots actual capital outflows against potential determinants. The initial net external position $\tilde{b}_0/\tilde{y}_0$ is positively correlated with actual capital outflows, which is consistent with the riskless approach, in particular with the trend component. The initial capital share ϕ_0 is also positively correlated, which is consistent with the portfolio approach, in particular with the portfolio component. However, each of these two determinants is specific to one approach, and is not exclusive of the other. The other determinants are more discriminant. The capital gap $(\tilde{k}^* - \tilde{k}_0)/\tilde{k}_0$ does not seem to be correlated in any way to actual capital flows, so it cannot be used, at least at this stage, to assess the predictive power of the model. However, the productivity catch-up π is positively correlated with capital outflows. This can be the case only in the portfolio approach ($\sigma > 0$). In the riskless approach ($\sigma = 0$), the correlation should be negative. The former approach should therefore be a better candidate to account for capital flows to developing countries. However, to confirm that, one should take into account all the determinants of capital flows together. Next section extends the calibration method used by Gourinchas and Jeanne (2007) in order to confront both approaches.

4 Capital flow accounting and calibration

In this section, we compare the two models' predictions in terms of capital flows to the data. Do developing countries with faster productivity growth and larger initial capital scarcity receive more capital flows, as the riskless approach predicts, or the opposite, as the portfolio approach suggests? More generally, we investigate whether the portfolio approach fares better than the riskless one in explaining capital flows to developing countries. This requires, for each country, estimates for the levels of initial capital scarcity and for productivity growth.

In order to facilitate the comparison with Gourinchas and Jeanne (2007), the same time span (1980-2000) and the same sample of 69 emerging countries is used[9]. The parameters which are common across countries also follow their paper: the discount rate ρ is set to 4%, the depreciation rate δ to 6%, the capital share of output α to 0.3 and the growth rate of world productivity g^* to 1.7%. Given these parameters values, the world's interest rate R^* is equal to 5.7% when $\sigma = 0$, that is when the riskless approach is used.

In the portfolio approach, the amount of risk σ is set to 0.3, which is an amount of entrepreneurial risk commonly reported by empirical studies in the US and the Euro area (Campbell et al., 2001; Kearney and Poti, 2006). The world's interest rate is then set so that the implied

[9]This sample includes: Angola, Argentina, Bangladesh, Benin, Bolivia, Botswana, Brazil, Burkina Faso, Cameroon, Chile, China, Colombia, the Republic of Congo, Costa Rica, Cyprus, Côte d'Ivoire, Dominican Republic, Ecuador, Egypt, Arab Republic, El Salvador, Ethiopia, Fiji, Gabon, Ghana, Guatemala, Haiti, Honduras, Hong Kong, India, Indonesia, Iran, Israel, Jamaica, Jordan, Kenya, Republic of Korea, Madagascar, Malawi, Malaysia, Mali, Mauritius, Mexico, Morocco, Mozambique, Nepal, Niger, Nigeria, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Rwanda, Senegal, Singapore, South Africa, Sri Lanka, Syrian Arab Republic, Taiwan,Tanzania, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Uganda, Uruguay and Venezuela.

steady-state ratio of capital to wealth ϕ^* matches the US capital share in 2000. This gives $\phi^* = 0.08$ and $R^* = 5.64\%$.

The country-specific data are the paths for output, capital, productivity and working-age population. These data come from Version 6.2 of the Penn World Tables (Heston et al., 2006). Following Gourinchas and Jeanne (2007) and Caselli (2004), the capital stock is constructed with the perpetual inventory method from time series data on real investment. The level of productivity A_t is calculated as $(y_t/k_t^{\alpha})^{1/(1-\alpha)}$ and the level of capital per efficient unit of labor $\tilde{k}_t$ as $(y_t/k_t)^{1/(1-\alpha)}$. The level of TFP A_t and the capital per efficient unit of labor $\tilde{k}_t$ are filtered using the Hodrik-Prescott method in order to suppress business cycles. The parameter n is measured as the annual growth rate of the working-age population. Under Assumption 1, the long-term catch-up π can be measured as $\ln(A_T) - \ln(A_0) - Tg^*$.

Finally, in order to determine the capital wedge τ, we proceed differently from Gourinchas and Jeanne (2007). They compute numerically a mapping from the average investment rate to the capital wedge τ, for given productivity catch-up π and population growth n. Their method cannot be extended easily to the portfolio approach, where the investment rate cannot be written explicitly as a simple function of the steady-state level of capital per efficient unit of labor $\tilde{k}^*$ but is contingent on its whole path $\{\tilde{k}_t\}_{t=1...T}$. More simply, τ is calibrated here in order to replicate the ratio of steady-state capital relative to the US, where $\tau^{US} = 0$. The capital wedge τ must therefore be interpreted in relative terms to the US. If τ is positive (negative), it means that the capital wedge is higher (lower) than in the US. Besides simplicity, this method has the advantage to yield identical capital wedges in both approaches, which facilitates the comparison. Indeed, in both cases, $\tilde{k}^*/\tilde{k}^{*US} = (1-\tau)^{1/(1-\alpha)}$. We use the fact that, assuming that $T = 20$ is a sufficiently large number, $\tilde{k}^*$ is approximately equal to $\tilde{k}_T$. We thus take $\tilde{k}^* = \tilde{k}_T$. This method assigns a high capital wedge to countries with low end-of-period capital per efficient unit of labor relative to the US. The introduction of τ shuts down the Lucas paradox since this parameter is used to adjust the private marginal return to capital to the world interest rate.

Jordan and Angola are removed from the sample because their working-age population does not satisfy $n < \rho$. The Syrian Arab Republic is also removed because it is an outlier: its predicted outflows according to the portfolio approach are well below the sample range. The sample is therefore reduced to 66 countries.

4.1 Some key parameters

Table 1 sums up some key parameters given by the calibration method. It presents the steady-state capital stocks per efficient unit of labor $\tilde{k}^*$, measured by their end-of-period value, and the levels of capital wedge τ compatible with these steady-state values. It also provides some potential determinants: the long-term productivity catch-up π, the beginning-of-period ratio of external position to output $\tilde{b}_0/\tilde{y}_0$, the beginning-of-period capital share in the portfolio ϕ_0, the initial level of capital $\tilde{k}_0$ and the growth rate of capital $(\tilde{k}^* - \tilde{k}_0)/\tilde{k}_0$. Countries are classified by income group (World Bank classification based on 2007 GNI per capita) and by region. Finally, for robustness checks, some potential outliers (China, India, Africa) are excluded.

Consider column (2) of Table 1. The average wedge τ on capital return is equal to 36%, which is consistent with the average wedge on capital return of 12% found in Gourinchas and Jeanne (2007). This is because the definition of capital return differs: they consider the *gross* capital return, but net of depreciation, while we consider the *net* capital return, but before depreciation.[10] Despite using a different method to compute the capital wedge, the results are comparable. The net return varies between 64% in low income countries and −9% in high income countries, which corresponds to 14% and −4% for the gross return. Notice that the capital wedge τ and the end-of-period level of capital $\tilde{k}^*$ (column (1)) are respectively decreasing and increasing with income, except for middle-income countries: upper-middle-income countries have a lower end-of-period level of capital than lower-middle-income. This is not inconsistent with the income classification since the revenue is not defined only by capital, but also by TFP. Generally, countries that achieved a higher level of income are those who maintained a higher end-of-period capital level $\tilde{k}^*$ thanks to a lower wedge τ. Africa, which has the smallest end-of-period capital level, has therefore the highest estimated capital wedge, while Asia's estimated capital wedge is the smallest, since it benefits from a high end-of-period capital level.

Consider now the long-run productivity catch-up π in column (3) of Table 1. On average, non-OECD countries have fallen behind in terms of productivity. When looking into details, only high income economies have caught up with the world productivity. Consistently, countries with intensive catch-up ended up richer at the end of period. In particular, upper-middle-income countries show a less negative productivity catch-up than the lower-middle-income group, which might have compensated for their lower end-of-period capital $\tilde{k}^*$. As for the geographical pattern, it seems that only Asia's productivity has caught up with the world's level. Both Africa and Latin America fell behind.

[10] This deviation from Gourinchas and Jeanne (2007) is due to the use of a continuous-time framework.

Table 1: Long-term capital per efficient unit of labor, capital wedge and potential determinants of capital flows

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\tilde{k}^*$	τ	π	$\frac{\tilde{b}_0}{\tilde{y}_0}$	ϕ_0	$\tilde{k}_0$	$\frac{\tilde{k}^*-\tilde{k}_0}{\tilde{k}_0}$	Obs.
Non-OECD†	1,88	36,37%	-6,87%	-29,91%	1,37%	2,24	-3,53%	66
Low income‡	0,80	64,44%	-15,69%	-31,72%	0,65%	0,98	1,28%	23
Lower middle income‡	2,24	26,76%	-12,75%	-35,23%	1,46%	2,56	-2,2%	22
Upper middle income‡	2,11	28,31%	-4,88%	-34,42%	1,66%	2,68	-13,61%	14
High income‡ (non-OECD†)	3,87	-9,49%	36,59%	1,82%	2,85%	4,51	-3,38%	7
Africa	1,27	52,66%	-13,04%	-39,75%	0,99%	1,77	-12,39%	27
Latin America	2,06	30,38%	-32,36%	-32,66%	1,95%	2,50	-3,9%	22
Asia	2,62	18,27%	35,92%	-10,71%	1,22%	2,65	11%	17
Excluding China and India	1,88	36,39%	-10,11%	-30,79%	1,39%	2,24	-3,16%	64
China and India	1,80	35,81%	96,6%	-1,57%	0,78%	2,20	-15,45%	2
Excluding Africa	2,31	25,1%	-2,6%	-23,09%	1,63%	2,57	2,6%	39

The figures are unweighted country averages.

†: Includes also Korea, Mexico and Turkey.

‡: World Bank classification based on 2007 GNI per capita.

It appears that the initial level of external position $\tilde{b}_0/\tilde{y}_0$ (column (4)) is negative on average: non-OECD countries started with an average debt of 30% of GDP. All regions started with debt, only Asia had a smaller initial level: 11% versus respectively 40% and 33% in Africa and Latin America.

Column (6) of Table 1 presents initial capital per efficient unit of labor $\tilde{k}_0$. The main observation is that the final stock of capital is usually close to the initial one. There is no change in hierarchy due to convergence: countries with low initial capital ended up with low capital. This also appears when considering the capital global growth rates in column (7): they are rather small in absolute value. Notice also that the capital stock *decreased* on average. This suggests that developing countries started with a stock of capital per efficient unit of labor *above* the steady-state, that is with *too much* capital. Consistently to Gourinchas and Jeanne (2006), emerging countries were not capital-scarce but capital-abundant. Among regions, only Asia increased its capital per efficient unit of labor.

The average initial share of capital in portfolio ϕ_0 (column (5)) is very low: less than 2%.

This is because human wealth accounts for an extremely large part of the household's portfolio: not only is it an infinite discounted sum, but it is also inflated both by labor and productivity growth. Additionally, the net external position is small in absolute value. All this results in a small share of capital in the portfolio. It appears that this share is increasing with income (from 0.65% to 2.85%). This can be explained by the fact that initial capital $\tilde{k}_0$ is increasing with income too, as column (6) shows. Among regions, Africa has a very low share of capital: 0.99% versus respectively 1.95% and 1.22% in Latin America and Asia. The lower share of capital in Latin America than in Asia can be explained by lower productivity catch-up and therefore lower human wealth in Latin America.

Importantly, ϕ_0 does not seem to be related to π. Everything equal, the share of initial capital in the portfolio should be decreasing in π, because it increases initial human wealth. But ϕ_0 does not show this pattern. Its main variation seems to be due to other factors such as income (through k_0). Therefore, it is likely that, in the portfolio approach, the effect of π on the catch-up and human wealth components is not going to be offset by the impact of ϕ_0 on the portfolio adjustment component. And since π has a positive effect on capital outflows through the catch-up and human wealth components, the portfolio approach should match the data better than the riskless approach.

4.2 Capital flows

We now turn to the confrontation of actual and predicted capital flows. In order to achieve this, actual capital flows are computed, using net foreign asset data from Lane and Milesi-Ferretti (2006). They provide estimates for the net external position in current US dollars. These estimates are calculated using the cumulated current account data and are adjusted for valuation effects. In order to be consistent with the PPP-adjusted data used here, a PPP deflator is extracted from the Penn World Table and is used to calculate a PPP-adjusted measure of net external position. Actual capital outflows during the period, as a share of initial output, are denoted $\frac{\Delta B}{Y_0}$. These estimates are confronted with the predicted values given by the riskless and portfolio approaches, respectively $\overline{\frac{\Delta B}{Y_0}}$ and $\widehat{\frac{\Delta B}{Y_0}}$, and to the components highlighted in the previous section.

4.2.1 The riskless approach

Table 2 reproduces the outcome of the riskless approach. Column (1) reports the actual net capital outflows as a share of initial output $\Delta B/Y_0$: their size is -54% on average, which means that emerging countries have received net capital inflows during the period. Column (2) reports the predicted capital outflows based on equation (18). These estimates are constructed under the hypothesis that the productivity catch-up follows a linear trend: $\pi_t = \pi \min\{t/T, 1\}$, as in Gourinchas and Jeanne (2007). Our results, despite the continuous time framework and the use of a different method to calibrate the capital wedge τ, are in line with Gourinchas and Jeanne (2007). According to the model, non-OECD countries should have received capital inflows on average, which is the case. However, here, contrary to Gourinchas and Jeanne (2007), average predicted flows (column (2)) in non-OECD countries are of the same order of magnitude as the actual ones (column (1)). This comes from the fact that consumption smoothing has a lower magnitude than in their calibration. Still, when excluding African countries, capital inflows seem to be strongly overestimated. They also seem to have the right sign, but if we exclude China and India, which account for a large part of negative outflows, the sign of predicted outflows becomes positive, while actual ones are negative on average. While unclear in terms of global trends, the model fails completely when considering the direction of flows inside the sample. According to the predictions, low-income countries should have exported capital while high-income countries should have received capital inflows. Actually, the opposite happened. Latin America and Africa should have invested abroad while Asia should have hosted capital inflows. But in fact Asia received less capital than the other regions. The origins of these discrepancies are examined by looking into the components of predicted capital flows.

After looking into components, it appears that the convergence term in column (3) of Table 2 contributes positively to the total predicted outflows. This can be explained by the fact that, as shown above, countries have started on average above their long-term level of capital, and thus have disinvested on average. As a consequence, they should have lent to the rest of the world. This is the case in Latin America and Africa which had too much capital and should have used their extra capital stock to invest abroad, whereas Asia should have received capital from abroad to finance its growth in capital stock.

The catch-up component, in column (4), has a negative contribution. This average result is mainly driven by Asia, which had a strong positive long-term productivity catch-up: it should have borrowed from the rest of the world in order to finance the extra investment. Other non-OECD countries have fallen behind world productivity on average, namely Africa and Latin America. This relative fall in productivity should have led households to disinvest and enabled them to lend to the rest of the world.

Table 2: Predicted and actual capital flows between 1980 and 2000 - Riskless approach

Capital flows (share of initial output)	(1) $\frac{\Delta B}{Y_0}$	(2) $\overline{\frac{\Delta B}{Y_0}}$	(3) $\overline{\frac{\Delta B^c}{Y_0}}$	(4) $\overline{\frac{\Delta B^i}{Y_0}}$	(5) $\overline{\frac{\Delta B^s}{Y_0}}$	(6) $\overline{\frac{\Delta B^t}{Y_0}}$	(7) Obs.
Non-OECD†	-0,54	-0,36	0,51	-0,14	-0,29	-0,43	66
Low income‡	-1,13	1,56	0,38	0,08	1,58	-0,49	23
Lower middle income‡	-0,68	-0,42	0,45	-0,10	-0,29	-0,47	22
Upper middle income‡	-0,03	0,12	0,81	0,005	-0,20	-0,50	14
High income‡ (non-OECD†)	0,82	-7,40	0,54	-1,29	-6,64	-0,02	7
Africa	-0,77	1,24	0,92	0,09	0,81	-0,58	27
Latin America	-0,46	4,81	0,55	0,41	4,31	-0,47	22
Asia	-0,29	-9,57	-0,20	-1,23	-8,00	-0,15	17
Excluding China and India	-0,55	0,70	0,51	-0,03	0,67	-0,45	64
China and India	-0,35	-34,30	0,57	-3,67	-31,19	-0,01	2
Excluding Africa	-0,38	-1,46	0,23	-0,30	-1,05	-0,33	39

$\frac{\Delta B}{Y_0}$ is the observed ratio of net capital outflows to initial output, predicted capital flows $\overline{\frac{\Delta B}{Y_0}}$ and its components $\overline{\frac{\Delta B^c}{Y_0}}$, $\overline{\frac{\Delta B^i}{Y_0}}$, $\overline{\frac{\Delta B^s}{Y_0}}$ and $\overline{\frac{\Delta B^t}{Y_0}}$ are given by (18).

The figures are unweighted country averages.

†: Includes also Korea, Mexico and Turkey.

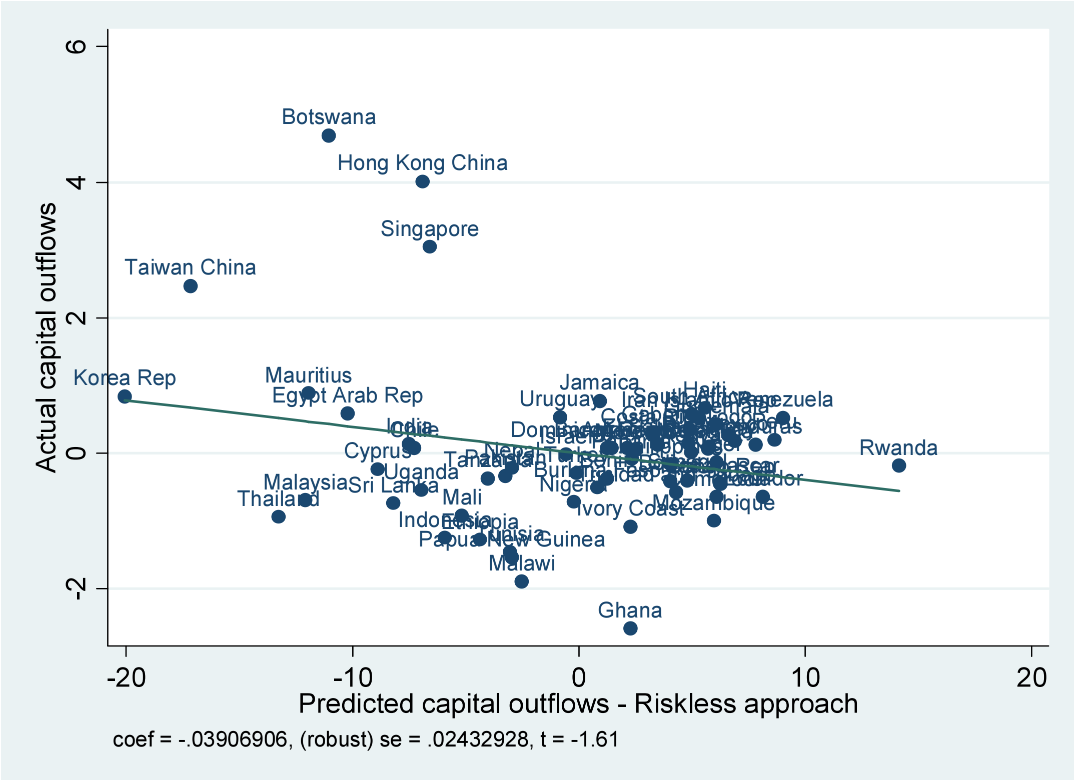
‡: World Bank classification based on 2007 GNI per capita.

The consumption smoothing component, in column (5), is negative on average despite the negative average productivity catch-up. This is because Asian countries, which have benefited from a positive productivity catch-up, contribute highly to the sample mean. When considering regions, it still appears that Latin America and Africa, which expected a fall in their revenue because of a negative catch-up, should have saved in order to smooth consumption. The contribution of the consumption smoothing term is therefore positive for those regions. On the opposite, Asiatic countries, which expected a relative rise in their productivity and therefore a relative rise in their revenue, should have dissaved in order to smooth consumption. Their consumption smoothing term is thus negative.

These three components (convergence, catch-up and consumption smoothing) are at odds with the data. They all imply capital inflows to Asia and capital outflows from Latin America and Africa, while actually Asia received less capital than the two other regions.

Only the last one, the trend component in column (6), is consistent with the data. Indeed, as

Figure 3: Actual capital outflows (as a share of initial GDP) against their predicted value, according to the riskless approach, 1980-2000



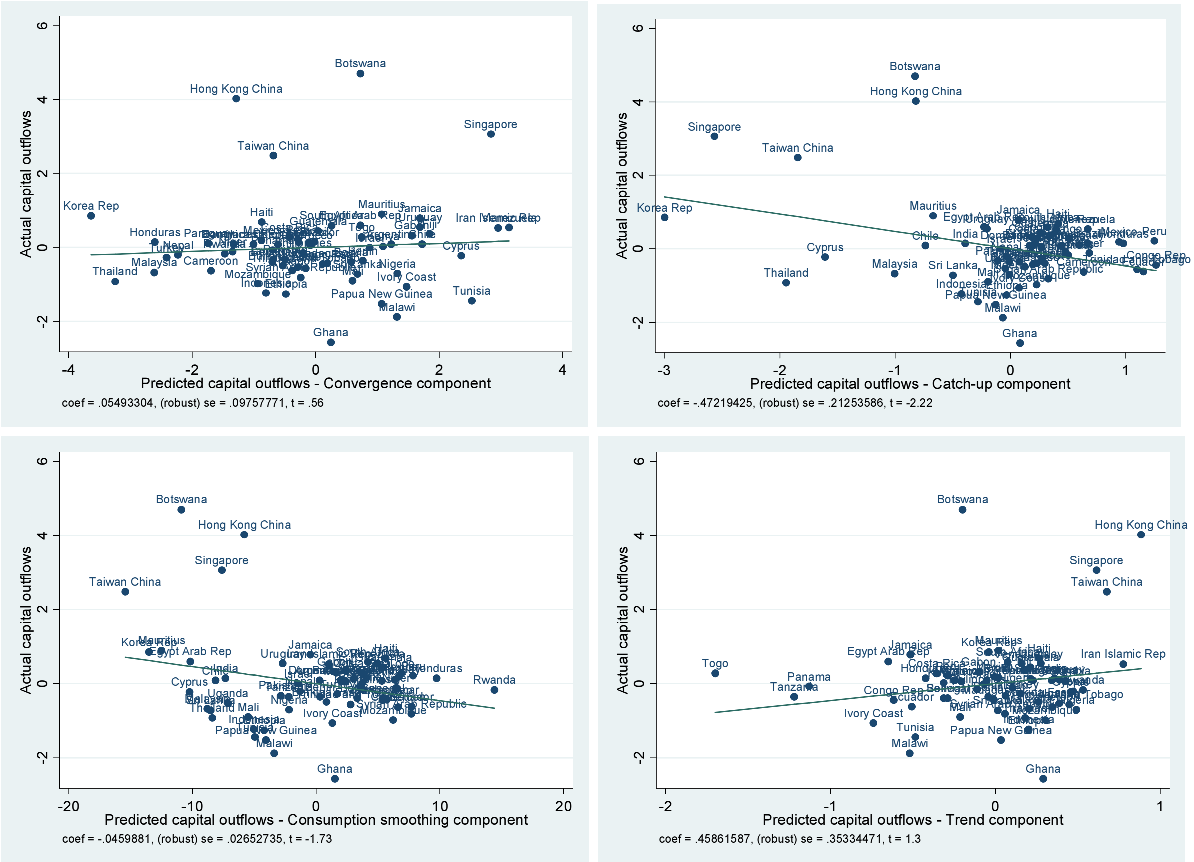
Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author's calculations.

observed capital inflows, it is decreasing with income. Also, according to this component, Asia should receive less capital than Latin America and Africa. However, its quantitative importance is not sufficient to counteract the other components.

On the whole, the puzzle of Gourinchas and Jeanne (2007) seems to be robust to the continuous-time approach and to the use of an alternative method to compute the capital wedge: capital seems to flow in the wrong direction, that is less to the more productive countries than to the less productive. Figure 3 sums up the puzzle by showing the scatter plot of actual versus predicted flows. The correlation seems, at best, non-significant and, at worst, negative.

Figure 4 presents the scatter plots of actual capital flows against each components, stressing the contribution of each of them to the overall correlation between predicted and capital flows. The component which is the most negatively correlated with actual flows is the catch-up component. This is consistent with Gourinchas and Jeanne (2007)'s findings. After comes the

Figure 4: Actual capital outflows (as a share of initial GDP), against the different components of predicted capital flows, 1980-2000 - Riskless approach



Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author's calculations.

consumption smoothing component, while the convergence component does not seem to be correlated. Also, as expected, the trend component is the only one which is positively correlated. The puzzle is therefore mainly due to the catch-up and consumption smoothing components. Long-run productivity catch-up, which is the main determinant of these two components, is thus at the core of the puzzle. Figure 2 showed that, indeed, long-run productivity catch-up is positively correlated with capital outflows, while the riskless approach predicted the opposite. This calibration analysis confirms that the wrong correlation with long-run productivity catch-up is responsible for the puzzle.

4.2.2 The portfolio approach

We have shown that the model without risk reproduces exactly the puzzle highlighted by Gourinchas and Jeanne (2007). We now turn to the extension with risk.

Table 3 sums up the results for the portfolio approach. Column (2) reports the estimated predicted net outflows according to equation (19). The estimates are computed under the Assumption that the productivity catch-up follows a linear path, as in the riskless approach. The path of capital per efficient unit of labor $\tilde{k}_t$ implied by the model is approximated by the following formula: $\tilde{k}_t = \tilde{k}_0 e^{g_k \min\{t/T,1\}}$, where $g_k = \ln(\tilde{k}^*) - \ln(\tilde{k}_0)$[11].

Note first that the magnitude of predicted flows (column (2)) is well above the actual ones (column (1)), by three to four orders of magnitude. This is a shortcoming of the portfolio approach that has been already highlighted in Kraay et al. (2005). But this shortcoming is accentuated here by the presence of potentially huge human wealth effects, due to labor and productivity growth. This human wealth can represent more than one hundred times current income and can enable the country to borrow enormous amounts, as long as it can pledge its future labor income and transfers.

When looking into details, it appears that the main origin of the discrepancy is the portfolio term, in column (5). The magnitude of this term can be explained by the fact that the initial share of capital in the portfolio is low as compared to the steady-state one, as pointed to when analyzing Table 1. This initial share is small because the beginning-of-period human wealth is large and the external debt is small. This results in a predicted reallocation of the portfolio in favor of capital in the long term. Provided some adjustments in human wealth, this implies that bond holdings should diminish in the long term, which is equivalent to capital inflows.

The human wealth adjustment term, in column (6) of Table 3, is positive in all country groups. This can be explained by the fact that human wealth falls on average between the beginning and the end of period. This fall in human capital contributes positively to capital outflows, since, holding portfolio shares constant, bond holdings are substituted to human wealth inside the safe portfolio. But this adjustment is not sufficient to compensate for the portfolio term. The magnitude of the other terms is not as striking, so the discrepancy between the data and the model comes mainly from the discrepancy between the beginning-of-period observed external position and human wealth, which results in a very negative portfolio term.

When abstracting from the magnitude issue, it appears that the predicted outflows in column (2) of Table 3 exhibit the right sign, which is negative, and, contrary to the riskless approach, the right ranking between country groups. Predicted capital inflows are now decreasing with

[11] The sequence of of $\{\tilde{k}_t\}_{t=1,..,T}$ could be inferred from the parameters, the initial values and the exogenous trend of productivity by solving the (9), (10), (11) and (12) system. However, the assumed trend is a good proxy for the capital dynamics since the theory predicts that it moves smoothly from $\tilde{k}_0$ to $\tilde{k}^*$.

Table 3: Predicted and actual capital flows between 1980 and 2000 - Portfolio approach

Capital flows (share of initial output)	(1) $\frac{\Delta B}{Y_0}$	(2) $\widehat{\frac{\Delta B}{Y_0}}$	(3) $\widehat{\frac{\Delta B^c}{Y_0}}$	(4) $\widehat{\frac{\Delta B^i}{Y_0}}$	(5) $\widehat{\frac{\Delta B^p}{Y_0}}$	(6) $\widehat{\frac{\Delta B^h}{Y_0}}$	(7) Obs.
Non-OECD†	-0,54	-101,54	-2,55	23,56	-138,42	15,80	66
Low income‡	-1,13	-130,56	-1,54	9,82	-152,33	13,37	23
Lower middle income‡	-0,68	-101,59	-2,17	26,28	-142,35	16,59	22
Upper middle income‡	-0,03	-89,53	-4,31	23,69	-125,79	16,85	14
High income‡ (non-OECD†)	0,82	-30,04	-3,51	59,85	-105,65	19,23	7
Africa	-0,77	-131,92	-4,17	13,65	-155,55	14,03	27
Latin America	-0,46	-90,32	-3,02	12,53	-114,53	14,65	22
Asia	-0,29	-67,80	0,63	53,55	-142,13	20,11	17
Excluding China and India	-0,55	-101,99	-2,52	21,32	-136,32	15,46	64
China and India	-0,35	-87,09	-3,38	94,95	-205,59	26,96	2
Excluding Africa	-0,38	-80,50	-1,43	30,41	-126,56	17,03	39

$\frac{\Delta B}{Y_0}$ is the observed ratio of net capital outflows to initial output, predicted capital flows $\widehat{\frac{\Delta B}{Y_0}}$ and its components $\widehat{\frac{\Delta B^c}{Y_0}}$, $\widehat{\frac{\Delta B^i}{Y_0}}$, $\widehat{\frac{\Delta B^p}{Y_0}}$ and $\widehat{\frac{\Delta B^h}{Y_0}}$ are given by (19).

The figures are unweighted country averages.

†: Includes also Korea, Mexico and Turkey.

‡: World Bank classification based on 2007 GNI per capita.

income, as the actual ones. Also, Africa is the region that receives the highest amount of capital inflows while Asia is the one that receives the smallest amount, as in the data.

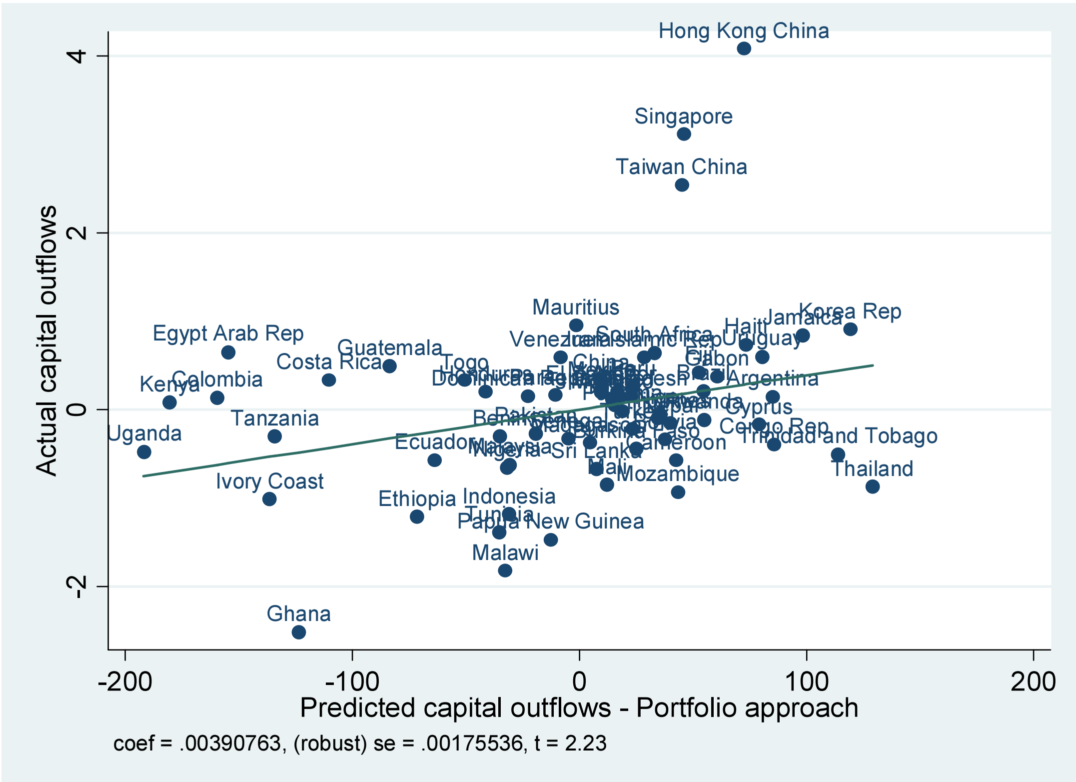
Since, on average, developing countries started from a high level of capital relative to the long-term one, the average contribution of the convergence term in column (3) of Table 3 to capital outflows is negative. This is because, holding the portfolio structure unchanged, a less capitalized country can hold less safe assets since it has to self-insure against less risk. The sign of the contribution of the convergence term in this specification is opposite to its sign in the riskless one. Asia is supposed to export and not import capital, while Latin America and Africa are supposed to import and not export capital. Regions are now correctly ranked in terms of capital outflows when considering the convergence component. Regions are also correctly ranked when considering the catch-up term in column (4). While, in the riskless approach, Asia was supposed to receive more inflows than the other non-OECD countries, the estimates here suggest that it should export more capital, which matches the data better, not in terms of the direction of flows but in terms of hierarchy between regions. To understand this predictions' reversal for the convergence and catch-up terms, note that both high productivity catch-up and positive convergence imply investment in domestic technology. In the riskless approach, more investment is financed through more borrowing from abroad while in the portfolio approach, more investment implies more safe assets to compensate for more risk-taking.

Coming back to the portfolio term in column (5) of Table 3, we can notice that it is negative in all income groups and all regions. Interestingly, it is increasing with income and as a consequence it reproduces the right income-group ranking in terms of flows. This effect is not originated in productivity catch-up, since we would expect it to vary negatively with the portfolio component. Indeed, high productivity growth implies a high beginning-of-period human wealth and therefore a low beginning-of-period capital share ϕ_0. The adjustment in the portfolio structure would then entail a diminution in the bond level and therefore lead to large capital inflows. In that case, high-income countries, which benefited from higher catch-up terms, should present a more negative portfolio component. But it is not the case. Rather, we should seek the explanation of this phenomenon in the size of the initial capital per efficient unit of labor $\tilde{k}_0$. This term has two counteracting effects. First, countries which are already highly capitalized in the beginning tend to have initially a large share of capital in their portfolio. The adjustment to a larger capital share in the long-term implies then a smaller diminution in bond holdings, and therefore less capital inflows. Second, the level of capital affects the scale of bond holdings adjustment consecutive to these adjustments in portfolio shares. This is simply because the magnitude of the bond adjustment consecutive to a change in portfolio structure depends on the size of the portfolio itself. The initial capital level $\tilde{k}_0$ has therefore a negative effect on capital inflows through its impact on the portfolio structure and a positive effect through the scale of the portfolio. Finally, it appears that the first effect dominates since high-income countries, which started on average with a higher level of capital, are supposed to get less capital inflows according to the portfolio term. This analysis is also consistent with the fact that Africa, which has the smallest level of initial capital among regions, has the more negative portfolio term. However, Asia and Latin America have a very close level of initial capital, while Asia is supposed to receive more capital. Here, higher productivity catch-up in Asia can explain the difference.

The last term, in column (6) of Table 3, which sums up the adjustment in human wealth, is positive on average and in all country groups. This comes from the fact that human wealth is higher in the beginning than in the end of period: for a given portfolio structure, bond holdings must rise in order to compensate for the decrease in human wealth. This term is increasing with income and is higher in Asia than in the other regions. This is because high income countries and Asian economies have experienced larger TFP gains than the others on average during the period. Therefore, their initial human wealth is higher, so the implied rise in bond holdings is also more important.

Therefore, all the components reproduce the right pattern of flows (in terms of groups ranking): the convergence, catch-up, portfolio and human wealth components.

Figure 5: Actual capital outflows (as a share of initial GDP) against their predicted value, according to the portfolio approach, 1980-2000

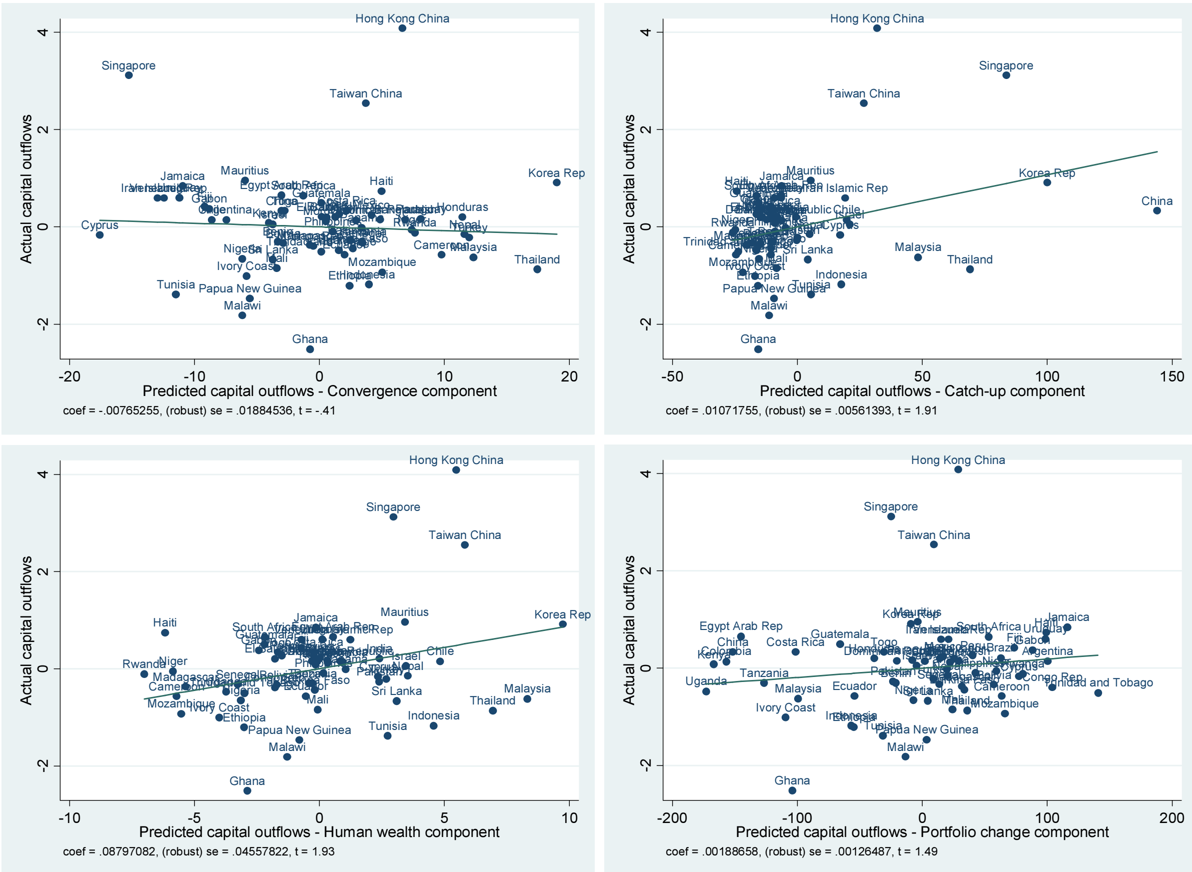


Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author's calculations.

To conclude, the portfolio approach seems to be a better predictor, if not of the magnitude of flows, at least of their direction. Figure 5 sums up this idea by plotting predicted flows against the actual ones. The upper and lower panels are constructed respectively by using the riskless and the portfolio approaches. While the upper graph shows a negative correlation between predicted and actual flows, a positive correlation is recovered in the lower graph. According to the previous analysis, this reversal may be due to the convergence, catch-up or portfolio terms.

To understand which components contribute to solving the puzzle, consider Figure 6, which plots actual capital flows against the different components of predicted capital flows. It appears here that, on the sample of non-OECD countries, the convergence component does not seem correlated with actual capital flows, so it does not contribute to the positive correlation between actual and predicted flows. However, the graph confirms that the catch-up, portfolio and human wealth components are positively correlated with actual flows and thus contribute to solving the puzzle.

Figure 6: Actual capital outflows (as a share of initial GDP), against the different components of predicted capital flows, 1980-2000 - Portfolio approach



Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author's calculations.

Two characteristics of the portfolio approach contribute therefore to the resolution of the puzzle. (i) Long-term productivity catch-up has a positive effect on capital outflows in the portfolio approach, through the catch-up and human wealth component. As a consequence, the positive correlation between productivity growth and capital outflows (see Figure 2), which was at the core of the puzzle in the riskless approach, contributes to solving the puzzle in the portfolio choice specification. (ii) A novel effect, specific to the portfolio approach, appears: the change in portfolio structure. Countries with a higher initial capital share in the portfolio are expected to receive less capital from abroad. This is the case in the data, as shown in Figure 2.

5 Extensions

The main shortcoming of the portfolio approach is that it overestimates the magnitude of flows by several orders of magnitude. This section aims at diminishing this discrepancy by providing extensions to the portfolio approach: (i) sovereign risk, (ii) differing amounts of production risk σ in emerging countries and the rest of the world and (iii) the presence of hand-to-mouth workers. These extensions all aim at diminishing the portfolio change component, which is the main source of the discrepancy, in particular by affecting human wealth. This section does not seek quantitative relevance but tries to propose some potential directions for further research aiming at reconciling the magnitude of flows in the data and in the portfolio approach.

5.1 Sovereign risk

Sovereign risk is introduced through a risk premium on the country's bond liabilities. If the country is indebted, then it faces an interest rate equal to $R^*(1+\epsilon)$, where $\epsilon > 0$. The parameter ϵ is supposed to be common across developing countries and reflects the level of sovereign risk, that is the probability that the economy defaults on its debt. If the country has positive bond holdings, then it faces the world's interest rate R^* without risk premium. This assumption is made to illustrate the fact that there is no default risk on the rest of the world's bond liabilities, since it is composed mainly of industrial countries with sound institutions. The risk premium is introduced in an ad hoc way and it is supposed moreover that it does not depend on the amount of debt. This is justified since this extension does not target quantitative relevance but aims rather at showing whether the magnitude of flows is indeed reduced when introducing sovereign risk through a fixed premium on bond liabilities. The idea behind this is that a higher interest rate might reduce human wealth and therefore limit the ability of households to hold huge amounts of debt.

This extension does not require to change the baseline parameters. ϵ only has to be defined. It is set at 1% in order to satisfy the constraint $R^*(1+\epsilon) - \rho - g^* > 0$.

Consider Table 4. It represents the actual and predicted net capital outflows and their components, for the baseline portfolio approach and for the various extensions presented in this

Table 4: Predicted and actual capital flows between 1980 and 2000 - Extensions of the portfolio approach

Capital flows	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(share of initial output)	$\frac{\Delta B}{Y_0}$	$\widehat{\frac{\Delta B}{Y_0}}$	$\widehat{\frac{\Delta B^c}{Y_0}}$	$\widehat{\frac{\Delta B^i}{Y_0}}$	$\widehat{\frac{\Delta B^p}{Y_0}}$	$\widehat{\frac{\Delta B^h}{Y_0}}$	Obs.
Baseline portfolio approach	-0,54	-101,54	-2,55	23,56	-138,42	15,80	66
Sovereign risk	-	-41,88	-8,65	79,97	-86,27	15,73	-
Asymmetric production risk	-	-57,00	-5,39	49,82	-117,24	15,80	-
HTM workers	-	-40,22	-0,82	7,54	-51,77	4,83	-
HTM workers (asymmetric)	-	-13,56	-0,82	7,54	-22,70	2,42	-

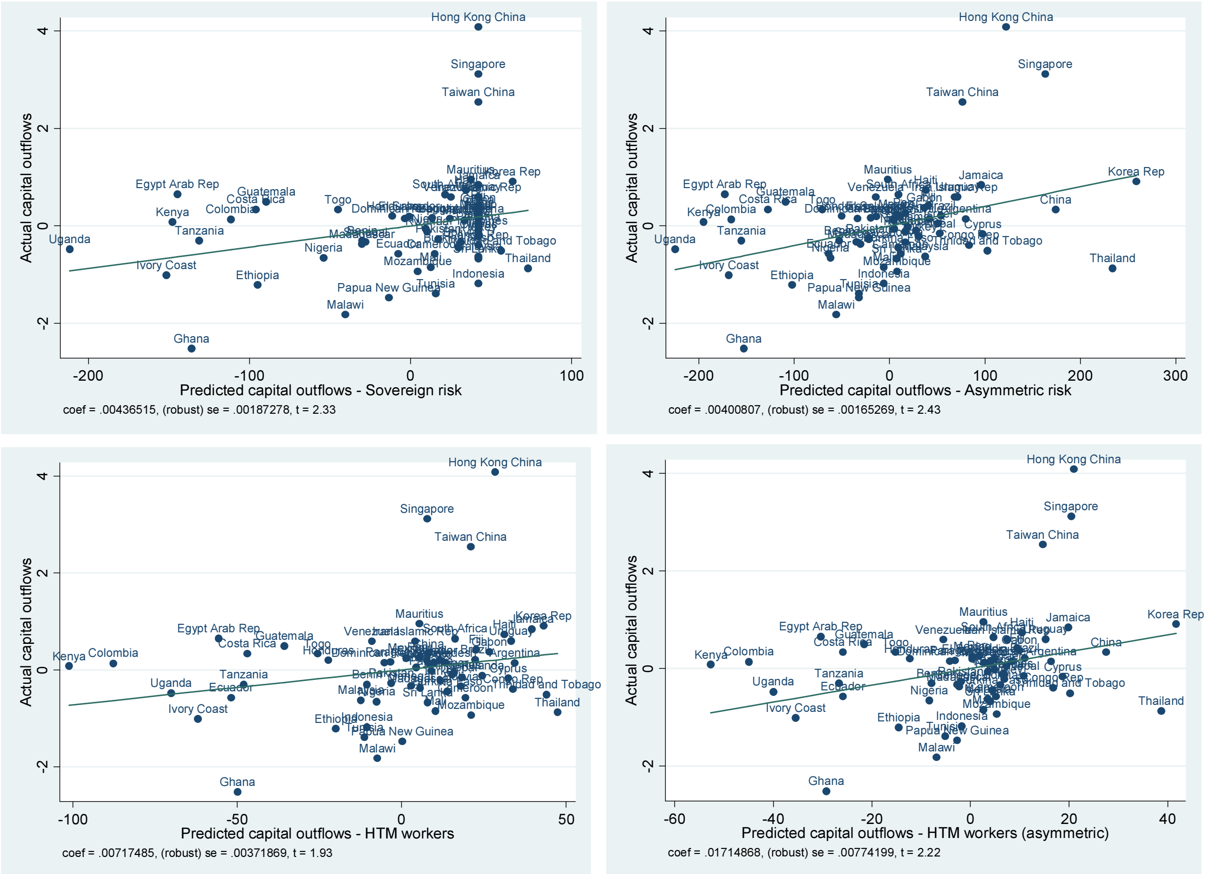
$\frac{\Delta B}{Y_0}$ is the observed ratio of net capital outflows to initial output, predicted capital flows $\widehat{\frac{\Delta B}{Y_0}}$ and its components $\widehat{\frac{\Delta B^c}{Y_0}}$, $\widehat{\frac{\Delta B^i}{Y_0}}$, $\widehat{\frac{\Delta B^p}{Y_0}}$ and $\widehat{\frac{\Delta B^h}{Y_0}}$ are given by (19).

The figures are unweighted country averages.Sample: Non-OECD countries, including also Korea, Mexico and Turkey.

section. Despite the low sovereign risk premium, the mean predicted inflows in column (2) diminish by more than a half as compared to the baseline specification. This comes mainly from the increase in capital outflows implied by productivity catch-up (that is the catch-up component in column (3)) and the diminution of the capital inflows implied by portfolio change (that is the portfolio component in column (5)). The increase in the perceived interest rate on bonds has two effects that could explain this. First, the steady-state share of capital ϕ^* diminishes, which means that households want to hold more safe assets in the end of period. This magnifies the impact of productivity catch-up on bond holdings, which explains the effects on the catch-up component: an increase in capital is matched by a further increase in bond assets. Second, the initial level of human wealth $\tilde{h}_0$ diminishes because the discount factor increases, so the estimated initial share of capital ϕ_0 is higher. As a result, the share of safe assets in the beginning-of-period is lower. This implies a smaller decrease in bond holdings during the period, holding human wealth constant, which explains the smaller capital inflows reported in the portfolio component.

However, if the magnitude of flows is lower on average, this is not true when looking into details. Figure 7 shows clearly that the dispersion of predicted flows is similar to the baseline case. This can be explained as follows. Sovereign risk might reduce the volatility of the portfolio term, but it increases the dispersion between countries with negative and positive catch-up because it magnifies the impact of capital accumulation on bond holdings.

Figure 7: Actual capital outflows (as a share of initial GDP), against the different components of predicted capital flows, 1980-2000 - Extensions of the portfolio approach



Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author's calculations.

5.2 Asymmetric production risk

In the baseline portfolio approach, the amount of idiosyncratic production risk has been assumed to be identical across countries. The parameter σ has been set to 0.3 in all countries, following microeconomic empirical studies in the US and Eurozone. The world interest rate R^* has been set in order to match the steady-state share of capital in the US ϕ^{*US}. Assuming identical risk implies that the share of capital in developing countries should catch-up with the US. However, financial markets are often less developed in developing than in industrial countries, and therefore less able to insure investors against their individual risk. We assume then, for illustrative purposes, that the amount of individual risk in developing countries is twice as high as in the US (that is, $\sigma = 0.6$). As a consequence, the corresponding steady-state share of capital in the portfolio ϕ^* is smaller, and the portfolio adjustment term, which was the main source of the excessive magnitude of predicted flows, should be less negative, because the adjustment in safe assets should be milder.

The increase in the level of idiosyncratic risk has similar effects as in the sovereign risk case. The diminution in ϕ^* has a positive impact on capital outflows due to productivity catch-up (column (4) of Table 4) and a negative effect on capital inflows due to portfolio change (column (5) of Table 4). As a result, the total predicted inflows are smaller than in the baseline case by a significant margin (a little more than one third). However, as in the sovereign risk extension, predicted outflows are still dispersed, as shown by Figure 7.

5.3 Hand-to-mouth workers

We have assumed so far that all households had access to financial assets, capital and bonds. However, a significant share of the population holds no assets and has a limited ability to borrow. As in Angeletos and Panousi (2007), hand-to-mouth workers are introduced to capture in a crude way this heterogeneity among households. The population is composed of two groups: "investors", who supply labor, invest in productive capital and have access to the bond market; and "hand-to-mouth workers", who supply labor but do not hold any asset, and consume their entire labor income at each period. A notable consequence of this extension is that investors hold only a fraction of the country's human wealth, which should reduce their amount of debt.

I follow Angeletos and Panousi (2007) in setting the proportion of hand-to-mouth workers in the US so that their share of aggregate consumption is equal to 50%. This gives a proportion of 70%: only 30% of the population have access to financial assets. This new calibration yields a higher initial and steady-state portfolio share of capital (respectively ϕ_0 and ϕ^*). Then, it is assumed in a first step that the proportion of investors in developing countries is identical to the US (symmetric case). In a second step, the proportion of investors is set at 15%, to represent the fact that financial markets are less accessible in developing countries (asymmetric case). This has no additional impact on the end-of-period share of capital in the portfolio ϕ^*. However, it will have an impact on the initial one, through the initial investors' human wealth. As a consequence, the portfolio change component should be less important in the asymmetric than in the symmetric case.

According to column (2) of Table 4, the decrease in the predicted level of capital inflows in the symmetric case is of the same magnitude as in the extension with sovereign risk. This time, the diminution comes mainly from the portfolio term in column (5). The key element is the increase in the initial share of capital in portfolio ϕ_0 due to the diminution in the amount of human wealth held by investors. The decrease in safe assets during the period is therefore mitigated. The diminution in the capital outflows due to human wealth adjustment (column (6)) comes simply from the fact that investors hold only a fraction of the country's human wealth. Contrary to the case with sovereign risk and asymmetric production risk, capital outflows due to productivity catch-up (column (4)) diminish, because ϕ^* increases. Households hold less bonds in their portfolio, so the impact of productivity catch-up on bond accumulation is alleviated. As a consequence, the dispersion of predicted flows is also diminished, while the correlation between actual and predicted flows is still significantly positive, as illustrated in Figure 7.

In the asymmetric case, the predicted inflows decrease further, and are now one order of magnitude lower than in the baseline specification. The origin of this decrease lies again in the portfolio term, thanks to the further increase in the initial share of capital in the portfolio ϕ_0. As a consequence of the decline in the share of human wealth held by investors, the human wealth component decreases. The catch-up and convergence components do not change as compared to the symmetric case, since the steady-state share of capital ϕ^* is unaffected. As shown by Figure 7, the cross-country dispersion of predicted flows is also diminished.

6 Conclusion

This paper develops an extension of the traditional neoclassical growth model to risky investment that changes the predictions in terms of capital flows, and therefore contributes to match the actual ones and to solve the puzzle highlighted by Gourinchas and Jeanne (2007). In particular, the puzzle is solved because the portfolio approach can account for two main facts: (i) countries that experienced higher productivity growth experienced more capital outflows; (ii) countries with larger initial shares of capital in their portfolio also experienced more capital outflows. While fact (i) is at the core of the puzzle inside the baseline model, it contributes to solve it inside the portfolio approach. Fact (ii) can be accounted for only inside the portfolio approach, where the long-run share of capital in the portfolio is determined.

The advantage of the portfolio approach is that it does not constitute a great departure from the textbook model and therefore allows the adoption of a similar development accounting approach to Gourinchas and Jeanne (2007). The portfolio approach appears then more promising than the riskless one in explaining the allocation of capital flows among developing countries. This shows that international financial markets have to be considered not only as a financing source, but also as a way to provide insurance in the presence of domestic investment risk.

However, while the portfolio approach explains better the *direction* of flows than the riskless one, it fails to account for their *magnitude*, which is overestimated by several orders of magnitude. From that point of view, the portfolio approach fares worse than the riskless one. Still, this problem of magnitude in the portfolio choice model is commonly come across in the literature. It can be related to the findings of Kraay et al. (2005) on the magnitude of North-South bond position. In this paper, some potential explanations have been proposed to enrich the model and account for the discrepancy between actual and predicted flows: sovereign risk, asymmetric production risk and the presence of hand-to-mouth workers. The extension with hand-to-mouth workers appears to be the more promising one because it reduces both the average amount of predicted capital inflows and their cross-country dispersion. A challenge for future research is to reconcile both the direction and the magnitude of flows. Another potential candidate is to model productivity growth as a random walk. This would have two effects: human wealth would be diminished since positive productivity shocks would be unanticipated, which would diminish the countries' ability to borrow; bonds would constitute a larger share of the portfolio since it would be the only safe asset. This is left for future research.

Another direction for research consists in checking whether the portfolio approach can also account for the composition of flows. Extending the model to the possibility to trade equity could lead to predictions in terms of equity holdings. According to portfolio choice models, the more productive assets should constitute a higher share both in the domestic and foreign portfolio, which would explain why direct foreign investment is still positively correlated with productivity growth, as shown in Gourinchas and Jeanne (2007).

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7 Appendix

Proof of Lemma 1

Maximizing U_t^i is equivalent to maximizing $V_t^i = U_t^i/N_t = E_t \int_t^\infty e^{-(\rho-n)(s-t)} \ln(c_s^i) ds$.

Indexes are now dropped for simplicity. Define ϕ such that $\phi = k/\omega$. The constraint of the maximization problem is therefore: $d\omega = [(r - R^*)\phi + R^*)\omega - c - n\omega]dt + \sigma\phi\omega dz$.

The Bellman equation for this problem is:

$$(\rho - n)V_t = \max_{c,\phi} \left\{ \ln(c_t) + \frac{\partial V_t}{\partial t} \right\}$$

Then, applying Ito's Lemma, we obtain:

$$(\rho-n)V(\omega,t) = \max_{c,\phi} \left\{ \ln(c) + \frac{\partial V(\omega,t)}{\partial t} + \frac{\partial V(\omega,t)}{\partial \omega} [((r - R^*)\phi + R^*)\omega - c - n\omega] + \frac{\partial^2 V(\omega,t)}{\partial \omega^2} \frac{1}{2} \phi^2 \omega^2 \sigma^2 \right\}$$

The first-order conditions of this problem are:

$$\frac{1}{c} - \frac{\partial V(\omega,t)}{\partial \omega} = 0$$

$$\frac{\partial V(\omega,t)}{\partial \omega}(r - R^*) + \frac{\partial^2 V(\omega,t)}{\partial \omega^2} \phi\omega\sigma^2 = 0$$

An educated guess for the general form of the value function is:

$$V(\omega,t) = \frac{\ln(\omega)}{\chi} + \psi$$

where χ and ψ have to be determined.

Substituting the derivatives of the value function into the first order conditions yields the solutions:

$$c = \chi\omega \qquad \text{and} \qquad \phi = \frac{r - R^*}{\sigma^2}$$

Plugging these expressions into the Bellman equation yields $\chi = \rho - n$ and $\psi = \ln(\rho - n) + (r - R^*)^2/2\sigma^2 + R^* - \rho$. This gives Equations (7) and (8).

Proof of Lemma 2

Proof of (i):

(9) is derived as follows. Lemma 1 states that $c_t^i = (\rho - n)\omega_t^i$. Since every family has the same number of members, then the country's average wealth per capita is equal to the average of families' wealth per capita: $\omega_t = \int_0^1 \omega_t^i$. As a consequence, the country's consumption per capita is equal to a fraction of the country's wealth per capita: $c_t = (\rho - n)\omega_t$. This implies that they grow at the same rate: $\dot{c}_t/c_t = \dot{\omega}_t/\omega_t$. When aggregating across households, risk disappears and

Equation (6) gives: $\dot{\omega}_t/\omega_t = \bar{R}_t - \rho$, where $\bar{R}_t = \phi r_t + (1-\phi)R^*$ the average return on wealth. As a consequence, we can derive the aggregate Euler condition in per capita terms:

$$\frac{\dot{c}_t}{c_t} = \bar{R}_t - \rho \tag{20}$$

Now, using the definition of π_t (2), the growth rate of productivity $\dot{A}_t/A_t$ is equal to $\dot{\pi}_t + g^*$. Then , applying the definition of $\tilde{c}_t$, we obtain:

$$\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = \frac{\dot{c}_t}{c_t} - \frac{\dot{A}_t}{A_t} = \bar{R}_t - \rho - (\dot{\pi}_t + g^*)$$

Equation (10) is obtained as follows. From aggregating Equation (6) and rearranging terms, using the fact that $r_t k_t = (1-\tau)\alpha F(k_t, A_t) - \delta k_t$, $w_t + z_t = (1-\alpha+\tau\alpha)F(k_t, A_t)$, one obtains the following resource constraint of the economy in per capita terms:

$$\dot{k}_t + \dot{b}_t = F(k_t, A_t) - \delta k_t + R^* b_t - c_t - n(k_t + b_t)$$

The Wiener process disappears from the aggregate resource constraint since by assumption $\int_0^1 dv_t^i = 0$. Then, using $\dot{k}_t/k_t = \dot{\tilde{k}}_t/\tilde{k}_t + \dot{A}_t/A_t$, we obtain:

$$\dot{\tilde{k}}_t + \dot{\tilde{b}}_t = f(\tilde{k}_t) - \delta\tilde{k}_t + R^*\tilde{b}_t - \tilde{c}_t - (n + \dot{\pi}_t + g^*)(\tilde{k}_t + \tilde{b}_t)$$

Equation (11) is derived by aggregating the investment policy rule when $\sigma > 0$ (8) across households and using the definition of ω_t: $\omega_t = k_t + h_t + b_t$. Then, we obtain

$$k_t = \frac{\phi_t}{1-\phi_t}(h_t + b_t)$$

Dividing each side by A_t yields Equation (11).

Equation (12) is derived in the same way as Equation (10), starting from the law of evolution of human wealth (4).

Proof of (ii):

Consider Equation (8). When σ goes to zero, ϕ goes to infinity as long as $r_t > R^*$. The only possible equilibrium outcome when $\sigma = 0$ is therefore $r_t = R^*$.

When $\sigma = 0$, the same consumption policy rule (7) and the same budget constraint (6) hold. Therefore, the evolution of consumption obeys to the Euler conditions in efficient unit of labor and in per capita terms (9) and (20), only with $\bar{R}_t = R^*$. However, the investment rule obeys to the arbitrage condition $r_t = R^*$. By using $r_t = R^*$ and the definition of r_t, we obtain the optimal value of capital (13).

Proof of Proposition 1

Proof of (i):

To obtain Equation (14), write Equation (9) for $\dot{\tilde{c}}/\tilde{c} = 0$ and $\dot{\pi} = 0$:

$$\bar{R} - \rho - g^* = 0$$

This implies, after replacing $\bar{R}$:

$$R^* - \rho - g^* + \phi(r - R^*) = 0$$

And after replacing ϕ and rearranging, we obtain:

$$\frac{(r - R)^2}{\sigma^2} = \rho + g^* - R^*$$

This implies that $R^* \leq \rho + g^*$ necessarily at steady state. Again, after replacing r and rearranging, we obtain Equation (14). Finally, in order to rule out $R^* = \rho + g^*$, notice that it would imply $\phi = 0$ at steady state, which means that $\tilde{k}^* = 0$. This is impossible since $\lim_{\tilde{k} \to 0} f'(\tilde{k}) = +\infty$, which contradicts Equation (14).

Equation (15) derives from the portfolio rule (11). We only have to determine $\tilde{h}$ at steady state. $\tilde{h}_t = \int_0^\infty e^{-R^* s} \frac{N_{t+s} A_t t+s}{N_t A_t t}(1 - \alpha + \tau\alpha) f(\tilde{k}_{t+s}) ds = \int_0^\infty e^{-(R^* - (n+g^*))s + \pi_s - \pi_t}(1 - \alpha + \tau\alpha) f(\tilde{k}_{t+s}) ds$. Equation (14) gives $\tilde{k}^*$, the steady-state value of $\tilde{k}$. We have also $\pi_t = \pi$ in the long run. Therefore,

$$\tilde{h}^* = \frac{(1 - \alpha + \tau\alpha) f(\tilde{k}^*)}{R^* - (n + g^*)}$$

Replacing $\tilde{h}^*$ in Equation (11) yields Equation (15).

Proof of (ii):

When $\sigma = 0$, Equation (9) becomes:

$$\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = R^* - \rho - (\dot{\pi}_t + g^*)$$

Applying the definition of steady state, this yields:

$$R^* = \rho + g^*$$

Equation (16) is only another way to write the arbitrage condition $r_t = R^*$ or (13).

Equation (17) is inferred from the budget constraint (10) and the Euler condition (20) with $\bar{R}_t = R^*$. Equation (10) can be rewritten as follows:

$$\dot{\tilde{k}}_t + \dot{\tilde{b}}_t = (1 - \alpha + \tau\alpha)) f(\tilde{k}_t) + R^*(\tilde{k}_t + \tilde{b}_t) - \tilde{c}_t - (n + \dot{\pi}_t + g^*)(\tilde{k}_t + \tilde{b}_t)$$

Write the Euler condition (20) when $R^* = \rho + g^*$:

$$\frac{\dot{c}_t}{c_t} = g^*$$

Therefore, $c_t = c_0 e^{g^* t}$, and $\tilde{c}_t = \tilde{c}_0 e^{g^* t} A_0 / A_t = \tilde{c}_0 e^{-\pi_t}$. As a consequence, we obtain at steady state: $\tilde{c}^* = \tilde{c}_0 e^{-\pi}$. We know also that $\tilde{k}_t = \tilde{k}^*$ always. We thus have at steady state:

$$\dot{\tilde{b}}_t = (1 - \alpha + \tau\alpha) f(\tilde{k}^*) + R^* (\tilde{k}^* + \tilde{b}_t) - \tilde{c}_0 e^{-\pi} - (n + g^*)(\tilde{k}^* + \tilde{b}_t)$$

Since $\rho > n$, then $R^* > n + g^*$, so the only non-explosive solution for $\tilde{b}_t$ is:

$$\tilde{b}_t = \tilde{b}^* = -\tilde{k}^* - \frac{(1 - \alpha + \tau\alpha) f(\tilde{k}^*) - \tilde{c}_0 e^{-\pi}}{R^* - (n + g^*)}$$

Hence the result. To derive $\tilde{c}_0$, we use the intertemporal budget constraint at $t = 0$:

$$\int_0^\infty e^{-R^* t} N_t c_t dt = \int_0^\infty e^{-R^* t} N_t (w_t + z_t) dt + N_0 (k_0 + b_0)$$

Replacing $w_t + z_t$ by $(1 - \alpha + \tau\alpha) A_t f(\tilde{k}^*)$, using the fact that N_t grows at rate n, that $A_t = A_0 e^{\pi_t + g^* t}$ and that $c_t = c_0 e^{g^* t}$, we obtain:

$$\int_0^\infty e^{-(R^* - (n+g^*))t} c_0 dt = \int_0^\infty e^{-(R^* - (n+g^*))t + \pi_t} A_0 (1 - \alpha + \tau\alpha) f(\tilde{k}^*) dt + k_0 + b_0$$

which implies:

$$\frac{c_0}{R^* - (n + g^*)} = A_0 \left((1 - \alpha + \tau\alpha) f(\tilde{k}^*) \int_0^\infty e^{-(R^* - (n+g^*))t + \pi_t} dt + k_0 + b_0 \right)$$

The final result is obtained by rearranging terms and replacing R^* by $\rho + g^*$.

Proof of Proposition 2

Notice that, with or without risk, when T is sufficiently large, the predicted capital flows must satisfy:

$$\frac{\Delta B}{Y_0} = e^{\pi + (n+g^*)T} \frac{\tilde{b}^*}{y_0} - \frac{\tilde{b}_0}{y_0} \tag{21}$$

Proof of (i):

Replacing the expression for $\tilde{b}^*$ (17) in Equation (21) and substituting for $\tilde{c}_0$, we obtain:

$$\frac{\overline{\Delta B}}{Y_0} = \frac{\tilde{k}_0 - \tilde{k}^*}{\tilde{k}_0^\alpha} e^{(n+g^*)T} + \left(e^{(n+g^*)T} - 1 \right) \frac{\tilde{b}_0}{\tilde{k}_0^\alpha} - (e^\pi - 1) \frac{\tilde{k}^*}{\tilde{k}_0^\alpha} e^{(n+g^*)T}$$

$$-e^{\pi + (n+g^*)T} (1 - \alpha + \alpha\tau) f(\tilde{k}^*) \left(\frac{1}{\rho - n} - \int_0^\infty e^{-(\rho - n)t + \pi_t - \pi} dt \right)$$

Since $1/(\rho - n) = \int_0^\infty e^{-(\rho - n)t} dt$, this expression leads to Equation (18).

Proof of (ii):

In order to derive Equation (19), we use Equation (21) where $\tilde{b}^*$ is replaced using Equation (15) and $\tilde{b}_0$ is given by the portfolio rule in $t = 0$:

$$\tilde{b}_0 = \frac{1-\phi_0}{\phi_0}\tilde{k}_0 - \tilde{h}_0$$

This yields:

$$\frac{\Delta B}{Y_0} = \frac{1-\phi}{\phi}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha} + \frac{1-\phi}{\phi}\frac{\tilde{k}^*}{\tilde{k}_0^\alpha}\left(e^{\pi+(n+g^*)T} - 1\right) + \frac{\tilde{k}_0}{\tilde{k}_0^\alpha}\left(\frac{1}{\phi} - \frac{1}{\phi_0}\right) - \frac{e^{\pi+(n+g^*)T}\frac{(1-\alpha+\tau\alpha)f(\tilde{k}^*)}{R^*-(n+g^*)} - \tilde{h}_0}{\tilde{k}_0^\alpha}$$

Besides:

$$\tilde{h}_0 = \int_0^\infty e^{-R^*t}(1-\alpha+\tau\alpha)\frac{N_tA_t}{N_0A_0}f(\tilde{k}_t)dt$$

$$= \int_0^T e^{-R^*t}(1-\alpha+\tau\alpha)\frac{N_tA_t}{N_0A_0}f(\tilde{k}_t)dt + \int_T^\infty e^{-R^*t}(1-\alpha+\tau\alpha)\frac{N_tA_t}{N_TA_T}\frac{N_TA_T}{N_0A_0}f(\tilde{k}_t)dt$$

$$= \int_0^T e^{-(R^*-(n+g^*))t+\pi_t}(1-\alpha+\tau\alpha)f(\tilde{k}_t)dt + \int_T^\infty e^{-(R^*-(n+g^*))t}(1-\alpha+\tau\alpha)e^{(n+g^*)T+\pi}f(\tilde{k}^*)dt$$

$$= \int_0^T e^{-(R^*-(n+g^*))t+\pi_t}(1-\alpha+\tau\alpha)f(\tilde{k}_t)dt + e^{(n+g^*)T+\pi}\frac{(1-\alpha+\tau\alpha)f(\tilde{k}^*)}{R^*-(n+g^*)}$$

Hence the final result.

Productivity and consumption smoothing

Under Assumption 3, $\overline{\frac{\Delta B^s}{Y_0}}$ can be written as follows:

$$\overline{\frac{\Delta B^s}{Y_0}} = -e^{(n+g^*)T}\frac{(1-\alpha+\alpha\tau)\tilde{k}^{*\alpha}}{\tilde{k}_0^\alpha}\int_0^T e^{-(\rho-n)t}(e^\pi - e^{\pi f(t)})dt$$

A sufficient condition for $\overline{\frac{\Delta B^s}{Y_0}}$ to be decreasing in π is that $e^\pi - e^{\pi f(t)}$ is increasing in π. We have:

$$\frac{\partial\left[e^\pi - e^{\pi f(t)}\right]}{\partial\pi} = e^\pi\left[1 - f(t)e^{\pi(f(t)-1)}\right]$$

Consider the term between brackets:

$$\frac{\partial\left[1 - f(t)e^{\pi(f(t)-1)}\right]}{\partial f(t)} = -e^{\pi(1-f(t))}[1+\pi f(t)]$$

A sufficient condition for this derivative to be negative is $\pi \geq -1$. If it is the case, then for $0 \leq f(t) \leq 1$:

$$1 - f(t)e^{\pi(f(t)-1)} \geq 0$$

which implies that $\frac{\partial\left[e^\pi - e^{\pi f(t)}\right]}{\partial\pi} \geq 0$. As a consequence, if $\pi \geq -1$, then $\overline{\frac{\Delta B^s}{Y_0}}$ is decreasing in π.