EconomiX
http://economix.u-paris10.fr/

Document de Travail
Working Paper
2008-28

The Spaceship Problem Re-Examined

Pierre-André Jouvet
Pierre Pestieau
Grégory Ponthière

UMR 7166 CNRS

Université Paris X-Nanterre
Maison Max Weber (bâtiments K et G)
200, Avenue de la République
92001 NANTERRE CEDEX

Tél et Fax : 33.(0)1.40.97.59.07
Email : secretariat-economix@u-paris10.fr

Université Paris X Nanterre

The Spaceship Problem Re-Examined

P.A. JOUVET*, P. PESTIEAU†, and G. PONTHIERE‡

October 28, 2008

Abstract

This paper re-examines the spaceship problem, i.e. the design of the optimal population under the environmental constraint of a fixed area available for life, by focusing on the dilemma between adding new beings and extending the life of existing beings. For that purpose, we characterize, under the assumption that individual lifetime welfare depends positively on the length of life but negatively on population density, the preference ordering of a utilitarian planner over lifetime-equal histories, i.e. pairs of initial population size and survival conditions yielding an equal number of life periods. It is shown that a Benthamite planner is not necessarily indifferent between lifetime-equal histories, and that a Millian utilitarian planner prefers lifetime-equal histories yielding the smallest population with the longest life. The solutions under Critical-level and Number-dampened utilitarianisms are also shown to differ significantly.

Keywords: environmental congestion, fertility, longevity, population ethics, utilitarianism.

JEL codes: D63, Q56, Q57.

*EconomiX, University Paris X and CORE.

†University of Liege, CREPP, CORE, PSE and CEPR.

‡PSE and Ecole Normale Supérieure, Paris. [corresponding author] Address: Ecole Normale Supérieure, Department of Social Sciences, boulevard Jourdan, 48, 75014 Paris, France. E-mail: gregory.ponthiere@ens.fr

1 Introduction

Given that the Earth is of finite size - and, thus, can be compared, following Boulding (1966), to a 'spaceship' -, the question of the optimal population size can be formulated as the 'spaceship problem': are there too few, or, on the contrary, too many people living on our bounded, resources-finite, spaceship? Undoubtedly, that question is an old issue, to which various answers were provided over time.

While Mercantilism was, during the 16th and 17th centuries, promoting a population as large as possible by means of various policies, it should not be deduced from this that the spaceship constraint was ignored by Mercantilist thought.[1] Actually, it is quite the opposite: a standard Mercantilist argument was that the finiteness of the living space, if coupled with a large population, would favour migrations, and, hence, the colonization of other countries.

The Italian philosopher Giovanni Botero (1589) is generally regarded as one of the first thinkers who argued against the Mercantilist populationism, and asked the question of the optimal population size. Botero argued that men have a tendency to multiply themselves as much as nature allows them. However, natural resources are limited, so that, according to Botero, one cannot escape the following adjusments: either people will modify their behaviours, or there will be some adjustment in numbers through famines, diseases or wars.

Within economic thought, Richard Cantillon (1755) provided another early study of the spaceship problem, by highlighting the existence of a quantity of life *versus* quality of life trade-off, which restraints feasible population sizes. According to Cantillon, Man's subsistance requires living space, whose amount depends on lifestyles, so that an arbitrage is to be made between the quantity and quality of life.[2] While Cantillon did not solve that trade-off, he showed that, contrary to Mercantilists' beliefs, more people is *not* always better.[3]

Another contribution to the spaceship problem was made by Thomas Malthus (1798), who argued that the population size is necessarily limited within some boundaries. A population would follow a geometric progression if left unchecked (i.e. in the absence of resources constraints), but the production of means of subsistance follows, at best, an arithmetical progression (because of the finiteness of land), so that the population must, at some point, be 'checked', either by a positive check (deaths), or by a preventive check (fewer children).[4]

Given that Botero, Cantillon and Malthus's positive theories of population, by underlining the constraints imposed by the finiteness of land, only restrict the set of *feasible* population sizes, these leave open the normative question of

[1] On this, see Schumpeter (1954, 1, pp. 352-356).

[2] See Cantillon's comparison of peasants living modestly in the South of France with grown-up *bourgeois* living in abundance (1755, I, 15 p. 25). Cantillon argued also that land-owners, by deciding the allocation of land between different uses, influence the set of feasible population sizes.

[3] See Cantillon (1755, I, 15, p. 30):

> 'It is also a question outside of my subject whether it is better to have a great multitude of inhabitants, poor and badly provided, than a small number, much more at their ease: a million who consume the produce of 6 acres per head or 4 million who live on the product of an acre and a half.'

[4] Note that, in later writings, Malthus (1830) will add a third type of check: the moral restraint.

the *optimal* population size.

That issue was widely studied within utilitarianism. While Jeremy Bentham's (1789) Classical utilitarianism recommends a number of people producing 'the greatest happiness of the greatest number', John Stuart Mill (1861), in his refined utilitarianism, recommends the maximization not of *total* welfare, but of welfare *on average* (i.e. average utility), which differs from Benthamite utilitarianism in different numbers choices. However, as shown by Parfit (1984), those two criteria of population ethics are unsatisfactory. Classical utilitarianism suffers from the Repugnant Conclusion, whereas Mill's utilitarianism faces the Mere Addition Paradox.[5] Hence, other population ethics criteria were developed, such as Critical-level utilitarianism (Blackorby and Donaldson, 1984) and Number-dampened utilitarianism (Ng, 1986). But although (partly) immunized against Parfit's criticisms, these suffer from other weaknesses. Imposing a low critical level still implies the Repugnant Conclusion, while a high critical level leads to what Arrhenius and Bykvist (1995) call the Sadistic Conclusion.[6] Moreover, Number-dampened utilitarianism might lack intuitive support.

Whereas those discussions focused on the optimal number of people to be brought to life, another dimension of the problem was recently explored by Broome (2004). In his *Weighing Lives*, Broome argued that the issue of the optimal *number* of people is not independent from the issue of the optimal *length* of life. True, making a person exist and making an existing person live more are, from a moral point of view, two distinct things, but these tend both to influence the number of existing people at a given point in time, so that population ethics could hardly focus on births and ignore deaths. Hence, in order to study the spaceship problem, the two ends of the 'demographic chain' - births and deaths - must be taken into account.

Although that survey suggests that much has been writen on the optimal population of our spaceship, this paper aims at casting a new light on that old problem. The specificity of this paper with respect to the existing literature is precisely to consider simultaneously the two ends of the demographic chain: *births and deaths*. Our motivation lies in the fact that longevity is not fixed, but endogenous, so that the spaceship problem is not exclusively about bringing new people to life. The maintenance of existing people in life is also relevant for the spaceship problem. This is why longevity will also be considered here.

Therefore, we propose a re-examination, from a utilitarian perspective, of the spaceship problem, in which both births and deaths are endogenous. The question raised is the following. Suppose that a population, which enjoys a long life in a large living space, has to share a space of finite size. What do the optimal population and the optimal longevity look like?

To answer that question, we shall make here some significant simplifications, so that the present study should only be regarded as a first step towards a

[5] The Repugnant Conclusion is defined as follows: for any population with some welfare per head, there exists a larger population of individuals with a very low welfare per head, but which is regarded as better. The Mere Addition Paradox consists of regarding as undesirable the addition of a group of people with an average welfare that is only slightly lower than the initial average welfare, even if additional people have a life worth living and do not affect the welfare of the initial group of people.

[6] The Sadistic Conclusion consists of the mere fact that fixing a high critical utility level may make one prefer a situation where a small population exhibits an extremely low utility level to a situation where a very large population is just below the critical level, which is counter-intuitive under a high critical level. See also Bykvist (2007).

characterization of the solution of the spaceship problem.

First, we shall leave aside the production of goods, which, since Malthus's work, has occupied a central place in the study of the spaceship problem. Actually, given the observed growth of production per head despite the large population growth, it is not obvious that production limitations due to the finiteness of land can still be justified.[7] Thus, in our model, the numbers's pressure will come only from population density, which affects welfare negatively through a direct effect (see Cramer *et al*, 2004), as well as through a limitation of individual longevity due to congestion.

Moreover, in order to formalize the longevity *versus* fertility trade-offs, we shall study the solutions of the spaceship problem by characterizing the preferences of a social planner on lifetime-equal histories, defined as pairs of initial population size and survival conditions yielding an equal number of life periods. Note that reasoning on the basis of a fixed total number of life periods is only an analytically convenient way to account for longevity *versus* fertility trade-offs, which will be complemented, in future works, by an explicit modelling of the transformation function characterizing such trade-offs.

Finally, given the intuitive weaknesses faced by the various criteria of population ethics (see Blackorby *et al*, 2005), it makes sense to re-examine the spaceship problem not on the basis of a single criterion, but, rather, by comparing the social optima under several ethical criteria. Because of space constraints, we shall here focus only on some widely used normative criteria, such as Classical utilitarianism, Millian (or Average) utilitarianism, Critical-level utilitarianism and Number-dampened utilitarianism.[8] Thus, this paper shall only compare the social planner's orderings on lifetime-equal histories under those ethical criteria, and neglect other criteria, which is also a significant simplification.

The rest of the paper is organized as follows. Section 2 formalizes the problem. Section 3 characterizes the preferences of a Benthamite utilitarian social planner on lifetime-equal histories. A similar task is carried out in Section 4 under a Millian social planner. Sections 5 and 6 concentrate on how Critical-level utilitarianism and Number-dampened utilitarianism solve the spaceship problem. Concluding remarks are drawn in Section 7.

2 The spaceship problem

2.1 Assumptions

The spaceship problem can be formulated as follows. Suppose that a planet of finite surface $\bar{Q}$ is available for life.

Axiom 1 *A surface of finite surface $\bar{Q}$ is available for the life of the population.*

For simplicity, it is assumed that only living persons occupy a significant space on that planet.[9]

[7] Actually, as Heilig (1994) rightly argues in the light of (mistaken) past demographic studies on the maximum carrying capacity of the Earth in terms of population, the finiteness of space does not seem to be a problem for production, so that we can abstract from that constraint.

[8] Naturally, this paper has no pretension to exhaustiveness, and will concentrate on some major normative criteria under varying population size. See Blackorby *et al* (2005) for a synthesis of population ethics.

[9] In other words, the dead are supposed to occupy no space.

Axiom 2 *Only living persons occupy a part of the available space* $\bar{Q}$.

Moreover, all living persons at a particular point in time $t = 0, 1, 2, ..., \infty$, whatever their age is, are assumed to occupy an *equal* amount of space, defined as the total space divided by the number of people alive at that time.

Axiom 3 *Each person alive at time* t *enjoys an equal share of the total available space:*

$$q_t = \frac{\bar{Q}}{L_t}$$

where L_t *denotes the population size at time* t*, whereas* q_t *denotes the space available per person.*

Thus we shall abstract here from problems of intragenerational distribution of space.

Here are the assumptions we shall make on individual welfare.

Axiom 4 *At each period, the welfare of a person who is alive depends on the space available per person, according to the function:*

$$u_t = u(q_t) = \left(\frac{\bar{Q}}{L_t}\right)^{\sigma} \quad \forall t < T$$

where $0 \leq \sigma \leq 1$ *and* T *denotes the time of death of the person.*

Axiom 5 *At each period, the welfare of a person who is dead equals 0:*

$$u_t = 0 \;\; \forall t > T$$

Axiom 6 *The lifetime welfare* w *of a person is the sum of the utilities associated to each period:*

$$w = \sum_{t=0}^{T} u\left(q_t\right)$$

where T *denotes the time of death of the person.*

The first assumption - exclusive focus on space - is made for analytical conveniency. The second assumption allows us to avoid difficulties associated with the possibility of infinite utility. The third assumption is, although largely questioned by Bommier (2005), still a standard one. This is used here for analytical conveniency.

In the present model, a high number of people - and thus a low space per head - can come not only from a large number of births, but, also, from a large length of life. To model this, we shall, for simplicity, assume that all births are concentrated on the first period.

Axiom 7 *All births are concentrated on the first period,* $t = 0$*, and form the initial population* L *of finite size.*

By doing so, we avoid discussions on whether all agents should be born at an early point in time or, on the contrary, should all be brought to life at the latest possible moment. This simplification is not problematic for the purpose at hand, as our goal is to focus on trade-offs between births and deaths, whatever the precise timing of births is.

Regarding the modeling of the survival process, we shall assume that, whereas the whole initial population L will enjoy the first period of life, only a proportion S of that population will survive up to the second period, and, then, only a proportion S of the surviving population will enjoy a third period of life, etc.

Axiom 8 *At each passage of time, a fraction S of the population alive at a period survives to the next.*

Thus, under those assumptions, the population at time t can be written as:

$$L_t = S^t L$$

where $0 < S < 1$ is the proportion of survivors within a cohort after the passage of a unitary period of time. Actually, Axiom 8 amounts to assume that the strength of mortality is constant over time.

Note that, under Axiom 8, the total number of periods P lived by a population of initial size L is:

$$\begin{aligned} P &= L + SL + S^2 L + ... \\ &= L\left(1 + S + S^2 + ...\right) \\ &= L\frac{1}{1-S} \end{aligned}$$

Thus, if S tends towards 0, P tends towards L: the number of periods lived equals the number of persons. However, if S tends towards 1, P tends towards $+\infty$: as there is no death, the number of periods lived by a group of strictly positive size is infinite.

Finally, regarding the definition of space per head q_t, it follows from the previous formulae that, if the proportion of survivors S tends to 1, the space available per person is constant over time, and equal to $\bar{Q}/L$, as:

$$\lim_{S\to 1} q_t = \lim_{S\to 1} \frac{\bar{Q}}{S^t L} = \frac{\bar{Q}}{1^t L} = \frac{\bar{Q}}{L}$$

In that case, the population that is alive initially will remain forever in the spaceship, and will enjoy the same space conditions during their whole life.

On the contrary, if S tends towards 0, the space available per person q_t tends to be infinite for any $t > 0$, as:

$$\lim_{S\to 0} q_t = \lim_{S\to 0} \frac{\bar{Q}}{S^t L} = \frac{\bar{Q}}{0^t L} = \frac{\bar{Q}}{0} = +\infty$$

Indeed, as S tends towards 0, the survivors can enjoy an infinitely large space at each period of survival.

2.2 Problem setting

In order to study how a social planner would solve the spaceship problem, defined here as the choice of a number of persons and a proportion of survivors under the space constraint, we shall, in the rest of this paper, assume that the social planner has to choose among a set of histories, defined as follows.

Definition 9 *A history is a pair $\{L, S\}$, where L is the size of the population at time 0 $(0 < L)$, and S is the proportion of survivors from a period t to a period $t+1$ $(0 < S < 1)$.*

In order to describe the planner's solution, we shall concentrate on a particular subset of the set of histories: *lifetime-equal* histories, defined as follows.

Definition 10 *Two histories $\{L, S\}$ and $\{L', S'\}$ are lifetime-equal if and only if these exhibit an equal total number of periods lived (i.e. $P = P'$).*

Thus, we shall concentrate on histories $\{L, S\}$ and $\{L', S'\}$ such that:

$$L\frac{1}{1-S} = L'\frac{1}{1-S'}$$

Obviously, under $P = P'$, $L \gtreqless L' \iff S \lesseqgtr S'$. In other words, for a given total number of periods lived, if one lifetime-equal history exhibits a higher initial population than another lifetime-equal history, it must also necessarily exhibit a lower survival probability, *et vice versa*.

Note that comparing two lifetime-equal histories $\{L, S\}$ and $\{L', S'\}$ amounts to compare two histories for which there is an equality of the ratios:

$$\frac{L}{L'} = \frac{1-S}{1-S'}$$

that is, there is an equality of the ratios of initial populations (LHS) and strengths of mortality (RHS) for histories $\{L, S\}$ and $\{L', S'\}$ having an equal number of periods lived (i.e. for which $P = P'$). Intuitively, if one history exhibits a larger initial population than another history with the same total number of periods lived (i.e. $L > L'$), it must also be characterized by a larger mortality (i.e. $1 - S > 1 - S'$).

3 The Classical utilitarian solution

According to Classical utilitarianism, as firstly stated by Bentham (1789), all actions - at the individual and institutional levels - should be chosen in such a way as to produce the 'greatest happiness of the greatest number', in conformity with the Principle of Utility. That principle must also govern the choice of histories in general, and, in particular, of lifetime-equal histories.[10]

[10] Note that the Benthamite objective function does not involve any discounting here, in conformity with Ramsey's (1928) views. Note also that the above formula presents social welfare by aggregating across time periods rather than persons (as usually done), but this is equivalent to aggregating across people, because of the additivity of lifetime welfare.

Definition 11 *A social planner is Benthamite if and only if, when facing two histories* $\{L,S\}$ *and* $\{L',S'\}$*, he prefers the history yielding the highest total welfare, that is,*

$$\begin{aligned}\{L,S\}\succeq\{L',S'\} &\iff \textstyle\sum_{t=0}^{\infty} L_t u(q_t) \geq \sum_{t=0}^{\infty} L'_t u(q'_t)\\ &\iff \textstyle\sum_{t=0}^{\infty} LS^t u(\frac{\bar{Q}}{LS^t}) \geq \sum_{t=0}^{\infty} L'S'^t u(\frac{\bar{Q}}{L'S'^t})\end{aligned}$$

What will be the attitude of a Benthamite social planner in front of the spaceship problem? Actually, it is tempting to believe that a Classical utilitarian social planner will be indifferent between lifetime-equal histories. The intuition behind that belief is that, from a Benthamite perspective, whether there exist a large number of people with short lives, or, on the contrary, a small number of people with long lives, should not matter that much. But such an intuition is untrue, as there is an additional dimension to the problem here: the impact of population density on individual welfare.

Contrary to what one may believe, a Benthamite social planner is not necessarily indifferent between lifetime-equal histories. His preferences are characterized in the following proposition.

Proposition 12 *Consider two lifetime-equal histories* $\{L,S\}$ *and* $\{L',S'\}$*, with* $L>L'$ *and* $S<S'$*. The preferences of the Benthamite social planner can be characterized as follows:*

$$\{L,S\}\succeq\{L',S'\} \iff \left(\tfrac{1-S}{1-S'}\right)^{1-\sigma} \geq \tfrac{1-S^{1-\sigma}}{1-S'^{1-\sigma}}$$

Proof. The Benthamite social planner prefers $\{L,S\}$ on $\{L',S'\}$ if and only if total welfare is larger under $\{L,S\}$ than under $\{L',S'\}$.

Social welfare is equal to

$$\begin{aligned}&L\left(\frac{\bar{Q}}{L}\right)^{\sigma}+SL\left(\frac{\bar{Q}}{SL}\right)^{\sigma}+S^2L\left(\frac{\bar{Q}}{S^2L}\right)^{\sigma}+...+...\\ =\;&L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\left[1+S^{1-\sigma}+S^{2(1-\sigma)}+...+...\right]\\ =\;&L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\left[1+s+s^2+...+...\right]\\ =\;&L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\frac{1}{1-s}\end{aligned}$$

where $s\equiv S^{1-\sigma}$.

Hence, social welfare is larger under $\{L,S\}$ than under $\{L',S'\}$ iff:

$$\begin{aligned}L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\frac{1}{1-S^{1-\sigma}} &\geq L'\left(\frac{\bar{Q}}{L'}\right)^{\sigma}\frac{1}{1-S'^{1-\sigma}}\\ L^{1-\sigma}\frac{1}{1-S^{1-\sigma}} &\geq L'^{1-\sigma}\frac{1}{1-S'^{1-\sigma}}\end{aligned}$$

Given that $\frac{L}{1-S}=\frac{L'}{1-S'}$, we have $\left(\frac{L}{L'}\right)^{1-\sigma}=\left(\frac{1-S}{1-S'}\right)^{1-\sigma}$. Thus social welfare is larger under $\{L,S\}$ than under $\{L',S'\}$ iff:

$$\left(\frac{1-S}{1-S'}\right)^{1-\sigma}\geq\frac{1-S^{1-\sigma}}{1-S'^{1-\sigma}}$$

■

Thus, contrary to what one expects *a priori*, the Benthamite social planner is not necessarily indifferent between lifetime-equal histories. Note also that the Benthamite planner's preferences on lifetime-equal histories are independent from the size of the available space $\bar{Q}$. The only parameter that determines the planner's preferences on two lifetime-equal histories $\{L, S\}$ and $\{L', S'\}$ is the parameter σ describing individual taste for space.

In the general case where $0 \neq \sigma \neq 1$, it is not obvious to see whether the Benthamite social planner prefers a high population with low survival or the opposite. This indeterminacy comes from the influence of population density on individual welfare. Initially, individual welfare per period is larger in $\{L', S'\}$, as the initial population is less numerous. However, after some periods, the higher mortality in $\{L, S\}$ makes population density fall there much more than under $\{L', S'\}$, so that the time profiles of utility per period differ significantly between $\{L, S\}$ and $\{L', S'\}$. Those different temporal utility paths explain why the preference parameter σ plays such an important role. Actually, the parameter σ, by governing the concavity of temporal welfare, is a major determinant of the spaceship problem regarded as a problem of intertemporal allocation of persons.

While it is not obvious at all to see whether the Benthamite social planner prefers lifetime-equal histories that exhibit a larger population and a lower survival or the opposite, it is possible to see what his preferences would be in the extreme case where $\sigma = 0$.

Corollary 13 *Under a constant temporal welfare - i.e. $\sigma = 0$ -, the Benthamite social planner is indifferent between any two lifetime-equal histories.*

Proof. *This corollary follows from the above condition. Fixing $\sigma = 0$ in*

$$\left(\frac{1-S}{1-S'}\right)^{1-\sigma} \geq \frac{1-S^{1-\sigma}}{1-S'^{1-\sigma}}$$

yields:

$$\frac{1-S}{1-S'} \geq \frac{1-S}{1-S'}$$

which is a strict equality, so that $\{L, S\} \sim \{L', S'\}$ for any lifetime-equal histories $\{L, S\}$ and $\{L', S'\}$. ■

The intuition behind that corollary is the following. If temporal welfare is constant, the quality of life periods does not matter at all: only the number of periods lived matters, so that lifetime-equal histories are regarded as equivalent. In other words, when $\sigma = 0$, the effect of population density is completely neutralized, so that having many people with short lives or few people with long lives is the same.

Finally, note also that, if $\sigma = 1$, the social planner cannot rank lifetime-equal histories, as the above condition does not yield any conclusion.

Corollary 14 *Under a linear temporal welfare - i.e. $\sigma = 1$ -, there can be no ranking of the Benthamite social planner over any two lifetime-equal histories.*

Proof. *Fixing $\sigma = 1$ in the above condition yields:*

$$1 \geq \frac{0}{0}$$

which cannot be said to be true or false, as the RHS is indeterminate. ■

Thus, if individual utility is linear in space, the spaceship problem has no solution under Classical utilitarianism.

4 Millian utilitarianism

Although accepted as an ethical basis in various issues, the classical utilitarian doctrine faces intuitive difficulties in the field of population ethics. A major critique of Classical utilitarianism in the context of population choices consists of Parfit's (1984) Repugnant Conclusion: for any population with a significant welfare per head, there exists a larger population of individuals with a very low welfare level, but which is ranked as better by Classical utilitarianism (the rise in the quantity of lives compensating the fall in the quality of lives).

One alternative criterion, which avoids the Repugnant Conclusion, is Millian utilitarianism, which recommends to maximize not total welfare, but *average* welfare. As this is well-known, this makes a difference when considering population issues. In the present context, the problem of a Millian social planner is to choose a history $\{L, S\}$ in such a way as to maximize the average utility within the population.

Definition 15 *A social planner is Millian if and only if, when facing two histories $\{L, S\}$ and $\{L', S'\}$, he prefers the history yielding the highest average welfare (i.e. the highest welfare per existing person), that is,*

$$\begin{aligned} \{L,S\} \succeq \{L',S'\} &\iff \tfrac{1}{L}\textstyle\sum_{t=0}^{\infty} L_t u(q_t) \geq \tfrac{1}{L'}\sum_{t=0}^{\infty} L'_t u(q'_t) \\ &\iff \textstyle\sum_{t=0}^{\infty} S^t u(\frac{\bar{Q}}{LS^t}) \geq \sum_{t=0}^{\infty} S'^t u(\frac{\bar{Q}}{L'S'^t}) \end{aligned}$$

How would a Millian social planner rank lifetime-equal histories? Is there here some indeterminacy as in the Benthamite case? As stated below, this is clearly not the case, as a Millian social planner, when facing lifetime-equal histories, chooses necessarily the history with the lower population and the higher survival.

Proposition 16 *Consider two lifetime-equal histories $\{L, S\}$ and $\{L', S'\}$, with $L > L'$ and $S < S'$. The preferences of the Millian social planner can be characterized as follows:*

$$\{L, S\} \preceq \{L', S'\}$$

Proof. Under Millian utilitarianism, social welfare is equal to average welfare

$$\begin{aligned} & \frac{L\left(\frac{\bar{Q}}{L}\right)^{\sigma} + SL\left(\frac{\bar{Q}}{SL}\right)^{\sigma} + S^2L\left(\frac{\bar{Q}}{S^2L}\right)^{\sigma} + ... + ...}{L} \\ = & \left(\frac{\bar{Q}}{L}\right)^{\sigma}\left[1 + S^{1-\sigma} + S^{2(1-\sigma)} + ... + ...\right] \\ = & \left(\frac{\bar{Q}}{L}\right)^{\sigma}\left[1 + s + s^2 + ... + ...\right] \\ = & \left(\frac{\bar{Q}}{L}\right)^{\sigma}\frac{1}{1-s} \end{aligned}$$

where $s \equiv S^{1-\sigma}$.

Hence, social welfare is larger under $\{L,S\}$ than under $\{L',S'\}$ iff:

$$\begin{aligned} \left(\frac{\bar{Q}}{L}\right)^{\sigma} \frac{1}{1-S^{1-\sigma}} &\geq \left(\frac{\bar{Q}}{L'}\right)^{\sigma} \frac{1}{1-S'^{1-\sigma}} \\ \left(\frac{L'}{L}\right)^{\sigma} &\geq \frac{1-S^{1-\sigma}}{1-S'^{1-\sigma}} \end{aligned}$$

Given that $\left(\frac{L'}{L}\right)^{\sigma} = \left(\frac{1-S'}{1-S}\right)^{\sigma}$, we have

$$\left(\frac{1-S'}{1-S}\right)^{\sigma} \geq \frac{1-S^{1-\sigma}}{1-S'^{1-\sigma}}$$

which is always false, as the LHS is smaller than 1, whereas the RHS exceeds 1. Hence we have always $\{L,S\} \preceq \{L',S'\}$. ■

Thus, contrary to what prevailed under Benthamite utilitarianism, the Average utilitarian criterion leads to the unambiguous selection of histories with few people and extremely long lives. That result is independent from the total surface available, and from the preference parameter σ.

The intuition behind that result goes as follows. Under a given number of periods lived, it is always better, for a Millian social planner, to concentrate these on the smallest number of individuals. True, the high survival implies that population density will hardly fall over time, so that temporal welfare per period will hardly grow. But the few existing persons will enjoy this quasi-constant temporal welfare during a long period of time, which is, on average, much better than the situation where more people would enjoy it over a shorter period.

5 Critical-level utilitarianism

Whereas Millian utilitarianism avoids the Repugnant Conclusion, it suffers from another weakness, which was called by Parfit (1984) the Mere Addition Paradox. Clearly, Average utilitarianism regards as undesirable the addition of a group of people with an individual welfare that is slightly lower than the initial average welfare, even if the additional people have a life worth living, and even if the added people do not influence in any way the lives of the initial group.

In order to avoid that counter-intuitive result - but without facing the Repugnant Conclusion -, one solution proposed by Blackorby and Donaldson (1984) consists of summing, instead of absolute utilities, utilities *net* of the utility level that makes a life neutral. This yields Critical-level utilitarianism: a rise in the population is desirable if the welfare of additional people exceeds the critical level, and undesirable when it is lower than it.[11]

Under Critical-level utilitarianism, the problem of the social planner is to choose a history $\{L,S\}$ in such a way as to maximize the sum of all cohorts's net welfares. In conformity with Blackorby and Donaldson's intuition, an agent's welfare is here defined not absolutely, but with respect to some critical level of welfare $\hat{u}$ ($\hat{u} \geq 0$) making life - as a whole - neutral. A life with a welfare higher

[11] As this is well-known, Benthamite utiltiarianism amounts to Critical-level utilitarianism under a zero critical level, while Millian utilitarianism equals Critical level utilitarianism when the critical level equals the average welfare.

than $\hat{u}$ is regarded as socially desirable, whereas a life with a lower welfare level is regarded as undesirable. A life with $u = \hat{u}$ is 'neutral'.

Definition 17 *A social planner is Blackorbian if and only if, when facing two histories* $\{L, S\}$ *and* $\{L', S'\}$*, he prefers the history yielding the highest net welfare, that is,*

$$\{L,S\} \succeq \{L',S'\} \iff \sum_{t=0}^{\infty} L_t u(q_t) - L\hat{u} \geq \sum_{t=0}^{\infty} L'_t u(q'_t) - L'\hat{u}$$
$$\iff L\left[\sum_{t=0}^{\infty} S^t u(\tfrac{\bar{Q}}{LS^t}) - \hat{u}\right] \geq L'\left[\sum_{t=0}^{\infty} S'^t u(\tfrac{\bar{Q}}{L'S'^t}) - \hat{u}\right]$$

where $\hat{u}$ *is the critical welfare level, making a whole life neutral.*

It is crucial here to distinguish the introduction of that neutral level for existence from the introduction of what Broome (2004) calls a 'neutral level for *continuing* existence' (i.e. a critical level defined for each period of life). The introduction of such a critical level for continuing existence would not alter the Benthamite condition for $\{L, S\} \succeq \{L', S'\}$, unlike what is the case under the introduction of a lifetime critical level $\hat{u}$.[12]

The preferences of a Blackorbian social planner on lifetime-equal histories are characterized by the following proposition.

Proposition 18 *Consider two lifetime-equal histories* $\{L, S\}$ *and* $\{L', S'\}$*, with* $L > L'$ *and* $S < S'$*. The preferences of the Blackorbian social planner can be characterized as follows:*

$$\{L,S\} \succeq \{L',S'\} \iff (\bar{Q})^{\sigma}\left[\tfrac{L^{1-\sigma}}{1-S^{1-\sigma}} - \tfrac{L'^{1-\sigma}}{1-S'^{1-\sigma}}\right] \geq \hat{u}(L-L')$$

Proof. The Blackorbian social planner prefers $\{L, S\}$ on $\{L', S'\}$ if and only if social welfare is larger in the former.

Social welfare is here equal to

$$\begin{aligned} & L\left[\left(\frac{\bar{Q}}{L}\right)^{\sigma} + S\left(\frac{\bar{Q}}{LS}\right)^{\sigma} + S^2\left(\frac{\bar{Q}}{LS^2}\right)^{\sigma} + \ldots\right] - L\hat{u} \\ = \; & L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\left[1 + S^{1-\sigma} + S^{2(1-\sigma)} + \ldots + \ldots\right] - L\hat{u} \\ = \; & L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\left[1 + s + s^2 + \ldots + \ldots\right] - L\hat{u} \\ = \; & L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\frac{1}{1-s} - L\hat{u} \end{aligned}$$

where $s \equiv S^{1-\sigma}$.

Hence, social welfare is larger under $\{L, S\}$ than under $\{L', S'\}$ iff:

$$\begin{aligned} L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\frac{1}{1-S^{1-\sigma}} - L\hat{u} \quad & \geq \quad L'\left(\frac{\bar{Q}}{L'}\right)^{\sigma}\frac{1}{1-S'^{1-\sigma}} - L'\hat{u} \\ L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\frac{1}{1-S^{1-\sigma}} - L'\left(\frac{\bar{Q}}{L'}\right)^{\sigma}\frac{1}{1-S'^{1-\sigma}} \quad & \geq \quad \hat{u}(L-L') \end{aligned}$$

[12] To see this, note that social welfare under such a critical level $\tilde{u}$ is: $L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\left[1 + S^{1-\sigma} + S^{2(1-\sigma)} + \ldots + \ldots\right] - L\tilde{u}\left[1 + S + S^2 + \ldots\right]$
$= L\left(\frac{\bar{Q}}{L}\right)^{\sigma}\frac{1}{1-s} - L\hat{u}\frac{1}{1-S}$.
Hence, $\{L,S\} \succeq \{L',S'\} \iff \frac{L^{1-\sigma}}{1-S^{1-\sigma}} - \frac{L'^{1-\sigma}}{1-S'^{1-\sigma}} \geq 0$
as under Benthamite utilitarianism.

Thus social welfare is larger under $\{L, S\}$ than under $\{L', S'\}$ iff:

$$(\bar{Q})^{\sigma} \left[\frac{L^{1-\sigma}}{1 - S^{1-\sigma}} - \frac{L'^{1-\sigma}}{1 - S'^{1-\sigma}} \right] \geq \hat{u} \left(L - L' \right)$$

■

Note that the condition for $\{L, S\} \succeq \{L', S'\}$ collapses to the Benthamite condition under $\hat{u} = 0$. Moreover, under $\hat{u}$ equal to average lifetime welfare, the condition for $\{L, S\} \succeq \{L', S'\}$ vanishes to

$$\begin{aligned}
(\bar{Q})^{\sigma} \left[\frac{L^{1-\sigma}}{1 - S^{1-\sigma}} - \frac{L'^{1-\sigma}}{1 - S'^{1-\sigma}} \right] &\geq \sum_{t=0}^{\infty} \left(S^t \left(\frac{\bar{Q}}{LS^t} \right)^{\sigma} \right) (L - L') \\
(\bar{Q})^{\sigma} \left[\frac{L^{1-\sigma}}{1 - S^{1-\sigma}} - \frac{L'^{1-\sigma}}{1 - S'^{1-\sigma}} \right] &\geq \left(\frac{\bar{Q}}{L} \right)^{\sigma} \frac{1}{1 - S^{1-\sigma}} (L - L') \\
\left[\left(L' \frac{1 - S}{1 - S'} \right)^{1-\sigma} - L'^{1-\sigma} \left(\frac{1 - S'}{1 - S} \right)^{\sigma} \frac{L - L'}{L'} \right] &\geq L'^{1-\sigma} \frac{1 - S^{1-\sigma}}{1 - S'^{1-\sigma}} \\
\left(\frac{1 - S'}{1 - S} \right)^{\sigma} &\geq \frac{1 - S^{1-\sigma}}{1 - S'^{1-\sigma}}
\end{aligned}$$

which is never satisfied, as under Average utilitarianism. One can thus interpret Critical-level utilitarianism as a generalization of both Benthamite and Millian utilitarianism.

Note also that, in comparison with the optimality condition under Benthamite utilitarianism, the RHS of the condition for $\{L, S\} \succeq \{L', S'\}$ is likely to be larger than zero, so that the Blackorbian social planner tends to exhibit a preference ordering on lifetime-equal histories that differs significantly from the one of the Benthamite social planner. More precisely, whereas the Benthamite social planner prefers $\{L, S\}$ over $\{L', S'\}$ if and only if $\frac{L^{1-\sigma}}{1-S^{1-\sigma}}$ exceeds $\frac{L'^{1-\sigma}}{1-S'^{1-\sigma}}$, the Blackorbian social planner requires $\frac{L^{1-\sigma}}{1-S^{1-\sigma}}$ to exceed $\frac{L'^{1-\sigma}}{1-S'^{1-\sigma}}$ to a larger extent, which makes the preference for histories with large populations and low longevity less likely. This is the natural corollary of the introduction of a critical utility level.

Thus, without surprise, the critical utility level $\hat{u}$ affects the planner's ordering of lifetime-equal histories: the larger $\hat{u}$ is, the more likely is the preference for histories with a small number of persons and a long life. This dependency suggests that how the spaceship problem is solved depends crucially on the selection of an adequate critical utility level. Recent debates on how the critical utility level should be fixed (see Crisp, 2007 and Broome, 2007) are thus not neutral as far as the spaceship problem is concerned.

One should also notice here that the available space $\bar{Q}$ does, under $\sigma \neq 0$, influence the social planner's ordering on lifetime-equal histories, unlike what used to be the case under Classical and Millian utilitarianisms. Clearly, for a given critical utility level, the larger the available space is, the more likely the preference for a lifetime-equal history with a larger population is. This dependency on the available space is quite surprising, because both L and S affect the space per person, so that one does not expect $\bar{Q}$ to influence the ranking between $\{L, S\}$ and $\{L', S'\}$, which exhibit, by construction, the same number of periods lived. But contrary to the intuition, the critical level does influence the solution of the spaceship problem.

6 Number-dampened utilitarianism

While Critical-level utilitarianism has become widely used as a criterion in population ethics, it can only avoid the Repugnant Conclusion and the Mere Addition Paradox for extremely-well selected critical levels $\hat{u}$. If the critical level is too low, the Repugnant Conclusion arises, whereas if it is too high, the Mere Addition Paradox or the Sadistic Conclusion prevails. This observation - and the difficulties to select a single value of $\hat{u}$ - encouraged the development of other criteria not relying on a critical utility level.[13]

One of those alternative criteria is Ng's (1986) Number-dampened utilitarianism.[14] The value function under Number-dampened utilitarianism is equal to the average utility multiplied by a positive-valued function of the population size. When that function is a multiple of the population size, that criterion is equivalent to Benthamite utilitarianism, whereas, if that function is a constant, number-dampened utilitarianism is equivalent to Millian utilitarianism.

Under number-dampened utilitarianism, the problem of a social planner is to choose $\{L, S\}$ in such a way as to maximize the sum of all cohorts's welfares. Following Ng's intuitions, the welfare of a cohort is defined as the product of the average utility in that cohort and a concave transform of its size.

Definition 19 *A social planner is Ngian if and only if, when facing two histories $\{L, S\}$ and $\{L', S'\}$, he prefers the history yielding the highest average welfare multiplied by a concave transform of the population brought into existence, that is,*

$$\begin{aligned}\{L, S\} \succeq \{L', S'\} &\iff L^{\gamma}\left[\tfrac{1}{L}\sum_{t=0}^{\infty} L_t u(q_t)\right] \geq L'^{\gamma}\left[\tfrac{1}{L'}\sum_{t=0}^{\infty} L'_t u(q'_t)\right] \\ &\iff L^{\gamma}\sum_{t=0}^{\infty} S^t u\left(\tfrac{\bar{Q}}{LS^t}\right) \geq L'^{\gamma}\sum_{t=0}^{\infty} S'^t u\left(\tfrac{\bar{Q}}{L'S'^t}\right)\end{aligned}$$

where $0 \leq \gamma \leq 1$.

Let us now consider how an Ngian social planner would rank lifetime-equal histories $\{L, S\}$ and $\{L', S'\}$. As stated in the proposition below, the preferences of an Ngian social planner on lifetime-equal histories can be characterized in a simple way.

Proposition 20 *Consider two lifetime-equal histories $\{L, S\}$ and $\{L', S'\}$,with $L > L'$ and $S < S'$. The preferences of the Ngian social planner can be characterized as follows: $\{L, S\} \succeq \{L', S'\} \iff \left(\frac{1-S}{1-S'}\right)^{\gamma-\sigma} \geq \frac{1-S^{1-\sigma}}{1-S'^{1-\sigma}}$.*

Proof. The Ngian social planner prefers $\{L, S\}$ on $\{L', S'\}$ if and only if social welfare is larger in the former.

[13] One solution is the adherence to Critical-band utilitarianism (see Balckorby *et al*, 2005), which allows some interval of individual welfare levels between which a life is neutral.

[14] On Number-dampened utilitarianism, see also Hurka (1983).

Social welfare is here equal to

$$\begin{aligned}
& L^{\gamma}\left[\sum_{t=0}^{\infty} S^t \left(\frac{\bar{Q}}{LS^t}\right)^{\sigma}\right] \\
= & L^{\gamma}\left(\frac{\bar{Q}}{L}\right)^{\sigma}\left[1+S^{1-\sigma}+S^{2(1-\sigma)}+\ldots+\ldots\right] \\
= & L^{\gamma}\left(\frac{\bar{Q}}{L}\right)^{\sigma}\left[1+s+s^2+\ldots+\ldots\right] \\
= & L^{\gamma}\left(\frac{\bar{Q}}{L}\right)^{\sigma}\frac{1}{1-s}
\end{aligned}$$

where γ is lower than 1. In conformity with Ng's intuitions, the marginal effect of each additional person in terms of welfare is decreasing with the number of persons.

Hence, social welfare is larger under $\{L,S\}$ than under $\{L',S'\}$ iff:

$$\begin{aligned}
L^{\gamma}\left(\frac{\bar{Q}}{L}\right)^{\sigma}\frac{1}{1-S^{1-\sigma}} &\geq L'^{\gamma}\left(\frac{\bar{Q}}{L'}\right)^{\sigma}\frac{1}{1-S'^{1-\sigma}} \\
L^{\gamma-\sigma}\frac{1}{1-S^{1-\sigma}} &\geq L'^{\gamma-\sigma}\frac{1}{1-S'^{1-\sigma}} \\
\left(\frac{1-S}{1-S'}\right)^{\gamma-\sigma} &\geq \frac{1-S^{1-\sigma}}{1-S'^{1-\sigma}}
\end{aligned}$$

■

Note that, under γ equal to 1, we are back to the condition prevailing under Benthamite utilitarianism. Alternatively, under γ equal to 0, we are back to the Average utilitarian condition for $\{L,S\} \succeq \{L',S'\}$. Number-dampened utilitarianism is thus an alternative generalization of standard utilitarian criteria, along with Critical-level utilitarianism.

However, a fundamental difference with respect to the latter lies in the fact that the ordering of an Ngian planner on lifetime-equal histories is *independent* from the total space available $\bar{Q}$, unlike what used to be the case under Critical-level utilitarianism. Such an independence is surprising, as one may expect that, as a generalization of standard utilitarian criteria alternative to Critical-level utilitarianism, Number-dampened utilitarianism should exhibit a dependency on the available space, exactly as Critical-level utilitarianism does. But this is not the case, and this suggests that the dependency of the planner's ranking on $\bar{Q}$ is specific to the introduction of a critical utility level.

That significant difference between Critical-level utilitarianism and Number-dampened utilitarianism can play in favour of one or the other criterion, depending on whether one regards the dependency on the available space as a desirable property or not.

7 Concluding remarks

The goal of this paper was to cast a new light on the spaceship problem, by paying a particular attention to the trade-offs between adding new persons and extending the life of existing persons. For that purpose, we assumed that individual lifetime welfare is additive over time, and that temporal welfare depends negatively on population density, and we characterized, under those assumptions, the preference orderings of a utilitarian social planner on lifetime-equal histories, which all exhibit, by construction, an equal number of periods lived.

We showed that, contrary to the intuition, a Classical utilitarian planner is not necessarily indifferent between lifetime-equal histories, because of the non-constancy of population density over time, which, under non-linear utility in space per head, makes the Classical utilitarian planner prefer one history over another even though in each case the number of life periods is exactly the same. We showed also that an Average utilitarian planner would always opt for the history with the smallest number of people and the largest longevity. The Critical-level utilitarian solution and the Number-dampened utilitarian solution were also contrasted, and, quite surprisingly, were shown to differ in their treatment of the available space, the former being sensitive to it, while the later is insensitive to whether there is a large or a small space available for life.

While our re-examination of the spaceship problem casts a new light on that old issue, it should be stressed, however, that some refinements of this analysis could be made, to explore the sensitivity of our conclusions to the postulates on which these rely. Assumptions at the individual level, such as the zero utility from death, and the additivity of lifetime welfare, may be questioned, and one may want to know how the spaceship problem is solved once those postulates are relaxed. Hence this work is only a first stage in the re-examination of the spaceship problem, which invites many others.

8 References

Arrhenius, G. and Bykvist, K. (1995): *Interpersonal compensation and moral duties to future generations: moral aspects of energy use*, Uppsala Prints and Preprints in Philosophy, 21, Department of Philosophy, Uppsala University.

Bentham, J. (1789): *Introduction to the Principles of Morals and Legislation*, in J.S. Mill: Utilitarianism, edited by M. Warnock (1962), Fontana Press.

Blackorby, C. and Donaldson, D. (1984): 'Social criteria for evaluating population change', *Journal of Public Economics*, 25, pp. 13-33.

Blackorby, C., Bossert, W., and Donaldson, D. (2005): *Population Issues in Social Choice Theory, Welfare Economics, and Ethics*, Econometric Society Monograph, Cambridge University Press.

Bommier, A. (2005): 'Uncertain lifetime and intertemporal choice: risk aversion as a rationale for time discounting', *International Economic Review*, 95(1), pp. 277-291.

Botero, G. (1589): *Della Ragione di Stato*, edited by G. D. Tarino (1596).

Boulding, K. (1966): 'The Earth as a spaceship', Washington State University Commitee on Space Sciences, available at

http://www.colorado.edu/econ/Kennneth.Boulding/spaceship-earth.html.

Broome, J. (2004): *Weighing Lives*, Oxford University Press.

Broome, J. (2007): 'Replies', *Economics and Philosophy*, 23, pp. 115-124.

Bykvist, K. (2007): 'The good, the bad and the ethically neutral', *Economics and Philosophy*, 23, pp. 97-105.

Cantillon, R. (1755): *Essay on the Nature of Trade in General*, (new edition, INED, Paris, 1952).

Cramer, V., Torgersen, S., and Kringlen, E. (2004): 'Quality of life in a city: the effect of population density', *Social Indicators Research*, 69, pp. 103-116.

Crisp, R. (2007): 'Neutrality and pleasure', *Economics and Philosophy*, 23, pp. 81-88.

Heiling, G.K. (1994): 'How many people can be fed on Earth? in W. Lutz (ed.): *The Future Population of the World. What can we assume today?*, IIASA, Earthscan, London.

Hurka, T. (1983): 'Value and population size', *Ethics*, 93, April, pp. 496-507.

Malthus, T.R. (1798): *Essay on the Principle of Population*, reprint by Pelican Books (1970).

Mill, J.S. (1861): *Utilitarianism*, edited by M. Warnock (1962), Fontana Press.

Ng, Y.-K. (1986): 'Social criteria for evaluating population change: an alternative to the Blackorby-Donaldson criterion', *Journal of Public Economics*, 29, pp. 375-381.

Parfit, D. (1984): *Reasons and Persons*, Oxford University Press.

Ramsey, F.P. (1926): 'A mathematical theory of saving', *Economic Journal*, 38, pp. 543-549.

Schumpeter, J.A. (1954): *A History of Economic Analysis*, George Allen & Unwin Ltd.