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### The Dynamic Properties of Alternative Assumptions on Price Adjustment in New Keynesian Models

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# The Dynamic Properties of Alternative Assumptions on Price Adjustment in New Keynesian Models

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## Abstract

This paper presents a classification of the different new Phillips curves existing in the literature as a set of choices based on three assumptions: the choice of the structure of price adjustments (Calvo or Taylor), the presence of backward indexation, and the type of price contracts (fixed prices or predetermined prices). The paper suggests study of the dynamic properties of each specification, following different monetary shocks on the growth rate of the money stock. We develop the analytical form of the price dynamics, and we display graphics for the responses of prices, output, and inflation. We show that the choice made for each of the three assumptions has a strong influence on the dynamic properties. Notably, the choice of the price structure, while often considered as unimportant, is indeed the most influential choice concerning the dynamic responses of output and inflation.

*JEL Codes: E31; E52.*

## *Key words:*

New Keynesian Phillips Curves, Taylor Price Rule, Calvo Price Rule, Fixed Prices, Predetermined Prices, Disinflation policy.

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## Introduction

One of the central questions in the recent literature concerning inflation dynamics has been the ability of the New Keynesian Phillips curve (NKPC thereafter) to correctly reproduce the empirical impact of monetary shocks. This specification is attractive for several reasons: among others, its tractability and the existence of microfoundations. Moreover, as shown by Roberts (1995), the different assumptions on price adjustment existing in the literature (Taylor, 1980, Rotemberg, 1982, and Calvo, 1983) lead to the same structural relation between inflation and output. For these reasons, this specification, based on the assumption of fixed prices (FP thereafter), has been for some years something close to a "standard specification" (McCallum, 1997). It represents inflation as a pure forward looking variable. However, since the work of Ball (1994a) and Fuhrer and Moore (1995), the empirical plausibility of the NKPC has been strongly questioned. The main reason is that the forward-looking dynamics of the NKPC imply that inflation behaves like a "jump-variable". As a consequence, this model predicts an absence of inflation persistence, and a real neutrality of disinflation policies, while in reality, inflation is persistent and nearly all disinflations are costly<sup>1</sup>.

One way to improve the empirical validation of the NKPC is to introduce lagged inflation in the dynamics, for example assuming the presence of backward-looking agents (Gali and Gertler, 1999), or a backward-looking indexation mechanism (Woodford, 2003, Christiano, Eichenbaum and Evans, 2005). Recently, these "hybrid" Phillips curves, combining forward and backward inflation terms, have replaced the forward-looking NKPC as the new standard specification. Although the theoretical foundations of this lagged inflation term are unclear, it is often considered as a necessary condition for the reproduction of plausible inflation dynamics.

Some authors, like Ball, Mankiw and Reis (2005) criticize all models based on FP, and prefer to give up this assumption. Deploring the "sorry state" of the literature, these authors say that forward-looking New Keynesian models are at odds with the facts, and that hybrid models are even worse. In response to the apparent failures, they propose to replace the FP assumption by the sticky information one (Mankiw and Reis, 2002), which formally is a resurgence of the Fischer (1977) predetermined prices assumption (PP thereafter). They argue that this alternative outperforms the FP hypotheses. Notably, the predetermination of prices prevents inflation from jumping immediately after shocks.

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<sup>1</sup> Critics about the inflation dynamics implied by the NKPC are extensive in the literature (see among others Roberts, 1998, Walsh, 1998, Mankiw, 2001, Mankiw and Reis, 2002, Rudd and Wheelan, 2005).

However, the superior performance of PP models has been questioned by some recent papers. Analyzing the responses of FP and PP models built on the structure of price adjustment of Calvo, some authors conclude that the Mankiw and Reis PP model displays indeed a performance similar to the forward-looking NKPC (Devereux and Yetman, 2003) and to the "hybrid" Phillips curve (Trabandt, 2005). Woodford (2003) also uses forward and "hybrid" versions of the NKPC and finds that these specifications correctly fit the facts. Collard and Dellas (2003), and Dupor and Tsuruga (2005) show that giving up the Calvo price structure significantly lowers the performance of the PP model.

Because the choice of the structural form of the economy is important for the recommendations of monetary policy, the current controversy and the presence of many contradicting declarations is embarrassing. Despite the apparent similarities of these models, their predictions in terms of responses to monetary shocks are quite different and there is no agreement on their relative performances. One reason that could explain these contrasted results is the absence in the literature of an exhaustive synthesis summarizing the dynamic properties of these different models. Some comparative works exist, but either they do not compare models within a common framework (Nelson, 1998, Walsh, 1998, Jondeau and LeBihan, 2001), giving hardly comparable results, or the comparison only takes in account a limited set of alternatives<sup>2</sup> (Jeanne, 1998, Perea, 2001, Mankiw and Reis, 2002, Devereux and Yetman, 2003, Trabandt, 2005). Another reason is the frequent use of simulation methods (for example Dupor and Tsuruga, 2005, or De Walque, Smets and Wouters, 2005). The absence of analytical results does not clearly highlight the properties of each assumption on price adjustment, and conclusions remain uncertain.

In this paper, we will try to evaluate the relative performance of the most important new Phillips curves within a common framework, for different money shocks. We present both an analytical representation of the price dynamics, and some graphical illustrations. This allows for a better understanding of the dynamic properties of the different Phillips curves.

To do this comparative work, we adopt a classification of the different hypotheses presented in the literature based on three points: the first is the choice of the nature of the nominal rigidity (either PP or FP), the second is the choice of the price adjustment rule (either Calvo or Taylor), and the third is the presence or the absence of indexation. This permits to obtain comparable Phillips curves and to attribute the differences of response of each specification exclusively to the assumptions made on the three parameters presented. The other

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<sup>2</sup> Most of the time, these papers only focus on one of the schemes of price rigidity (Calvo or Taylor). The justification for this limitation comes from a general feeling of homogeneity in the reduced form of the models using different assumptions on price rigidity (Roberts, 1995).

elements of the economy are voluntarily simplified in order to highlight the properties of the price equation.

In addition to the survey, our work shows the following elements:

a) The choice of the price structure (Calvo or Taylor) has very important implications for the price and output dynamic. The influential paper by Roberts (1995) concluding to the unimportance of the specific type of price rigidity is misleading. For example, in the forward-looking FP models, the structure of Taylor implies structural expectation errors and a positive cost of disinflation absent from the structure of Calvo.

b) Except for the disinflation case, the introduction of indexation does not by itself add significantly more persistence. The choice of the price structure (Calvo or Taylor) is more important. In other words, hybrid sticky-prices models have very different dynamic properties concerning persistence. Under the Taylor pricing rule, indexation does not add much persistence. Under the Calvo pricing rule, it can raise significantly the degree of persistence.

c) Contrary to the affirmation of Ball, Mankiw and Reis (2005), the hybrid sticky price model built on the structure of Calvo produces plausible responses to all the shocks we consider. The PP assumption needs the presence of the Calvo pricing rule to generate sufficient persistence.

This paper is organized as follows. Section 1 presents the derivation of the different Phillips curves. Section 2 presents the responses of the model to an auto-correlated shock on money growth. Section 3 is a study of the dynamic properties of each model consecutively to a disinflation policy. Section 4 suggests a comparison of our results with those of others important papers in the literature, and in the last section, we conclude.

## 1 PRESENTATION OF THE PHILLIPS CURVES

### 1.1 *The common set-up*

We follow a standard two-step procedure. In the first step, we present the level at which prices would be set if they were entirely flexible. We note  $p_t^*$  as the profit-maximizing price of a firm during the period  $t$  (all firms are identical). Following Romer (2001) or Kiley (2002a), in a monopolistic competition setup, absent wage rigidities, this price is given by:

$$p_t^* = p_t + \phi y_t \tag{1}$$

where  $p_t$  is the overall price level,  $y_t$  is the output gap and  $\phi$  is a measure of real rigidities. All variables are in logarithms.

In the second step we introduce nominal rigidities, with the standard assumption that firms can take new decisions on prices only when they receive signals of price changes. Each time a firm receives a signal, it sets an entire path of future prices. As signals of price changes are infrequent, we note  $\lambda_j$  the expected probability of having a new signal of price change  $j$  periods after setting the price path. We note  $x_{t,t+j}$  the price set at time  $t$  for the period  $t+j$ . The objective of the firm setting new prices is to minimize the sum of the differences between  $x_{t,t+j}$  and  $p_{t+j}^*$ . We also assume that prices can be indexed to past inflation (Woodford, 2003). We note  $\gamma$  the degree of indexation. Up to a linear approximation, a firm having a signal of price change during period  $t$  tries to minimize the following loss function<sup>3</sup>:

$$\underset{x_{t,t}, \dots, x_{t,t+j}}{\text{Min}} L_t = (x_{t,t} - p_t^*)^2 + \sum_{j=1}^{\infty} (1 - \lambda_j)^j E_t \left( x_{t,t+j} - p_{t+j}^* + \gamma \sum_{i=0}^{j-1} \pi_{t+i} \right)^2 \quad (2)$$

The price gap of period  $t+i$  is weighted by the probability of not having a new price signal until this date.

Most of the models presented in the literature can be seen as imposing some restrictions on three elements of equation (2):  $\lambda_j$ ,  $x_{t,t+i}$  and  $\gamma$ .

### 1.2 The choice of the price adjustments structure ( $\lambda_j$ )

The first distinction is about the choice of the rule governing price path adjustments. Each time a firm receives a new signal of price change, it can immediately set a new path of prices. We assume that the expected average duration between two signals is equal to  $N$  periods of the model (i.e. a firm changing its price path at the beginning of period  $t$  should have on average a new signal at the beginning of period  $t+N$ ). Two assumptions have retained much attention in the literature. The first is based on the staggered prices structure of Taylor (1980). In this specification, each price lasts exactly  $N$  periods, with an individual probability of price change of 1 every  $N$  periods and 0 otherwise. Firms in the economy are divided between  $N$  cohorts of equal size, each cohort being differentiated by the date of its price change. The alternative, which is more used, is based on the partial adjustment structure of Calvo (1983). A firm has a constant probability equal to  $1/N$  to change its

<sup>3</sup> The discount rate is equal to one. Its introduction is straightforward but unimportant for our analysis.

price each period. This probability is independent of the date of the last price change. The possible values of  $\lambda_j$  are given in Table 1.

Structure	Probabilities of price signal
<i>CALVO</i>	$j \in [1; +\infty[ \quad \Rightarrow \lambda_j = \frac{1}{N}$
<i>TAYLOR</i>	$\left\{ \begin{array}{l} j \in [1; N - 1[ \quad \Rightarrow \lambda_j = 0 \\ j = N \quad \Rightarrow \lambda_j = 1 \end{array} \right\}$

TAB. 1. Assumptions on the arrival of price signals

This presentation highlights the fact that the main difference between the Calvo and Taylor pricing rule is that the price signal probability in Calvo is time independent whereas under Taylor it is time dependent.

### 1.3 The nature of price rigidity ( $x_{t,t+j}$ )

The second distinction concerns the restrictions imposed on the value of the prices set for each period of the price contract. We call a price contract the vector of prices set between two adjustments of the price path. In the literature, the prices specified in price contracts have taken two forms: fixity or predetermination. Following Blanchard and Fischer (1989), a price is predetermined from  $t$  to  $t + j$  if its path from  $t$  to  $t + j$  is predetermined as of time  $t$ . A price is fixed if it is predetermined and constant during that length of time. This is equivalent to imposing  $x_{t,t+j} = x_t$  during all the duration of the contract.

### 1.4 The indexation degree ( $\gamma$ )

Some authors have introduced an indexation mechanism for the price set ( $x_t$ ) to lagged inflation. Each period, a fraction  $\gamma$  of the inflation rate observed during the last period can be added to the value of the price if there is no new signal of price change. Several  $\gamma$  values are considered in the literature. Christiano et al. (2005) assume that indexation is complete ( $\gamma = 1$ ), and Woodford (2003) uses partial indexation ( $0 < \gamma < 1$ ).

The motivation for indexation in FP models (apart from raising the degree of persistence) is that for a positive trend inflation, there is a growing gap between the optimal price and the price effectively set, which is inefficient. However, as under PP, all expected inflation is integrated in the price path.

This scheme of indexation is of little interest<sup>4</sup>. Consequently, we shall only study the impact of indexation in FP models. We shall also restrict our analysis to the following cases: full indexation (noted I thereafter), and no indexation (noted FL, for forward-looking, thereafter).

### 1.5 Presentation of inflation dynamics

Combining the different assumptions, it is possible to derive the most important Phillips curves discussed in the recent literature.

#### 1.5.1 Fixed prices in the Calvo model

First, we consider the most popular combinations: the FP version of the Calvo model (with and without indexation). The loss functions reduce to the following forms (respectively for the FL model, then the model with full indexation):

$$\begin{aligned} \underset{x_t}{\text{Min}} L_t &= (x_t - p_t^*)^2 + \sum_{j=1}^{\infty} \left(\frac{N-1}{N}\right)^j E_t (x_t - p_{t+j}^*)^2 \\ \underset{x_t}{\text{Min}} L_t &= (x_t - p_t^*)^2 + \sum_{j=1}^{\infty} \left(\frac{N-1}{N}\right)^j E_t \left( x_t - p_{t+j}^* + \sum_{i=0}^{j-1} \pi_{t+i} \right)^2 \end{aligned}$$

FOC give the following optimal prices (respectively with, then without indexation):

$$x_t = \left(\frac{1}{N}\right) \sum_{j=0}^{\infty} \left(\frac{N-1}{N}\right)^j E_t p_{t+j}^* \quad (3)$$

$$x_t = \left(\frac{1}{N}\right) \left[ \left( \sum_{j=0}^{\infty} \left(\frac{N-1}{N}\right)^j E_t p_{t+j}^* \right) - E_t \left( \sum_{j=0}^{\infty} \left(\frac{N-1}{N}\right)^{j+1} \sum_{i=0}^j \pi_{t+i} \right) \right] \quad (4)$$

Up to an approximation, the aggregate price level is a weighted average of the prices coexisting at time  $t$ , given that a fraction  $(1/N)$  of the prices is modified each period:

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<sup>4</sup> It is possible to imagine an indexation rule that would introduce a correction mechanism if realized inflation differs from expected inflation, but this issue is beyond the scope of our paper.

$$p_t = \left(\frac{1}{N}\right) x_t + \left(\frac{N-1}{N}\right) p_{t-1} \quad (5)$$

$$p_t = \left(\frac{1}{N}\right) x_t + \left(\frac{N-1}{N}\right) (p_{t-1} + \pi_{t-1}) \quad (6)$$

Given these equations and the value of  $p_{t+i}^*$  (equation 1), it is possible to derive a Phillips curve linking the current inflation to its expected value (and its lagged value in the case of indexation), and the output gap:

$$\pi_t = E_t \pi_{t+1} + \frac{\phi}{N(N-1)} y_t \quad (7)$$

$$\pi_t = \frac{1}{2} (\pi_{t-1} + E_t \pi_{t+1}) + \frac{\phi}{2N(N-1)} y_t \quad (8)$$

Equation (7) corresponds to the widely used Calvo forward-looking Phillips curve, and equation (8) represents a simplified version of the hybrid Phillips curve presented by Christiano et al. (2005).

### 1.5.2 Fixed prices in the Taylor model

We consider now the structure of the Taylor price adjustments, under the FP hypotheses. Given the probabilities of price adjustment, the loss functions are reduced to the following forms (respectively for the FL version, then with full indexation):

$$\begin{aligned} \underset{x_t}{\text{Min}} L_t &= (x_t - p_t^*)^2 + \sum_{j=1}^{N-1} E_t (x_t - p_{t+j}^*)^2 \\ \underset{x_t}{\text{Min}} L_t &= (x_t - p_t^*)^2 + \sum_{j=1}^{N-1} E_t \left( x_t - p_{t+j}^* + \sum_{i=0}^{j-1} \pi_{t+i} \right)^2 \end{aligned}$$

The FOC respectively give the following optimal prices:

$$x_t = \left(\frac{1}{N}\right) \sum_{j=0}^{N-1} E_t p_{t+j}^* \quad (9)$$

$$x_t = \left(\frac{1}{N}\right) \left[ \left( \sum_{j=0}^{N-1} E_t p_{t+j}^* \right) - E_t \left( \sum_{j=0}^{N-2} \sum_{i=0}^j \pi_{t+i} \right) \right] \quad (10)$$

The price level is again approximated by an average of the existing prices:

$$p_t = \frac{1}{N} \sum_{j=0}^{N-1} (x_{t-j}) \quad (11)$$

$$p_t = \frac{1}{N} \left[ \sum_{i=0}^{N-1} x_{t-j} + \sum_{j=1}^{N-1} \sum_{i=1}^{N-1-j} \pi_{t-i} \right] \quad (12)$$

Given these equations and the value of  $p_{t+i}^*$ , it is possible to obtain a Phillips curve. However, while with the Calvo structure, the form of the Phillips curve does not depend on the assumption made on  $N$  (the average length of contracts), this is not the case under the Taylor structure. In most of the literature, two-period contracts ( $N=2$ ) are considered. We retain this assumption for numerical applications (in a semi-annual model, it represents a length of contracts equal to one year on average), which is standard (see Taylor, 1999). For this length of contracts, the differences between the Phillips curves resulting from the structure of Taylor and Calvo are minimized. This gives the following Phillips curves (respectively for the FL version, then with full indexation):

$$\pi_t = \frac{1}{2} (E_{t-1}\pi_t + E_t\pi_{t+1}) + \frac{\phi}{2} \tilde{y}_t \quad (13)$$

$$\pi_t = \frac{1}{3} (\pi_{t-1} + E_{t-1}\pi_t + E_t\pi_{t+1}) + \frac{2\phi}{3} \tilde{y}_t \quad (14)$$

where  $\tilde{y}_t = y_t + y_{t-1} + E_{t-1}y_t + E_t y_{t+1}$ .

Equation (13) corresponds to the Phillips curve of the Taylor model, and equation (14) to the Phillips curve of the Fuhrer and Moore model. This presentation of inflation dynamics is important because it differs from the well-known presentations of the same models made by Roberts (1995), Fuhrer and Moore (1995) or Walsh (1998). In those papers, equations (13) and (14) are presented with the same form as equations (7) and (8), representing their equivalent under the Calvo structure, adding a neglected expectation error. This error term is neglected in most papers and the choice concerning the Calvo or Taylor structure is often considered as equivalent (Roberts, 1995, Mankiw, 2001). However, as Ben Aïssa and Musy (2007) and Musy (2006) have shown, this error term is crucial to properly understand the dynamic properties of the Taylor price structure. This point will be clearly illustrated in the following sections.

The origin of these terms indeed just reflects the interactions between the cohorts of price setters. The price level is an average of the prices set by cohorts in the past. As the prices are expected to be constant over several periods, price setters have to expect the future economic situation. The current price level (and then inflation dynamics) then contains these past expectations. When the number of staggering price cohorts rises, the number of these lags also rises, because firms have to take into account a higher number of future

prices. As an example, if we have 3 cohorts of firms in the model of Taylor, we obtain the following Phillips curve :

$$\begin{aligned} \pi_t = & \frac{1}{3} (E_t \pi_{t+1} + E_{t-1} \pi_t + E_{t-2} \pi_{t-1}) \\ & + \frac{1}{6} (E_t \pi_{t+2} + E_{t-1} \pi_{t+1} + E_{t-2} \pi_t) - \frac{1}{2} \pi_{t-1} \\ & + \frac{\phi}{6} \left[ \begin{aligned} & y_t + y_{t-1} + y_{t-2} + E_t (y_{t+1} + y_{t+2}) \\ & + E_{t-1} (y_t + y_{t+1}) + E_{t-2} (y_{t-1} + y_t) \end{aligned} \right] \end{aligned}$$

In a staggered framework, such terms appear because they resume the past fixations of prices. They do not appear only in cases such that all the past history of price decisions is resumed by the last index of the price level ( $p_{t-1}$ ). It is precisely the case if we follow the rule of Calvo. But is it of course a particular case, resulting from the use of a Poisson process, where the probability of changing the price is always the same for all the firms.

One should also note that the assumption leading to the presence of lagged inflation in the Fuhrer and Moore model is not based on the presence of indexation, but rather on an assumption of relative contracting. We show in the Appendix 1 that for two-period contracts, the Phillips curve resulting from the indexation assumption in the Taylor model and the Phillips curve derived by Fuhrer and Moore are exactly equivalent. However, we prefer to use indexation because the assumptions used by Fuhrer and Moore are hard to reconcile with microeconomic foundations (Holden and Driscoll, 2003)

### 1.5.3 Predetermined Prices

As indicated below, we only consider the case with zero indexation. The loss functions of a firm setting predetermined prices are the following (respectively for the Calvo and Taylor structures):

$$\begin{aligned} \underset{x_{t,t}, \dots, x_{t,t+j}}{\text{Min}} L_t = E_t \sum_{j=0}^{\infty} \left( \frac{N-1}{N} \right)^j (x_{t,t+j} - p_{t+j}^*)^2 \\ \underset{x_{t,t}, \dots, x_{t,t+j}}{\text{Min}} L_t = E_t \sum_{j=0}^{N-1} (x_{t,t+j} - p_{t+j}^*)^2 \end{aligned}$$

In both cases, the optimal price sequence is the same:

$$x_{t,t+j} = E_t p_{t+j}^*$$

The price level is an average of current prices predetermined at different dates, i.e., with different sets of information. We have (respectively under the Calvo and Taylor hypotheses):

$$p_t = \frac{1}{N} \sum_{j=0}^{\infty} \left( \frac{N-1}{N} \right)^j x_{t-j,t}$$

$$p_t = \frac{1}{N} \sum_{j=0}^{N-1} x_{t-j,t}$$

Even if the sequences of prices are identical in both cases, the different definitions of the price level lead to very different Phillips curves. To be in accordance with the previous section, we assume two-period contracts for the Taylor price structure :

$$\pi_t = \frac{1}{N} \sum_{j=0}^{\infty} \left( \frac{N-1}{N} \right)^j E_{t-1-j} (\pi_t + \phi \Delta y_t) + \frac{\phi}{N-1} y_t \quad (15)$$

$$\pi_t = \pi_{t-1} + E_{t-1} \pi_t - E_{t-2} \pi_{t-1} + \hat{y}_t \quad (16)$$

where  $\Delta y_t = y_t - y_{t-1}$  and  $\hat{y}_t = \phi y_t - \phi y_{t-1} + E_{t-1} y_t - E_{t-2} y_{t-1}$ . Equation (15) is equivalent to the Phillips curve derived by Mankiw and Reis, and equation (16) represents a Phillips curve closely related to the Fischer model (1977). Contrary to the models with FP, current inflation does not include any expectations of future variables. The expectation terms are related only to past expectations of current variables. The number of expectation lags is equal to maximum length of contracts, that is,  $N$  under the structure of Taylor, and infinity under the Calvo structure.

### 1.6 Summary of the Phillips Curves

In Table 2, we summarize the different Phillips curves presented. We now assume two-period contracts for all specifications. We note C for the Calvo hypotheses, T for Taylor, FL for the forward-looking version (i.e. zero indexation) and I for full indexation. For each combination, we indicate the main paper of the literature using these hypotheses, and the associated Phillips curve.

Hypotheses	Associated Model	Phillips Curve
C/FP/FL	Calvo	$\pi_t = E_t \pi_{t+1} + \frac{\phi}{2} y_t$
C/FP/I	Christiano, Eichenbaum and Evans	$\pi_t = \frac{1}{2} (E_t \pi_{t+1} + \pi_{t-1}) + \left(\frac{\phi}{4}\right) y_t$
T/FP/FL	Taylor	$\pi_t = \frac{1}{2} (E_{t-1} \pi_t + E_t \pi_{t+1}) + \phi \tilde{y}_t$
T/FP/I	Fuhrer and Moore	$\pi_t = \frac{1}{3} (\pi_{t-1} + E_{t-1} \pi_t + E_t \pi_{t+1}) + \left(\frac{2\phi}{3}\right) \tilde{y}_t$
C/PP	Mankiw and Reis	$\pi_t = \frac{1}{2} \sum_{j=0}^{\infty} \left(\frac{1}{2}\right)^j E_{t-1-j} (\pi_t + \phi \Delta y_t) + \kappa y_t$
T/PP	Fischer	$\pi_t = \pi_{t-1} + E_{t-1} \pi_t - E_{t-2} \pi_{t-1} + \hat{y}_t$

TAB. 2. The Phillips Curves

where  $\tilde{y}_t = y_{t-1} + E_{t-1} y_t + y_t + E_t y_{t+1}$  and  $\hat{y}_t = \phi y_t - \phi y_{t-1} + E_{t-1} y_t - E_{t-2} y_{t-1}$ .

We assume a simple output function, which depends on the level of real balances:

$$y_t = m_t - p_t \quad (17)$$

The money path is exogenous and we study different money shocks in the following sections. As our aim is to understand the implications of each assumption for price rigidity, the simplified economy we study has the virtue to derive all dynamics from the nominal rigidity<sup>5</sup>.

## 2 AN AUTO CORRELATED SHOCK ON MONEY GROWTH

### 2.1 Presentation

We assume that the growth of the money stock follows an AR(1) process:  $\Delta m_t = \rho \Delta m_{t-1} + \varepsilon_t$ , where  $\Delta m_t \equiv m_t - m_{t-1}$ . The main criteria of evaluation in the literature is the presence of a delayed and gradual response of output and inflation to the monetary innovation (Mankiw, 2001, Mankiw and Reis, 2002). As stated by Kiley (2002b), the inflation response should also be more delayed than the output response. Other evaluation elements, such as the reproduction of the "acceleration phenomenon", are also used to assess the relative performance of inflation dynamics models (Mankiw and Reis, 2002, and Trabandt, 2005).

<sup>5</sup> Trabandt (2005) studies the implications of some alternative assumptions of price rigidity in dynamic stochastic general equilibrium models.

## 2.2 Price dynamics

Economy begins on its steady-state. The money shock occurs at the beginning of period  $t = 1$ . We assume that the initial value of  $m$  is equal to  $m_0$ . The price dynamics<sup>6</sup> consecutive to the shock are presented in Table 3.

Hypotheses	Price Dynamics
Calvo/Fixed Prices Forward-looking	$p_t = \theta p_{t-1} + (1 - \theta) m_t + \frac{(1-\theta)\theta\rho}{1-\rho} \Delta m_t$
Calvo/Fixed Prices Indexation	$p_t = (\theta_1 + \theta_2) p_{t-1} - \theta_1 \theta_2 p_{t-2} + \frac{B}{\theta_3 - 1} m_t + \frac{B\rho}{(\theta_3 - 1)(\theta_3 - \rho)} \Delta m_t$
Taylor/Fixed Prices Forward-looking	$p_t = \theta p_{t-1} + \frac{(1-\theta)}{2} (m_t + m_{t-1}) + \frac{(1-\theta)(1+\theta)\rho}{4(1-\theta\rho)} (\Delta m_t + \Delta m_{t-1})$
Taylor/Fixed Prices Indexation	$p_t = (\theta_1 + \theta_2) p_{t-1} - \theta_1 \theta_2 p_{t-2} + \frac{A}{\theta_3 - 1} (m_t + m_{t-1}) + \frac{A(1+\theta_3)\rho}{2(\theta_3 - 1)(\theta_3 - \rho)} (\Delta m_t + \Delta m_{t-1})$
Calvo/Predetermined Prices	$p_t = m_0 + \sum_{j=0}^{\infty} \left[ \frac{\phi(1-\lambda^{j+1})(1-\rho^{j+1})}{(1-\rho)[1-(1-\lambda^{j+1})(1-\phi)]} \right] \varepsilon_{t-j}$
Taylor/Predetermined Prices	$p_t = p_{t-1} + \frac{1}{1+\phi} [\phi \Delta m_t + (1 + \rho) \Delta m_{t-1} - \rho \Delta m_{t-2}]$

TAB. 3. Price dynamics (auto correlated shock on money growth)

with  $A = [4\phi / (1 - 2\phi)]$  and  $B = \phi/4$ .  $\theta$  is the stable root of forward-looking FP models. In models with indexation,  $\theta_1$  and  $\theta_2$  are two complex roots, with a modulus below 1, and  $\theta_3 > 1$  is a real root.

We focus on price dynamics instead of inflation dynamics because it is helpful for a better understanding of the price adjustment mechanics. This will be more evident in the next section on disinflations.

Before seeing graphical illustrations, one can see the properties of each assumption on price adjustment.

- In FP models: - Indexation introduces a second price lag, with a negative

<sup>6</sup> The PP model resolution under the Calvo structure is given by Mankiw and Reis (2002). The Taylor model with PP is straightforward to resolve once the values of expectations are given. The FP models resolution is obtained by the method of factorization (see Romer, 2006, or Mankiw and Reis, 2002).

influence;

- The Taylor price structure implies, even in its purely forward-looking form, the presence of one lag of the money stock value, and one lag of the money growth value<sup>7</sup>. This results from the predetermination at time  $t$  of one cohort of firms' expectations.

- In PP models, dynamics are very dependent on the choice of the price adjustment structure. A unique and permanent shock on  $\Delta m$  has a very short impact in the PP model built on the Taylor structure. With the Calvo structure, dynamics include an infinity of predetermined expectations. Consequently, persistence is larger.

### *2.3 The response of output and inflation*

For numerical applications, we arbitrarily assume that  $\rho = 0.5$ <sup>8</sup>. We also assume that  $\phi = 0.1$ , which corresponds to an important degree of strategic interactions between firms. While there is a strong debate, Woodford (2003) shows that values around  $\phi = 0.1$  are possible. This allows to make easier comparisons with the literature<sup>9</sup> (see Taylor, 1999).

In Figure 1, we present the reaction of inflation to the shock.

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<sup>7</sup> When  $N$  rises, the number of these lags also rises.

<sup>8</sup> As this value is the same for all models, and as we focus essentially on qualitative aspects of the dynamics, this particular value is unimportant.

<sup>9</sup> Kiley (2002a) and Dixon and Kara (2005b) study the impact of variations of  $\phi$  on persistence.

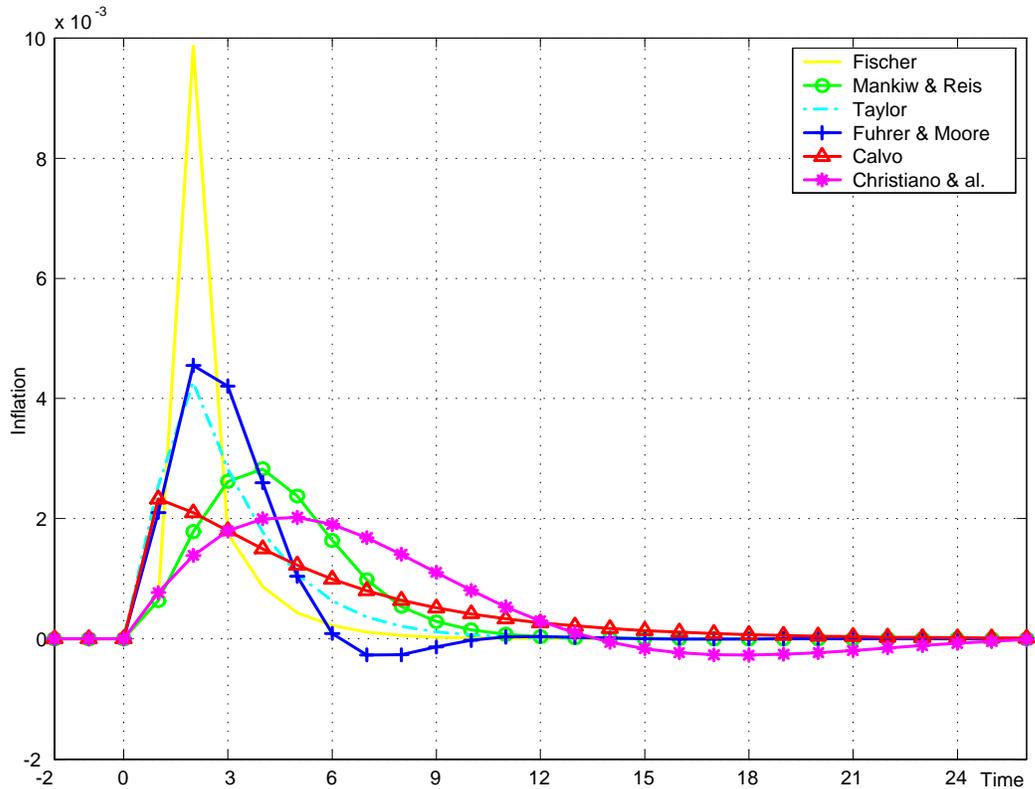


FIG. 1. Response of inflation (autocorrelated shock)

The three models built on the Calvo structure display a hump-shaped response of output. None of the models built on the Taylor structure can reproduce such output behavior<sup>10</sup>. Since they reproduce a hump-shaped response of inflation, they all respect the condition of a more delayed response of inflation than output, a criterion advanced by Kiley (2002b). The only model that reproduces a faster response of inflation (the peak of inflation occurs before the peak of output) is the Calvo forward-looking model and the inflation response of the Christiano et al. model is even more delayed than the response of the Mankiw and Reis PP model (2002).

In Figure 2 we present the response of the output.

<sup>10</sup> This result is dependent of the use of 2-periods contracts. With longer contracts, hump shaped dynamics can be reproduced.

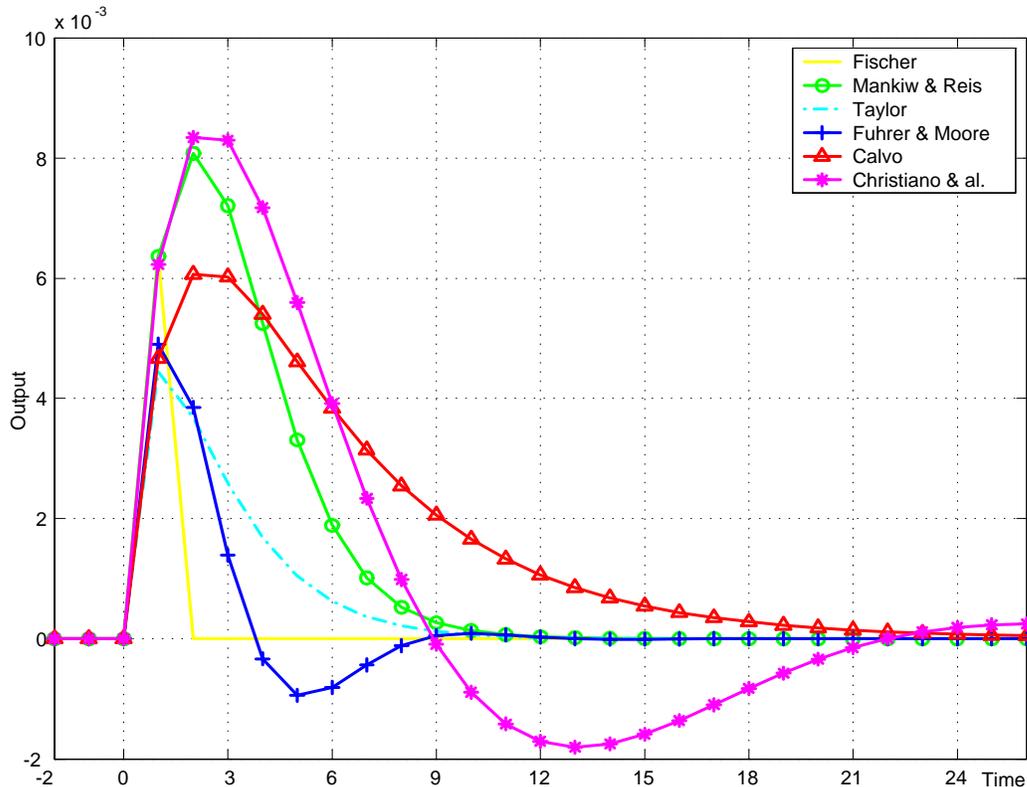


FIG. 2. Response of output (autocorrelated shock)

The three models built on the Calvo structure display a hump-shaped response of output. None of the models built on the Taylor structure can reproduce such output behavior<sup>11</sup>. Since they reproduce a hump-shaped response of inflation, they all respect the condition of a more delayed response of inflation than output, a criterion advanced by Kiley (2002b). The only model that reproduces a faster response of inflation (the peak of inflation occurs before the peak of output) is the Calvo forward-looking model.

The behavior of output is of interest because it can summarize the degree of persistence implied by nominal rigidities. If such rigidities were absent, we would have each period  $m = p$  (and then  $y = 0$ ). The degree of persistence can be measured as the cumulative deviation between  $m$  and  $p$ , which is equal to  $y$ . If we adopt this measure of persistence, the strongest degree of persistence is displayed by the FL-FP model of Calvo (see Table 4), which is surprising given all the debate on the lack of persistence of this model.

<sup>11</sup> This result is dependent of the use of 2-periods contracts. With longer contracts, hump shaped dynamics can be reproduced.

Model	C/FP/FL	C/PP	C/FP/I	T/FP/FL	T/FP/I	T/PP
Deviation	0.046	0.034	0.030	0.015	0.008	0.006

TAB. 4. Cumulative deviation between  $m$  and  $p$  (over 25 periods)

The forward-looking versions of the FP models imply a more important total output deviation than their alternatives with indexation. This results from the cyclical responses of the models with indexation. If we consider only the absolute value of deviations, this strongly raises the cumulative deviation of the Calvo FP model with indexation (from 0.03 to 0.057), but the impact is low with the Taylor model (from 0.008 to 0.013), which is still lower than with the forward-looking version.

One could note that the degree of persistence is lower with the Taylor structure. However, as shown by Dixon and Kara (2006), the difference should be lower if we allow for a motivated change in the calibration of  $\lambda_i$ . The value of this parameter should be higher in the models using the Calvo structure, which will lower the persistence of output and inflation. Despite the persuasive arguments of Dixon and Kara, we do not adopt this calibration here to facilitate the comparisons between our results and previous papers in the literature. We also note that our results are independent from this choice, and adopting the formulation by Dixon and Kara would reinforce our argument. Indeed, from a qualitative viewpoint, we show that the implications of the forward-looking Taylor contracts are in accordance with all the stylized facts, while their main weakness is to produce a lower degree of persistence than the models using the Calvo structure (the reason is shown by Kiley, 2002). As Dixon and Kara show, using their calibration simply reduces this quantitative difference. This would only reinforce the interest of Taylor contracts.

#### 2.4 *The acceleration phenomenon*

As stated by Mankiw and Reis (2002), the acceleration phenomenon, which represents the correlation between the output gap  $y_t$  (log from trend) and one-year inflation change ( $\pi_{t+1} - \pi_{t-1}$ ), focused on the observation date is a well-documented macroeconomic fact.

In Figure 3, we present the correlation between output gap  $y_t$  and the annual change of inflation ( $\pi_{t+1} - \pi_{t-1}$ ) (the acceleration phenomenon), assuming that the output gap and the output coincide (real shocks are held constant).

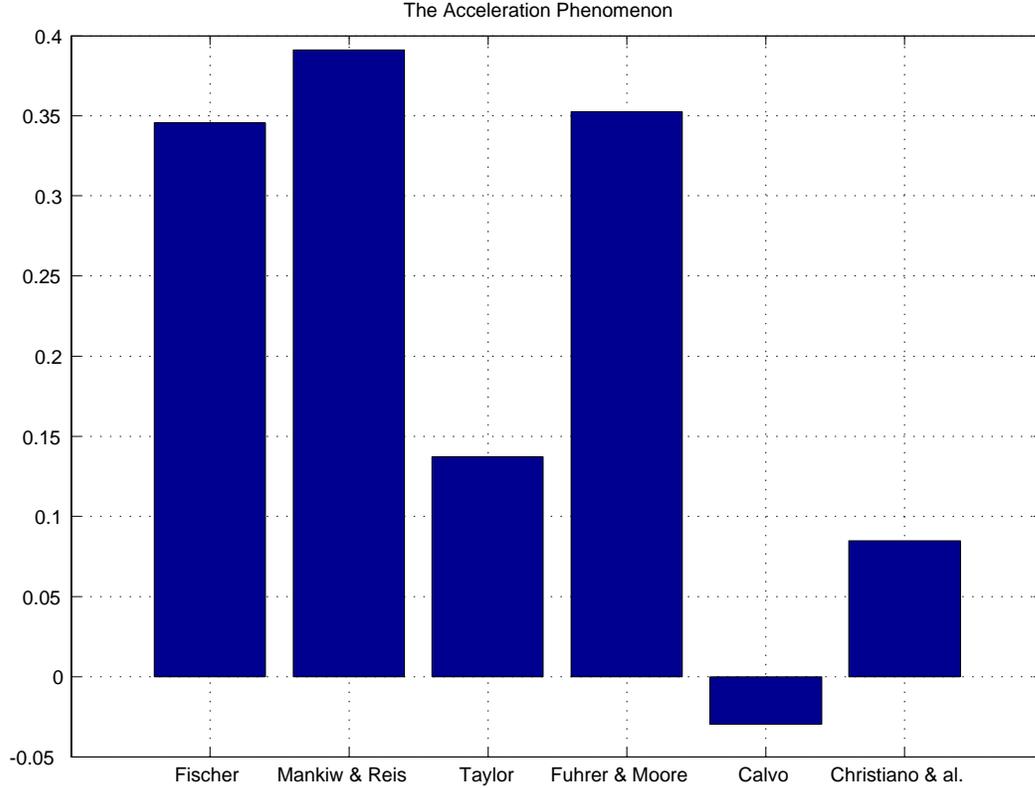


FIG. 3. The acceleration phenomenon

All correlations are positive except the one of the Calvo FL model. Contrary to the assertion of Mankiw and Reis (2002), a FP forward looking model can reproduce a positive correlation if we depart from the Calvo assumption. Models with backward looking components predict a higher correlation than the initial models, but the low value for the Taylor model is not necessarily a weakness since it is in line with the empirical values found using semi-annual US data<sup>12</sup> (Table 5).

Periods	1960:1-2005:2	1970:1-2005:2	1980:1-2005:2	1990:1-2005:2
The acceleration phenomenon	0.1472	0.1853	0.2017	0.3392

TAB. 5. The empirical acceleration phenomenon

<sup>12</sup> For this calculation, output  $y_t$  is the deviation of log real GDP from trend, where trend is calculated using the Hodrick-Prescott filter. The US inflation is based on the Consumer Price Index (CPI) from the FRED II Database of the Saint Louis Federal Reserve. As we use a semi-annual model, we consider  $corr(y_t, \pi_{t+1} - \pi_{t-1})$ , and not  $corr(y_t, \pi_{t+2} - \pi_{t-2})$  as in Mankiw and Reis, who use a quarterly model.

### 3 A DISINFLATION POLICY

#### 3.1 Presentation

A special case of the previous money process, where  $\rho = 1$ , is also often regarded as a criterion of evaluation. This particular value of  $\rho$ , while counterfactual, is of great interest because, with a negative monetary innovation, it corresponds to the case of a "cold-turkey" disinflation policy engineered by the central bank. This kind of disinflation policy has often been used as a test to dismiss the validity of the forward-looking New Keynesian inflation dynamics model (Ball, 1994a, Fuhrer and Moore, 1995, Mankiw and Reis, 2002). This model is supposed to be unable to reproduce the presence of a positive output cost of the disinflation (Ball, 1994b), and the delayed response of inflation following the slowing of money growth (Mankiw and Reis, 2002).

#### 3.2 The money process

The central bank sets the money growth rate  $\Delta m_t$  to a level compatible with its inflation target  $\pi_t^*$ , that is, each period  $\Delta m_t = \pi_t^*$ . Then, expectations are given by:  $E_t \Delta m_{t+i} = \pi_t^* = \Delta m_t, \forall i \geq 0$ .

The economy is initially at its steady-state, defined by  $\Delta m_t = \pi_t = \pi_1^*$  and  $y_t = 0$ . A sudden and unexpected disinflation policy is equivalent to a negative shock on the inflation target, e.g. from  $\pi_1^*$  to  $\pi_2^*$  with  $\pi_2^* < \pi_1^*$ . Let the disinflation begin in period 1. This policy is credible and permanent, but is not expected by the agents. For numerical applications, we assume that  $m_0 = 0$ , and the evolution of  $m_t$  is given by  $m_t = m_{t-1} + \pi_t^*$ , with the following path:

$$\left\{ \begin{array}{l} t \in [-\infty, 0], \pi_t^* = 0.025 \\ t \in [1, +\infty], \pi_t^* = 0 \end{array} \right\}$$

#### 3.3 Price dynamics

Given the expectations of the money path, we can derive the analytic form of price dynamics, which correspond to the ones presented in Table 1, assuming  $\rho = 1$  (the resolution of the Calvo model with PP is different, and we use the method presented by Mankiw and Reis, 2002). They are displayed in the Appendix 2.

### 3.4 Graphical illustrations

Figure 4 represents the reaction of the price level.

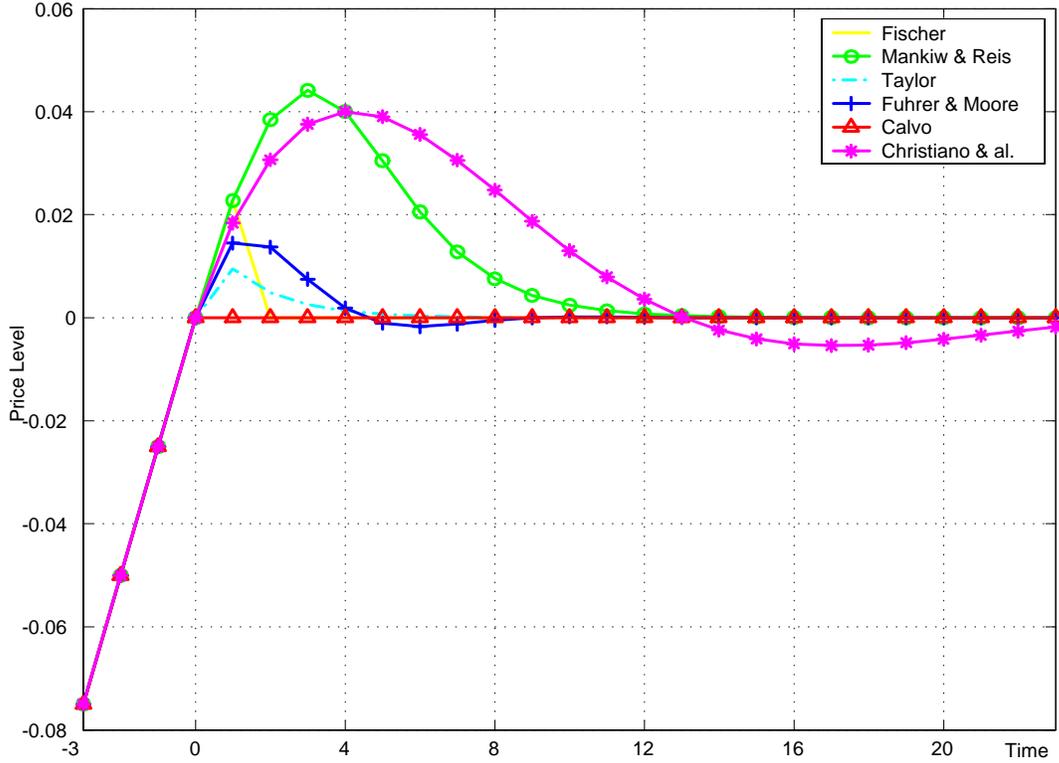


FIG. 4. Response of the price level (unexpected disinflation)

Although interest variables are mostly inflation and output, we also present price dynamics because they are important to understand clearly the origins of the real costs implied by disinflation. Given equation demand (17), in order to reproduce a positive disinflation cost, models have to display a mechanism of price over-shooting (i.e., the price level must continue to rise despite the stability of the money growth). The only model unable to reproduce this over-shooting mechanism is the forward-looking Calvo FP model. This is not a general feature of forward-looking FP models, but only a characteristic of the very specific rule of Calvo adjustment. Even in its simplest form, the Taylor structure can reproduce this price over-shooting. We also could notice that when considering this kind of shock, the equivalence between PP and FP in the Calvo model when  $\phi = 1$  (showed by Devereux and Yetman, 2003) does not hold.

Figure 5 represents the reaction of inflation.

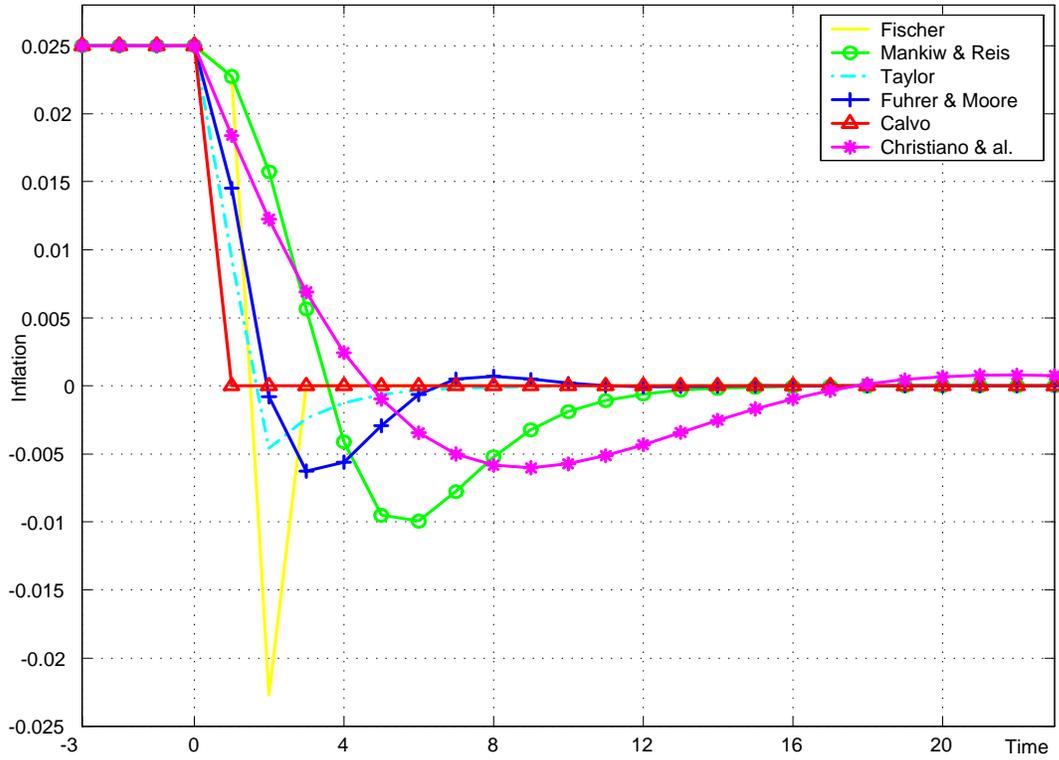


FIG. 5. Response of inflation (unexpected disinflation)

The behavior of prices implies that after reaching a peak, the price level has to fall. Consequently, inflation also displays an over-shooting response. For all models built on the structure of Taylor, this mechanism begins very quickly<sup>13</sup> (the second period consecutive to the shock), while for the models built on the Calvo structure (excepted the FL version), the peak response of inflation is more delayed.

Figure 6 represents the reaction of output.

<sup>13</sup> The very quick response presented here is dependent on the very short length of contracts assumed. For the forward-looking version, the peak response of the price level occurs  $N$  periods after the shock, where  $N$  is the length of contracts.

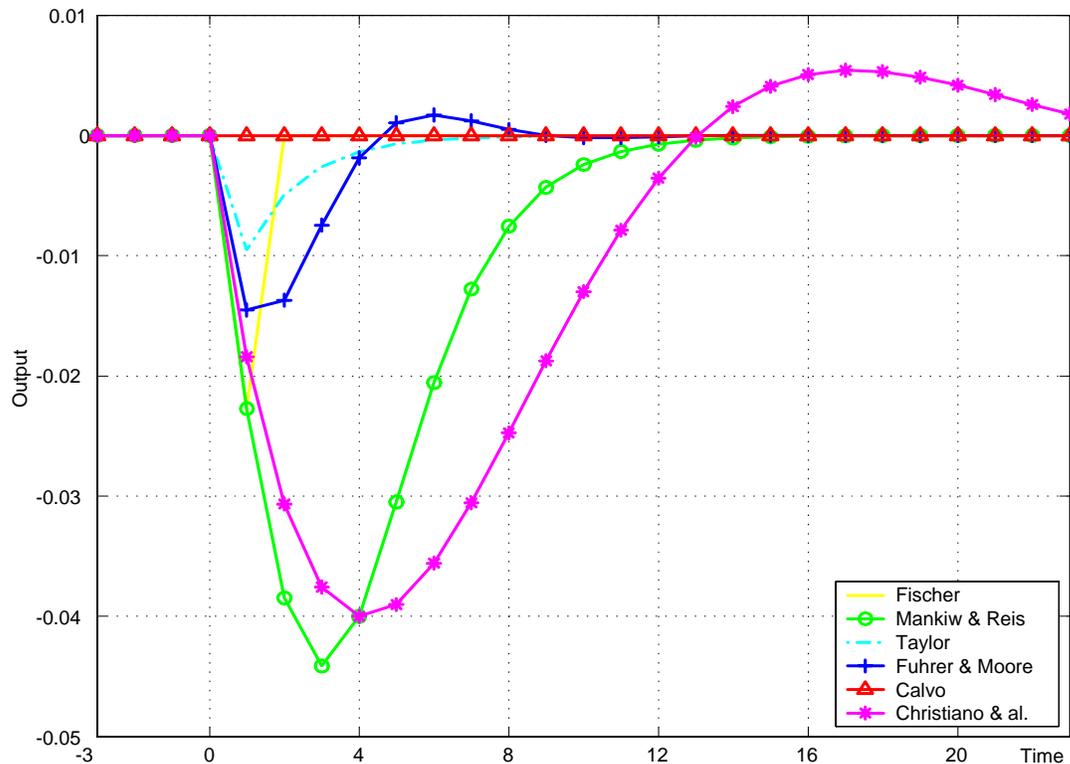


FIG. 6. Response of output (unexpected disinflation)

The magnitude of the output response depends on the degree of over-shooting presented in Figure 1. The Christiano, Eichenbaum and Evans (Calvo hybrid) and Mankiw Reis (Calvo and PP) models display comparable responses. The two models with indexation (Fuhrer and Moore, and Christiano, Eichenbaum and Evans) reproduce oscillatory dynamics due to the presence of complex roots. It is important to note that the introduction of indexation in the Taylor structure does not significantly increase the output cost of disinflation. This can be more clearly illustrated by the calculation of sacrifice ratios<sup>14</sup> (Figure 7).

<sup>14</sup>The sacrifice ratio is defined as the cumulative reduction in output required to achieve a one percentage point reduction in the inflation rate.

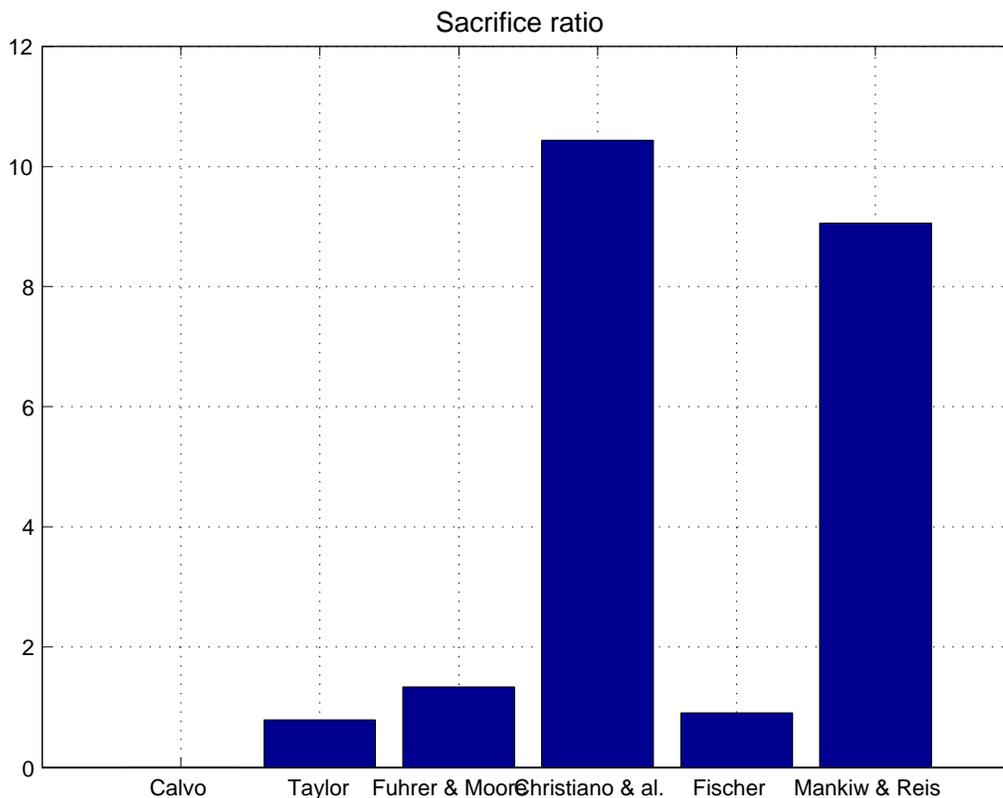


FIG. 7. Sacrifice ratio (unexpected disinflation)

To have a comparison order, Ball (1994b) estimates sacrifice ratios ranging from 0.0 to 3.6, with quarterly data. Of course, the values obtained depend on the value chosen for parameters. Figure 7 has only some illustrative value.

To summarize the results, as it is well known, the forward-looking Calvo model does not produce any inflation persistence or output costs. However, contrary to a common view (among others Roberts, 1998, Walsh, 1998), this is a special feature of the probabilistic Calvo price structure and not a property shared by all FP forward-looking models. Even with the simplest deterministic structure (Taylor uniform contracts for two periods), inflation displays some persistence, and the disinflation output cost is positive. This result is general and does not depend on the specific value chosen for  $\phi$ . The sacrifice ratio in the presented Taylor model is always positive and is equal to  $(1/4\sqrt{\phi})$  (see next section for a demonstration). While often neglected, the expectation errors present in the Taylor structure have very important implications for disinflations. When looking at the Phillips curves in the Taylor and Mankiw and Reis models, one can see that the sources of persistence are indeed similar: both models rely on the presence of predetermined expectations to reproduce a delay in the inflation adjustment. Even in the sticky prices model, past information sets of agents have an influence on current variables because they are included

in the prices which are still in effect in the current period. Then, as in the sticky-information model, past money growth has an influence on current output. There is one exception to this case: the Calvo assumption of a constant probability of price changes among all firms (see next section).

The introduction of indexation in the Taylor structure does not significantly raise persistence. As shown by Ben Aïssa and Musy (2007), when taking into account the expectation errors, the Taylor, Fuhrer and Moore models have close dynamics properties, the latter produces only a little more persistence than the former. However, introducing indexation in the Calvo model significantly alters the dynamics. Inflation is very persistent and the output cost is the strongest of all the specifications considered. The corollary of the previous remark is that FP hybrid models have very different dynamic properties.

The properties of PP models are also sensitive to the choice of the pricing structure. As it is well known for this price rigidity, a monetary shock has real effects only as long as all contracts have not been modified (Blanchard and Fischer, 1989, p. 393). In the particular Calvo structure, some contracts have an infinite length, which produces a strong persistence and an asymptotic convergence. When the length of contracts is finite, persistence is much lower and there is strict convergence. Another presentation of this point is made by Collard and Dellas (2003), and Dupor and Tsuruga, (2005). The properties of Mankiw and Reis model, which depend on the values chosen for the parameters, are discussed extensively by Coibion (2006).

### 3.5 *The sacrifice ratio in fixed price forward-looking models*

The presence of a positive real cost following a disinflation is possible even with fully rational forward-looking agents facing sticky prices. This is a result often contested in the literature. We propose a general demonstration of this result for a "cold-turkey" disinflation, considering only the two Taylor and Calvo models, with fixed prices but no indexation. Let disinflation begin in period  $j$ . We derive the general form of inflation dynamics in the Calvo model<sup>15</sup>, given the output equation (17):

$$\pi_t = \theta^C \pi_{t-1} + (1 - \theta^C) \Delta m_t + (1 - \theta^C) \theta^C \sum_{i=0}^{\infty} (\theta^C)^i (E_t \Delta m_{t+i+1} - E_{t-1} \Delta m_{t+i})$$

where  $\theta^C$  is the stable root of the dynamics. Inflation dynamics in the Taylor model are given by:

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<sup>15</sup> To make the comparison simpler, we index by  $C$  the variables relative to the Calvo model, and we index by  $T$  the variables relative to the model of Taylor.

$$\begin{aligned}\pi_t = & \theta^T \pi_{t-1} + \left( \frac{1 - \theta^T}{2} \right) (\Delta m_t + \Delta m_{t-1}) + \frac{(1 - \theta^T)(1 + \theta^T)}{4} (E_t \Delta m_{t+1} - E_{t-2} \Delta m_{t-1}) \\ & + \frac{\theta^T (1 - \theta^T)(1 + \theta^T)}{4} \left[ \sum_{i=0}^{\infty} (\theta^T)^i (E_t \Delta m_{t+i+2} - E_{t-2} \Delta m_{t+i}) \right]\end{aligned}$$

where  $\theta^T$  is the stable root of the dynamics. Expected terms on money growth are different in the two structures. Given the money process, expectations are given by:  $E_t \Delta m_{t+i} = \pi_t^* = \Delta m_t$ ,  $\forall i \geq 0$ . Inflation dynamics are reduced in both cases to the following forms:

$$\pi_t = \theta^C (\pi_{t-1} - \Delta m_{t-1}) + \Delta m_t \quad (18)$$

$$\pi_t = \theta^T \pi_{t-1} + \frac{1 - \theta^T}{2} (\Delta m_t + \Delta m_{t-1}) + \frac{1 + \theta^T}{4} (\Delta m_t - \Delta m_{t-2}) \quad (19)$$

We note  $\pi_1^*$  the inflation rate before the disinflation. The initial state of the economy is given by:  $\Delta m_t = \pi_t = \pi_1^*$ , and  $y_t = 0$ . When the disinflation policy begins, the inflation target is changed permanently to  $\pi_2^*$ , with  $\pi_2^* < \pi_1^*$ . Let the disinflation begin during period  $j$ .

In the Calvo model, equation (18) implies that as soon as the economy begins on its steady-state ( $\pi_{j-1} = \Delta m_{j-1}$ ), we have  $\pi_{j+i} = \Delta m_{j+i} = \pi_2^*$ , and  $y_{j+i} = 0$ ,  $\forall i \geq 0$ . In the Taylor model, due to the presence of lagged expectations, we have by contrast:

$$\begin{aligned}\pi_j &= \frac{3 - \theta^T}{4} \pi_2^* + \frac{1 + \theta^T}{4} \pi_1^* \\ \pi_{j+1} &= \frac{5 - \theta^T}{4} \pi_2^* - \frac{1 - (\theta^T)^2}{4} \pi_1^* \\ \pi_{j+i} &= \theta^T \pi_{j+i-1} + (1 - \theta^T) \pi_2^*, \quad \forall i \geq 2\end{aligned}$$

The condition  $\pi_j > \pi_2^*$  is always verified<sup>16</sup>. This means that there is some inflation persistence at the beginning of the disinflation. After, the inflation rate overshoots its long run value ( $\pi_{j+1} < \pi_2^*$ ). During subsequent periods, the inflation rate converges to this long-run value. Convergence is gradual and monotone if  $\phi < 1$ , immediate if  $\phi = 0$ , and oscillatory if  $\phi > 1$ .

In the Taylor model, output is always negative during the process, and the

<sup>16</sup> Because  $\phi$  is always positive and  $\theta^T$  is given by:  $\theta^T = (1 - \sqrt{\phi}) / (1 + \sqrt{\phi})$ .

dynamics are given by:

$$y_{j+i} = -\frac{(\theta^T)^i (1 + \theta^T)}{4} [\pi_1^* - \pi_2^*]$$

for  $i \geq 0$ . The total disinflation cost (TDC), given by  $\sum_{i=0}^{\infty} y_{j+i}$ , is equal to (respectively in the Calvo and in the Taylor models):

$$\begin{aligned} TDC^C &= 0 \\ TDC^T &= -\frac{(1 + \theta^T)}{4(1 - \theta^T)} [\pi_1^* - \pi_2^*] \end{aligned}$$

As  $\theta^T \in ]-1; 1[$ , we have  $TDC^T < 0$ . In the forward-looking Taylor model, a cold-turkey disinflation always implies a positive real cost of disinflation. This result does not depend on the value of  $\phi$ . This is an important point, as there are some debates on the plausible values of  $\phi$  (see Chari, Kehoe and McGrattan, 2000 and Woodford, 2003). However, the sacrifice ratio (SR) is decreasing in  $\phi$ :

$$SR^T = \frac{1}{4\sqrt{\phi}}$$

A presentation of the mechanics underlying this positive cost is given by Musy (2006). The source of this cost is the presence of expectation errors. Consider first the price dynamics of the FL Calvo model (see Table 1). Assume for simplicity that when the disinflation takes place,  $\Delta m_j = 0$  (*i.e.*  $m_j = m_{j-1}$ ). It comes that  $p_j = p_{j-1}$  and  $y_j = 0$ . Thus, in the Calvo model, there is no inflation persistence and disinflation is costless. By contrast, in the price dynamics of the FL Taylor model, the forward looking component at time  $j$  is driven by the expectations made both in  $j$  and  $j - 1$ . Since half of the firms have set their prices without expecting the disinflation, one has  $E_{j-1} \Delta m_j > 0$ , which implies  $p_j > p_{j-1}$ . Thus, in the Taylor model, there is a positive rate of inflation and a real cost of disinflation.

The difference in the models' properties comes from the difference in the distribution of contracts and in the timing of price changes. In the Calvo model, the probability of a price change is independent from the date of the last price modification. Thus, the price changes are equally distributed among the firms. During period  $j$ , the average unmodified price is equal to  $p_{j-1}$ . This implies that  $\pi_j$  depends only on the price changes made by those firms that set their prices at time  $j$ . These firms fully respond to the monetary change. In the Taylor model, a cohort of firms charges the older prices and only this cohort changes its prices. For positive inflation rates, the average price of the firms keeping their prices fixed is greater than  $p_{j-1}$ . At the beginning of the disinflation, in period  $j$ , the only prices that are modified are those set in  $j - 2$ . The

index of the unmodified prices is then greater than  $p_{j-1}$ . Given the presence of real rigidities (equation 1 can be rewritten as  $p_t^* = (1 - \phi) p_t + \phi m_t$ ), the new prices have to be close to the unmodified prices and will take a value between the index of the unchanged prices and the new equilibrium price level. In the following periods, the price level will continue to be higher than the money stock (see Figure 4)

If the disinflation is announced in advance, these error terms disappear and the mechanisms presented do not play any role, and disinflation is not costly. But in this case, PP combined with the Taylor structure also implies a costless disinflation (see Taylor, 1983).

## 4 COMPARISON WITH THE LITERATURE

Our paper is close in spirit to the one by Nelson (1998), who shows that many alternative sticky-price specifications may be written as special cases of a log-linear model of prices dynamics. Among others, Nelson (1998) concludes that the hybrid Fuhrer and Moore model is better suited to reproduce empirical inflation serial correlations than the forward-looking Taylor model. However, his conclusions are distorted for two reasons: the first one is the absence of expectations errors in the dynamics, while the results presented below show the importance of these errors. The second reason is the absence of a common framework to derive the implications of the alternative assumptions on price adjustment. Consequently, in his presentation, the numerical value of some parameters appears to artificially differs from one model to another. As an example, the value of  $\phi$  is estimated when he considers the Fuhrer and Moore model, which gives a very small value of 0.008. This estimated value is common in empirical studies. But when he analyzes the Taylor model, he uses the calibration of  $\phi$  proposed by Chari, Kehoe and McGrattan (2000), who argue that  $\phi$  cannot be smaller than 1.2 in a general equilibrium framework. As we know, the lower the value of  $\phi$ , the greater the model persistence. The higher value of  $\phi$  in his representation of the Taylor model can explain most of his contrasted conclusions about the relevance of the two models. As we have shown, Furher and Moore contracts do not add much more persistence.

Another relevant paper is Kiley (2002a). He considers a shock on the level of the money stock, and shows that the different price distributions between the two models have an impact in terms of output persistence, the Calvo model being more persistent. In this paper, we focus on shocks on the money growth rate. The different assumptions on price adjustment have other implications. Notably, in the Taylor model, price changes are made by the firms charging the oldest prices in the economy. As Kiley only considers shocks on the level of the money stock, the effects presented in the previous section do not play

any role in the dynamics presented in his paper, as in this case, before the shock, all contracts are set at the same value.

The paper by Mankiw and Reis (2002) is also very relevant, because we adopt their criteria consisting in the reproduction of stylized facts. Their conclusion is that the sticky information model is consistent with "accepted views of how monetary policy works", while the forward-looking fixed price model fail on three points: 1- Disinflations are not contractional; 2- Money shocks have their maximum effect on inflation with a substantial delay (this concerns also the hybrid model, according to Ball, Mankiw and Reis, 2005); 3- The model cannot explain the acceleration phenomenon that vigorous economic activity is positively correlated with rising inflation (Mankiw and Reis, 2002, p. 1318). They study only the Calvo version of the forward-looking fixed price model. Interestingly, we show that the Taylor model can successfully reproduce all these three points, even in its entire forward-looking version : the sacrifice ratio is positive, the correlation between the output gap and the annual change of inflation is positive (the acceleration phenomenon), and the maximum impact of the money shock on inflation does not occur immediately. The conclusion by Mankiw and Reis is biased toward the sticky information model because they only compare it to the forward-looking Calvo model, which implies very specific dynamic properties. Even simple departures from this special rule can greatly improve the fit of the macro implications of the sticky price assumption to the "accepted views of how monetary policy works".

Mankiw and Reis also use the "disinflation criterion"<sup>17</sup> to discard the new Keynesian model. We show that indeed, all models, excepted the FL Calvo model, can fill this criterion. However, the analysis of price level dynamics we conducted in this paper shows that a clear comprehension of the costs of disinflations in those models, indeed leads to the following question: "why do firms continue to raise prices even if the money stock, and then optimal prices, are stable?". Our opinion is that the most interesting response is given by the FL Taylor model : it results from the interactions between decisions of fully rational and informed firms which can change their prices, with firms which cannot. The sticky price mechanism is indeed a good mechanism to reproduce the effects of disinflation. All the alternatives rely on assumptions either of a lack of rationality from firms, or on a limited use of information. If we consider the new current standard model of inflation dynamics (the Calvo model with indexation), disinflation is costly only because some firms use an ad hoc rule of past inflation indexation. There is nothing to explain why and when firms should use indexation, and if they use it, what should be the appropriate degree of indexation (which is a free parameter, as in Woodford, 2003). If we consider that the New Neoclassical Synthesis emphasized the importance of

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<sup>17</sup> The disinflation criterion represents the ability of the model to reproduce correctly the costs of disinflations.

optimization and rational expectations (Goodfriend, 2004), its current leading model provides all but a satisfactory explanation to the understanding of disinflations, and more generally on the phenomenon of inflation persistence<sup>18</sup>.

## 5 CONCLUSION

Our main conclusion is that the most important element for dynamic properties is the choice of the price structure. In this paper, we have considered only the simplest forms of the Calvo and Taylor structures, but even with these restrictions, differences are important. This result is surprising because, since the influential paper by Roberts (1995), this choice has often been considered as unimportant. As a consequence, the conclusion that there is a total absence of sources of inflation inertia in forward-looking FP models, as stated by Ball, Mankiw and Reis (2005), is not exact. The frequent negligence of expectation errors inherent to the Taylor structure explains this result. Indeed, the absence of disinflation costs is a very special characteristic attributable to the Calvo structure. For critics finding that the simplest Taylor structure can produce only a small degree of inflation persistence, the introduction of a small part of longer contracts seems able to significantly rise persistence [see Dixon and Kara, 2005]. Then, when using the New Keynesian Phillips curve, it seems more important not to systematically use the Calvo model, rather than to apologize in a footnote, as suggested by Mankiw (2005). Another surprising result is the fact that the presence of lagged inflation in the dynamics improves only slightly the degree of inflation persistence produced by each price structure, except in the case of the Calvo structure submitted to a disinflation shock. Concerning the PP versus FP choice, we show that this choice by itself is not sufficient to determine dynamic inflation properties and output. Both hypotheses can produce an important degree of persistence. In PP models, the choice of the price structure is even more important than in FP models, because under the Taylor structure, the degree of persistence is very low.

An interesting extension would be to go deeper in the study of the foundations of these price rules. The assumption of exogenous time-dependent price rigidities is frequent in the literature, but for a comparative work, it would be interesting to associate output and inflation dynamics at a macro level with the microeconomic nature of the adjustment costs preventing firms from changing their prices (menu costs, contracting costs, rational inattentiveness, indexation decision etc. . .). Yetman (2007) provided an interesting paper in this way of research, and it seems to be a natural way to prolong our work.

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<sup>18</sup>Of course, the costs of disinflations can come from other equations than the inflation equation. As we use a simple model, we do not consider these effects.

**APPENDIX 1: Equivalence between equation (14) and the Phillips curve of Fuhrer and Moore (1995)**

The general formulation of the Fuhrer and Moore model consists in introducing a relative term in the Taylor structure. Equations (1), (9) and (11) of the Taylor model can be re-written under the following form:

$$x_t = \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (x_{t+i-j}) + \frac{\phi}{N} \sum_{i=0}^{N-1} E_t (y_{t+i}) \quad (20)$$

There is no reference to the price level. Fuhrer and Moore (1995) criticize this point and argue that contracts should be written in relative terms. They propose the following rule for the evolution of  $x_t$ :

$$x_t - p_t = \left( \frac{1}{N} \right) \sum_{i=0}^{N-1} E_t (v_{t+i} + \phi y_{t+i}) \quad (21)$$

where  $v_t$  is the mean real price of other contracts coexisting during period  $t$ :

$$v_t = \left( \frac{1}{N} \right) \sum_{i=0}^{N-1} (x_{t-i} - p_{t-i}) \quad (22)$$

Then, (21) becomes:

$$x_t - p_t = \sum_{i=1}^{N-1} \frac{N-i}{N(N-1)} (x_{t-i} - p_{t-i}) + \sum_{i=1}^{N-1} \frac{N-i}{N(N-1)} E_t (x_{t+i} - p_{t+i}) + \frac{\phi}{N-1} E_t y_{t+i} \quad (23)$$

For  $N = 2$ , this gives:

$$x_t - p_t = \frac{1}{2} (x_{t-1} - p_{t-1} + E_t x_{t+1} - E_t p_{t+1}) + \phi (y_t + E_t y_{t+1}) \quad (24)$$

The corresponding Phillips curve is:

$$\pi_t = \frac{1}{2} (\pi_{t-1} + E_t \pi_{t+1}) + \phi (\tilde{y}_t) + (1/2) \eta_t \quad (25)$$

where  $\tilde{y}_t = y_t + y_{t-1} + E_{t-1} y_t + E_t y_{t+1}$  and  $\eta_t^\pi = E_{t-1} \pi_t - \pi_t$ . When we take into account explicitly the expectation error, the last equation corresponds exactly to the equation (14) of the text:

$$\pi_t = (1/3) (\pi_{t-1} + E_{t-1} \pi_t + E_t \pi_{t+1}) + (2\phi/3) (\tilde{y}_t)$$

## APPENDIX 2: Price dynamics for the disinflation policy

Hypotheses	Price Dynamics
Calvo/Fixed Prices Forward-looking	$p_t = \theta p_{t-1} + (1 - \theta) m_t + \theta \Delta m_t$
Calvo/Fixed Prices Indexation	$p_t = (\theta_1 + \theta_2) p_{t-1} - \theta_1 \theta_2 p_{t-2} + \frac{B}{\theta_3 - 1} m_t + \frac{B}{(\theta_3 - 1)^2} \Delta m_t$
Taylor/Fixed Prices Forward-looking	$p_t = \theta p_{t-1} + \frac{(1-\theta)}{2} (m_t + m_{t-1}) + \frac{(1+\theta)}{4} (\Delta m_t + \Delta m_{t-1})$
Taylor/Fixed Prices Indexation	$p_t = (\theta_1 + \theta_2) p_{t-1} - \theta_1 \theta_2 p_{t-2} + \frac{A}{\theta_3 - 1} (m_t + m_{t-1}) + \frac{A(1+\theta_3)}{2(\theta_3 - 1)^2} (\Delta m_t + \Delta m_{t-1})$
Calvo/Predetermined Prices	$p_t = \frac{\phi [1 - (1/2)^{t+1}] + 0.025 (1 + t) (1/2)^{t+1}}{1 - (1 - \phi) [1 - (1/2)^{t+1}]}$
Taylor/Predetermined Prices	$p_t = p_{t-1} + \frac{1}{1 + \phi} (\phi \Delta m_t + 2 \Delta m_{t-1} - \Delta m_{t-2})$

TAB. 6. Price dynamics (Disinflation)

## LITERATURE CITED

BALL L. (1994a), "Credible Disinflation with Staggered Price Setting", *American Economic Review*, 84-1, pp. 282-289.

BALL L. (1994b), "What Determines the Sacrifice Ratio ?", in *Monetary Policy*, Ed. by G. Mankiw, University of Chicago Press, Chicago.

BALL L., G. MANKIW AND R. REIS (2005), "Monetary Policy for Inattentive Economies", *Journal of Monetary Economics*, 52, pp. 703-725.

BEN AISSA M. S., AND O. MUSY, (2007), "Persistence with Staggered Price Setting in Nominal and Real Terms", *Applied Economics Letters*, 14-4, pp. 233-237.

BLANCHARD O. AND S. FISCHER (1989), *Lectures on Macroeconomics*, MIT Press, Cambridge.

CALVO G. (1983), "Staggered Prices in a Utility Maximizing Framework", *Journal of Monetary Economics*, 12, pp. 383-398.

CHARI V. V., P. KEHOE, AND E. McGRATTAN (2000), "Sticky Prices Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem ?", *Econometrica*, 68, pp. 1151-1175.

CHRISTIANO L., M. EICHENBAUM, AND C. EVANS (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, 113-1, pp. 1-45.

COIBION O. (2006), "Inflation Inertia in Sticky Information Models", *B.E. Journals, Contributions to Macroeconomics*, 6-1.

COLLARD F. AND H. DELLAS (2003), "Sticky Information", Working paper, University of Toulouse.

DEVEREUX M. AND J. YETMAN (2003), "Predetermined Prices and the Persistent Effects of Money on Output", *Journal of Money Credit and Banking*, 35-5, pp. 729-741.

DIXON H. AND E. KARA (2005) "Persistence and Nominal Inertia in a Generalized Taylor Economy: How Longer Contracts Dominate Shorter Contracts", European Central Bank WP No 489.

DIXON H. AND E. KARA (2006) "How to compare Taylor and Calvo contracts: a comment on Michael Kiley", *Journal of Money, Credit and Banking*,

38, pp. 1119-1126.

DUPOR B. AND T. TSURUGA (2005), "Sticky Information: The Impact of Different Updating Assumptions", mimeo, Ohio State University.

FISCHER S. (1977), "Long-term Contracts, Rational Expectations and the Optimal Money Supply Rule", *Journal of Political Economy*, 85, pp. 191-205.

FUHRER J. AND G. MOORE (1995), "Inflation persistence", *Quarterly Journal of Economics*, 110-1, pp. 127-160.

GALÍ J., AND M. GERTLER (1999), "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics*, 44, pp. 195-222.

GOODFRIEND, M. (2004), "Monetary Policy in the New Neoclassical Synthesis: A Primer", Federal Reserve of Richmond Economic Quarterly, 90-3, pp. 21-45.

HOLDEN S. AND DRISCOLL, J. (2003), "Inflation Persistence and Relative Contracting", *American Economic Review*, 93-4, pp. 1369-1372.

JEANNE O. (1998), "Generating Real Persistent Effects of Monetary Shocks: How Much Nominal Rigidity Do we Really Need ?", *European Economic Review*, 42, pp. 1009-1032.

JONDEAU E. AND H. LE BIHAN (2001), "Testing for a Forward-Looking Phillips Curve Additional Evidence From European and US Data", Notes d'Études et de Recherches No 86, Banque de France.

KILEY M. (2002a), "Partial Adjustment and Staggered Price Setting", *Journal of Money, Credit and Banking*, 34-2, pp. 283-298.

KILEY M. (2002b), "The lead of output over inflation in sticky price models", *Economics Bulletin*, 5.

MANKIW G. (2001), "The Inexorable and Mysterious Trade-off Between Inflation and Unemployment", *Economic Journal*, 111, pp. C45-C61.

MANKIW G. (2005), "Comment on Cecchetti and Kim", in B. Bernanke and M. Woodford (Ed.), *The Inflation-Targeting Debate*, University of Chicago Press, Chicago.

MANKIW G., AND R. REIS (2002), "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve", *Quarterly Journal of Economics*, 117, pp. 1295-1328.

MUSY O. (2006), "Inflation Persistence and the Real Costs of Disinflation in

Staggered Prices and Partial Adjustment Models", *Economics Letters*, 91, pp. 50-55.

NELSON E. (1998), "Sluggish Inflation and Optimizing Models of the Business Cycle", *Journal of Monetary Economics*, 42, pp. 303-322.

PEREAU J.C. (2001), "Staggered Wage Setting in Normal and Real Terms: A Comparative Analysis", *Bulletin of Economic Research*, 53-2, pp. 143-151.

ROBERTS J. (1995), "New Keynesian Economics and the Phillips Curve", *Journal of Money, Credit and Banking*, November, pp. 975-984.

ROBERTS J. (1998), "Inflation Expectations and the Transmission of Monetary Policy", Finance and Economics Discussion Series No 1998-43, *Board of Governors of the Federal Reserve System*, Washington.

ROMER D. (2006), *Advanced Macroeconomics*, 3rd edition, McGraw-Hill Irwin, New York.

RUDD J., AND K. WHELAN, (2005), "Modelling Inflation Dynamics: A Critical Review of Recent Research", Federal Reserve Board Finance and Economics Discussion Series, No 2005-66.

TAYLOR J. (1980), "Aggregate Dynamics an Staggered Contracts", *Journal of Political Economy*, 88, pp. 1-22.

TAYLOR J. (1983), "Union Wage Settlements during a Disinflation", *American Economic Review*, pp. 981-993.

TAYLOR J. (1999), "Staggered Price and Wage Setting in Macroeconomics", in J. Taylor et M. Woodford (Ed.), *Handbook of Macroeconomics*, Elsevier, Amsterdam.

TRABANDT M. (2005), "Sticky Information vs. Sticky Prices: A Horse Race in a DSGE Framework", *Working paper*, Humboldt University.

WALSH C. (1998), *Monetary Theory and Policy*, MIT Press, Cambridge.

WOODFORD M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

YETMAN J. (2007), "Explaining Hump-shaped Inflation Responses to Monetary Policy Shocks", *Managerial and Decision Economics*, 28-6, pp. 605-617.