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Document de Travail
Working Paper
2012-08

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Abstract

This paper tackles the increasingly significant problem of irrigation-induced soil salinity within a groundwater management model. Irrigation can result not only in heavier salt concentrations, but also in the removal of salt from the soil through return flows. Given these contradictory observations, we are interested in the effects on soil salt concentration if irrigation efficiency is improved. We develop a model of salt concentration patterns in both soil and groundwater. We introduce a negative externality to the production process by assuming that soil degradation due to higher soil salinity affects total factor productivity. Within this framework, we show that in the presence of this externality, increasing irrigation efficiency can lead to higher or lower soil salt concentration, depending on the social cost of transferring salt from one reservoir to another.

Keywords: *Groundwater Management, Optimal Control of Water Consumption, Soil Salinity*

JEL: Q24, Q25, C61, D61

Introduction

Groundwater management is an important issue, especially in arid regions. There is increasing demand for water in a context of climate change and uncertain future water supplies, rendering more efficient water use crucial. The concept of irrigation efficiency is frequently misunderstood because of an over-restrictive definition. The classical approach to irrigation efficiency is based on the definition proposed by Israelsen [9], i.e. "the ratio of the amount of water that is stored by the irrigator in the soil root zone and ultimately consumed to the amount of water delivered to the farm", which does not take into account return flows of water, water recovery and leaching (Keller and Keller [17]). Against this background, this paper develops a model based on the literature on groundwater economics and the literature on soil salinity and replacement flows.

The literature on groundwater economics is quite well developed along two lines. One strand of work focuses exclusively on pumping patterns and strategic behavior, based on the dynamic model proposed by Gisser and Sanchez [11], Provencher [23], Provencher and Burt [24] and Rubio and Casino [27]. This work takes account of the externalities from private exploitation of a common good by comparison with the socially optimal solution. The other strand of the literature tackles the problem of polluted groundwater due to non-point pollution. Most of the work in this group of studies is based on a pollution control perspective and tends to ignore the relationship between contamination and water pumping decisions (Hellegers et *al.* [13]). However, some authors do address the so-called quantity-quality problem. Among others, Xepapadeas [33] provides an empirical analysis showing that pollution generates production externalities. He proposes the idea of deep percolation caused by irrigation generating groundwater pollution, which negatively affects water quality and therefore agricultural output. Vickner et *al.* [30] and Larson et *al.* [19] develop models that use pesticides and water withdrawal as control variables. However, both of these studies are based on empirical relationships in a dynamic model rather than optimal control theory. Roseta -Palma ([25] and [26]) proposes alternative models for joint quantity-quality management. She shows that intervention can reduce quantity but improve quality and vice versa, and she derives different optimal taxes from those in the existing quantity-only or quality-only models. Hellegers et al. discuss socially optimal agricultural shallow groundwater extraction patterns with mutual pollution interaction. They emphasize an interesting dilution effect and point to the importance of studying the relationship between water quantity and quality. However, these studies focus only on the degradation of groundwater quality and do not address the important issue of the interaction of this natural resource with the whole natural system. The literature on coastal aquifers addresses the quantity-quality question but also takes account of the fact that freshwater is ultimately connected to seawater, which means that freshwater quality may decline if withdrawals become excessive, allowing greater intrusion of seawater. Moreaux and Reynaud [21] go a step further and include certain relevant hydrogeological processes to capture the change in the hydraulic properties of groundwater under saline intrusion and displacement of the interface between freshwater and seawater. However, like some other previous works, they focus on only one aspect of the interaction, namely groundwater quality, whereas there is a bi-directional interaction.

This paper investigates quantity-quality interactions in a resource management model. It also integrates both soil quality and groundwater quality. Degradation of the land through increased soil salinity is an increasing problem, and in many parts of the world it is having a negative effect on food production. To the best of our knowledge, there are no studies that investigate this kind of interaction within a dynamic model. Wichelns [32] examines the economic causes of waterlogging and salinization in arid regions to determine the possibility of economic incentives to reduce these problems. The present paper helps to fill a gap in the literature: it focuses on soil salinity dynamics and analyzes the effect of an incremental increase in water efficiency on long-term soil salt concentrations.

Soil salinity refers to the salt content of soil. Salt is a naturally occurring element in soil and water, and soil salinization can be due to the original soil material or irrigation water being rich in soluble salt. Because much irrigation water is rich in salt, salinization of irrigated land may build over time. The process of accumulation is simple: as plants take up the water, the salt is left behind and accumulates in the root zones in the soil. Excess salt in soil has a negative impact. The higher the accumulations of salt, the less permeable the soil and the more crops are destroyed. IPTRID [7] estimates that 10% to 15% of irrigated areas suffer from salinization, 0.5% to 1% of crops are lost each year, and nearly half of all irrigated areas are threatened by excess salinization in the long term. This problem is jeopardizing food security in many parts of the world: Umali [29] reports that in the US, China and Pakistan, 28%, 23% and 21% of land respectively is affected by salinization. One way to control this problem is to remove the salt from the soil. Salinity makes it more difficult for plants to absorb soil moisture. The salt can be leached out from the plant root zone by applying more water to the land than can be retained by the soil in the crop root zone, so that the excess water drains out below the root system, carrying the salt with it. Thus, the more water that is applied, in excess of the crop requirements, the lower the salinity in the root zone - despite the fact that more salt will be added as a result of this irrigation. Over-irrigation is the most effective way to remove salt from the root zone area in soils. The requirement for additional irrigation is even more crucial in arid regions with low levels of precipitation. In other words, this solution is especially relevant if the soil moisture content is low and the groundwater table is deep. Generally, irrigation should ideally take place in winter when there is more water available. However, in some parts of India, for example, leaching out of salt is most effective during the summer months when the water table is at its lowest and the soil is very dry. Thus, control of salinity by leaching out is a relevant solution (under certain conditions), but it also raises some questions. For example, policies for water conservation could have contradictory effects: their aim may be to improve irrigation efficiency and use less water, which could result in reduced leaching effects and therefore higher soil salinity levels. On the other hand, policies for water conservation could result in lower levels of soil salinity as a result of lower volumes of salt-carrying irrigation water being applied to the soil. This paper investigates whether a water conservation technology could have a perverse impact on soil salinity.

Water conservation can be achieved through irrigation technology that reduces the amount of water that percolates below the root system. In this context, water conservation is associated with improving efficiency. In Israelsen et *al.* [9], efficiency is achieved when the quantity of water used by the plants is higher than the amount of water delivered. However, this leads to a reduction in deep water percolation that removes salt from the root zone and sustains productivity over time. This definition of efficiency overlooks the benefit of soil irrigation for controlling soil salinity. Jensen [16] points out that use of water to control soil salinity should be considered beneficial. According to this author, irrigation efficiency should be defined as the ratio of the water consumed by plants plus the water necessary for leaching out salt to the volume of water applied to achieve sustainable irrigation.

This paper analyzes soil-groundwater interactions in a general setting. We introduce a quality variable in a typical groundwater extraction model. We assume that this quality variable is affected by resource withdrawals and water stocks. Following Xepapadeas [33], we analyze a quality-quantity problem in which soil salinity generates negative externalities on production. We introduce a negative externality on the production process by assuming that soil degradation (due to higher soil salt concentrations) affects total factor productivity (TFP) in the sense of Barro. First, within this framework we show that in the long run, for a saddle-point equilibrium to exist, the soil salt concentration and the groundwater salt concentration must be equivalent in order for a dilution process to lead to the transfer of salt from one reservoir to another until concentrations are balanced. Our main result is that higher irrigation efficiency can lead to increased soil degradation, or the opposite, depending on the social cost of salinization. This means that, contrary to the results in the literature showing that increased irrigation efficiency leads to higher stocks of water , the reverse may also be true, depending on the level of externality costs. More generally, our analysis shows that we need to adopt a wider definition of water efficiency to take account of the role played by water as a vehicle. By taking account of the soil salt externality of irrigation, water efficiency can be seen as the speed of transfer of salt between the two water reservoirs.

The paper is organized as follows. Section 2 introduces the setting of our joint quantity-quality soil-groundwater model and discusses how soil salt and groundwater salt concentrations are linked and evolve over time. Section 3 introduces a socially optimal solution. Section 4 studies the existence and stability of the equilibrium and its properties. Section 5 discusses the impact of improved irrigation efficiency on soil salt concentrations and water stocks. Section 5 provides some concluding comments.

1 The setting

This section presents the overall framework of water and salt dynamics and farmers' behavior.

1.1 Water dynamics

We assume a "bathtub" type aquifer, flat-bottomed with perpendicular sides. The stock of groundwater at time t, $S(t)$, declines because of the extraction flow $w(t)$, or the water resource increases as the result of a constant natural recharge R and return flows $(1-e)w(t)$, i.e. the share of irrigation water that returns to the aquifer. The coefficient of return flows, $0 < e < 1$, assumes that return flows reach the water table almost instantaneously. Under real conditions, return flows may take years to soak into the soil. However, for the sake of simplicity, we consider a standard approximation of a more complex dynamic process. To this end, our water dynamics can be written as follows:

$$\dot{S} = R - ew_t \tag{1}$$

1.2 Salt concentration dynamics

Salt concentrations in the soil and the aquifer are based on three facts: (i) salt is a natural occurring element in soil and water; (ii) the salt content of the soil depends on the volume and salinity of water; and (iii) water serves as the vehicle by which salt is transported into and out of both the soil root zone and the aquifer. Thus, the salt content in the root zone varies according to the quantity and the salt concentration of the water supplied which mixes with the soil water content, and the salt concentration of the water discharged from the same area. Similarly, the salt content in the aquifer depends on the amount of water it contains and the salt concentrations of the extracted water and the total recharge. The soil and the aquifer are two reservoirs of salt that are interdependent, via exchanges of water in the form of irrigation from the aquifer and deep percolation from the soil. We try to capture these interactions through the following laws of motion (see below).

To address the main question in this paper, for simplicity, we assume that soil moisture, theta, is constant over time, as is the total stock of salt, i.e. the sum of the salt quantities diluted in the groundwater and in the soil. By allowing variations in these salt quantities, we can capture the consequences of irrigation-induced salinity on production. This assumption is plausible in that the water-holding capacity of soil is such that there is a constant soil moisture status, or there is an irrigation scheme that allows soil moisture to remain constant over time. In other words, we assume a perfect balance between the total amount of water infiltrating the soil, i.e. the sum of the natural recharge and the applied water $(R + w(t))$, and the quantity of water lost by the soil, i.e. the amount used by plants and the amount that percolates through the ground $(ew(t) - Percolation)$: this deep percolation corresponds to the total recharge: $(R + (1 - e)w(t))$.

We also assume that the land area is located above the aquifer and deep percolation corresponds to a perfect vertical movement (Hoffman [14]). This assumption allows us to have a given total stock of salt M that is the sum of the salt in the soil moisture and in the groundwater:

$$M = C_t^{\theta}\theta + C_t^S S_t \tag{2}$$

with C_t^S and C_t^{θ}, respectively groundwater salt concentration and the soil salt concentration per unit of water at time t.

This means that we only need to integrate one salt concentration dynamics into the analysis. We choose the soil salt dynamics arbitrarily. As already mentioned, the variation in the concentration over time depends on the difference between the inflow of salt diluted in irrigation water and the natural recharge, which is salt-free, i.e. $C_t^S w(t)$, and the outflows due to deep percolation, i.e. $C_t^\theta[(1-e)w_t + R]$. Formally, this can be written:

$$\begin{aligned} \dot{C^\theta}\theta &= C_t^S w_t - C_t^\theta \left[(1-e)w_t + R\right] \\ \Leftrightarrow \dot{C^\theta} &= \frac{\left[C_t^S - C_t^\theta(1-e)\right] w_t - C_t^\theta R}{\theta} \end{aligned} \tag{3}$$

1.3 Net farm profit

The net farm profit corresponds to the value of production net of production costs.

1.3.1 The production function

Water, denoted w, is used as a single input in a standard production function $F(\cdot)$ which is increasing and concave, $F'(\cdot) > 0$ and $F''(\cdot) < 0$. We ignore all other possible inputs in order to highlight the interaction between the soil and the water systems.

We need to account for the difference between the actual volume of water used by the crop, i.e. the effective water, and the crop's water requirement, $e \cdot w$, and the amount of applied water w. As already mentioned, only a part of the irrigation water is absorbed by the crop; we assume that the remaining part returns to the aquifer in its entirety. In fact, application efficiency, e, depends on various exogenous parameters, such as land quality, and the technology available (Caswell and Zilberman [5]; Burness and Brill [4]; Chakravorty and Umetsu [6]). Therefore, for a given crop production, the crop water requirement is fixed and the representative farmer has to decide how much water must be applied to meet this crop requirement given the technology available.

Finally, we assume that output is reduced by soil salinity. We assume that salt concentration hinders TFP, i.e. the part of growth not explained by the inputs used in crop production. As in endogenous growth models (Barro [3]), we assume that TFP depends on the soil salt concentration: $A(C^{\theta})$. This allows us to capture the agronomic process, i.e. the fact that energy is required for the plant to extract water from the soil. This has an impact on economic production in addition to water input. However, as salt concentrations increase, it becomes increasingly difficult for the plant to absorb water, because of the energy required to access it. This extra energy used by the crops detracts from growth, inevitably leading to reduced production and sometimes plant death. However, we assume that the marginal effect of salt concentration is decreasing, meaning that a higher salt concentration when salt concentration is already high will have a lower impact.

Assumption 1 $A'(C^{\theta}) < 0$; $A''(C^{\theta}) < 0$.

Given our definition of TFP, we interpret $A(C_t^{\theta})$ differently to the standard approach in the growth literature (Barro [3], Solow [28]). In our case, TFP is a negative externality in the production process. It contributes to reducing production and, in that sense, represents the share of production from which the farmer benefits . Thus, in the case of soil that contains no salt, TFP means that the farmer reaps the benefits of his entire production; in the case of high salt concentrations, TFP is affected so negatively that production is nil. Formally, we can express this as:

Assumption 2 $\lim\limits_{C_t^{\theta} \to 0} A(C_t^{\theta}) = 1$; $\lim\limits_{C_t^{\theta} \to \infty} A(C_t^{\theta}) = 0$

Plants have a wide range of responses to soil salinity. Reduced growth is usually progressive once the level rises above the plant's tolerance threshold. This threshold will vary from crop to crop, but we can identify certain typologies. There are some plants, such as apple trees or red fruit bushes, that can be described as salt tolerant, i.e. they can thrive in saline soils; these contrast with very salt-sensitive crops, such as olive trees and asparagus plants, which do not thrive in saline soils. For the sake of clarity in our analysis and without any loss of generality, we do not include different thresholds in our model. We analyze the system as if the soil salt concentration were already above the tolerance level, in order to investigate the equilibrium.

1.3.2 Groundwater extraction cost

Following the literature (Gisser [10], Koundouri [18], Rubio and Casino [27]), groundwater use involves a stock-dependent extraction cost. We denote the unit pumping cost by $c(S)$, which depends on the water stock S. This cost function is decreasing and convex, which means that at lower stock levels, it is more costly to extract water because the resource must be pumped over longer distances, and as the aquifer nears exhaustion, this unit cost increases rapidly.

Assumption 3 $c'(S) < 0; c''(S) > 0.$

We also assume that the unit pumping cost is zero when the aquifer is full, but infinitely high when there is no groundwater. Finally, we claim intuitively that the marginal pumping cost externality decreases infinitely as the water resource is exhausted, and tends to zero when the aquifer is full. This yields:

Assumption 4 $C(\overline{S}) = 0; \quad \lim_{S \to 0} c(S) = +\infty; \quad \lim_{S \to 0} c'(S) = -\infty; \quad \lim_{S \to \overline{S}} c'(S) = 0.$

1.3.3 Net farm benefit

Incorporating earlier discussions on production and cost, the time t profit, π_t, of the representative farmer is given by:

$$\pi_t = A(C_t^\theta) \cdot F(ew_t) - c(S_t)w_t \tag{4}$$

2 The optimal management

In this context, the objective of the social planner is to maximize the sum of discounted net agricultural benefits with respect to $w(t)$ and subject to the state equations (1) and (3) and the static relation (2). Formally, the social planner's problem is given by:

$$\max_{\{w_t\}} \int_0^\infty \left(A(C_t^\theta) \cdot F(ew_t) - c(S_t)w_t\right) \exp^{-\rho t} dt$$
$$\text{w.r.t} \left| \begin{array}{l} \dot{S} = R - ew_t \\ \dot{C^\theta} = \frac{\left[C_t^S - C_t^\theta (1-e)\right] w_t - C_t^\theta R}{\theta} \\ C^S\left(C_t^\theta, S_t\right) = \frac{M - C_t^\theta \theta}{S_t} \\ \lim\limits_{t\to\infty} \lambda_t \geq 0 \quad \lim\limits_{t\to\infty} \lambda_t S_t = 0 \\ \lim\limits_{t\to\infty} \mu_t \geq 0 \quad \lim\limits_{t\to\infty} \mu_t C_t^\theta = 0 \\ S_0,\ C_0^\theta \text{ given and :: } S(\infty) \text{ free} \end{array} \right. \tag{5}$$

The transversality conditions require that the two co-state variables λ_t and μ_t are not negative and the value of groundwater $\lambda_t S_t$ and the cost of the salt per unit of resource $\mu_t C_t^\theta$ are driven to zero at the end of the planning period.

The current value Hamiltonian for the optimal management problem is:

$$H = A\left(C_t^\theta\right) \cdot F(ew_t) - c(S_t)w_t + \lambda_t(R - ew_t) - \mu_t \left(\frac{\left[C_t^S(\cdot) - C_t^\theta(1-e)\right] w_t - C_t^\theta R}{\theta}\right)$$

where $\lambda_t, \mu_t \geq 0$, are respectively the shadow price associated with the stock of water and the shadow cost associated with the concentration of salt.

We then obtain the following first-order conditions (for an interior solution):

$$eA\left(C_t^{\theta}\right)\cdot F'(ew_t) = c(S_t)+e\lambda_t+\mu_t\frac{\left(C_t^S(\cdot)-C_t^{\theta}(1-e)\right)}{\theta} \tag{6}$$

$$\frac{\dot{\lambda}}{\lambda_t} = \rho+\frac{c'(S_t)w_t}{\lambda_t}+\frac{\mu_t}{\lambda_t\theta}\frac{\partial C_t^S}{\partial S_t}w_t \tag{7}$$

$$\frac{\dot{\mu}}{\mu_t} = \rho+\frac{1}{\theta}\left[\left(-\frac{\partial C_t^S}{\partial C_t^{\theta}}+(1-e)\right)w_t+R\right]+\frac{A'(\cdot)\,F(ew_t)}{\mu_t} \tag{8}$$

Equation (6) represents the usual optimality result, which yields a marginal benefit in each period equal to the sum of the total marginal extraction cost (the sum of actual extraction cost and the opportunity cost of removing 1 unit of water from the ground), the opportunity cost of changing the salt concentration through the addition of 1 unit of groundwater and the shadow cost of the salt-soil concentration adjusted to the difference in salt concentration between the two reservoirs. Both opportunity costs reflect the future impact on profits: the first is the effect of extracting 1 unit of resource today rather than at some time in the future, and the second is the future impact of the new salt-soil concentration due to the application of groundwater.

Equations (7) and (8) describe the behavior of the two shadow prices. By using equation (2), equation (7)can be rewritten as:

$$\frac{\dot{\lambda}}{\lambda_t}=\rho+\frac{c'(S_t)w_t}{\lambda_t}-\frac{\mu_t}{\lambda_t}\cdot\frac{w_t}{\theta}\cdot\frac{\partial C_t^S}{\partial C_t^{\theta}}\cdot\frac{\partial C_t^{\theta}}{\partial S_t} \tag{9}$$

The above equation shows that the growth rate of the water scarcity rent depends on three effects. It depends on the discount factor; the change in future costs due to the variation in water stock, and a negative effect that is a combination of a dilution effect and an interdependence effect. The dilution effect, $\frac{w_t}{\theta}$, is represented by the share of applied irrigation water in soil moisture. This is the part of the water that is added and is therefore mixed with the soil water content. We refer to this as the dilution effect. The more water that is applied, the more the soil salt content is diluted. The second term, the interdependence effect, $\frac{\partial C_t^S}{\partial C_t^\theta} \cdot \frac{\partial C_t^\theta}{\partial S_t} > 0$, captures the interaction between soil and aquifer through the interdependence of salt concentration in the two reservoirs, and the impact of a larger water stock on the salt concentration in the soil. On the one hand, as the salt concentration in the soil increases, the salt concentration in the aquifer decreases. Given the total salt stock M and soil moisture θ, an increase in soil salt concentration means that a quantity of salt is transferred from the aquifer to the soil, leading to a reduction in the groundwater salt concentration. On the other hand, a larger water stock reduces the soil salt concentration . Consequently, the water being applied will be less salty than the soil water content, which should reduce the salt concentration in the soil.

Based on equation (2), equation (8) can be rewritten as follows:

$$\frac{\dot{\mu}}{\mu_t} = \rho + \frac{A'(\cdot)\, F(ew_t)}{\mu_t} + \left(\frac{w_t}{S_t} + \frac{(1-e)w_t + R}{\theta}\right) \tag{10}$$

The rate of change of soil salt concentration over time depends on the sum of three effects: (1) the discount rate; (2) the TFP effect, which is negative, meaning that a higher soil salt concentration reduces production and therefore reduces the soil quality value; (3) a dilution effect, which is characterized by the exchange of water between the two reservoirs, i.e. the share of withdrawals and the share of water lost by the soil. This last term represents the role of water in salt transfer and increases the value of soil quality.

3 Existence and Stability of the steady-state

This section analyzes the existence of a steady state(s) and investigates the stability properties.

3.1 Existence of a steady-state

A steady state, if it exists, can be investigated by setting the time derivatives of equations (1), (3), (7) and (8) respectively, to zero. We first compute the time derivative of equation (6):

$$\begin{aligned} \dot{w}eA(\cdot)F''(\cdot) &= \dot{S}\left(\frac{c'(S_t)}{e}+\frac{\mu}{e\theta}\frac{\partial C^S}{\partial S}\right)-\dot{C}^\theta\left[A'(\cdot)F'(\cdot)-\frac{\mu_t}{e\theta}\left(\frac{\theta}{S}+(1-e)\right)\right] \\ &+ \dot{\lambda}+\dot{\mu}\frac{\left(C_t^S-C_t^\theta(1-e)\right)}{e\theta} \end{aligned} \tag{11}$$

Equation (11) shows that the change in the shadow value of water $\dot{\lambda}$ and the change in the shadow cost $\dot{\mu}$, are equal to zero, based on our knowledge that at the steady state, $\dot{w}=\dot{S}=\dot{C}^\theta=0$. Using equations (6) and (7), we can rewrite equation (11) as follows:

$$\begin{aligned} \dot{w}eA(\cdot)F''(\cdot) &= \dot{S}\left(\frac{c'(S_t)}{e}+\frac{\mu}{e\theta}\frac{\partial C^S}{\partial S}\right)-\dot{C}^\theta\left[A'(\cdot)F'(\cdot)-\frac{\mu_t}{e\theta}\left(\frac{\theta}{S}+(1-e)\right)\right] \\ &+ \rho\left(A(\cdot)F'(\cdot)-\frac{c(S_t)}{e}-\mu_t\frac{(C_t^S-C_t^\theta(1-e))}{e\theta}\right)+c'(S_t)w_t \\ &+ \frac{\mu_t}{\theta}\frac{\partial C_t^S}{\partial S_t}w_t+\dot{\mu}\frac{\left(C_t^S-C_t^\theta(1-e)\right)}{e\theta} \end{aligned} \tag{12}$$

This allows us to reduce the dimensionality of the system from a set of five variables $\{w;S;C^\theta;\lambda;\mu\}$ to a set of four variables $\{w;S;C^\theta;\mu\}$.

We can then investigate the steady state. From equation (1), we can directly derive the steady state level of extraction:

$$w^* = \frac{R}{e} \tag{13}$$

As usual, the rate of extraction depends on the natural recharge and the irrigation efficiency.

From equation (2) and (3), the steady state of salt concentration can be computed as a function of the stock of groundwater:

$$C^{\theta^*}(S^*) = C^{S^*}(S^*) = \frac{M}{S^* + \theta} \tag{14}$$

The salt concentration, at the steady state, is simply the ratio of the total salt stock M over the sum of the water contained in both reservoirs. Also, in the long-run, the mixing of both water sources leads to the same salt concentration.

Using condition (8), we obtain that the steady state for the co-state variable of the salt concentration as a function of the stock of groundwater is:

$$\mu^*(S^*) = -A'\left(\frac{M}{S^* + \theta}\right) \cdot F(R) \cdot \frac{1}{\left(\rho + \frac{R}{eS^*} + \frac{R}{e\theta}\right)} \tag{15}$$

The cost of salt concentration depends on the negative impact on productivity relative to the share of the extracted amount of water and the part of the applied water that is in soil moisture.

Equation (12) gives the condition required for the level of water stock at the steady state. On the basis of previous observations, investigation of the existence of a steady state can be reduced to a study of the following condition:

$$\begin{aligned}\phi(S^*) &\equiv \rho\left[A\left(\frac{M}{S^*+\theta}\right)\cdot F'(R)-\frac{c(S^*)}{e}\right]+c'(S^*)\frac{R}{e}\\ &+ \frac{A'\left(\frac{M}{S^*+\theta}\right)\cdot F(R)\cdot M}{Z\left(S^*\right)\cdot\left(S^*+\theta\right)}\left[\rho S^* e+R\right]=0 \end{aligned} \tag{16}$$

with $Z\left(S^*\right)=\rho e S_t^*\theta+R(S_t^*+\theta)>0$.

The analysis of this condition is straightforward and allows us to assert that:

Proposition 1 *Under assumptions 1-4, there exists a unique stationary equilibrium.*

Proof 1 (for the proof, see appendix)

3.2 Stability of the dynamic system

Next, we examine the local dynamics of the system. Using the method proposed by Dockner [8] to compute in a simple way the stability properties of a four-dimensional dynamic system, we find that:

Proposition 2

(i) *For a stationary state* $\left(w^*, C^{\theta^*}, S^*, \mu^*\right)$ *, if the marginal TFP is small enough, i.e.* $A'(\cdot)\rightarrow 0$*, then the stationary state is a saddle point.*

ii) *We have the sufficient conditions for real eigenvalues ensuring local monotonicity, i.e. a monotonic approach to equilibrium on a stable two-dimension manifold.*

Proof 2 (for the proof, see appendix)

4 Impact of long-run irrigation efficiency

We can now investigate whether improved on-farm irrigation efficiency e will lead to an increase in soil salt concentrations. We first compute the implicit derivative of the salt concentration equilibrium:

$$\frac{\partial C^{\theta^*}}{\partial e} = \frac{\partial S^*}{\partial e} \cdot \frac{C^{\partial\theta^*}}{\partial S} = -\frac{\partial S^*}{\partial e} \cdot \frac{M}{(S^*+\theta)^2} \tag{17}$$

Since we know that $\phi(S,e)$ given in equation (16) is such that $\phi(S,e) = 0$ and that its first derivative with respect to S is strictly positive, we can use the implicit function theorem to deduce the sign of $\frac{\partial S^*}{\partial e}$. Formally, we know that:

$$\frac{\partial S^*}{\partial e} = -\frac{\frac{\partial \phi(S,e)}{\partial e}}{\frac{\partial \phi(S,e)}{\partial S}} \text{ with } \frac{\partial \phi(S,e)}{\partial S} > 0$$

Therefore, we can say that the impact of an increase in on-farm irrigation efficiency is given by the impact of an increase in e on the function $\phi(S,e) = 0$. That is:

$$sign\left(\frac{\partial C^{\theta^*}}{\partial e}\right) = sign\left(\frac{\partial \phi(S,e)}{\partial e}\right) \tag{18}$$

Let us compute the derivative of interest:

$$\frac{\partial \phi(S,e)}{\partial e} = \frac{1}{e^2}\left(\rho c(S^*) - c'(S^*)R + \frac{\rho e^2}{Z^2(S+\theta)} \cdot A'(\cdot)F(R) \cdot M \cdot S^2 \cdot R\right)$$

Rearranging this equation based on the steady state value of the salt concentration C^{S^*} and the co-state μ^* given by equations (14) and (15), we obtain:

$$\frac{\partial \phi(S,e)}{\partial e} = \frac{\rho}{e^2}\left(c(S^*) - \frac{c'(S^*)R}{\rho} - \frac{eS^*}{\theta Z} \cdot \mu^* \cdot C^{S^*}R\right)$$

We notice first that, if $A'(\cdot) = 0$, i.e. there is no negative externality on productivity because of high soil salinity, then the social cost of this pollution is logically zero: $\mu^* = 0$. In this case, the sign of $\frac{\partial \phi(S,e)}{\partial e}$ is straightforward and positive. Our result is the same as in the literature (e.g. Ahmad [2], Huffaker and Whittlesey [15], Lichentenberg [20], Pfeiffer and Lin [22], Ward and Pulido-Velasquez [31]) i.e. that an increase in irrigation efficiency leads to a decline in water stock. This is because a more efficiency technology may have perverse effects. On the one hand, a more efficient irrigation system typically sparks an upward shift in the crop production function. This implies a higher rate of crop water consumption. On the other hand, more efficient technology causes a switch from the production of crops with low water requirement to water-intensive crops. Here, the fall in groundwater reserves leads to an increase in salt concentration. As irrigation systems become more efficient, the amount of water applied tends to match more and more precisely the crop water requirements, and this negates the need to leach the soil. However, this result holds because there is no externality on production caused by saline soil.

If we turn to the case where there are some negative externalities, that is $A'(\cdot) < 0$, then the sign is ambiguous. Recall that water serves as a vehicle for the salt in our system, inasmuch as some salt is added to the soil through irrigation while some salt is leached out to the ground through return flows. Thus, two cases can be distinguished: the positive leaching externality of irrigation water, and the irrigation-induced salinity externality. Depending on the circumstances, an improved irrigation system can have two opposite impacts.

4.1 Alienation of the positive leaching externality of irrigation water

If the social cost of transferring salt from the aquifer to the soil is sufficiently low (compared with the full marginal cost of extraction), then an improved irrigation system will lead to a decline in groundwater stocks and an increase in the salt concentration in the soil:

$$c(S^*) - \frac{c'(S^*)R}{\rho} > \frac{eS^*}{\theta Z} \cdot \mu^* \cdot C^{S^*} R$$

The left-hand side of the inequality represents the full marginal cost of extraction, which is the sum of the marginal pumping cost and the marginal pumping cost externality. Recall that this externality arises because the cost of extraction increases with the pumping depth. This is represented in our case by the capitalized value of future increases in cost resulting from a reduction in water stock equal to the recharge. The right-hand side inequality reflects the value of the flow of salt from the aquifer to the soil. At the steady state, this flow is given by $C^{S^*}R$. Since μ^* is the optimal social cost associated with the change in the soil salinity, we can interpret $\mu^* \cdot C^{S^*} \cdot R$ as the optimal social cost of transferring salt from the aquifer to the soil. This makes this transfer costly, because it increases the concentration of salt in the soil, which reduces productivity for the representative farmer. However, when this social cost is low, the impact on productivity is also low. Therefore, an improvement in irrigation efficiency should result in higher crop production per unit of water and should imply an upward movement in production similar to the basic case without any externality. In addition, in the presence of a negative externality, the higher the evapotranspiration due to higher levels of production, the higher the amount of salt left in the soil. If we recall Jensen's definition [16], the measurement of efficiency cannot be based only on crop water requirement; it must include the need to leach the soil with extra volumes of water to prevent an increase in salt concentration. According to this point of view, an "augmented" efficiency approach should allow to control both soil salinity and water consumption to achieve sustainable irrigation.

4.2 Irrigation-induced salinity externality

The scenario changes when the social cost of transferring salt from the aquifer to the soil is so high that an improvement in irrigation efficiency leads to a reduction in the soil salt concentration.

$$c(S^*) - \frac{c'(S^*) \cdot R}{\rho} < \frac{e \cdot S^*}{\theta Z} \cdot \mu^* \cdot C^{S^*} \cdot R$$

This means that the impact on production of the concentration of salt in the upper reservoir is sufficient . In this case, more efficient irrigation means that productivity decreases more rapidly . In other words, although more efficient irrigation increases production per unit of water, this is counterbalanced by the negative impact of TFP. Recall that higher salt concentrations require plants to expend more energy to absorb water, which reduces the amount of energy that can be devoted to plant growth. In contrast to the first case, where higher irrigation efficiency increases total water consumption through a higher rate of evapotranspiration, in this case, the water actually used by the crop is reduced, which leaves an amount of water that leaches out some of the salt. Consequently, an increase in irrigation efficiency increases water stocks and removes salt from the soil.

We have shown that a broader definition of water efficiency is needed, to take account of the role played by water as a vehicle for salt. Efficiency can be defined as the "speed" of transfer of salt between aquifer and soil.

5 Conclusion and future directions for research on salinity abatement

This paper contributes to the literature and introduces a quality dimension into the resource management model. It differs from most previous studies, which focus on the impact of groundwater withdrawal on water quality, by investigating the effect of groundwater extraction and quality on another ecosystem, the soil. We have studied the dynamics of socially optimal water pumping with respect to the optimal dynamics of soil salt concentration and discussed the impact of improved irrigation efficiency on soil salinity. We depicted two scenarios, in which groundwater extraction either increases or reduces soil salinity. These results point to the importance of adopting a different approach to irrigation efficiency. The most common definition is not useful in that it does not take account of other beneficial water uses, including the fact that demand for a water-based ecosystem is as important as demand for soil leaching.

However, this paper focuses on just one problem of salt-affected soil, namely the insufficient volume of irrigation water to leach away accumulated salt in the soil. Extensions to this research could include investigation of a number of other features. A first extension could be to integrate the upward movement of salt from groundwater to the soil due to capillary action. Over time, the water table may rise due to excessive irrigation and deep percolation, favouring the build-up of salt in the root zone and the surface soil. Modeling the effect of capillary water action in a resource management model would be relevant when two water sources are being used simultaneously.

Another extension would be to introduce explicit trade-offs between investment in irrigation technology, reducing water loss, and investment in drainage technology, reducing yield losses caused by inadequate drainage. This investigation would be relevant in the context of waterlogging problems or possible water transfers from upstream to downstream users.

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Appendix : proof of proposition 1

By construction, the steady state $\{w^*; S^*; C^{\theta *}; \mu^*\}$ satisfies :

$$\begin{cases} w^* = \frac{R}{e} \\ C^{\theta^*}(S^*) = C^{S^*}(S^*) = \frac{M}{S^*+\theta} \\ \mu^*(S^*) = -\frac{A'\left(\frac{M}{S^*+\theta}\right)F(R)eS^*\theta}{\rho e S^*\theta + R(S^*+\theta)} \\ \phi(S^*) = \rho\left[A\left(\frac{M}{S^*+\theta}\right)\cdot F'(R) - \frac{c(S^*)}{e}\right] + c'(S^*)\frac{R}{e} + \frac{A'\left(\frac{M}{S^*+\theta}\right)\cdot F(R)\cdot M}{Z\cdot(S^*+\theta)}\left[\rho S^* e + R\right] = 0 \end{cases}$$

We only have to verify that $\phi(S)$ admits at least one solution. First, we observe that:

$$\begin{aligned} \lim_{S\to 0}\phi(S) &= \rho\left(A\left(\frac{M}{\theta}\right)F'(R) - \lim_{S\to 0} c(S_t)\right) + \frac{A'\left(\frac{M}{\theta}\right)\cdot F(R)\cdot M}{\theta^2} = -\infty \\ \lim_{S\to +\infty}\phi(S) &= \rho F'(R) > 0 \text{ with Assumption 2, 4 and } \lim_{S\to +\infty} A'(\cdot) = 0 \end{aligned}$$

We also observe that the function is monotonic:

$$\begin{aligned} \phi'(S) &= \rho\left[-\frac{M}{(S+\theta)^2}A'\left(\frac{M}{S+\theta}\right) cdot F'(R) - \frac{c'(S)}{e}\right] + c''(S)\frac{R}{e} \\ &+ \frac{u\left[\rho S^* e + R\right] - \frac{A'\left(\frac{M}{S^*+\theta}\right)\cdot F(R)\cdot M}{Z\cdot(S^*+\theta)}\rho e}{\left[\rho S^* e + R\right]^2} > 0 \end{aligned}$$

with $u = \frac{-\frac{M}{(S+\theta)^2}A''\left(\frac{M}{S+\theta}\right)\cdot F'(R)Z(S+\theta) - A'\left(\frac{M}{S+\theta}\right)\cdot F'(R)M[Z+(\rho e\theta+R)(S+\theta)]}{Z^2(S+\theta)^2} > 0$ and given $A''(\cdot) \leq 0$.

Since $\phi(S)$ is strictly increasing from $-\infty$ to $\lim_{S \to +\infty} \phi(S) = \rho F'(R) > 0$, there is at least one S^* such that $\phi(S^*) = 0$.

Appendix : proof of proposition 2

As usual, focusing on the local stability of the dynamic system, we can derive the following Jacobian matrix.

$$J = \begin{pmatrix} \frac{\partial \dot{w}}{\partial w} & \frac{\partial \dot{w}}{\partial C^\theta} & \frac{\partial \dot{w}}{\partial S} & \frac{\partial \dot{w}}{\partial \mu} \\ \frac{\partial \dot{C}^\theta}{\partial w} & \frac{\partial \dot{C}^\theta}{\partial C^\theta} & \frac{\partial \dot{C}^\theta}{\partial S} & \frac{\partial \dot{C}^\theta}{\partial \mu} \\ \frac{\partial \dot{S}}{\partial w} & \frac{\partial \dot{S}}{\partial C^\theta} & \frac{\partial \dot{S}}{\partial S} & \frac{\partial \dot{S}}{\partial \mu} \\ \frac{\partial \dot{\mu}}{\partial w} & \frac{\partial \dot{\mu}}{\partial C^\theta} & \frac{\partial \dot{\mu}}{\partial S} & \frac{\partial \dot{\mu}}{\partial \mu} \end{pmatrix}_{(w^*, C^{\theta *}, S^*, \mu^*)}$$

To find the properties of this matrix, we shall apply a method developed by Dockner [8] to investigate in a simple way the stability properties of a linearized four-dimensional dynamic system. Using this method, the eigenvalues of the system can easily be computed according to the following simple formula:

$$p_{1,2,3,4} = \frac{\rho}{2} \pm \sqrt{\left(\frac{\rho}{2}\right)^2 - \frac{\Omega}{2} \pm \frac{1}{2}\sqrt{\Omega^2 - 4detJ}}$$

with

$$\Omega = \begin{pmatrix} \frac{\partial \dot{w}}{\partial w} & \frac{\partial \dot{w}}{\partial S} \\ \frac{\partial \dot{S}}{\partial w} & \frac{\partial \dot{S}}{\partial S} \end{pmatrix} + \begin{pmatrix} \frac{\partial \dot{C}^\theta}{\partial C^\theta} & \frac{\partial \dot{C}^\theta}{\partial \mu} \\ \frac{\partial \dot{\mu}}{\partial C^\theta} & \frac{\partial \dot{\mu}}{\partial \mu} \end{pmatrix} + 2 \begin{pmatrix} \frac{\partial \dot{w}}{\partial C^\theta} & \frac{\partial \dot{w}}{\partial \mu} \\ \frac{\partial \dot{S}}{\partial C^\theta} & \frac{\partial \dot{S}}{\partial \mu} \end{pmatrix}$$

According to this method, the equilibrium of the system is saddle path if $detJ > 0$ and $\Omega < 0$.

However, to study the stability of the dynamic system, we will also assume that the marginal TFP is small enough in the long run, i.e. $A'(\cdot) \to: 0$. This means that the negative impact of salt concentration in the soil does not hardly reduce crop production.

Within this assumption, the Jacobian matrix can be rewritten as:

$$J = \begin{pmatrix} \frac{\partial \dot{w}}{\partial w} & 0 & \frac{\partial \dot{w}}{\partial S} & \frac{\partial \dot{w}}{\partial \mu} \\ \frac{\partial \dot{C}^\theta}{\partial w} & \frac{\partial \dot{C}^\theta}{\partial C^\theta} & \frac{\partial \dot{C}^\theta}{\partial S} & 0 \\ -e & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial \dot{\mu}}{\partial \mu} \end{pmatrix}_{(w^*, C^{\theta*}, S^*, \mu^*)}$$

with

$$\begin{aligned}
\frac{\partial \dot{w}}{\partial w} &= \frac{1}{eA(\cdot)F''(\cdot)} \left(\rho e A(\cdot)F''(\cdot) + c'(S) + c'(S)\right) = \rho > 0 \\
\frac{\partial \dot{w}}{\partial S} &= \frac{1}{eA(\cdot)F''(\cdot)} \left(-\rho \frac{c'(S)}{e} + c''(S)\right) < 0 \\
\frac{\partial \dot{w}}{\partial \mu} &= -\frac{1}{eA(\cdot)F''(\cdot)} \left(\frac{e\rho}{\theta} C^{\theta^*} + \frac{R}{\theta^2} C^{S^*} > 0\right) \\
\frac{\partial \dot{C}^\theta}{\partial C^\theta} &= -\frac{1}{\theta} \left[\frac{R}{e}\left(\frac{\theta}{S^*} + (1-e)\right) + R\right] < 0 \\
\frac{\partial \dot{\mu}}{\partial \mu} &= \rho + \frac{R}{e}\left(\frac{1}{S^*} + \frac{1}{\theta}\right)
\end{aligned}$$

We therefore can deduce the following results

(i) First, we can easily calculate $\det J = e \cdot \frac{\partial \dot{\mu}}{\partial \mu} \cdot \frac{\partial \dot{w}}{\partial S} \cdot \frac{\partial \dot{C}^\theta}{\partial C^\theta} > 0$. In a similar way, we find that $\Omega = e\frac{\partial \dot{w}}{\partial S} + \frac{\partial \dot{C}^\theta}{\partial C^\theta} \cdot \frac{\partial \dot{\mu}}{\partial \mu} < 0$.Thus, the steady state is a saddle point (this shows part (i) of proposition 2).

(ii) Besides, we can also show that we always have $\frac{1}{4}\Omega^2 > det(J) > 0$. This condition is sufficient for real eigenvalues to exist and thus for local monotonicity to hold (this shows part (ii) of proposition 2)..