Modeling the horizon-dependent risk premium in the forex market: evidence from survey data

Georges PRAT
Remzi UCTUM
Modeling the horizon-dependent risk premium in the forex market: evidence from survey data

Georges PRAT
EconomiX, Centre National de la Recherche Scientifique (CNRS) and University of Paris West Nanterre La Défense, 200 avenue de la République, 92001 Nanterre, France.
Email: prat@u-paris10.fr

Remzi UCTUM
EconomiX, Centre National de la Recherche Scientifique (CNRS) and University of Paris West Nanterre La Défense, 200 avenue de la République, 92001 Nanterre, France.
Email: uctum@u-paris10.fr. Phone (+33) 1 40 97 78 48. Fax (+33) 1 40 97 77 84.

Abstract – Using Consensus Economics survey data on experts’ expectations, we aim to model the 3- and 12-month ahead ex-ante risk premia on the Yen/USD and the British Pound/USD exchange markets. For each market and at a given horizon, we show that the risk premium is well determined by the conditional expected variance of the change in the real exchange rate, agents’ real net market position in assets and a constant composite risk aversion coefficient, as suggested by a two-country portfolio asset pricing model. The expected variance depends on the past values of the observed variance and the unobservable real net market position is estimated as a state variable using the Kalman filter methodology. We found that the trends of our estimated horizon-specific net market positions are consistent with the ones of the observed short term aggregate net market positions calculated using the U.S. Treasury International Capital System dataset. Moreover, we show that the ex-post premia tend to adjust towards the ex-ante values, suggesting that experts’ beliefs provide a relevant information to the market. These results bring new responses to the difficulties reported by the widespread ex-post risk premium literature and enhances the usefulness of survey data in modelling the risk premium.

1 Corresponding author.
Keywords: risk premium – foreign exchange market – international asset pricing model – survey data

Classification J.E.L.: D84, F31, G14

1. Introduction

Since the beginning of the floating exchange rates in 1973, the asset approach to the exchange rate has become the dominant theoretical model of exchange rate determination. According to the class of portfolio balance models, the risk premium is an important factor of the exchange rate. Under the risk-neutrality hypothesis, domestic and foreign assets are perfect substitutes, and the forward exchange rate equals the expected exchange rate: in this case the uncovered interest rate parity (UIRP) is equivalent to the covered interest rate parity (CIRP). But in the general case when agents are risk adverse, domestic and foreign bonds are imperfect substitutes, so that the open positions taken by speculative agents in the foreign exchange market lead them to take into account the uncertainty associated with the future value of the spot rate. In this case, the spread between the expected and the forward exchange rates represents the risk premium required by agents to hold foreign assets in place of domestic assets.

While the risk premium plays an essential role in exchange rate dynamics, the way to model it is still an open issue for research. Most of empirical analyses are based on the ex-post risk premium$^2$ where the exchange rate expected at time $t$ for $t+1$ is replaced by the one observed at time $t+1$. The main drawback of this approach is that agents do not refer to the ex-post premium to decide their financial choices at time $t$ because at this time the future

---

$^2$ See Baillie and MacMahon (1989, §7.7), MacDonald (1990), Lewis (1995) and Engel (1996) for surveys of the literature on ex-post risk premium models.
exchange rate is not known to them. Under the rational expectation hypothesis (REH), the ex-post risk premium corresponds to the rational ex-ante premium plus a white noise forecast error. This ex-ante premium remains unknown to the investigator since the rational expectations of exchange rate are unknown. However, the estimators are not biased since the white noise error is captured by the residuals of the ex post premium model. Of course, if the investigator uses ex-post premium in place of ex-ante premium when the market is not rational, estimation biases cannot be avoided. One way to overcome these difficulties is to use ex-ante risk premium calculated with survey-based expectations, provided that the survey sample is representative of the market.

Studies attempting to model the ex-post premium raise numerous difficulties which can be summarized as follows. First, the failure for the forward exchange rates to predict future values of the spot rates suggests that at least one of the REH and the risk neutrality hypothesis is to be rejected. Second, under his so-called predicted excess return puzzle, Fama (1984) showed that the variance of excess returns (i.e. the ex-post rational premium) is larger than the one of the ex-post change in exchange rate, implying that the predictive power of the spot exchange rate is higher than the one of the forward rate, which is rather counter-intuitive. Third, although the ex-post risk premium exhibits strong time variability, empirical analyses have depicted rather weak volatility effects (ARCH effects) and this result makes the ex-post premium a questionable concept. Fourth, although general equilibrium models related to the international CCAPM predict the existence of a risk premium in the foreign exchange market,

3 Note that, under the perfect foresight hypothesis, the ex-post premium is equal to the ex-ante premium required at the time \( t \) of the decision, so that the ex-post premium becomes a behavioural concept. However, under this hypothesis, there is no risk premium!

4 See MacDonald and Taylor (1989) and Baillie and MacMahon (1989, Chapter 6).

these models are not empirically validated.\textsuperscript{6} Fifth, partial equilibrium models based on the international CAPM do not do better. When the ex-post premium is indeed assumed to depend on a vector of \textit{ad-hoc} instrumental variables (among them, past predictive errors), these models fail to represent the risk premium in the foreign exchange market.\textsuperscript{7}

Overall, empirical studies based on ex-post risk premia have proved unsuccessful in identifying significant factors of the premia in the foreign exchange market. In fact, under the market efficiency hypothesis, the models mentioned above lead to a single equilibrium value of the risk premium whatever the time horizon of investment. It will be shown in this study that the partially predictable feature of returns\textsuperscript{8} allows for a set of premia depending on the time horizon of the investment, and that these horizon-dependent premia are to be modelled.

These difficulties led some authors to focus on ex-ante rather than ex-post risk premia. To measure the ex-ante risk premium, that is the difference between the expected exchange rate and the forward rate, some studies used survey data to represent experts’ exchange rate expectations. This approach has the advantage of avoiding arbitrary hypotheses about expectation representation. Note that, contrary to the ex-post premium, such an ex-ante

\textsuperscript{6} For theoretical aspects, see Lucas (1978) and Hansen and Hodrick (1983). Models including money have been proposed later by Lucas (1982) and Svensson (1985) under flexible price hypothesis and by Obstfeld and Rogoff (1995) and Devreux and Engel (1998) under sticky price hypothesis. For empirical approaches, see among others, Mark (1985), Hodrick (1989), Kaminsky and Peruga (1990). For models introducing habits in the consumption behavior, see Backus et al. (1993) and Sibert (1996).

\textsuperscript{7} Since the seminal paper of Hansen and Hodrick (1983), many studies have confirmed this general result (see among others, Campbell and Clarida (1987) and Cumby (1988); see also Lewis (1990) who considers different holding periods and regimes.

\textsuperscript{8} In particular, see Fama (1984) and MacDonald and Taylor (1994) who have successfully estimated error correction models for the U.S.dollar-Sterling and the U.S. dollar-Mark exchange rates. These models are shown to have good forecasting properties when long-run solutions are given by the monetary and real interest differential models. See Mark (1995) who shows that the accuracy of the forecast increases with the horizon when forecasts are based on fundamentals.
premium is an opinion variable that is formed at the moment the decision is made. A common finding of these studies is that the REH is systematically rejected by survey data, and this possibly explains why ex-post premium models lead to weak empirical evidence, thus stressing the relevance of the ex-ante premium. Although the studies by Frankel and Froot (1989, 1990) using survey data showed evidence of significant but unchanging ex-ante risk premia, MacDonald and Torrance (1988, 1990), Liu and Maddala (1992), Cavaglia et al. (1993) and Verschoor and Wolff (2001) showed the existence of time-varying ex-ante premia. Attention has then been focused on the question of the stationarity of these premia (Liu and Maddala, 1992; Cavaglia et al., 1993, 1994; Chionis and MacDonald, 2002). Authors generally conclude that risk premia are stationary variables. However, this approach remains somewhat questionable. First, it seems difficult to state the stationarity hypothesis when conditional volatility effects are present. Second, rather than examining the stationarity of the risk premia, it seems to us more relevant to check if one can identify a vector of variables which is cointegrated with these premia. By regressing the survey-based expected change in exchange rate on the spread between the forward rate and the spot rate, some studies confirm the existence of an ex-ante risk premium although no factors are identified (Cheung, 1993; Verschoor and Wolff, 2001; Chinn and Frankel, 2002). Using Consensus Economics individual survey data on 3-month risk premia in three foreign exchange markets, Chionis and MacDonald (2002) show that these premia depend on the conditional variances of domestic

---

9 Among others, see McDonald and Torrance (1990), Prat and Uctum (2007) find similar results for 6 European currencies. Ruelke et al. (2010) report the rejection of REH using panel survey data on Yen/Dollar expectations from the Wall Street Journal forecast poll. Surveys on the empirical rejection of the REH in the foreign exchange market are proposed by MacDonald (2000) and Benassy and Raymond (1997).

10 If the regression coefficient is different from 1, then a risk premium exists.
and foreign fundamentals (such as money supplies and inflation rates)\textsuperscript{11} and on idiosyncratic effects, hence explaining a significant part of the ex-ante time-varying premia. Comparing aggregate (consensus), individual and sector-averaged measures of the risk premia for GBP/USD, DEM/USD and JPY/USD exchange rates from October 1989 to March 1995 and using an ARCH-M approach, the authors find that the volatilities of the individual survey-based risk premia are much larger than the volatility of the consensus risk premium and are often close to the volatility of the ex-post rational risk premium. According to the authors, these findings would imply that aggregate measures of the risk premium “average out much of the heterogeneity and richness of the individual survey expectations” (p.67).

It is of course of great interest to seek the determinants of the risk premia at the individual level in order to detect the sources of heterogeneity. However, it should be noted that individual survey-based expectations may imbed large measurement errors, thus leading to the high volatility of the premia. These individual errors, in turn, are offset within the average risk premium. On the other hand, as noted above, the ex-post premium contains both the true but unknown market ex-ante premium and the forecast error. This implies that the variance of the market ex-ante premium is lower than the variance of the ex-post premium, until the former be eventually equal to the variance of the consensus. This leaves open the possibility that the consensus is a good proxy of the market risk premium, which is the appropriate concept in a macroeconomic framework. Especially, in the international portfolio choice model presented below, the risk premium depends on the net market position of foreign assets, and this makes sense only at the aggregate level. Finally, when expectations are not rational, it is relevant to model the premium under different time-horizons.

\textsuperscript{11} In fact, the volatility of the fundamentals determines the volatility of the exchange rate, which is the most commonly considered determinant of the risk premium. Our model presented below includes such a relationship between exchange rate volatility and risk premium.
Overall, significant time varying ex-ante risk premia are evidenced by the studies mentioned above. However, several issues deserve further work. The most salient lack of the literature is the empirical identification of the determinants of the ex-ante market premium within a theoretical framework. Another important feature of the ex-ante risk premium ignored by the literature is the time horizon of the underlying investment. Using GBP/USD and JPY/USD exchange rate expectations over the 3- and 12-month horizons from Consensus Economics (CE) survey data, we aim to contribute simultaneously on these two issues. The paper is organized as follows. Section 2 is devoted to the presentation of a two-country portfolio choice model. Section 3 presents the data and the empirical results. Section 4 concludes.

2. The theoretical framework

Define the ex-ante risk premium required at time \( t \) for horizon \( \tau \) as:

\[
\delta_{t,\tau} = \ln E_t S_{t+\tau} - \ln F_{t,\tau}
\]  

where \( S_t \) is the spot exchange rate at time \( t \) (expressed in units of domestic currency per foreign currency), \( F_{t,\tau} \) the forward exchange rate at time \( t \) with a maturity date at \( t+\tau \), \( \delta_{t,\tau} \) the ex-ante risk premium required at time \( t \) for horizon \( \tau \) and where \( E_t \) stands for the conditional expectation operator.\(^{12}\) Figures 1a and 1b exhibit the dynamics of the 3 and 12-month ahead ex-ante risk premia based on financial experts’ JPY/USD and GBP/USD exchange rate expectations provided by CE surveys. On each market, it can be seen that, despite obvious common trends, the two risk premia are characterized by substantial discrepancies. This paper precisely aims to explain why premia are not only time-varying but also horizon-dependent.

\(^{12}\) Defining the risk premium as in (1) or as \( \ln F_{t,\tau} - \ln E_t S_{t+\tau} \) is arbitrary. As will be shown below (equation 3), our definition implies that a positive (negative) risk premium implies that the risk supported by the domestic agent, here the Japanese or the British agent, is larger (lower) than the one supported by the foreign agent, here the American agent.
This issue can be adequately analyzed by using the two-country portfolio choice model first introduced by Lewis (1995), where the domestic and foreign representative agents maximize their expected utilities in a partial equilibrium framework. This model has been empirically rejected under REH by Lewis (1995), Engel (1996) and Andrade and Bruneau (2002) (hereafter AB). AB expand the model so as to account for heterogeneity of expectations and regime shifts. According to the AB model, the risk premium is the product of three factors: a risk aversion coefficient, the expected variance of the rate of change in the real exchange rate, and the difference between the domestic agent’s real position in foreign currency denominated assets and the foreign agent’s real position on domestic currency denominated assets expressed in foreign currency, namely the net market position (NMP) of foreign assets. The authors assume that expectations are described by a process combining chartist and fundamentalist traders’ behaviors (Frankel and Froot, 1988) and that the expected variance and the fundamental level of the exchange rate are constant. Performing cointegration tests with endogenous breaks on monthly data from the JPY/USD exchange rate over the sample period 1980-1998, they show that a long run relationship exists between the risk premium and its factors.

Beside its innovating aspects, AB’s study contains however three questionable hypotheses that we aim to relax. First, their constant expected variance assumption contradicts the most widely accepted stylized fact that the variance is time-varying. Second, the assumption of heterogeneous expectations implies that the market is not rational, and this in turn should imply that the expected variance depends on the horizon time-span whereas a

\[13\] Note that the risk premium, defined as the difference between the expected change in the real exchange rate and the spread between home and foreign real interest rates, equals the difference between the nominal values of the two components since the expected inflation terms in real exchange rate and in real interest rates vanish.
unique one-period horizon is considered in AB’s model. We relax the assumption of a constant expected variance and specify the model for any horizon. Third, the net market position is very roughly measured by the authors as the difference between the Japanese cumulated long-term capital exports and the Japanese cumulated current accounts supposed to proxy the American cumulated long-term capital exports. In fact, we need to measure precisely the NMP between the United States and the domestic country for a specific horizon and this requires the knowledge of the monthly data of the current accounts relating these two countries for this horizon. Such data are clearly not observable. The difficult task of measuring the NMP leads us to estimate it within an unobservable-component model framework. The relaxation of AB’s three restrictive hypotheses seems all the more important as they may call into question their cointegration test results. Moreover, AB’s findings are conditional on the restriction that the forecasters are split into two groups of agents with unchanging proportions over time. By using aggregate survey data, we do not condition our analysis of the risk premia on a presupposed structure of heterogeneity of expectations.

We show now why the expected variance, and thus the premium, is horizon-dependent. Let $s_t$ denote the logarithm of the spot exchange rate at time $t$ and $\Delta$ the 1-period change operator. If the foreign exchange market is efficient, then the spot rate conveys all available information about the future rate and is expected rationally. The return $\Delta s_t$ is thus a white noise plus possibly a constant drift.\footnote{Even if we introduce a discount rate with constant variance which is independent of the white noise forecast error, this conclusion remains valid.} In this case we have $E(s_{t+\tau} - s_t) = \tau E(\Delta s_{t+1})$ and $V(s_{t+\tau} - s_t) = \tau V(\Delta s_{t+1})$, $\tau \geq 1$, that is, the first two moments increase in the same proportion with $\tau$. Because the risk premium depends on the expected variance, the premium averaged per period may be time-varying if the variance is so but does not depend on $\tau$, so that there is
one single premium. Conversely, if returns are partially predictable on the basis of their past values and/or macroeconomic variables, the foreign exchange market is not efficient and agents do not require a unique risk premium but a set of premia scaled by the time horizon. As a simple example, suppose that the one period return is related to the variable $\Delta X$, according to the relation $\Delta s_{t+1} = \alpha \Delta X_t + \eta_{t+1}$, where $\Delta X_t$ stands for any regressor possibly including $\Delta s_t$ such that $\text{Cov}(\Delta X_{t+1}; \Delta X_t) = \kappa \forall t$, and where $\eta_{t+1}$ is a white noise uncorrelated with $\Delta X_t$ for all lags with $V(\eta_t) = \omega^2$. Assuming further, for the sake of simplicity of our illustration, that $V(\Delta X_t) = \theta^2$ and $\text{Cov}(\Delta X_{t+\tau}; \Delta X_t) = 0 \quad \forall \tau > 1$, it is then easy to write the variances averaged per period for different time horizons:

1 period: $V(\Delta s_{t+1}) = \alpha^2 \theta^2 + \omega^2$

2 periods: $\frac{1}{2} V(s_{t+2} - s_t) = \frac{1}{2} V(\Delta s_{t+1} + \Delta s_{t+2}) = V(\Delta s_{t+1}) + \alpha^2 \kappa$

3 periods: $\frac{1}{3} V(s_{t+3} - s_t) = \frac{1}{3} V(\Delta s_{t+1} + \Delta s_{t+2} + \Delta s_{t+3}) = V(\Delta s_{t+1}) + \frac{4}{3} \alpha^2 \kappa$

or, more generally: $\frac{1}{\tau} V(s_{t+\tau} - s_t) = V(\Delta s_{t+1}) + 2 \left(1 - \frac{1}{\tau}\right) \alpha^2 \kappa$

Note that the case $\alpha = 0$ corresponds to the efficiency hypothesis according to which returns are a white noise. When $\kappa > 0$, the variance and thus the required premium increase with the horizon, while when $\kappa < 0$, the variance and the premium decrease with the horizon. Cochrane (1999) assumes the special case $\Delta X_{t-1} = \Delta s_{t-1}$ and argues that a sufficient condition

---

15 See Merton (1969) and Samuelson (1969).

16 This result is evidenced by Barberis (2000) in an optimal portfolio model framework composed by U.S. stocks and bonds. Given that stock returns can be predicted on the basis of past values of the dividend/price ratio, the author shows that the structure of this portfolio is very sensitive to the time horizon of the investment.

17 We checked that several such autocorrelated variables exert significant predictive powers on both exchange rates (in first differences), such as the lagged dependent variable, the lagged difference between domestic and US inflation rates, the lagged change in the forward premium.
to generate a horizon dependent variance is the existence of a serial correlation in returns. More generally, if the sign or the magnitude of the covariance is time-varying, the slope of the term structure of the premia is also time-varying. Now relax the hypothesis \( V(\Delta X_t) = \theta^2 \) and let \( V(\Delta X_t) \) be an AR(p) process, as usually observed with returns data. It can then be shown that \( V(s_{t+\tau} - s_t) \) follows an autoregressive structure with order p when \( \tau = 1 \) and greater than p when \( \tau > 1 \).\(^{18}\) The latter result will be useful later when we will specify the expected variance.

The AB model implicitly assumes that the risk premium is the same for all asset maturities and defines an aggregate net market position which comprises assets of all maturities. According to the stylized facts exhibited on Figures 1a and 1b, we choose a horizon-dependent premia framework where we allow the expected variance and the net market positions to be horizon-dependent. The investors’ problem is then to determine at time \( t \) the optimal share of his/her wealth to be invested respectively in the domestic and in the foreign assets which allows to maximize the expected utility of his/her future real wealth at time \( t + \tau \). To this end, we consider a two-country portfolio choice model for a given horizon \( \tau \). Let \( W_t \) the real wealth held by the domestic agent at time \( t \) in the form of the \( \tau \)-month asset (expressed in units of foreign currency), \( W^*_t \) the real wealth held by the foreign agent at time \( t \) in the form of the \( \tau \)-month asset (expressed in units of foreign currency), \( x_{t,\tau} \) the share of \( W_t \) held by the domestic agent in the form of foreign \( \tau \)-month assets (1- \( x_{t,\tau} \) is then the share held in the form of domestic assets), and \( x^*_{t,\tau} \) the share of \( W^*_t \) held by the foreign agent in the form of domestic \( \tau \)-month assets (1- \( x^*_{t,\tau} \) is the share held in the form of foreign assets).

As in the AB model, a CARA utility function \( U(W_t) = -e^{-\lambda t} \cdot W_t \ (U^* > 0 \text{ and } U^* < 0) \) is supposed for the domestic agent and a similar function \( U(W^*_t) = -e^{-\lambda^* \cdot \tau \cdot W^*_t} \) is

\(^{18}\) Proof available upon request.
considered for the foreign agent, where coefficients $\lambda_\tau$ and $\lambda_\tau^*$ represent the absolute risk aversion coefficients for the horizon $\tau$ and for the two agents, respectively. Each agent is assumed to choose the optimal share $x_{t,\tau}$ and $x_{t,\tau}^*$ of his/her real wealth in order to maximize the expected utility of the end-of-period real wealth conditionally on the information known at time t. The representative agents’ programs for the $\tau$-month investments may be written in the mean-variance form as follows:

**Domestic agent’s program**:  
Max $E_t[\tau, W_{t+\tau}(x_{t,\tau})] - \frac{1}{2} \lambda_\tau V_t[\tau, W_{t+\tau}(x_{t,\tau})]$

**Foreign agent’s program**:  
Max $E_t[\tau, W_{t+\tau}^*(x_{t,\tau}^*)] - \frac{1}{2} \lambda_\tau^* V_t[\tau, W_{t+\tau}^*(x_{t,\tau}^*)]$

(2)  

$s.t. 0 \leq x_{t,\tau}, x_{t,\tau}^* \leq 1$

where $V_t[.]$ denotes the expected variance operator conditional on time t. The first order conditions in (2) allow to determine the optimal positions of both agents and lead to the corresponding set of equilibrium risk premia $\delta_{t,\tau}$ for given values of t and $\tau$ (see Appendix A):

$$\delta_{t,\tau} = \phi_{\tau}(\hat{x}_{t,\tau} - \tilde{x}_{t,\tau}^*, \hat{W}_t - \tilde{W}_t^*) \tilde{\sigma}_{t,\tau}^2$$

(3)

where $\hat{x}_{t,\tau}$ and $\tilde{x}_{t,\tau}^*$ are the optimal values of $x_{t,\tau}$ and $x_{t,\tau}^*$, $\tilde{\sigma}_{t,\tau}^2$ is the $\tau$ months ahead conditional expected variance of the real rate of change in the exchange rate, $\phi_{\tau}$ is a composite risk aversion coefficient defined as the half of the harmonic mean of the absolute risk aversion coefficients ($\phi_{\tau} = \frac{\lambda_\tau \lambda_\tau^*}{\lambda_\tau + \lambda_\tau^*} > 0$) and the term in brackets stands for the real net market position, labeled $NMP_{t,\tau}$. 


Equation (3) says that the risk premium $\delta_{t,t}$ is determined as the product of the risk aversion, the real net market position and the expected volatility. It can be seen that the sign of $\delta_{t,t}$ is determined by the sign of $NMP_{t,t}$. When $NMP_{t,t}>0$, that is when the domestic agent’s position in foreign assets ($\hat{x}_{t,t}W_t$) is greater than the foreign agent’s position in domestic assets ($\hat{x}^{*}_{t,t}W^*_t$), all positions being expressed in units of foreign currency, the premium remunerates domestic investor for the risk supported when they hold foreign assets. Conversely, when $NMP_{t,t}<0$, the foreign investor is remunerated for the risk supported.

3. **Empirical issues**

In this section we examine, for each of the 3- and 12-month horizons available from our survey data, whether the expected variance and the net market position explain the ex-ante risk premium according to equation (3).

3.1. **Data and stylized facts**

Let $S_t$ stand for the JPY/USD or GBP/USD exchange rate and the Japanese or the British agent represents the domestic investor while the American agent represent the foreign investor. The values of the variables $E_tS_{t+t}$ and $F_{t,t}$ are needed to be known to measure the ex-ante premium $\delta_{t,t}$. Over our sample period, at the beginning of each month, *Consensus Economics* asks about 200 economists, foreign exchange operators and executives in various institutions (commercial and investment banks, forecasting agencies and industrial corporations) in over 30 countries to forecast future values of principal macroeconomic
variables for the three and the twelve month horizons. The rate of response for each
exchange rate exceeds 50%. The respondents answer only when they think they have a good
knowledge about the variable of interest, and this allows assuming that those who respond are
informed agents. Since the individual answers are confidential (only the consensus is
disclosed to the public with a time lag) and since each individual is negligible within the
consensus, it is difficult to claim that, for reasons which are inherent to speculative games,
individuals might not reveal their « true » opinion. Note that these considerations only suggest
that the responses are not distorted but they do not imply that the consensus represents an
unbiased proxy of the market expectations. However, regarding the existence of the forward
market for the two horizons, one can argue that there is an incentive for experts to compare
their expected rate to the forward rate. This means that their expectations should capture a
market reference but should also be distinguished from their assessment of the market risk.
Figures 1a and 1b show that the risk premia defined by Equation (1) do not take zero values
and this implies that experts make a clear distinction between their expectations and the
forward rates. Moreover, to interpret the consensus expectation as a market expectation, we
only need to suppose that the latter equals the former plus an intercept and a white noise,
representing the systematic and the random components of the measurement error,
respectively. For all these reasons, we can assume that the expectations provided by the
respondent experts are representative of the market expectations.

The CE newsletter gives every month the “consensus” corresponding to the individual
expected values of exchange rates (arithmetic averages). These consensus time series are

---

19 Since the beginning of 1996, 1 month and 24 month time horizons are also included in the survey and
published in the special bulletin named “Foreign exchange Consensus Forecasts”.

20 This “consensus” is made up by more than the half of the 200 experts questioned.
used in this paper and are denoted $E_t S_{t,\tau}$ ($\tau = 3,12$ months).\textsuperscript{21} The CE requires a very specific day for the answers. As a rule, this day is the same for all respondents.\textsuperscript{22} Accordingly, we consider the forward exchange rates $F_{t,\tau}$ ($\tau = 3,12$ months) and the spot rate $S_t$ at the same day as the expected values (these series are issued from Datastream). Our empirical analysis covers the period November 1989 – December 2008.

Insert Table 1

Table 1 provides the main statistics related to the two premia on each exchange market, both expressed in percent per month. For each exchange rate, the means between the two horizons are close to each other but the standard deviation of the 3-month premium is much larger than the one of the 12-month premium. The unit root test shows that the series are I(0), which implies that they do not contain a long run drift.

Another preliminary issue is to examine whether or not the consensus provides indication of rationality. Indeed, if the REH were not rejected, the use of the rational ex post premia concept would be appropriate. We thus implemented the unbiasedness test over the sample period by regressing the $\tau$-month ahead expected change $\ln E_t S_{t,\tau} - \ln S_t$ on the ex-

\textsuperscript{21} It is easy to show that, if the expected returns on the market sum to zero, the consensus of speculators’ expectations is the relevant variable allowing for representing an indicator of « the » expected value in foreign exchange market. Recall that AB assume the existence of fundamentalists and chartists on the market. In our approach, if such heterogeneity exists, it is imbedded in the exchange rate expectations provided by survey data.

\textsuperscript{22} This day is the first Monday of the month until March 1994, and the second Monday since April 1994, except closed days (in this last case, the survey is dated at the following day). The effective horizons however always remain equal to 3 and 12 months. If, for instance, the answers are due on the 3rd of May (which was the case in May 1993), the future values are asked for August 3, 1993 (3 months ahead expectations) and for January 3, 1994 (12 months ahead expectations). The individual responses are then concentrated on the same day.
post rate of change \( \ln S_{t+\tau} - \ln S_t \). To capture the possible overlapping data bias which may arise from the use of monthly data with any horizon \( \tau \) longer than 1 month, we applied two methods. We first implemented the Newey-West method which is robust to residual autocorrelation and heteroskedasticity. Second, following Hansen and Hodrick (1980), a MA(\( \tau - 1 \)) process for residuals was included to capture this bias. In this case the relationship tested is:

\[
\ln E_t S_{t+\tau} - \ln S_t = \alpha (\ln S_{t+\tau} - \ln S_t) + \theta + e_t
\]

\[e_t = \xi_t + \lambda_1 \xi_{t-1} + \ldots + \lambda_{\tau-1} \xi_{t-\tau+1}\]

Table 2 provides the test results. The t-values as well as the Wald joint tests on coefficients show that the null of unbiasedness \( (\alpha = 1, \theta = 0) \) and therefore the REH are systematically rejected, confirming with our data the findings of the literature. This justifies to focusing the analysis on the ex-ante risk premia.

\textit{Insert Table 2}

Rejection of the REH is in accordance with the economically rational expectations hypothesis proposed by Feige and Pearce (1976) who state that agents do not use all the relevant information because of the information costs they face. Instead, they essentially form their forecast using a set of information limited to the present and past values of the forecasted variable. These biased expectations, which contain in particular the well-known extrapolation bias, are the ones that drive agents’ decisions that we aim to model.

3.2. Methodology and empirical results

In Equation (3), the expected variance and the net market position must be determined. Representing \( \tilde{\sigma}_{t,\tau}^2 \), the \( \tau \)-month ahead expected variance of the change in the real exchange rate, by an ARCH-M model would not be relevant for two reasons: first, generally speaking,
this class of models is appropriate for frequencies higher than monthly data and second, the
expected conditional volatility would then represent the variance of the residuals of the risk
premium equation and not the variance of the change in the real exchange rate as required.
This variance could be estimated from an ARCH model where the mean equation specifies
the change in the real spot rate as a constant term plus an error term, but such an estimation
would be disconnected from the estimation of the portfolio model.\(^{23}\) For each horizon, the
expected variance is assumed to be represented as an \(m\)-order weighted average of the past
monthly variances of the change in the real exchange rate \(\Delta q_t\), which is expressed in percent
per month:

\[
\tilde{\sigma}^2_{t,\tau} = \sum_{i=0}^{m} \gamma_{i,\tau} \sigma^2_{t-i} / \sum_{i=0}^{m} \gamma_{i,\tau} , \quad \gamma_{0,\tau} = 1
\]

with \(\sigma^2_t = (\Delta q_t)^2\) and \(q_t = s_t + p_t^* - p_t\), where \(q_t\), \(p_t^*\) and \(p_t\) are the logarithms of the real
exchange rate, the foreign CPI and the domestic CPI, respectively.\(^{24}\) This assumption is in
accordance with the developments in section 2 where the overall covariances have been
represented by a constant term. The latter has not proved to be significant, reflecting an
offsetting effect between covariances. We posit \(\gamma_{0,\tau} = 1\) so that the contemporaneous variance
is captured totally; relaxing this restriction did not improve the model. We define the
conditional volatility \(\sigma^2_t\) as the squared return of the real exchange rate rather than the
squared difference to the mean because the latter is found to be zero even on different

\(^{23}\) An approach in terms of implicit volatility could be a possible alternative, but it is well known that this
indicator is a weak predictor of the future volatility. It is therefore unlikely that agents use the implicit volatility
as a measure of the forecasted exchange rate volatility.

\(^{24}\) Note that the \(\tau\)-month expected variance should be \(\tau\) times the expected variance expressed in monthly
basis, as given by the right hand side of equation (4). Since the \(\tau\)-month premia are expressed in percent per
month, we write the expected variance on a monthly basis.
subsamples. Note that \( m \) depends on \( \tau \) although it has not been indexed accordingly for convenience.

The second variable in (3) which is to be represented is the real net market position between US and Japan. Since this variable is not observable (see section 2), we estimate a state-space model where for each horizon a signal (or measurement) equation describes the risk premium and a state (or transition) equation generates the unobservable product \( \varphi_{NMP_{\tau,NMP}} \). This state variable is supposed to follow a simple AR(1) process. We attempted to augment the standard AR(1) process with observed macroeconomic variables, but none of them was found to be significant.\(^\text{25}\) The state equation is then:

\[
\omega_{\tau,t} = \beta_0 \omega_{\tau-1,t} + \kappa_{0,\tau} + \varepsilon_{\tau,t}, \quad 0 \leq \beta_\tau \leq 1, \quad \forall \tau = 3, 12. \tag{5}
\]

where we define \( \omega_{\tau,t} = \varphi_{\tau,NMP_{\tau,t}} \) as the sensitivity of the risk premium to the expected variance and where \( \varepsilon_{\tau,t} \) is a zero-mean \textit{Niid} error term. The sign of the drift \( \kappa_{0,\tau} \) is undetermined a priori.

Adding an error term to equation (2) yields the signal equation:

\[
\hat{\delta}_{\tau,t} = \tilde{\sigma}_{\tau,t}^2 \omega_{\tau,t} + \upsilon_{\tau,t} \quad \forall \tau = 3, 12 \tag{6}
\]

where the state variable \( \omega_{\tau,t} \) is given by (5) and the expected variance \( \tilde{\sigma}_{\tau,t}^2 \) by (4). The innovation \( \upsilon_{\tau,t} \) is supposed to be a zero-mean \textit{Niid} error term independent of the error term \( \varepsilon_{\tau,t} \) of the state variable.\(^\text{26}\) We expect these signal innovations to be contemporaneously

\(^{25}\) These were the differences between domestic and US observed values of the change in CPIs, the change in real GNPs, the change in real investments, the current account and government budget imbalances, the change in M1 and M2 money supplies and the stock returns. These series were extracted from DATASTREAM.

\(^{26}\) Note that we did not find any significant MA (\( \tau - 1 \)) process characterizing \( \upsilon_{\tau,t} \), which suggests that there is no overlapping bias resulting from the difference between the horizons and the monthly observations in the
correlated since the two premia are themselves correlated (see Figure 1). To account for this possible correlation between \( \nu_{t,3} \) and \( \nu_{t,12} \), we estimate jointly the 4-equations formed by (5) and (6) using the Kalman filter methodology (see Appendix B for a formal presentation of the state-space model and of the recurrent equations used in the estimation method). As like the vector of hyperparameters, the state variables have been given initial values by minimizing the Akaike, Schwarz and Hannan-Quinn criteria of information of the system.

**Insert Table 3**

Table 3 presents the empirical results. A grid search over the index \( m \) of the expected variance lag structure (Equation (4)) led to the optimal values 3 and 8 in the JPY/USD case and 2 and 6 in the GBP/USD case for the 3- and 12-month horizons, respectively. Thus, compared to the 3-month premium, the 12-month premium is influenced by the variance over a longer time span. For each horizon, the impacts of the lagged variances tend to decrease with the time-lags. Note that the variance of the GBP/USD real exchange rate in (4) exhibits an outlier at October 1992 that corresponds to the exit of this currency out of the EMS because of the speculative attacks initiated by George Soros. We accounted for this event by adding to \( \sigma_t^2 \) a constant times a dummy variable (labeled \( D9210 \)) which equals 1 at this date and zero elsewhere. We estimated a specific constant for each horizon to allow for an unconstrained correction to the expected variance at the horizon considered.

Figures 2a and 2b compare the two horizon-specific expected variance patterns: for each currency, around similar trends, the 3-month variance exhibits higher volatility that the 12-month variance. This partially explains why the 3-month premium is more volatile that the 12-month premium, as shown in Figures 1a and 1b. For the two horizons, all the structural measurement of the risk premia. This result is not surprising since when forming expectations forecasters fully revise their information from one month to the following one.
parameters are significant both in the signal and the state equations and have the expected signs. The intercepts $\kappa_{0,r}$ in the state equations were not found to be significant and therefore have been removed at the final stage of estimation. As expected, the estimates of $\beta_r$ fall into the interval $[0,1]$. For each of the two currencies, the significantly positive value of the covariance $\rho$ between the two signal residuals according to horizons result from the interdependences between the two premia (Figures 1a and 1b), between the two expected variances (Figures 2a and 2b) and between the two state variables (Figures 3a and 3b). The covariance between the two state residuals is found to be insignificantly different from zero and this is why this parameter has been removed from the estimations.27

Insert Figures 2a and 2b

Insert Figures 3a and 3b

Since this paper is concerned by a structural model, the state variable is estimated conditionally on the whole sample (smoothed inference) rather than using only the past observations at each point in time (predicted inference) or actual and past observations (filtered inference). Figures 3a and 3b exhibit substantial correlations between the two smoothed state variables $\omega_{3,t}$ and $\omega_{12,t}$ on each exchange market.

In 1998, Japan has gone through the worse economic recession in the Post-War period, leading to a record number of bankruptcies. The large peak drawn by the expected variance at this date reflects this crisis (Figure 2a). At the aftermath of the crisis, Japan has initiated a banking reform aiming to bring independence and transparency into the Japanese banking and financial system. The subsequent structural change is represented by the decline of the state variables after the early 2000s (Figure 3a). The rehabilitation consisted notably in making available huge amounts of government funds to recapitalize fifteen major banks and to write

\[27 \text{ We also found a zero covariance between the signal residuals and the state residuals for each horizon. This was a condition underlying the updating equations (B5) and (B6) used and presented in Appendix B.}\]
off the bad loans of nationalized or bankrupted banks, introducing profound changes in Japan's financial system (Hoshi and Patrick, 2000). As such, the reform is likely to have increased the relative preference for the Japanese asset, leading the Japanese agent to hold less US assets and the US agent to hold more Japanese assets. Overall, the reform seems thus to have contributed to the fall in the net market positions. Surprisingly, the subprime mortgage crisis, which broke out mid-2007, seems not to have affected noticeably the expected variances (Figure 2a). One can also observe that it has not reversed the real NMPs, as we would expect (Figure 3a). Indeed, one can think that the crisis would make the NMPs fall because of the Japanese agents’ distrust regarding the US toxic assets joint to the US agents’ growing interest to the Japanese assets. The rise in the NMPs can be explained by the fact that during the economic turmoil of 2007 and 2008, Japanese bank indebtedness was much lower than the one in Europe and in USA, so that Japanese banks and financial institutions resisted better to the crisis and even were incited to purchase more assets at reduced prices as the crisis deepened. 28 Figures 4a and 5a show that the crisis has not either altered the quality of the fits.

Insert Figures 4a and 5a

Recall that our real NMP variables are assessed as unobserved components because they refer to negotiable assets for two specific short term maturities. These precise data are not available but it seems however interesting to compare our NMP estimates to an aggregate short term NMP calculated using series from U.S. Treasury International Capital Reporting System (TIC) dataset. The series used are the total short term US liabilities to Japan and the total short term US claims on Japan, both provided in quarterly frequency (end of period) and expressed in millions of US Dollars. The first series proxies the Japanese agents’ holdings in

28 This can indeed be checked regarding the evolution of the short term US liabilities to Japan provided by the U.S. Treasury International Capital Reporting System dataset.
US assets while the second one proxies the American agents’ holdings in Japanese assets. Since the distinction between negotiable and non-negotiable assets is not available for claims, we selected the total amounts both for liabilities and claims. This aggregate real NMP is thus calculated as the difference between these two series expressed in 1990Q3 US Dollars. Figure 6a displays our estimated risk premia sensitivities to the conditional expected variances $\omega_{t,\tau} = \varphi_{t,NMP_{t,\tau}} (\tau = 3, 12)$ and the aggregate real NMP calculated using the TIC dataset. 29 Although each of the estimated risk premium sensitivities compares to one single component of the aggregate real NMP in the fields of maturity and negotiability, we can observe similar trends that are especially remarkable after 1995. In particular, the fall after 2002 due to the Japanese banking reform and the recovery towards the end of the period are well evidenced in all NMPs.

Concerning GBP/USD premia, we can see from Figure 3b that the 3-month maturity NMP is clearly more volatile than the 12-month maturity NMP, thus explaining the higher fluctuations of the 3-month premium with respect to the 12-month premium (Figure 1b). Both sensitivity curves draw meaningful common trends. The decline from 1994 to 2002 is attributable to the sharp devaluation of the Pound following the exit of the UK from the SME, which resulted in a decrease in the demand of the British agent for the American assets and symmetrically an increase in the demand of the American agent for the British assets, bringing the NMP from positive values to negative values. After 2002, all is reversed because of the persistent recovery of GBP/USD until the Global Crisis broke out in 2008, which led the Pound to steeply depreciate. Conversely to the Japanese case, the large increase in expected variances and the notable fall in real NMPs show that these variables have been

29 For comparison purposes, the monthly risk premium sensitivities have been converted to quarterly frequency (last month of each quarter).
impacted by the 2008 Global Crisis (Figures 2b and 6b). The fall in the real NMPs reflects some disinvolvevement of the British agent in American toxic assets and a growing preference by the American agent for the British assets. Overall, Figures 4b and 5b show that our risk premium model leads to good fits for each horizon. Interestingly, here again, the aggregate short term NMP calculated using UK-related series from U.S. TIC dataset is characterized accurately by the same trends (Figure 6b).

Insert Figures 4b, 5b and 6b

We further checked the goodness of the fits by using the conventional coefficient of determination \( R^2 \) and a modified measure, \( R_D^2 \), assessing the goodness of the fit with respect to the simple random walk plus drift model.\(^30\) The \( R_D^2 \) values (Table 3) indicate that, in the case of JPY/USD, the residual variance of the signal equation is 0.42 and 0.50 times the one of the random walk model for the 12-month and 3-month horizons, respectively. As for GBP/USD, the corresponding values are 0.21 and 0.40. These results imply that our unobserved component model (3) to (5) strongly outperforms the random walk.

We now examine the statistical properties of the residuals of the signal equations (innovations). The diagnostic tests we refer to are presented in Appendix C. The appropriate Ljung-Box Q test by Harvey (1992) based on the first 15 autocorrelations applied to the signal standardized smoothed residuals showed that no significant autocorrelation is to be reported for either horizon at the 5% level of significance. According to Harvey’s (1992)

\(^30\) The two measures of goodness of fit are defined by

\[
R^2 = 1 - \frac{SSR}{\sum_{t=1}^{T} (y_t - \bar{y})^2}
\]

and

\[
R_D^2 = 1 - \frac{SSR}{\sum_{t=2}^{T} (\Delta y_t - \Delta \bar{y})^2}
\]

where \( y_t = \delta_t \) and \( SSR \) is the sum of the squared residuals of the signal equation. A negative \( R_D^2 \) implies that the estimated model is beaten by a simple random walk plus drift (Harvey, 1992).
heteroskedasticity test, the null of homoskedasticity of these residuals is not rejected for both horizons at the 5% level. Overall, these test results show that the innovations of our two-horizon state-space model are well-behaved for either currency.

Overall, these results tend to assess the relevance of the Consensus Economics survey data used to measure the ex-ante risk premium $\delta_{t,\tau}$ (see Equation (1)). Moreover, they suggest that the latter provides a good representation of the equilibrium value of the risk premium (3) derived by the maximization of the expected utility of the real wealth held by each of the domestic and foreign agents. Let $\delta_{t,\tau}^M$ be the ex-post (market) risk premium. We now examine whether the ex-post premium adjusts towards the ex-ante premium, in other words whether factors such as transaction costs or heterogeneity of expectations disrupt the immediacy of the adjustment. To this end, we estimate the following error-correction model for each currency:

$$\Delta \delta_{t,\tau}^M = \alpha + \beta (\delta_{t-1,\tau} - \delta_{t-1,\tau}^M) + \sum_{i=0}^{\tau} c_i \Delta \delta_{t-i,\tau} + \sum_{j=1}^{12} d_j \Delta \delta_{t-j,\tau}^M + \sum_{k=0}^{\tau} e_k x_{t-k} + \eta_{t,\tau}$$  (7)

where $x_{t-k}$ is a vector of macroeconomic variables at lag $k$, $e_{k,\tau}$ a vector of the associated parameters at lag $k$ and horizon $\tau$, $\beta > 0$ and $\tau = 3,12$. We estimate Equation (7) as a two-horizon system for each currency using the Seemingly Unrelated Regression (SUR) method. Table 4 provides the estimation results. We find that the error-correcting term, the change in the target and the lagged ex-post premium are all significant. Among various variables tested for $x_{t-k}$ (3- and 12-month forward premia, inflation differential and industrial production growth rate differential), only the 3-month forward premium was found to be significant in the case of the two 3-month models. Interestingly, in most cases, the estimate of the change in the target is close to one, which implies that in the long run the ex-post premium moves as the ex-ante premium. This suggests that, beyond the fact that the equilibrium premium model (6) fits the survey data, the latter also help in modelling the dynamics of the market ex-post premium. Note that we also tested the hypothesis of an adjustment of the ex-ante premium towards the ex-post one, meaning that the experts would gradually form their beliefs on the
basis of the premium revealed by the market. For none of the currencies the error-correction model could be validated as the mean-reverting component was systematically insignificant and had the wrong sign.

4. Conclusion

Using financial experts’ JPY/USD and GBP/USD exchange rate forecasts provided by Consensus Economics surveys, the rational expectation hypothesis in exchange rates is found to be rejected for the 3 and 12-month horizons. This implies that the ex-post and the ex-ante risk premia, which we measure as the difference between the survey-based forecasted exchange rates and forward exchange rates, must be distinguished. We choose to model the ex-ante premium because it is a decision-making concept described by the expected utility maximizing framework. According to a two-country portfolio choice model, the ex-ante equilibrium risk premium required by the representative domestic and foreign investors for a given horizon is determined as the product of a constant composite risk aversion coefficient, the real net market position in assets and the conditional expected variance of the change in the real exchange rate. Under the condition of predictability of the latter, the expected variance is horizon-dependent, and so is the net market position by construction. This explains why, at any time, there exists a set of exchange rate premia scaled by the time horizon of the investment. The time-varying real net market positions being unobservable for a given horizon, they have been estimated through a state space model using the Kalman filter methodology. Our results show that the two-country portfolio asset pricing model considered in this paper is capable of explaining most of the common movements and of the specific patterns of the 3- and 12-month ex-ante premia for the two currencies.

Our empirical findings contribute to the existing literature in the following points. First, the ex-ante risk premia calculated using Consensus Economics survey data exhibit
noticeable differences according to the horizons. These premia are well explained by the horizon-dependent equilibrium model, which confers them the status of required equilibrium premia. Second, our estimated horizon-specific real net market positions share the same trends as the observed aggregate short term net market positions given by the U.S. Treasury International Capital Reporting System dataset. These similarities increase the reliability of the method used to estimate our unobservable variables. Third, the convergence of the ex-post premium towards the ex-ante premium shows that the ex-ante premium partially explains the market premium dynamics. Fourth, it seems to us that the difficulties reported by the literature concerning the modeling of the ex-post premium in an equilibrium framework result from the hypothesis of rational expectations joint to the one of the rationality of intertemporal choices. We show that only the latter rationality is acceptable, presumably because it fits better the cognitive abilities of the agents. Overall, our findings show that the Consensus Economics surveys provide reliable data to model the risk premium in the foreign exchange market.

Appendix A. Determination of the theoretical risk premium

Write the real wealth as \( W_{t+\tau} = W_t (1 + \bar{r}_{t,\tau}) \) and \( W_t^* = W_t (1 + \bar{r}_t^*) \), where \( \bar{r}_{t,\tau} \) and \( \bar{r}_t^* \) are the real interest rates defined as the weighted averages of the domestic and foreign real rates on deposits \( \tau \) months to maturity, that is, \( \bar{r}_{t,\tau} = (1 - x_{t,\tau}) r_{t,\tau} + x_{t,\tau} (r_{t,\tau}^* + \Delta q_{t+\tau}^*) \) and \( \bar{r}_t^* = (1 - x_t^*) r_t^* + x_t^* (r_t^* - \Delta q_{t+\tau}) \). Here, \( r_{t,\tau} = i_{t,\tau} - \pi_{t,\tau} \) and \( r_t^* = i_t^* - \pi_t^* \) are the real rates and \( \Delta q_{t+\tau} = \Delta s_{t+\tau} + \pi_{t+\tau}^* - \pi_{t,\tau} \) stands for the change in the real exchange rate, \( \pi_{t,\tau} \) standing for the inflation rate between \( t \) and \( t + \tau \). Using these elements, expand the conditional means \( E_t[W_{t+\tau}(x_{t,\tau})] \) and \( E_t[W_t^*(x_t^*)] \) and variances \( V_t[W_{t+\tau}(x_{t,\tau})] \) and \( V_t[W_t^*(x_t^*)] \) and replace in (2). Solving the two equations of (2) with respect to \( x_t \) and \( x_t^* \) respectively, and
combining the two solutions assuming Covered Interest Rate Parity, we obtain the expression of the risk premium as stated in (3).

Appendix B. The risk premia model and the Kalman filter equations

The system formed by the equations (6) and (5) can be written in the following state-space form (see Harvey, 1992, Ch. 3; Hamilton, 1994, Ch.13):

Measurement or signal equations:
\[ y_t = F_t \alpha_t + \nu_t \quad t = 1, \ldots, T \]  \hspace{1cm} \text{(B1)}

Transition or state equations:
\[ \alpha_t = M \alpha_{t-1} + c + \epsilon_t \quad t = 1, \ldots, T \]  \hspace{1cm} \text{(B2)}

where \( y_t = \begin{bmatrix} \delta_{t,3} \\ \delta_{t,12} \end{bmatrix}, \alpha_t = \begin{bmatrix} \omega_{t,3} \\ \omega_{t,12} \end{bmatrix}, F_t = \begin{bmatrix} \widetilde{\sigma}_{t,3}^2 & 0 \\ 0 & \widetilde{\sigma}_{t,12}^2 \end{bmatrix}, M = \begin{bmatrix} \beta_3 & 0 \\ 0 & \beta_{12} \end{bmatrix}, c = \begin{bmatrix} \kappa_{o,3} \\ \kappa_{o,12} \end{bmatrix}, \]
\[ \nu_t = \begin{bmatrix} \nu_{t,3} \\ \nu_{t,12} \end{bmatrix} \text{ and } \epsilon_t = \begin{bmatrix} \epsilon_{t,3} \\ \epsilon_{t,12} \end{bmatrix}. \]

The magnitudes \( \widetilde{\sigma}_{t,\tau}^2 (\tau = 3, 12) \) in \( F_t \) depend on the lag parameters \( \gamma_{t,\tau} \) (see equation (4)), and \( \omega_{t,\tau} = \phi_\nu NMP_{t-1,\tau} \) in \( \alpha_t \). \( F_t, M \) and \( c \) are matrices containing fixed and unknown parameters to be estimated. \( \alpha_t \) is a vector of time-varying unobservable components, with initial value \( \alpha_o \) assumed to have a mean \( a_o \) and a covariance matrix \( P_o \). The disturbances \( \nu_t \) and \( \epsilon_t \) are serially uncorrelated with mean zero and covariance matrices \( V(\nu_t) = U \) and \( V(\epsilon_t) = Q \). They are moreover mutually uncorrelated, that is \( E(\nu_t, \epsilon_{t'}) = 0 \) for all \( t, t' \), and also uncorrelated with \( \alpha_o \). Let \( \hat{\alpha}_{t/t} \) be the optimal estimator (or the update, see below) of \( \alpha_t \) based on all available information up to \( t \), denoted \( \Omega_t \). Let \( P_{t/t} = E[(\alpha_t - \hat{\alpha}_{t/t})(\alpha_t - \hat{\alpha}_{t/t})'] \) be the covariance matrix of the estimation error.

The optimal predictor of \( \alpha_t \) conditional on \( \Omega_{t-1} \), is given by:

\[ P_{t-1/t} = \text{...} \]

31 Note that \( E(\nu_t, \epsilon_{t'}) \) may be equal to some non-zero matrix \( G \) if \( t = t' \) and 0 otherwise, that is, the residuals may be contemporaneously correlated. In this case the prediction equations (B3) and (B4) are unaltered but the updating equations (B5) and (B6) are modified as described in Harvey (1992, sub-section 3.2.4).
\[ \hat{\alpha}_{t/-1} = M\hat{\alpha}_{t-1/-1} + c \]  
\[ \text{(B3)} \]

and it can be shown that the covariance matrix of the forecast error,

\[ P_{t/-1} = E[(\alpha_{t} - \hat{\alpha}_{t/-1})(\alpha_{t} - \hat{\alpha}_{t/-1})'] \],
can be written as:

\[ P_{t/-1} = MP_{t-1/-1}M' + Q \]  
\[ \text{(B4)} \]

The equations (B3) and (B4) are the prediction equations of the Kalman filter. From (B1) we get the forecast error on \( y_t \) and its covariance matrix given by

\[ H_{t} = E[(y_t - \hat{y}_{t/-1})(y_t - \hat{y}_{t/-1})'] = F_{t}P_{t/-1}F'_{t} + U \].
The linear projection of \( \alpha_t \) on \( \Omega_t \) leads to the following updating equations:

\[ \hat{\alpha}_{t,t} = \hat{\alpha}_{t/-1} + K_{t}(y_{t} - F_{t}\hat{\alpha}_{t/-1}) \]  
\[ \text{(B5)} \]

\[ P_{t,t} = P_{t/-1} - K_{t}F_{t}P_{t/-1} \]  
\[ \text{(B6)} \]

where \( K_{t} = P_{t/-1}F'_{t}H_{t}^{-1} \) is a correction term, known as the gain matrix of the Kalman filter, applied in (B5) to the forecast error in \( y_t \) and in (B6) to the covariance matrix between the forecast errors in \( y_t \) and \( \alpha_t \), namely \( F_{t}P_{t/-1} = E[(y_{t} - \hat{y}_{t/-1})(\alpha_{t} - \hat{\alpha}_{t/-1})'] \). If \( \nu_t, \varepsilon_t \) and \( \alpha_o \) are multivariate Gaussian, then \( y_t \) is \( N(F_{t}\hat{\alpha}_t/-1, H_t) \). The parameters in equations (B1) and (B2) can then be estimated by the maximization of the log-likelihood function

\[ L = \sum_{t=1}^{T} \log(f(y_{t})) \], where \( f(y_{t}) = (2\pi)^{-1/2}|H_{t}|^{-1/2} \exp\left(-\frac{1}{2}(y_{t} - F_{t}\hat{\alpha}_{t/-1})'H_{t}^{-1}(y_{t} - F_{t}\hat{\alpha}_{t/-1})\right) \) is the pdf of \( y_t \).

**Appendix C. Diagnostic tests for the Kalman filter inference**

We describe Harvey’s (1992) autocorrelation and heteroskedasticity tests for the standardized signal residuals \( \hat{\eta}_t \) resulting from the smoothed inference over our sample size \( T = 221 \). Let \( \gamma_{\theta} \) be the sample autocorrelations in \( \hat{\eta}_t \) at lag \( \theta = 0, ..., p \). We set \( p = \sqrt{T} \approx 15 \) (see Harvey (1992, p.259)). The null of no serial autocorrelation in the residuals can be tested by using the
Ljung-Box Q statistic \( Q^* = T^* (T^* + 2) \sum_{\theta=1}^{p} \frac{\gamma^2_{\theta}}{(T^* - \theta)} \), where \( T^* = T - d \) (\( d \) is the number of non-stationary elements of the state vector that are associated to a signal equation, equal to 1 in our case). Under the null, \( Q^* \) is a \( \chi^2(p - n) \), where \( n \) is the number of hyperparameters to be estimated minus one, equal to 11 and 6 in the JPY/USD case and to 10 and 6 for the GBP/USD case for the 12-month and 3-month horizon models, respectively. The author suggests to calculate the test for heteroskedasticity as \( H(h) = \sum_{t=T-h+1}^{T} \frac{\eta^2_{t}}{\sum_{t=d+1}^{d+1+h} \eta^2_{t}} \), where \( h \) is the nearest integer to \( T^* / 3 \), equal to 73 with our sample size. The asymptotic distribution of the statistic \( hH(h) \) is then \( \chi^2(h) \).

**REFERENCES**


FIGURE TITLES

Figure 1a : Ex-ante JPY/USD exchange rate risk premia

Figure 1b : Ex-ante GBP/USD exchange rate risk premia

Figure 2a : Expected variances of the change in the JPY/USD real exchange rate

Figure 2b : Expected variances of the change in the GBP/USD real exchange rate

Figure 3a : The sensitivity of the JPY/USD risk premium to the expected variance
Figure 3b : The sensitivity of the GBP/USD risk premium to the expected variance

Figure 4a : Observed and fitted values of the 12-month JPY/USD ex-ante risk premium

Figure 4b : Observed and fitted values of the 12-month GBP/USD ex-ante risk premium

Figure 5a : Observed and fitted values of the 3-month JPY/USD ex-ante risk premium

Figure 5b : Observed and fitted values of the 3-month GBP/USD ex-ante risk premium

Figure 6a : Risk premia sensitivities to expected variances and the aggregate short term real net market position between Japan and US

Figure 6b : Risk premia sensitivities to expected variances and the aggregate short term real net market position between UK and US

Table 1. Risk premia : descriptive statistics

<table>
<thead>
<tr>
<th>horizon</th>
<th>JPY/USD</th>
<th>GBP/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 months</td>
<td>12 months</td>
</tr>
<tr>
<td>Mean</td>
<td>0.215</td>
<td>0.182</td>
</tr>
<tr>
<td>Median</td>
<td>0.215</td>
<td>0.159</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.55</td>
<td>1.45</td>
</tr>
<tr>
<td>Minimum</td>
<td>-2.23</td>
<td>-0.65</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.88</td>
<td>0.39</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.26</td>
<td>0.43</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.07</td>
<td>3.21</td>
</tr>
<tr>
<td>Jarque-Bera statistic (probability)</td>
<td>13.69</td>
<td>7.52</td>
</tr>
<tr>
<td>ADF t-stat</td>
<td>-3.66</td>
<td>-2.40</td>
</tr>
</tbody>
</table>

Notes: The risk premia are expressed in percent per month following Equation (1). The sample period is 1989.11-2008.12 (230 observations). The asymptotic critical values for the ADF test statistic are -2.58 and -1.94 at the 1% and 5% levels, respectively (no significant intercept).
<table>
<thead>
<tr>
<th>horizon $\tau$</th>
<th>JPY/USD</th>
<th>GBP/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 months</td>
<td>12 months</td>
</tr>
<tr>
<td>overlapping</td>
<td>NW</td>
<td>MA</td>
</tr>
<tr>
<td>bias correction method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.034</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td>(-1.03)</td>
<td>(2.47)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>-0.039</td>
<td>-0.018</td>
</tr>
<tr>
<td></td>
<td>(-0.37)</td>
<td>(-0.16)</td>
</tr>
<tr>
<td>MA(n), $n=1,...,\tau-1$</td>
<td>-</td>
<td>The two lags are significant</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.00</td>
<td>0.35</td>
</tr>
<tr>
<td>$DW$</td>
<td>0.78</td>
<td>1.75</td>
</tr>
<tr>
<td>Wald test ($\alpha = 1, \theta = 0$)</td>
<td>$F(2,225) = 512, p=0.00$</td>
<td>$F(2,223) = 387, p=0.00$</td>
</tr>
<tr>
<td>Sample size</td>
<td>227</td>
<td>227</td>
</tr>
</tbody>
</table>

Notes: Numbers in brackets represent $t$-values. Estimations cover the period 1989.11–2008.12. NW and MA indicate Newey-West and Moving Average methods.
Table 3: Estimating the risk premia model
Notes. Estimations cover the period 1990.08-2008.12 (221 observations). For each currency, the two signal equations
\[ \delta_{t,t} = \beta_{t} + \mathbf{c}_{2,t}, \] (where the expected variance \( \mathbf{c}_{2,t} \) is given by (4)) and the two state equations
\[ \omega_{t,t} = \beta_{t} + \kappa_{0,t} + \epsilon_{t,t} \] (\( \tau = 3, 12 \)), with \( \omega_{t,t} = \varphi \mathbf{NMP}_{t-1,t} \), have been estimated as a system of equations using the Kalman filter methodology. \( \rho \) stands for the contemporaneous covariance between the two signal residuals. The estimates are obtained by setting to zero the insignificant covariance between the two state residuals and the insignificant intercepts \( \kappa_{0,t} \). To ensure positivity, the variances of \( \epsilon_{t,t} \) and \( \mathbf{c}_{2,t} \) are estimated as exp(\( \epsilon_{1,t} \)) and exp(\( \mathbf{c}_{2,t} \)), respectively. AIC, SC and HQC stand for Akaike, Schwarz and Hannan and Quinn information criteria for the system estimation. According to the minimum information criteria, the state vector \( (\omega_{t-1,t}, \epsilon_{t,t})' \) has been given optimal initial values \((-0.04, -0.03)'\) and \((0.05, 0.05)'\) in the case of the JPY/USD and GBP/USD models, respectively. All the estimates are significant at the 1% level except \( \gamma_{8,12} \) in the JPY/USD model and \( \gamma_{6,12} \) in the GBP/USD model, which are significant at the 5% and 10% levels, respectively. \( R^2 \) and \( R^2_D \) are two goodness-of-fit measures (see footnote 30) while \( Q \) and \( hH \) represent Ljung-Box serial correlation and heteroskedasticity test statistics (see Appendix C for a presentation of these statistics). In the case of the 12-month horizon, the asymptotic critical values of the \( Q \)-statistics are 7.78, 9.49 and 13.28 for a \( \chi^2 \) with 4 d.f. (JPY/USD) and 9.24, 11.07 and 15.09 for a \( \chi^2 \) with 5 d.f. (GBP/USD) for the 10%, 5% and 1% levels of significance, respectively. In the case of the 3-month horizon and for both currencies, the critical values

<table>
<thead>
<tr>
<th></th>
<th>JPY/USD</th>
<th>GBP/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \tau = 3 ) months</td>
<td>( \tau = 12 ) months</td>
</tr>
<tr>
<td>( \beta_{t} )</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>(49.4)</td>
<td>(67.0)</td>
</tr>
<tr>
<td>( c_{2,t} )</td>
<td>-9.31</td>
<td>-9.87</td>
</tr>
<tr>
<td></td>
<td>(-15.0)</td>
<td>(-49.3)</td>
</tr>
<tr>
<td>( \gamma_{1,t} )</td>
<td>0.96</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>(3.6)</td>
<td>(5.7)</td>
</tr>
<tr>
<td>( \gamma_{2,t} )</td>
<td>0.50</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>(2.8)</td>
<td>(5.5)</td>
</tr>
<tr>
<td>( \gamma_{3,t} )</td>
<td>0.71</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>(2.7)</td>
<td>(5.7)</td>
</tr>
<tr>
<td>( \gamma_{4,t} )</td>
<td>-</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(5.8)</td>
</tr>
<tr>
<td>( \gamma_{5,t} )</td>
<td>-</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(4.9)</td>
</tr>
<tr>
<td>( \gamma_{6,t} )</td>
<td>-</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(4.7)</td>
</tr>
<tr>
<td>( \gamma_{7,t} )</td>
<td>-</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(3.7)</td>
</tr>
<tr>
<td>( \gamma_{8,t} )</td>
<td>-</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(2.1)</td>
</tr>
<tr>
<td>( \mathbf{c}_{1,t} )</td>
<td>-1.17</td>
<td>-3.86</td>
</tr>
<tr>
<td></td>
<td>(-10.4)</td>
<td>(-34.8)</td>
</tr>
<tr>
<td>( D9210 )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(-17.8)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.076</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(9.48)</td>
<td>(12.7)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.63</td>
<td>0.78</td>
</tr>
<tr>
<td>( R^2_D )</td>
<td>0.50</td>
<td>0.58</td>
</tr>
<tr>
<td>( Q )</td>
<td>27.34</td>
<td>25.91</td>
</tr>
<tr>
<td>( hH )</td>
<td>167.35</td>
<td>148.16</td>
</tr>
<tr>
<td>AIC</td>
<td>0.338</td>
<td>-</td>
</tr>
<tr>
<td>SC</td>
<td>0.615</td>
<td>-</td>
</tr>
<tr>
<td>HQC</td>
<td>0.450</td>
<td>-</td>
</tr>
</tbody>
</table>


for a $\chi^2$ with 9 d.f. are 14.68, 16.92 and 21.70. The asymptotic critical values of the $hH$ statistics for a $\chi^2$ with 73 d.f. (both currencies and both horizons) are 89, 94 and 104 for 10%, 5% and 1% levels of significance, respectively.

Table 4. Error-correction model estimation results

<table>
<thead>
<tr>
<th></th>
<th>JPY/USD $\tau = 3$ months</th>
<th>JPY/USD $\tau = 12$ months</th>
<th>GBP/USD $\tau = 3$ months</th>
<th>GBP/USD $\tau = 12$ months</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.21 (3.84)</td>
<td>0.05 (2.22)</td>
<td>0.23 (5.83)</td>
<td>0.06 (2.27)</td>
</tr>
<tr>
<td>$c_{0,\tau}$</td>
<td>0.80 (5.29)</td>
<td>1.36 (10.96)</td>
<td>1.02 (8.38)</td>
<td>1.08 (8.92)</td>
</tr>
<tr>
<td>$c_{1,\tau}$</td>
<td>-</td>
<td>0.38 (2.41)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$d_{1,\tau}$</td>
<td>0.23 (3.35)</td>
<td>0.13 (1.75)</td>
<td>0.32 (6.14)</td>
<td>0.20 (3.69)</td>
</tr>
<tr>
<td>$d_{2,\tau}$</td>
<td>0.15 (2.38)</td>
<td>1.16 (2.53)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$d_{3,\tau}$</td>
<td>-0.20 (-2.46)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$d_{4,\tau}$</td>
<td>0.06 (0.91)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$d_{5,\tau}$</td>
<td>-0.15 (-2.57)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$d_{6,\tau}$</td>
<td>-0.19 (-3.00)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$e_{0,\tau}$</td>
<td>-1.98 (-2.74)</td>
<td>1.14 (2.24)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$e_{1,\tau}$</td>
<td>1.89 (2.61)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$D9110$</td>
<td>-</td>
<td>-</td>
<td>1.60 (6.47)</td>
<td>-</td>
</tr>
<tr>
<td>$D9207$</td>
<td>-</td>
<td>6.05 (6.62)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.46</td>
<td>0.40</td>
<td>0.45</td>
<td>0.39</td>
</tr>
<tr>
<td>Q-stat p-value(*)</td>
<td>0.096</td>
<td>0.40</td>
<td>0.201</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Notes. The estimated equation is:

$$\Delta \delta_{t,\tau}^{M} = a + b_{\tau} (\delta_{t-1,\tau} - \delta_{t-\tau}^{M}) + \sum_{i=0}^{\tau} c_{i,\tau} \Delta \delta_{i-1,\tau}^{M} + \sum_{j=1}^{\tau} d_{j,\tau} \Delta \delta_{j-1,\tau}^{M} + \sum_{k=0}^{\tau} e_{k,\tau} x_{t-k} + \eta_{t,\tau},$$

with $b_{\tau} > 0$ and $\tau = 3, 12$. For each currency, a two-horizon system is estimated over the period 1990.01–2008.12 using the SUR methodology. Estimates are obtained by removing the intercept $a$ that was systematically found to be insignificant. Numbers in brackets represent $t$-values. (*) System residual portmanteau test Q-stat p-values for testing the null of no residual autocorrelation: the test is valid only for lags larger than the system lag order, thus the p-values provided are those of the lags indicated. The variable $x_{t}$ stands for the forward premium with 3-month maturity.
Figure 1a. Ex-ante JPY/USD exchange rate risk premia

Figure 1b. Ex-ante GBP/USD exchange rate risk premia
Figure 2a. Expected variances of the change in the JPY/USD real exchange rate

Figure 2b. Expected variances of the change in the GBP/USD real exchange rate
Figure 3a. The sensitivity of the JPY/USD risk premium to the expected variance

Figure 3b. The sensitivity of the GBP/USD risk premium to the expected variance

Note: the sensitivity of the risk premium to the expected variance = constant risk aversion times the real net market position
Figure 4a. Observed and fitted values of the 12-month JPY/USD ex-ante risk premium

Figure 4b. Observed and fitted values of the 12-month GBP/USD ex-ante risk premium
Figure 5a. Observed and fitted values of the 3-month JPY/USD ex-ante risk premium

Figure 5b. Observed and fitted values of the 3-month GBP/USD ex-ante risk premium
Figure 6a. The sensitivities of the risk premia to the expected variances and the real aggregate short term net market position between Japan and USA.

Figure 6b. The sensitivities of the risk premia to the expected variances and the real aggregate short term net market position between UK and USA.

Note: sensitivity of the risk premium = constant risk aversion times real NMP.