Does Bayesian Shrinkage Help to Better Reflect What Happened during the Subprime Crisis?

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ABSTRACT

We study the contagion effects of a U.S. housing shock on OECD countries over the period of the subprime crisis. Considering a large database containing national macroeconomic, financial, and trade dynamic variables for 17 OECD countries, we evaluate forecasting accuracy, and perform a structural analysis exercise using VAR models of different sizes: a standard VAR estimated by OLS and a MEDIUM and LARGE VARs estimated by a Bayesian shrinkage procedure.

Our main findings are that: First, the largest specification outperforms the smallest one in terms of forecast accuracy. Second, the MEDIUM VAR outperforms both the LARGE BVAR and the SMALL VAR in the case of structural analysis. So the MEDIUM VAR is sufficient to provide plausible impulse responses, and reproduce more realistically what happened during the subprime crisis. Third, the Bayesian shrinkage procedure is preferable to the standard OLS estimation in the case of an international contagion study.

Keywords: Contagion, subprime crisis, OECD housing markets, VAR/ BVAR models and Bayesian shrinkage.

JEL Classification Codes: F47, C11, C32.
1. INTRODUCTION

The subprime crisis has gripped the U.S and spread out to many countries all over the world. The current financial crisis is nothing but the result of the American mortgage crisis that leads to many adjustments in housing markets. However, the subprime crisis is not the first turmoil event occurred in the housing markets. Yet, the housing prices boom during the early 2000s has raised many questions and since that many studies focused on a possible international transmission of housing shocks across countries (Otrok and Terrones, 2004). So, the last subprime crisis has confirmed the fears of possible contagion effects due to liberalization of markets. In fact, since 2006, the decrease of U.S. housing prices as a result of collapsing residential investments has been followed by a wave of crises and a sharp decrease in housing prices in other economies. This last crisis, which emerged in the USA in the summer of 2006, was followed by a sharp fall in housing prices in Ireland, New Zealand, Spain and the United Kingdom. These almost parallel developments provide evidence in favor of a significant correlation across national housing markets.

There appears to be a large degree of co-movements between very different and distant countries. It seems that the U.S. housing boom and bust has spread to other parts of the world and so confirm that the United States continues to retain its place as the world’s principal leading country. Nowadays, there are several possible different explanations of an international transmission of housing prices shocks. The housing prices may be driven by economic or financial fundamentals\(^1\), the wealth effect\(^2\), external news\(^3\), etc.

In this paper, we aim at going further beyond questions of international transmission of housing prices to examine "contagion effects" which are directly associated with crisis events. Here by contagion we mean, as in the case of a pandemic, a process where a sudden change at an important date in local prices in one country affects global prices in other countries. We formulate this idea by saying that contagion may occur when a local shock affects the propagation mechanism of a large number of OECD variables. In other words, the investigation of contagion effects is to focus on the changes in the transmission mechanisms of shocks at a critical date. Our analysis is important in terms of policy implications, or even for international investors.

The scope of our paper is to contribute to the literature on international transmission. The idea is to examine the contagion effects of a U.S. housing price index shock on OECD countries, and to explore which VAR model better reflects the propagation and magnitude of the changes in the transmission processes, so as to reproduce what happened during the subprime crisis. To our knowledge, no paper in the literature has used such comparison to test and model the contagion effects.

Specifically, in our study, we address the following questions:

1. Is it necessary to deal with a large panel of data when studying contagion effects during the subprime crisis?
2. Which VAR model specification (SMALL, MEDIUM or LARGE) does reflect most faithfully what happened during the subprime crisis, in terms of forecast and structural analysis?
3. In practice, is Bayesian shrinkage a valid alternative to Ordinary Least Squares (OLS) estimation in the case of international transmission study?

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\(^1\) The idea is that housing prices are likely to co-move if housing prices are driven by fundamentals, and if the cycles of fundamentals are correlated. See Goodhart and Hoffman (2008).

\(^2\) Decrease in house prices will induce households to reduce their consumption since 5% of the household income is from real estate; e.g. Lettau and Ludvigson (2004) and Case and al. (2005).

\(^3\) News on housing prices in some countries may lead investors and (potential) house buyers to revise their expectations on housing prices in other countries. These revised expectations can be unrelated to changes in fundamentals akin to contagion effects in exchange rates and stock markets during the Asian crisis (Kaminsky and Reinhart, 2000).
To answer these questions we choose different sizes of VAR models to characterize such dynamics for the OECD countries in our panel. Also, we build on the results obtained by De Mol, Giannone, and Reichlin (2008) and Banbura, Giannone and Reichlin (2010) by setting the degree of Bayesian shrinkage in relation to the cross-sectional dimension of the model, so as to compare the three models.

Our main contributions to the existing literature are: First, the novelty of dealing with a Bayesian shrinkage procedure allows us to study the impact of U.S. housing price index shock on the many OECD financial and economic variables included in our dataset in the form of impulse responses. This is particularly relevant given the most recent crisis which has been characterized by sudden shocks of large magnitude. Researchers, investors, and policy analysts are focusing on robust models allowing the reflection of negative effects on many variables in crisis period.

Second, many economic concepts need more than one variable such as the real activity. So, dealing with a Bayesian shrinkage approach, i.e. the MEDIUM/ LARGE Bayesian VARs may be, a priori, a solution to capture many concepts.

Third, another advantage of using this type of VAR models specifications is that impulse responses can be observed both for variables included only in a small VAR, and for large key variables. If housing markets are contagious, economic policy should focus on structural reforms ensuring a stable domestic market in order to limit the amplification of shocks between housing markets.

Fourth, an alternative to analyse relatively large data sets is to define a small set of factors (indicators), or group of variables, at a time, FAVAR models, for instance, see e.g. Christiano, and al. (1996), Kim (2001) & Kaabia and Abid (2012). However, comparison of impulse responses across models is problematic. Our approach may be a not neglecting solution for that.

Finally, using different VARs specifications allows us to compare our results to previous findings in terms of forecast and structural analysis, in the case of international transmission process.

Dealing with a huge database, we will study the impact of the U.S. housing price index shock and analyze the contagion effects on OECD countries. Our database includes the 204 monthly following variables: real GDP, personal consumption, short-term and long-term interest rates, all share price index, effective exchange rates, housing price index, unemployment rate, export and import prices for each of the seventeen considered OECD countries, over the period of 1980: M1 - 2006: M6. It is worth emphasizing that this sample is larger and more international than related studies.

The main results are: First, the largest specification outperforms the smallest one in terms of forecast accuracy. Second, the MEDIUM VAR outperforms the LARGE BVAR and the SMALL VAR in the case of structural analysis. So the MEDIUM VAR is sufficient to provide plausible impulse responses and reproduce more realistically what happened during the subprime crisis. This result is interesting in proving that a LARGE Bayesian VAR estimated over a hundred variables is not needed and produces worse forecasting, and structural analysis results than the MEDIUM VAR which has not yet been considered in the literature. Third, the Bayesian shrinkage procedure is preferable to the standard OLS estimation, in the case of an international contagion study.

The rest of the paper is structured as follows: Section 2 presents the literature review. Section 3 exposes the empirical framework. Section 4 describes the data and presents the results in terms of forecast and structural analysis. Section 5 draws the appropriate conclusions.

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4 It is considered as a broad measure of financial and economic co-movements.
5 For example, that of Stock and Watson (2005), or even that of De Bandt and Malik (2010).
2. LITERATURE REVIEW

Most of studies on contagion effects have focused on global financial markets, in general, and on asset market linkages, in particular. These studies can be broadly classified in several categories, as there is not only one definition of contagion. Surprisingly, the economists are not unanimous on a single definition of contagion. From all the proposed definitions, we retain that of Favero and Giavazzi (2002) who focus on financial linkages and transmission process. They used Vector Auto-regressions (VAR) models developed by Sims (1980) to offer an alternative to simultaneous equation models, and to detect contagion effects. Initially, Sims had emphasized the use of unrestricted VAR models as a means of modeling economic relationships.

Nowadays, VAR models are standard tools in macroeconomics, and are widely used for structural analysis. They have the advantage of not imposing restrictions on the parameters, and, hence, provide a very general representation allowing the capture of complex data relationships. Other VAR-type models have been proposed as Structural VARs; however, we always notice the same criticism of the VAR approach, according to the relatively small amount of information used in VARs. This issue has been especially addressed by Bernanke and al. (2005).

In that sense, many researches propose to add more than eight variables in a VAR model; for instance, the marginal approach proposed by Christiano, Eichenbaum and Evans (1996), or Kim (2001). They define a core set of indicators and add one variable, or a group of variables, at a time; however, comparison of impulse responses across models is problematic. Also, let mention Leeper, Sims, and Zha (1998) who increased the number of variables included by applying Bayesian priors. However, in their study, the VAR systems still contain less than 20 variables. Also, Stock and Watson (2005), and Bernanke and al. (2005) introduced the FAVAR models. The idea is that if a small number of estimated factors effectively summarizes large amounts of information about the economy, then a natural solution to the degrees-of-freedom problem in VAR analyses — which have to be of limited dimensions — is to augment the standard VAR with estimated factors. However, many issues remain unsolved concerning the estimation approach, the number, and the nature of the factors.

So, our analysis starts from the great criticism of the sparse information set used in the VAR model, which normally does not include more than eight variables to conserve degrees of freedom. Generally, central banks use a large information set to analyze the state of the economy before making any decision. In that sense, the VAR approach may exclude important information considered pertinent in the transmission process.

3. EMPIRICAL FRAMEWORK

We propose to study contagion effects in the case of simulating the last subprime crisis, using different sizes of VAR models. So, we evaluate forecasting accuracy, and perform a structural exercise on the effect of a U.S. housing shock using different sizes of VARs: a SMALL one, estimated by OLS as well as a MEDIUM VAR and, a LARGE one estimated by Bayesian shrinkage.

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6 For a more complete review, the reader can refer to the study of Dungey and al. (2003, 2005). They compare the correlation analysis approach popularized in this literature by Forbes and Rigobon (2002), the VAR approach of Favero and Giavazzi (2002), the probability model of Eichengreen and al. (1995, 1996) and the approach of Bae and al. (2003). They showed that the different definitions used to test for contagion are minor, and under certain conditions, are even equivalent.

7 The focus on relations between the transmissions of shocks through fundamental linkages has been primarily studied by Masson (1999), and called "Pure contagion".

8 Introduced by Sims (1980).

9 To conserve degrees of freedom, standard VARs rarely employ more than six to eight variables.
In this section, first, we follow standard recommendations in the Bayesian literature, and build on the results of De Mol, Giannone, and Reichlin (2008), and Banbura, Giannone and Reichlin (2010) by coping with the curse of dimensionality using Bayesian shrinkage via the imposition of priors. Second, we evaluate the forecast performance of different VAR model sizes. And finally, we deal with a structural analysis and make the impulse responses.

3.1 Setting Priors

Let a VAR model with \( p \) lags, \( \text{VAR} (p) \), be as follows:

\[
Y_t = c + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + u_t
\]

where \( Y_t = (y_{1,t}, \ldots, y_{n,t})' \), \( c = (c_1, \ldots, c_n)' \) is a vector of constants; \( A_1, \ldots, A_p \) are the autoregressive \((n \times n)\) matrices and \( u_t \) are independent \( N(0, \psi) \) errors.

The Bayesian methods combine likelihood function with prior may lead, a priori, to a valid posterior density even if some parameters are not identified in the likelihood function. However, prior information becomes increasingly important as the number of parameters increases relatively to sample size. In this case, priors on the parameters and the residual covariance matrix, \( \psi \), should be set. In the literature, many priors suggested. For a complete review of the existing priors, the reader can refer to Koop (2010).

We follow the standard procedure developed by Litterman (1986) for the VAR coefficients priors. Then, we take into account the modifications proposed by Kadiyala and Karlsson (1997), and Sims and Zha (1998) for the residual covariance matrix priors.

First, according to Litterman (1986), a \( \text{VAR} (p) \) can be considered as “centered” equations around the random walk with drift as follows:

\[
Y_t = c + Y_{t-1} + u_t
\]

The idea suggested by Litterman is to shrink all VAR coefficients towards zero except for coefficients on own lags of each dependent variable. The latter are either set to one (for variables which exhibit substantial persistence), or zero (for variables which do not). So Litterman assumes that the Minnesota prior beliefs are:

\[
E \left[ (A_k)_{ij} \right] = \begin{cases} \delta_i & \text{if } j = i, k = 1 \\ 0 & \text{otherwise} \end{cases} \quad V \left[ (A_k)_{ij} \right] = \begin{cases} \frac{2}{k^2} & \text{if } j = i \\ \frac{2}{k^2} \frac{\sigma_i^2}{\sigma_j^2} & \text{otherwise} \end{cases}
\]

The coefficients \( A_1, \ldots, A_p \) are assumed to be a priori independent and normally distributed. Also, the hyper parameters \( \lambda \) and \( \sigma^2 \) control the overall tightness of the prior distribution around the random walk or white noise, and govern the relative importance of the prior beliefs with respect to the information contained in the data.

More precisely, if \( \lambda = 0 \), the posterior equals the prior and the data do not influence the estimates. If \( \lambda = \infty \), the posterior expectations coincide with the OLS estimates. In the rest of the paper, we will choose \( \lambda \) in relation to the size of the VAR model.

\[\text{\textsuperscript{10}} \quad \text{The prior on the intercept, } c, \text{ is diffuse.}\]

\[\text{\textsuperscript{11}} \quad \text{The reader can refer to Litterman (1986) for more details concerning the setting and hypothesis of each parameter.}\]
of variables increases, the parameters should be shrunk some more in order to avoid overfitting.

The parameter $k$ is the lag length and the ratio $1/k^2$ gives the rate at which prior variance decreases with increasing lag length. Also, $\sigma_i^2 / \sigma_j^2$ reflects the different scale and variability of the data.

Second, according to Kadiyala and Karlsson (1997), and Robertson and Tallman (1999), Litterman's assumption of fixed, and diagonal residual covariance matrix is somewhat unrealistic; that is why they impose a Normal Inverted Wishart prior which retains the principles of the Minnesota prior. Also, this assumption is problematic in the case of the structural analysis, where it is necessary to take into account possible correlation among the residual of different variables.

Consequently, we follow Kadiyala and Karlsson (1997), and Robertson and Tallman (1999) who propose to deal with the matrix form of a $VAR(p)$ model given by:

$$Y = X \beta + U$$

where $Y = (Y_1, ..., Y_T)'$ is a $(T \times n)$ matrix. $X = (X_1, ..., X_T)'$ is a $(T \times K)$ matrix, with $X_t = (1, y_{t-1}', ..., y_{t-p}')'$ and $K = (1 + np)$ since each row contains $p$ lags for each dependent variable, and an intercept. Also, $\beta = (c, A_1, ..., A_p)'$ is the $(K \times n)$ matrix of coefficients and $U = (u_1, ..., u_T)'$ is a $(T \times n)$ residual matrix. The Normal Inverted Wishart prior has the form:

$$\text{vec}(\beta) | \psi \sim N(\text{vec}(\beta_0), \psi \otimes \Omega_0) \quad \text{and} \quad \psi \sim iW(S_0, \alpha_0)$$

where the prior parameters $\beta_0$, $\Omega_0$, $S_0$ and $\alpha_0$ are chosen so that prior expectations, and variances of $\beta$ coincide with those implied by equation (3), and the expectation of $\psi$ is equal to the fixed residual covariance matrix $\Sigma$ of the Minnesota prior\textsuperscript{12}.

In order to match the Minnesota moments in equation (3), and implement the prior of equation (5), it is necessary to add dummy variables as follows:

$$Y^* = X^* \beta + U^*$$

where $Y^* = (Y', Y_d)'$; $X^* = (X', X_d)'$; $U^* = (U', U_d)'$ and the dummy observations $Y_d$ and $X_d$ are:

$$Y_d = \begin{pmatrix} \text{diag} (\delta \sigma_1, ..., \delta \sigma_n) \\ \lambda \\ 0_{n(p-1)\times n} \\ \cdots \cdots \cdots \cdots \cdots \\ \text{diag} (\sigma_1, ..., \sigma_n) \\ \cdots \cdots \cdots \cdots \cdots \\ 0_{b\times n} \end{pmatrix}$$

$$X_d = \begin{pmatrix} \text{diag} (1, ..., p) \otimes \text{diag} (\sigma_1, ..., \sigma_n) \\ \lambda \\ \cdots \cdots \cdots \cdots \cdots \\ 0_{n\times p} \\ 0_{n\times b} \\ \cdots \cdots \cdots \cdots \cdots \\ 0_{b\times p} \\ 0_{b\times n} \\ \eta \end{pmatrix}$$

\textsuperscript{12} See Kadiyala and Karlsson (1997) for more details.
So we obtain that \( \beta_0 = (X' X_d)^{-1} X'_d Y_d, \) \( \Omega_0 = (X' X_d)^{-1}, \) \( S_0 = (Y_d - X_d \beta_0)' (Y_d - X_d \beta_0) \) and \( \alpha_0 = T_d - K. \)

And in that case, the posterior has the form\(^\text{13}\):

\[
\begin{align*}
vec(\beta) | \psi, Y &\sim N\left(vec(\tilde{\beta}), \psi \otimes (X' X')^{-1}\right), \text{ and } \psi | \tilde{\Sigma}, T, + 2 + T - k \sim IW(\tilde{\Sigma}, T_d + 2 + T - k) \\
\end{align*}
\]

with \( \tilde{\beta} = (X' X')^{-1} X' Y \) and \( \tilde{\Sigma} = (Y^* - X^* \tilde{\beta})' (Y^* - X^* \tilde{\beta}) \)

So we impose a Normal Inverted Wishart prior which retains the principles of the Minnesota prior under the condition that \( \varphi = 1 \)\(^\text{14}\).

The posterior expectation of the coefficients coincides with the OLS estimates of the regression of \( Y^* \) on \( X^* \). This expression is common in the Bayesian literature, and coincides with the posterior mean for the Minnesota prior.

After setting the priors, we will move to explain how to make the forecast analysis resulting from different VAR specifications.

### 3.2 Forecast Analysis

We compute point forecasts using the posterior mean of the parameters. Let \( \hat{A}_j^{(\lambda, m)} \) (\( \forall j = 1, \ldots, p \)) and \( \hat{c}^{(\lambda, m)} \) be, respectively, the posterior mean of the autoregressive coefficients, and the constant term of a given model (\( m = \text{SMALL}, \text{MEDIUM} \) or \( \text{LARGE} \)) and parameter \( \lambda \). The point estimate of the one-step-ahead forecast is computed as:

\[
\hat{Y}_{t+h}^{(\lambda, m)} = \hat{c}^{(\lambda, m)} + \hat{A}_1^{(\lambda, m)} Y_t + \cdots + \hat{A}_p^{(\lambda, m)} Y_{t-p+1}
\]

Also, the other forecasts \( h \)-steps ahead are computed recursively as follows:

\[
\hat{Y}_{t+h}^{(\lambda, m)} = \left( \hat{\gamma}_{1,t+h}^{(\lambda, m)}, \ldots, \hat{\gamma}_{n,t+h}^{(\lambda, m)} \right)
\]

where \( n \) is the number of variables included in the \( m \) model and \( h \) is the forecast horizon.

Note that in the case of the benchmark model (random walk with drift), the prior restriction is imposed exactly, that is \( \lambda = 0 \) and the corresponding forecasts are denoted by \( \hat{Y}_{t+h}^{(0)} \), and are the same for all the specifications.

In our analysis, we compute \( h \)-step-ahead forecasts, \( \hat{Y}_{t+h}^{(\lambda, m)} \), using only the information up to time \( T \). For a given forecast horizon \( h \), in each period \( T = T_0 + H - h, \ldots, T_1 - h \) where \( H \) denotes the longest forecast horizon to be evaluated. \( T_0 \) and \( T_1 \) are respectively the beginning and the end of the evaluation sample.

As for the out-of-sample forecast accuracy, we compute the Mean Squared Forecast Error (MSFE) for the variable, \( i \), and a horizon \( h \) as:

\(^{13}\) To insure the existence of the prior expectation of \( \psi \), it is necessary to add an improper prior \( \psi \sim |\psi|^{-(n+3)/2} \). See De Mol, Giannone, and Reichlin (2008), and Bańbura, Giannone and Reichlin (2010), for more details.

\(^{14}\) See Kadiyala and Karlsson (1997) for more details.
\[
MSFE_{i,h}^{(Δ,m)} = \frac{1}{T_1 - T_0 - H + 1} \sum_{t=I_0 + H-h}^{T_0 + h} \left( \hat{y}_{i,T+h}^{(Δ,m)} - y_{i,T+h} \right)^2
\] (11)

And to compare the different specifications, we use the MSFE relative (RMSFE) to the benchmark as follows:

\[
RMSFE_{i,h}^{(Δ,m)} = \frac{MSFE_{i,h}^{(Δ,m)}}{MSFE_{i,h}^{(0)}}
\] (12)

We follow De Mol, Giannone, and Reichlin (2008) and Bańbura, Giannone and Reichlin (2010), and set the overall tightness, \( \lambda \), to yield a desired average "Fit" for the key variables of interest during the pre-evaluation period, and then keep it fixed for the entire evaluation period. So, \( \lambda \) is chosen for a desired "Fit", and is given by:

\[
\lambda^{(m)}(\text{Fit}) = \arg \min_{\lambda} \left| \text{Fit} - \frac{1}{I} \sum_{i \in I} msfe_{i}^{(Δ,m)} \right|
\] (13)

In which \( I \) represents the variables included in the SMALL VAR model, \( msfe_{i}^{(Δ,m)} \) is an in-sample one-step-ahead mean squared forecast error evaluated using the training sample \( t = 1, \ldots, T_0 - 1 \) and is given for the number of lags \( p \) as follows:

\[
msfe_{i}^{(Δ,m)} = \frac{1}{T_0 - p - 1} \sum_{t=p}^{T_0-2} \left( \hat{y}_{i,T+1}^{(Δ,m)} - y_{i,T+1} \right)^2
\] (14)

More precisely, the desired "Fit" coincides with the one obtained by OLS estimation on the VAR model that is for:

\[
\text{Fit} = \frac{1}{I} \sum_{i \in I} msfe_{i}^{(Δ,m)} \bigg|_{\lambda=\arg \min \lambda MSFE}^{\lambda=\arg \min \lambda MSFE}
\] (15)

In the next section, we will explain how to make the structural analysis resulting from the VAR models.

3.3 Structural Analysis

We follow Stock and Watson (2005) and Bernanke and al. (2005), and divide the variables in the data into two categories: slow and fast-moving variables. This distinction is crucial because it implies that slow-moving variables do not respond contemporaneously to an initial shock. This hypothesis is equivalent to ranging the variable in an exogeneity order.

So, if we note \( S_t \) as representing the slow variables, \( r_t \) is the shocked variable and \( Z_t \) is the fast-moving variable, we can write \( Y_t = (S_t, r_t, Z_t) \).

The Structural VAR is written as follows:

\[
\tilde{\lambda}_0 Y_t = v + \tilde{\lambda}_1 Y_{t-1} + \cdots + \tilde{\lambda}_p Y_{t-p} + e_t
\] (16)

with \( e_t \sim WN (0, D) \) and where \( v = C^{-1}c \), \( \tilde{\lambda}_0 = C^{-1} \), \( \tilde{\lambda}_j = C^{-1}A_j \), \( \forall j = 1, \ldots, p \) and \( e_t \) is the linear transformation of the VAR residuals: \( e_t = (e_{1t}, \ldots, e_{nt})' C^{-1}u_t \).

Let the lower diagonal Cholesky matrix of the covariance of the residuals of the reduced form of a VAR be noted \( B = CD^{1/2} \), with \( CDC' = E[u_t u_t'] = \psi \) and \( D = \text{diag}(\psi) \).
We follow Gordon and Leeper (1994) by generating draws from the posterior of \( \left\{ A_1, \ldots, A_p, \psi \right\} \). So for each draw, \( \psi \), we compute \( B, C \) and even \( \lambda_j \), \( \forall j = 0, \ldots, p \).

4. DATA and RESULTS

4.1 Data

Our large international dataset is drawn from Datastream, Eurostat and the Federal Reserve website, FRED - Saint Louis Fed. The data consists of monthly variables from the period 1981M1-2006M6 for 17 OECD countries, namely United States (U.S.), Canada (CAN), Finland (FIN), France (FRA), Germany (ALL), Ireland (IRL), Italy (ITA), Netherlands (NLD), Spain (ESP), Denmark (DNK), Norway (NOR), Sweden (SWE), Switzerland (SWI), United Kingdom (UK), Australia (AUS), Japan (JPN), and New-Zealand (NZL).

Our dataset includes 204 variables, with 12 variables for each of the 17 countries, encompassing a wide range of financial variables (3-Month Interest Rates, 10-Year Interest Rates, Stock Indices and Housing Price Indices, HPI), variables related to real economy (GDP, Personal Consumption, Industrial Production, Unemployment Rates), aggregate price variables (CPI), trade variables (Import and Export of Goods and Services), and Effective Exchange Rates.

All the data are seasonally adjusted, and the variables are measured at constant national prices. As in the literature, interest rates are differenced, and activity variables are logarithmized. More detailed description is provided in appendix A.

In our analysis, we will consider the three following VAR specifications:

- SMALL: This is a VAR including five OECD housing price indices. The U.S. housing price index, HPI_USA, is included because we assume that it will impact the other OECD housing markets. Due to the number limitation, we choose to consider the most representative country for each of the four considered regions. So Canada will represent North America (NA); France, the European Monetary Union (EMU); the U.K, the Non European Monetary Union (NEMU); and Japan, Asian Pacific (AP). Thus, the SMALL VAR represents an international model.
- MEDIUM: This VAR includes all the seventeen housing price indices of the OECD considered countries.
- LARGE: This VAR contains all the considered 204 variables in our database.

4.2 Results

4.2.1 Forecast Analysis

In this section, we evaluate the forecast performance of the five considered OECD housing price indices included in the three VAR specifications: HPI_USA, HPI_CAN, HPI_FRA, HPI_UK and HPI_JPN, respectively the housing price indices of USA, Canada, France, United Kingdom and Japan, over the period going from mid-2006 until the end of 2010.

Note that for the SMALL VAR, we implement information criteria for lag selection and take the optimal lag according to the Bayesian Information Criterion (BIC). The number of lags retained is \( p = 11 \). This choice confirms the one made by Bernanke and al. (2005), indicating that the series are very persistent.
The estimation is based on the sample from 1982:2 to 2006:6, and the results reported are for the same overall shrinkage obtained by: 
\[ \hat{\lambda}^{(m)}(\text{Fit}) = \arg \min_{\lambda} \left[ \text{Fit} - \frac{1}{5} \sum_{i} \frac{msfe_{i}^{(\hat{\lambda},m)}}{msfe_{i}^{(0)}} \right] \] and as given in Table 1 in appendix B.

Moreover, to compare models of different sizes, we choose the Bayesian shrinkage hyper parameter in relation with the models dimensions by ensuring that the in-sample "Fit" is computed by 
\[ \lambda \in \sum \lambda_{\text{VAR}} \] so is constant and equal to 0.9572.

As the dimension increases, we set the tightness of the prior so that all models have the same in-sample "Fit" as the smallest VAR estimated by OLS.

Also, we report, in Table 2, the RMSFE for the five OECD housing price indices for the three different VARs.

So, according to this table, we notice that the MEDIUM and LARGE VARs outperform the SMALL one for the OECD countries except for the USA. For the U.S and for all lags, the SMALL VAR outperforms the other VARs. This confirms that the USA continues to retain its place as the world’s principal leader and does not depend on the other OECD housing markets. It seems, clearly, that for the USA, the smaller the model, the better the forecast.

For the other OECD housing markets, the MEDIUM and the LARGE VARs provide better performance than the SMALL one. Except for the USA, using a large data set helps to better forecast the OECD housing markets. This is an important result denoting that adding information, e.g. dealing with the MEDIUM or LARGE VARs, helps to improve the forecast in the case of the OECD housing markets except for the USA.

Our results are interesting and confirm the previous findings of Bernanke and al. (2005) who criticize the SMALL VAR model due to its limited number of variables.

With this simple forecast exercise, we show that employing five variables produces worse forecasting results than using seventeen or 204 ones except for the U.S.

### 4.2.2 Structural Analysis

In this section, we present the results of the impulse response functions, and the variance decompositions to the U.S. housing price index shock in mid-2006 for the VARs.

We display the impulse response functions for the three models under consideration only for the five housing price indices (HPI_US, HPI_CAN, HPI_FRA, HPI_UK and HPI_JPN) included in the different VARs.

We divide the variables in the data into slow and fast-moving variables: the slow-moving ones are GDP, consumption, housing price index, CPI, industrial production, export and import of goods and services, and unemployment rate. The rest of the variables e.g. stock price index, effective exchange rates, short-term and long-term interest rates are the fast-moving ones. Also, \( r \) is the U.S. housing price index.

The dotted lines indicate the posterior coverage intervals corresponding to 90% and 68% confidence levels as mentioned in the legend.

At first sight, we notice that the impulse responses maintain the expected sign for all specifications, and that according to the considered information and also to the VAR specification, impulse response functions change in shape as predicted.
For the SMALL VAR, the initial shock is less persistent and become nil about a four or five-month horizon. The impulse responses are significant only for a two-month horizon. We can conclude that the positive U.S housing shock is very brief. We remark that the more significant the impulse responses are, the tighter the 90% and 68% confidence intervals are, and vice versa.

For the MEDIUM VAR and except for Japan, the impulse responses have the same shape. Also, it appears that a positive American housing prices shock affects immediately and significantly the Canada, France, the U.K and Japan. The downward trend of the persistent responses for over a one-year horizon seems to be a realistic figure. It denotes that the subprime crisis effects spread over the five considered OECD countries at least for a one-year horizon which was really the case. Moreover, the tightness of the confidence bounds, compared to the ones of the SMALL VAR, is clear and denotes that the responses of the MEDIUM model are more reliable than those of the SMALL one.

For the LARGE VAR, the U.S impulse response is significant for the whole one-year horizon considered, unlike the French impulse responses which are not significant. The Canadian and Japanese impulse responses are significant only for the first month; however, the most significant impulse response is that of the U.K becoming nil in a four-month horizon. Unfortunately, the confidence bounds are wide denoting that the impulse responses are unreliable.

The relative tight confidence bounds of the SMALL and the MEDIUM VARs indicate the precision of the response of the U.S. housing price index positive shock on the other five considered OECD housing markets, which is not the case for the LARGE VAR. In the case of the last model, the 90 and 68% confidence bounds are very wide. So, it seems that the MEDIUM model provides better results in terms of impulse response functions. We can conclude that adding information related to the other OECD countries is a better choice and is sufficient when making an international study on OECD housing markets.

At this stage, combining those conclusions with the previous ones made in the case of forecasting analysis, leads us to say that the MEDIUM VAR is preferable to the SMALL and LARGE ones.

Also, to complete our analysis, we report the impulse response functions for the seventeen OECD housing prices in the case of the MEDIUM and LARGE VARs, respectively in figures 2 and 3. In fact, the advantage of the MEDIUM and LARGE VARs, compared to the SMALL one, is that impulse responses can be observed for all the OECD countries.

[Figure 2 about here]

[Figure 3 about here]

So, just as a confirmation of our previous findings, it appears, clearly, that the MEDIUM VAR is preferable to the LARGE one. This last model has very wide confidence bounds. The MEDIUM VAR provides realistic response functions reflecting what happened during the subprime crisis. The OECD did not react in the same way. It seems that the effect of the positive U.S. housing shock on OECD housing markets is significant and persistent over a one-year horizon.

Moreover, the same features can be seen from the variance decomposition analysis below:

[Table 4 about here]

The results show that the size of the positive U.S. housing price index shock is clearly more pronounced for the MEDIUM model than for the SMALL or for the LARGE VARs. Besides, for the U.S. housing price index, and since the 6-month horizon, we remark that the size of the initial shock in the MEDIUM VAR is bigger than that in the LARGE VAR, and both remain bigger than that in the SMALL VAR. This suggests that the U.S. housing shock is more and more persistent, even after a 36-month horizon.
At this stage, it appears that the MEDIUM VAR is preferable to the LARGE and SMALL ones; however, we notice, that the SMALL and MEDIUM VARs produce qualitatively and quantitatively similar results. So as to complete our analysis and decide which one of the estimation methods (Bayesian Shrinkage or OLS) is more suitable in our study, we estimate the SMALL VAR by Bayesian shrinkage. Therefore, we repeat forecasting and structural analyses for the SMALL VAR using the Bayesian approach. We get exactly the same results for the SMALL whether it is estimated both by the two methods in terms of forecasting or structural exercises. So the Bayesian shrinkage is a not neglecting alternative to the OLS when dealing with more than 10 variables.

All the results confirm that the MEDIUM VAR is preferable to the SMALL and LARGE ones. Our findings are in accordance with those of Bańbura, Giannone and Reichlin (2010). Moreover, we confirm that the Bayesian shrinkage procedure is preferable to the OLS estimation while studying the contagion effects during the subprime crisis. Besides and since the SMALL VAR is criticized due to its limited number of variables, the MEDIUM VAR is a suitable model for international study.

5. CONCLUSION

This paper assesses the performance in terms of forecast and structural analysis of different sizes of VAR models while studying contagion effects during the subprime crisis. The great criticism addressed especially by Bernanke and *et al.* (2005) is the sparse information set of the VAR model which can normally only include a few variables to conserve degrees of freedom. So, we propose to study different-sized VAR models (SMALL, MEDIUM and LARGE). In other words, we consider the following specifications: 5, 17 OECD housing price indices and 204 international macroeconomic and financial variables.

We examine both forecasting accuracy and structural analysis of the effect of a positive U.S. housing price index shock so as to study what has happened during the subprime crisis. We follow the results of De Mol, Giannone and Reichlin (2008) and Bańbura, Giannone and Reichlin (2010) by setting the degree of Bayesian shrinkage in relation to the cross-sectional dimension of the model. So, we build on standard recommendations in the Bayesian literature; as the model becomes larger, we increase the overall shrinkage so as to maintain the same in-sample "Fit" across models, and guarantee a meaningful model comparison.

Our findings show that a MEDIUM VAR estimated by Bayesian shrinkage outperforms the standard VAR and a large panel of data. So, the MEDIUM Bayesian VAR estimated over the OECD housing markets produces better forecasting and structural analysis results than the standard VAR or the LARGE one.

Consequently, the MEDIUM VAR is sufficient to provide a plausible impulse response and so reproduces what has happened during the subprime crisis. Moreover, our results prove that the Bayesian shrinkage procedure is preferable to the OLS estimation while dealing with an international study.

REFERENCES

Appendix A: Data Transformations Details

As in Stock and Watson (2005), we use the following transformation codes: 1 - no transformation (levels); 2 - first difference; 3 - second difference; 4 –logarithm; 5 – first difference of logarithm and 6 – second difference of logarithm.

In the transformation line, ln denotes logarithm, Δln and Δ²ln denote the first and second difference of the logarithm, lv denotes the level of the series, and Δlv denotes the first difference of the series.

Also, following Bernanke and al. (2005), we divide the variables into slow moving (denoted by an asterisk: * next to the variable) and fast moving variables.

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**Notes:** The abbreviations GDP (Gross Domestic Product), CONS (Personal Consumption), HPI (Housing Price Index), CPI (Consumer Price Index), PI (Industrial Production), iTB_3 (3-Month Interest Rates), iBond 10 (10-year Government Bond Index), tx change (Effective Exchange rate), SI (Stock Index) and Unemp (Unemployment rate).
Appendix B: Tables and figures

Table 1: The Value of the Shrinkage Hyper Parameter $\lambda$ for the Three Considered VARs

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Table 2: The $RMSFE$ for Forecast Horizons $h = 1, 3, 6$ and $12$ for the Three VARs

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Figure 1: Impulse Response Functions for the five considered OECD housing price indices (HPI_USA, HPI_CAN, HPI_FRA, HPI_UK and HPI_JPN) for the Three Models (SMALL, MEDIUM and LARGE VARs)
Figure 2: Impulse Response Functions of the Seventeen OECD Housing Price Indices for the MEDIUM VAR
Figure 3: Impulse Response Functions for the Seventeen OECD Housing Price Indices for the LARGE VAR

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- FIN
- FRA
- ALL
- IRL
- ITA
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