Forecasting US growth during the Great Recession: Is the financial volatility the missing ingredient?
Forecasting US growth during the Great Recession: Is the financial volatility the missing ingredient?

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Abstract — The Great Recession endured by the main industrialized countries during the period 2008-2009, in the wake of the financial and banking crisis, has pointed out the major role of the financial sector on macroeconomic fluctuations. In this paper, we reconsider macro-financial linkages by assessing the leading role of the daily volatility of two major financial variables, namely commodity and stock prices, in their ability to anticipate US GDP growth. For this purpose, an extended MIDAS model is proposed that allows the forecasting of the quarterly growth rate using exogenous variables sampled at various higher frequencies. Empirical results show that using both daily financial volatilities and monthly industrial production is helpful at the time of predicting quarterly GDP growth over the Great Recession period.

JEL codes — C53, E37
Keywords — GDP forecasting, financial volatility, MIDAS approach

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INTRODUCTION

In the wake of the financial and banking crisis, virtually all industrialized countries experienced a very severe economic recession during the years 2008 and 2009, generally referred to as the Great Recession. This recession has shed light on the necessary re-assessment of the contribution of financial markets to the business cycle. In this respect, there is growing volume of work in the literature that underlines their leading role in the forecasting of macroeconomic fluctuations. For example, Kilian (2008) reviewed the impact of energy prices shocks, especially oil prices, on macroeconomic fluctuations; Hamilton (2003) put forward a non-linear Markov-Switching model to predict the US Gross Domestic Product (GDP) growth rate using oil prices. Stock and Watson (2003) have proposed a review on the role of asset prices for predicting the GDP while Claessens et al. (2012) have empirically assessed interactions between financial and business cycles. Bellégo and Ferrara (2012) have also proposed a factor-augmented probit model enabling to summarize financial markets information into few synthetic factors in order to anticipate euro area business cycles.

Nevertheless, there are only very few studies in the literature dealing with the impact of financial volatility on macroeconomic fluctuations. Among the rare existing references, Hamilton and Lin (1996) have shown evidence of relationships between stock market volatility and US industrial production through non-linear Markov-Switching modeling; Ahn and Lee (2006) have estimated bivariate VAR models with GARCH errors for both industrial production and stock indices in five industrialized countries. Chauvet et al. (2012) have recently analyzed the predictive ability of stocks and bonds volatilities over the Great Recession using a monthly aggregated factor. Indeed, they estimate a monthly volatility common factor based on realized volatility measures for stock and bond markets. They show that this volatility factor largely explains macroeconomic variables during the 2007-2009 recession, both in-sample and out-of-sample.

When dealing simultaneously with daily financial variables and quarterly macroeconomic variables, a standard way to proceed is to temporally aggregate the high frequency variable in order to assess dependence at the same frequency between both types of variables. Alternatively, instead of aggregating data, the MIxed DAta Sampling approach (MIDAS) introduced by Ghysels and his coauthors has proved to be useful (see Ghysels et al. (2004) and Ghysels et al. (2007)). In the forecasting framework, several empirical papers have shown the pertinence of financial information at the time of predicting macroeconomic fluctuations using a MIDAS-based approach, mainly in
the US economy (see for example Clements and Galvão (2008)) but also as regards the euro area (see Marcellino and Schumacher (2010) for Germany or Ferrara and Marsilli (2013) for euro area countries). Indeed in the presence of various sampling frequencies, the MIDAS approach avoids the need for data temporal aggregation with the associated loss of information.

In this paper, our objective is twofold. First, we propose an extended MIDAS model enabling to mix daily and monthly explanatory variables in order to predict quarterly US GDP growth. We focus on two well-known daily financial ingredients, namely, commodity and stock prices, combined with a monthly industrial production index. Then, we assess through this extended MIDAS model the gain in forecasting, for various forecasting horizons, stemming from financial volatility variables estimated on a daily basis. We empirically prove that this approach would have increased the predictive accuracy during a period that includes the last Great Recession.

1 THE ECONOMETRIC MODEL

In this paper, we aim at assessing the predictive content of the daily volatility of financial variables regarding the broadest output growth measure (GDP) by avoiding the temporal aggregation step. In addition, it is well known that hard data, generally sampled at monthly frequency, convey additional information to anticipate GDP. To achieve this goal we propose an extended MIDAS model that allows us to explain a quarterly sampled variable using both monthly and daily sampled variables.

Let \( Y_{tQ} \) be a quarterly sampled stationary variable that we aim at predicting, \( X_{tM}^M \) is a vector of \( N_M \) stationary monthly variables, and \( X_{tD}^D \) is a vector of \( N_D \) stationary daily variables. We propose the following extended MIDAS model enabling the mixing of daily and monthly information:

\[
Y_{tQ} = \alpha + \sum_{i=1}^{N_D} \beta_i \text{midas}^{K_D}(\theta_i) X_{i,t}^D + \sum_{j=1}^{N_M} \gamma_j \text{midas}^{K_M}(\omega_j) X_{j,t}^M + \phi Y_{t-1} + \varepsilon_t, \tag{1}
\]

where \( \varepsilon_t \) is a white noise process with constant variance and \( \alpha, \beta, \theta, \gamma, \omega \) are the regression parameters to be estimated. We also include a first order autoregressive term in the expression (1) as it has been showed that it generally improves forecasting accuracy based on leading indicators (see for example Stock and Watson (2003)). Andreou et al. (2013) has proved that this extended
MIDAS mode that integrates different frequencies in the right hand side of the equation is useful in
the context of macroeconomic forecasting.

The \( \text{midas}^K(\cdot) \) function in equation (1) prescribes the polynomial weights that allow the frequency
mixing. The main idea behind the MIDAS specification consists of smoothing the past values of
each covariate \( \{X_{i,t}^\kappa\} \) by using polynomials \( \text{midas}^K(b) \) of the form

\[
\text{midas}^K (b) := \sum_{k=1}^K \frac{f\left(\frac{k}{K}, b\right)}{\sum_{k=1}^K f\left(\frac{k}{K}, b\right)} L^{(k-1)/\kappa},
\]

where \( K \) is the cardinality of the data set window on which the regression is based and \( \kappa \) is the
number of realizations of \( (X_{i,t}^\kappa) \) during the period \([t-1, t]\); for example, in equation (1), \( \kappa = M \equiv 3 \)
for \( X_{j,t}^M \). Also \( L \) is the lag operator such that \( L^{s/\kappa} X_{t}^\kappa = X_{t-(s/\kappa)}^\kappa \), and \( f(\cdot) \) is the weight function
that can be chosen out of various parametric families. As in Ghysels et al. (2007), \( f(\cdot) \) is given in
our case by

\[
f(z, b) = b (1 - z)^{b-1}.
\]

The window size \( K \) is in general chosen by the user but the parameter \( \theta \) is part of the estimation
problem. By construction, equation (2) is only influenced by the last \( K \) sample values. Other
parametrizations of the weight function have been used in the literature, but we have chosen the
particular specification for \( f(\cdot) \) given by equation (3) because it constitutes a parsimonious solution
for which the weights are automatically positive and show a slow decrease over time.

As one of the main objectives of our work consists of providing evidence of the macroeconomic
predictive content of financial volatilities, a crucial issue is the estimation of volatility. Given that
volatility is not directly observable, several methods have been developed in the literature to estimate
it. The most straightforward approach to this problem relies in the use of the absolute value of the
returns as a proxy for volatility; unfortunately, the results obtained this way are generally very noisy
(see Andersen and Bollerslev (1998)). This difficulty can be partially fixed by using an average of this
noisy proxy over a given period; this method yields one of the most widely used notion of volatility,

namely the \textit{realized} volatility (as used, for example, in Chauvet et al. (2012)). For example, for a
given quarter \( t \), the realized volatility \( RV_t \) can be estimated by

\[
RV_t = \left(\sum_{s=1}^{n_t} r_s^D z_s^2\right)^{1/2},
\]

(4)
where \((r_i^D)\) are the daily returns and \(n_t\) is the number of days for the quarter \(t\).

However, when using daily financial volatility in the prediction of quarterly economic growth, the most convenient approach appears to be the volatility filtered out of a GARCH-type parametric family (Engle (1982), and Bollerslev (1986)). The AR\((p)\)-GARCH\((1,1)\) specification is given by

\[
\begin{align*}
 r_i^D &= \psi_0 + \psi_1 r_{i-1}^D + \cdots + \psi_p r_{i-p}^D + w_t, \\
 w_t &= v_t^D \eta_t, \\
 (v_t^D)^2 &= c + a (v_{t-1}^D)^2 + b w_{t-1}^2,
\end{align*}
\]

where \(\psi_0\) is a constant, where \(\psi = (\psi_1, \ldots, \psi_p)\) is a \(p\)-vector of autoregressive coefficients and where \(\{\eta_t\} \sim \text{WN}(0,1)\). In order to ensure the existence of a unique stationary solution and the positivity of the volatility, we assume that \(a > 0, b \geq 0\) and \(a + b < 1\). Estimated daily volatilities \((\hat{v}_t^D)\) stemming from equation (5) will be considered as explanatory variables of the macroeconomic fluctuations using the MIDAS regression equation (1), with \(X_{i,t}^D = \hat{v}_{i,t}^D\).

Finally, when using general regression models for forecasting purposes at a given horizon \(h > 0\), forecasters can either predict covariates or implement direct multi-step forecasting (see for example Chevillon (2007) for a review on this point). The idea behind direct multi-step forecasting is that the potential impact of specification errors on the one-step-ahead model can be reduced by using the same horizon both for estimation and for forecasting at the expense of estimating a specific model for each forecasting horizon. In our work we adopt the direct multi-step forecasting approach and assume that the predictor \(Y_{t+h|t}^Q\) of the GDP quarterly growth rate, for any forecasting horizon \(h\), is given by

\[
Y_{t+h|t}^Q = \hat{\alpha}^{(h)} + \sum_{i=1}^{N_D} \hat{\beta}_i^{(h)} \text{midas}^{K_D} (\hat{\theta}_i^{(h)}) \hat{v}_{i,t}^D + \sum_{j=1}^{N_M} \hat{\gamma}_j \text{midas}^{K_M} (\hat{\omega}_j^{(h)}) X_{j,t}^M + \hat{\phi}^{(h)} Y_{t}^Q, \tag{6}
\]

where \((\hat{\alpha}^{(h)}, \hat{\beta}_1^{(h)}, \ldots, \hat{\beta}_{N_D}^{(h)}, \hat{\theta}_1^{(h)}, \ldots, \hat{\theta}_{N_D}^{(h)}, \hat{\gamma}_1^{(h)}, \ldots, \hat{\gamma}_{N_M}^{(h)}, \hat{\omega}_1^{(h)}, \ldots, \hat{\omega}_{N_M}^{(h)}, \hat{\phi}^{(h)})\) are the non-linear least squares estimators (this yields \(2N_D + 2N_M + 2\) parameters that need to be estimated).
2 Empirical results

In this section we focus on the US GDP growth prediction. We implement the model previously introduced in equation (6) in order to assess the forecasting ability of the volatility of two financial variables, namely commodity and stock prices (i.e. $N_D = 2$) in comparison with the monthly industrial production (i.e. $N_M = 1$).

The variable that we want to predict is the quarterly growth rate of the real US GDP (expressed in percentage), denoted $(GDP_t)$, as released by the BEA in May 2012. We consider as explanatory daily variables the CRB index of commodity prices and the S&P500 as an index of stock prices. For both daily returns of financial variables, we estimate their volatility over the period January 1967–December 2010 by using the AR-GARCH specification given by the equation (5). Given the low autocorrelation exhibited by these time series at lags higher than 2, a first order AR model is sufficient to whiten residuals. Since we are using a standard maximum likelihood estimator for the GARCH process and not a robust one we have smoothed out the Black Monday outlier in the S&P500 series occurring the 19th October 1987 (see for example, Charles and Darné (2005), or Carnero et al. (2012)). Estimates of daily volatility for both variables, denoted $(\hat{v}_{t,CRB}^D)$ and $(\hat{v}_{t,SP}^D)$, are presented in Figure 1. It is worth noting that as it usually happens with financial time series, periods of high volatility are clustered in time; nevertheless, the high volatility clusters do not occur at the same time for both time series. The volatility of stock prices presents a huge peak during the recent financial crisis, as well as several smaller peaks related to specific events (first oil shock in 1974-75, Asian crisis, burst of the internet bubble, etc.). The commodity volatility exhibits two main peaks; the first one corresponds to the first oil shock while the second one is also related to the recent financial crisis. Some specific events also drive commodity volatility dynamics such as the second oil shock in the early 1980s. The information conveyed by both volatilities does not seem redundant in spite of a recent increase in their correlation (see e.g. Creti et al. (2013)) and both variables are potentially useful in explaining GDP growth.
As monthly explanatory variable in the MIDAS regression (6), we use the Industrial Production Index (IPI), well known by practitioners to reflect macroeconomic evolutions as a whole, with a high degree of correlation. We consider the monthly IPI growth rate, denoted \((IPI_t)\). The time series dataset used in this empirical study is described in Table 2.

In our study we carry out an in-sample analysis over the period 1967q1-2006q4 and then we implement an out-of-sample experience over the period 2007q1 - 2010q4 that includes the Great Recession. Though we do not carry out a true real-time analysis, we take into account the specific lag structure in data publication. Indeed, while financial variables are available at the end of each month, we assume that IPI figures are available with a lag of one month and that GDP figures of the previous quarter are only available four months before the target date. Concerning the forecasting experiment, given that financial data are always available the last working day of any given month, we suppose that forecasts for a specific quarter are computed at the end of each month, for 12 horizons that range from \(h = 0\) (nowcasts computed at the end of the last month of the reference quarter) to \(h = 11/3\) (forecasts computed 11 months before the end of the reference quarter). For any time \(t\) either in the in-sample or in the out-of-sample periods, the MIDAS regression optimally takes advantage of the fluctuations of the last \(K_M = 12\) for the monthly series \((IPI)_t\) and \(K_D = 200\) for
financial covariates \((\hat{v}^{D}_{t,CRB})\) and \((\hat{v}^{D}_{t,SP})\). As we have chosen a direct multi-step forecasting approach, model parameters are estimated separately for each prediction horizon \(h\), as in equation (6).

In a first step, we assess the specific impact of both financial volatilities on the GDP growth process through a standard MIDAS model that relates daily variables with a quarterly variable. Thus, the first model that we estimate, denoted **Model M\(_d\)**, contains as regressors the daily volatilities of both financial series, namely \(\hat{v}^{D}_{t,CRB}\) and \(\hat{v}^{D}_{t,SP}\):

\[
\text{GDP}_{t+h|t} = \alpha(h) + \beta_1(h) \text{midas}^{K_D}(\hat{\theta}_1^{(h)}) \hat{v}^{D}_{t,CRB} + \beta_2(h) \text{midas}^{K_D}(\hat{\theta}_2^{(h)}) \hat{v}^{D}_{t,SP} + \phi^{(h)} \text{GDP}\_t. \quad (M_d)
\]

Parameter estimates are presented in Table 3. It turns out that the slope of the commodity volatility tends to be remarkably stable as \(h\) varies, being always negative. This means that, as expected, an increase in financial volatility has a negative impact on economic growth. The slope of the stock price volatility is higher in absolute value, but goes from positive to negative values for \(h \leq 2\). This reflects the negative short-term influence of stock volatility on output. Also, the autoregressive coefficient clearly increases for \(h \leq 4/3\). In fact, this corresponds to the moment when the GDP realization of the previous quarter is included in the regression model.

The second model, denoted **Model M\(_m\)**, contains only as regressors the monthly growth rate of the IPI:

\[
\text{GDP}_{t+h|t} = \alpha^{(h)} + \gamma^{(h)} \text{midas}^{K_M}(\hat{\omega}^{(h)}) \text{IPI}\_t + \phi^{(h)} \text{GDP}\_t. \quad (M_m)
\]

Explaining GDP growth using industrial production is standard in the empirical literature on short-term macroeconomic forecasting, especially when using bridge equations (see for example Diron (2008), or Barhoumi et al. (2012)). However, the monthly IPI series is generally aggregated before using it in quarterly equations. Here, by using a standard MIDAS equation, we allow for different weights concerning the contribution of monthly IPI to GDP growth, adding thus more flexibility to the model. Parameter estimates in Table 3 reveal that the industrial production strongly contributes to anticipate GDP growth, especially in the short-run, \(\gamma_{IPI}\) being greater than one for \(h = 0\) and \(h = 1/3\). At the same time, the contribution of the autoregressive part is less important.
The third model, denoted Model $M_{dm}$, contains as regressors both daily volatilities and the monthly IPI:

$$\text{GDP}_{t+h|t} = \hat{\alpha}^{(h)} + \hat{\beta}_1^{(h)} \text{midas}^{K_D}(\hat{\theta}_1^{(h)}) \hat{\nu}_{t,CRB}^D + \hat{\beta}_2^{(h)} \text{midas}^{K_D}(\hat{\theta}_2^{(h)}) \hat{\nu}_{t,SP}^D + \hat{\gamma}^{(h)} \text{midas}^{K_M}(\hat{\omega}^{(h)}) \text{IPI}_t + \hat{\phi}^{(h)} \text{GDP}_t.$$  

(M$_{dm}$)

Parameter estimates of this Model $M_{dm}$ in Table 3 are consistent with those of the previous two models, which can be seen as a proof of robustness. Indeed, slope parameters present the same sign as in the previous two models. We only note that the impact of the stock prices volatility has been reduced in the short run.

To assess the forecasting accuracy of each model, we compute the usual root mean square forecasting error (RMSFE), for all forecasting horizons $h$, based on differences between realized values GDP$_{t+h}$ and forecasted values GDP$_{t+h|t}$, from 2007q1 to 2010q4. For each model ($Model M_d$, $Model M_m$, $Model M_{dm}$) and each forecast horizon $h$, RMSFE($h$) values are presented in Table 1. In addition, RMSFE($h$) values, for $h$ ranging from zero to 11/3, are also plotted in Figure 2.

<table>
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<th>RMSFE($h$)</th>
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Table 1: RMSFE($h$) for quarterly GDP growth. The forecasting horizon $h$ is measured in quarters.

As expected, RMSFE($h$) for all models decrease when $h$ tends to zero, reflecting the use of an information set of increasing size. Indeed, RMSFE($h$) are more than halved when $h$ goes from 8/3 to zero. Especially, when $2/3 \leq h \leq 4/3$, we observe a strong negative slope, visible on all models. This is due to the integration of the newly available GDP growth figure for the previous quarter. From Figure 2, we note that the Model $M_{dm}$, based on daily financial volatilities and monthly IPI, provides the best outcomes for all horizons $h$. This result underlines the fact that combining the information coming from both macroeconomic and financial sources appears to be a good strategy.
when forecasting GDP, for all horizons. On this latter point we are in agreement with much of the literature on short-term macroeconomic forecasting and nowcasting that points out the usefulness of either combining information (for example through dynamic factor models, see e.g. Giannone et al. (2008)) or combining forecasts (see e.g. Timmermann (2006)). In fact, the gain of using the Model $M_{dm}$ in forecasting becomes important as soon as $h \leq 1$. We also note that between $h = \frac{4}{3}$ and $h = \frac{7}{3}$, the contribution of financial volatilities to the forecasting accuracy is remarkable, as RMSFE$(h)$ stemming from Model $M_{dm}$ and Model $M_d$ are similar. This result is interesting for practitioners in the sense that using industrial production to predict GDP with a lead of four to seven months does not appear useful; only financial volatilities help in this range of horizons. When we are close to the target date, that is during the quarter before the release (i.e. $h \leq 1$), the IPI tends to increase its impact on the forecast. This stylized fact has been also observed in empirical papers pointing out the increasing role of hard variables on macroeconomic forecasts when we are close to the release date, while financial variables have a stronger impact for longer horizons (we refer for example to Angelini et al. (2011)). Our study shows that the information contained on the industrial output series can not replace the one associated to financial volatility; both sources of information are playing an important role, but at different forecast horizons.
CONCLUSION

In this paper, we assess the predictive content of financial volatility on the US economic growth during the 2007-2010 period, that includes the Great Recession. In this respect, we implement an extended MIDAS model that integrates explanatory variables at both daily and monthly frequencies to predict the quarterly GDP growth. We show that adding daily financial volatility of stock and commodity prices increases the forecasting accuracy in comparison with a benchmark model that includes only industrial production as an explanatory variable. Moreover, our results indicate that an extended MIDAS model that includes both financial volatilities and industrial production is the most adequate approach for all forecasting horizons. Additionally, we note that between 4 and 7 months before the target date, there is no predictive gain to include industrial production, the information conveyed by financial volatilities predominates.

REFERENCES


A Description of variables involved in the MIDAS regressions

Real output

| GDP | US GDP growth (Bureau of Economic Analysis) | Quarterly growth rate |

Daily series

| CRB | CRB Spot index, commodities price index | Daily volatility |
| SP500 | S&P500 index (Standard & Poors) | Daily volatility |

Monthly indicator

| IPI | US Industrial Production Index growth (Federal Reserve) | Monthly growth rate |

Table 2: Description of indicators and covariates

B Estimated parameters

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Table 3: Estimated parameters of the three MIDAS models $M_d$, $M_m$ and $M_{dm}$, over the period 1967-2006.