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Testing for the Systemically Important Financial Institutions: a Conditional Approach *

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We introduce in this paper a testing approach that allows checking whether two financial institutions are systemically equivalent, with systemic risk measured by CoVaR (Adrian and Brunnermeier, 2011). The test compares the difference in CoVaR forecasts for two financial institutions via a suitable loss function that has an economic content. Our testing approach differs from those in the literature in the sense that it is conditional, and helps evaluating in a forward-looking manner, the extent to which statistically significant differences in CoVaR forecasts can be attributed to lag values of market state variables. Moreover, the test can be used to identify systemically important financial institutions (SIFIs). Extensive Monte Carlo simulations show that the test has desirable small sample properties. With an application on a sample including 70 large U.S. financial institutions, our conditional test using market state variables such as VIX and various yield spreads, reveals more (resp. less) heterogeneity in the systemic profiles of these institutions compared to its unconditional version, in crisis (resp. non-crisis) period. It also emerges that the systemic ranking provided by our testing approach is a good forecast of a financial institution's sensitivity to a crisis. This is in contrast to the ranking obtained directly using CoVaR forecasts which has less predictive power because of estimation uncertainty.

Key words: Systemic Risk, SIFIs, CoVaR, Estimation Uncertainty, Conditional Predictive Ability Test

1. Introduction

The U.S. financial market turmoil that began in August 2007 following the bankruptcy of Lehman Brothers has spread to the global financial market and the impact on the worldwide real economy remains current. The magnitude of the crisis and the social costs in most countries raise the need for new macro-prudential devices for a more efficient stabilization of systemic risk in the financial sector. In response to the crisis, a number of reforms have been initiated by the Financial Stability Board (FSB) and the Basel Committee on Banking Supervision (BCBS) to improve the quality of banking supervision worldwide. A main measure from these reforms addresses the identification of the so-called global systemically important financial institutions (G-SIFIs). The objective is to allocate G-SIFIs into buckets according to their required level of additional loss absorbency. In a country-level, a set of principles has also been established to allow national authorities to identify domestic systemically important financial institutions (D-SIFIs).

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From a methodological point of view, the identification of SIFIs requires above all a deeper understanding of the nature of systemic risk and the development of suitable tools for its measurement. In this line the Basel Committee introduced an indicator-based approach for the assessment of the systemic importance of financial institutions. The retained indicators (quantitative and qualitative) which are mostly non-publicly available are related to the key driving forces of the systemic fragility of financial institutions, such that size, contagion or interconnectedness, lack of substitute financial products, global cross-jurisdictional activity, and complexity of business models.

In contrast to the indicator-based approach, the academic literature has evolved significantly in recent years, offering a model-based approach to measure systemic risk based on publicly available market data. The main contribution can be broadly divided into two groups. The first group including the contributions of Huang et al. (2009), Acharya et al. (2010), Brownlees and Engle (2012), and Acharya et al. (2012), among others, attempts to evaluate the systemic importance of a financial institution by the expected losses it generates in the case of a global systemic event. A different but closely related approach is the Conditional Value-at-Risk (CoVaR) of Adrian and Brunnermeier (2011) which measures the systemic risk of a given institution, by the downside risk its distress imposes to the system as whole. All these papers share the common feature that the institution-level systemic risk they produce is based on the dependence structure between negative large variations of the institutions market values. The second group of papers captures another dimension of systemic risk, scrutinizing the systemic linkages among financial institutions given by the network topology of their asset returns (Hautsch et al., 2011; Billio et al., 2012; Barigozzi and Brownlees, 2013).[1]

Although the evaluation of systemic risk provided by the model-based approach can be useful in determining the level of capital surcharges that should be imposed on the SIFIs, it is worth mentioning that the ranking of SIFIs from this approach can be inconsistent due to estimation uncertainty. Indeed, the different contributions in the model-based approach are based on econometric models for the dynamics and the interrelationships between market data (equity or credit default swap returns). Because these models are possibly misspecified, the provided measures of systemic risk are noisy, and the associated rankings of SIFIs are misleading. The impact of estimation uncertainty on systemic risk models has been discussed by Danielsson et al. (2011). The authors show that most common systemic risk models such as marginal expected shortfall (Acharya et al., 2010) or CoVaR (Adrian and Brunnermeier, 2011) contain a high degree of uncertainty with

[1] Our decomposition into two groups of the contributions about systemic risk measurement based on market data is not exhaustive. Indeed there are some papers that focus on other aspects of systemic risk such as (*i*) probability of failure derived from contingent-claims analysis (Capuano, 2008; Gray and Jobst, 2010) and (*ii*) equity market illiquidity (Getmansky et al., 2004; Khandani and Lo, 2011; Billio et al., 2012).

the consequence that the signal they produce is not useful. They suggest the development of a set of evaluation criteria (confidence intervals, backtesting, and robustness analysis) to be applied to systemic risk models based on market data.

The impact of estimation uncertainty on the ranking of SIFIs can be alleviated using robust inferential procedures to compare the systemic risk profiles of financial institutions. This approach is followed by Castro and Ferrari (2012) who introduce a dominance test to compare the CoVaR of a pair of financial institutions. Applying the test to all pairs of institutions allows providing a ranking of SIFIs. Our article is related to the work of Castro and Ferrari (2012), as we introduce a statistical procedure to check for the equality between the CoVaRs of two financial institutions. Formally, our test based on the conditional predictive ability (CPA) test of Giacomini and White (2006) compares the difference in CoVaR forecasts for two financial institutions via a suitable loss function that has a meaningful economic interpretation. For a given financial institution, our loss function measures at each date the incurred system loss in exceedance of its VaR when the financial institution in question is in distress. For systemically equivalent financial institutions with the same CoVaRs, the expected difference in the loss functions is close to zero, and deviates from zero otherwise. Our test is robust to estimation uncertainty and helps providing a consistent ranking of SIFIs. Extensive Monte Carlo simulations show that the test has desirable small sample size and power properties.

Although closely related to the paper of Castro and Ferrari (2012), our work differs from this contribution in two aspects. First, we consider testing for the statistical difference in systemic risk measures for two financial institutions in an out-of-sample environment. Second, and more importantly, our test is conditional and helps evaluating in a forward-looking manner, the extent to which statistically significant differences in CoVaR forecasts can be attributed to lag values of market state variables. Hence in our framework, rejection of the null hypothesis of statistical equality between CoVaR forecasts will occur because market state variables help predict out-of-sample, the statistical differences in systemic risks for both institutions. An unconditional version of the test is also considered and corresponds to a Diebold and Mariano (1995) type test.

An empirical application is conducted using a sample of 70 U.S. financial institutions. We observe two things. First, our conditional test using market state variables such as VIX and various yield spreads, reveals more (resp. less) heterogeneity in the systemic profiles of these institutions compared to its unconditional version, in crisis (resp. non-crisis) period. Hence, given the information conveyed by the market state variables over the crisis period, the conditional test is able to discriminate financial institutions much more in comparison to the unconditional test. Second, we observe that the evaluation of the systemic importance of financial institutions provided by our conditional testing approach seems consistent. Indeed, the systemic ranking obtained through the conditional testing approach appears as a good forecast of a financial institution's sensitivity to a crisis. More precisely, institutions which are identified as systemically important in the period preceding the 2007-2008 financial crisis are those that experience large losses in the crisis period. This is in contrast to the ranking obtained directly using CoVaR forecasts which has less predictive power because of estimation uncertainty.

The remainder of the paper is organized as follows. In the second section, we present our conditional test for the statistical equality between CoVaR forecasts. Section 3 analyzes the small sample properties of the test via extensive Monte Carlo simulations. Section 4 is devoted to the empirical application, and the last section concludes the paper.

2. Testing for the statistical equality of CoVaR forecasts

This section is devoted to the main contribution of our paper. We introduce a testing procedure to check whether two financial institutions are systemically equal, with systemic risk quantified by CoVaR. The section is divided into two parts. In the first part we briefly review the concept of CoVaR as introduced by Adrian and Brunnermeier (2011) to measure systemic risk of financial institutions, and present the intuition underlying our test. In the second part we give an overview of our testing approach, introducing the test statistic and its asymptotic properties.

2.1. A brief review of CoVaR

CoVaR is based on the concept of Value-at-Risk (VaR), a risk measure often used by financial analysts and risk managers to measure and monitor the risk of loss for a trading or investment portfolio. The VaR of an instrument or portfolio of instruments is the maximum dollar loss within the q%-confidence interval (Jorion, 2007). Moving from the portfolio level to the global level for a financial institution i, the VaR at the confidence level q is defined as:

$$\Pr\left(r_{i,t+1} \leq \mathrm{VaR}^{i}_{q,t+1}\right) = q, \tag{1}$$

with $r_{i,t+1}$ the equity return of the financial institution i at time $t+1$, and $\mathrm{VaR}^{i}_{q,t+1}$ the corresponding VaR. Hence, the VaR is equal to the q-quantile of the returns distribution for the financial institution i.

CoVaR as defined by Adrian and Brunnermeier (2011) is built on the VaR as risk measure, and evaluates the systemic importance of a financial institution by the risk encountered by the financial system as whole, when the institution in question is in distress. The distress of the financial institution occurs when its return or market loss is equal to its VaR. More precisely, if we denote $r_{m,t+1}$ the return at time $t+1$ of the market, the CoVaR of the financial institution i is defined as:

$$\Pr\left(r_{m,t+1} \leq \mathrm{CoVaR}^{i}_{q,t+1} \left| r_{i,t+1} = \mathrm{VaR}^{i}_{q,t+1}\right.\right) = q, \tag{2}$$

where $\mathrm{CoVaR}^i_{q,t+1}$ is the CoVaR of the financial institution i. From (2), it appears that the CoVaR of the financial institution i is the q-quantile of the conditional probability distribution of market returns, with the conditioning event given by the distress of the institution i. Note that both $\mathrm{VaR}^i_{q,t+1}$ and $\mathrm{CoVaR}^i_{q,t+1}$ in (1) and (2) are typically negative numbers. Therefore, for two financial institutions i and j, a configuration with $\mathrm{CoVaR}^i_{q,t+1} < \mathrm{CoVaR}^j_{q,t+1}$ means that the financial institution i is more systemic than the financial institution j, because the distress of the former has a larger consequence in the left tail of the market returns distribution.

Regarding estimation, the standard method to estimate CoVaR is the parametric quantile regression used by Adrian and Brunnermeier (2011) in their application of the measure. The parametric quantile regression links the returns of the market to the returns of the financial institution as follows:

$$r_{m,t+1} = \alpha^i_q + \beta^i_q r_{i,t+1} + \varepsilon_{i,t+1}, \tag{3}$$

where α^i_q and β^i_q are parameters, $\varepsilon_{i,t+1}$ is an innovation having the q-quantile equal to zero. The parameters α^i_q and β^i_q are usually estimated by minimizing the "check" loss function of Koenker and Bassett (1978). By the definition of quantile regression, the fitted value of the dependent variable in (3) corresponds to the q-quantile of the market returns, *i.e.*, the estimated q-VaR of the market:

$$\mathrm{VaR}^m_{q,t+1} = \widehat{\alpha}^i_q + \widehat{\beta}^i_q r_{i,t+1}. \tag{4}$$

This expression provides the VaR of the system for any value of the return of the financial institution i. Replacing $r_{i,t+1}$ by $\mathrm{VaR}^i_{q,t+1}$ in the last expression yields the estimated value of CoVaR for the financial institution i:

$$\mathrm{CoVaR}^i_{q,t+1} = \widehat{\alpha}^i_q + \widehat{\beta}^i_q \mathrm{VaR}^i_{q,t+1}. \tag{5}$$

Remark that in order to compute the value of $\mathrm{CoVaR}^i_{q,t+1}$, we need to estimate $\mathrm{VaR}^i_{q,t+1}$, the VaR of the financial institution i at time $t+1$. This estimation can be accomplished in several ways using either non-parametric, semi-parametric or parametric models for the probability distribution function of $r_{i,t+1}$. For a review of models for the estimation of VaR, see Engle and Manganelli (2001). It is worth noting that the fitted value of CoVaR at time t depends on estimated parameters, *i.e.*, $\widehat{\alpha}^i_q$ and $\widehat{\beta}^i_q$ obtained through the quantile regression, but also a vector $\widehat{\eta}_i$ of parameters related to the estimation of $\mathrm{VaR}^i_{q,t+1}$. Hence throughout the paper we will use if needed the notation $\mathrm{CoVaR}^i_{q,t+1}\left(\widehat{\theta}_i\right)$ to denote the forecast CoVaR at time $t+1$ for the financial institution i, with:

$$\widehat{\theta}_i = \left(\widehat{\alpha}^i_q, \widehat{\beta}^i_q, \widehat{\eta}'_i\right)'. \tag{6}$$

If we consider a second financial institution j, its CoVaR is similarly obtained through the following quantile regression:

$$r_{m,t+1} = \alpha_q^j + \beta_q^j r_{j,t+1} + \varepsilon_{j,t+1}, \tag{7}$$

with the estimated CoVaR at time $t+1$ equal to:

$$\text{CoVaR}_{q,t+1}^j \left(\widehat{\theta}_j\right) = \widehat{\alpha}_q^j + \widehat{\beta}_q^j \text{VaR}_{q,t+1}^j. \tag{8}$$

Equations (3) and (7) show that the CoVaRs of both financial institutions are computed through a quantile regression where the dependent variable is the market return. The only difference between the two regressions lies in the choice of the factor or covariate that affects the dependent variable. In other words, the CoVaRs of the two financial institutions are quantiles of the same variable estimated through two different conditional models. Hence, a simple way to compare the two CoVaRs is to compare the predictive ability of both models in estimating the q-quantile of the market returns. To achieve this task, we rely on the conditional predictive ability test of Giacomini and White (2006). Their framework allows for the comparison of the predictive power of two competing models under a general econometric environment: (i) it can handle forecasts based on both nested and nonnested models, (ii) allows the forecasts to be produced by general estimation methods, (iii) and authorizes the use of general loss functions, either economic or statistical. In the next subsection, we develop our testing approach to compare CoVaR forecasts.

2.2. Comparing CoVaR forecasts across institutions

The environment of our testing procedure is as follows. At each date t the information set available corresponds to $\mathcal{F}_t = \{W_s, s \leq t\}$. The observed vector W_t has the following partition $W_t = (r_{i,t}, r_{j,t}, r_{m,t}, z_t')'$, where $r_{i,t}$ and $r_{j,t}$ are the returns at time t of two financial institutions i and j respectively, and $r_{m,t}$ is the return at time t of the market as whole. The vector z_t of length k contains the values at time t of k exogenous variables, measuring the state of both stocks and credit markets, with predictive power on the variations of systemic risk measures. Examples of such variables are VIX, short term treasury rate, slope of the yield curve and corporate default spreads (Adrian and Brunnermeier, 2011).

Now, consider that we use a rolling-window forecasting scheme to produce out-of-sample CoVaR forecasts for the financial institution i. Recall that a rolling-window procedure consists in partitioning the whole sample of size T into an in-sample part of size m and an out-of-sample part of size n, with $T = m + n$. The first m observations are used to produce the first out-of-sample forecast. The $n-1$ subsequent forecasts are obtained moving each time the estimation window (including a new data and dropping the earliest data) until the last observation. At the end of the rolling-window procedure, we obtain $n = T - m$ out-of-sample CoVaR forecasts for the financial institution i, *i.e.*, $\mathrm{CoVaR}^{i}_{q,t+1}\left(\widehat{\theta}_{i,t}\right)$, $t=m,...,T-1$. The dependence to time of the estimated vector of parameters $\widehat{\theta}_i$ comes from the fact that through the rolling-window procedure, this vector is re-estimated. One can instead use a fixed forecasting scheme, in which θ_i is estimated only once on the first m observations. For a second financial institution j, we can similarly produce $n=T-m$ out-of-sample CoVaR forecasts given by $\mathrm{CoVaR}^{j}_{q,t+1}\left(\widehat{\theta}_{j,t}\right)$, $t=m,...,T-1$.

We evaluate at each date $t=m,...,T-1$, the out-of-sample CoVaR forecast for the financial institution i, using the following one-sided loss function

$$\mathcal{L}_{t+1}\left(r_{m,t+1},\mathrm{CoVaR}^{i}_{q,t+1}\left(\widehat{\theta}_{i,t}\right)\right)=e_{i,t+1}\mathcal{I}\left(e_{i,t+1}<0\right), \tag{9}$$

with

$$e_{i,t+1}=r_{m,t+1}-\mathrm{CoVaR}^{i}_{q,t+1}\left(\widehat{\theta}_{i,t}\right), \tag{10}$$

where $\mathcal{I}\left(.\right)$ is the usual indicator function. Since $\mathrm{CoVaR}^{i}_{q,t+1}\left(\widehat{\theta}_{i,t}\right)$ is the q-VaR of the market conditional on the institution i being in financial distress, this loss function measures the excess shortfall at date $t+1$, and hence has a meaningful economic interpretation, *i.e.*, the incurred market loss in exceedance of its VaR. The above loss function can be similarly computed for the financial institution j over the same out-of-sample period, with

$$\mathcal{L}_{t+1}\left(r_{m,t+1},\mathrm{CoVaR}^{j}_{q,t+1}\left(\widehat{\theta}_{j,t}\right)\right)=e_{j,t+1}\mathcal{I}\left(e_{j,t+1}<0\right). \tag{11}$$

Whereas the loss function in (9) gives the excess shortfall of the market when the institution i is in financial distress, the loss function in (11) measures the same quantity, but when institution j is in distress. Relying on the conditional predictive ability test of Giacomini and White (2006), our main idea is to test the following null hypothesis

$$\mathbb{H}_0:\mathbb{E}\left[\Delta\mathcal{L}_{t+1}\left|\mathcal{F}_t\right.\right]=0,\ \text{ almost surely }\quad t=m,...,T-1, \tag{12}$$

where

$$\begin{aligned}\Delta\mathcal{L}_{t+1}=&\ \mathcal{L}_{t+1}\left(r_{m,t+1},\mathrm{CoVaR}^{i}_{q,t+1}\left(\widehat{\theta}_{i,t}\right)\right)-\\&-\mathcal{L}_{t+1}\left(r_{m,t+1},\mathrm{CoVaR}^{j}_{q,t+1}\left(\widehat{\theta}_{j,t}\right)\right),\end{aligned} \tag{13}$$

and $\mathcal{F}_t=\{W_s,s\leq t\}$ is the set of information available at time t.

Our null hypothesis invokes two main comments. First, comparing the predictive power of both models via our loss function is equivalent to testing whether the two financial institutions are systemically equivalent. The systemic nature of each financial institution corresponds to the expectation of the loss function, which is equal for each date to the expected excess shortfall of the market when the financial institution is in distress. Note that for systemically equivalent financial institutions, the expected difference in the loss functions is close to zero. Otherwise, the expectation deviates from zero positively (resp. negatively) when the financial institution i (resp. j) is more systemic. Second, the expectation in (12) is conditional on the information set $\mathcal{F}_t$ which includes market state variables (slope of the yield curve, corporate default spreads, etc.) with predictive power on systemic risk. This conditional setup helps to account for the effect of these state variables, in comparing the systemic nature of the two financial institutions. In other words, rejection will occur because state variables help predict out-of-sample, the statistical differences in systemic risks for both institutions.

With h_t a given $d \times 1$ $\mathcal{F}_t$-measurable vector, we build on Giacomini and White (2006) using the following hypothesis to test for the equality of systemic risk measures

$$\mathbb{H}_{0,h} : \mathbb{E}\left[h_t \Delta \mathcal{L}_{t+1}\right] = 0, \text{ almost surely } \quad t = m, ..., T-1, \tag{14}$$

which is an implication of the null hypothesis in (12). A GMM-type test can thus be conducted to make inference.

PROPOSITION 1. *Under the null hypothesis of conditional equality between CoVaR forecasts,*

$$T^h_{m,n} = n\overline{Z}'_{m,n}\widehat{\Omega}_n^{-1}\overline{Z}_{m,n} \underset{n\to\infty}{\overset{d}{\to}} \chi^2(d),$$

with

$$\overline{Z}_{m,n} = n^{-1}\sum_{t=m}^{T-1} Z_{m,t+1} = n^{-1}\sum_{t=m}^{T-1} h_t \Delta \mathcal{L}_{t+1},$$

where $\chi^2(d)$ is a chi-square distribution with d degrees of freedom. The matrix $\widehat{\Omega}_n$ is a consistent estimate of the variance of $Z_{m,t+1}$

$$\widehat{\Omega}_n = n^{-1}\sum_{t=m}^{T-1} Z_{m,t+1} \times Z'_{m,t+1}.$$

See Giacomini and White (2006) for the proof. Assumptions for the validity of the asymptotic distribution are weak, allowing for heterogeneity and dependence in the loss functions differences. It is worth mentioning that the asymptotic framework requires $n \to \infty$, while m is kept fixed, with m the estimation sample size and n the out-of-sample size. This authorizes forecasting schemes based on fixed or rolling-windows, and rules out the use of an expanding window. Because m is fixed, estimation errors which result from approximating the unknown vectors θ_i and θ_j by their respective estimators, do not vanish asymptotically. This ensures that the test captures the impact of estimation uncertainty on the equality of the loss functions (see Giacomini and White, 2006 for a complete discussion on this issue).

As in the general framework of Giacomini and White (2006), an unconditional version of the test corresponds to the following null hypothesis

$$\mathbb{H}_0 : \mathbb{E}\left[\Delta\mathcal{L}_{t+1}\right] = 0, \quad t = m, ..., T-1, \tag{15}$$

which can be tested using a Diebold and Mariano (1995) type test.

Proposition 2. *Under the null hypothesis of unconditional equality between CoVaR forecasts, we have*

$$t_{m,n} = \frac{\overline{\Delta}\mathcal{L}_{m,n}}{\widehat{\sigma}\,/\sqrt{n}} \underset{n\to\infty}{\overset{d}{\rightarrow}} N\left(0,1\right),$$

with

$$\overline{\Delta}\mathcal{L}_{m,n} = n^{-1}\sum_{t=m}^{T-1}\Delta\mathcal{L}_{t+1},$$

and $\widehat{\sigma}^2$ an HAC estimator of the long run variance of the loss functions differences.

The interested reader should refer to Giacomini and White (2006) for the proof, and the regularity assumptions. The unconditional version of the test checks whether the two CoVaR forecasts are equal or not on average, while the conditional version asks whether there is current information that can help predict the statistical difference between the CoVaR forecasts for the next period. The HAC estimator $\widehat{\sigma}^2$ of the long run variance has the expression

$$\widehat{\sigma}^2 = n^{-1}\sum_{t=m}^{T-1}\Delta\mathcal{L}_{t+1}^2 + 2\left[n^{-1}\sum_{j=1}^{p_n} w_{n,j} \times \sum_{t=m+j}^{T-1}\Delta\mathcal{L}_{t+1}\Delta\mathcal{L}_{t+1-j}\right], \tag{16}$$

where $\{p_n\}$ is a sequence of integers with $p_n \to \infty$ as $n \to \infty$, and $p_n = o(n)$. The weights function $w_{n,j}$ are such that $|w_{n,j}| < \infty$, $w_{n,j} \to 1$ as $n \to \infty$.

Remark that in the case of rejection of the null hypothesis of the unconditional test, a positive (resp. negative) sign for $\overline{\Delta}\mathcal{L}_{m,n}$ means that the financial institution i (resp. j) is more systemic. For the conditional test the decision rule in the case of rejection of the null hypothesis is not straightforward. Indeed, the null hypothesis in (14) is formulated as $\mathbb{E}\left[h_t\Delta\mathcal{L}_{t+1}\right] = 0$, where 0 is the null vector of length d. Deviation from the null hypothesis occurs when at least one entry of the vector $\mathbb{E}\left[h_t\Delta\mathcal{L}_{t+1}\right]$ is not equal to zero. In the case where those entries which are not equal to zero are all positive (resp. negative), we can conclude that the financial institution i is more (resp. less) systemic than the financial institution j. These configurations should be rare in empirical applications as they correspond to the case where all of the conditioning variables in the vector h_t impact $\Delta\mathcal{L}_{t+1}$ in the same direction. For the more plausible cases where some entries are negative and others positive, there is no guidance about the ranking of the systemic nature of the two financial institutions. Nevertheless, Giacomini and White (2006) in their general framework suggest a procedure for selecting at time t a forecasting method for $t+1$. We adopt this procedure to the present framework. The main idea is to recognize that rejection occurs because variables in the vector h_t can predict the loss differences $\Delta\mathcal{L}_{t+1}$ out of sample. Hence, variables in h_t can be used to predict which CoVaR forecast will yield lower loss at $t+1$. The procedure works as follows:

- Step 1: regress $\Delta\mathcal{L}_{t+1}$ the difference in loss functions at time $t+1$ on h_t over the out-of-sample period $t=m,...,T-1$ and let $\widehat{\delta}_n$ denote the regression coefficients.
- Step 2: the approximation $\widehat{\delta}_n' h_t \approx \mathbb{E}\left[\Delta\mathcal{L}_{t+1} \left| \mathcal{F}_t \right.\right]$ motivates the following decision rule: the financial institution i is more (resp. less) systemic than the financial institution j at time $t+1$ when $\widehat{\delta}_n' h_t > 0$ (resp. $\widehat{\delta}_n' h_t < 0$).

Note that this decision rule is applied for each date over the out-of-sample period. Therefore we can identify at each date $t+1$ over the out-of-sample period, which institution is the most systemic given the information available at time t. This is another advantage of the conditional test over its unconditional counterpart. For the whole out-of-sample period, the financial institution i will be selected as the most (resp. the least) systemic financial institution when the proportion of the dates the above decision rule concludes that i is more (resp. less) systemic is higher than 50%.

3. Small sample properties

In this section, we investigate the performance of our testing procedure in finite samples. We perform the evaluation along two dimensions: the size and the power of the tests (unconditional and conditional).

3.1. Analysis of the size

Our Monte Carlo simulation design can be described as follows. First, we simulate n VaR forecasts for the financial institution i assuming that its returns follows a GARCH(1,1) model

$$\begin{cases} r_{i,t+1} = \sigma_{i,t+1}\varepsilon_{i,t+1} \\ \sigma^2_{i,t+1} = \omega_{i,0} + \omega_{i,1} r^2_{i,t} + \omega_{i,2}\sigma^2_{i,t}, \quad t=0,...,n-1, \end{cases} \tag{17}$$

with $\varepsilon_{i,t+1}$ a Gaussian *i.i.d.* innovation process with zero mean and unit variance. Hence, over the out-of-sample period, the n VaR forecasts at the risk level q are equal to

$$VaR^i_{t+1}(q) = \Phi^{-1}(q)\,\sigma_{i,t+1}, \quad t=0,...,n-1, \tag{18}$$

where Φ is the cumulative distribution function of a standard normal random variable. The predicted values of $\text{CoVaR}^i_{q,t+1}$ are obtained via the following formula

$$\text{CoVaR}^i_{q,t+1} = \alpha^i_q + \beta^i_q VaR^i_{t+1}(q), \quad t=0,...,n-1. \tag{19}$$

Recall that these simulated CoVaRs are theoretically the q-VaR of the market when the financial institution i is in distress, and then satisfy the relationship

$$\Pr\left(r_{m,t+1} \leq \mathrm{CoVaR}_{q,t+1}^{i} \left| r_{i,t+1} = \mathrm{VaR}_{q,t+1}^{i}\right.\right) = q, \quad t = 0, ..., n-1. \tag{20}$$

We simulate market returns consistent with (20) using the following data generating process (DGP)

$$r_{m,t+1} = \mathrm{CoVaR}_{q,t+1}^{i} + u_{t+1}, \tag{21}$$

with

$$u_{t+1} \sim \mathcal{N}\left(-\lambda\Phi^{-1}(q), \lambda^2\right). \tag{22}$$

This specification ensures that $\mathrm{CoVaR}_{q,t+1}^{i}$ is indeed the q-VaR of the market, because the qth quantile of u_{t+1} is equal to zero. We set the value of the parameter λ to $\lambda = 0.1$.[2]

We follow the same procedure described in (17-22) to simulate the n forecasts of $\mathrm{CoVaR}_{q,t+1}^{j}$ for the financial institution j. The market returns consistent with these forecasts are also generated according to (21-22)

$$r_{m,t+1} = \mathrm{CoVaR}_{q,t+1}^{j} + u_{t+1}. \tag{23}$$

The market returns series we consider in the testing procedure is a simple weighted average of the simulated returns in (21) and (23). Note that in our simulation setup, the forecast values of CoVaR are directly generated over the out-of-sample period. This supposes that the in-sample size m is zero and $T = n$. This is not restrictive as the asymptotic analysis in the framework of Giacomini and White (2006) requires only that $n \to \infty$. More precisely, the conditional predictive ability test of Giacomini and White (2006) is shown to be robust to estimation uncertainty. Hence, we proceed here as if the true parameters are known and generate the CoVaR forecasts directly.

In order to simulate data under the null hypothesis of statistical equality between the CoVaR forecasts for the two financial institutions, we suppose that their returns follow the same GARCH process, *i.e.*,

$$(\omega_{i,0}, \omega_{i,1}, \omega_{i,2}) = (\omega_{j,0}, \omega_{j,1}, \omega_{j,2}) = (0.002, 0.2, 0.6). \tag{24}$$

The profile of the n VaR forecasts are thus identical for the two financial institutions. Moreover, we simulate the CoVaR forecasts assuming that

$$\left(\alpha_q^i, \beta_q^i\right) = \left(\alpha_q^j, \beta_q^j\right) = (0.0, 0.5). \tag{25}$$

[2] See Giacomini and Komunjer (2005) for a similar simulation design in the context of conditional quantile forecasts encompassing test.

It is worth noting that although these restrictions are particular, they ensure that the simulated CoVaR forecasts for the two financial institutions are statistically equal.[3] The sizes of the unconditional and conditional versions of our test are displayed in Table 1 over 5,000 replications for the nominal risk level $\eta = 5\%$. The downside risk level q is set to 5%. We report results for different configurations of the out-of-sample length $n \in \{500, 1000, 2000, 5000\}$. For the unconditional test, the truncation parameter p_n is set to $p_n \in \{0, 5, 10, 15, 20, 25, 50\}$. For the conditional test, we let the h_t measurable vector to be $h_t = (1, S_t)'$, where S_t is a variable measuring the state of the economy at time t, which follows an *i.i.d.* Bernoulli distribution with parameter 0.5. Because, S_t is simulated independently to the two CoVaR forecasts, the null hypothesis of the conditional test is also valid. Indeed, recall that the null hypothesis in the conditional version of the test requires the difference in loss functions to be uncorrelated to the conditioning variable, we denote here S_t. Results in Table 1 show that both tests are well-sized. Indeed, the rejection frequencies are overall close to 5%. The only exceptions occur for the smallest sample sizes where the conditional (resp. unconditional) test appears undersized for $n = 500$ and $n = 1000$ (resp. $n = 500$). Nevertheless for both tests, the rejection frequencies converge to the nominal size when the sample size increases.

3.2. Analysis of the power

We consider two different simulation setups. In the first one, the difference in CoVaR forecasts is not related to the conditioning variable S_t. In the second setup, the difference in CoVaR forecasts is related to the lagged value of the conditioning or state variable S_t. In this last case, we expect the conditional test to be more powerful than its unconditional counterpart.

3.2.1. Experiment 1 - independence between CoVaRs and S_t: We rely on the simulation design used for the analysis of the size, except that the responses of the CoVaR forecasts to the VaRs differ across the two financial institutions. More precisely, we suppose that the returns of each financial institution follow the GARCH(1,1) model calibrated above. However, we set the parameters β_q^i and β_q^j as follows

$$\left(\alpha_q^i, \beta_q^i\right) = (0.0, 0.4)\,, \tag{26}$$

$$\left(\alpha_q^j, \beta_q^j\right) = (0.0, 0.6)\,. \tag{27}$$

Hence, although the VaRs are statistically equivalent for the two financial institutions, their CoVaR forecasts are not statistically equal, due to the difference between the slope coefficients β_q^i and β_q^j. In other words, the CoVaR forecasts of the financial institution i are on average lower in absolute value than the CoVaR forecasts of the financial institution j. For the unconditional test,

[3] Indeed, it is conceptually difficult to simulate data under the null hypothesis, assuming different processes for the returns of the two financial institutions, and (ii) different responses of the VaR of the market to the VaR of each financial institution.

the truncation parameter p_n is set to $p_n \in \{0, 5, 10, 15, 20, 25, 50\}$. To implement the conditional test, the conditioning variable S_t is generated independently to the CoVaR forecasts and follows once again an *i.i.d.* Bernoulli distribution with parameter 0.5. Results computed over $5,000$ replications are displayed in Table 2. The unconditional test has good small sample power properties. For instance, with $p_n = 5$ the test rejects the null hypothesis 81% (resp. 98%) of time when the out-of-sample length n is equal to 500 (resp. 1000). The rejection frequencies are equal to one for the largest sample length ($n = 5000$) and this result holds whatever the value of the truncation parameter p_n. Although the conditional test has fairly good power properties, the reported rejection frequencies are lower to those for the unconditional test, especially for the smallest sample size ($n = 500$). This is expected because as underlined by Giacomini and White (2006), the inclusion of a conditioning variable that is either uncorrelated or weakly correlated with the loss functions will lower the power of the conditional test. This is the case here, since the state variable S_t is not correlated to the CoVaR forecasts.[4] Remark that in empirical applications, plausible conditioning variables are market state variables with predictive power on systemic risk, such as short-term treasury rate, the slope of the yield curve, corporate default spreads, etc. (Adrian and Brunnermeier, 2011). Hence, the case of a conditioning variable not related to the loss functions is for an illustration purpose. Our goal is to show that in this case, the conditional test would have less power. Hence, one should take care about the choice of the conditioning variables.

3.2.2. Experiment 2 - dependence between CoVaRs and S_t: The simulation setup is similar to the one described above, with the exception that we condition the difference in the loss functions to the state variable S_t which gives insight on the state of the economy. Formally, we suppose that the returns of each financial institution follows the GARCH(1,1) model in (17), with the calibrated values in (24). The n forecast values of VaR over the out-of-sample period are generated using (18), for each financial institution. The n forecast values of CoVaR for the financial institution i are obtained as

$$\text{CoVaR}^i_{q,t+1} = \begin{cases} \alpha^i_q + \beta^i_{1,q} VaR^i_{t+1}(q) & \text{if} \quad S_{t-1} = 1 \\ \alpha^i_q + \beta^i_{2,q} VaR^i_{t+1}(q) & \text{if} \quad S_{t-1} = 0, \end{cases} \tag{28}$$

with the state variable S_t following an *i.i.d.* Bernoulli distribution with parameter 0.5, and $\beta^i_{1,q} \neq \beta^i_{2,q}$. Hence, according to the state prevailing in the economy at date $t-1$, the responses of the CoVaR forecasts to the VaRs of the financial institution i at time t are different. For the financial institution j we simulate the n CoVaR forecasts similarly, yielding

$$\text{CoVaR}^j_{q,t+1} = \begin{cases} \alpha^j_q + \beta^j_{1,q} VaR^j_{t+1}(q) & \text{if} \quad S_{t-1} = 1 \\ \alpha^j_q + \beta^j_{2,q} VaR^j_{t+1}(q) & \text{if} \quad S_{t-1} = 0, \end{cases} \tag{29}$$

[4] This also explains why the conditional test is undersized in small samples (see Table 1).

where again $\beta^j_{1,q} \neq \beta^j_{2,q}$. Finally as above, the market returns used in the testing procedure are generated as the simple average of the returns consistent with the generated CoVaR forecasts (see equations (21) and (23)).

We simulate the empirical power under the assumption that the CoVaR forecasts for the two financial institutions are unconditionally equal, but each financial institution has statistically lower CoVaR forecasts in a given state of the economy. This simulation framework is of great importance, as it helps to show the relevance of the conditional test over its unconditional counterpart. To generate data consistent with this assumption, we consider that the returns of each financial institution follow the GARCH(1,1) model, and their respective CoVaR forecasts are such that

$$\left(\alpha^i_q, \beta^i_{1,q}, \beta^i_{2,q}\right) = (0.0, 0.4, 0.6)\,, \tag{30}$$

$$\left(\alpha^j_q, \beta^j_{1,q}, \beta^j_{2,q}\right) = (0.0, 0.6, 0.4)\,. \tag{31}$$

This means that in the first state of the economy, the CoVaR forecasts of the financial institution i is on average lower in absolute value than the CoVaR forecasts of the financial institution j. The reverse holds in the second state of the economy. Hence, conditionally to the state of the economy prevailing at time $t-1$, the two CoVaR forecasts at time t are statistically different. Nevertheless, since they dominate each other to the same extent across the two states, and the latters occur equally[5] through the out-of-sample period, they are unconditionally equal. Consequently, the unconditional test would fail to reject the null hypothesis, whereas the conditional test should do.[6] Table 3 displays the rejection frequencies of the two tests for $n \in \{500, 1000, 2000, 5000\}$ and for the nominal risk level $\eta = 5\%$. We observe that the conditional test has appealing power properties. For instance, the rejection frequencies of the null hypothesis are equal to 54% for $n = 500$ and 96% for $n = 1000$. Hence, the conditional test has power to detect differences in CoVaR forecasts in the different states, while the rejection frequencies for the unconditional test are close to the nominal risk level $\eta = 5\%$.

4. Empirical application

In this section, we apply our testing approach to a sample of financial institutions and analyze their statistical differences in term of CoVaR forecasts. Our goal is to identify the systemically important financial institutions, stressing the relevance of our testing procedure.

[5] The state variable S_t has indeed a Bernoulli distribution with success probability set to 0.5.

[6] This framework is adapted from Giacomini and White (2006).

4.1. Data

Our data set contains daily returns of 70 large U.S. financial institutions with a market capitalization greater than \$5 billion as of end of June 2007. The data set covers the period from June 2, 2003 to December 31, 2008, with a total of $T = 1408$ observations. This data set is a part of the larger data set used by Acharya et al. (2010) and Brownlees and Engle (2012) that includes 94 large U.S. financial institutions (depositories, insurance, broker-dealers and others). Among the 94 financial institutions considered by these authors, we exclude 24 institutions due to the lack of complete history. All data are extracted from CRSP. Table 4 presents the list of all of the 70 financial institutions.

4.2. The methodology

For each financial institution i under investigation, we use a rolling-window approach to produce one-step out-of-sample CoVaR forecasts. More precisely, we set the estimation sample length to $m = 504$, which corresponds to two years of daily data. The first m observations corresponding to returns data from June 2, 2003 to May 31, 2005 are used to generate the first out-of-sample CoVaR forecast for the financial institution i. The remaining CoVaR forecasts are computed moving each day the estimation sample, including a new data and dropping the earliest data, until the last observation. Therefore, we obtain for each financial institution i a time series of length $n = 904$ of CoVaR forecasts over the out-of-sample period ranging from June 1, 2005 to December 31, 2008.

To generate out-of-sample CoVaR forecasts, we rely on the parametric quantile regression. For the financial institution i the CoVaR forecast for time $t+1$ using the set of information available at time t is equal to

$$\text{CoVaR}^i_{q,t+1}\left(\widehat{\theta}_{i,t}\right) = \widehat{\alpha}^i_q + \widehat{\beta}^i_q VaR^i_{t+1}\left(q, \widehat{\eta}_i\right), \quad t = m, ..., T-1,$$

where $\widehat{\theta}_{i,t} = \left(\widehat{\alpha}^i_q, \widehat{\beta}^i_q, \widehat{\eta}^\prime_i\right)^\prime$ is the vector of estimated parameters, with $\widehat{\eta}_i$ a vector of parameters related to the estimation of the VaR of this financial institution. For the estimation of the latter, we rely on a semi-parametric model. We make the assumption that the returns of the financial institution i follows a GARCH(1,1) model

$$r_{i,t+1} = \sigma_{i,t+1}\varepsilon_{i,t+1}, \tag{32}$$

$$\sigma^2_{i,t+1} = \omega_{i,0} + \omega_{i,1} r^2_{i,t} + \omega_{i,2}\sigma^2_{i,t}, \tag{33}$$

$$\varepsilon_{i,t+1} \sim \text{m.d.s.}\left(0,1\right), \tag{34}$$

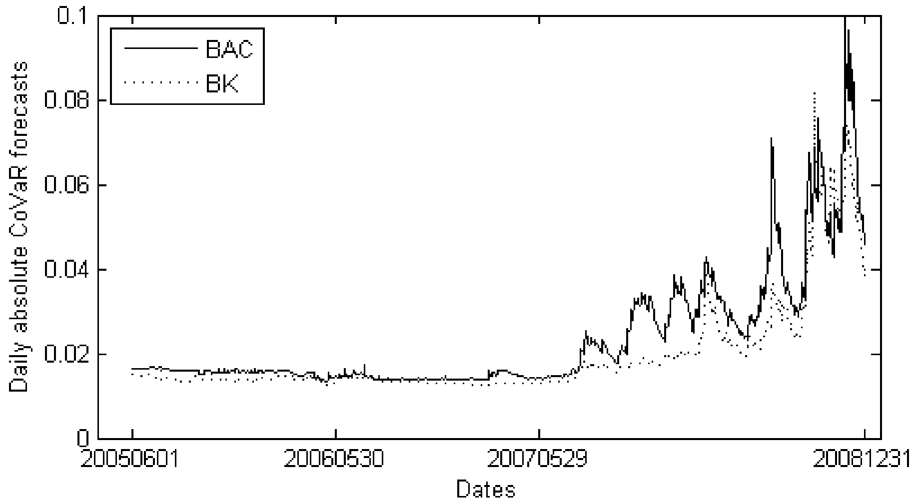
with $\eta_i = (\omega_{i,0}, \omega_{i,1}, \omega_{i,2})$. The GARCH model is estimated by quasi maximum likelihood and the forecast value of VaR for time $t+1$ is equal to

$$VaR^i_{t+1}\left(q, \widehat{\eta}_i\right) = \text{quantile}\left(\widehat{\varepsilon}, q\right)\widehat{\sigma}_{i,t+1}, \quad t = m, ..., T-1, \tag{35}$$

$$\widehat{\sigma}^2_{i,t+1} = \widehat{\omega}_{i,0} + \widehat{\omega}_{i,1} r^2_{i,t} + \widehat{\omega}_{i,2} \widehat{\sigma}^2_{i,t}, \tag{36}$$

where quantile$(\widehat{\varepsilon}, q)$ is the empirical quantile of order q of the estimated standardized innovations. Figure 1 displays over the out-of-sample period the profile of the n CoVaR forecasts in absolute value for two selected financial institutions: Bank of America (BAC) and Bank of New York Mellon (BK). We observe that daily CoVaR forecasts are highly dynamic and increase significantly (in absolute value) during the 2007-2008 financial crisis.

Figure 1 Absolute values of daily CoVaR forecasts for two selected financial institutions.



For each couple (i, j) of financial institutions, we test for the statistical equality of CoVaR forecasts using our testing approach. We use both the unconditional and the conditional versions of the test. Recall that the unconditional test is a Diebold-Mariano-type test that relies on the following test statistic

$$t_{m,n} = \frac{\overline{\Delta}\mathcal{L}_{m,n}}{\widehat{\sigma}/\sqrt{n}}, \tag{37}$$

with $\overline{\Delta}\mathcal{L}_{m,n}$ the average value of the differences in loss functions, and $\widehat{\sigma}$ its long-run standard deviation. This statistic follows under the null hypothesis of statistical equality between CoVaR forecasts a standard Gaussian distribution. As for the conditional version of the test, the appropriate test statistic is given by

$$T^h_{m,n} = n\overline{Z}'_{m,n}\widehat{\Omega}^{-1}_n\overline{Z}_{m,n},$$

with

$$\overline{Z}_{m,n} = n^{-1}\sum_{t=m}^{T-1} Z_{m,t+1} = n^{-1}\sum_{t=m}^{T-1} h_t \Delta\mathcal{L}_{t+1},$$

where $\Delta\mathcal{L}_{t+1}$ is the difference in loss functions at time $t+1$, h_t a given $d \times 1$ vector of conditioning variables at time t, and $\widehat{\Omega}$ the estimate of the long-run covariance matrix of $Z_{m,t+1}$. The test statistic $T_{m,n}^{h}$ has under the null hypothesis of statistical equality between CoVaR forecasts a chi-square distribution with d degrees of freedom. For our application of the conditional test, we set the vector h_t to $h_t = (1, z_t)$, where z_t is a vector of given market state variables at time t.[7] As market state variables, we consider four different variables with known predictive power on systemic risk as measured by CoVaR (see Adrian and Brunnermeier, 2011). The considered variables are:

- the daily values of the VIX. The VIX measures the implied volatility in the stock market. Daily data for the VIX were obtained from Bloomberg.
- the daily values of the slope of the yield curve. The slope of the yield curve is defined as the spread between ten-year and three-month Treasury constant maturity rates. This variable is a forward-looking indicator of real economic activity (Estrella and Trubin, 2006). Daily data for the ten-year and the three-month Treasury rates were obtained from the website of the Federal Reserve Bank of New York.
- the daily values of the default spread. The latter is defined as the yield differential between Moody's seasoned Aaa-rated and Baa-rated corporate bonds, and is an aggregate measure of credit risk not affected by bond market frictions such as taxes and liquidity (Chen et al., 2008). Daily data for the default spread were obtained from the website of the Federal Reserve Bank of New York.
- the daily values of the TED (Treasury-Eurodollar) spread. The TED spread is defined as the difference between the three-month LIBOR rate and the three-month Treasury bill rate. This variable is a measure of credit risk in the financial system. The historical daily values of the LIBOR were obtained from Economagic.

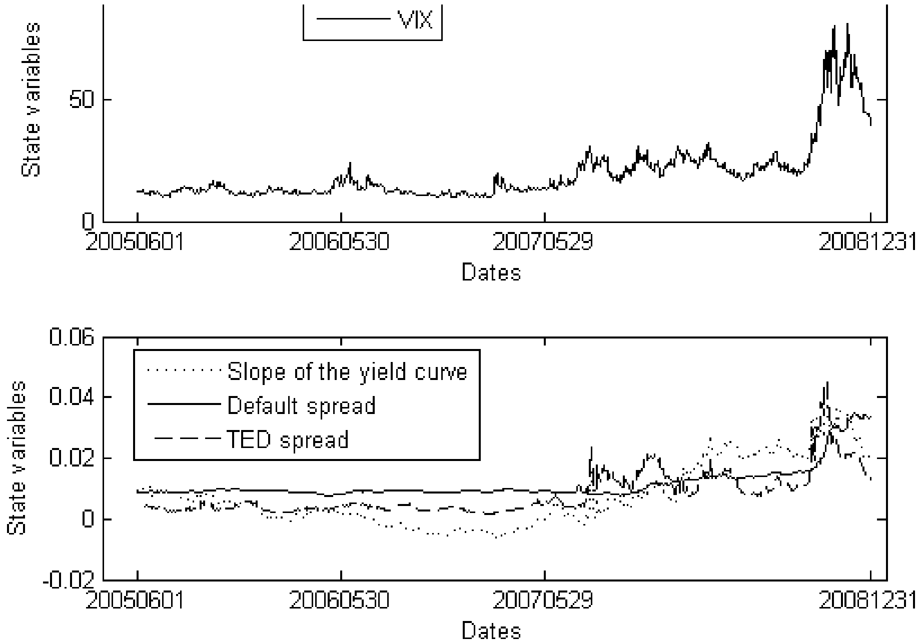
Figure 2 displays the time series behavior of the four variables over the out-of-sample period. We observe that over the 2007-2008 financial crisis period, the daily values of all four variables increase.

4.3. Results

In this section, we present the results of the tests (unconditional and conditional) when applied to each couple of financial institutions across our data set. We will show how these results can be used to identify systemically important financial institutions.

[7] To take into account possible dependence in the difference of loss functions, $\Delta\mathcal{L}_{t+1}$, we also include in z_t the lagged value of the latter, i.e., $\Delta\mathcal{L}_t$

Figure 2 Time series behavior of the four state variables over the out-of-sample period



4.3.1. Results of the unconditional test: Table 5 summarizes the outcomes of the unconditional test of equality between CoVaR forecasts. We divide the out-of-sample period into two parts and give results for the pre-crisis period (ranging from June 1, 2005 to June 29, 2007) and for the 2007-2008 crisis period (July 2, 2007 to December 31, 2008). The column labeled "%H1+" (resp. "%H1−") gives for a given financial institution i, the percentage of the other institutions j that are statistically less (resp. more) systemic than the financial institution i, with systemic risk measured by CoVaR forecasts. For an illustration, in the pre-crisis period, Bank of America (BAC) is statistically more systemic than 43 out of the 69 other financial institutions, which leads to the value of 62.32% for the column "%H1+". Symmetrically, this institution is statistically less systemic than only 1 out of the 69 other financial institutions, which corresponds to the proportion reported in the column "%H1−", that is 1.45%. The third column labeled "%diff" that gives the difference between the first and the second columns is an indicator of the systemic nature of an institution. Indeed, large positive (resp. negative) values correspond to more (resp. less) systemic institutions. Finally, the last column reports the ranking of the financial institutions from the most systemic (rank 1) to the least systemic. To save space we report results for the 10 most (resp. least) systemic institutions in Panel A (resp. B).

Over the pre-crisis period, results in Table 5 show that the top-five systemic institutions are Goldman Sachs (GS), Huntington Bancshares (HBAN), Eaton Vance (EV), Cincinnati Financial Corp. (CINF), and Northern Trust (NTRS). For the same period the least systemic institutions are Aon

Corp. (AOC), Berkshire Hathaway (BRK), TD Ameritrade (AMTD), Legg Mason (LM) and Aflac (AFL). For the crisis-period, the most systemic institutions are Lincoln National (LNC), Torchmark (TMK), Citigroup Inc. (C), New York Community Bancorp (NYB) and Principal Financial (PFG), whereas the least systemic are Humana (HUM), Aon Corp. (AOC), Unum Group (UNM), Berkshire Hathaway (BRK) and Huntington Bancshares (HBAN). For all of the 70 financial institutions, the correlation of the rankings for the two periods is equal to 38.59%. This suggests fairly significant changes in the relative systemic nature of these institutions.

Figure 3 **Scatter-plot of the ranking of systemic importance: pre-crisis versus crisis periods**

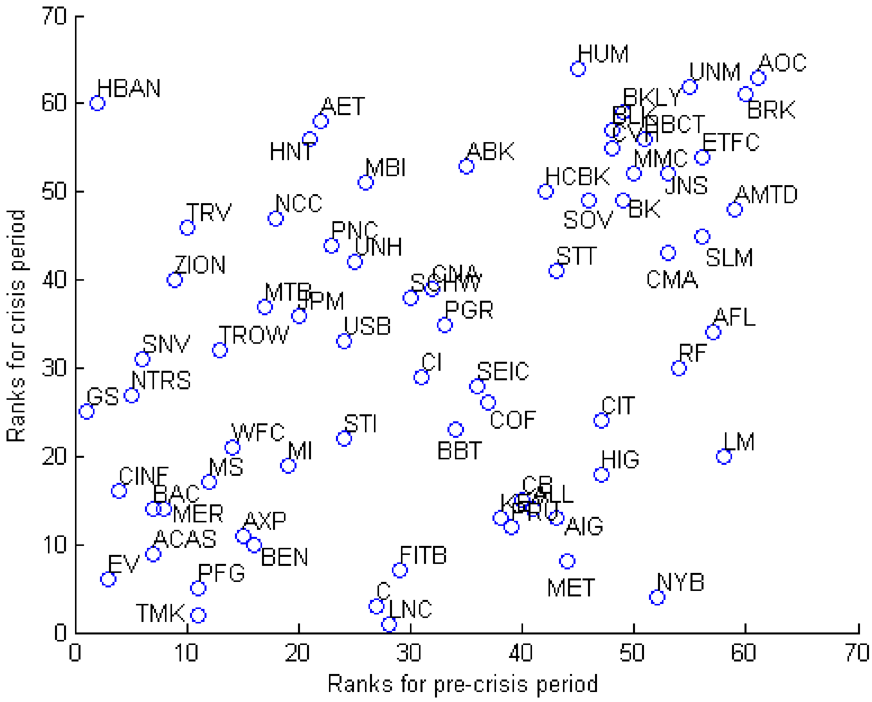


Figure 3 displays the scatter-plot of the ranks of the financial institutions for the two periods. The ranks over the pre-crisis (resp. crisis) period are on the horizontal (resp. vertical) axis. Financial institutions which are located in the south-west corner of the plot are those that remain the most systemic across the two periods. These include among others Eaton Vance (EV), Torchmark (TMK), Principal Financial (PFG) and American Capital (ACAS). Aon Corp. (AOC), Berkshire Hathaway (BRK) and Unum Group (UNM), to cite a few, located in the north-east corner are the least systemic financial institutions across the two periods. New York Community Bancorp (NYB), Metlife (MET) and American Inter. Group (AIG) appear as the institutions that are severely affected by the 2007-2008 financial crisis. Indeed, while the statistic %diff is equal to -47.83% for New York Community Bancorp in the pre-crisis period, the same statistic increases up to 66.67%

in the crisis period, with a switch of the ranking from 52 to 4. There are some financial institutions (HBAN for example) located in the north-west corner, which are less systemic (relatively to the other financial institutions) in the crisis period compared to the pre-crisis period.

Lastly, the last row of Table 5 labeled "All" displays for all couples of institutions the proportion of rejections of the null hypothesis of statistical equality between systemic risks. It can be considered as an overall measure of systemic risk heterogeneity across institutions. The reported values are equal to 57.47% and 54.28% for the pre-crisis and the crisis periods, respectively. Therefore from the viewpoint of systemic risk, these financial institutions are heterogeneous to the same extent over both periods (crisis and non-crisis).

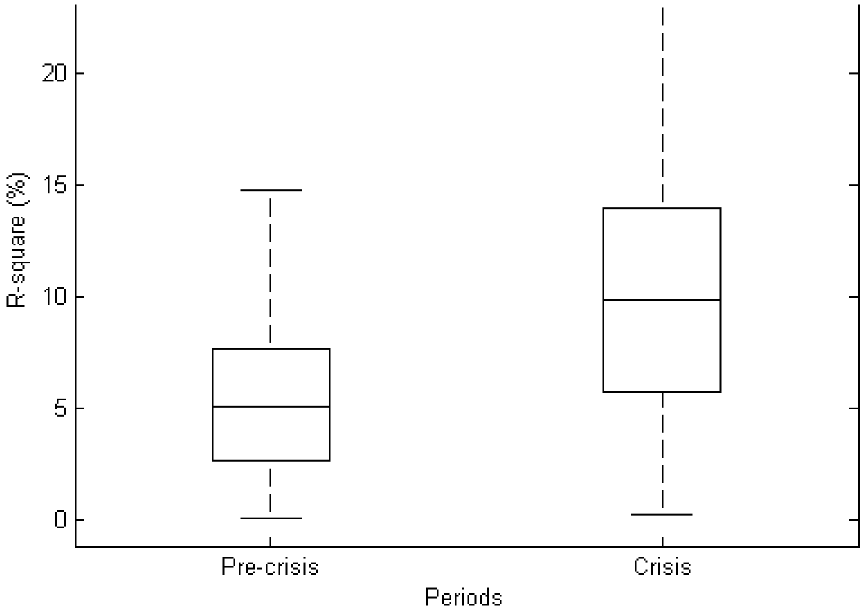
4.3.2. Results of the conditional test: Table 6 presents the results of our conditional test applied to each couple of financial institutions, where the conditioning variables are the market state variables displayed in Figure 2. The presentation is similar to Table 5. Focusing on the last row of the Table which reports the level of systemic risk heterogeneity across institutions (proportion of systemically different institutions), we observe that the value for the pre-crisis period is equal to 12.33% and is much lower than the one reported in Table 5 which is equal to 57.47%. Hence, the conditional test suggests less heterogeneity between the 70 institutions in the pre-crisis period. This means that at each date t across the pre-crisis period, the four market state variables have little power in explaining statistical divergences between the systemic risk profiles of the institutions. However, for the crisis period we obtain opposite results. Indeed, the level of systemic risk heterogeneity between institutions is equal to 64.51% for the conditional test, whereas for the unconditional test it is equal to 54.28%. Therefore, given the information conveyed by the market state variables over the crisis period, the conditional test is able to discriminate financial institutions much more in comparison to the unconditional test. To confirm this result, we run for each couple of financial institutions (i, j), $i \neq j$ the following time-series regression over the out-of-sample period

$$\Delta\mathcal{L}_{t+1} = \gamma_0 + \sum\nolimits_{k=1}^{4} \gamma_k X_{k,t} + u_t, \quad t = m, ..., T-1,$$

with $X_{k,t}$, $k = 1, ..., 4$, the value at time t of the four market state variables, and u_t the regression error term. Figure 4 gives the box-plot of the multiple R-square (in percentage) from all these regressions for the pre-crisis and the crisis periods. We observe that the explanatory power of the regressions over the crisis period is much higher than those over the pre-crisis period.

The difference between the two versions of the test can also be stressed comparing their respective ranking of the institutions. Figures 5 and 6 in Appendix display the scatter-plot of the ranks of the institutions for the pre-crisis and the crisis periods, respectively. The correlation between the

Figure 4 **Boxplot of the multiple R-square (in%) from the regression of loss function dierences on market state variables**



ranks is equal to 85% for the pre-crisis period, whereas it is equal to 86.4% for the crisis-period. Both correlations are statistically significant at the 5% level. In both Figures, institutions which are far below or above the 45-degree line are those that are differently ranked by the two tests. For instance, in Figure 6 (crisis period), Comerica Inc. (CMA) is identified as more systemically risky by the conditional test.

4.3.3. Performance Predictability of the testing approach As stressed by Sedunov (2011), an institution-level measure of systemic risk should be a good forecast of a financial institution's performance in crisis period. In other words, any consistent measure of the systemic risk profile of an institution should be an early-warning indicator of losses in the case of a systemic event. In this subsection, we check whether this characteristic is fulfilled by our measure of systemic risk given by the statistic %diff. More precisely, we focus on the crisis period, *i.e.*, the period ranging from July 2, 2007 to December 31, 2008, with a total of $T_c = 380$ daily observations. Over the crisis period, we compute for each financial institution its performance given by the average of negative or downside returns. For a given financial institution $i = 1, ..., 70$, the performance is given by

$$\text{Perf}_i = \frac{1}{m} \sum_{t=1}^{T_c} r_{i,t} Z_{i,t}, \quad t = 1, ..., T_c, \tag{38}$$

where $r_{i,t}$ is the equity return of the financial institution i at time t, $Z_{i,t}$ is the downside indicator variable at time t defined as

$$Z_{i,t} = \begin{cases} 1 & \text{if} \quad r_{i,t} < 0 \\ 0 & \text{else.} \end{cases} \tag{39}$$

The parameter m is the number of times $Z_{i,t}$ takes value one over the crisis period, *i.e.*,

$$m = \sum_{t=1}^{T_c} Z_{i,t}. \tag{40}$$

The performance measure in (38) gives the average value of the losses experienced by the financial institution i in the crisis period.[8] Since our goal in this section is to check whether institutions with high level of systemic risk perform more poorly (out-of-sample) than institutions with low level of systemic risk, we consider the following regression

$$\text{Perf}_i = \gamma_0 + \gamma_1 \%\text{diff}_i + u_i, \quad i = 1, ..., 70, \tag{41}$$

where $\%\text{diff}_i$ is the measure of systemic importance[9] (derived from our testing approach) for the financial institution i in the pre-crisis period ranging from June 1, 2005 to June 29, 2007. We extend this benchmark regression into two directions. First, remind that our inferential measure of institution-level systemic risk is built on the pitfall of the estimated CoVaRs in ranking institutions. Indeed, we argued in the line of Danielsson et al. (2011) that financial institutions ranking based on estimated CoVaRs are misleading due to estimation uncertainty. Therefore, to evaluate the relevance of this statement, we rather consider the regression

$$\text{Perf}_i = \gamma_0 + \gamma_1 \%\text{diff}_i + \gamma_2 \text{CoVaR}_i + u_i, \tag{42}$$

with CoVaR_i the average value of CoVaR forecasts over the pre-crisis period. Second we further extend the regression (42) to avoid misspecification, including two firm characteristics (leverage and size) which are known as relevant factors for the cross-sectional variations in systemic risk. For instance, leverage defined as the relative amount of debt versus equity is related to systemic risk, because when a shock hits an institution, the ability of the latter to contain the adverse effect of the shock is determined by how much debt it has relative to equity. Institutions with higher leverage ratio will struggle to manage the consequences of the shock due to the lack of sufficient equity. The associated financial distress can impact other financial institutions leading to a global

[8] We focus here only on the downside performance, as CoVaR is about events in the left tail distribution both for the financial institutions and the system.

[9] We will consider the two versions of this statistic, separately. The one based on the unconditional version of our testing procedure, and the one derived from the conditional version.

systemic event. In the same line, the size of a firm related to the notion of "too big to fail" is a key component of systemic risk, because the distress of a large firm is likely to impact the system as whole. Following Acharya et al. (2010), we approximate the leverage by the ratio of quasi-market value of assets to market value of equity. The data available at a quarterly frequency from CRSP-Compustat merged dataset are averaged over the pre-crisis period. As for the size, we measure it by the natural logarithm of total book assets, and consider the average value over the pre-crisis period. Lastly, we also include in the regression industry dummies (insurance, broker-dealers and others).

Table 7 displays the results of the regressions. We consider four (04) different estimations. First results of regressions (2) and (3) show that our statistic %diff obtained from the conditional test is statistically significant at the 5% nominal risk level, whereas the same statistic computed using the unconditional test is not significant. In the regression (2) the adjusted R-square is equal to 15.67%, whereas it is equal to 17.56% in the regression equation (3). Figure 7 in Appendix which displays the scatter plot of the conditional version of %diff versus the performance measure (and the associated least squares regression line) shows that higher values of the former statistic are associated with worse performances in the crisis-period. Second, outcomes from the regression (1) indicate that evaluating the systemic nature of financial institutions using CoVaR forecasts directly is not useful, since the latters have no predictive power on an institution's performance in crisis period. The associated coefficient is not significant, and the adjusted R-square is the smallest. Lastly, the regression (4) that includes all regressors confirms all these findings. Remark that while the leverage appears highly significant in predicting an institution average loss in crisis period, the size is insignificant in all regressions. Moreover, the dummy 'Others' is significant in all regressions, implying that ceteris paribus the performance is on average lower for this group.

5. Conclusion

The recent financial crisis has reinforced the need for a better regulation of financial institutions. One of the major aspects of the new face of the regulation is the imposition of additional loss absorbency for the so-called systemically important financial institutions (SIFIs), which should be related to the level of systemic risk. In this paper, we propose a testing procedure useful for the identification and ranking of SIFIs. Our test based on the conditional predictive ability (CPA) test of Giacomini and White (2006) compares the difference in systemic risk, as measured by CoVaR (Adrian and Brunnermeier, 2011), for two financial institutions via an economic loss function. Applied to all couples of financial institutions in a system, one can derive a simple and intuitive statistic that helps ranking SIFIs.

Unlike the naive approach that consists in identifying and ranking SIFIs using estimated CoVaRs, our testing approach has two main advantages. First, as the test is built on the CPA framework of

Giacomini and White (2006), it is robust to estimation uncertainty. Second, the test is conditional and helps incorporating the role of market state variables (VIX and various yield spreads) in the statistical differences in systemic risks. In other words, rejection of the null hypothesis will occur because market state variables help predict out-of-sample the difference in systemic risks as measured by CoVaR. An unconditional version of the test is also considered. We show via Monte Carlo simulations that the test has good small sample properties.

We conduct an empirical application using a sample of 70 U.S. financial institutions, and observe two things. First, the systemic profiles of these institutions derived from our conditional test reveal more (resp. less) heterogeneity, compared to the unconditional test, in crisis (resp. non-crisis) period. Hence, over the pre-crisis (resp. crisis) period, market state variables have little (resp. high) explanatory power for the statistical divergences between the systemic risk profiles of the institutions. Second, it appears that our inferential procedure to identify SIFIs leads to a ranking which appears as an early-warning indicator of losses in crisis period. More precisely, institutions which are identified as systemically risky in the pre-crisis period are those that experienced large losses in the crisis period.

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Appendix. Tables and Figures

Figure 5 Scatter-plot of the ranking of systemic importance over the pre-crisis period: unconditional test versus conditional test

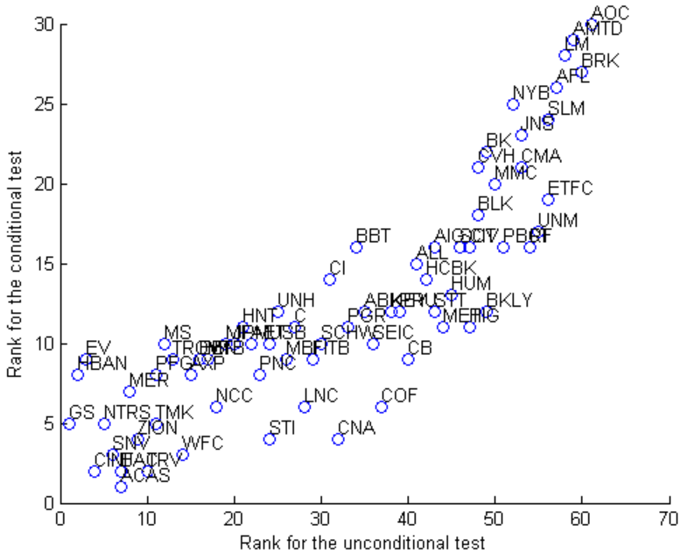

Figure 6 Scatter-plot of the ranking of systemic importance over the crisis period: unconditional test versus conditional test

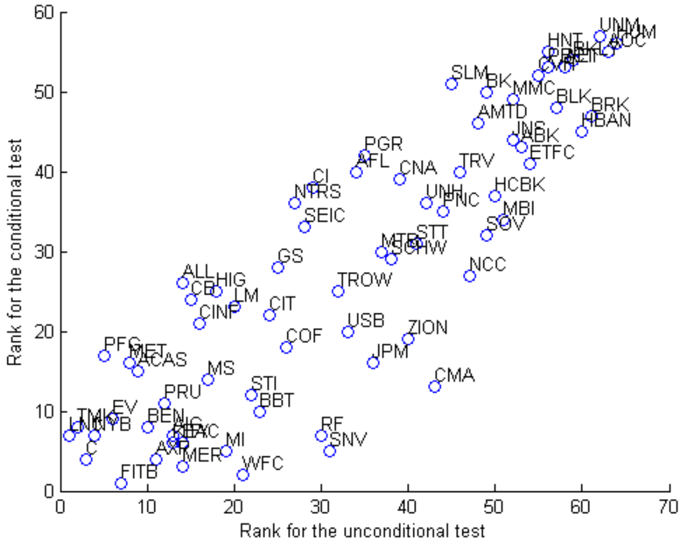

Figure 7 **Scatter-plot of performance in crisis period versus %diff (conditional test) in pre-crisis period**

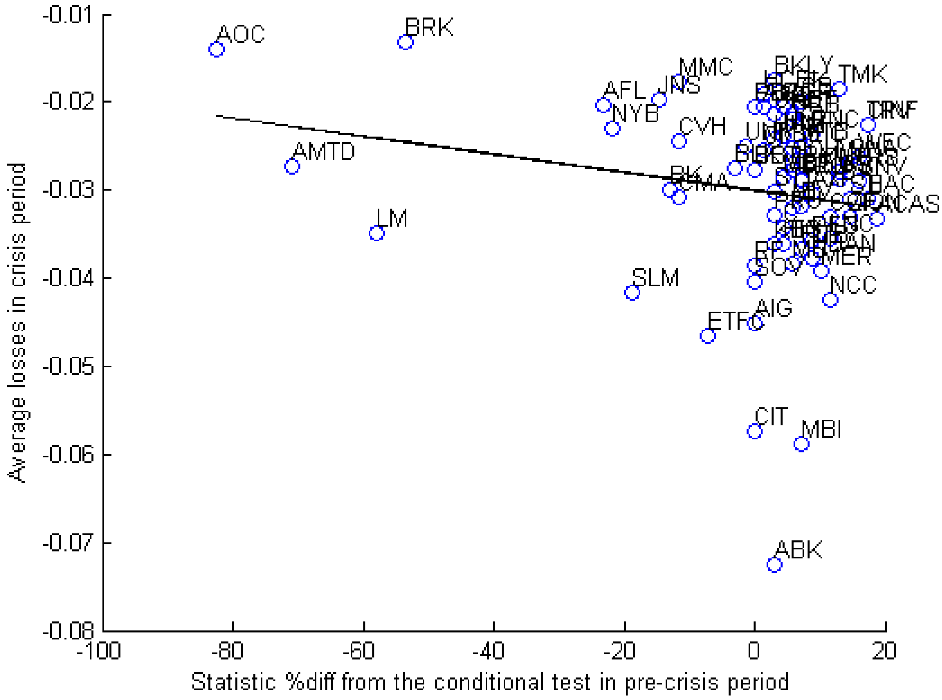

Table 1 **Empirical sizes of the tests of equality between CoVaR forecasts**

	$n=500$	$n=1000$	$n=2000$	$n=5000$
	Panel A: Conditional test			
	0.0278	0.0326	0.0450	0.0556
	Panel B: Unconditional test			
$p_n=0$	0.0316	0.0500	0.0616	0.0668
$p_n=5$	0.0304	0.0436	0.0550	0.0560
$p_n=10$	0.0306	0.0418	0.0510	0.0528
$p_n=15$	0.0326	0.0416	0.0498	0.0510
$p_n=20$	0.0344	0.0424	0.0502	0.0490
$p_n=25$	0.0372	0.0448	0.0494	0.0498
$p_n=50$	0.0500	0.0514	0.0536	0.0492

Notes: The table displays the empirical sizes of the test of equality between the predicted values of CoVaR for two financial institutions. Results are reported for both the unconditional and the conditional versions of the test. Rejection frequencies are reported over 5000 simulations for the nominal risk levels $\eta = 5\%$, with n the out-of-sample size. For the unconditional test, different values of the truncation parameter p_n are considered. The conditional test is implemented using as conditioning variable a random i.i.d. Bernoulli variable which is independent to the CoVaR forecasts.

Table 2 Empirical powers of the tests of equality between CoVaR forecasts: simulations under experiment 1

	$n=500$	$n=1000$	$n=2000$	$n=5000$
	Panel A: Conditional test			
	0.5498	0.9638	0.9998	1.0000
	Panel B: Unconditional test			
$p_n=0$	0.8196	0.9878	0.9998	1.0000
$p_n=5$	0.8112	0.9854	0.9996	1.0000
$p_n=10$	0.8092	0.9840	0.9996	1.0000
$p_n=15$	0.8084	0.9820	0.9996	1.0000
$p_n=20$	0.8098	0.9824	0.9994	1.0000
$p_n=25$	0.8086	0.9818	0.9994	1.0000
$p_n=50$	0.8170	0.9798	0.9994	1.0000

Notes: The table displays the empirical powers of the test of equality between the predicted values of CoVaR for two financial institutions. Results are reported for both the unconditional and the conditional versions of the test. Rejection frequencies are reported over 5000 simulations for the nominal risk levels $\eta=5\%$, with n the out-of-sample size. For the unconditional test, different values of the truncation parameter p_n are considered. The conditional test is implemented using as conditioning variable a random i.i.d. Bernoulli variable which is independent to the CoVaR forecasts.

Table 3 Empirical powers of the tests of equality between CoVaR forecasts: simulations under experiment 2

	$n = 500$	$n = 1000$	$n = 2000$	$n = 5000$
	Panel A: Conditional test			
	0.5426	0.9680	1.0000	1.0000
	Panel B: Unconditional test			
$p_n = 0$	0.0468	0.0536	0.0558	0.0636
$p_n = 5$	0.0450	0.0514	0.0518	0.0588
$p_n = 10$	0.0468	0.0506	0.0516	0.0568
$p_n = 15$	0.0508	0.0514	0.0516	0.0566
$p_n = 20$	0.0534	0.0540	0.0518	0.0558
$p_n = 25$	0.0568	0.0554	0.0522	0.0564
$p_n = 50$	0.0726	0.0622	0.0550	0.0560

Notes: The table displays the empirical powers of the test of equality between the predicted values of CoVaR for two financial institutions. Results are reported for both the unconditional and the conditional versions of the test. Rejection frequencies are reported over 5000 simulations for the nominal risk levels $\eta = 5\%$, with n the out-of-sample size. For the unconditional test, different values of the truncation parameter p_n are considered. The conditional test is implemented using as conditioning variable a random i.i.d. Bernoulli variable which is dependent to the CoVaR forecasts.

Table 4 Tickers and Company Names by Sector Groups

Depositories (24)		Insurance (27)		Broker-Dealers (6)		Others (13)	
BAC	Bank of America Corp.	ABK	Ambac Financial Group	EFTC	E*Trade Financial	ACAS	American Capital
BBT	B B & T Corp.	AET	Aetna	GS	Goldman Sachs	AMTD	TD Ameritrade
BK	Bank New York Inc.	AFL	AFLAC	MER	Merill Lynch	AXP	American Express
C	Citigroup Inc.	AIG	American Inter. Group	MS	Morgan Stanley	BEN	Franklin Resources
CMA	Comerica Inc.	ALL	Allstate Corp.	SCHW	Nymex Holdings	BLK	BlackRock
HBAN	Huntington Bancshares	AOC	Aon Corp.	TROW	T. Rowe Price	CIT	CIT Group
HCBK	Hudson City Bancorp	BKLY	W.R. Berkley Corp.			COF	Capital One Financial
JPM	JPMorgan Chase & Co.	BRK	Berkshire Hathaway			EV	Eaton Vance
KEY	Keycorp New	CB	Chubb Corp.			FITB	Fifth Third Bancorp
MI	Marshall & Ilsley	CI	CIGNA Corp.			JNS	Janus
MTB	M&T Bank Corp.	CINF	Cincinnati Financial Corp.			LM	Legg Mason
NCC	National City Corp.	CNA	CNA Financial Corp.			SEIC	SEI Investment Comp.
NTRS	Northern Trust	CVH	Coventry Health Care			SLM	SLM Corp.
NYB	New York Community Bancorp	HIG	Hartford Financial Group				
PBCT	Peoples United Financial	HNT	Health Net				
PNC	PNC Financial Services	HUM	Humana				
RF	Regions Financial Corp. New	LNC	Lincoln National				
SNV	Synovus Financial	MBI	MBIA				
SOV	Sovereign Bancorp	MET	Metlife				
STI	Suntrust Banks	MMC	Marsh & McLennan				
STT	State Street	PFG	Principal Financial				
USB	US Bancorp	PGR	Progressive				
WFC	Wells Fargo & Co	PRU	Prudential Financial				
ZION	Zions	TMK	Torchmark				
		TRV	Travelers				
		UNH	UnitedHealth Group				
		UNM	Unum Group				

Table 5 Results of the unconditional test of equality between CoVaR forecasts over the out-of-sample period

	Pre-crisis period					Crisis-period			
	%H1+	%H1−	%diff	rank		%H1+	%H1−	%diff	rank
					Panel A : the 10 most systemic institutions				
GS	88.41	0.00	88.41	1	LNC	84.06	0.00	84.06	1
HBAN	75.36	0.00	75.36	2	TMK	71.01	0.00	71.01	2
EV	73.91	0.00	73.91	3	C	69.57	0.00	69.57	3
CINF	71.01	0.00	71.01	4	NYB	66.67	0.00	66.67	4
NTRS	69.57	0.00	69.57	5	PFG	65.22	0.00	65.22	5
SNV	66.67	0.00	66.67	6	EV	65.22	1.45	63.77	6
BAC	62.32	1.45	60.87	7	FITB	60.87	0.00	60.87	7
ACAS	60.87	0.00	60.87	7	MET	60.87	1.45	59.42	8
MER	57.97	0.00	57.97	8	ACAS	57.97	0.00	57.97	9
ZION	59.42	1.45	57.97	8	BEN	59.42	1.45	57.97	10
					Panel B : the 10 least systemic institutions				
JNS	7.25	57.97	−50.72	53	PBCT	2.90	66.67	−63.77	56
RF	4.35	57.97	−53.62	54	HNT	2.90	66.67	−63.77	56
UNM	2.90	62.32	−59.42	55	BLK	2.90	68.12	−65.22	57
ETFC	4.35	76.81	−72.46	56	AET	4.35	71.01	−66.67	58
SLM	1.45	73.91	−72.46	56	BKLY	1.45	73.91	−72.46	59
AFL	2.90	76.81	−73.91	57	HBAN	0.00	75.36	−75.36	60
LM	2.90	84.06	−81.16	58	BRK	1.45	78.26	−76.81	61
AMTD	1.45	91.30	−89.86	59	UNM	0.00	81.16	−81.16	62
BRK	0.00	95.65	−95.65	60	AOC	0.00	88.41	−88.41	63
AOC	0.00	98.55	−98.55	61	HUM	0.00	95.65	−95.65	64
ALL		57.47					54.28		

Notes: The column labeled "%H1+" (resp. "%H1-") gives for a given financial institution i, the percentage of the other institutions j that are statistically less (resp. more) systemic than the financial institution i, with systemic risk measured by CoVaR forecasts. The third column that gives the difference between the first and the second columns is an indicator of the systemic nature of an institution. Large positive (resp. negative) values correspond to more (resp. less) systemic institutions. The last column reports the ranking of the financial institutions from the most systemic (1) to the least systemic. We give results for the pre-crisis period (ranging from June 1, 2005 to June 29, 2007) and for the 2007-2008 crisis period (July 2, 2007 to December 31, 2008). The last row labeled "All" displays for all couples of institutions the proportion of rejections of the null hypothesis of statistical equality between systemic risks. It can be considered as an overall measure of systemic risk heterogeneity across institutions.

Table 6 Results of the conditional test of equality between CoVaR forecasts over the out-of-sample period

	Pre-crisis period					Crisis-period			
	%H1+	%H1−	%diff	rank		%H1+	%H1−	%diff	rank
					Panel A : the 10 most systemic institutions				
ACAS	18.84	0.00	18.84	1	FITB	72.46	1.45	71.01	1
BAC	17.39	0.00	17.39	2	WFC	72.46	5.80	66.67	2
CINF	17.39	0.00	17.39	2	MER	69.57	2.90	66.67	3
TRV	17.39	0.00	17.39	2	C	57.97	0.00	57.97	4
SNV	15.94	0.00	15.94	3	AXP	57.97	0.00	57.97	4
WFC	15.94	0.00	15.94	3	MI	56.52	0.00	56.52	5
STI	14.49	0.00	14.49	4	SNV	63.77	7.25	56.52	5
ZION	14.49	0.00	14.49	4	BAC	53.62	0.00	53.62	6
CNA	14.49	0.00	14.49	4	KEY	56.52	2.90	53.62	6
NTRS	13.04	0.00	13.04	5	NYB	50.72	0.00	50.72	7
					Panel B : the 10 least systemic institutions				
CVH	1.45	13.04	−11.59	21	BK	10.14	75.36	−65.22	50
BK	2.90	15.94	−13.04	22	SLM	10.14	79.71	−69.57	51
JNS	1.45	15.94	−14.49	23	CVH	5.80	76.81	−71.01	52
SLM	2.90	21.74	−18.84	24	PBCT	4.35	76.81	−72.46	53
NYB	4.35	26.09	−21.74	25	AET	4.35	76.81	−72.46	53
AFL	0.00	23.19	−23.19	26	BKLY	2.90	76.81	−73.91	54
BRK	0.00	53.62	−53.62	27	AOC	1.45	88.41	−86.96	55
LM	0.00	57.97	−57.97	28	HNT	4.35	91.30	−86.96	55
AMTD	0.00	71.01	−71.01	29	HUM	1.45	92.75	−91.30	56
AOC	0.00	82.61	−82.61	30	UNM	0.00	98.55	−98.55	57
ALL		12.33					64.51		

Notes: The column labeled "%H1+" (resp. "%H1-") gives for a given financial institution i, the percentage of the other institutions j that are statistically less (resp. more) systemic than the financial institution i, with systemic risk measured by CoVaR forecasts. The third column that gives the difference between the first and the second columns is an indicator of the systemic nature of an institution. Large positive (resp. negative) values correspond to more (resp. less) systemic institutions. The last column reports the ranking of the financial institutions from the most systemic (1) to the least systemic. We give results for the pre-crisis period (ranging from June 1, 2005 to June 29, 2007) and for the 2007-2008 crisis period (July 2, 2007 to December 31, 2008). The last row labeled "All" displays for all couples of institutions the proportion of rejections of the null hypothesis of statistical equality between systemic risks. It can be considered as an overall measure of systemic risk heterogeneity across institutions.

Table 7 Performance Predictability in crisis period

	(1)	(2)	(3)	(4)
constant	−0.0129	−0.0220***	−0.0224***	−0.0585
	(−0.6554)	(−5.9397)	(−6.2229)	(−1.3461)
Dummy				
insurance	−0.0002	−0.0003	−0.0005	−0.0001
	(−0.1018)	(−0.1110)	(−0.1970)	(−0.0604)
broker-dealers	−0.0012	−0.0012	−0.0014	−0.0021
	(−0.3190)	(−0.3197)	(−0.3956)	(−0.6560)
others	−0.0073***	−0.0075***	−0.0079***	−0.0081***
	(−2.6209)	(−2.6547)	(−2.7970)	(−2.7302)
leverage	−0.0014***	−0.0014***	−0.0013***	−0.0013***
	(−4.8588)	(−4.8900)	(−4.7094)	(−4.7012)
size	0.0000	0.0000	0.0001	0.0002
	(0.0542)	(0.0385)	(0.1470)	(0.2231)
CoVaR	0.6456			−2.5215
	(0.4778)			(−0.8001)
%diff (unconditional)		−0.0010		−0.0003
		(−0.5203)		(−0.0567)
%diff (conditional)			−0.0078**	−0.0139**
			(−2.0850)	(−2.3484)
Adj. R^2	15.63%	15.67%	17.56%	15.91%
#obs	70	70	70	70

Notes: The table displays results for the regressions of the performance indicator in crisis period on some systemic risk measures and firms characteristics (leverage and size). %diff refers to our systemic risk indicator derived from the unconditional or conditional tests in the pre-crisis period. CoVaR is the average value of CoVaR forecasts over the pre-crisis period. We also consider three industry dummies (insurance, broker-dealers and others). The pre-crisis period ranges from June 1, 2005 to June 29, 2007, and the crisis period from July 2, 2007 to December 31, 2008. Adj. R^2 is the adjusted R-square. ***, **, * mean statistically significant at 1%, 5% and 10%, respectively.