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Quorum Rules and Shareholder Power*

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Abstract

This paper characterizes the equilibria of a costly voting game in which shareholders heterogeneous in both size and preferences strategically vote for or against a proposed resolution or withhold. It is shown that a minimum quorum generates (1) equilibria in which one or several shareholders form voting coalitions in favor of the resolution that is adopted (2) an equilibrium in which shareholders strategically abstain from voting and the resolution is rejected. The size of blockholders and their preferences (in favor or against the resolution) play a crucial role in the existence of equilibria, their nature, the size and the number of voters in coalitions. We derive conditions under which the dominant shareholder controls the meeting. We also examine how large shareholders influence the result of the vote. In particular, we analyze the interaction between blockholders and discuss the situations in which large shareholders jointly control annual meetings or form coalitions to counter the dominant shareholder.

Keywords: Shareholder Meeting, Strategic voting, Coalitions, Quorum rule, Dominant, Controlling and Reference shareholders.

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1 Introduction

Recent years have seen regulatory developments aiming at increasing the power of shareholders in annual meetings, balancing what is sometimes considered a board-centric governance model.

Access to information before and after such a meeting has been made easier. The "record date" is now the rule in most countries. Under this rule, persons who can demonstrate they are shareholders at a specified date – usually three days before the meeting – can take part in the vote and are not required to block their shares. Voting has also been facilitated through the development of electronic voting and the use of proxy advisors' services. Overall, the direct and indirect costs (finding the relevant information and losing liquidity around the date of the meeting) of exercising voting rights have decreased.

The scope of resolutions that have to be put to a vote has widened. The Say On Pay rule, now effective in many countries, is an example. It requires public companies to provide their shareholders with an advisory or binding vote on the compensation of the most highly compensated executives.

Although the objective of "empowering" shareholders is advocated by many practitioners and academics (see for example Bebchuk (2005)), the relevance of a shareholder democracy is questioned on two different grounds. First, shareholders are presumably less informed than directors and may make decisions contrary to their own collective interest. Second, shareholders are heterogeneous. Under conflicts of interest between shareholders, the outcome of the vote possibly contradicts the majority view. When turnout is low, a majority of favorable votes may actually represent less than a majority of capital.

For these reasons, a minimum quorum is generally required for all or some decisions (typically special or extraordinary resolutions). Until 2013, the minimum quorum requirement was at least 50% for companies listed on the New York Stock Exchange (NYSE). Noting that listed companies are subject to quorum requirements under the laws of their state of incorporation and that bylaws frequently include more stringent voting requirements

imposed by state law, the NYSE removed the requirement. However, it gives "careful consideration to provisions in a listed company's bylaws that fixes a quorum for a shareholders' meeting at less than a majority of the outstanding shares".[1]

In Europe, the legal quorum varies across countries. Respondents to a consultation by the European Commission[2] suggested that it should be identical across the EU. In France, it amounts to 25% of capital for special general meetings (first call, 20% on the second call). In the United Kingdom, two shareholders – regardless of the level of their aggregate share ownership – are a quorum. There is no legal quorum in Germany. However, companies generally introduce provisions in their articles requiring a minimum quorum. In many countries (e.g. Italy, the United Kingdom, etc.), quorum rules are subject to the provisions of the company's articles and to the provisions of the law. The quorum defined in the corporate charter may be modified in the meetings.

In this paper, we study how a quorum rule impacts the outcome of annual meetings. We focus on the case in which shareholders differ both in size and opinion, incur a (small) voting cost but have access to the same information. Our results show that requiring a quorum, in addition to a majority, has important consequences for the results of the vote.

The introduction of a minimum quorum increases both the number and types of Nash equilibria. First, it generates an equilibrium in which shareholders strategically do not vote and the resolution is not adopted due to the lack of a quorum. Thus, the rule does not necessarily increase voter turnout. Second, the minimum quorum creates an incentive for coalition formation of shareholders voting in favor of the resolution to reach the quorum. Depending on the size of the largest opponent, the voting coalition comprises

[1] See SEC (Release No.34-69970, July 11, 2013). The NYSE does not list a company with a quorum requirement of less than one-third of outstanding shares, which is the required quorum under NASDAQ rules.

[2] See the Synthesis of the comments on the third consultation document of the internal market and services Directorate General, 2007, "Fostering an appropriate regime for shareholders' rights".

exclusively blockholders or possibly very small shareholders.

Quorum rules supposedly increase the representativity of the vote. The legitimacy of the meeting may be questioned when the result is controlled by the largest or dominant shareholder, possibly in contradiction to the preference of the majority. We show that a resolution supported by the largest or dominant shareholder is systematically adopted if his ownership reaches the quorum. In all other cases, the result of the vote may oppose the preference of the largest shareholder. Moreover, the outcome – whether rejection or adoption of the resolution – does not need to coincide with the preference of the majority.

The two opposing results may coexist for high values of the minimum quorum. To better predict the result of the meeting, we apply the coalition-proof refinement from Bernheim, Peleg and Whinston (1987), which selects Nash equilibria immunized from a mutually beneficial deviation by a coalition of shareholders. Indeed, receiving the agenda of the meeting sufficiently in advance, shareholders – especially blockholders who know each other – have the opportunity to discuss the meeting's items. Agreement on a voting strategy can reasonably be expected from coalitions of shareholders before the meeting. The coalition-proof equilibria in which the resolution is accepted exhibit a nice property because they demand a majority of favorable shareholders, in line with the objective of increasing the representativity of the vote through a minimum quorum. In contrast, rejection may occur despite a favorable majority.

In this setting, equilibria in which resolutions are not adopted exist under restrictive conditions. As a consequence, it is easier for a "dissident" shareholder to propose and pass a shareholder resolution than to oppose a board resolution. Shareholder proposals are therefore an important component of corporate governance.

Finally, we study in greater depth the role of large shareholders in meetings. We provide natural interpretations of the notions of controlling and reference shareholders in this context. Although often used in practice, they lack precise definitions, and we contribute to fill the gap. Specifically, control is defined as the power to pass and to block resolutions. We derive sufficient

conditions related to the ownership structure for control by the dominant shareholder alone or necessarily involving an alliance of shareholders. Furthermore, we identify the conditions under which a coalition of shareholders can successfully counter the power of the dominant one. Reference shareholders are usually vaguely defined as large enough to influence firms' strategies. In the context of annual meetings, we thus describe precisely how this influence can be exerted over the outcome of the vote.

The paper is organized as follows. We present the related literature in section 2. Section 3 describes the basic model, and section 4 details shareholders' equilibrium strategies. Section 5 analyzes the consequences of a quorum rule, fully characterizes the equilibria of the voting game and discusses the role of large shareholders. In section 6, we define reference, dominant and controlling shareholders within our framework. Section 7 concludes.

2 Related literature

Many collective decisions, such as those made in shareholder meetings, are made through a vote on two alternatives. Referenda are important examples. A satisfactory voting process addresses two important issues. First, when participants do not share the same objective, the result of the vote is expected to reflect the view of the majority. Second, when the participants have access to different information but share the same objective, the voting mechanism should favor the best decision given the information structure.

The literature on voting in public economics deals with both issues. In particular, it questions the efficiency of rules aiming at encouraging participation for better representativity, such as quorum rules. The existing but very limited theoretical analysis of voting in shareholder meetings largely draws on this literature.

Quorum rules

High participation presumably increases the legitimacy of the vote, particularly when electors disagree on the best alternative. In this regard, the

observed turnout in elections, although positive, is often considered too low.[3]

Minimum quorums are widely used in general meetings, committees or referenda with the objective of increasing participation. The reform is adopted if it obtains a majority of votes and the quorum is met; otherwise, the status quo prevails. The advocates of the rule argue that it constitutes a protection against the risk that an active minority imposes its view on a passive majority. A few articles analyze the consequences of the introduction of quorum rules on voter behavior, turnout and the outcome of the vote in referenda.

Herrera and Mattozzi (2010) develop a simple group turnout model where one party supports the reform while the other is for the status quo. Both spend campaign funds to mobilize voters. The two parties interact strategically when deciding how much to spend, given that the participation of the two groups of citizens simply increases with their party's spending. They find that the equilibrium expected turnout may actually be lower with a participation quorum than without it. Indeed, the reform party has an incentive to mobilize voters to push the turnout above the quorum threshold, but the status quo party has little or no incentive to do so, as it may win through a lack of participation. This asymmetry distorts the outcome away from the preference of the majority. However, it does not necessarily favor the status quo, as the demobilization of opponents may result in an increased probability that the reform is adopted.

Aguiar-Conraria and Magalhães (2010a) analyze the effect of a participation quorum on the incentive to vote in a simple model with no voting cost. While favorable electors unambiguously vote in favor, opposed electors choose to withhold if they believe there is a marked majority of electors in favor but a high probability that the turnout will be below the quorum; therefore, opposed electors have a chance to swing the election by not voting. Therefore, they predict that the turnout will be lower with a minimum quorum. These theoretical results are tested using data for all (99) referenda held in the EU from 1970 to 2007. Quorums are found to increase abstention

[3]Note however that, on one hand, even with a small voting cost, most people should not vote since the probability of affecting the result in large elections is infinitesimal. On the other hand, with no voting cost, all electors should vote for their preferred alternative.

by 11%.

Voters are not strategic in the aforementioned models. Building on Börgers (2004)[4], Aguiar-Conraria and Magalhães (2010b) analyze the consequences of a quorum in a pivotal voter model with costly voting. Citizens rationally anticipate that their vote will be pivotal and cast a vote if the expected benefit outweighs the cost of voting. Relying on numerical methods under various scenarios, the authors reach conclusions similar to Herrera and Mattozzi (2010): Quorum requirements possibly reduce the turnout since the opponents of the reform have more incentive to withhold, they do not introduce a systematic bias for the status quo, and they may favor active minorities against a passive majority.[5]

Following Côrte-Real and Pereira (2004), another strand of the literature analyzes the welfare properties of different quorum requirements.[6] Their main result is an impossibility one: no rule can ensure an accurate representation of the preferences of the citizens while abstention is possible unless restrictive assumptions are made on the preferences of abstainers.

Voting in shareholder meetings

The annual meeting is the only regular occasion in which all shareholders have the opportunity to express themselves directly on important issues regarding the company. The other case in which they have a direct say is a tender offer, which is a rare and disruptive event in the company's life. Despite the importance of shareholder voting for corporate governance, most studies are empirical, and theoretical analyses of annual meetings remain very scarce.

[4]Börgers (2004) shows that different voting costs may explain the voting strategy of electors who disagree on the best alternative in the case of small electorates. Participation is then driven by the probability of being pivotal. Under the simple majority rule, he concludes that all, part, or none of the electors vote in equilibrium depending on the voting cost. This is an explanation to the observed positive but limited turnout in elections.

[5]In a closely related paper, Hizen and Shinmyo (2011) characterize the Bayesian Nash equilibria of a pivotal model in a referendum with a participation quorum. They provide numerical examples that show the same kind of perverse effects.

[6]See Houy (2009), Maniquet and Morelli (2010) and Pauly (2013).

Maug and Rydqvist (2009) examine the role of annual meetings when stockholders share the same objective – to raise the value of their shares – but have access to different information. The focus is on how dispersed information is aggregated in the vote. Based on a variant of Feddersen and Pesendorfer (1998), they analyze the strategic voting of shareholders under the following assumptions: (1) each shareholder has one vote and voting entails no cost; (2) the only two possibilities are to vote for or against a resolution proposed by the management; (3) information is asymmetric: although they share the same initial priors, stockholders receive a private signal regarding the resolution. In line with Feddersen and Pesendorfer (1998)[7], they show that shareholders may vote against their private information; they also find an interesting result concerning the effects of majority voting rules. More stringent majority rules (for example, a majority of 2/3 of votes rather than 1/2 to pass a resolution) induce more shareholders to vote in favor of the resolution. Indeed, understanding that a higher majority may prohibit the adoption of a good resolution, shareholders compensate this bias by voting more often in favor. As a result, their model predicts that the number of votes in favor increases with the required majority and that the adoption rate is independent of the rule in equilibrium. This prediction is tested using the voting outcome of annual meetings in the United States for the period from 1994 to 2003. The observed relation between the majority rule and the outcome confirms that the strategic voting behavior of shareholders counteracts the conservatism of supermajority rules.

The aforementioned model assumes all stockholders have one vote, share the same objective and that voting is costless. In many countries, the presence of large voting blocks in addition to smaller ones is the rule, and shareholders differ in their voting power (Becht and Roell (1999) or Becht and Mayer (2002)). Additionally, shareholders often have conflicting interests

[7] When some voters are better informed about the best alternative, as explained in Feddersen and Pesendorfer (1996), abstention may be strategic even when voting is costless: it is rational to delegate the choice to the better informed and to abstain. This is the second main direction been pursued in order to explain the observed limited turnout. See Feddersen (2004) for a survey.

(e.g., the State versus hedge funds or employees).[8] Ritzberger (2005) analyzes strategic voting in annual meetings when stockholders with different voting shares disagree on the resolutions, with some being in favor of the proposal and others against (or equivalently for the status quo). Information is symmetric and voting entails a small cost. The focus is therefore on the predicted voting outcome when unanimity fails among shareholders owning different amounts of stock rather than on efficient information aggregation. He concludes that an equilibrium exists if and only if the largest (or dominant) shareholder supports the resolution (assuming that, when nobody votes, the status quo prevails). In this case, only one shareholder votes, and the resolution is adopted. The outcome therefore always corresponds to the dominant shareholder's preference. This result is easily explained. Since voting is costly, a shareholder votes only if (1) his vote is necessary to obtain his preferred outcome; (2) no opponent shareholder may change the result. This can happen only when the shareholder voting for the resolution (not necessarily the largest) commands more votes than any partisan of the status quo. It should be noted that, in equilibrium, the turnout is the share of the only voter, and the resolution passes with a majority of 100%. Thus, the majority rule plays no role.

In a recent contribution, Matsusaka and Ozbas (2017) examine the role of the largest shareholder in annual meetings. Specifically, their model has three actors: a manager, a non-controlling blockholder, and a set of identical small shareholders considered as a block. The manager and the large shareholder receive private benefits depending on the voting outcome. In a first stage, the manager sets the agenda of the meeting. The large shareholder may or may not make a counterproposal. With no counterproposal, small shareholders simply approve the management's proposal as long as it matches the status quo, which limits the private benefits enjoyed by management at the expense of small shareholders. In case of a counterproposal, small shareholders vote either for the management proposal or for the large shareholder proposal. As a consequence, the manager accommodates the large investor in setting the agenda to discourage him from making a coun-

[8] See Matvos and Ostrovski (2010).

terproposal, which benefits small shareholders if interests of all – both small and large – investors are close, and harms them otherwise. Depending on the direction of the compromise, the right to propose a resolution may thus help or hurt small shareholders. The emphasis of Matsusaka and Ozbas (2017) is on the process rather than on the voting behavior, as small shareholders are not strategic.

Our approach is closest to Ritzberger's, as we analyze strategic voting by shareholders who are heterogeneous in both size and opinion. However, our results differ substantively, particularly regarding the role of the dominant shareholder and the possibility of rejecting resolutions in equilibrium. It is also related to the literature on quorum rules. In the context of annual meetings, we show that a minimum quorum does not necessarily increase the turnout and instead may favor the adoption of resolutions rather than create a bias towards rejection. Finally, we provide precise content regarding the concepts of dominant, controlling and reference shareholders and discuss the situations in which shareholders control annual meetings – whether completely, partially or jointly – in relation to the minimum quorum.

3 The model

The main role of annual meetings is to adopt or reject resolutions on the agenda that is usually sponsored by the board of directors and sometimes by one or several shareholders. We analyze the result of corporate meetings in a context where in which among shareholders does not hold and in the presence of large voting blocks in addition to smaller ones.

It is often assumed that the common objective of shareholders is to adopt resolutions increasing the value of the company.[9] Actually, even in the case in which shareholders' interest is limited to their financial wealth, they may disagree on the relevance of resolutions in several instances. When a stockholder holds shares in two business-related companies simultaneously, he may be favorable to a value-decreasing resolution in a company that has a pos-

[9] See for example Maug and Rydqvist (2009).

itive impact on the value of his holdings in the other company, while other shareholders disagree with the resolution.[10] Other types of private benefits (ethical considerations,...) may explain opposite views.

Throughout the article, F (resp. A) denotes "for the resolution" (resp. "against the resolution"). We represent a shareholder i who supports the resolution (resp. opposes the resolution) by his voting share α_i^F (resp. α_i^A). There are N^F partisans of the resolution belonging to the grand coalition for $\mathcal{P}^F$=$\{\alpha_1^F, \alpha_2^F, ..., \alpha_{N^F}^F\}$, and N^A opponents belonging to the grand coalition against $\mathcal{P}^A$=$\{\alpha_1^A, \alpha_2^A, ..., \alpha_{N^A}^A\}$, where $\alpha_1^K \geq \alpha_2^K \geq \alpha_3^K \geq ... \geq \alpha_{N^K}^K > 0$ for $K = F, A$. Some shareholders may also be indifferent to the resolution.

Shareholders vote strategically and choose the best action given their preferences and expectations about other shareholders' strategies. They incur a small voting cost and are not required to vote. In practice, shareholders usually have four possibilities: if they vote, they may approve (vote for), disapprove (vote against) or abstain; they may also decide to withhold from voting.[11]

Two conditions must be verified for a resolution to be adopted. First, a minimum number of shareholders must be present or represented (quorum rule) in the annual meeting. We call Q the minimum proportion of shareholders who must cast a vote for a resolution to be adopted. When the minimum quorum Q is not reached, the resolution cannot pass. Second, the resolution must obtain a minimum of favorable votes; we assume that a resolution cannot pass unless the total number of votes in favor is strictly higher than the

[10] Matvos and Ostrovsky (2008) show, for example, that "in mergers with negative acquirer announcement returns, cross-owners are significantly more likely to vote for the merger". Charléty, Fagart and Souam (2009) endogenize such private benefits in the case of horizontal partial acquisitions.

[11] Sometimes the vote is limited to "approve" or "reject" with no possibility to "abstain". In other cases, the only option is "for"; as an example, shareholders may vote "for" up to a limited number of directors among a proposed list, with no possibility to vote "against" a director. Under this plurality voting system, the candidates who receive more votes are elected and need not obtain a majority. See Hewitt (2011).

total number of votes against (simple majority rule). When both conditions are met, the "for" side – or F – wins. If either the quorum or the majority are not met, the resolution is rejected, and A prevails. Thus, the result of an insufficient turnout is considered to be equivalent to a majority vote against, and may be interpreted as the "Status Quo".

We model the annual meeting as a simultaneous game in which each shareholder decides to vote (for, against, or abstain) or to withhold based on his expectations about others' strategies. All shareholders are assumed to know all others' voting shares and preferences (information is perfect). We look for the pure strategy Nash equilibria of this game. Throughout the paper, equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$) refers to a pure strategy Nash equilibrium in which F (resp. A) passes. "Inexistence" refers to a situation where no pure strategy equilibrium exists.[12]

We investigate the role played by the quorum rule in the strategies adopted by shareholders and the result of the annual meeting, given these hypotheses.

4 Shareholders' equilibrium strategies

We first present preliminary results regarding shareholders' equilibrium strategies. These considerations enable us to restrict the examination of the Nash equilibrium conditions to the only admissible strategies under a quorum rule. To better predict the result of the meeting, we then apply the concept of a Coalition-Proof Nash Equilibrium (CPNE), which is appropriate in games in which players can privately communicate before choosing their strategy, as is the case for annual meetings.

Preliminary results

Abstaining may not change the outcome. It is therefore strictly dominated by either voting in line with preferences or withholding, as voting is costly. In

[12] When no pure strategy Nash equilibrium exists, the outcome of the meeting cannot be predicted. The outcome of the vote is random in that case. Since the game is finite, we know (Nash) that a mixed strategy equilibrium always exists.

particular, withholding is a strictly dominant strategy for indifferent shareholders. We therefore consider only three possible strategies for shareholders: vote for (F) the proposed resolution, vote against (A) the resolution, or do not participate in the vote.

Since voting is slightly costly, the best outcome for any shareholder in favor of the resolution $\alpha_i^F \in \mathcal{P}^F$ (resp. any shareholder against $\alpha_i^A \in \mathcal{P}^A$)) is F (resp. A) without participation, the second-best is F (resp. A) with participation, which is better than A (resp. F) without participation, and the worst is outcome A (resp. F) with participation.

Consequently, two properties[13] hold in equilibrium:

(P1) No partisan of A (resp. F) votes in equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$).

Indeed, suppose a non-voting partisan who is against α_i^A (resp. α_i^F) expects F (resp. A) to result from the vote. If, given the others' actions, he can change this outcome, his best strategy is to vote against (resp. for), which means that the initial set of shareholders' actions was not an equilibrium. Therefore, in equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$), no partisan of A (resp. F) is able to change the result and does not vote, since voting is costly.

(P2) In equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$), a shareholder for $\alpha_i^F \in P^F$ (resp. a shareholder against $\alpha_i^A \in P^A$) participates in the vote if and only if he is pivotal, i.e., his vote is necessary to obtain his preferred outcome.

Effectively suppose a voting partisan for α_i^F (resp. α_i^A) expects the meeting to decide F (resp. A). If, given the others' actions, the result remains F (resp. A) if he does not vote, his best strategy is to withhold, since voting is costly; this means that the initial set of shareholders' actions was not an equilibrium. Therefore, in equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$), no shareholder participates when his preferred outcome F (resp. A) emerges without his vote.

[13] These properties result from costly voting. If some shareholders do not incur a voting cost or when voting is mandatory for some of them, the mechanisms at work in our setting remain with a higher turnout, as equilibria $\mathcal{F}$ (resp. $\mathcal{A}$) may exist with votes against (resp. for).

To summarize, with a small voting cost, a shareholder votes in equilibrium only if he anticipates that his vote is "useful," which is expressed by properties (P1) and (P2).

Consequences of a Quorum Rule

Casting a vote is optional for all shareholders. However, a quorum rule specifies that no resolution may be adopted unless a minimum proportion Q of all equity capital is present or represented in the meeting. We analyze the consequences of this rule on the outcome of the meeting and the nature of the equilibrium. Our analysis thus concentrates on the strategies of partisans of both sides, recalling that, in equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$), partisans of A (resp. F) do not vote.

Let Δ^F represent the set of all groups of partisans V^F voting in favor of the resolution that cannot be defeated by any – including the largest – partisan against (we assume no cooperation between shareholders). Thus, $\Delta^F = \{V^F \subset \mathcal{P}^F \mid \sum_{V^F} \alpha_i^F > \alpha_1^A\}$. Define

$$V_m^F = Arg \underset{V^F \in \Delta^F}{Min} \left\{ \sum_{V^F} \alpha_i^F - \underset{V^F}{Min}(\alpha_j^F) \right\}$$

$$\text{and } \alpha_m^F = \sum_{V_m^F} \alpha_i^F - \underset{V_m^F}{Min}(\alpha_j^F).$$

α_m^F represents the minimum, among all coalitions in Δ^F, of total partisan votes in favor minus the share of the smallest partisan in the coalition.

Proposition 1 fully characterizes the conditions of existence and the nature of the equilibria (for or against; involving the vote of some partisans or not) with a quorum rule.

Proposition 1 *Suppose that a quorum Q is required for the adoption of resolutions. In that case,*

(1) *There exists a unique equilibrium $\mathcal{A}$ where no shareholder votes if and only if $\alpha_1^F < Q$,*

(2) *There always exists at least one equilibrium* $\mathcal{F}$ *when* $\alpha_1^A < Q$*; when* $\alpha_1^A \geq Q$*, equilibria* $\mathcal{F}$ *exist if and only if* $\alpha_m^F < Q$*.*

The first result can be easily explained. Indeed, partisans against are satisfied with a zero turnout, as the resolution is rejected at no cost, and no shareholder in favor is in a position to overturn the outcome, as the largest shareholder in favor of the resolution does not reach the quorum. Thus, the situation in which no shareholder votes – resulting in A – is stable. This non-voting equilibrium is the only possible equilibrium against. Indeed, suppose several partisans vote against and reach Q. All of them have an incentive to withhold since the outcome would not be affected (partisans in favor never vote in equilibrium $\mathcal{A}$). Thus, there is no voting equilibrium against.[14] Conversely, the situation in which no partisan votes is never an equilibrium when $\alpha_1^F \geq Q$; the largest partisan in favor should vote since, being above the quorum, the resolution would pass. Note that the necessary and sufficient condition for the existence of an equilibrium $\mathcal{A}$ is independent of both the size of the grand coalition $\sum_{P^A} \alpha_i^A$ and the size of the group of partisans against. The resolution may be rejected even when shareholders opposed to the resolution together do not reach the minimum quorum. The minimum quorum Q therefore does not increase the representativity of the vote or the turnout in this case.

The second result indicates that voting equilibria in favor may coexist. Three conditions must hold:

(i) partisans' aggregated votes in favor must reach the quorum, $\sum_{V^F} \alpha_i^F \geq Q$,

[14] Again, the main results of our analysis remain unchanged if we assume that some shareholders do not incur a voting cost. In this case, essentially, equilibria for (resp. against) would occur with some votes against (resp. for), and the turnout would be higher. While equilibria with votes against may exist, strategic withholding to prevent adoption remains. The main difference would be the possibility that a vote against from opponents incurs a cost in equilibrium, with the favorable costless votes providing an incentive to vote against.

(ii) and exceed the share of the largest partisan against, $\sum_{V^F} \alpha_i^F > \alpha_1^A$,

(iii) every voting shareholder and the smallest one in particular must be pivotal, i.e., necessary to meet the quorum, $\sum_{V^F} \alpha_i^F - \underset{V^F}{Min}(\alpha_j^F) < Q$.

Hereafter, we suppose that the grand coalition of partisans in favor of the resolution represents at least the minimum quorum, $\sum_{\mathcal{P}^F} \alpha_i^F \geq Q$, and is larger than the largest partisan against, or $\sum_{\mathcal{P}^F} \alpha_i^F > \alpha_1^A$ (Δ^F is not empty), for non-triviality.

When $\alpha_1^A < Q$, the first condition actually implies the second one. As we assumed the whole set of partisans in favor represents at least the quorum, a voting equilibrium always exists in that case. Simply add up partisans in favor in a coalition according to decreasing size until Q is reached (i). As the smallest voter is pivotal by construction, so are other voters in the coalition since they represent a larger share of equity (iii). Additionally, the coalition cannot be overturned because the largest partisan against does not reach the quorum (ii).

When $\alpha_1^A \geq Q$, coalitions in favor that cannot be overturned meet the quorum, and the second condition implies the first one. If, for at least one of those coalitions in Δ^F, the turnout falls below the quorum after removing the smallest partisan, then this coalition is stable, as all partisans are pivotal. Consequently, a voting equilibrium $\mathcal{F}$ exists. V_m^F is defined as the coalition minimizing the total shares minus the lowest one among non-contestable coalitions. Therefore, equilibrium $\mathcal{F}$ exists if and only if V_m^F verifies this stability condition, as formally stated by $\alpha_m^F < Q$ (see Appendix).

Example 1 illustrates the consequences of a quorum for the existence and nature of equilibria (for vs. against) in the presence of blockholders.

Example 1 *Consider the following shareholding structure:*

$$\mathcal{P}^A = \{14\%, 12\%, ...\}$$

$$\mathcal{P}^F = \{11\%, 9\%, 8\%, 7\%, 4\%, 3\%, 2\%, 1\%, 1\%, 1\%, ...\}$$

With a quorum $Q = 25\%$, there exists an equilibrium $\mathcal{A}$ where no shareholder votes because the largest partisan's vote for the resolution cannot change the outcome ($\alpha_1^F = 11\% < Q$). Simultaneously, many coalitions (e.g., $\{11\%, 9\%, 8\%\}$, $\{9\%, 8\%, 7\%, 4\%\}$, $\{9\%, 8\%, 7\%, 1\%\}$, $\{9\%, 8\%, 3\%, 2\%, 1\%, 1\%, 1\%\}$, ...) support a voting equilibrium $\mathcal{F}$ since no coalition above the quorum can be overturned by the largest partisan against ($\alpha_1^A = 14\% < Q$).

With a quorum $Q = 10\%$, coalitions $\{9\%, 8\%\}, \{9\%, 7\%\}$ $\{8\%, 7\%\}$ support an equilibrium for ($\alpha_m^F = 8\% < Q$). No equilibrium against exists, the largest partisan for could change the result from rejection with no turnout to adoption when he alone votes ($\alpha_1^F = 11\% \geq Q$).

Finally, with no quorum requirement ($Q = 0\%$), no equilibrium in pure strategies exists ($\alpha_1^F = 11\% \geq Q, \alpha_m^F = 8\% \geq Q$).

As shown in the above example, requiring a quorum has two consequences. On one hand, it generates an equilibrium against, and no shareholder votes. Indeed, with a high enough quorum, shareholders in favor of the resolution may believe that the quorum will not be reached, which makes their vote useless; the same is true for shareholders opposed to the resolution. On the other hand, a minimum quorum works as a coordination device for partisans of the resolution who believe that their vote is necessary to win. It creates an incentive to form voting coalitions, gathering possibly very small shareholders in favor of the resolution; this never happens with no quorum requirement in which at most one large-enough (share greater than α_1^A) shareholder votes in favor in equilibrium and no equilibrium against ever exists (see Ritzberger 2005).

Predicting the result of the vote

In some cases, several Nash equilibria coexist. In example 1, with $Q = 25\%$, rejection ($\mathcal{A}$) and acceptance of the resolution, ($\mathcal{F}$) are equilibrium results. Moreover, different voting coalitions support $\mathcal{F}$. In these cases of multiplicity of equilibria, which (is) are the most likely? Among voting equilibria, it is natural to select those gathering fewer shareholders, as they involve fewer

coordination problems and a lower cost. When equilibria $\mathcal{A}$ and $\mathcal{F}$ coexist, what is the predicted outcome of the meeting?

In order to answer these questions, we apply a refinement of the set of Nash equilibria that seems appropriate in our setting. Namely, we use the CPNE concept introduced by Bernheim, Peleg and Whinston (1987). It is reasonable to believe that, before they vote, shareholders discuss and agree on a voting strategy. Because they vote separately, agreements are meaningless unless they are self-enforcing, which is true for Nash equilibria. However, the individual best response property of Nash equilibria may not be sufficient. If a coalition of shareholders can arrange a mutually beneficial deviation from a Nash agreement, the enforceability of this original agreement appears to be weak. The CPNE criterion considers such a possibility. A Nash equilibrium is said to be CPNE when there exists no profitable self-enforcing joint deviation of a coalition of shareholders. A deviation is defined as self-enforcing when no subcoalition has an incentive to initiate a new deviation, with the strategies of other voters being fixed.

The following proposition provides the necessary and sufficient conditions under which the Nash equilibria are coalition-proof.

Proposition 2 *Among the Nash equilibria,*

(1) *Equilibrium $\mathcal{A}$ is CPNE if and only if it is the unique Nash equilibrium (in pure strategies) of the voting game.*

(2) *An equilibrium $\mathcal{F}$ is CPNE if and only if it cannot be overturned by the grand coalition of shareholders against the resolution.*

Proof:

Since shareholders' strategies are very simple, it is relatively easy to check whether the different Nash equilibria of our voting game are CPNE.

Part 1 of the proposition is straightforward. On one hand, when $\mathcal{A}$ is the unique equilibrium, no coalition prefers deviating from not voting. Shareholders against the resolution obtain their best possible outcome. Moreover, every coalition of shareholders in favor of the resolution gathering more than the minimum quorum is intrinsically not self-enforcing since there exists no

Nash equilibrium in favor: at least one shareholder would find it profitable to deviate from the deviation. On the other hand, suppose that at least one equilibrium $\mathcal{F}$ coexists with $\mathcal{A}$. Shareholders in the coalition supporting this equilibrium $\mathcal{F}$ would then agree to deviate from not voting (equilibrium $\mathcal{A}$) to jointly vote in favor. This deviation is self-enforcing since every voting shareholder in this coalition is pivotal. Therefore, none of them (a fortiori no sub-coalition) would like to deviate from the deviation.

Let us turn to part 2 of the proposition. On one hand, consider an equilibrium $\mathcal{F}$ that cannot be overturned by the grand coalition of shareholders against. Obviously, no coalition of shareholders against would like to jointly change their strategy and vote against since this is both costly and useless. Similarly, no non-voting shareholder in favor of the resolution would profitably deviate and vote since he gets his best outcome without incurring the cost of voting. Only shareholders belonging to the voting coalition could possibly benefit from a change in strategy. However, since $\mathcal{F}$ is a Nash equilibrium and all voters are pivotal, no individual shareholder and a fortiori no coalition of shareholders would profitably deviate from voting to withholding. On the other hand, suppose that the grand coalition of shareholders against can overturn a coalition supporting equilibrium $\mathcal{F}$. Among all coalitions against able to overturn this equilibrium $\mathcal{F}$, consider the coalition representing the least total votes. By construction, the vote of each shareholder is necessary to beat the coalition supporting $\mathcal{F}$. A joint deviation by this smallest coalition from withholding to voting against is thus self-enforcing: no individual shareholder – and, a fortiori, no sub-coalition – may profitably deviate from the deviation coalition against since this would change the outcome from $\mathcal{A}$ to $\mathcal{F}$. Therefore, the coalition supporting equilibrium $\mathcal{F}$ is not robust to this self-enforcing coalition deviation. The Nash equilibrium $\mathcal{F}$ is not CPNE. *Endproof.*

Let us return to example 1 with $Q = 25\%$ to illustrate Proposition 2. The non-voting equilibrium against is not CPNE. Indeed, the sub-coalition $\{11\%, 9\%, 8\%\}$ of favorable shareholders who withhold in the Nash equilibrium $\mathcal{A}$ jointly benefit from deviating to vote in favor and pass the resolution.

This voting Nash equilibrium $\mathcal{F}$ is itself coalition-proof if the turnout outreaches the grand coalition against. In this example, the grand coalition $\mathcal{P}^A = \{14\%, 12\%, ...\}$ represents at least 26% of votes. Thus, all equilibria $\mathcal{F}$ representing less than 26% are not CPNE. The Nash equilibrium $\mathcal{F}$ supported by $\{11\%, 9\%, 8\%\}$ is coalition-proof when the grand coalition $\mathcal{P}^A$ represents less than 28% of voting rights.

Two important results are direct consequences of Proposition 2. First, a coalition supporting an equilibrium $\mathcal{F}$ is CPNE only when – among shareholders with strict preferences – a majority are in favor of the resolution. In this case, the outcome coincides with the preference of the majority of shareholders. Second, when there does not exist an equilibrium $\mathcal{F}$, the unique equilibrium $\mathcal{A}$ is CPNE, while the majority could be in favor of the resolution.

The conditions for existence of Nash equilibria and their nature (for or against, CPNE or not) are summarized in Table 1.

Table 1. The different types of equilibria

	$\alpha_1^A \geq Q$	$\alpha_1^A < Q$
$\alpha_1^F < Q$	eq. $\mathcal{A}$ (CPNE if it is the unique eq.) eq. $\mathcal{F}$ if $\alpha_m^F \leq Q$ (possibly CPNE)	eq. $\mathcal{A}$ (not CPNE) eq. $\mathcal{F}$ (possibly CPNE)
$\alpha_1^F \geq Q$	eq. $\mathcal{F}$ if $\alpha_m^F \leq Q$ (possibly CPNE)	eq. $\mathcal{F}$ (possibly CPNE)

5 Consequences of the shareholding structure

The shareholding structure plays a key role in the determination of the outcome of the vote. As summarized in Table 1, the size of the largest equityholder favorable to a resolution relative to the quorum conditions the existence of equilibria against this resolution. Similarly, the size of the largest opponent to the resolution is central for the existence of equilibria $\mathcal{F}$ both directly and indirectly. Indeed, when $\alpha_1^A \geq Q$, α_m^F is determined in relation to

α_1^A. The size of shares held by voters in favor happens to be crucial for the formation of equilibrium coalitions favorable to the resolution when the largest opponent is above the quorum, as shown in example 1 (with $Q = 10\%$).

Widely held companies

In some companies, virtually all shareholders are small. This situation is common in the UK and is also observed in continental Europe, where ownership is, on average, rather concentrated.

Proposition 3 *Nash equilibria $\mathcal{A}$ and $\mathcal{F}$ coexist in widely held companies for $Q > 0$. Turnout coincides with the minimum quorum in equilibria $\mathcal{F}$; they are CPNE if and only if the grand coalition against is below Q, and no CPNE exists otherwise.*

The first part of Proposition 3 follows directly from Propositions 1. In the particular case where all shareholders have one vote and are below the quorum, both $\mathcal{A}$ and $\mathcal{F}$ are equilibria. The voting coalitions in $\mathcal{F}$ just match the quorum because it is necessary and sufficient to pass the resolution. $\mathcal{A}$ is never CPNE because any coalition of favorable shareholders just reaching Q would profitably deviate and vote in favor. Similarly, $\mathcal{F}$ is CPNE if no coalition of opposed shareholders can profitably deviate and vote against; therefore, all opponents together must represent less than the quorum.

In the following, we first look in greater detail at the role played by the largest partisan and the largest opponent when the company is not widely held. We then examine the size of shareholders and the number of partisans in voting coalitions. In light of the previous results, we finally compare the efficiency of the two mechanisms available to shareholders in meetings when they do not agree with the board: voting against resolutions proposed by the board or offering resolutions directed against the board.

The role of large shareholders

Major shareholders play a decisive role in meetings. However, given the asymmetry between equilibria in favor and against, their importance varies

depending on whether they support or oppose a resolution.

Large shareholders supporting the resolution

The presence of a large shareholder in favor of a resolution facilitates the adoption of this resolution in two complementary ways.

First, when the largest shareholder is in favor of the resolution ($\alpha_1^F > \alpha_1^A$), or α_1^F is the dominant shareholder, the existence of voting equilibria in favor of the resolution is guaranteed. Partisans of a resolution in a coalition are added according to decreasing size until Q is reached (we assumed the whole set of partisans for represents at least the quorum). As the smallest voter is pivotal by construction, so are other voters in the coalition since they represent a larger share of equity. Moreover, the coalition cannot be contested because it includes $\alpha_1^F > \alpha_1^A$.

Second, this process prevents the event of rejection due to the absence of a quorum. Indeed, a quorum below the share of the largest partisan for ($\alpha_1^F \geq Q$) excludes any opposition to resolutions in equilibrium, as the largest supporting shareholder reaches the quorum alone (see Proposition 1).

Proposition 4 summarizes these results.

Proposition 4 *When the dominant shareholder is favorable to the resolution* ($\alpha_1^F > \alpha_1^A$),

(1) *The resolution is always adopted in equilibrium if* $\alpha_1^F \geq Q$.

(2) *If* $\alpha_1^F < Q$, *a voting equilibrium* $\mathcal{F}$, *necessarily involving a coalition of shareholders, and a non-voting equilibrium against coexist.*

Example 2 illustrates the major role played by the dominant shareholder when he supports the resolution.

Example 2 *Consider the following shareholding structure:*

$$\mathcal{P}^A = \{14\%, 13\%, ...\}$$
$$\mathcal{P}^F = \{18\%, 8\%, 2\%, 1\%, 1\%, 1\%, ...\}$$

From Proposition 4, resolutions are always adopted in equilibrium when the dominant shareholder is favorable *and* represents at least the quorum ($\alpha_1^F \geq \max(Q, \alpha_1^{A+})$ where α_1^{A+} represents α_1^A plus one vote). In particular, an equilibrium $\mathcal{F}$ where he (or another large shareholder) votes alone exists (as in Ritzberger, 2005). Consider a quorum $Q = 15\%$ in example 2. When the dominant shareholder votes alone, no other partisan of the resolution has an incentive to vote since the quorum is reached, and the vote cannot be opposed successfully since the largest opponent commands fewer votes. As no equilibrium against exists in that case ($\alpha_1^F \geq Q$), the result of the meeting conforms to the dominant shareholder's preference.[15] It is possible that coalitions of small shareholders vote in equilibrium: for instance, a subset of $\mathcal{P}^F$ comprising fifteen shareholders each owning 1% of equity.

Now suppose $Q = 25\%$. The dominant shareholder in favor of the resolution does not reach the quorum alone. At least two shareholders must cast a vote in favor of the resolution. The situation where the two largest supporters of the resolution – representing a turnout of 26% – vote is an equilibrium: if either one does not vote, the quorum is not met, and no shareholder against can successfully oppose the vote.[16] Rejection of the resolution due to the absence of a quorum is also an equilibrium, as $Q > \alpha_1^F$. This situation would never occur without a quorum, as any supporter of the resolution would find it useful to vote and pass the resolution. Thus, the equilibrium result of the meeting does not always conform the dominant shareholder's preference even though he is for the resolution.[17]

[15] Indeed, when $\alpha_1^F \geq Q > \alpha_1^A$ from Proposition 1, there exists no equilibrium against, and an equilibrium in favor always exists. This is also the case when $\alpha_1^F > \alpha_1^A \geq Q$. In both cases, the singleton $\{\alpha_1^F\} \in \Delta^F$ and $\alpha_m^F = 0 < Q$.

[16] There exist many other equilibrium coalitions for, e.g., $\{18\%, 2\%, 1\%, 1\%, 1\%, 1\%, 1\%\}$.

[17] However, it follows from Proposition 2 that rejection of the resolution is not CPNE. It can be checked easily that no CPNE exits in example 2.

Large shareholders opposed to the resolution

The presence of a dominant shareholder in favor of the resolution guarantees the existence of an equilibrium $\mathcal{F}$. This does not apply symmetrically to the case of a dominant shareholder opposed to the resolution. From Proposition 1, equilibrium rejection depends solely on the equity ownership of the largest partisan of the resolution relative to the quorum. Thus, even if the largest opponent commands more votes than the largest partisan, $(\alpha_1^A \geq \alpha_1^F)$, coalitions of shareholders may still pass the resolution in equilibrium.

Example 3 *Consider the following shareholding structure:*

$$\mathcal{P}^A = \{30\%, 3\%, 3\%, ...\}$$
$$\mathcal{P}^F = \{26\%, 16\%, 15\%, 1\%, 1\%, 1\%, ...\}$$
$$Q = 25\%$$

In this example, there is no equilibrium against since $\alpha_1^F \geq Q$, and the coalition of the two partisans for $\{16\%, 15\%\}$ successfully approves the resolution.

Size and number of voters in equilibria $\mathcal{F}$

Related important questions have not yet been completely addressed: Who belongs to voting coalitions – large or small shareholders? Discussions before the meeting are certainly easier and less costly if the coalition gathers a smaller number of large rather than many atomistic shareholders.

Our results show that the size and number of supporters of the resolution actually casting a vote are directly linked to the share of the largest opponent relative to the quorum. This is the purpose of the following proposition.

Proposition 5

(1) *When $\alpha_1^A < Q$, an equilibrium coalition for the resolution may gather shareholders of any size without limitation as to the number of shareholders present in the coalition.*

(2) *When $\alpha_1^A \geq Q$, the partisans forming an equilibrium coalition for the resolution are never atomistic; their shares are necessarily larger than $\alpha_1^A - Q$. The number of voters n is bounded: $\frac{\alpha_1^A}{\alpha_1^F} < n < \frac{\alpha_1^A}{\alpha_1^A - Q}$. Moreover, the largest partisan for α_1^F never votes in equilibrium when $\alpha_1^A \geq \alpha_1^F \geq Q$, and votes alone when $\alpha_1^F > \alpha_1^A \geq Q$.*

When $\alpha_1^A < Q$, as explained following Proposition 1, voting coalitions gather shareholders of any size as long as the quorum is met ($\sum_{V^F} \alpha_i^F \geq Q$). They possibly encompass many small shareholders if the firm is widely held, and the turnout matches the minimum quorum in that case.

When $\alpha_1^A \geq Q$, coalitions that cannot be overturned ($\sum_{V^F} \alpha_i^F > \alpha_1^A$) automatically reach the quorum. Moreover, all shareholders are pivotal in equilibrium ($\sum_{V^F} \alpha_i^F - \underset{V^F}{Min}(\alpha_j^F) < Q$). As a consequence, $\underset{V^F}{Min}(\alpha_j^F) > \alpha_1^A - Q$. Therefore, partisans in a winning coalition are never atomistic.

In this case, the number of voters n is necessarily bounded. In fact, from the above inequalities, it follows

(i) $n\alpha_1^F \geq \sum_{V^F} \alpha_i^F > \alpha_1^A$; which gives $n > \frac{\alpha_1^A}{\alpha_1^F}$.

(ii) $(n-1)(\alpha_1^A - Q) < \sum_{V^F} \alpha_i^F - \underset{V^F}{Min}(\alpha_j^F) < Q$; thus, $n < \frac{Q}{\alpha_1^A - Q} + 1 = \frac{\alpha_1^A}{\alpha_1^A - Q}$.

Therefore, the number of voting shareholders is bounded as follows:

$$\frac{\alpha_1^A}{\alpha_1^F} < n < \frac{\alpha_1^A}{\alpha_1^A - Q}.$$

Finally, the largest supporter does not necessarily belong to the coalition. He never votes in equilibrium if his share is above the quorum but below the largest opponent stake ($\alpha_1^A \geq \alpha_1^F \geq Q$). Indeed, no other partisan in favor would join him in a coalition because the quorum is met, whereas the largest partisan against would successfully oppose him. Clearly, the largest partisan votes alone when $\alpha_1^F > \alpha_1^A \geq Q$.

In examples 1 and 2 ($\alpha_1^A = 14\%$), with $Q = 25\%$, since $\alpha_1^A < Q$ voters can be of any size; any coalition, as long as it reaches the quorum, supports

$\mathcal{F}$. In addition to the coalitions previously mentioned for the two examples, a group of 25 shareholders, if they exist, each representing 1% of equity and voting would support an equilibrium $\mathcal{F}$.

In example 1 with $Q = 10\%$, ($\alpha_1^A = 14\% \geq Q$) partisans voting in favor must hold more than 14% of shares together and more than $\alpha_1^A - Q = 4\%$ each. Indeed, a shareholder with less than 4% shares has an incentive to leave a coalition representing more than 14% of equity since the quorum requirement remains satisfied. In this case, the number of voters is bounded, $\frac{14}{10} < n < \frac{14}{4}$. Thus, $n \in \{2, 3\}$. This example (with $\alpha_1^A = 14\%$, $\alpha_1^F = 11\%$ and $Q = 10\%$) illustrates the case in which the largest shareholder for never votes in equilibrium ($\alpha_1^A \geq \alpha_1^F \geq Q$).

In example 3, since $\alpha_1^A = 30\% > Q = 25\%$,, pivotal shareholders hold more than $5\% = \alpha_1^A - Q$. Moreover, with $\alpha_1^F = 26\%$, the number of voting shareholders in the equilibrium verifies $\frac{30}{26} < n < \frac{30}{30-25} = 6$. Thus, $n \in \{2, 3, 4, 5\}$. Moreover, α_1^F never votes in equilibrium.

The right to oppose and the right to propose

The board of directors is responsible for organizing the annual meeting, and resolutions are essentially proposed by the board. Most of these resolutions pass[18] by a very large majority; approval rates higher than 95% prevail.

Shareholders also have the right to initiate resolutions. They may nominate directors and propose changes in bylaws and stock-option plans for employees. The power of shareholders may be quite important in some countries in which they can sponsor resolutions in favor of the replacement of management (Charléty, Chevillon and Messaoudi, 2009) and where the vote is binding. Although the threshold varies across countries, a shareholder needs to own a minority stake of around 5% to have the right to put a resolution on the proxy statement. Empirical evidence shows that these "external" propositions often pass, especially when they are corporate governance re-

[18] For example, the rejected resolutions in the French AGM in 2014 represent less than 1% of proposed resolutions (see Proxinvest Annual Report on French AGM, December 2014).

lated, as directors' removal/election (about 25% of such external resolutions are adopted in the UK, see Renneboog and Szilagyi, 2011).

When they disagree with management, shareholders therefore have the right to oppose resolutions submitted by the board and the right to put resolutions to the vote. Given the fundamental asymmetry between equilibria $\mathcal{F}$ and $\mathcal{A}$ in our analysis, we argue that the right to propose is more effective from an individual shareholder's point of view.[19] Indeed, even a large dissident shareholder has no power to reject resolutions. Only a high quorum requirement constitutes a credible threat to the status quo. However, even when relatively small, a dissident shareholder has the power to pass resolutions thanks to the vote of other shareholders united in a coalition, without coordination. This is the case in example 4.

Example 4 *Consider the following shareholding structure:*

$$\mathcal{P}^A = \{30\%, 13\%, 13\%...\}$$
$$\mathcal{P}^F = \{7\%, 6\%, 6\%, 6\%, 6\%, 1\%, ...\}$$
$$Q = 25\%$$

There is a non-voting equilibrium against ($\alpha_1^F < Q$). Simultaneously, the coalition of partisans $\{7\%, 6\%, 6\%, 6\%, 6\%\}$ successfully approves the resolution in equilibrium. In this example, $\alpha_1^F = 7\%$ with $\alpha_1^A = 30\% > Q = 25\%$. From Proposition 5, a winning coalition necessarily involves 5 shareholders with at least 5% voting rights.[20] Thus, $\{7\%, 6\%, 6\%, 6\%, 6\%\}$ is the unique equilibrium voting in favor. Note that not only the largest but also the majority of shareholders are against the resolution in this example. Whereas uncoordinated coalitions of supporters voting for resolutions emerge easily,

[19] Matsusaka and Ozbas (2017) reach a similar conclusion: they show that the right to propose is more efficient than the right to oppose, as the board will accomodate the blockholder.

[20] The number of voting shareholders verifies $\frac{\alpha_1^A}{\alpha_1^F} = \frac{30}{7} < n < \frac{\alpha_1^A}{\alpha_1^A - Q} = \frac{30}{30-25} = 6$. Thus, $n = 5$. Moreover the share of each voter in the coalition must exceed $\alpha_1^A - Q = 5\%$. In this case, there are only 5 shareholders with more than 5% shares.

our results suggest that opponents need to establish a mechanism to vote against resolutions.

6 Dominant, controlling and reference shareholders

With the exception of the United Kingdom, capital structures with several large owners are frequent in Europe, where firms usually have at least one blockholder. The ownership structure and the influence of large shareholders can take different forms.

In some cases, the largest shareholder – or dominant shareholder – actually controls the company. By law, a controlling shareholder owns enough voting shares so that no other shareholder or group of shareholders can successfully oppose him. Under the majority rule, 50% plus one voting share is a controlling interest. However, minority but dominant shareholders often control companies, particularly when the remaining equity is in the hands of small shareholders, which is common. For example, it is generally agreed upon that the French state controls Orange, a major French telecom company, with around 25% of capital. Other large shareholders also influence the outcome of meetings; they may agree on a voting strategy or threaten the power of the dominant shareholder.

Except for the case of a dominant shareholder controlling a company with more than 50% equity, neither the notion of effective control and nor that of significant influence are well defined.

From our results, the power of shareholders in annual meetings is determined not only by their ownership relative to others but also by the minimum quorum and their position regarding resolutions: for a given stake, the ability to secure the adoption of resolutions put to the vote does not imply the power to block resolutions. By control, we mean that the outcome of the vote is *systematically* in line with the dominant shareholder's preference. In the following, we analyze the conditions under which the dominant shareholder controls the meeting. We also provide an interpretation of who may

be considered as a reference shareholder. Indeed, a reference shareholder is considered as large enough to influence the policy of the company in which he has a stake. This notion of a reference shareholder lacks operational content. We partly fill this gap by proposing a precise working definition.

Controlling dominant shareholder

So far, we considered an ownership structure in which a shareholder is defined by both size and preference. Actually, the same shareholder may favor some resolutions while being opposed to others. Thus, a shareholder is controlling when the unique outcome is to accept all resolutions he supports and to reject all resolutions he opposes. The following proposition provides a sufficient condition for the largest shareholder to control the outcome of the meeting.

Proposition 6 *Let α_1 represent the largest share and α_2 the second-largest share. The dominant shareholder controls the meeting if $\alpha_1 \geq Q$ and $Min(Q, \alpha_1 - Q) > \alpha_2$.*

Indeed, if the dominant shareholder favors a resolution and represents more than the quorum ($\alpha_1 = \alpha_1^F \geq Q$), the resolution is always adopted. In particular, when he votes alone, no other partisan of the resolution has an incentive to vote since the quorum is reached, and the vote cannot be opposed successfully since the largest possible opponent commands fewer votes. Because no equilibrium against exists in that case, resolutions are systematically adopted when the dominant shareholder is in favor.

Conversely, when the largest shareholder is against a resolution ($\alpha_1 = \alpha_1^A \geq Q$), rejection is always an equilibrium, as the second-largest shareholder does not reach the quorum ($\alpha_1^F \leq \alpha_2 < Q$). Moreover, as stated in Proposition 5, an equilibrium coalition in favor of the resolution necessarily gathers sufficiently large shareholders (representing at least $\alpha_1^A - Q$). Since there is no such shareholder, there does not exist an equilibrium in favor of the resolution.

Thus, the equilibrium outcome of the vote is systematically consistent with the preference of the largest shareholder, even if he holds less than 50% of the common voting stock .

Example 5 illustrates this situation, in which the dominant shareholder controls the meeting. The shareholding structure in this example is actually similar to the case of Orange, where other identified shareholders represent 4.64% (France Telecom SA Employee Stock Ownership Plan), 1.52% (Norges Bank Investment Management), 1.40% (The Vanguard Group, Inc.), 1.19% (Amundi Asset Investment Management), and other shareholders owning less than 1% equity. The minimum quorum of 20% is the French legal quorum for ordinary general meetings, and it is 25% for special general meetings.

Example 5 *Consider the following shareholding structure of partisans $\mathcal{P}$ who may be for or against resolutions:*

$$\mathcal{P} = \{25\%, 4\%, 2\%, 2\%, 1\%, 1\%, 1\%, ...\}$$
$$Q = 20\%$$

When the dominant shareholder is in favor of a resolution, with a stake of 25%, his vote is sufficient to pass it, and no shareholder is in a position to oppose him. Other equilibria $\mathcal{F}$ may exist; for instance, the case in which small favorable shareholders representing together 20% of equity vote, the resolution is an equilibrium $\mathcal{F}$.[21]

Conversely, the unique Nash Equilibrium is $\mathcal{A}$ when the dominant shareholder is opposed to the resolution. Being below $Q = 20\%$, no shareholder in favor of the vote is in a position to overturn the result. Moreover, no shareholder except α_1 has a stake greater than $\alpha_1 - Q = 5\%$, the minimum share to be part of a voting coalition in favor of the resolution.

With 25% of equity, the largest shareholder controls the meeting. In example 5, the meeting is under the control of the dominant shareholder. The result is *systematically* in accordance with his preference.

[21] From Proposition 5, any coalition gathering just enough shareholders in favor of the resolution supports $\mathcal{F}$, as the quorum exceeds the share of the largest shareholder against.

The conditions stated in Proposition 6 guarantee the existence of Nash equilibria of the voting game, where the outcome always corresponds to the preferences of the dominant shareholder. From Proposition 2, $\mathcal{A}$ being unique is CPNE. However, equilibrium $\mathcal{F}$ is not necessarily CPNE. To be coalition-proof, the equilibrium turnout must exceed the total share of partisans against the resolution. In our example, equilibrium $\mathcal{F}$ where the controlling shareholder votes alone is CPNE if shareholders against the resolution represent less than 25% of all shares. The largest shareholder may be considered as exercising strong control over the meeting in this case.

Weak control For some ownership structures, the outcome of the meeting coincides with the dominant shareholder's preferences, but not systematically. We interpret this as a weak control of the meeting. Weak control is guaranteed under the following condition.

Proposition 7 *The dominant shareholder weakly controls the meeting if* $\alpha_1 \geq Q > \alpha_2 \geq \alpha_1 - Q$.

Effectively, when $\alpha_1 \geq Q > \alpha_2$, all resolutions supported by α_1 are accepted and no equilibrium against coexists. If he is against, $\mathcal{A}$ is an equilibrium, consistent with the largest shareholder's preferences. However, the resolution may also be accepted even though the largest shareholder opposes it. This is the case when at least two pivotal shareholders in favor of the resolution vote in favor and represent together more than α_1. The following example illustrates this case of weak control.

Example 6 *Consider the following shareholding structure:*

$$\mathcal{P} = \{25\%, 13\%, 13\%, 4\%, 2\%, 2\%, 1\%, 1\%, ...\}$$
$$Q = 20\%$$

When the dominant shareholder is opposed to a resolution, as in example 6, rejection of the resolution is an equilibrium outcome. Simultaneously,

the resolution is accepted if the two second-largest shareholders, who are favorable to the resolution, vote. When $\mathcal{F}$ and $\mathcal{A}$ coexist, only $\mathcal{F}$ may be coalition-proof (Proposition 2). Actually, in this example, $\mathcal{F}$ is CPNE only if shareholders against represent less than 26%. If just one shareholder owning 1% equity is against, the coalition constituted by this small and the dominant shareholder representing 26% equity benefits from deviating and voting against.

To block a resolution, the dominant shareholder should agree with one or several shareholders with the same preferences, representing more than 8% equity in this example. With more than 33% of votes, the alliance meets the condition $\alpha_{alliance} - Q = \alpha_{alliance} - 20\% > \alpha_2 = 13\%$ needed for strong control (Proposition 6). With 13% equity, the second-largest shareholder seems to be a potential ally. The role of such influential shareholders is discussed in the following paragraphs.

Non-controlling reference shareholders

A reference shareholder is considered large enough to exercise some power even though he does not (necessarily) control a company. Typically, a stake representing at least 5% of voting rights is sufficient to impact the company's governance. Laeven and Levine (2008) document that one-third of publicly listed firms in Europe have multiple large equityholders holding more than 10% of voting rights each. Moreover, they show that, in more than 80% of the multiple large shareholder cases, there are actually two large owners.[22] In the following, we discuss the level of influence exercised by large minority shareholders, from joint control to the ability to counter a dominant shareholder.

[22] Maury and Pajuste (2005) document that, in the Finnish case, where ownership is concentrated, the largest shareholder is a family with 33.5% of the cash-flow rights on average, and it almost always has a representative on the board or management. The average stakes of the second- and third-largest shareholders are 11.6% and 5.9%, respectively. The second-largest owner category is corporations, then financial institutions.

Joint control Two or a few blockholders may share control of a company. Gomes and Novaes (2006) suppose that two shareholders control by deciding jointly on investments; if one of them does not agree, the investment is simply not undertaken. However, in their model, shared control requires ownership of an exogenously given stake of equity.

In our setting, joint control by two (or more) shareholders with the same preferences can be interpreted as an explicit agreement to vote as a larger controlling block. The commitment to vote in the same direction can take the form of exercising the proxy of other shareholders. Allied shareholders may also sign a voting pact. Indeed, voting pacts between two shareholders are frequently used in countries such as Italy or France. Volpin (2002) reports that 15% of firms listed on the Milan Stock Exchange from 1987 to 1996 were controlled by large shareholders through voting syndicates. Such voting agreements usually involve shareholders of the same nature regardless of whether they are individuals from the same family or other types of shareholders. From above, an alliance block between the two largest shareholders α_1 and α_2 satisfying $\alpha_1 + \alpha_2 \geq Q$ and $Min(Q, \alpha_1 + \alpha_2 - Q) > \alpha_3$ is controlling. As an illustration, from 2008 to 2015, the two largest shareholders of Accor (a major French hospitality firm), Colony and Eurazeo entered in a voting pact. With approximately 15% of voting rights each and the remaining equity being widely held, the two hedge funds jointly controlled Accor. Moreover, the third-largest shareholder owned less than 3%, $\alpha_1 + \alpha_2 - Q = 10\% > 3\% = \alpha_3$. According to our results, the ownership structure of Accor leads to the control by the two funds sharing the same vision and behaving as a single blockholder.

Powerful allied reference shareholders The power of the two reference shareholders in the previous example remains strong with no voting pact or any specific commitment.

When in favor of a resolution, the situation in which they both vote is an equilibrium. Whereas $\mathcal{A}$ coexits, from Proposition 2, it is not CPNE, as the two largest owners together gain from voting. Shareholders can easily implement such informal agreements when there are only a few and know

each other. The legal obligation to inform the Financial Markets Authority when thresholds (the lowest often being 5%) of the share capital or voting rights are crossed facilitates this process.

Definition 1 *Powerful allied reference shareholders are defined as reference shareholders sharing the same preferences and having the power to pass together the resolutions they support. As an application, in the case of two allied reference shareholders* α_1 *and* α_2*, this is the case when* $\alpha_1 < Q$*,* $\alpha_1 + \alpha_2 \geq Q$ *and* $Min(Q, \alpha_1 + \alpha_2 - Q) > \alpha_3$*.*

If the two reference shareholders disagree with the proposed resolution, it is rejected in equilibrium, as α_3, the largest possible supporter of the resolution does not reach the quorum alone. However, equilibria in favor also exist if favorable shareholders all together reach the quorum. Again, from Proposition 2, only an equilibrium $\mathcal{F}$ is possibly CPNE. However, the deviating coalition necessarily gathers many small favorable shareholders and seems more difficult to coordinate in this case.

Compared with joint control, allied reference shareholders have the power to pass resolutions and have less power to reject them.[23] They should enter into a credible agreement to counter resolutions. If not through a legal form such as a long-term voting pact or voting others' shares, it may also be implemented through shareholder activism. It is common for institutional investors to write a joint letter to the management when they disagree with a resolution and announce that they will not support it. Very often, the resolution is actually withdrawn before the meeting.

Reference shareholders as a counterpower The two reference shareholders at Accor – Colony and Eurazeo – are both investment firms with the same preferences. Blockholders do not necessarily agree. In the presence

[23] Cai, Hillier and Wang (2016) argue that the second- and third-largest shareholders may either collude with the largest one at the expense of other shareholders or play a monitoring role for the dominant shareholder. Based on listed Chinese companies, they present evidence that there are costs to having a second (and third) largest shareholder, consistent with the collusion hypothesis, when the voting shares are similar in size.

of conflicts of interest between the largest and other reference shareholders, the latter may form coalitions to counter the former. Individuals and families often own the largest stake when they are present, and institutions and corporations are the second- and third-largest shareholders in that case. Moreover, Edmans and Holderness (2016) document that 86% of individual blockholders are directors (62% are both directors and officers), and inside (directors or officers) blockholders own 26.8% of the stock in the United States on average.

As an illustration, in 2015, the largest shareholder of Bouygues, a French diversified industrial group, is the Bouygues family (with approximately 21% equity), who also manages the company, followed by Amundi (16% equity) and Eagle (6% equity), both investment companies. With a legal quorum of 20%, from Proposition 4, resolutions proposed by the Board controlled by Bouygues are always adopted. However, the two funds are large enough to be part of a voting coalition in favor of a resolution ($\alpha_i > \alpha_1 - Q = 1\%$ for $i = 2, 3$). The investment companies can counter the largest shareholder (and management) by putting a resolution on the proxy, such as proposing a representative on the board and modifying the compensation policy.

As a consequence of Proposition 5, reference shareholders can credibly contest a large powerful shareholder ($\alpha_1 \geq Q$) when they represent a sufficiently high stake each ($\alpha_i > \alpha_1 - Q$) and together form a coalition V^F representing more votes than the largest shareholder ($\sum_{V^F} \alpha_i > \alpha_1$).[24]

Definition 2 *Reference shareholders α_i, united in a coalition V^F, have the power to counter the weak controller ($\alpha_1 \geq Q > \alpha_2$) when $\alpha_i > \alpha_1 - Q$ for all α_i in V^F and $\sum_{V^F} \alpha_i > \alpha_1$.*

Once again, the importance of rules enabling large-enough reference shareholders to put resolutions to the vote is recognized. It represents a credible

[24] Attig, El Ghoul and Guedhami (2009) examine the impact of multiple large shareholders on the valuation of 1,252 firms from East Asian economies. They find that a greater voting power of the second-largest shareholder relative to the largest one increases firm value. They interpret this result as evidence of an effective monitoring of the former by the latter.

way to challenge the largest shareholder in meetings and, hence, the board, which is often dominated by the latter.

The conditions under which the dominant and reference shareholders can exert power in annual meetings are summarized in Table 2.

Table 2. Control of the meeting by the dominant and reference shareholders

Case 1	Strong Control by the dominant shareholder	Weak Control by the dominant shareholder
$\alpha_1 \geq Q$	$\alpha_2 < Min(Q, \alpha_1 - Q)$ All outcomes are aligned with the dominant shareholder's preference	$\alpha_1 - Q \leq \alpha_2 < Q$ The dominant shareholder can be countered by a coalition of reference shareholders
Case 2	Joint Strong Control	Joint Weak Control
$\alpha_1 < Q$	$\alpha = \alpha_1 + \alpha_2 > Q$ and $\alpha_3 < Min(Q, \alpha - Q)$ Two reference shareholders have a credible commitment to vote as one shareholder	$\alpha_1 + \alpha_2 > Q$ and $\alpha_3 < Min(Q, \alpha_1 + \alpha_2 - Q)$ Powerful allied reference shareholders have the power to pass resolutions but not to oppose them

7 Conclusion

Minimum quorums matter. They encourage the vote among small shareholders, even when the vote entails a cost. Our analysis indeed predicts that resolutions should be adopted in widely held firms, provided that favorable shareholders altogether represent at least the minimum quorum, with the latter working as a coordination mechanism. This result contradicts the widespread view that small shareholders should not vote when they incur a voting cost, but consistent with recent evidence: Appel, Gormley and Keim (2016), suggest that index funds, which should receive the least direct benefit from voting, vote. It can be argued that institutional shareholders

choose to vote to maintain their reputation and therefore actually gain from voting. We provide an alternative explanation by showing that the minimum quorum gives an incentive to vote. At the same time, we demonstrate that minimum quorums lead to abstention from voting to prevent the adoption of resolutions. Rejection due to the lack of votes is possible when the largest opponent reaches the quorum and the largest supporter is below the quorum. This outcome is rarely observed in practice. However, in a straightforward extension of our model, the board of directors should withdraw the concerned resolutions before the meeting. Indeed, the agenda is frequently modified with a new version including less resolutions, following discussions with major shareholders.

Shareholders are heterogeneous in both size and opinion. According to our model, the power of large shareholders, alone or allied, depends on where they stand regarding the proposals and, more importantly, on their size relative to the minimum quorum.

Legally, 50% of outstanding shares plus one is a controlling interest. However, de facto control often requires less than a majority. What percentage stake gives a blockholder effective control? Our results indicate that the dominant shareholder systematically imposes his view when his stake is at least as large as the required quorum provided that the second-largest shareholder does not reach the quorum (when the minimum quorum is 50%, legal control coincides with effective control). In other cases, non-dominant shareholders influence the outcome of the meeting. We find that coalitions of large shareholders can act as an effective counterweight to the dominant shareholder. In particular, they may propose and pass a resolution against the will of the latter, which is consistent with the monitoring role of the second- and third-largest shareholders discussed in the literature (e.g., Cai, Hillier and Wang, 2016).

To conclude, we show that quorum rules are important in the corporate setting. Depending on the shareholder structure, they induce small shareholders to vote, reinforce the control of the dominant shareholder, and facilitate coordination between blockholders to counter the power of the dominant shareholder. It is not surprising that ISS, the largest proxy advisory

firm, systematically provides recommendations in their guidelines regarding resolutions to amend quorum requirements and that these recommendations differ across countries: whereas a case-by-case vote is recommended for Europe or Japan, where large and heterogeneous shareholders prevail, the firm recommends to "vote against proposals to reduce quorum requirements for shareholder meetings below a majority of the shares outstanding unless there are compelling reasons to support the proposal" in the United States, where blockholders are smaller and mostly institutional shareholders.[25]

8 Appendix

Equilibrium conditions

We successively examine the conditions under which the (Nash equilibrium) result of the meeting is to adopt (equilibrium for or $\mathcal{F}$) or reject (equilibrium against or $\mathcal{A}$) the resolution.

We reiterate the properties (P1) and (P2) described in section 4 necessary for a Nash equilibrium.

(P1) No partisan of A (resp. F) votes in equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$).

(P2) In equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$), a shareholder in favor $\alpha_i^F \in P^F$ (resp. a shareholder against $\alpha_i^A \in P^A$) participates in the vote if and only if he is pivotal, i.e., his vote is necessary to obtain his preferred outcome.

Equilibrium $\mathcal{F}$

Let $V^F \subset \mathcal{P}^F$ represent the subset of partisans of F who vote in favor in equilibria $\mathcal{F}$.

Four conditions must hold for an equilibrium $\mathcal{F}$ to exist:

(F1) At least one shareholder in favor of F votes (**simple majority rule, ties are broken in favor of the Status Quo**):

[25] ISS (Institutional Shareholder Services) Proxy Voting Guidelines, 2016 (Brazil, Canada, Europe, Japan, United States).

$$V^F \neq \varnothing$$

As no partisan of A ever votes in equilibrium $\mathcal{F}$ (P1), if no partisan of F votes either, the tie is broken in favor of the status quo and "for" never passes.

(F2) The minimum quorum must be reached (**quorum rule**):

$$\sum_{V^F} \alpha_i^F \geq Q$$

(F3) The winning coalition may not be overturned by any partisan of A; therefore, by the largest α_1^A (**non-contestability condition** for a Nash Equilibrium),

$$\sum_{V^F} \alpha_i^F > \alpha_1^A$$

(F4) All voting partisans α_j^F of the coalition must be pivotal (**pivotal voting partisan condition** for a Nash Equilibrium):

$$\text{either (a)} \sum_{V^F} \alpha_i^F - \alpha_j^F < Q \text{ for any } \alpha_j^F \in V^F \text{ for } Q > 0$$

$$\text{or (b) } V^F = \left\{\alpha_i^F\right\} \text{ for } Q = 0$$

The minimum quorum is not reached, and the Status Quo, which is equivalent to against, prevails if any shareholder $\alpha_j^F \in V^F$ (therefore the smallest) in the coalition does not participate in the vote (a). In the particular case where $Q = 0$ (no minimum quorum), voting coalitions must be composed of only one shareholder in favor so that the turnout is zero when he does not vote (ties are in favor of the Status Quo) (b). Condition (F4) can be rewritten as

$$Max(0, Q^-) \geq \sum_{V^F} \alpha_i^F - \underset{V^F}{Min}\left(\alpha_j^F\right)$$

where Q^- represents the minimum quorum minus one vote.

Since (F3) implies (F1), there remain three conditions (F2) to (F4) for the existence of an equilibrium $\mathcal{F}$. These necessary and sufficient conditions are summarized in (F5):

$$Max(0, Q^-) \geq \sum_{V^F} \alpha_i^F - \underset{V^F}{Min}\left(\alpha_j^F\right) > Max(\alpha_1^A, Q^-) - \underset{V^F}{Min}\left(\alpha_j^F\right)$$

where Q^- represents the minimum quorum minus one vote.

Remarks and additional assumptions Note that (F2) can never hold if $\sum\limits_{\mathcal{P}^F} \alpha_i^F < Q$; similarly, (F3) can never hold if $\alpha_1^A \geq \sum\limits_{\mathcal{P}^F} \alpha_i^F$. We therefore assume that $\sum\limits_{\mathcal{P}^F} \alpha_i^F > Max(\alpha_1^A, Q^-)$ for non-triviality.

Existence of equilibrium $\mathcal{F}$ Let Δ^F represent the set of coalitions of partisans for that cannot be challenged, i.e., that satisfy condition (F3), $\Delta^F = \{V^F \subset \mathcal{P}^F \mid \sum\limits_{V^F} \alpha_i^F > \alpha_1^A\}$ and

$$V_m^F = Arg \underset{V^F \in \Delta^F}{Min} \left\{ \sum_{V^F} \alpha_i^F - \underset{V^F}{Min}(\alpha_j^F) \right\}$$

Among all coalitions in Δ^F, V_m^F is one of the coalitions that gather the least votes when removing its smallest member. We also define $\alpha_m^F \equiv \sum_{V_m^F} \alpha_i^F - \underset{V_m^F}{Min}(\alpha_j^F)$.

Depending on the value of Q, three cases emerge.

***Case 1* - $Q = 0$** As stated above, with no quorum, if an equilibrium coalition exists, it contains only one partisan (F4-b). Condition (F2) is always met (no quorum requirement). Therefore, there exists an equilibrium $\mathcal{F}$ if and only if $\alpha_i^F > \alpha_1^A$ (F3) for some i; in other words, at least one partisan of F commands more votes than the largest partisan of A. V_m^F is the set of singletons of such shareholders. The necessary and sufficient condition simplifies to $\alpha_1^F > \alpha_1^A$. The situation where α_1^F votes alone is an equilibrium; there exist as many equilibria as the number of partisans of F in V_m^F.

Case 2 **-** $\alpha_1^A \geq Q > 0$ The necessary and sufficient conditions for the existence of an equilibrium voting coalition V^F simplify to

$$\alpha_1^A - \underset{V^F}{Min}(\alpha_j^F) < \sum_{V^F} \alpha_i^F - \underset{V^F}{Min}(\alpha_j^F) < Q.$$

Suppose there exists a coalition V^F that supports an equilibrium $\mathcal{F}$.

By definition of V_m^F, we have

$$\alpha_m^F \equiv \sum_{V_m^F} \alpha_i^F - \underset{V_m^F}{Min}(\alpha_j^F) \leq \sum_{V^F} \alpha_i^F - \underset{V^F}{Min}(\alpha_j^F).$$

Therefore, $\alpha_m^F < Q$. Moreover, V_m^F is non-contestable since it belongs by definition to Δ^F. Thus, coalition V_m^F supports an equilibrium $\mathcal{F}$.

Now, suppose $\alpha_m^F < Q$. This means every member of V_m^F is pivotal. In addition, V_m^F is, by definition, non-contestable. Thus, V_m^F is an equilibrium coalition: at least one equilibrium coalition exists (others may coexist).

Case 3 **-** $\alpha_1^A < Q$ As shown in the text, a voting equilibrium $\mathcal{F}$ always exists in this case.

Remarks: - When there exists a coalition V^F s.t. $\sum_{V^F} \alpha_i^F = Q$ (this is the case for a widely held company with atomistic shareholders), the voting coalition may match exactly the minimum quorum Q in equilibrium.

- If $\alpha_1^F > \alpha_1^A$ and $\alpha_1^F \geq Q$, there exists a voting equilibrium where only the largest shareholder in favor votes.

Equilibrium $\mathcal{A}$

$\mathcal{A}$ prevails whenever voters against obtain a majority and the minimum quorum is reached (voting equilibrium) or the minimum quorum is not reached (non-voting equilibrium).

Voting equilibrium Let $V^A \subset \mathcal{P}^A$ represent the set of partisans who vote against in the first case.

For an equilibrium $\mathcal{A}$ to exist, among other conditions, all voting partisans α_j^A in the coalition must be pivotal (**pivotal voting partisan condition** for a Nash Equilibrium), which implies that the resolution is adopted if any shareholder in the coalition does not vote. Because no partisan of F ever votes in equilibrium $\mathcal{A}$ (property (P1)) and ties are in favor of the Status Quo, which is equivalent to against, this can never be the case (the share of the shareholders remaining in the coalition is at least zero).

Non-voting equilibrium In the second case, where equilibrium $\mathcal{A}$ prevails because the minimum quorum is not reached, the turnout is necessarily zero; indeed, from (P1), no partisan of F votes in equilibrium $\mathcal{A}$ since voting is useless, and from (P2), no partisan of $\mathcal{A}$ votes either since voting against is never necessary to prevent the adoption of the resolution when no partisan of F votes. For this situation, where no partisan votes to be a Nash equilibrium, it must be the case that no partisan of F (therefore the largest α_1^F) may change the result, (**non-contestability condition** for a Nash Equilibrium).

Necessary and sufficient condition for existence of an equilibrium $\mathcal{A}$ The necessary and sufficient for existence of a necessarily non-voting equilibrium for the three cases is simply

$$\alpha_1^F < Q.$$

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