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Welfarism and segregation in endogenous jurisdiction formation models

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Abstract

This paper analyses how welfarism affects the segregative properties of endogenous jurisdiction formation, in a model where local jurisdictions produce a local public good and distribute an allowance to their households, both financed by a proportional tax based on the households' wealth. A jurisdiction is composed of all the households that live in the same place. Local wealth tax rates and the level of the allowance are determined to maximize a social welfare function. Households can "vote with their feet", which means that they can choose to move to the jurisdiction that offers the package "tax rate - amount of public good - allowance" that provides the highest utility level. The main result of this article is the proof that the maximin criterium is more segregative than the utilitarian one.

JEL Classification: C78; D02; H73; R13

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1 Introduction

Since the beginning of the XX^{th} century, the share of local public spedings in the overall public spending has benn increasing in many countries. For instance, the total local public spendings in the US are almost equal to the federal spendings. As a consequence, the literature about local public economics has significantly risen. The first article dealing with local public goods is probably Tiebout's intuitions [11], in response to Samuelson's article [10].

Samuelson stated, among other significant results, that preferences over the public good are not observable, thus, a public good can not be financed through volontary contributions, or through contributions that depends on the marginal utility procured by the public good.

Tiebout answered that preferences over public goods can be observable, as households can choose their jurisdiction based on a trade-off between local tax rates and amounts of public goods (vote with one's feet). Consequently, households whose marginal utility with respect to the public good is high will live in a jurisdiction where the tax rate and the amount of public good is high, while households whose preferences weakly depend on the public good will rather settle in jurisdictions wwith a low tax rate. Such a jurisdiction formation is therefore endogenous. Tiebout's article was mostly a non-formal intuition, but has laid the foundations of many formalized articles, either identifying the conditions ensuring the existence of an equilibrium, or examining the segregative properties of endogenous jurisdiction formation.

Westhoff [12] considered a model based on Tiebout's intuitions, in which jurisdictions produce a local public good, financed by a local tax on wealth, and consumed by its inhabitants. Households' utility depends on the amount of public good available in their jurisdiction, and on the net-of-tax wealth. He found a sufficient condition for the existence of an equilibrium: the slopes of individuals' indifference curves in the "tax rate-amount of public good" space must be monotoc with respect to their private wealth. In addition, he proved that, under this condition, a stable jurisdiction structure will always be segregated.

Later, a necessary and sufficient condition has been identified by Gravel and Thoron [5] to ensure the wealth-stratification of any stable jurisdiction structure, as per Greenberg and Weber's definition [6]. This condition is called the Gross Substitutability/Complementarity (GSC) condition. It is respected if and only if the Marshallian demand for the public good is always monotonic with respect to the private good price. Furthermore, the GSC condition is equivalent to the monotonicity of the favorite tax rate function with respect to the private wealth, for any level of prices and wealth.

Oddou [9] examined the robustness of this condition when the public good may suffer from congestion, and households may benefit from other jurisdictions' local public services. In such a framework, the GSC condition is affected neither by the existence of spillovers across jurisdictions, nor the congestion (at least if it is not too strong). Those articles prove the robustness of the GSC condition for several generalizations of Gravel and Thoron's model.

However, as proven by Gravel and Oddou ([4]), two generalizations of the basic model mitigate the segregative properties : the existence of different kinds of public goods within each jurisdiction, and the presence of a housing market (if the preferences over the private good and the housing are not homothetic). These two elements make the GSC condition not sufficient to ensure segregation.

The closest model of this article is Biswas, Gravel and Oddou's article [1]. They added a central government to Gravel and Thoron's model, whose objective is to implement a policy to pursue a welfarist objective. A central government policy is characterized by equalization payments between jurisdiction. Those equalization payments can be either vertical (the government taxes households and redistributes the revenues to jurisdictions), horizontal (the government redistributes local tax revenues between the jurisdictions), or mixed. Two types of social welfare functions are considered in turn: the maximin and the generalized utilitarism. Though a maximin government radically affects the results, they proved that the GSC condition remains necessary and sufficient to ensure the segregation of stable jurisdiction structure with a generalized utilitarian central government. However, their model only allows for redistribution among jurisdictions by the central government, not among households by the jurisdictions. This is the point that this article is generalizing upon.

In this article, we assume that jurisdictions can give their households an allowance in a model framework *à la* Gravel and Thoron. Jurisdictions choose their policy, i.e. the amount of allowances and their tax rates, in order to maximize a social welfare function. We require that the budgets of jurisdictions be balanced.

There is no cost of mobility and households have perfect information, so once all jurisdictions have chosen their policies, households move to the jurisdiction whose policy will satisfy their utility to the highest possible level. Equilibrium is reached as soon as there is no household left who can increase its utility by unilaterally leaving its jurisdiction for another one.

We prove that, in the presence of jurisdictional allowance, whose amount is determined by the local utilitarian government, the Gross Substitutability Complementarity Condition remains necessary, but is no longer sufficient to have all stable jurisdiction structure segregated.

The article is organized as follows: the next section introduces the formal model. Section 3 provides an example of how congestion and spillovers can modify a jurisdiction structure, section 4 states and proves the results. Finally, section 5 concludes.

2 The formal model

We consider an economy with a set of households $I \subset \mathbb{N}$, the number of household in I is given by N. An economy is composed of three elements.

The first element is the preferences. All households share identical preferences, represented by a twice differentiable, increasing and concave utility function

$$U : \left| \begin{array}{ccc} \mathbb{R}^2_{++} & \longrightarrow & \mathbb{R}_+ \\ (Z,x) & \longmapsto & U(Z,x) \end{array} \right.$$

where

1. Z is the available amount of public good,
2. x is the amount of a composite private good.

We denote $Z^M(p_Z, p_x, R)$ and $x^M(p_Z, p_x, R)$ the Marshallian demands for the public good and the private good (respectively), when the public good price is p_Z, the private good price, p_x, and the revenue, R. We also define $MRS(Z, x)$ as the Marginal Rate of Substitution of the public good to the private good.

We add two extra conditions : the **regularity condition** and the Inada condition.

Condition 1. *If, for some public good price p_Z, some income level R and some non degenerated interval I of positive private good price, one has $Z^M(p_Z, p_x, R) = Z^M(p_Z, p'_x, R)$, $\forall (p_x, p'_x) \in I^2$, then, one must have, $\forall \bar{p_x} \in \mathbb{R}_+, Z^M(p_Z, p_x, R) = Z^M(p_Z, \bar{p_x}, R)$*

Condition 2. *The preferences satisfies the Inada condition if and only if, $\forall x > 0$, $\lim\limits_{Z \to 0} U(Z, x) = \infty$ and, $\forall Z > 0$, $\lim\limits_{x \to 0} U(Z, x) = \infty$*

$\mathbb{U}$ representes the set of all functions satisfying the properties defined above.

The second element is the wealth distribution, given by $\omega : \left| \begin{array}{ccc} I & \longrightarrow & \mathbb{R}_+^* \\ i & \longmapsto & \omega_i \end{array} \right.$ - household i is endowed with a wealth $\omega_i \in \mathbb{R}_+$ - with ω being an increasing and bounded from above function.

The third element is the set of conceivable locations. Households choose their place of residence among the finite set of locations, represented by $\mathbb{L} \subset \mathbb{N}$. The possibility for some locations to be empty is allowed. Households living at the same location form a jurisdiction. We denote $J \subseteq \mathbb{L}$ the set of jurisdictions. The subset of households living in j is denoted I_j. As every household must live in one and only one jurisdiction, one has $\forall (j, j') \in J^2$, $\bigcup\limits_{j \in J} I_j = I$ and $j \neq j', I_j \cap I_{j'} = \emptyset$.

Jurisdictions have two purposes: producing a public good, that will be consumed only by the households that compose it, and distributing an allowance to households. To finance those two functions, the jurisdictions raise a tax proportional to the households' wealth. The tax rate in jurisdiction j is denoted t_j, the amount of allowance provided by jurisdiction j is denoted G_j.

The amount of local public good produced by jurisdiction j is given by

$$Z_j = t_j \varpi_j - n_j G_j$$

with :

- $n_j = card(I_j)$ being the number of households in j,
- $\varpi_j = \sum\limits_{i \in I_j} \omega_i$ being the aggregated wealth in j.

The amount of the composite private good that household i living in jurisdiction j is given by

$$x_{ij} = (1 - t_j)\omega_i + G_j$$

The preferences and the production of the public good having been introduced, we can now define the favorite tax rate function.

Definition 1. $\forall(\varpi, n, G, \omega_i) \in \mathbb{R}_+^3 \times \mathbb{N}$, *we define*

$$t^* : \left| \begin{array}{ccl} \mathbb{R}_+^3 \times \mathbb{N} & \longrightarrow & [0;1] \\ (\varpi, n, G, \omega_i) & \longmapsto & t^*(\varpi, n, G, \omega_i) = \underset{t\in[0;1]}{\operatorname{argmax}} U(t\varpi - nG, (1-t)\omega_i + G) \end{array} \right.$$

as the favorite tax rate function, eg the tax rate that maximizes the utility of a household endowed with private wealth ω_i, in a jurisdiction with an aggregated wealth ϖ, a number of inhabitants equal to n, that grants an amount G of allowance.

Under the standard assumptions, $\forall(\varpi, n, G, \omega_i) \in mathbbR_+^3 \times \mathbb{N}$, the function $U(t\varpi - nG; (1-t)\omega_i + G)$ is single-peaked with respect to t, i.e. $\forall \bar{t} < t*(\varpi, n, G, \omega_i)$ (resp. $>$), $\forall t \in]\bar{t}; t^*(\varpi, n, G, \omega_i)[$ (resp. $\forall t \in]t^*(\varpi, n, G, \omega_i); \bar{t}[$), $U(\bar{t}\varpi - nG, (1-\bar{t})\omega_i + G) < U(t\varpi - nG, (1-t)\omega_i + G) < U(t^*(\varpi, n, G, \omega_i)\varpi - nG, (1-t^*(\varpi, n, G, \omega_i))\omega_i + G)$. Consequently, this "favorite tax rate function" always exists. For $G = 0$ (as in Gravel and Thoron's article), the monotonicity of the favorite tax rate function is equivalent to the GSC condition, but it is not the case otherwise. However, the next lemma will define the relation that exists between the favorite tax rate function and the Marshallian demand for the public good. Some conditions have to be respected for t^* to exist. For instance, clearly, the function will not exist if $\varpi < nG$, because even if all the wealth was taxed, it would not be sufficient to finance the allowance.

The next lemma will define the relation between the favorite tax rate function and the Marshallian demand for the public good.

Lemma 1. *For any preferences belonging to $\mathbb{U}$, one has, $\forall(\omega_i, \varpi, \mu, G) \in \mathbb{R}_+^4$,:*

$$t^*(\varpi, n, G, \omega_i) \equiv \frac{Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 + G(\frac{1}{\omega_i} + \frac{n}{\varpi})) + nG}{\varpi}) \tag{1}$$

Proof. At the optimum, the MRS is equal to the price ratio. Hence, one has:

$$MRS = \frac{p_Z}{p_x}] \tag{2}$$

$$\tag{3}$$

The first order condition (FOC) implies that:

$$\frac{U_Z(t^*\varpi - nG; (1-t^*)\omega_i + G)}{U_x(t^*\varpi - nG; (1-t^*)\omega_i + G)} = \frac{\omega_i}{\varpi} \tag{4}$$

$$\tag{5}$$

Consequently, combining equations (2) and (4), we know that:

$$t^*(\varpi, n, G, \omega_i)\varpi - \mu G = Z^M(p_Z, p_x, R) \tag{6}$$

$$(1 - t^*(\varpi, n, G, \omega_i)\omega_i + G = x^M(p_Z, p_x, R) \tag{7}$$

$$\tag{8}$$

when $p_Z = \frac{1}{\varpi}, p_x = \frac{1}{\omega_i}$ and $R = 1 + G(\frac{1}{\omega_i} + \frac{n}{\varpi})$. which leads to the result. □

We can now formally provide the definition of an economy.

Definition 2. *An economy is composed of 3 elements:*

- *Preferences represented by the utility function* $U \in \mathbb{U}$
- *A wealth distribution* ω
- *A set of locations* $\mathbb{L} \in \mathbb{N}$

We denote Δ as the set of all economies that respect the assumptions presented above. Let us now define the notion of jurisdiction structure in the economy $(U, \omega, \mathbb{L})$.

Definition 3. *A jurisdiction structure in the economy* $(U, \omega, \mathbb{L})$ *is a vector* $\Omega = (J, \{I_j\}_{j\in J}; (\{t_j\}_{j\in J}); (\{G_j\}_{j\in J})$.

Literally, a jurisdiction structure is characterized by the set of jurisdictions, the partition of households among the different jurisdictions, and the vectors of the tax rates and amounts of allowance implemented in every jurisdiction.

Before presenting the notion of equilibrium, we must introduce the notion of **welfarist local government**. A welfarist local government chooses the policy that will maximize a certain social welfare function. As in [1], we will only consider two kinds of objective for the local governments : the *maximin* and the *utilitarism.* First, we will present the objective of a maximin government.

Definition 4. *A local government of a jurisdiction* j *with a set of households* I_j *and a aggregate wealth* ϖ_j *pursues a maximin objective if and only if its policy* (t_j, G_j) *is such that,* $\forall (t, G) \in [0; 1] \times \mathbb{R}_+, \min\limits_{i\in I_j} U(t_j\varpi_j - n_jG_j; (1 - t_j)\omega_i + G_j) \geq \min\limits_{i\in I_j} U(t\varpi_j - n_jG; (1 - t)\omega_i + G)$.

In words, a maximin local government chooses the policy that provides the highest possible level of utility to the poorest household of the jurisdiction. Alternatively, a maximin government of the jurisdiction j chooses a policy (t^{max}, G^{max}) that solves the maximization program

$$\max_{(t,G)\in[0;1]\times\mathbb{R}_+} \inf_{i\in I_j} U(t\varpi_j - n_jG, (1 - t)\omega_i + G) \tag{9}$$

We can now provide a definition of a **stable** jurisdiction structure **with maximin local governments**.

Definition 5. *A jurisdiction structure* $\Omega = (J, (\{I_j\}_{j\in J}); (\{t_j\}_{j\in J}); (\{G_j\}_{j\in J})$ *is* ***stable*** *in the economy* $(U, \omega, \mathbb{L})$ *if and only if*

1. $\forall j, j' \in J, \forall i \in I_j, U(Z_j, (1 - t_j)\omega_i + G_j) \geq U(Z_{j'}, (1 - t_{j'})\omega_i + G_{j'})$,
2. $\forall j \in J, Z_j \leq \frac{t_j\varpi_j - n_jG_j}{p_Z}$
3. $\forall j \in J, \forall (t, G) \in [0; 1] \times \mathbb{R}_+, \min\limits_{i\in I_j} U(t_j\varpi_j - n_jG_j; (1 - t_j)\omega_i + G_j) \geq \min\limits_{i\in I_j} U(t\varpi_j - n_jG; (1 - t)\omega_i + G)$.

In words, a jurisdiction structure is stable if and only if :

1. No household can increase its utility by modifying its consumption bundle or by leaving its jurisdiction,
2. Every jurisdiction's budget is balanced,
3. Every jurisdiction applies a policy that maximizes the utility of its poorest inhabitant.

We can now switch to the definition of a utilitarian local government.

Definition 6. *A local government of the jurisdiction j with a set of households I_j and an aggregate wealth equal to ϖ_j is utilitarian if and only if it chooses the policy that maximizes the following program:*

$$\max_{(t,G)\in[0;1]\times\mathbb{R}_+} \sum_{i\in I_j} U(t\varpi_j - n_jG, (1-t)\omega_i + G) \tag{10}$$

In words, a utilitarian local government chooses the bundle "tax rate-allowance" that maximizes the sum of household utilities. Thus, we can now define the notion of equilibrium **with utilitarian local governments**.

Definition 7. *A jurisdiction structure $\Omega = (J, (\{I_j\}_{j\in J}); (\{t_j\}_{j\in J}); (\{G_j\}_{j\in J})$ is **stable** in the economy $(U, \omega, \mathbb{L})$ if and only if*

1. $\forall j, j' \in J, \forall i \in I_j, U(Z_j, (1-t_j)\omega_i + G_j) \geq U(Z_{j'}, (1-t_{j'})\omega_i + G_{j'})$,
2. $\forall j \in J, Z_j \leq \frac{t_j\varpi_j - n_jG_j}{p_Z}$
3. $\forall j \in J, \forall (t,G) \in [0;1]\times\mathbb{R}_+, \sum\limits_{i\in I_j} U(t_j\varpi_j - n_jG_j; (1-t_j)\omega_i + G_j) \geq \sum\limits_{i\in I_j} U(t\varpi_j - n_jG; (1-t)\omega_i + G)$.

Let us now formally express the definition of the segregation, which is the same definition as in [12].

Definition 8. *A jurisdiction structure $\Omega = (J, (\{I_j\}_{j\in J}); (\{t_j\}_{j\in J}); (\{G_j\}_{j\in J})$ in the economy $(\omega, U, \mathbb{L})$ is **segregated** if and only if $\forall(\omega_h, \omega_i, \omega_k) \in \mathbb{R}^3_+$ such that $\omega_h < \omega_i < \omega_k$, $(h,k) \in I_j$ and $i \in I_{j'} \Rightarrow Z_j = Z_{j'}, G_j = G_{j'}$ and $t_j = t_{j'}$*

In words, a jurisdiction structure is wealth-segregated if, except for groups of jurisdictions offering the same available amount of public good, the same amount of allowance and the same tax rate, the poorest household of a jurisdiction with a high per capita wealth is (weakly) richer than the richest household in a jurisdiction with a lower per capita wealth.

In the next section, we examine the robustness of the GSC condition to ensure segregation.

3 Results

This article proves that, with maximin local governemnts, the stable jurisdiction structure will always be segregated, but, with utilitarian local governments, the GSC condition is no longer sufficient to have all stable jurisdiction structures segregated. Let us first prove the first part of this statement.

3.1 With maximin local governments

First, we need to provide a solution to a maximin local government maximization program.

Lemma 2. *Under a maximin local government, a solution of the program (9) is $t^{max} = 1$ and $G^{max} = \underset{G\in[0;\frac{\varpi_j}{n_j}]}{\text{argmax}}(\varpi_j - n_jG; G)$*[1]*.*
This solution is unique as soon as at least two strictly positive numbers of households in the jurisdiction j are endowed with different levels of wealth, while, if a

[1] $G^{max} = \frac{(1-t^*(\varpi_j, n_j, G, \omega))\varpi_j}{n_j}$ with $G = 0$ and $\omega = \frac{\varpi_j}{n_j}$

jurisdiction only contains households endowed with the same level of wealth ω_i, then, $\forall \alpha \in [0;1], t^{max} = \alpha + (1-\alpha)t^*(\varpi_j, n_j, 0, \omega_i)$ and $G^{max} = \alpha G^{max}$ are solutions of the program (9).

Proof. The proof of this lemma is quite intuitive. Consider a jurisdiction j with a maximin government, whose poorest household is i. Obviously, the poorest household's private wealth is less than or equal to the average wealth of the jurisdiction, i.e. $\omega_i \leq \frac{\varpi_j}{n_j}$. This inequality becomes strict as soon as there is a non-null number of households strictly richer than households of type i in the jurisdiction.

Deriving $U(t\varpi_j - n_jG, (1-t)\omega_i + G)$ with respect to t and to G, one obtains:

$$\varpi_j \frac{U(t\varpi_j - n_jG, (1-t)\omega_i + G)}{\partial Z} - \omega_i \frac{U(t\varpi_j - n_jG, (1-t)\omega_i + G)}{\partial x} \quad (11)$$

$$-n_j \frac{U(t\varpi_j - n_jG, (1-t)\omega_i + G)}{\partial Z} + \frac{U(t\varpi_j - n_jG, (1-t)\omega_i + G)}{\partial x} \quad (12)$$

Let us suppose, first, that there exist at least one household in jurisdiction j that is richer than households i. Then, $\frac{\varpi_j}{n_j} > \omega_i$, if, for some $(\bar{t}, \bar{G}) \in [0;1] \times \mathbb{R}_+$, one has

$$-n_j \frac{U(\bar{t}\varpi_j - n_j\bar{G}, (1-\bar{t})\omega_i + \bar{G})}{\partial Z} + \frac{U(\bar{t}\varpi_j - n_j\bar{G}, (1-\bar{t})\omega_i + \bar{G})}{\partial x} = 0$$

then one has

$$\varpi \frac{U(\bar{t}\varpi_j - n_j\bar{G}, (1-\bar{t})\omega_i + \bar{G})}{\partial Z} - \omega_i \frac{U(\bar{t}\varpi_j - n_j\bar{G}, (1-\bar{t})\omega_i + \bar{G})}{\partial x} > 0$$

If one fixes $t = 1$, then, thanks to the Inada condition, we know that there exists $G^{max} \in]0; \frac{\varpi_j}{n_j}[$ such that $-n_j \frac{U(\varpi_j - n_jG^{max}, G^{max})}{\partial Z} + \frac{U(\varpi_j - n_jG^{max}, G^{max})}{\partial x} = 0$.

As t is bounded from above to 1, and given that

$$\varpi_j \frac{U(\varpi_j - n_jG^{max}, G^{max})}{\partial Z} - \omega_i \frac{U(\varpi_j - n_jG^{max}, G^{max})}{\partial x} > 0$$

we can conclude that $t^{max} = 1$ and $G^{max} = \underset{G \in [0; \frac{\varpi_j}{n_j}]}{\text{argmax}}(\varpi_j - n_jG; G)$ is the unique solution of the program (9).

Let us now suppose that all households in jurisdiction j are endowed with the same wealth. The solution $t^{max} = 1$ and $G^{max} = \underset{G \in [0; \frac{\varpi_j}{n_j}]}{\text{argmax}}(\varpi_j - n_jG; G)$ remains a valid solution, but is no longer unique.

One can observe that (11) $= -\omega_i$(12). So, if (11) $= 0$, then (12) $= 0$. By definition, $t = t^*(\varpi_j, n_j, 0, \omega_i$ and $G = 0$ is another solution for the local government maximization program. Furthermore, $\forall \alpha \in [0;1], t^{max} = \alpha + (1-\alpha)t^*(\varpi_j, n_j, 0, \omega_i)$ and $G^{max} = \alpha G^{max}$ will generate the exact same amounts of public good and private good, thus there are solutions for the maximin local government maximization program. □

Literally explained, a maximin local government can, by fixing the tax rate to 1 and redistributing a fixed allowance, equalize all households' wealth to the average wealth, which is the ultimate objective for a maximin government. Consequently, at equilibrium, every jurisdiction structure will be segregated, as stated by the following proposition.

Theorem 1. *A stable jurisdiction structure with maximin local governements is always segregated.*

Proof. Suppose that there exists a non segregated jurisdiction. Hence, there exists (at least) two jurisdictions 1 and 2, and (at least) three households (with a positive integer) with private wealth $\omega_h < \omega_i < \omega_k$ such that:

$$U(t_1\varpi_1 - n_1G_1, (1-t_1)\omega_h + G_1) \geq U(t_2\varpi_2 - n_2G_2, (1-t_2)\omega_h + G_2) \tag{13}$$

$$U(t_1\varpi_1 - n_1G_1, (1-t_1)\omega_i + G_1) \leq U(t_2\varpi_2 - n_2G_2, (1-t_2)\omega_i + G_2) \tag{14}$$

$$U(t_1\varpi_1 - n_1G_1, (1-t_1)\omega_k + G_1) \geq U(t_2\varpi_2 - n_2G_2, (1-t_2)\omega_k + G_2) \tag{15}$$

$$\tag{16}$$

with at least one strict inequality.

Thanks to lemma 2, we know that, at equilibrium, $t_1 = 1$. Hence, all households living in jurisdiction 1 have the same utility level, thus, $U(t_1\varpi_1 - n_1G_1, (1-t_1)\omega_i + G_1) = U(t_1\varpi_1 - n_1G_1, (1-t_1)\omega_k + G_1)$. In addition, as $\omega_i < \omega_k$, we know that $U(t_2\varpi_2 - n_2G_2, (1-t_2)\omega_i + G_2) < U(t_2\varpi_2 - n_2G_2, (1-t_2)\omega_k + G_2)$.

Combining this inequality and equation ((14) and (15), one obtains:

$$U(t_1\varpi_1 - n_1G_1, (1-t_1)\omega_k + G_1) \geq U(t_2\varpi_2 - n_2G_2, (1-t_2)\omega_k + G_2)$$

$$>$$

$$U(t_2\varpi_2 - n_2G_2, (1-t_2)\omega_i + G_2) \geq U(t_1\varpi_1 - n_1G_1, (1-t_1)\omega_k + G_1)$$

which is impossible, as $U(t_1\varpi_1 - n_1G_1, (1-t_1)\omega_i + G_1) = U(t_1\varpi_1 - n_1G_1, (1-t_1)\omega_k + G_1)$.

□

To be more specific, the only stable jurisdiction structure that can emerge are either the grand jurisdiction, or, if, at equilibrium, there is more than one jurisdiction, then all jurisdictions that contains at least two types of households with a non-null number implement the same policy and offer the same amount of public good. In this second situation, there can exist jurisdictions containing only households having the same wealth, who are richer than the richest household of any jurisdiction containing two different kinds of households. The following example present such a jurisdiction structure.
B

Let us consider households' preferences represented by the following utility function

$$U(Z,x) = \begin{cases} \ln(Z) + 4x - x^2 & \text{if } x \leq 2 \\ \ln(Z) + 4 & \text{otherwise} \end{cases}$$

Such an utility function is continuous, twice differentiable, strictly increasing as long as $x < 2$ and concave with respect to every argument. Furthermore, such preferences violates the GSC condition. Consider an economy with three jurisdictions j_1, j_2 and j_3 and four types of households a, b, c and d with private wealth $\omega_a = 0.4$, $\omega_b = 2-\sqrt{2}$, $\omega_c = 1$ and $\omega_d = 2$, and with number $n_a = 1$, $n_b = 4$, $n_c = 2$ and $n_d = 1$.

As long as $x \leq 2$ and $G = 0$, the favorite tax rate function is given by

$$t^*(\varpi, \omega_i) = \frac{\omega_i - 2 + \sqrt{(\omega_i - 2)^2 + 2}}{2\omega_i}$$

Here, the favorite tax rate function depends only on the private wealth, and not on the aggregate wealth, nor, obviously, on the number of households, as $G = 0$. Determining the preferred tax rate function if $x > 2$ will not be required, no household will be endowed with more than 2 units of private good.

One possible stable jurisdiction structure is where households of type a and b are placed in jurisdiction 1, households of type c, in jurisdiction 2, and households d, in jurisdiction 3. Hence, one will have $\varpi_1 \approx 2.7431$ and $n_1 = 5$, $\varpi_2 = \varpi_3 = 2$ and $n_2 \approx 2$ and $n_3 = 1$.

According to lemma 2, the local government of jurisdiction 1 will set $t_1 = 1$ and $G_1 \approx 0.2835$. As jurisdictions 2 and 3 respectively contains households endowed with the same wealth, they can implement different policies, that would maximize the maximin social welfare function. Let us suppose that they both fix $G = 0$. Accordingly, the jurisdictions will set their tax rate to the favorite tax rate of their households when $G = 0$, i.e. $t_2 = t^*(2; 2; 0; 1) \approx 0.36603$ and $t_3 = t^*(2; 1; 0; 2) \approx 0.35355$.

Therefore, households' utility level (rounded to three digits after the decimal point) in every jurisdiction would be:

	j_1	j_2	j_3
a	1.336	0.638	0.621
b	1.336	1.036	1.025
c	1.336	1.822	1.821
d	1.336	3.152	3.153

So, this jurisdiction structure is stable. We can make two remarks. First, if jurisdictions 2 or 3 had implemented another policy, for instance, by setting their tax rate to 1, and their allowance to $G_2 = 0.6340$ and $G_3 = 1.293$, households would have kept the same utility level in their own jurisdiction, but the jurisdiction structure would not have been stable anymore, as the new utility level would have been, in this case:

	j_1	j_2	j_3
a	1.336	1.822	3.153
b	1.336	1.822	3.153
c	1.336	1.822	3.153
d	1.336	1.822	3.153

So every household would have an incentive to move to jurisdiction 3.

The second remark is that, in such an economy, with such preferences, there exist two other stable jurisdiction structures. One, obviously, is the grand jurisdiction, containing all households, and the second one is where households of type a, b and c are placed in jurisdiction 1 and households d, in jurisdiction 3. Hence, one will have $\varpi_1 = 4.7431$ and $n_1 = 7$, $\varpi_3 = 2$ and $n_3 = 1$. Local government of jurisdiction 1 would set $t_1 = 1$ and $G_1 \approx 0.2061$. Jurisdiction 3 would keep $t_3 = t^*(2; 1; 0; 2) \approx 0.35355$ and $G = 0$. The utility level would then be:

	j_1	j_3
a	1.976	0621
b	1.976	1.025
c	1.976	1.822
d	1.976	3.153

So this jurisdiction structure would be stable as well. As a consequence, as this example reinforces the proof of theorem 1, with maximin local governments, no stable jurisdiction structure can be segregated when local governments pursue a maximin objective.

We can now switch to the results with another welfarist objective: the utilitarism.

3.2 With utilitarian local governments

Let us start this subsection by defining the GSC condition.

Definition 9. *The GCS condition holds if and only if one has either* $\frac{\partial Z^M(p_Z,p_x,R)}{\partial p_x} \leq 0 \forall (p_Z, p_x, R)$ *(if* Z *is a gross complement to* x*) or* $\frac{\partial Z^M(p_Z,p_x,R)}{\partial p_x} \geq 0 \forall (p_Z, p_x, R)$ *(if* Z *is a gross substitute for* x*)*

If $G = 0$, then the GSC condition and the monotonicity of the favorite tax rate function with respect to the private wealth are equivalent (the proof provided by Gravel and Thoron holds in this very case). However, if $G > 0$, the only relation between the Marshallian demand for the public good and the favorite tax rate function is that the favorite tax rate function will be decreasing with respect to the private wealth **if** the public good is a normal good and a gross substitute of the private good.

Lemma 3. *If the public good is a normal good and a gross substitute of the private good, then the favorite tax rate function is decreasing with respect to the private wealth.*

Proof. Deriving equation (1) with respect to ω_i, one has:

$$\frac{\partial t*(\varpi, n, G, \omega_i)}{\partial \omega_i} \tag{17}$$

$$= \frac{-1}{p_x^2 \varpi}[\frac{\partial Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 + G(\frac{1}{\omega_i} + \frac{n}{\varpi})}{\partial p_x} + G\frac{\partial Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 + G(\frac{1}{\omega_i} + \frac{n}{\varpi})}{\partial R}] \tag{18}$$

$$\tag{19}$$

As $\frac{\partial Z^M(\frac{1}{\varpi},\frac{1}{\omega_i},1+G(\frac{1}{\omega_i}+\frac{n}{G}\varpi)}{\partial p_x} > 0$ if the public good is a gross substitute of the private good, and $\frac{\partial Z^M(\frac{1}{\varpi},\frac{1}{\omega_i},1+G(\frac{1}{\omega_i}+\frac{n}{G}\varpi)}{\partial R} > 0$ if the public good is normal, then the favorite tax rate function will be decreasing **if** the public good is a gross substitute. □

We can now formally state the main result of this article.

Theorem 2. *For all economies belonging to* Δ*, the GSC condition is not sufficient to have any stable jurisdiction structure segregated.*

Proof. To prove this proposition, we consider, in turn, two different economies (with two different preferences), the public good being a gross complement to the private good with the first one, and a gross substitute for the second one, and, for each preference, we construct a stable and yet non-segregated jurisdiction structure.

Let us consider the following utility function : $U(Z, x) = ln(Z) - \frac{1}{x}$

Such a utility function respects the assumptions presented above. The public good is a gross complement to the private good, as the Marshallian demand for the public good is given by:

$$Z^M(p_Z, p_x, R) = \frac{2R + p_x - \sqrt{p_x^2 + 4Rp_x}}{2p_Z}$$

which is decreasing with respect to p_x. Let us consider an economy with two jurisdictions j_1 and j_2 and three types of households a, b, c with private wealth $\omega_a = 0.1$, $\omega_b = 1$ and $\omega_c = 20$, with $n_a = 1$, $n_b = 100,000$ and $n_c = 5,000$.

A stable jurisdiction structure would be households of type a and c living in one jurisdiction, denoted j_1, with $G_1 = 0.15171$ and $t_1 \approx 0.7973665$, hence one has $Z_1 \approx 78,978$, and households of type b, living in a second jurisdiction j_2, with $G_2 = 0$ and $t_2 \approx 0.2680$, hence one has $Z_2 = 26,795$.

Consequently, one has the following utility level (rounded to four digits after the decimal point):

	j_1	j_2
a	5.4621	−3.4643
b	8.4548	8.8299
c	11.0391	10.1277

Futhermore, one can check that the first order conditions, given by:

$$\sum_{i \in I_j} \frac{\varpi}{t\varpi - nG} - \frac{\omega_i}{[(1-t)\omega_i + G]^2} = 0 \tag{20}$$

$$\sum_{i \in I_j} \frac{n}{t\varpi - nG} - \frac{1}{[(1-t)\omega_i + G]^2} = 0 \tag{21}$$

$$\tag{22}$$

are respected for both jurisdictions.

Hence, the above jurisdiction structure is stable and yet, non-segregated.

Let us now consider the second economy, with a utility function for which the public good is a gross substitute of the private good, such as: $U(Z, x) = ln(Z) + \sqrt{x}$

This function also respects the assumptions developed in the previous section. The Marshallian demand for the public good is given by :

$$Z^M(p_Z, p_x, R) = \frac{2(\sqrt{p_x^2 + Rp_x} - p_x)}{p_Z}$$

which is increasing with respect to p_x

We can construct a stable jurisdiction structure by placing 44 households of type a and 15 households of type c in jurisdiction j_1, with $G_1 = 0.7275$ and $t_1 = 0.4805$, hence one has $Z_1 \approx 122.3695$, and households of type b, living in a second jurisdiction j_2, with $G_2 = 0$ and $t_2 = 0.7320$, hence one has $Z_2 = 222.5280$.

One will, therefore, have the following utility level (rounded to four digits after the decimal point):

	j_1	j_2
a	5.9237	5.9227
b	6.1361	6.1372
c	8.1413	7.7202

As previously, the first order conditions, given by:

$$\sum_{i \in I_j} \frac{\varpi}{t\varpi - nG} - \frac{\omega_i}{2\sqrt{(1-t)\omega_i + G}} = 0 \tag{23}$$

$$\sum_{i \in I_j} \frac{n}{t\varpi - nG} - \frac{1}{2\sqrt{(1-t)\omega_i + G}} = 0 \tag{24}$$

$$\tag{25}$$

are respected.

Hence, the above jurisdiction structure is also stable and yet, non-segregated, which proves the proposition. □

4 Conclusion

The main conclusion of this paper is that, when jurisdictions can distribute an allowance to its inhabitants, endogenous jurisdiction formation is more segregative with maximin local governments than with utilitarian local governments. The effect of this allowance is remarkable, as endogenous jurisdiction formation without allowances is, in terms of segregative properties, between the two welfarist approaches taken into account in this article.

Summed in one sentence, this article allows to state that struggling against wealth inequalities favors wealth segregation. Such intuitions were already expressed in [1], with a welfarist central government, but they are reinforced with welfarist local governments.

However, one must note that our analysis is reduced to models *à la* Westhoff, and that we do not specify whether the GSC condition is necessary or not with utilitarian local governments. Also, we only consider two types of social welfare functions. Observing the segregative properties of other social welfare functions, such as the generalized utilitarism would be an interesting topic. Finally, we assume that all local governments share the same welfarist objective. For further researches, examining what would happen in a jurisdiction structure where both maximin and utilitarian local governments co-exist would be worthy of interest.

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