"Taking Diversity into Account": Real Effects of Accounting Measurement on Asset Allocation

Document de Travail
Working Paper
2018-28

Gaëtan Le Quang
"Taking Diversity into Account": Real Effects of Accounting Measurement on Asset Allocation

Gaëtan Le Quang*

May 2018

Abstract: Following a request made by the G20, the IASB begins to work in 2009 on a new accounting standard meant to replace IAS 39: IFRS 9. Among other things, IFRS 9 puts forward a new way of classifying financial instruments that rests on a two-step procedure: a business model assessment and a contractual cash flow characteristics test. We develop a theoretical model that assesses the relevance of this procedure, specifically that of the business model assessment. We show that a mixed accounting regime where financial institutions whose time horizon is short resort to fair value accounting while those whose time horizon is longer resort to historical cost accounting provides a better asset allocation than a pure accounting regime where all FIs resort to the same accounting rule. In other words, business models are worth being taken into consideration when deciding whether an asset should be evaluated at its fair value or at its historical cost, which is in line with the framework presented in IFRS 9.


*EconomiX (UMR7235), GATE-LSE (UMR5824) and Université Paris-Nanterre, 200 Avenue de la République, 92001 Nanterre cedex, France. E-mail: glequang@parisnanterre.fr
d’actifs que des régimes où les institutions financières sont soumises à une règle comptable qui ne tient pas compte de leur modèle économique. En conclusion, notre modèle propose une justification théorique à l’évaluation du modèle économique telle que proposée par IFRS 9.

**Keywords:** IFRS 9; Fair Value Accounting, Historical Cost Accounting; Asset Allocation; Real Effects of Accounting; Banks; Insurers

**JEL codes:** G11; G21; G22; M41

1 Introduction

Since the global financial crisis, accounting issues, particularly those related to fair value accounting,\(^1\) have become more and more popular among economists, as suggested by the growing literature on the matter (see Beatty and Liao, 2014). In particular, fair value accounting is accused of intensifying and synchronizing financial institutions’ responses to changes in their economic environment and consequently to further harm the financial system in times of crisis (Plantin *et al.*, 2008). This is the reason why the IASB decided to amend IAS 39 in October 2008 by making it possible for financial institutions (FIs) to reclassify financial instruments from a class resorting to fair value to a class resorting to historical cost. From this time on, the IASB has worked on a new accounting standard that would optimally allow FIs to choose either fair value accounting (FVA) or historical cost accounting (HCA) to evaluate their financial instruments. This led to the implementation of IFRS 9 in 2018, which puts forward a mixed accounting regime based on an evaluation of FIs’ business model.

The idea behind the business model assessment is to make it possible for FIs to choose the accounting method that best suits their activities. For instance, the financial instruments found on the liability side of a long-term investor – such as a young pension fund or a life insurer – are mainly long-term instruments whereas those found on the liability side of a bank generally have a shorter maturity. Consequently a long-term investor is able to invest safely in long-term assets since the maturities of his liabilities

\(^{1}\)The first occurrence of the expression ‘fair value’ is to be found in the 1975 U.S. standard FAS 12. More precisely, FAS 13, which was published in 1976, defines fair value as ‘[the] normal selling price, reflecting any volume or trade discounts that may be applicable’. As for Europe, the first accounting standard to mention fair value was IAS 32, which was issued in 1995. The definition currently used in the European Union is that of 2011 IFRS 13.
make unexpected liquidity needs in the short-term unlikely. On the contrary, since banks are engaged in maturity transformation, they are constantly vulnerable to unexpected liquidity needs, which lie at the very center of bank runs as described by Diamond and Dybvig (1983). Banks should therefore invest in short-term liquid assets in order to cover themselves against unexpected liquidity needs, this is the very logic underlying the Liquidity Coverage Ratio (LCR) put forward by the Basel Committee on Banking Supervision. In short, from an accounting point of view, long-term investors would be better off resorting to HCA since short-term market fluctuations are irrelevant to them due to the maturities of their liabilities, while banks would be better off resorting to FVA. This is precisely the logic of the classification of financial instruments put forward by IFRS 9, which is summarized in Figure 1.

This article aims at assessing the relevance of FIs’ business model as a key variable to determine whether a FI should resort to FVA or to HCA to evaluate its financial instruments. To do so we develop a theoretical model where FIs have two successive decisions to make: each FI first chooses the composition of its portfolio of assets by allocating an initial endowment between a risky and a riskless asset and then each FI decides whether to hold its risky asset until maturity or not. Business models are introduced through a parameter that accounts for FIs’ time preference. Specifically we assume that FIs whose business model consists in collecting contractual cash flows and selling financial assets are more short-sighted than those whose business model consists in holding assets to maturity to collect contractual cash flows. We compare two cases: one where FIs are subject to FVA and another where FIs resort to HCA. We then design a mixed accounting regime where FIs are allowed to use either FVA or HCA depending on their business model. Specifically, in this mixed accounting regime long-sighted FIs are allowed to use HCA while short-sighted FIs resort to FVA. Our model allows us to exhibit the following results:

- When FIs are long-sighted, both FVA and HCA achieve the first-best and accounting does not have real effects on FIs’ decisions.

- When FIs’ time horizon is shorter than the duration of the risky asset they invest in, real effects associated with accounting arise and are as follows:
– When FIs are subject to FVA and when the expected return associated with the risky asset is low enough, they tend to overreact to expected drops in the price of the risky asset by underinvesting in it. On the contrary, when the expected return associated with the risky asset is high enough and provided that the market is not too illiquid, FIs underestimate the risk associated with a high-yield risky asset and overinvest in it.

– When FIs are subject to HCA, they always overinvest in the risky asset. In particular, the more short-sighted FIs are, the more they tend to overinvest in the risky asset.

• The mixed accounting regime we design reaches the first-best allocation between the risky and the riskless asset when FIs are discriminated regarding their time horizon. Especially, this mixed accounting regime is consistent with that put forward by IFRS 9.

Our paper belongs to the burgeoning literature on the real effects of accounting regimes (Kanodia, 2006). This literature starts from the hypothesis that measurement and disclosure rules have real effects on the economy in the sense that they have a direct influence on the way managers behave. Those effects have to be taken into consideration while comparing accounting regimes. In particular, the comparison between HCA and FVA cannot rest only on the study of the value relevance of fair value but has to take into account the idea that 'how accountants measure and disclose a firm’s economic transactions changes those transactions' (Kanodia and Sapra, 2016, p. 624). In this context, FVA does not a priori outperforms HCA solely because it supposedly increases the quality of the information displayed in financial statements.

Theoretical works indeed demonstrate that, in some cases, FVA distorts the behavior of banks’ managers in such a way that HCA can be preferable. For instance, O’Hara (1993) shows that market value accounting incentivizes banks to shorten the maturity of their portfolio of assets, which can prove detrimental to the funding of the economy. Freixas and Tsomocos (2004) demonstrate that HCA makes it possible for banks to perform a better intertemporal smoothing than FVA. Plantin et al. (2008) point out that FVA is a bad option for financial institutions that manage long-lived, illiquid and senior assets. Allen and Carletti (2008) show that, because of the existence of contagion mechanisms between the insurance and the banking sector, mark-to-market accounting can be detrimental
to financial institutions when markets are illiquid. Yet, their main point is to insist on what would be the advantages of a mixed accounting regime that would make it possible to combine the best features of FVA and those of HCA. Heaton et al. (2010) show that some of the problems that have arisen with the introduction of FVA are in fact due to its interaction with capital requirements. Plantin and Tirole (forthcoming) show that mark-to-market accounting can have deleterious effects in terms of liquidity. Otto and Volpin (2017) show that mark-to-market accounting can make banks take inefficient investment decisions if the behavior of banks’ managers is driven by a reputational motive.

Empirical evidence on the real effects of accounting are relatively sparse (Leuz and Wysocki, 2016). Some papers provide evidence that banks’ behavior responds to changes in accounting rules if those changes are expected to have an influence on regulatory capital ratios (Beatty, 1995; Hodder, Kohlbeck and McAnally, 2002; Bens and Monahan, 2008). Beatty (2006) goes further by showing that accounting changes do affect banks’ behavior even when regulatory capital calculations are not affected. Recently, Ellul et al. (2015) provide empirical evidence that HCA can induce insurers to engage in gains trading to shore up capital.

Our model addresses two problems that have to our knowledge always been treated separately in the literature. Indeed, as in O’Hara (1993) and Otto and Volpin (2017) we are interested in the impact of accounting rules on FIs’ investment decisions and as in Plantin et al. (2008) and Plantin and Tirole (forthcoming) we are interested in the question of assets’ sales that occur because of accounting measurement. We build a bridge between the two problems through the idea that when FIs choose the composition of their portfolio of assets, they try to anticipate the amount of the risky asset that will be sold. The rationale is that if a lot of FIs decided to sell their risky asset, its market price would decrease sharply and it would have been less interesting to have invested in this asset in the first place. Therefore, the more a FI expects the others to sell their risky asset, the less it invests in this asset. By building a bridge between those two problems, our framework allows us to reconcile some of the previous results that could have seemed contradictory. In line with O’Hara (1993) we show that FVA can deter FIs from investing in risky assets especially when the duration of these assets is long. In line with the empirical evidence provided by Ellul et al. (2015), we show that HCA is no more than FVA a panacea since it does not provide FIs with the right incentives to assess properly the risk associated
with long-term assets. We therefore agree with the conclusion of Allen and Carletti (2008) and Otto and Volpin (2017) that neither FVA nor HCA is the panacea and that the 'one-size-fits-all' approach does not suit well accounting issues. Furthermore we show that a mixed accounting regime as that put forward by IFRS 9 can soften the negative effects associated with both accounting rules and ensure an asset allocation close to that that would arise in the first-best world. This paper therefore provides a new rationale for a mixed accounting regime close to that put forward by IFRS 9.

We resort to a theoretical framework that is close to that developed by Plantin et al. (2008). We however introduce two features in our model that are not present in the latter: a variable that accounts for the business model of FIs and a proper modelling of the portfolio optimization problem that FIs face by allowing them to choose between a risky and a riskless asset. We therefore borrow from the literature on portfolio optimization. In particular we make great use of the mean-variance analysis put forward by Markowitz (1952) and of its application to the analysis of the behavior of financial institutions developed by Pyle (1971) and Hart and Jaffee (1974).

The model is presented in the next section and our main results are presented in section 3. Section 4 concludes.

2 Model

2.1 General Framework

We present here the general framework of our model. We consider a unit continuum of FIs. There are three dates.

In $t = 0$, each FI chooses the composition of its portfolio of assets. In particular, we make the assumption that each FI has to allocate an initial endowment of 1 between a risky asset that yields $v \sim \mathcal{N}(\mu, \sigma)$ per unit at an uncertain date (it yields in $t = 1$ with probability $1 - d$ and in $t = 2$ with probability $d$) and a riskless asset that yields 0. If we define the duration of an asset as the weighted average of the times until fixed cash flows of this asset are received, we can interpret $d$ as the duration of the risky asset: since $d$ is the probability that the risky asset pays in $t = 2$, the bigger $d$, the longer the duration of this asset. We assume that FIs do not directly observe $v$ but are granted a noisy signal such that FI $i$ receives a signal $x_i = v + \eta \varepsilon_i$ where $\varepsilon_i \sim \mathcal{N}(0, 1)$ and $\eta > 0$. We focus on
the case where $\eta \to 0$. In this case, even if FIs have an accurate estimate of the value of the fundamental $v$, the fundamental is not common knowledge. This assumption will make it possible to derive the unique equilibrium of the model. We denote by $\alpha$ the weight of the risky asset in the portfolio of a FI. We denote by $v_0$ the initial price of the risky asset and we assume that $v_0$ is endogenously determined as the $t = 0$ rational expectation of the return of the risky asset. We make the assumption that the time horizon of FIs does not necessarily match the duration of the risky asset: FIs are assumed to maximize a weighted average of their short-term (ST) book-value and of their long-term (LT) earning. Specifically, the ST book-value of the risky asset (i.e. its $t = 1$ value) is given by the probability that the asset pays in $t = 1$ (i.e. $1 - d$) times the payoff of the asset (i.e. $v$) plus the probability that the asset pays in $t = 2$ times its accounting value, which depends on the accounting rule chosen (i.e. either FVA or HCA). The ST book-value of the risky asset is therefore given by the following expression:

$$\ (1 - d)v + dv_a, \tag{1}$$

where $v_a$ denotes the $t = 1$ accounting value of the risky asset. The LT earning associated with the risky asset is simply its payoff $v$ and FIs consequently seek to maximize the following weighted average:

$$\ (1 - \rho)[(1 - d)v + dv_a] + \rho v, \tag{2}$$

where $\rho \in [0, 1]$ accounts for the weight FIs grant to their LT earning. $\rho$ can therefore be interpreted as the time horizon of FIs: the larger $\rho$, the longer the time horizon of FIs. In particular, when $\rho$ equals 0, FIs are short-sighted since they only focus on their ST book-value, while they are long-sighted when $\rho = 1$ since in this case they maximize their LT earning.

Between $t = 0$ and $t = 1$, each FI has to decide whether to sell the proportion it has invested in the risky asset or to hold it to maturity. We make the assumption that there is no market price for this asset and FIs consequently need to resort to an intern model to price it. In particular, intern models yield the following price per unit of the risky asset:

$$p(v) = \delta v - \gamma \alpha s, \tag{3}$$
where $\delta$ is a positive constant that can be interpreted as the liquidity risk associated with the asset, $\gamma$ is a positive constant that captures market liquidity (the larger $\gamma$ is, the less liquid the market is), $s$ is the proportion of FIs that have sold their portfolio. We assume that $\delta \leq 1$. In this case, sales of the risky asset are only driven by the short-sightedness of FIs. When $\delta \leq 1$ we indeed know that $v \geq p(v)$: the LT earning associated with the risky asset is always above its market price and a FI that seeks to maximize its LT earning would therefore never find it interesting to sell it. In other words, sales of the risky asset are only motivated by the interaction between the incentives coming from accounting rules and those coming from the short-sightedness of FIs. Table 1 summarizes the timing of the model.

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>Between $t = 0$ and $t = 1$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIs are given a signal of $v$ such as FI $i$ is granted a signal $x_i = v + \eta \varepsilon_i$. FIs allocate a proportion $\alpha$ of their portfolio to the risky asset.</td>
<td>FIs can sell their risky asset or hold it to maturity.</td>
<td>The risky asset pays $v$ with probability $1 - d$.</td>
<td>The risky asset pays $v$ with probability $d$.</td>
</tr>
</tbody>
</table>

Table 1: Timing of the Model

### 2.2 Benchmark Case

We define the benchmark case as that where FIs’ time horizon is longer than or equal to the duration of the risky asset. In this case, FIs only focus on the LT earning associated with the risky asset (i.e. $\rho = 1$) and never decide to sell it when $\delta \leq 1$. Since FIs only focus on the return of the risky asset, accounting rules do not play any part in their decision-making process. We assume that each FI behaves like a portfolio manager when deciding the proportion of its portfolio of assets to allocate to the risky asset. In particular, we resort to the mean-variance analysis to compute this proportion: we assume that a FI chooses the composition of its portfolio by maximizing a utility function $U(\mu_p, \sigma_p^2)$ where $\mu_p$ is the expected value of the portfolio and $\sigma_p^2$ its variance. We assume that $\frac{\partial U(\cdot)}{\partial \mu_p} > 0$ and that $\frac{\partial U(\cdot)}{\partial \sigma_p^2} < 0$, meaning that FIs are risk-averse. Let us denote by $\hat{\alpha}$ the proportion of their portfolio FIs invest in the risky asset in the benchmark case. The random value of
the portfolio of a FI (denoted by $\tilde{\pi}_p$) consists in the proportion $\hat{\alpha}$ it invests in the risky asset times its return plus the proportion invested in the riskless asset (i.e. $1 - \hat{\alpha}$). Since FIs are assumed to maximize their LT earning, we can express $\tilde{\pi}_p$ as follows:

$$\tilde{\pi}_p = \hat{\alpha}(v - 1) + 1.$$  

(4)

Following the mean-variance assumption, the value of $\hat{\alpha}$ is given by the following first-order condition:

$$\frac{\partial U}{\partial \mu_p} \frac{\partial \mu_p}{\partial \hat{\alpha}} + \frac{\partial U}{\partial \sigma^2_p} \frac{\partial \sigma^2_p}{\partial \hat{\alpha}} = 0,$$

(5)

The value of $\hat{\alpha}$ is consequently as follows:

$$\hat{\alpha} = \lambda \frac{\mu - 1}{\sigma^2},$$

(6)

where $\lambda = -\frac{1}{2} \frac{\partial U}{\partial \mu_p} \left[ \frac{\partial U}{\partial \sigma^2_p} \right]^{-1}$ can be interpreted as the risk-aversion coefficient: the greater $\lambda$ is, the less risk-averse the agent is. $\hat{\alpha}$ is therefore the proportion of their portfolio of assets FIs would decide to invest in the risky asset if their time horizon was such that they only focused on the LT payoff associated with the risky asset. From now on, this allocation between the risky and the riskless asset will consequently be referred to as the first-best allocation.

2.3 Fair Value Accounting

We now assume that FIs’ time horizon does not perfectly match the duration of the risky asset. In this case, FIs are no longer only interested in the LT earning associated with the risky asset but focus on a weighted average of its ST book-value and of its LT earning. Accounting therefore plays a role in FIs’ decision-making since different accounting rules induce different ST book-values. Contrary to the benchmark case, real effects of accounting measurement therefore arise and are the consequence of the mismatch between FIs’ time horizon and the duration of the risky asset. In this section we assume that FIs resort to FVA to determine the book-value of their portfolio. The model is solved by backward induction: we first determine when sales of the risky asset occur and then we solve the portfolio optimization problem.
2.3.1 Step 1: Selling versus Holding

As previously mentioned, each FI can either sell its risky asset or hold it to maturity. A FI holds its risky asset to maturity if the weighted average of its ST book-value and of its LT earning is larger than its estimated market price. The ST book-value of the risky asset is equal to $v$ if the asset pays in $t = 1$. Yet, if the asset does not pay in $t = 1$, FIs resort to a model to estimate its accounting value, that is equation (3) when FIs are subject to FVA. Since the market is not perfectly liquid (i.e. $\gamma > 0$), the price of the risky asset depends on the proportion of FIs that decide to sell their risky asset. There consequently exists an uncertainty concerning the price a FI would face if it decided to sell its asset between $t = 0$ and $t = 1$. Indeed, since the price responds immediately to the proportion of FIs that decide to sell, the price faced by a particular FI depends on the date on which it decides to sell. The idea is that FIs do not take the decision to sell exactly at the same time but continuously decide to sell or hold their risky asset between $t = 0$ and $t = 1$. The price expected in $t = 0$ by a particular FI consequently depends on its expected position in the sellers’ line – i.e. on the expected proportion of FIs that will already have sold their asset when the FI will decide to do so. We make the assumption that the expected position of a particular FI in the sellers’ line follows a uniform distribution on $[0, s]$. Let us denote by $s_i$ the random variable that accounts for the position of FI $i$ in the sellers’ line. The price FI $i$ would face if it decided to sell its risky asset between $t = 0$ and $t = 1$ is consequently given by the following equation:

$$p_i(v) = \delta v - \gamma \alpha_F \mathbb{E}[s_i] = \delta v - \gamma \alpha_F \frac{s}{2},$$

(7)

where $\mathbb{E}[-]$ denotes the expectation operator and $\alpha_F$ the proportion of their portfolio FIs invest in the risky asset when they are subject to FVA. Finally, a FI holds its risky asset to maturity if:

$$\frac{(1 - \rho)}{(1 - \delta)}[(1 - d)v + d(\delta v - \gamma \alpha_F s)] + \rho v \geq \delta v - \gamma \alpha_F \frac{s}{2},$$

(8)

which can be rewritten as follows:

$$v \geq \frac{\gamma s \alpha_F}{(1 - \delta)(1 - d)} \left[\frac{(1 - \rho)d - \frac{1}{2}}{1 - d(1 - \rho)}\right].$$

(9)
We notice that sales of the risky asset only occur when the time-horizon of FIs (i.e. \( \rho \)) is too short by comparison to the duration of the risky asset (i.e. \( d \)), that is whenever \( \rho < 1 - \frac{1}{2d} \).

In this case, according to (9), when \( v > \frac{\gamma_{FV}(1-\rho)d-\frac{1}{2}}{(1-\delta)(1-d(1-\rho))} \) FIs always hold their risky asset to maturity. Conversely, if \( v < 0 \), all FIs sell their asset. When \( v \in \left[ 0, \frac{\gamma_{FV}(1-\rho)d+\frac{1}{2}}{(1-\delta)(1-d(1-\rho))} \right] \), there are two equilibria in the case where \( v \) is common knowledge: one where all FIs sell their risky asset and one where they all hold it until maturity. In this case, the impossibility to select \( \text{ex ante} \) the equilibrium that will be reached \( \text{ex post} \) is the consequence of the strategic complementarities that exist between players. Yet, as we assumed that \( v \) is not common knowledge, we can get rid of the multiple equilibria by using the global game technique.2

**Lemma 1.** In the limiting case where \( \eta \to 0 \), there exists a unique threshold value of \( v \), denoted \( v^* \equiv \frac{\gamma_{FV}(1-\rho)d-\frac{1}{2}}{2(1-\delta)(1-d(1-\rho))} \), such that when \( v < v^* \) all FIs sell their risky asset and when \( v \geq v^* \) they all decide to hold it to maturity.

**Proof.** Appendix 5.1.1.

According to Lemma 1, when there is a mismatch between FIs time horizon and the duration of the risky asset (i.e. when \( \rho < 1 - \frac{1}{2d} \)), sales of the risky asset occur whenever \( v \) is below \( v^* \). When FIs’ time horizon is, on the contrary, long enough (i.e. when \( \rho \geq 1 - \frac{1}{2d} \)), sales of the risky asset never occur between \( t = 0 \) and \( t = 1 \).

### 2.3.2 Step 2: Portfolio Optimization

Each FI has to determine the proportion of its portfolio it invests in the risky asset. As in the previous section, we resort to the mean-variance assumption. The random value of the portfolio of a FI (denoted by \( \tilde{\pi}_{FV} \)) consists in the proportion \( \alpha_{FV} \) it invests in the risky asset times its return plus the proportion invested in the riskless asset. The return associated with the risky asset consists in the weighted average of its ST book-value and of its LT earning. Therefore \( \tilde{\pi}_{FV} \) is given by the following expression:

\[
\tilde{\pi}_{FV} = \alpha_{FV} \{(1-\rho)(1-d)v + d(\delta v - \gamma_{FV}s) + \rho v\} + 1 - \alpha_{FV}.
\]  

---

2Global games were introduced by Carlsson and Van Damme (1993) and had been applied to many economic issues (see for instance Morris and Shin, 1998 and Morris and Shin, 2003).
To compute $\alpha_{FV}$ we have to make a distinction between situations where FIs decide to sell their risky asset (i.e. $s = 1$) and those where they decide to hold it to maturity (i.e. $s = 0$). We denote by $\bar{\alpha}_{FV}$ the proportion of its portfolio of assets a FI invests in the risky asset in the former case and by $\alpha_{FV}$ this proportion in the latter. The expected value of $\alpha_{FV}$ is therefore given by the following expression:

$$E[\alpha_{FV}] = \int_0^v \alpha_{FV} f(v)dv + \int_v^{+\infty} \bar{\alpha}_{FV} f(v)dv,$$

(11)

where $f(\cdot)$ denotes the probability density function of $v$. We focus first on the case where $v \geq v^*$. In this case, $s = 0$ (see Lemma 1) and (10) can be rewritten as follows:

$$\tilde{\pi}_{FV} = \bar{\alpha}_{FV}\{(1 - \rho)((1 - d)v + d\delta v] + \rho v - 1\} + 1.$$  

(12)

The value of $\bar{\alpha}_{FV}$ can therefore be computed as follows (see Appendix 5.1.2):

$$\bar{\alpha}_{FV} = \lambda \frac{(1 - d + d\delta + \rho d(1 - \delta))\mu - 1}{(1 - d + d\delta + \rho d(1 - \delta))^2\sigma^2}.$$ 

(13)

Similarly, following the mean variance assumption we compute the value of $\alpha_{FV}$:

$$\alpha_{FV} = \frac{(1 - d + d\delta + \rho d(1 - \delta))\mu - 1}{2(1 - \rho)d\gamma + \lambda^{-1}(1 - d + d\delta + \rho d(1 - \delta))^2\sigma^2}.$$ 

(14)

We notice that $\alpha_{FV} \leq \bar{\alpha}_{FV}$. This means that when short-sighted FIs expect that sales of the risky asset will take place between $t = 0$ and $t = 1$, they invest less in the risky asset in $t = 0$. This is because they expect that the fair value of the risky asset will be low and therefore that the ST book-value of the risky asset will be low too. In our model this is the main channel through which the real effects associated with FVA materialize.

**Proposition 1.** When FIs are long-sighted (i.e. $\rho = 1$), FVA achieves the first-best: no sales of the risky asset occurs and $\alpha_{FV} = \hat{\alpha}$.

**Proof.** We indeed notice that $\hat{\alpha}_{FV} = \alpha_{FV} = \lambda \frac{\mu - 1}{\sigma^2} = \hat{\alpha}$ when $\rho = 1$.

□

**Proposition 1** states that when FIs’ time horizon is greater than or equal to the duration of the risky asset, FVA does not distort the behavior of FIs and it therefore reaches the first-best allocation.
2.4 Historical Cost Accounting

We now assume that FIs resort to HCA. As in the previous section, we solve the model by backward induction.

2.4.1 Step 1: Selling versus Holding

When FIs resort to HCA, the ST book-value of the risky asset is given by:

\[(1 - d)v + dv_0,\]

where \(v_0\) is the initial value of the risky asset. We assume that \(v_0\) is determined in \(t = 0\) as the rational expectation of the weighted average of the ST book-value of the risky asset and of its LT earning:

\[v_0 = E[(1 - \rho)((1 - d)v + dv_0) + \rho v] \leftrightarrow v_0 = \mu.\] (16)

In order to decide whether to sell the proportion of its portfolio it has invested in the risky asset or not, a FI compares the weighted average of the ST book-value of the risky asset and of its LT earning to its expected market price between \(t = 0\) and \(t = 1\). In particular, a FI holds the proportion of its portfolio invested in the risky asset until maturity if:

\[
(1 - \rho)[(1 - d)v + dv_0] + \rho v \geq \delta v - \frac{\gamma \alpha_{HC}}{2} \tag{17}
\]

where \(\alpha_{HC}\) denotes the proportion of its portfolio a FI invests in the risky asset when it is subject to HCA.

**Lemma 2.** When \(\rho \geq 1 - \frac{1-\delta}{d}\), sales of the risky asset never occur when FIs resort to HCA.

When \(\rho < 1 - \frac{1-\delta}{d}\) and no matter what other FIs do, a FI decides to hold its risky asset to maturity when \(v < \frac{(1 - \rho)dv_0}{d(1 - \rho) + \delta - 1}\) and to sell it when \(v > \frac{(1 - \rho)dv_0 + \gamma \alpha_{HC}}{d(1 - \rho) + \delta - 1}\). When \(v \in \left[\frac{(1 - \rho)dv_0}{d(1 - \rho) + \delta - 1}, \frac{(1 - \rho)dv_0 + \gamma \alpha_{HC}}{d(1 - \rho) + \delta - 1}\right]\), a FI sells its asset with probability \(\frac{2}{\gamma \alpha_{HC}}[(d(1 - \rho) + \delta - 1)v - (1 - \rho)dv_0]\). The proportion of FIs that sell their risky asset as a function of \(v\) is
therefore given by:

\[
s(v) = \begin{cases} 
0 & \text{if } v < v_d \equiv \frac{(1-\rho)d\nu_0}{d(1-\rho)+\delta-1}, \\
\frac{2}{\gamma \alpha_{HC}}((d(1-\rho)+\delta-1)v-(1-\rho)d\nu_0) & \text{if } v_d \leq v < v_u \equiv \frac{(1-\rho)d\nu_0+\gamma \alpha_{HC}}{d(1-\rho)+\delta-1}, \\
1 & \text{if } v \leq v_u.
\end{cases}
\]

(18)

2.4.2 Step 2: Portfolio Optimization

As previously we resort to the mean-variance analysis to determine the proportion of their portfolio of assets FIs invest in the risky asset in \( t = 0 \). The random value of the portfolio of a representative FI that is subject to HCA (i.e. \( \tilde{\pi}_{HC} \)) is given by:

\[
\tilde{\pi}_{HC} = \alpha_{HC} \{(1-\rho)[(1-d)v + d\nu_0] + \rho v - 1\} + 1.
\]

(19)

Following the mean-variance assumption and replacing \( v_0 \) by its value, we compute the value of \( \alpha_{HC} \):

\[
\alpha_{HC} = \lambda \frac{\mu-1}{(1-d(1-\rho))^2\sigma^2}.
\]

(20)

**Proposition 2.** When FIs are long-sighted (i.e. \( \rho = 1 \)), HCA achieves the first-best: no sales of the risky asset occurs and \( \alpha_{HC} = \hat{\alpha} \).

**Proof.** We indeed have \( \alpha_{HC} = \lambda \frac{\mu-1}{\sigma^2} = \hat{\alpha} \) when \( \rho = 1 \).

\( \square \)

3 Taking Diversity into Account

In the previous section, we noticed that when FIs only focus on the LT earning associated with the risky asset (i.e. when \( \rho = 1 \)) accounting does not matter since both FVA and HCA reach the first-best allocation between the risky and the riskless asset (Propositions 1 and 2). In this case, accounting does not have real effects on FIs’ behaviors. When FIs’ time horizon is on the contrary shorter than the duration of the risky asset, accounting does have real effects\(^3\) since the allocation between the risky and the riskless asset is not

\(^3\)Such a mismatch is indeed what gives birth to what is referred to as the real effects of accounting: 'The assumption that is critical to the real effects studies that we describe is not the absence of incentive
the same whether FIs resort to FVA or to HCA. In addition, none of these allocations matches that found in the benchmark case. Inefficiencies therefore arise when FIs’ time horizon is shortened. In this section, we inquire what the real effects associated with accounting rules are and design a mixed accounting regime that makes it possible to reach the first-best allocation between the risky and the riskless asset even when some FIs are short-sighted. We show that this mixed accounting regime is close to that put forward by IFRS 9 through the business model assessment.

3.1 Real Effects of Accounting Measurement on Financial Institutions’ Behaviors

3.1.1 Fair Value Accounting and Asset Allocation

FVA incentivizes FIs to focus on the fluctuations of short-term assets’ price. In particular, when FIs are short-sighted and the market is illiquid, they anticipate that sales of the risky asset will occur and that its price will therefore decrease. This expected drop in the price of the risky asset causes its fair value to decrease: short-sighted FIs that focus much on the ST book-value of their portfolio will consequently grant less value to the risky asset and invest less in it. In other words, FVA makes FIs overreact to others’ behaviors, which gives rise to self-fulfilling sales. Those expected sales incentivize in turn FIs to underinvest in the risky asset. However, this relationship between self-fulfilling sales of the risky asset and underinvestment in this asset because of the expectation of those sales only occur as far as the expected return associated with the risky asset does not exceed a certain threshold. When the expected return associated with the risky asset raises above this threshold – and provided that the market is not too illiquid –, FVA makes FIs understate the risk associated with their portfolio. This is particularly the case for the most short-sighted FIs that tend to underestimate the most the risk associated with high-yield assets. When the expected return associated with the risky asset increases, the benchmark case indicates that the proportion invested in the risky asset should increase as follows:

$$\frac{\partial \hat{\alpha}}{\partial \mu} = \frac{\lambda}{\sigma^2}.$$  \hspace{1cm} (21)

contracts, but rather the assumption that shareholders obtain their rewards through short-term price movements in the capital market, rather than through the long-term accumulation of cash flows in the firm.” (Kanodia and Sapra, 2016, p. 635).  

15
In other words, in the benchmark case, when the expected return associated with the risky asset increases, the proportion with which FIs increase the amount they invest in the risky asset only depends on their risk-aversion (i.e. $\lambda$) and on the risk associated with the risky asset (i.e. $\sigma^2$). When FIs resort to FVA, their reaction to an increase in the expected return of the risky asset is strongly related to their time-horizon. In particular, when $\gamma \leq \bar{\gamma}$ (see Appendix 5.1.3), we know for sure that $\frac{\partial \hat{\alpha}}{\partial \mu} \leq \frac{\partial \alpha_{FV}}{\partial \mu}$. In this case, FVA makes FIs understate the risk associated with the risky asset. In addition, we notice that the smaller $\rho$ is, the larger $\frac{\partial \alpha_{FV}}{\partial \mu}$ is, which confirms the idea according to which the most short-sighted FIs are those who underestimate the most the risk associated with assets whose expected return is increasing.

**Proposition 3.** Provided that the market is not too illiquid (i.e. $\gamma \leq \bar{\gamma}$), when short-sighted FIs (i.e. $\rho < 1$) are subject to FVA:

- when the expected return associated with the risky asset is small enough (i.e. $\mu < \bar{\mu}$), FIs overreact to expected drops in the price of the risky asset by underinvesting in it (i.e. $\alpha_{FV} < \hat{\alpha}$),
- when the expected return associated with the risky asset is large enough (i.e. $\mu > \bar{\mu}$), FIs underestimate the risk associated with a high-yield risky asset and overinvest in it (i.e. $\alpha_{FV} > \hat{\alpha}$).

**Proof.** See Appendix 5.1.3.

Proposition 3 provides some support to one of the main criticisms addressed to FVA: its procyclicality. During expansion phases – i.e. when assets yield high returns – FVA indeed encourages FIs to overinvest in risky assets, which makes their prices increase further. Yet, in depression phases – i.e. when assets’ expected returns are low – FVA deters FIs from investing in risky assets, which causes their prices to decrease further. In all cases, when FIs get more short-sighted, the allocation that arises in the FVA case gets further away from that arising in the benchmark case.

Figure 2 plots the proportion of their portfolio of assets FIs invest in the risky asset when they are subject to FVA (dotted line) and in the benchmark case (solid line) as functions of $\rho$ and for some values of $\mu$. We indeed notice that FIs tend to underinvest in the risky asset for the two smallest values of $\mu$: the dotted line is below the solid line.
when $\mu = 1.5$ and when $\mu = 2$. On the contrary, when the expected return associated with the risky asset is higher (i.e. when $\mu = 2.5$ and $\mu = 3$), FIs overinvest in the risky asset: the dotted line is above the solid line.

[Figure 2 about here]

3.1.2 Historical Cost Accounting and Asset Allocation

When they are subject to HCA, FIs do not take markets’ fluctuations into account. For long-sighted FIs whose business model makes it possible to focus on long-term earning, the distorting impact of HCA on the allocation between the risky and the riskless asset is therefore expected to be smaller than that for short-sighted FIs. This is what PROPOSITION 4 states.

**Proposition 4.** When $\rho < 1$, HCA makes FIs overinvest in the risky asset.

In particular, the more short-sighted FIs are (i.e. the smaller $\rho$ is), the more they overinvest in the risky asset (i.e. $\frac{\partial \alpha_{HC}}{\partial \rho} \leq 0$).

**Proof.** When $\rho < 1$, we indeed have $\alpha_{HC} = \lambda \frac{\mu - 1}{[1 - d(1 - \rho)]^2 \sigma^2} > \hat{\alpha} = \frac{\mu - 1}{\sigma^2}$ and $\frac{\partial \alpha_{HC}}{\partial \rho} = -\frac{2d\lambda(\mu - 1)}{\sigma^2(d(\rho - 1) + 1)^2} \leq 0$ when $\mu \geq 1$.

□

When short-sighted FIs resort to HCA, they focus much on the historical value of the risky asset. This makes them understate the real risk associated with the risky asset, which induces them to overinvest in it. Recall indeed that:

$$\alpha_{HC} = \lambda \frac{\mu - 1}{[1 - d(1 - \rho)]^2 \sigma^2} \quad \text{and that} \quad \hat{\alpha} = \frac{\mu - 1}{\sigma^2}. \quad (22)$$

The drift between the first-best allocation and the one that arises when FIs are subject to HCA is measured by $d(1 - \rho)$: the bigger $d(1 - \rho)$, the bigger the drift. In particular, we notice that $d(1 - \rho)$ is an increasing function in $d$ and a decreasing function in $\rho$, which means that when the mismatch between FIs’ time horizon and the duration of the risky asset increases, the allocation between the risky and the riskless asset that arises when FIs are subject to HCA goes further away from the first-best allocation.

[Figure 3 about here]
Figure 3 plots the proportion FIs invest in the risky asset when they are subject to HCA (dotted line) and in the benchmark case (solid line) as functions of $\rho$ for some values of $d$. We notice that the dotted line is always above the solid line. In addition, the distance between the two lines is indeed bigger for smaller values of $\rho$ – i.e. when FIs are the most short-sighted. When the duration of the asset increases, this distance grows furthermore and the most short-sighted FIs blindly invest all their initial endowment in the risky asset.

### 3.2 The Mixed Accounting Regime

According to the results presented in the previous sections, neither FVA nor HCA manage to reach the first-best allocation when FIs are short-sighted. Yet, we notice that in some cases, the effects of FVA and those of HCA on FIs’ investment behavior go the opposite way: the former deters FIs from investing in the risky asset while the latter provides incentives that make FIs overinvest in the risky asset. A mixed accounting regime that would allow some FIs to use HCA while the others would resort to FVA could therefore make it possible to reach the first-best allocation. Such a mixed accounting regime is precisely what has been put forward by IFRS 9 through what is referred to as the business model assessment. IFRS 9 indeed allows FIs whose purpose is to buy assets in order to hold them to maturity to use HCA, while FIs that may be forced to sell their assets in the short-run are subject to FVA. In our model, FIs whose time horizon is long-enough can be considered as those for which the HCA option was introduced in IFRS 9. In this section, we show that the mixed accounting regime introduced by IFRS 9 through the business model assessment makes it possible in some cases to reach the first-best allocation.

We assume here that all FIs have a different time horizon. In particular, the time-horizon of FI $i$ is characterized by a random variable $\rho_i$ such that $\rho_i \sim U[0, 1]$. In addition, we define $\bar{\rho}$ as the threshold value of $\rho$ such that FIs whose time horizon is shorter than $\bar{\rho}$ are subject to FVA while those whose time horizon is bigger than $\bar{\rho}$ resort to HCA. In this respect, the expected total investment in the risky asset – denoted $\mathbb{E}[\alpha_d]$ – is given by the following expression:

$$
\mathbb{E}[\alpha_d] = \Pr(\rho_i \leq \bar{\rho})\mathbb{E}[\alpha_{FV}] + \Pr(\rho_i > \bar{\rho})\mathbb{E}[\alpha_{HC}]
$$

$$
= \bar{\rho}\mathbb{E}[\alpha_{FV}] + (1 - \bar{\rho})\mathbb{E}[\alpha_{HC}].
$$

**Proposition 5.** There exists at least one threshold $\bar{\rho} \in [0, 1]$ such that the
first-best allocation between the risky and the riskless asset is reached if FIs whose $\rho$ is above $\bar{\rho}$ are subject to HCA while FIs whose $\rho$ is below $\bar{\rho}$ are subject to FVA.

**Proof.** See Appendix 5.1.4.

Figure 4 plots the expected total investment in the risky asset in the benchmark case (solid line) and in the mixed accounting regime (dotted line) as functions of $\bar{\rho}$ – i.e. the threshold from which FIs resort to HCA instead of resorting to FVA in the mixed accounting regime. The vertical dotted line represents the value of $\rho$ from which sales of the risky asset do not occur when FIs are subject to HCA (see Lemma 2). This threshold is therefore that introduced by IFRS 9 through the business model assessment: under IFRS 9, FIs whose $\rho$ is above the threshold defined by the vertical dotted line are allowed to use HCA instead of FVA. We notice that, in line with Proposition 5, the allocation between the risky and the riskless asset that arises under the mixed accounting regime indeed reaches the first-best for at least one value of $\bar{\rho}$. It is worth pointing out that the value of $\bar{\rho}$ for which the mixed accounting regime reaches the first-best is always above that derived from the IFRS 9 framework, meaning that the business model assessment put forward by IFRS 9 could lead to the first-best allocation. In addition, we notice that the longer the duration of the risky asset is, the more efficient the allocation that arises under the mixed accounting regime is. When $d$ is big ($d = 0.7$ and $d = 0.9$ in Figure 4), the allocation that arises under IFRS 9 – i.e. that read at the intersection between the vertical dotted line and the dotted curb – is very close to the first-best as defined in the benchmark case. IFRS 9 is therefore well-designed to limit the impact of the inefficiencies that arise from the mismatch between FIs’ time horizon and the duration of the risky asset, especially when this mismatch is important.

[Figure 4 about here]

## 4 Conclusion

This paper presents a theoretical model that assesses the impact of the new way of classifying financial instruments put forward by IFRS 9. We show that the mismatch between financial institutions’ time horizon and the duration of the risky asset they invest
in gives rise to real effects of accounting. Those real effects are not the same whether FIs resort to FVA or to HCA. FVA makes FIs overreact to the expectations they forge concerning the behavior of the others. Strategic complementarities arise and they give birth to self-fulfilling sales of the risky asset whose expectation incentivizes FIs to underinvest in the risky asset when its expected return is low. When the expected return associated with the risky asset is on the contrary high enough – and provided that the market is not too illiquid – FVA makes FIs understate the risk associated with the risky asset, which induces them to overinvest in it. When FIs are subject to HCA, they always overinvest in the risky asset. We design a mixed accounting regime where FIs are allowed to use either FVA or HCA depending on their time horizon. Precisely the most short-sighted FIs resort to FVA while the most long-sighted FIs are allowed to use HCA, which is consistent with the framework put forward by IFRS 9 through the business model assessment. We show that such a mixed accounting regime can reach the first-best allocation.

These results are in line with the idea that accounting rules should be designed in accordance with the nature of the activities led by financial institutions. In this perspective, since insurers are engaged in long-term activities, they should be offered the possibility to use HCA while banks are better off resorting to FVA. However, those two types of FIs are subject to capital requirements that are in both cases computed as a function of risk-weighted assets. The way capital requirements are calculated strongly interact with accounting measurement as pointed out by the empirical literature on the matter (see for instance Ellul et al., 2015). It would be of great interest to inquire through a theoretical model the interaction between accounting measurement and capital requirements to study how the main features of our model are affected by the necessity for FIs to reach regulatory capital requirements. We leave such applications for future work.
5 Appendix

5.1 Proofs

5.1.1 Proof of Lemma 1

We define \( v^* = \gamma s(v^*)\alpha_{FV} \frac{(1 - \rho)d - \frac{1}{2}}{(1 - \delta)(1 - d(1 - \rho))} \) the threshold value of \( v \), meaning the value of \( v \) from which a FI decides to hold its risky asset to maturity instead of selling it. Each FI is granted a noisy signal of \( v \) such that FI \( i \) observes \( x_i = v + \eta \varepsilon_i \) where \( \eta > 0 \) and \( \varepsilon_i \sim \mathcal{N}(0, 1) \). We assume that each FI resorts to a threshold strategy, meaning that a FI \( 'i' \) sells its risky asset when \( x_i < x^* \) and holds it to maturity when \( x_i \geq x^* \). Consequently, the proportion of FIs that sell their risky asset when \( v = v^* \) is given by:

\[
 s(v^*) = \Pr(x_i \leq x^* | v^*) = \Pr\left(\frac{x_i - v^*}{\eta} \leq \frac{x^* - v^*}{\eta}\right). \tag{24}
\]

We know that \( \Pr\left(\frac{x_i - v^*}{\eta} \leq \frac{x^* - v^*}{\eta}\right) = \frac{1}{2} \) because \( x_i \) is centered on \( v \) so when \( v = v^* \), the probability of observing a signal below \( x^* \) is the same as the probability of observing a signal above \( x^* \). We can therefore compute the value of \( v^* \):

\[
v^* = \alpha_{FV} \frac{[(1 - \rho)d - \frac{1}{2}]}{2(1 - \delta)(1 - d(1 - \rho))}. \tag{25}
\]

□

5.1.2 Computation of \( \bar{\alpha}_{FV} \)

According to the mean-variance assumption FIs maximize the following utility function:

\[
 U(\mu_p, \sigma_p^2), \tag{26}
\]

where \( \mu_p \) is the mean and \( \sigma_p^2 \) the variance of the random return associated with the portfolio (equation (10)). When \( s = 0 \) the random return of the portfolio is given by:

\[
 \tilde{\pi}_{FV} = \bar{\alpha}_{FV} \{(1 - \rho)[(1 - d)\bar{v} + d\delta\bar{v}] + \rho\bar{v} - 1\} + 1. \tag{27}
\]
We therefore have:
\[ \mu_p = \bar{\alpha}_{FV} \{ (1 - \rho)(1 - d)\mu + d\delta\mu + \rho\mu - 1\} + 1 \tag{28} \]
and
\[ \sigma_p^2 = \bar{\alpha}_{FV}^2 \{ (1 - \rho)(1 - d) + d\delta\}^2 \sigma^2. \tag{29} \]

The value of \( \bar{\alpha}_{FV} \) is given by the following first-order condition:
\[
\frac{\partial U}{\partial \mu_p} \frac{\partial \mu_p}{\partial \bar{\alpha}_{FV}} + \frac{\partial U}{\partial \sigma_p^2} \frac{\partial \sigma_p^2}{\partial \bar{\alpha}_{FV}} = 0
\]
\[
\Leftrightarrow \frac{\partial U}{\partial \mu_p} \{ [1 - d + d\delta + \rho d(1 - \delta)]\mu - 1\} + 2\bar{\alpha}_{FV} \frac{\partial U}{\partial \sigma_p^2} \{ [1 - d + d\delta + \rho d(1 - \delta)]^2 \sigma^2 = 0. \tag{30} \]

We can consequently compute the value of \( \bar{\alpha}_{FV} \) as follows:
\[
\bar{\alpha}_{FV} = -\frac{\partial U}{\partial \mu_p} \{ [1 - d + d\delta + \rho d(1 - \delta)]\mu - 1\} \times 2\bar{\alpha}_{FV} \frac{\partial U}{\partial \sigma_p^2} \{ [1 - d + d\delta + \rho d(1 - \delta)]^2 \sigma^2. \tag{31} \]

Let us define the risk-aversion coefficient as
\[
\lambda = -\frac{1}{2 \frac{\partial U}{\partial \mu_p}} \left[ \frac{\partial U}{\partial \sigma_p^2} \right]^{-1}. \]
Since we know by assumption that \( \frac{\partial U}{\partial \mu_p} > 0 \) and that \( \frac{\partial U}{\partial \sigma_p^2} < 0 \), we know that \( \lambda > 0 \). Therefore, the value of \( \bar{\alpha}_{FV} \) is given by:
\[
\bar{\alpha}_{FV} = \lambda \frac{[1 - d + d\delta + \rho d(1 - \delta)]\mu - 1}{[1 - d + d\delta + \rho d(1 - \delta)]^2 \sigma^2.} \tag{32} \]

\[\square\]

5.1.3 Proof of Proposition 3

Let us define the following function:
\[ g(\mu) = \hat{\alpha} - E[\alpha_{FV}]. \tag{33} \]

Recall that \( E[\alpha_{FV}] \) is given by the following expression:
\[ E[\alpha_{FV}] = F(v^*)\underline{\alpha}_{FV} + (1 - F(v^*))\bar{\alpha}_{FV}, \tag{34} \]
where \( F(\cdot) \) is the cdf of \( v \). When \( \mu \leq \frac{1}{[1 - d + d\delta + \rho d(1 - \delta)]} \), we have by definition \( E[\alpha_{FV}] = 0 \) (since in this case both \( \underline{\alpha}_{FV} \) and \( \bar{\alpha}_{FV} \) are equal to 0) while \( \hat{\alpha} = \lambda \frac{\mu - 1}{\sigma} > 0 \) since
$1 < \frac{1}{1-d+d\delta+\rho d(1-\delta)}$ when $d$ is strictly positive. For such values of $\mu$ we therefore have $\mathbb{E}[\alpha_{FV}] < \hat{\alpha}$ and $g(\mu) > 0$.

Since we know that $\hat{\alpha}_{FV} \geq \alpha_{FV}$, a sufficient condition for $\mathbb{E}[\alpha_{FV}] > \hat{\alpha}$ is $\alpha_{FV} \geq \hat{\alpha}$. Therefore we know for sure that $g(\mu) < 0$ when the following condition holds true:

$$\begin{align*}
\hat{\alpha}_{FV} &> \hat{\alpha} \\
\Leftrightarrow \mu [A\sigma^2(1-A) - 2\lambda(1-\rho)d\gamma] > \sigma^2(1-A) - 2\lambda(1-\rho)d\gamma,
\end{align*}
$$

where $A = 1 - d + d\delta + \rho d(1-\delta)$. The condition stated in (35) can be rewritten by the two following conditions:

$$\begin{align*}
\mu &> \frac{\sigma^2(1-A)-2\lambda(1-\rho)d\gamma}{A\sigma^2(1-A)-2\lambda(1-\rho)d\gamma} \\
\text{and} \\
\gamma &< \bar{\gamma} = \frac{\sigma^2 A(1-A)}{2\lambda(1-\rho)d^2}.
\end{align*}
$$

When $\gamma \in \left[ \frac{\sigma^2 A(1-A)}{2\lambda(1-\rho)d}, \frac{\sigma^2(1-A)}{2\lambda(1-\rho)d} \right]$, (35) never holds true and we always have $\hat{\alpha}_{FV} < \hat{\alpha}$. In this case we cannot derive a sufficient condition allowing us to compare $\hat{\alpha}$ and $\mathbb{E}[\alpha_{FV}]$. When $\gamma > \frac{\sigma^2(1-A)}{2\lambda(1-\rho)d}$, (35) holds true if $\mu < \frac{\sigma^2(1-A)-2\lambda(1-\rho)d\gamma}{A\sigma^2(1-A)-2\lambda(1-\rho)d\gamma} \leq 1$. For such values of $\mu$ we know by definition that both $\mathbb{E}[\alpha_{FV}]$ and $\hat{\alpha}$ are equal to 0. The only case where we know for sure that $\hat{\alpha}_{FV} > \hat{\alpha}$ is therefore that defined by the conditions stated in (36).

In addition, we have:

$$\frac{\partial \hat{\alpha}_{FV}}{\partial \mu} = \frac{\lambda}{1-d+d\delta+\rho d(1-\delta)\sigma^2} \geq \frac{\partial \hat{\alpha}}{\partial \mu} = \frac{\lambda}{\sigma^2},$$

and

$$\frac{\partial \hat{\alpha}_{FV}}{\partial \mu} = \frac{\lambda[1-d+d\delta+\rho d(1-\delta)]}{2\lambda(1-\rho)d\gamma + [1-d+d\delta+\rho d(1-\delta)]^2\sigma^2} \geq \frac{\partial \hat{\alpha}}{\partial \mu} = \frac{\lambda}{\sigma^2},$$

provided that $\gamma \leq \bar{\gamma}$. In this case, we know for sure that:

$$g'(\mu) = \frac{\partial \hat{\alpha}}{\partial \mu} - \frac{\partial \mathbb{E}[\alpha_{FV}]}{\partial \mu} \leq 0.$$

In sum, we know for sure that $g(\mu) > 0$ when the expected return associated with the risky asset is sufficiently small and that $g(\mu) < 0$ when the expected return associated with the risky asset is large enough, provided that the market is not too illiquid. In this case, we also know that $g'(\mu) \leq 0$. The intermediate value theorem therefore allows us
to state that there exists a unique \( \bar{\mu} \) such that \( g(\bar{\mu}) = 0 \) and \( g(\mu) > 0 \) when \( \mu < \bar{\mu} \) and \( g(\mu) < 0 \) when \( \mu > \bar{\mu} \).

5.1.4 Proof of Proposition 5

We want to show that there exists at least a \( \bar{\rho} \in [0, 1] \) such that:

\[
\mathbb{E}[\alpha_d] = \hat{\alpha} \leftrightarrow \bar{\rho}\mathbb{E}[\alpha_{FV}] + (1 - \bar{\rho})\mathbb{E}[\alpha_{HC}] = \hat{\alpha}.
\] (40)

Let us define the following function:

\[
\Delta(\bar{\rho}) = \bar{\rho}\mathbb{E}[\alpha_{FV}] + (1 - \bar{\rho})\mathbb{E}[\alpha_{HC}] - \hat{\alpha}.
\] (41)

When \( \bar{\rho} = 0 \) we know that:

\[
\Delta(0) = \mathbb{E}[\alpha_{HC}] - \hat{\alpha} \geq 0. \quad \text{(Proposition 4)}
\] (42)

When \( \bar{\rho} = 1 \) and provided that the market is not too illiquid we have:

\[
\Delta(1) = \begin{cases} 
\mathbb{E}[\alpha_{FV}] - \hat{\alpha} < 0 & \text{if } \mu < \bar{\mu}, \\
\mathbb{E}[\alpha_{FV}] - \hat{\alpha} > 0 & \text{if } \mu > \bar{\mu}.
\end{cases} \quad \text{(Proposition 3)}
\] (43)

If \( \mu \) is small enough we have \( \Delta(0) \leq 0 \) and \( \Delta(1) \geq 0 \), which ensures that there exists at least one value of \( \bar{\rho} \) in \([0, 1]\) such that \( \bar{\rho}\mathbb{E}[\alpha_{FV}] + (1 - \bar{\rho})\mathbb{E}[\alpha_{HC}] = \hat{\alpha} \).

\[\square\]
5.2 Figures

Figure 1: Classification of financial instruments under IFRS 9 (taken from EY, 2015)

Figure 2: Proportions of portfolios invested in the risky asset in the FVA case and in the benchmark case
Figure 3: Proportions of portfolios invested in the risky asset in the HCA case and in the benchmark case

Figure 4: The mixed accounting regime and the first-best allocation


References


[10] Ernst and Young. 2015. *Classification of Financial Instruments under IFRS 9*


American Economic Review