Corporate Social Responsibility and workers' motivation at the industry equilibrium

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Corporate Social Responsibility and workers’ motivation at the industry equilibrium

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Abstract

We consider an industry in which firms compete at two levels, the labor market and the product market. On the labor market two types of workers co-exist, socially responsible or not. Firms may strategically use CSR investments in order to screen and to elicit more effort from responsible workers. By doing so, virtuous firms lower their production costs and display a competitive advantage on the product market. As a consequence, CSR strategies by firms shape the toughness of competition on that market. In turn, incentives that firms have to invest in CSR are dampened when competition becomes harsher. Hence, we identify a twofold relationship between CSR and competition. Due to feedback effects on the competitive pressure, an increase in workers’ social awareness may reduce the overall level of CSR. We also show that an exogenous increase of competition may affect positively or negatively the corporate social performance depending on the proportion of responsible workers.

Keywords: Corporate Social Responsibility, Moral Motivation, Screening, Market Competition, Industry Equilibrium

JEL Codes: D64; D86; L13; M14; Q50.

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1 Introduction

More than ever, companies dedicate important resources to socially responsible activities, well beyond the requirements of the law, and broadly communicate about their virtuous behavior. The unprecedented development of these business practices has given rise to a growing interest in the determinants of firms’ investments in Corporate Social Responsibility (CSR). Nowadays, the strategic view of CSR constitutes the main line of explanation that has been put forward to figure out why firms embrace CSR. Along this line, CSR is often pointed out as a mean to boost consumers’ willingness to pay for the firm’s products (Bagnoli and Watts 2003 or Fisman et al. 2006), to preempt future activists’ actions (Baron 2001) or to foretell the scope and the stringency of new regulations (Maxwell et al. 2000 or Maxwell and Decker 2006). One additional business reward firms can potentially expect from their investment in CSR relies on the intrinsic motivations that some employees may have to work at a firm that addresses social or environmental issues. A recent body of papers have found empirical support for the idea that socially responsible firms can pay lower wages while attracting more motivated (productive) employees. The role played by CSR in the formation of a corporate culture enhancing employees’ motivation has also been put forward by several studies in the business and management literature. At the same time, a rapidly growing literature incorporates behavioral considerations in agency problems, offering theoretical

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1. The vitality of this field of research is attested by the large number of recent surveys devoted to such issues both in the economic and business literature (see, among others, Margolis and Walsh 2003, McWilliams et al. 2006, Kitzmueller and Shimshack 2012, Crifo and Forget 2015 or Schmitz and Schrader 2015).


tools to analyze labor contracts and incentive schemes when employees may be morally motivated, i.e. when they exhibit intrinsic motivations to work hard when employed in an organization that pursues social goals (see Benabou and Tirole 2010, Besley and Ghatak 2005, Brekke and Nyborg 2008, Delfgaauw and Dur 2007 and 2008, Francois 2000).

The first contribution of the present paper is to build a bridge between this theoretical literature on morally motivated agents and the strategic view of CSR. Indeed, a firm that voluntarily reduces harmful polluting emissions, ensures that subcontractors do not exploit child labor or contributes to charities may be perceived as virtuous by its employees even though the company belongs to the for-profit sector. Then, the development of CSR activities may be strategically used by a firm in order to attract intrinsically motivated workers. Our second contribution consists in reassessing the relationship between CSR and competition in the presence of morally motivated agents. An important literature has recently developed on the CSR-competition nexus (see Fernandez-Kranz and Santalo 2010 and references therein). Compared to this body of research, our framework has the advantage to simultaneously emphasize two natural dimensions through which firms interact: the labor market where they compete for motivated employees and the product market where they sell their goods. This allows us to highlight a two-way relationship between CSR and the toughness of competition.

Our economy comprises a large number of firms, each of them producing a variety of a horizontally differentiated good and hiring one skilled employee to manage the production process. Production costs are negatively related to a non-contractible effort chosen by the employee. Workers can be of two types: Socially responsible or not. Non-responsible employees have standard pecuniary preferences while responsible ones enjoy non-pecuniary benefits when working hard for a firm that is deemed to be sufficiently responsible. Both
workers’ efforts and preferences are unobservable but firms may use CSR activities in order to screen and incentivize responsible employees. Hence, it may be profitable for firms to invest in CSR since, by doing so, they elicit more effort, are better managed and are more likely to display low production costs.

At the labor market level, firms compete for a limited pool of skilled workers. At the separating equilibrium, a fraction of firms does not engage in responsible actions and is poorly managed by non-responsible employees while the remaining fraction adopts virtuous behaviors and is competently managed by responsible employees. Moreover, the intensity of CSR investments by virtuous firms is positively related to the incremental benefits associated with higher managerial efforts. This is directly due to the screening purpose of CSR in a context where firms compete for talents: when incremental benefits rise, CSR investments must increase in order to ensure that the separation condition and the zero profit condition are still simultaneously satisfied.

At the product market level, we consider a Salop-like model of monopolistic competition with cost heterogeneity (Aghion and Schankerman 2004). As discussed just above, the production costs of each individual firm are related to its investment in CSR. Those costs being given, firms compete in price. At the equilibrium, the profit differential between high-cost and low-cost firms is negatively affected by the toughness of cost competition, measured by the proportion of low-cost firms within the industry.

At the industry equilibrium, the cost structure derived from CSR choices made by firms must be consistent with the profit differential derived from pricing strategies on the product market. Hence, we point out a two-way relationship between CSR and competition on the product market. On the one hand, when firms invest in CSR they elicit more efforts from
their employees and the cost competition becomes tougher. On the other hand, when cost
competition is harsh, the profit differential between well managed and poorly managed firms
is depressed and firms have less incentive to invest in CSR. Our setting also enables us to
provide explicit expressions for the equilibrium levels of the corporate social performance
(CSP) and the Herfindhal-Hirschman Index (HHI) at the industry level.

Given this, we assess how does the spread of social awareness among workers affect the
CSP. At a first glance, we expect a positive relationship. Nonetheless, we show that a
rise in the proportion of responsible employees displays two opposite effects. First, a direct
positive extensive margin effect: At the separating equilibrium, more firms do invest in CSR.
This rises the toughness of competition on the product market which, in turn, generates a
second and negative intensive margin effect: Each virtuous firm reduces the intensity of its
CSR activities. In some configurations, the intensive margin effect overwhelms the extensive
margin one such that an increasing social consciousness among workers may be harmful for
the CSP of the industry.

Then, we wonder whether competition promotes or erodes virtuous behaviors on the
market. The existing theoretical literature on the CSR-competition nexus has found con-
flicting results regarding the sign of that linkage.\textsuperscript{4} Our model, by encompassing two effects
of competition on the profit differential, offers a unified framework to justify these ambigu-
ous results. In particular, we distinguish: i) a rent dissipation effect that reduces the profit
margin for all firms but disproportionately affects the low-cost ones and slackens CSR ; and
ii) a selection effect that increases the market shares of low-cost firms while it reduces the

\textsuperscript{4} For instance, Bagnoli and Watts (2003) conclude to a negative relationship while in Fisman et al. (2006),
the link is positive (see the enlightening discussion in Fernandez-Kranz and Santalo 2010 on this point).

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market shares of high-cost ones and favors CSR. We show that one or the other dominates, depending on the prevailing market conditions like the proportion of responsible employees. A similar conclusion may be reached when considering the impact of market integration on the aggregate social performance.

Our comparative statics also provide some interesting insights for empirical studies devoted to the CSR-competition relationship. These works consist in estimating the impact of changes in the HHI on measures of pro-social activities (see, for instance, Gupta and Krishnamurti 2016 or Simon and Prince 2016). Our results highlight that these analysis should be interpreted with caution due to the endogenous nature of the HHI with respect to CSR. In particular, the causality - from competition to social performances - could go the other way around.

Our article is related to different strands of the economic literature. First, we are in line with a growing body of papers dealing with agency problems when agents do not only respond to standard pecuniary incentives but also to non-monetary aspects of their jobs. Most of these papers consider those aspects as given while, in our set-up, CSR is a strategic variable used by the principal as both a screening and an incentive device. As far as we know, Brekke and Nyborg (2008) is the unique paper that considers CSR investments as a mean to attract and incentivize morally motivated agents. However, in their model, CSR is treated as a yes or no while the intensity of CSR activities is a central element for us. It allows to put forward an intensive margin of CSR which is at the hearth of a potentially non-monotonous relationship between workers’ social consciousness and the CSP. Moreover,

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Brekke and Nyborg (2008) assume perfect competition on the product market and, by the way, have nothing to say about the CSR-competition nexus. To that extent, our article is related to a second strand of the literature, which deals with the strategic CSR and its relationship with competition (see Bagnoli and Watts 2003 or Fisman et al. 2006). Nevertheless, none of these articles consider CSR as a mean to attract motivated employees. In addition, they take the competitive structure as given while, in our framework, the HHI is endogenous and the relationship between CSR and competition goes in both directions. Along this line, García-Gallego and Georgantzís (2009) theoretically investigate the impact of a change in consumers’ social responsibility on the market structure. However, they focus on discrete changes in the market structure and are interested in the impact of consumers’ willingness to pay for products sold by socially responsible companies on CSR. Finally, the recent paper by Manna (2017) proposes a model in which firms may hire either self-interested or customer-oriented employees and then compete in price and in quality. Nevertheless, she does not introduce CSR investments as a strategic choice variable for firms.

The remainder of the paper is organized as follows. Section 2 presents the model and characterizes the equilibrium of the industry. Section 3 discusses comparative statics. Finally, Section 4 concludes.

2 The Model

The industry comprises a larger number \((L)\) of consumers, a large number \((N)\) of potentially active firms (she hereafter) and a large number \((M)\) of skilled workers (he hereafter) with \(N > M\). Each firm (the principal) employs a skilled worker (the agent) in order to produce a variety of a horizontally differentiated good. This worker should supervise the production
process and, in addition to his managerial tasks, he may exert some extra efforts that could, to some extent, improve the quality of the production process and thereby reduce the unit cost of production. The latter could be either high or low and is low with a higher probability when the worker makes more efforts.

Two types of skilled workers co-exist: the responsible and the non-responsible ones. Responsible employees are not only interested in their net payoff (wage minus disutility of effort) but also in the type of firm they work in. More precisely, they can be intrinsically motivated depending on the firm’s pro-social behavior. Hence, the intensity of CSR activities may be strategically used by firms in order to attract and to incentivize responsible managers. In the following, we will refer to responsible employees as “green” (indexed by $g$) and non-responsible ones as “brown” (indexed by $b$) while CSR activities will be, most of the time, termed “abatement”. This terminology is adopted for the sake of realism since environmentally friendly actions are big part of overall CSR initiatives. Obviously, our framework applies for all other aspects of CSR.

Because $N > M$, firms compete for a limited pool of talented workers some of them being responsible. They can use two types of instruments: the wage to the employee and a level of CSR investment. The precise timing of the events is as follows:

i  Firms simultaneously decide to enter.

ii  Firms who enter the market choose a level of abatement and a wage rate. Each worker chooses whether to accept or not an offer and, if so, which one.

iii Workers choose their level of effort and production costs are revealed.

iv  Active firms compete in price. Wages are paid to workers.
We solve the model backward. Stage iv (product market level) is solved in Section 2.1, the distribution of production costs (cost structure) being given. Then stages i, ii and iii (labor market level) are solved in Section 2.2. In Section 2.3, we analyze how competition at the labor market level interacts with competition at the product market level and we characterize the equilibrium of the industry. Finally, in Section 2.4, we provide analytical expressions for the aggregate level of abatement (Corporate Social Performance of the industry) and the Herfindhal-Hirschman Index at the industry equilibrium.

### 2.1 Product market equilibrium

On the product market, we consider a Salop-like model of monopolistic competition where $L$ consumers are uniformly located on a circle of length one. When $n$ firms are active in the industry, they are equidistantly distributed on the unit circle. The location of a firm may be interpreted as the specific variety of the good that she produces. Consumers wish to buy one unit of good exactly and derive a utility $v$ from this consumption. This utility is high enough to ensure that all consumers purchase the good. The individual surplus of a consumer who buys the good to firm $i$ located at a distance $x$ from his ideal variety equals $v - p_i - xd$, with $p_i$ the price charged by firm $i$. The parameter $d$ accounts for the distaste of the consumer when moving away from his ideal variety. It can also be viewed as a measure of the competition intensity on the product market.

We depart from the standard circular model of spatial differentiation by introducing cost heterogeneity. Each firm has a probability $\rho \in [0,1]$ to be highly efficient (with low unit costs $c^A$) and a probability $1 - \rho$ to be poorly efficient (with high unit costs $c^B$). In order to ensure that, at the equilibrium, both types of firms are active we assume that the cost
differential \((\Delta c \equiv c^B - c^A > 0)\) is small enough. Formally, the following condition must hold:

**Assumption 1** \(\Delta c < 2d/n\)

Following Aghion and Schankerman (2004), we determine the Nash equilibrium in price under the following assumptions:

i When choosing their price, firms know their own type and the distribution of types \((\rho)\) but do not know the precise type of her neighbors.

ii We restrict our attention to the symmetric equilibrium of the game: All firms of a same type \(j\) charge the same price \(p^j\) at the equilibrium.

A firm \(i\) has only two relevant competitors: Her immediate left-hand side neighbor \(i - 1\) and her immediate right-hand side neighbor \(i + 1\), each of whom is charging a price \(p_{i-1}\) and \(p_{i+1}\) respectively. Let us define \(\bar{x}_{i-1}\) (resp. \(\bar{x}_{i+1}\)) the distance between the firm \(i\) and the consumer who is indifferent between consuming the good produced by \(i\) and \(i - 1\) (resp. \(i + 1\)): \(p_i + d\bar{x}_{i-1} = p_{i-1} + d(1/n - \bar{x}_{i-1})\) and \(p_i + d\bar{x}_{i+1} = p_{i+1} + d(1/n - \bar{x}_{i+1})\). The demand addressed to firm \(i\) can be expressed as:

\[
D(p_i; p_{i-1}, p_{i+1}) = L[\bar{x}_{i-1} + \bar{x}_{i+1}] = L \left[\frac{1}{n} + \frac{p_{i-1} + p_{i+1}}{2d} - \frac{p_i}{d}\right] \quad (1)
\]

When choosing \(p_i\), firm \(i\) does not observe neither \(p_{i+1}\) and \(p_{i-1}\) nor the production costs of firms \(i + 1\) and \(i - 1\). However, as we focus our analysis on the symmetric equilibrium of the

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6. See Aghion and Schankerman (2004) for an extensive discussion of these assumptions. Note that, in their baseline model, Aghion and Schankerman (2004) consider the cost structure \((c^A, c^B, \rho)\) as given. In an extended version of their framework, they endogenize \(c^A\) and \(c^B\) by assuming that firms can engage cost-reduction investments before the price competition stage. In the present model, we propose an alternative way to endogenize the cost structure since we consider that the probability \(\rho\) to be a low-cost firm depends on the managerial incentives scheme chosen by firms.
game, we can derive the expression of the expected demand\textsuperscript{7} of firm $i$ as a function of $p_i$:

$$\bar{D}(p_i) = \rho^2 D(p_i; p^A, p^A) + (1 - \rho)^2 D(p_i; p^B, p^B) + 2\rho(1 - \rho) D(p_i; p^A, p^B)$$ \hfill (2)

Finally, the expected benefits of firm $i$ (equal to profits when abstracting from both the salary costs and CSR investments) stand to be: $\pi_i(c^j) = (p_i - c^j)\bar{D}(p_i)$. Let us define $p(c^j; p^A, p^B) = \arg\max_{p_i} \pi_i(c^j)$ as the best-response of a firm of type $j$ to the equilibrium strategy $(p^A, p^B)$. The equilibrium pricing rule\textsuperscript{8} is solution of the system $p(c^A; p^A, p^B) = p^A$ and $p(c^B; p^A, p^B) = p^B$, that yields:

$$p^A = \frac{d}{n} + c^A + \frac{1 - \rho}{2} \Delta c \quad \text{and} \quad p^B = \frac{d}{n} + c^B - \frac{\rho}{2} \Delta c$$ \hfill (3)

Replacing those equilibrium prices (3) into equation (2), we obtain the expected demand, $D^j \equiv \bar{D}(p^j)$, and expected benefits, $\pi^j \equiv (p^j - c^j)\bar{D}(p^j)$, for a firm of type $j$:

$$D^A = L \left[ \frac{1}{n} + \frac{1 - \rho}{2d} \Delta c \right] \quad \text{and} \quad D^B = L \left[ \frac{1}{n} - \frac{\rho}{2d} \Delta c \right]$$ \hfill (4)

$$\pi^A = \frac{L}{4dn^2} [2d + (1 - \rho)n \Delta c]^2 \quad \text{and} \quad \pi^B = \frac{L}{4dn^2} [2d - \rho n \Delta c]^2$$ \hfill (5)

Finally, we deduce the expected benefit differential between a low-cost and a high-cost firm:

$$\pi^A - \pi^B = \frac{L}{n} \Delta c + \frac{L}{4d} (1 - 2\rho)(\Delta c)^2$$ \hfill (6)

An increase in $\rho$ reduces the profit for both types of firms, but this reduction is proportional to the market share. Hence, low-cost firms suffer more from an increase in competition and the profit differential is decreasing in the share of efficient firms (see equation (6)).

\textsuperscript{7} To the extent that the number of managers - and thus active firms - is finite, this expected demand is in fact an approximation. Nevertheless, it becomes accurate when the number of firms on the market is large enough.

\textsuperscript{8} From now on, we neglect the index $i$ when referring to prices and profits as we focus on the symmetric equilibrium configuration.
As previously discussed, at the product market level, the cost structure is taken as given by firms who compete in price. However, the parameter $\rho$ turns out to be endogenous, since it is shaped by the incentives offered by firms on the labor market. This is what we investigate in the following section.

2.2 Labor market equilibrium

As described above, the benefits of a firm depend on her production costs as well as the cost structure within the industry. Let us first clarify how the production costs of a given firm are related to the behavior of the worker employed in that firm. We consider that the unit cost depends on an extra effort that the employee can choose to achieve or not. Formally, once employed, the worker chooses a level of effort $e \in \{e^L, e^H\}$, with $e^L$ (normalized to zero) the baseline effort and $e^H > 0$ an extra effort. This effort $e \in \{e^L, e^H\}$ parametrizes the probability for a firm to produce at low cost: Costs are low ($c^A$) with a probability $\lambda(e) \in (0, 1)$ and high ($c^B$) with a probability $1 - \lambda(e)$, with $\lambda(e)$ increasing in $e$. As developed in the previous section, to a production cost $c^j$ corresponds a value of expected benefits $\pi^j$. Then, we can define the expected benefits of a firm conditional to the effort $e$ provided by the worker she employs:

$$\bar{\pi}(e) \equiv \lambda(e)\pi^A + (1 - \lambda(e))\pi^B$$  \hspace{0.5cm} (7)

We also define $\lambda^H \equiv \lambda(e^H)$, $\lambda^L \equiv \lambda(e^L)$, $\bar{\pi}^H \equiv \bar{\pi}(e^H)$, $\bar{\pi}^L \equiv \bar{\pi}(e^L)$, and the expected incremental benefits associated with high effort:

$$\Delta\bar{\pi} = (\lambda^H - \lambda^L)(\pi^A - \pi^B)$$  \hspace{0.5cm} (8)
In the following we rely on Brekke and Nyborg (2008), Delfgaauw and Dur (2008) or Benabou and Tirole (2015) by assuming that $e$ is not measurable and the contribution of this effort to the output of the firm hard to figure out, so that this contribution cannot be part of a formal compensation scheme. This allows us to drastically simplify the moral hazard side of the model since, in this set-up, wages are only used to ensure workers participation.

**Profits and preferences.** The expected profits of a firm are simply given by the difference between the expected benefits and the production costs. Those costs include the wage $w$ paid to the worker she employs and the price of CSR activities the firm can choose to afford. Those business practices are captured by a level of pollution abatement $a \in \mathbb{R}^+$ that induces abatement costs denoted $\phi(a)$, with, $\phi(0) = 0$, $\phi'(a) > 0$ and $\phi''(a) \geq 0$. Abatement costs are assumed to be independent of the firm’s production level and may encompass many types of CSR activities (e.g. planting trees in the rain forest or investing to preserve the coral reef, cleaning-up coastal areas and the like). Thus, the expected profits of a firm choosing the abatement level $a$, proposing the wage rate $w$ and hiring a worker who chooses a level of effort $e$ are:

$$\Pi(w, e, a) = \bar{\pi}(e) - w - \phi(a) \quad (9)$$

Two types of workers co-exist: a proportion $q$ of responsible workers and a proportion $1-q$ of non-responsible ones. Brown workers are considered to be standard *homoeconomicus*:

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9. This assumption may be justified by the fact that, before a court, output variation cannot be imputable to a higher involvement of the worker. Alternatively, it could be the case that workers themselves are not fully aware of the induced effects by their efforts on the probability distribution over outputs such that they do not react to monetary incentives.

10. Whenever the managerial effort may be related to the firm’s output, it becomes relevant to introduce monetary incentives in order to solve the moral hazard problem. The existing trade-off between monetary and non-monetary incentives is clearly beyond the scope of the present paper (on that issue, see Hiller and Verdier, 2014).

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They are only interested in the income \((y)\) they earn and the effort \((e)\) they spent:

\[
U_b(y,e,a) = y - e
\]  

(10)

Green workers may exhibit some intrinsic motivations to exert extra efforts if the firm they work in is deemed to be sufficiently socially responsible. Formally, the intensity of CSR activities affects the disutility that a green worker associates with hard working:

\[
U_g(y,e,a) = y - e + v(e,a)
\]  

(11)

with \(v(0,a) = 0\), \(v'_e(e,a) > 0\), \(v'_a(e,a) > 0\), \(v''_{aa}(e,a) < 0\) and \(v''_{ea}(e,a) > 0\).

The idea according to which some workers, in specific contexts, may enjoy exerting effort, has been already put forward by Benabou and Tirole (2003) or Delfgauuw and Dur (2007, 2008). In Delfgauuw and Dur (2007, 2008), this intrinsic motivation effect interacts with the type of job or the sector (public vs. private)\(^{11}\) just as it interacts with abatement in our framework. Said differently, by investing in CSR, firms tend to achieve socially oriented missions potentially aligned with the own goals of a responsible agent. Our formulation of the green worker’s utility is also close to the one adopted by Brekke and Nyborg (2008). One important point of departure though lies in the continuous feature of the intrinsic motivation effect (firms choose the intensity of their CSR activities) while, in Brekke and Nyborg (2008), it is only dichotomous (either positive or null since firms choose to invest or not in CSR). The existence of this intensive margin effect allows us to highlight a new purpose of CSR, the screening of green workers.

The game between firms and skilled workers at the labor market level may be decomposed

\(^{11}\) Similarly, in Besley and Ghatak (2005), some agents value their personal contribution to the output of the firm when their preferences are aligned with the mission of the organization they work for.
into two steps (steps ii and iii in the timing of the events): First, firms who enter the market choose a level of abatement $a$ and offer a wage rate $w$ while workers select their preferred offer. Second, workers choose their level of effort. The equilibrium of this game, when principals do not observe the precise type of agents but only know the proportion of green ones, is described below.

**Characterization of the labor market equilibrium.** First, since the level of output is non-contractible, the wage rate cannot be used in order to incentivize workers. In addition, since workers are on the short side of the market ($N > M$), this wage rate simply adjusts to ensure that firms’ profits are pinned down to zero. Second, a brown worker always prefers to exert low effort whatever the level of abatement. On the contrary, a green worker chooses to exert high effort if and only if the intrinsic motivation effect is at least equal to the disutility of effort. From (11), it is the case when the investment in CSR is a above a threshold $\bar{a}$, such that $v(e^H, \bar{a}) = e^H$. Let us also define $a^*$ as the value of abatement such that the marginal cost of abatement exactly equals the marginal utility of abatement for a hard working green worker: $\phi'(a^*) = v_a'(e^H, a^*)$. Suppose that the type of workers were observable, it would be optimal for a firm, aiming at inducing high-effort, to propose a level of abatement $a^*$ to a green worker, where:

$$a^* = \max\{a^*, \bar{a}\}$$

We refer to $a^*$ as the first best level of abatement. In addition, for the sake of simplicity, we assume that:

**Assumption 2** $\phi'(\bar{a}) < v_a'(e^H, \bar{a})$

It implies that $a^* > \bar{a}$, then $a^* = a^*$, and we define the induced-abatement costs $\phi^*$ as well.

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12. If a worker is indifferent between different offers, he accepts each of them with equiprobability.
as the net intrinsic motivation component of utility $\nu^*$ as follows:

$$
\phi^* \equiv \phi(a^*) > 0 \quad \text{and} \quad \nu^* \equiv v(e^H, a^*) - e^H \geq 0
$$

Having those elements in mind, the following proposition describes the labor market equilibrium:

**Proposition 1** At the labor market equilibrium (when it exists), $M$ firms enter the market, a proportion $q$ of those firms offers a pair $(\hat{w}_g, \hat{a}_g)$ while a proportion $1 - q$ offers a pair $(\hat{w}_b, \hat{a}_b)$ with:

$$(\hat{w}_b, \hat{a}_b) = (\bar{\pi}^L, 0) \quad \text{and} \quad (\hat{w}_g, \hat{a}_g) = \begin{cases} 
(\bar{\pi}^L, 0) & \text{if } \Delta \bar{\pi} < \phi^* - \nu^* \\
(\bar{\pi}^L, 0) \cup (\bar{\pi}^H - \phi^*, a^*) & \text{if } \Delta \bar{\pi} = \phi^* - \nu^* \\
(\bar{\pi}^H - \phi^*, a^*) & \text{if } \Delta \bar{\pi} \in (\phi^* - \nu^*, \phi^*) \\
(\bar{\pi}^L, \phi^{-1}(\Delta \bar{\pi})) & \text{if } \Delta \bar{\pi} > \phi^*
\end{cases}
$$

(12)

Green workers choose $(\hat{w}_g, \hat{a}_g)$ and an effort $\hat{\epsilon}_g$ while brown ones choose $(\hat{w}_b, \hat{a}_b)$ and an effort $\hat{\epsilon}_b$ with:

$$
\hat{\epsilon}_b = 0 \quad \text{and} \quad \hat{\epsilon}_g = \begin{cases} 
0 & \text{if } \Delta \bar{\pi} < \phi^* - \nu^* \\
0 \cup e^H & \text{if } \Delta \bar{\pi} = \phi^* - \nu^* \\
e^H & \text{if } \Delta \bar{\pi} > \phi^* - \nu^*
\end{cases}
$$

(13)

Finally, there exists $\hat{q} \in (0, 1)$ such that, when $\Delta \bar{\pi} > \phi^*$ and $q > \hat{q}$, the labor market equilibrium does not exist.

**Proof.** See Appendix A

As previously discussed, a firm whose goal is to attract a brown worker does not invest in abatement ($\hat{a}_b = 0$) and the brown worker chooses $e_L$. Consequently, the zero-profit condition implies that $\hat{w}_b = \bar{\pi}^L$. The offer made to attract a green worker may be intuitively illustrated thanks to Figure 1 that describes how $\hat{a}_g$ evolves with incremental benefits $\Delta \bar{\pi}$.

Let us recall that, when offering the first best level of abatement $a^*$ to a green worker, a firm expects two types of economic gains: i) since $a^* > \bar{a}$, the green worker is induced to work hard and expected benefits are enhanced (this effect is measured by $\Delta \bar{\pi} > 0$); and ii) the green worker is prone to accept a lower wage in exchange for the level of abatement $a^*$.
Fig. 1. Evolution of CSR activities as a function of $\Delta \bar{\pi}$

(this effect is measured by $\nu^*$ that could be interpreted as the willingness to pay for working at a firm who invests $a^*$ in CSR). Hence, when $\Delta \bar{\pi} < \phi^* - \nu^*$ it is not beneficial for a firm to induce her green employee to work hard. In that case, the equilibrium is pooling since all workers (green and brown) are employed by firms who do not invest in abatement. On the contrary, as soon as $\Delta \bar{\pi} > \phi^* - \nu^*$, the costs induced by the first best level of abatement are more than compensated by the incremental benefits associated with the extra effort they provide. Then, investing in CSR increases firms’ profitability. Some firms aim at attracting and incentivizing green managers by offering them the first best level of abatement $a^*$. Since the extra managerial effort generates higher benefits equal to $\bar{\pi}^H$, while the level of abatement $a^*$ induces a cost $\phi^*$, the zero profit condition implies that $\hat{w}^g = \bar{\pi}^H - \phi^*$. However, when $\Delta \bar{\pi}$ becomes higher than $\phi^*$, $\bar{\pi}^H - \phi^* > \bar{\pi}^L$ so that the separation condition does not hold anymore: brown workers are willing to pick the offer initially designed for green ones. In that configuration, CSR investments serve as a screening device. Indeed, to select a true green employee firms must increase abatement in order to pin down $\hat{w}^g$ to $\bar{\pi}^L$ such that brown workers are discouraged to apply. Thus, the level of abatement chosen by pro-social firms is

17
given by the condition: \( \bar{\pi}^H - \phi(a) = \bar{\pi}^F \) implying that \( \hat{a}_g = \phi^{-1}(\Delta \bar{\pi}) \).

The positive relationship between \( \Delta \bar{\pi} \) and CSR investments by firms somewhat echoes the result obtained by Brekke and Nyborg (2008). Nevertheless, in their model, firms do not choose the intensity of their investment so that the increasing pattern depicted in Figure 1 is fully driven by an extensive margin effect. On the contrary, in our set-up, this pattern emerges at the firm level and stems from the presence of an intensive margin effect. When \( \Delta \bar{\pi} > \phi^* \) this intensive margin effect is driven by a new purpose of CSR investments i.e. the screening of green employees.

At the labor market equilibrium\(^\text{14}\), the intensity of CSR activities crucially depends on the incremental benefits associated with high effort (\( \Delta \bar{\pi} \)). Then, this intensity affects the managerial incentives granted to green employees (equation (13)) that determine the probability for a given firm to be efficient. In turn, at the industry level, it shapes the cost structure parameter \( \rho \) that is a key determinant of the profit differential between low and high-cost firms and thus \( \Delta \bar{\pi} \). These mutual interplay between CSR decisions by firms and the toughness of competition on the product market, make it crucial to precisely analyze the competitive equilibrium at the industry level.

### 2.3 Industry equilibrium

The cost structure in the industry is directly related to the competitive equilibrium reached at the labor market level. From (13), we obtain the following expression for the probability

\[^{13}\] As detailed in Appendix A, the proof of this result follows the classical Rotshild-Stiglitz arguments (see Rotshild and Stiglitz 1976).

\[^{14}\] As claimed in Proposition 1, in some configurations, the labor market equilibrium does not exist. Nevertheless, as it will become clear in the next section, some parameters condition may ensure that this case never arises at the industry equilibrium. Then, we investigate the case where a competitive equilibrium exists.
\( \rho \) of being a low-cost firm:

\[
\rho \left\{ \begin{array}{ll}
= \lambda^L & \text{if } \Delta\bar{\pi} < \phi^* - \nu^* \\
\in [\lambda^L, q\lambda^H + (1 - q)\lambda^L] & \text{if } \Delta\bar{\pi} = \phi^* - \nu^* \\
= q\lambda^H + (1 - q)\lambda^L & \text{if } \Delta\bar{\pi} > \phi^* - \nu^*
\end{array} \right. 
\]  

(14)

When incremental benefits are small, firms do not have any incentive to invest in CSR and thus, a unique contract is offered to both types of workers: \((\bar{\pi}^L, 0)\). Workers do not provide any extra effort and the probability to be a low-cost firm is pinned-down to \(\lambda^L\). When incremental benefits overcome the net cost of CSR, it becomes profitable to motivate green employees through the spread of pro-environmental activities. Following Proposition 1, a proportion \(q\) (resp. \(1 - q\)) of firms has a probability \(\lambda^H\) (resp. \(\lambda^L\)) to exhibit low production costs. When \(\Delta\bar{\pi}\) exactly equals \(\phi^* - \nu^*\), firms are indifferent between investing in CSR or not: Some of them offer the pair \((\bar{\pi}^L, 0)\) that does not incentivize green workers and the residual fraction offers \((\bar{\pi}^H - \phi^*, a^*)\) that does encourage responsible employees to work hard. Consequently, the proportion of low-cost firms belongs to the interval \([\lambda^L, q\lambda^H + (1 - q)\lambda^L]\).

In turn, expected benefits \((\bar{\pi}^H\) and \(\bar{\pi}^L\)) and incremental benefits \((\Delta\bar{\pi})\) are determined on the product market as the results of the price-competition stage and vary with the endogenous cost structure parameter \(\rho\), which value is deduced from labor market competition.

Combining expressions (5) with expressions (7) and (8) yields:

\[
\bar{\pi}^H = \frac{L}{4dM^2} \left[ 4d^2 + (\lambda^H(1 - 2\rho) + \rho^2)M^2(\Delta c)^2 + 4dM(\lambda^H - \rho)\Delta c \right] \equiv \bar{\pi}^H(\rho) 
\]  

(15)

\[
\bar{\pi}^L = \frac{L}{4dM^2} \left[ 4d^2 + (\lambda^L(1 - 2\rho) + \rho^2)M^2(\Delta c)^2 + 4dM(\lambda^L - \rho)\Delta c \right] \equiv \bar{\pi}^L(\rho) 
\]  

(16)

\[
\Delta\bar{\pi} = \frac{L(\lambda^H - \lambda^L)\Delta c}{4dM} [(1 - 2\rho)M\Delta c + 4d] \equiv \Delta\bar{\pi}(\rho) 
\]  

(17)
At the industry equilibrium, the values of $\bar{\pi}_H$, $\bar{\pi}_L$ and $\Delta\bar{\pi}$ must be consistent with the equilibrium set of offers $\{(\hat{w}_b, \hat{a}_b), (\hat{w}_g, \hat{a}_g)\}$ and accordingly with the probability for a given firm to be highly efficient, $\rho$. Formally, the competitive equilibrium of the industry is defined as follows:

**Definition 1** An industry equilibrium is a list $(\bar{\pi}_H, \bar{\pi}_L, \Delta\bar{\pi}, \{(\hat{w}_b, \hat{a}_b), (\hat{w}_g, \hat{a}_g)\}, \rho)$ that solves (12), (14), (15), (16) and (17).

In order to characterize that equilibrium, let us assume that the following conditions apply:

**Assumption 3** $v'_e(e^H, a^*) \geq 1$

**Assumption 4** $\phi^* - \nu^* < \Delta\bar{\pi}(\lambda^H) \leq \phi^* \leq \Delta\bar{\pi}(\lambda^L)$

Assumptions 3 and 4 are made in order to abstract from cases which are irrelevant for our purpose and to ensure the co-existence of the two types of firms at the equilibrium.\(^\text{15}\) In particular, we focus our analysis on the separating equilibrium by assuming that equilibrium incremental benefits are larger than $\phi^* - \nu^*$.

Then, we define the threshold $\bar{\rho}$ which corresponds to the value of $\rho$ such that $\Delta\bar{\pi}(\bar{\rho}) = \phi^*$:

$$\bar{\rho} = \frac{1}{2} + \frac{2d}{M\Delta c} - \frac{2d\phi^*}{L(\lambda^H - \lambda^L)(\Delta c)^2} \quad \text{and} \quad \bar{q} = \frac{\bar{\rho} - \lambda^L}{\lambda^H - \lambda^L}$$

(18)

and we claim:

**Proposition 2** Under Assumptions 1-4 and for $\phi^*$ sufficiently close to $\Delta\bar{\pi}(\lambda^L)$, an industry

\(^{15}\) Defining $e^H_{\min}$ (resp. $e^H_{\max}$) as the value of $e^H$ such that $\phi^* = \Delta\bar{\pi}(\lambda^H)$ (resp. $\phi^* = \Delta\bar{\pi}(\lambda^L)$), Assumption 4 is satisfied for $e^H \in [e^H_{\min}, e^H_{\max}]$. Assumption 3 ensures that $\phi^* - \nu^* < \Delta\bar{\pi}(\lambda^H)$ for this set of values of $e^H$ (see Appendix B).
equilibrium exists, is unique, and is characterized by:

\[
\begin{align*}
\rho &= q \lambda^H + (1 - q) \lambda^L \\
\Delta \bar{\pi} &= \Delta \bar{\pi} \left( q \lambda^H + (1 - q) \lambda^L \right) \\
\bar{\pi}^L &= \bar{\pi}^L \left( q \lambda^H + (1 - q) \lambda^L \right) \\
\bar{\pi}^H &= \bar{\pi}^H \left( q \lambda^H + (1 - q) \lambda^L \right) \\
(\hat{w}_b, \hat{a}_b) &= (\bar{\pi}^L, 0) \\
(\hat{w}_g, \hat{a}_g) &= \begin{cases} 
(\bar{\pi}^L, \phi^{-1}(\Delta \bar{\pi})) & \text{if } q < \bar{q} \\
(\bar{\pi}^H - \phi^*, a^*) & \text{if } q \geq \bar{q}
\end{cases}
\end{align*}
\]

**Proof.** See Appendix B ■

The industry equilibrium is merely defined by the fact that the incremental benefits associated with hard working (\(\Delta \bar{\pi}\)) must be consistent with the probability to be a highly efficient firm (\(\rho\)).\(^{16}\) We draw on Figure 2, in the plan (\(\rho, \Delta \bar{\pi}\)), the two equilibrium relationships given by equations (14) and (17), each of them representing the labor market equilibrium (LL curve) and the product market equilibrium (PP curve) respectively. The industry equilibrium corresponds to the crossing points between these two curves.\(^{17}\) Under Assumptions 3 and 4, the industry equilibrium is a separating equilibrium in which a proportion \(q\) of firms offers (\(\hat{w}_g, \hat{a}_g\)) and the others offer (\(\hat{w}_b, \hat{a}_b\)). Nevertheless, according to the value of \(q\), these offers might differ. We successively address each of the two possible configurations.

When \(q \geq \bar{q}\) (see Figure 2(a)), the equilibrium value of \(\Delta \bar{\pi}\) belongs to the interval \((\phi^* - \nu^*, \phi^*)\) and the sole role of CSR is to elicit high effort from green workers: “green firms” offer \((\bar{\pi}^H - \phi^*, a^*)\). If \(q < \bar{q}\) (Figure 2(b)), the toughness of cost competition at the product market level is quite low. Consequently, equilibrium incremental benefits are large

\(^{16}\)As claimed in Proposition 1 the labor market equilibrium may fail to exist when \(q > \hat{q}\) and \(\Delta \bar{\pi} > \phi^*\). Nevertheless, we know from Proposition 2 that \(\Delta \bar{\pi}\) overcomes \(\phi^*\) if and only if \(q < \bar{q}\). Hence, a sufficient condition for the equilibrium to exist is that \(\bar{q} \leq \hat{q}\). As shown in the proof of Proposition 2 this condition is satisfied for \(\phi^*\) sufficiently close to \(\Delta \bar{\pi}(\lambda^L)\).

\(^{17}\)Once \(\Delta \bar{\pi}\) and \(\rho\) have been determined, the equilibrium values of \(\bar{\pi}^L\) and \(\bar{\pi}^H\) may be deduced, see equations (15) and (16); while the (wage/abatement) offers fully depend on \(\Delta \bar{\pi}, \bar{\pi}^L\) and \(\bar{\pi}^H\).
and the wage gap on the labor market widens. As a direct consequence, brown workers may be tempted to pick the offer \((\bar{\pi}^H - \phi^*, a^*)\) in order to benefit from a higher wage. Hence, green firms optimally choose to invest more in abatement \((\hat{a}_g = \phi^{-1}(\Delta \bar{\pi}))\) in order to lower wages and to discourage brown managers. In that case, CSR is both a screening and a motivation device.

Interestingly, the interactions between the competition on the labor market (captured by the LL curve) and the competition on the product market (captured by the PP curve) transit through the abatement choices made by firms. On the one hand, “green firms” invest in CSR all the more that incremental benefits are high (labor market level). On the other hand, abatement strategies shape the cost competition on the product market. Hence, crucial to our results, the aggregate social performance and the competitive pressure of the industry are jointly determined.

Fig. 2. The industry equilibrium
2.4 Social performance and competitive pressure at the industry equilibrium

At the industry level, we measure the corporate social performance (CSP) by the aggregate level of abatement denoted \( A \) and the competitive pressure by the Herfindhal-Hirschman Index (HHI) denoted \( H \). The equilibrium values of \( A \) and \( H \) are provided in Corrolaries 2 and 3.\(^{18}\)

**Corollary 2** Under Assumptions 1-4 and for \( \phi^* \) sufficiently close to \( \Delta \bar{\pi}(\lambda^L) \):

\[
A = \begin{cases} 
q \phi^{-1} \left( \Delta \bar{\pi} \left( \lambda^L + q[\lambda^H - \lambda^L] \right) \right) & \text{if } q < \bar{q} \\
a^* & \text{if } q \geq \bar{q}
\end{cases} 
\tag{25}
\]

At the industry level, the CSP depends on two components: the number of firms who do engage in CSR (extensive margin) and the intensity of their investment (intensive margin). The extensive margin equals \( q \), because the economy is at the separating equilibrium. The intensive margin equals either \( a^* \), when abatement is only used as a motivation device, or 
\( \phi^{-1} \left( \Delta \bar{\pi} \left( \lambda^L + q[\lambda^H - \lambda^L] \right) \right) \) when CSR also plays the role of a screening device. What is relevant for our purpose is that this intensive margin is driven by market conditions through the equilibrium value of \( \Delta \bar{\pi} \). This novel feature of our model, with respect to the existing literature, is more deeply investigated in the next section when studying the relationship between the CSP and workers’ preferences.

The HHI is defined as the sum of the squared market shares: 
\( H = n \rho (D^A)^2 + n (1 - \rho) (D^B)^2 \). Using (4), the HHI can be expressed as the following function of \( \rho \):

\[
H(\rho) = \frac{L}{n} \left[ 1 + \left( \frac{n \Delta c}{2d} \right)^2 \rho (1 - \rho) \right] 
\tag{26}
\]

From Proposition 2, we get that:

\(^{18}\) Proofs of Corollaries 2 and 3 are omitted since they can directly be derived from Proposition 2.
Corollary 3 Under Assumptions 1-4 and for $\phi^*$ sufficiently close to $\Delta \bar{\pi}(\lambda^L)$:

$$H = \frac{L}{M} \left\{ 1 + \left( \frac{M \Delta c}{2d} \right)^2 \left[ \lambda^L + q(\lambda^H - \lambda^L)] \left[ 1 - \lambda^L - q(\lambda^H - \lambda^L) \right] \right\}$$ (27)

Because the CSP and the HHI are jointly determined at the equilibrium, our framework is particularly appropriate to investigate the CSR-competition nexus. The endogenous feature of the HHI and the consequences on this specific issue will be more deeply discussed in Section 4. Before that, we focus our analysis on some determinants of the virtuous firms’ behaviors at the industry level.

3 Comparative statics

In this section we investigate how does the CSP vary with the proportion $q$ of green workers or let us say the green consciousness among skilled workers (Section 3.1). Then, in order to isolate the impact of competition on corporate social responsibility, we focus our analysis on the environmental outcomes induced by an exogenous variation of the competitive environment, captured by a change in the transport cost parameter $d$ (Section 3.2) or a change in markets size (Section 3.3).

3.1 The distribution of workers’ preferences

When $q \geq \bar{q}$, since $\Delta \bar{\pi} < \phi^*$, the purpose of CSR expenditures is only to incentivize green workers such that “green firms” choose the first best level of abatement $a^*$ which is invariant with $q$. Hence, $A$ is linearly increasing in $q$ through the extensive margin effect. When $q < \bar{q}$, CSR activities (equal to $\Psi(q)$) are strategically used as a screening device and thus are positively related to $\Delta \bar{\pi}$. Moreover, each additional motivated worker exacerbates competition on the product market which, in turns, reduces $\Delta \bar{\pi}$. Hence, the green consciousness among
workers has also a countervailing effect on the individual effort of CSR. Put it differently, in this configuration, the global effect of $q$ on the CSP of the industry depends on the relative strength of the positive extensive margin effect with regards to the negative intensive margin one. To determine under which condition one effect dominates the other, let us focus our analysis on the two levels at which firms interact (the product and the labor market) and claim that:

$$\text{sign} \left\{ \Psi'(q) \right\} = \text{sign} \left\{ |\epsilon_{\phi,a}(q)| - |\epsilon_{\Delta \pi,q}(q)| \right\}$$

(28)

with $\epsilon_{x,y}(q)$ the elasticity coefficient between $x$ and $y$ expressed as a function of $q$. As highlighted in (28) the sign of $\Psi'(q)$ depends on two prominent effects:

i At the product market level, $|\epsilon_{\Delta \pi,q}(q)|$ measures the magnitude of the fall in $\Delta \pi$ triggered by the arrival of an additional green worker;

ii At the labor market level, $|\epsilon_{\phi,a}(q)|$ accounts for the sensitivity of abatement costs to a change in $a$. If this sensitivity is low, a given fall in $\Delta \pi$ induces a large cut in abatement since a substantial drop in $a$ is necessary to ensure that the equality $\Delta \pi = \phi(a)$ is still satisfied.

Overall, if $|\epsilon_{\Delta \pi,q}(q)| > |\epsilon_{\phi,a}(q)|$, the drop in CSR investment at the individual-firm level (intensive margin effect) is sufficiently large to overcome the supplementary abatement involved by the arrival of an additional “green firm” (extensive margin effect) such that $\Psi'(q) < 0$.

Our analysis leads to the conclusion that, if $|\epsilon_{\phi,a}(q)|$ is large enough (for all $q \leq \bar{q}$) the extensive margin effect always dominates and $A$ is increasing in $q$. On the contrary, if for some $q \in [0, \bar{q}]$, $|\epsilon_{\phi,a}(q)|$ is too low, the intensive margin effect dominates and the CSP of

19. See Appendix C.
the industry may be depressed despite the spread of a green consciousness among workers.

More precisely, we can establish that:

**Proposition 3** Under Assumptions 1-4 and for \( \phi^* \) sufficiently close to \( \Delta \pi(\lambda^L) \):

i. when \( \epsilon_{\phi,a}(\bar{q}) \geq \frac{\Delta \pi(\lambda^L) - \phi^*}{\sigma} \): \( A \) is weakly increasing in \( q \);

ii. when \( \epsilon_{\phi,a}(\bar{q}) < \frac{\Delta \pi(\lambda^L) - \phi^*}{\sigma} \): \( A \) is a N-shaped function of \( q \).

**Proof.** See Appendix D □

Figure 3 depicts the results stated in Proposition 3. In each configuration, when \( q \geq \bar{q} \), \( A \) is linearly increasing in \( q \), through the sole extensive margin effect. For smaller values of \( q \), a negative intensive margin effect also arises. Figure 3(a) illustrates the case where this effect is always dominated while Figure 3(b) the case where the intensive margin effect overcomes the extensive margin one when \( q \) becomes sufficiently close to \( \bar{q} \).²⁰ In order to give insights to

![Graph](image)

this counter-intuitive pattern, let us emphasize that an increase in \( q \) weakens the extensive margin effect while it strengthens the intensive margin one. Indeed, on the one hand, as \( q \) grows, each green firm invests a smaller amount in CSR. Thus, the marginal impact of an

²⁰ We demonstrate in Appendix D that the condition \( \epsilon_{\phi,a}(\bar{q}) < \frac{\Delta \pi(\lambda^L) - \phi^*}{\sigma} \) may be satisfied for parameters values compatible with Assumptions 1-4. To ensure that the alternative configuration (point i) is satisfied we could consider an isoelastic cost function characterized by a sufficiently high elasticity parameter.
additional green worker on aggregate abatement is decreasing in \( q \). On the other hand, since abatement costs are convex, the fall in \( a \) required to compensate for a given reduction in \( \Delta \bar{\pi} \) is increasing in \( q \). Moreover, this intensive margin effect is magnified when the elasticity of the abatement costs function is low. It explains why the \( N \)-shaped pattern arises only for small values of \( \epsilon_{\phi,a}(\bar{q}) \).

With regards to the existing literature on the strategic use of CSR, the framework we propose has the advantage to take into account \( i) \) both the number of firms who invest in CSR and the intensity of their own investment ; and \( ii) \) the industry equilibrium effects. These two dimensions may help to figure out the feed-back effects that may arise when firms become more virtuous. In particular, this justifies the unexpected conclusion according to which the spread of green consciousness among workers might induce a lower corporate social performance.

### 3.2 Transport costs

In this section, we aim at answering the question: “Does competition erode social responsibility?” To do so, we explore the changes in the CSP involved by a shock in transport costs \((d)\). We proceed in two steps: First, we wonder how does the CSP vary with the incremental benefits? Second, how is \( \Delta \bar{\pi} \) impacted by a change in \( d \)?

In step one, we argue that \( A \) should be positively related to \( \Delta \bar{\pi} \) since a rise in incremental benefits leads firms to invest more in CSR (see Proposition 1). This positive relationship transits through an adjustment of the intensive margin when \( q < \bar{q} \). Otherwise, the intensive margin is constant as well as aggregate abatement. Let us now handle step two. Using
expressions (6) and (8), the differentiation of $\Delta \bar{\pi}$ with respect to $d$ yields:

$$\frac{\partial \Delta \bar{\pi}}{\partial d} = (\lambda^H - \lambda^L) \left\{ (D^A)^2 - (D^B)^2 + 2d \left[ \frac{\partial D^A}{\partial d} D^A - \frac{\partial D^B}{\partial d} D^B \right] \right\} \tag{29}$$

The decrease in $d$ affects $\Delta \bar{\pi}$ through two channels:

(1) **Rent dissipation effect:** The profit margin reduces for both types of firms, but proportionally to the demand. As a consequence, the effect on low-cost firms is larger and $\Delta \bar{\pi}$ goes down.

(2) **Selection effect:** More competition allows highly efficient firms to capture a larger part of the product market. Then, the market shares of each low-cost firm grow while the ones of each high-cost firm reduce. Consequently, $\Delta \bar{\pi}$ increases.

Through a simple inspection of equation (8), we conclude that the selection (resp. rent dissipation) effect dominates when $\rho < 1/2$ (resp. $\rho > 1/2$). Thus, a fall in $d$ makes the profit differential ($\Delta \bar{\pi}$) widen when $\rho < 1/2$ and narrow when $\rho > 1/2$. Since, at the equilibrium $\rho = q\lambda^H + (1-q)\lambda^L$, the selection (resp. rent dissipation) effect dominates for $q\lambda^H + (1-q)\lambda^L$ higher (resp. lower) than one half. This reasoning allows us to claim that:

**Proposition 4** Under Assumptions 1-4, for $\phi^*$ sufficiently close to $\Delta \bar{\pi}(\lambda^L)$ and $q < \bar{q}$:

i If $q < \frac{1/2 - \lambda^L}{\lambda^H - \lambda^L}$: $A$ is decreasing in $d$;

ii If $q \geq \frac{1/2 - \lambda^L}{\lambda^H - \lambda^L}$: $A$ is increasing in $d$.

Whether competition favors or erodes virtuous behaviors by firms depends on prevailing market conditions. In particular, the toughness of the cost competition captured by the proportion of green workers matters. If $q$ is sufficiently high the CSP is positively related
to \( d \). Otherwise, a cut in transport costs increases aggregate abatement. As underlined by Fernandez-Kranz and Santalo (2010), in the existing literature, a rise in the competitive pressure displays an unequivocal effect on CSR investments, be it positive like in Fisman et al. (2006) or negative like in Bagnoli and Watts (2003). Our set-up offers a unified framework that clarifies the conditions under which one configuration or the other occurs.

### 3.3 Markets size

Finally we assess how changes in the relative sizes of the product and the labor market affect the CSP of the industry. As in the previous section, these changes may affect incentives firms have to adopt virtuous behaviors through their impact on \( \Delta \bar{\pi} \) and only in the case where \( q < \bar{q} \). Consequently, we focus our analysis on this configuration.

Let us first examine the impact generated by a variation of the product market size \( L \). Recall that the demands and the profits are proportional to the product market size, see equations (4) and (5). Hence, when \( L \) grows profits for both types of firms are enhanced but this effect is larger for low-cost firms in comparison with high-cost ones. We conclude that \( \Delta \bar{\pi} \) is increasing in \( L \) so that firms perform better in terms of social responsibility in industries characterized by a larger product market size.

Let us now fix the product market size and assess the impact of a change in \( M \) (the proportion \( q \) being given). A larger \( M \) intensifies competition on the product market and therefore erodes profit margins. Low-cost firms are obviously disproportionately affected such that \( \Delta \bar{\pi} \) diminishes as well as the CSP of the industry.

It is interesting to note that, though seemingly similar, the effect of the number of workers is not symmetric to that of the product market size. Indeed, the demand addressed to a
firm is proportional to \( L \) implying that the impact of an increase in \( L \) is larger when the cost competition \((\rho)\) is low. Conversely, the impact of a change in \( M \) may be isolated from the cost competition effect. To see this more formally, consider the case where \( L \) and \( M \) increases in the same proportion such that \( L/M \) remains constant. In particular, consider that \( L = \kappa L_0, M = \kappa M_0 \) and let \( \kappa \) varies. From equation (17), incremental benefits \( \Delta \bar{\pi} \) rewrite:

\[
\Delta \bar{\pi} = \frac{L_0 (\lambda^H - \lambda^L) \Delta c}{4dM_0} [(1 - 2\rho)\kappa M_0 \Delta c + 4d]
\]  

(30)

The impact of \( \kappa \) on \( \Delta \bar{\pi} \) depends on the value of \( \rho \): it is negative when the cost competition is too harsh \( (\rho > 1/2) \) and positive otherwise.

An interesting implication of this asymmetric effect of \( L \) and \( M \) relates to the impact of market integration on aggregate social performance of an industry. Let us consider two identical countries each of them with \( L \) consumers, \( M \) workers and a proportion \( q \) of these workers being green. In autarky, in each of these two countries, \( qM \) firms invest \( \phi^{-1}(\Delta \bar{\pi}_a) \). Hence, the global CSP equals \( A_a = 2qM\phi^{-1}(\Delta \bar{\pi}_a) \) with \( \Delta \bar{\pi}_a \) given by (17). Suppose now that there is a perfect integration between the two economies. Under integration, we can consider a unique industry with \( 2L \) consumers, \( 2M \) workers, a proportion \( q \) of whom being green. Hence, the global CSP equals \( A_i = 2qM\phi^{-1}(\Delta \bar{\pi}_i) \), \( \Delta \bar{\pi}_i \) being given by (30) with \( L_0 = L, M_0 = M, \kappa = 2 \) and \( \rho = q \). It is straightforward that \( A_i > A_a \) if \( q < 1/2 \) while \( A_i < A_a \) if \( q > 1/2 \). Thus, the impact of market integration crucially depends on the pre-existing toughness of cost competition that is shaped, at the equilibrium, by the prevailing proportion of green workers. In particular, market integration could increase the global CSP only if green consciousness is relatively scarce among the workforce.
4 Discussion and Conclusion

More and more firms make efforts, sometimes heavily costly, in order to appear socially responsible. The present paper contributes to our understanding of CSR decisions by firms. In particular, we investigate how these decisions relate with the competitive environment. To that end, we emphasize two market dimensions through which firms compete: the labor market and the product market.

At the labor market level, firms compete to hire employees who are heterogeneous in their intrinsic motivation like in Manna (2017). However, our two models differ along two important dimensions. First, Manna (2017) does not consider strategic CSR investments; second, she assumes that firms are on the short side of the market while, in our model, the pool of talented employees is small with respect to number of competitors. The first point is obviously central for our purpose but the second one proves to be crucial too. Indeed, in Manna (2017), only one type of worker is hired at the equilibrium and their reservation utility is pinned down to zero. Conversely, in our setting the two types of workers are employed and their reservation utility is derived from the zero-profit condition. These two features involve the presence of both an extensive and an intensive margin of CSR as well as a positive relationship between the intensive margin and the profit differential.

At the product market level, firms compete in price to sold an horizontally differentiated good. One specificity of this price competition stage lies in the fact that firms are heterogeneous (low-cost firms co-exist with high-cost firms) and this cost structure is endogenous since the proportion of low-cost firms is determined by CSR investment strategies. Aghion and Shankerman (2004) provide a particularly convenient framework to model this situation.
Connected to our firms’ decision model it allows us to obtain a two-way relationship between CSR decisions by firms and the toughness of competition on the product market, making our model well suitable to investigate the CSR-competition nexus. Brekke and Nyborg (2008), who also consider the role that CSR could play to attract and motivate employees, abstract from this issue since they assume perfect competition on the product market.

By offering analytical expressions for the CSP and the HHI of the industry, we point out interesting insights for the empirical analysis of the CSR-competition relationship. Empirical studies mainly assess this relationship by estimating the impact of the HHI on the CSP at the sectoral level. Our results indicate that this should be cautiously interpreted as a causal relationship from competition to CSP due to an endogeneity issue. Furthermore, our theoretical framework allows us to re-visit this issue. In particular, we emphasize the role played by the social consciousness of the workforce as a determinant of both the HHI and the CSP of an industry. Some empirical analysis of the CSR-competition relationship overcome the endogeneity issue thanks to exogenous variations in the competitive environment (Fernandez-Kranz and Santalo 2010 and Flammer 2015). Our framework is also appropriate for analyzing the outcome of this kind of exogenous shocks (through a change in the transport cost parameter or in markets size). In particular, we find that it dramatically depends on the other characteristics of the industry like the proportion of responsible managers. These results deserve to be empirically investigated.

The framework we propose is sufficiently tractable to be extended in several directions. First of all, at the firm level, we could introduce monetary incentives in addition of those already exploited in the model. It would allow us to take into account the trade-off between pecuniary and non-pecuniary (CSR) incentives and explore how this trade-off is affected by
the market structure. Second, at the industry level, we could consider that a fraction of consumers is also socially responsible. It would enable to assess the joint impact of social consciousness of both consumers and workers on firms’ CSR performances.

Appendices

A Proof of Proposition 1

The proof is in two steps. First, Lemma 1 describes the labor market equilibrium when types are observable. Then we address the case with unobservable types and Proposition 1 is proven.

**Lemma 1** When the type of workers is observable, a unique labor market equilibrium exists where firms make the offer \((w_b^*, a_b^*) = (\bar{\pi}^L, 0)\) to brown managers who accept it and the offer

\[
(w_g^*, a_g^*) = \begin{cases} 
    (\bar{\pi}^L, 0) & \text{if } \Delta \bar{\pi} < \phi^* - \nu^* \\
    (\bar{\pi}^L, 0) \cup (\bar{\pi}^H - \phi^*, a^*) & \text{if } \Delta \bar{\pi} = \phi^* - \nu^* \\
    (\bar{\pi}^H - \phi^*, a^*) & \text{if } \Delta \bar{\pi} > \phi^* - \nu^*
\end{cases}
\]

to green managers who accept it.

**Proof.** Let us show that \((\bar{\pi}^L, 0)\) is the unique equilibrium contract proposed to green workers when \(\Delta \bar{\pi} < \phi^* - \nu^*\). By contradiction, assume that there is a profitable deviation such that a firm chooses the first best level of abatement \(a^*\) and reduces the wage, offering the pair: \((\bar{\pi}^L - \epsilon, a^*)\). Profits for the deviating firm must be:

\[
\bar{\pi}^H - (\bar{\pi}^L - \epsilon) - \phi^* = \Delta \bar{\pi} + \epsilon - \phi^*
\]

Moreover, such a contract would attract a green worker if and only if: \(\bar{\pi}^L - \epsilon + \nu^* \geq \bar{\pi}^L\) implying \(\epsilon < \nu^*\). Then under the condition \(\Delta \bar{\pi} < \phi^* - \nu^*\), the deviating firm makes negative profits and the deviation cannot be profitable. A symmetric reasoning applies for the case
\[ \Delta \bar{\pi} > \phi^* - \nu^*. \]

Let us now consider the case where the workers’ type is unobservable. In that case, each worker chooses the offer designed for him (we denote by \((w_i, a_i)\) the offer designed for a worker of type \(i \in \{b, g\}\)) if the two incentive compatibility constraints are satisfied:

\[
V_g(w_g, a_g) \geq V_g(w_b, a_b) \quad (A.1)
\]
\[
V_b(w_b, a_b) \geq V_b(w_g, a_g) \quad (A.2)
\]

where \(V_i(w_j, a_j) \equiv U_i(w_j, e_i(w_j, a_j), a_i)\) and \(e_i(w_j, a_j)\) the effort chosen by a worker of type \(i\) who has accepted the offer \((w_j, a_j)\). Simple inspections of \(V_b(w_b^*, a_b^*), V_g(w_b^*, a_b^*), V_b(w_g^*, a_g^*)\) and \(V_g(w_g^*, a_g^*)\) allow us to conclude that the set of equilibrium offers when types are observable \((w_i^*, a_i^*)\) also satisfies the two incentive compatibility constraints \((A.1)\) and \((A.2)\) if and only if \(\Delta \bar{\pi} \leq \phi^*\). Hence, as long as \(\Delta \bar{\pi}\) is lower than \(\phi^*\), firms propose the same offers as in Lemma 1 and workers self-select the offer designed for them. It remains to explore the case where \(\Delta \bar{\pi} > \phi^*\). In that case, we can claim that:

i. no pooling equilibrium exists;

ii. in any separating equilibrium, brown workers receive the offer: \((\bar{\pi}^L, 0)\);

iii. in any separating equilibrium, green workers receive the offer: \((\bar{\pi}^L, \phi^{-1}(\Delta \bar{\pi}))\);

iv. a separating equilibrium exists whenever \(q < \hat{q}\) and does no longer exist when \(q > \hat{q}\).

We now successively prove the points listed above.
i. Suppose that there is a pooling equilibrium offer \((w^p, a^p)\) such that green workers choose the high effort \(e^H\). The zero-profit condition writes:

\[
q\bar{\pi}^H + (1 - q)\bar{\pi}^L - w^p - \phi(a^p) = 0 \quad \text{(A.3)}
\]

We deduce the labor income \(w^p = q\Delta\bar{\pi} + \bar{\pi}^L - \phi(a^p)\). Suppose that a firm deviates by proposing the following offer \((w^p - \epsilon, a^p + \eta)\), with \(\epsilon, \eta > 0\) and such that the two following conditions are satisfied:

\[
\epsilon < v(e^H, a^p + \eta) - v(e^H, a^p) \quad \text{(A.4)}
\]

\[
\phi(a^p + \eta) - \phi(a^p) < (1 - q)\Delta\bar{\pi} + \epsilon \quad \text{(A.5)}
\]

The first condition induces that the deviating firm attracts a green worker and the second one implies that when it is the case, the deviating firm exhibits positive profits. Let us set \(\eta\) small enough such that \(\phi(a^p + \eta) - \phi(a^p) < (1 - q)\Delta\bar{\pi}\). Then, condition (A.5) is satisfied and, for a small enough value \(\epsilon\), condition (A.4) also holds. Hence, there is a profitable deviation.

ii. Suppose a separating equilibrium where the offer \((w_b, a_b)\), with \(a_b > 0\), is proposed to brown managers. Suppose that a firm deviates by proposing the following contract \((w_b + \epsilon, 0)\) so that she reduces abatement and simultaneously increases wages. For \(\epsilon < \phi(a_b)\), this deviation is profitable as the firm attracts a brown worker and gets positive profits. Hence, at the equilibrium, \(a_b\) must equal 0 and the zero-profit condition yields to \(w_b = \bar{\pi}^L\).

iii. Let us define a contract \((\bar{w}_g, \bar{a}_g) = (\bar{\pi}^L, \phi^{-1}(\Delta\bar{\pi}))\), where \(V_b(\bar{\pi}^L, 0) = V_b(\bar{\pi}^H - \phi(\bar{a}_g), \bar{a}_g)\).

By contradiction, suppose a separating equilibrium where the contract \((w_g, a_g) \neq (\bar{w}_g, \bar{a}_g)\) is proposed to green managers. The zero-profit condition yields: \(w_g = \bar{\pi}^H - \phi(a_g)\), with \(w_g < \bar{\pi}^L = \bar{w}_g\), otherwise the equilibrium looses its separating property. In addition, it
implies that \( a_g > \tilde{a}_g \) and further \( a_g > \tilde{a}_g > a^* \) since \( \phi(\tilde{a}_g) = \Delta \tilde{\pi} > \phi^* = \phi(a^*) \).

Suppose now that a firm deviates by proposing \((w_g + \epsilon, a_g - \eta)\), with \( \epsilon, \eta > 0 \) and such that: (i) the firm attracts a green worker; (ii) no brown worker accepts this contract; (iii) the firm exhibits positive profits, so that the deviation is profitable. Item (i) is verified if \( V^g(w_g + \epsilon, a_g - \eta) > V^g(w_g, a_g) \). It involves:

\[
\epsilon > v(e^H, a_g) - v(e^H, a_g - \eta) \tag{A.6}
\]

Item (ii) holds when \( w_g + \epsilon < \bar{\pi}^L \). Using the zero profit condition, it yields:

\[
\phi(a_g) - \phi(\tilde{a}_g) > \epsilon \tag{A.7}
\]

Item (iii) induces that \( \bar{\pi}^H - \phi(a_g - \eta) - (w_g + \epsilon) > 0 \) and the zero profit condition yields:

\[
\phi(a_g) - \phi(a_g - \eta) > \epsilon \tag{A.8}
\]

Let us set a small enough value of \( \eta \) so that \( \phi(a_g - \eta) > \phi(\tilde{a}_g) \). Moreover, since \( a_g > a_g^* \), \( \phi'(a_g) > v'_a(e^H, a_g) \) and \( \phi(a_g) - \phi(a_g - \eta) > v(e^H, a_g) - v(e^H, a_g - \eta) \). Then, we can choose a value of \( \epsilon \) satisfying the following inequalities:

\[
\phi(a_g) - \phi(\tilde{a}_g) > \phi(a_g) - \phi(a_g - \eta) > \epsilon > v(e^H, a_g) - v(e^H, a_g - \eta) \tag{A.9}
\]

such that conditions (A.6), (A.7) and (A.8) simultaneously hold.

iv. Suppose a separating equilibrium where the offer \((\tilde{w}_g, \tilde{a}_g)\) is proposed to green workers and consider the following deviation: \((\tilde{w}_g + \epsilon, \tilde{a}_g - \eta)\). This new contract obviously attracts brown workers. It also attracts green workers, who choose to exert high effort, if:

\[
\epsilon > v(e^H, \tilde{a}_g) - v(e^H, \tilde{a}_g - \eta) \tag{A.10}
\]
Hence, the firm exhibits positive profits if:

$$\epsilon < q\phi(\tilde{a}_g) - \phi(\tilde{a}_g - \eta)$$  \hspace{1cm} (A.11)

These two inequalities may be simultaneously satisfied if and only if:

$$q > \frac{\phi(\tilde{a}_g - \eta)}{\phi(\tilde{a}_g)} + \frac{v(e^H, \tilde{a}_g) - v(e^H, \tilde{a}_g - \eta)}{\phi(\tilde{a}_g)} \equiv q(\eta)$$  \hspace{1cm} (A.12)

To ensure that green workers choose high effort $\eta$ must belong to $(0, \tilde{a}_g - a^*)$ so that $q'(\eta) < 0$.

We define $\hat{q} = q(\tilde{a}_g - a^*)$:

$$\hat{q} \equiv \frac{\phi^* + v(e^H, \phi^{-1}(\Delta\bar{\pi})) - v(e^H, a^*)}{\Delta\bar{\pi}} \in (0, 1)$$  \hspace{1cm} (A.13)

If $q \leq \hat{q}$, then $q \leq q(\eta)$ for all $\eta \in (0, \tilde{a}_g - a^*)$ such that there is no profitable deviation. If $q > \hat{q}$, there exists a value of $\eta$ close enough to $\tilde{a}_g - a^*$ such that $q(\eta) < q$. This concludes the proof.

**B Proof of Proposition 2**

**Characterization.** The equilibrium values of $\rho$ and $\Delta\bar{\pi}$ are simply given by the crossing points between the curves respectively described by equations (14) and (17). Through (17), $\Delta\bar{\pi}$ is linearly decreasing in $\rho$ while through (14) $\rho$ is a step function equals to $\lambda^L$ when $\Delta\bar{\pi} < \phi^* - \nu^*$, $q\lambda^H + (1 - q)\lambda^L$ when $\Delta\bar{\pi} > \phi^* - \nu^*$ and any value between the two when $\Delta\bar{\pi} = \phi^* - \nu^*$. Moreover, according to Assumption 4, $\Delta\bar{\pi}(\lambda^L)$ is above $\phi^*$ while $\Delta\bar{\pi}(\lambda^H)$ is in between $\phi^* - \nu^*$ and $\phi^*$. Hence, $\Delta\bar{\pi}(q\lambda^H + (1 - q)\lambda^L) > \phi^* - \nu^*$ such that the two curves cross in $\rho = q\lambda^H + (1 - q)\lambda^L$. The equilibrium values of $\bar{\pi}^L$ and $\bar{\pi}^H$ are obtained by putting the equilibrium value of $\rho$ into expressions (15) and (16).
As for the equilibrium offers (equations (23) and (24)), we apply the results obtained in Proposition 1 using the fact that, under Assumption 4, \( \bar{q} \) belongs to \([0,1]\). Hence, when \( q < \bar{q} \), the equilibrium value of \( \Delta \bar{\pi} \) is above \( \phi^* \).

**Existence.** As claimed in Proposition 1 such an equilibrium may fail to exist when \( q > \hat{q} \) and \( \Delta \bar{\pi} > \phi^* \). Nevertheless, we know from Proposition 2 that \( \Delta \bar{\pi} \) overcomes \( \phi^* \) if and only if \( q < \bar{q} \). Hence, the industry equilibrium exists for all \( q \in [0,1] \) if and only if \( \bar{q} \leq \hat{q} \). Let us first show that \( \hat{q} \) is decreasing in \( \Delta \bar{\pi} \). The differentiation of (A.13) with respect to \( \Delta \bar{\pi} \) yields:

\[
\frac{\partial \hat{q}}{\partial \Delta \bar{\pi}} = \frac{v_a'(e^H, \phi^{-1}(\Delta \bar{\pi})) \phi^{-1}'(\Delta \bar{\pi}) \Delta \bar{\pi} - \left[ \phi^* + v(e^H, \phi^{-1}(\Delta \bar{\pi})) - v(e^H, a^*) \right]}{\phi^{-1}(\Delta \bar{\pi})^2} \tag{B.1}
\]

Hence, \( \frac{\partial \hat{q}}{\partial \Delta \bar{\pi}} < 0 \) if the numerator, that we denote \( H(\Delta \bar{\pi}) \), is negative. Since, \( \phi^{-1}(\phi^*) = a^* \) and \( \phi'(a^*) = v_a'(e^H, a^*) \), \( H(\phi^*) = \phi'(a^*) \phi^{-1}'(\phi^*) \phi^* - \phi^* = 0 \). Then, we can easily see that \( H(\Delta \bar{\pi}) \) is decreasing in \( \Delta \bar{\pi} \). Thus, \( H(\Delta \bar{\pi}) < 0 \) for any values of \( \Delta \bar{\pi} > \phi^* \) and \( \hat{q} \) is decreasing in \( \Delta \bar{\pi} \). As a consequence, we can define \( \hat{q}_{\text{min}} \) as the value of \( \hat{q} \) such that \( \Delta \bar{\pi} = \Delta \bar{\pi}(\lambda L) \) and it remains to demonstrate that \( \bar{q} \leq \hat{q}_{\text{min}} \). More precisely, we show that there exist some values of \( e^H \) that satisfy this condition and are compatible with Assumption 4.

Applying the implicit function theorem, we define \( a^* \) as an increasing function of \( e^H \), denoted \( g(e^H) \). Then, \( \hat{q}_{\text{min}} \) may be expressed as the following function of \( e^H \):

\[
\hat{q}_{\text{min}} = \frac{\phi(g(e^H)) + v(e^H, \phi^{-1}(\Delta \bar{\pi}(\lambda L))) - v(e^H, g(e^H))}{\Delta \bar{\pi}(\lambda L)}
\]
The differentiation of $\hat{q}_{\min}$ with respect to $e^H$ yields:

$$\frac{\partial \hat{q}_{\min}}{\partial e^H} = \frac{v'_e(e^H, \phi^{-1}(\Delta\bar{\pi}(\lambda^L))) - v'_e(e^H, g(e^H))}{\Delta\bar{\pi}(\lambda^L)}$$

which is positive since $\phi^{-1}(\Delta\bar{\pi}(\lambda^L)) > g(e^H)$ and $v''_e(e, a) > 0$. Moreover, when $\phi^* = \Delta\bar{\pi}(\lambda^L)$, $\hat{q}_{\min} = 1$.

Applying a similar reasoning, $\bar{q}$ may be expressed as an implicit function of $e^H$. The differentiation of this function with respect to $e^H$ yields:

$$\frac{\partial \bar{q}}{\partial e^H} = -\frac{-2d\phi'(g(e^H))g'(e^H)}{L(\lambda^L - \lambda^L)^2(\Delta e)^2} < 0$$

Moreover, when $\phi^* = \Delta\bar{\pi}(\lambda^H)$, $\bar{q} = 1$; while when $\phi^* = \Delta\bar{\pi}(\lambda^L)$, $\bar{q} = 0$.

Thus, there exists $\bar{e}^H$ such that $\phi(g(\bar{e}^H)) \in [\Delta\bar{\pi}(\lambda^H), \Delta\bar{\pi}(\lambda^L)]$ and $\bar{q} \leq \hat{q}_{\min}$ for $e^H \geq \bar{e}^H$. Hence, for $\phi^*$ between $\phi(g(\bar{e}^H))$ and $\Delta\bar{\pi}(\lambda^L)$ the industry equilibrium exists.

Finally, Assumption 3 excludes configurations for which $\phi^* - \nu^* > \Delta\bar{\pi}(\lambda^H)$. Indeed, under this Assumption:

$$\frac{\partial(\phi^* - \nu^*)}{\partial e^H} = 1 - v'_e(e^H, g(e^H)) < 0$$

### C Derivation of the sign of $\Psi'(q)$

Using equation (25), the differentiation of $A$ with respect to $q$ yields:

$$\Psi'(q) = \phi^{-1}(\cdot) + q \frac{\partial \phi^{-1}(\cdot)}{\partial \Delta\bar{\pi}}$$

(C.1)

Since $\frac{\partial \Delta\pi}{\partial q} < 0$, $\Psi'(q)$ is positive if and only if:

$$\frac{\partial \phi^{-1}(\cdot)}{\partial \Delta\bar{\pi}} \frac{\Delta\bar{\pi}(\cdot)}{\phi^{-1}(\cdot)} < -\frac{\Delta\bar{\pi}(\cdot)}{q} \frac{\partial \Delta\bar{\pi}(\cdot)}{\partial q} \Leftrightarrow -\epsilon_{\Delta\bar{\pi}, q}(q) < \frac{1}{\epsilon_{\phi^{-1}, \Delta\bar{\pi}}(q)}$$

(C.2)
By the properties of the inverse function and since $\epsilon_{\Delta\pi,q}(q) < 0$ while $\epsilon_{\phi,a}(q) > 0$, the latter condition may be rewritten as:

$$|\epsilon_{\Delta\pi,q}(q)| < |\epsilon_{\phi,a}(q)|$$  \hspace{1cm} (C.3)

### D Proof of Proposition 3

For $q \geq \bar{q}$, $A$ is linearly increasing. Let us focus on the case where $q < \bar{q}$ in which $A = \Psi(q)$.

By (C.1) we must have $\Psi'(0) = \phi^{-1}(\Delta\pi(\lambda^L)) > 0$ and

$$\Psi''(q) = 2 \frac{\partial \phi^{-1}(\cdot)}{\partial \Delta\pi} \frac{\partial \Delta\pi(\cdot)}{\partial q} + q \frac{\partial^2 \phi^{-1}(\cdot)}{\partial \Delta\pi^2} \left( \frac{\partial \Delta\pi(\cdot)}{\partial q} \right)^2$$  \hspace{1cm} (D.1)

which is negative since $\phi^{-1}(\cdot)$ is increasing and concave in $\Delta\pi$ while $\Delta\pi$ is linearly decreasing in $q$. Hence, on the interval $[0, \bar{q})$, $\Psi'(q)$ is either always positive or firstly positive and then negative. The former case occurs if and only if the inequality (C.3) is satisfied in $\bar{q}$. Moreover, by definition of $\bar{q}$, we must have $\Delta\pi(\lambda^L + \bar{q}(\lambda^H - \lambda^L)) = \phi^*$, such that:

$$-\epsilon_{\Delta\pi,q}(\bar{q}) = \frac{L}{2a} (\lambda^H - \lambda^L)^2 (\Delta c)^2 = \frac{\Delta\pi(\lambda^L) - \phi^*}{\phi^*}$$

Putting these elements together we obtain the results stated in Proposition 3.

Finally, let us show some conditions, compatible with Assumptions 1-4 and $\phi^*$ sufficiently close to $\Delta\pi(\lambda^L)$, ensuring that the relationship between $A$ and $q$ is $N$-shaped. To that end, let us consider the following functional forms: $\phi(a) = \varphi a$ and $v(e^H, a) = e^H a^\beta$ with $\varphi > 0$ and $\beta \in (0, 1)$, such that $a^* = (\beta e^H / \varphi)^{\frac{1}{1-\beta}}$, $\phi^* = (\beta e^H / \varphi^\beta)^{\frac{1}{1-\beta}}$ and both Assumptions 2 and 3 are satisfied for $\varphi < \beta e^H$. In that configuration we have:

$$\epsilon_{\phi,a} < \frac{\Delta\pi(\lambda^L)}{\phi^*} \quad \Leftrightarrow \quad \phi^* < \frac{\Delta\pi(\lambda^L)}{2} \quad \Leftrightarrow \quad e^H < \frac{\varphi^\beta}{\beta} \left( \frac{\Delta\pi(\lambda^L)}{2} \right)^{1-\beta} = \bar{e}^H$$
We know from the proof of Proposition 2, $\hat{q}_{\min} \geq \bar{q}$ if and only if $e^H > \bar{e}^H$. Then, if $\bar{e}^H > \tilde{e}^H$ there exists a range of values for $e^H$ (between $\bar{e}^H$ and $\tilde{e}^H$) such that $\hat{q}_{\min} \geq \bar{q}$ and $\epsilon_{\phi,a} < \frac{\Delta H(\lambda L) - \phi^*}{\phi^*}$. It is equivalent to demonstrate that $\hat{q}_{\min}|_{e^H = \bar{e}^H} > \bar{q}|_{e^H = \bar{e}^H}$. After some algebra we obtain:

$$\hat{q}_{\min}|_{e^H = \bar{e}^H} = \frac{1}{2} \left[ 1 + \frac{1}{\beta} (2^\beta - 1) \right]$$

and

$$\bar{q}|_{e^H = \bar{e}^H} = \frac{1}{\lambda^H - \lambda^L} \left( \frac{1}{4} + \frac{d}{M \Delta c} - \frac{\lambda^L}{2} \right)$$

Hence, the condition $\hat{q}_{\min}|_{e^H = \bar{e}^H} > \bar{q}|_{e^H = \bar{e}^H}$ writes as:

$$\frac{2^\beta - 1}{2^\beta} > \frac{1}{\lambda^H - \lambda^L} \left( \frac{1}{2} + \frac{2d}{M \Delta c} - \lambda^H \right)$$

It is straightforward to verify that there exist some sets of parameters, compatible with Assumption 1, and that satisfy the inequality above.

References


