Cobb-Douglas preferences and pollution in a bilateral oligopoly market

Document de Travail
Working Paper
2018-48

Anicet Kabre

Cobb-Douglas preferences and pollution in a bilateral oligopoly market*

Anicet B. KABRE

EconomiX-CNRS UMR 7235
Université Paris-Nanterre
Bureau G209
200 Avenue de la République
92001 Nanterre Cedex.
Mail: kabre.anicet@parisnanterre.fr
Tel: +33 (0) 1 40 97 59 22

Abstract

In this note, we introduce pollution and examine its effects in a finite bilateral oligopoly model where agents have asymmetric Cobb-Douglas preferences. We define two strategic equilibria: the Stackelberg-Cournot equilibrium with pollution (SCEP) and the Cournot equilibrium with pollution (CEP). While the supplied quantities of the polluting and the non-polluting good depend on the preferences of all economic agents in the case of symmetric preferences, we show that when preferences are asymmetric, *i)* at both equilibria, each polluter's equilibrium supply depends only on the non-polluters' preferences for the non-polluting good; *ii)* at the CEP and the SCEP, the elasticity of the polluters emissions is greater when non-polluters preferences for the non-polluting good increase, compared to an increase in their own preferences for this good; *iii)* firm's emissions'elasticity decreases with the market power if their marginal cost is lower than their competitor.

Keywords: Bilateral oligopoly; Pollution; Cobb-Douglas preferences.
JEL Classification: D43, D51, Q52

*I'm grateful to Professors Ludovic A. Julien and Louis de Mesnard for their remarks. All remaining deficiencies are my own.

1 Introduction

In this paper, we investigate the consequences of asymmetric Cobb-Douglas preferences on equilibrium strategies and pollution emissions level, in a bilateral oligopoly model with Stackelberg competition. To this end, we extend Julien and Tricou (2012)'s bilateral oligopoly model based on Gabszewicz and Michel (1997) by introducing a polluting good, and assuming asymmetric preferences. In this framework, we study two strategic equilibria in an exchange economy with production and pollution emissions. In the Stackelberg-Cournot model, we assume that a Stackelberg leader and one follower produce a non-polluting good, while other followers supply a polluting good. By contrast, in the Cournot equilibrium model, all firms set their decisions simultaneously.

Studies dealing with pollution mostly focus on partial equilibrium and are devoted to pollution permit markets. In a seminal paper, Hahn (1984) pioneered the analysis of strategic interactions in pollution permit markets. He considers two different scenarios. In the first, it is assumed that firms sell their products on perfectly competitive markets. In the second scenario, one dominant firm is assumed to face a competitive fringe. In this case, it is shown that the permit market is cost-effective if the dominant firm's initial endowment of permits is such that he chooses not to trade. Westskog (1996) extends Hahn (1984)'s model by considering several dominant firms and a competitive fringe in the permit market. In line with Hahn (1984), he finds that the permit market is cost-effective if the dominant firms endowments of permits are such that they don't need to be exchanged. All the above mentioned studies share the assumption that the dominant firm behaves non-strategically in the final product market, and the economy embodies competitive fringe. Some firms are therefore sentenced to be price-takers.

When the input market is strategic and the final good market is competitive, Salop and Scheffman (1983), Misiolek and Elder (1989) show that the dominant firm would manipulate the price of the input in order to increase the production costs of its rival. Thus, these studies do not provide an answer to the strategic markets of the final goods and, consequently, do not integrate the fact that a firm use its market power on the permit market to increase its efficiency on the final goods market. Montero (2009) relax that assumption by allowing firms to compete on both the permit markets and the product market. Kolstad and Wolak (2008) show that electric utilities in California used the NO_x market to enhance their ability to exercise (unilateral) market power in the electricity market. More recently, Dickson and Mackenzie (2018) study strategic trade in pollution permit market where firms decide endogenously to be buyers or sellers. They investigate the interplay between market power in the product market and the permit market equilibrium, and examine the effect of increased demand in the product market by assuming that firms have symmetric market power. They show that there is a unique equilibrium in which trade in permits takes place. Their assumption is not always appropriate because there are economies where agents display heterogeneous market powers

The oligopoly models with a finite number of traders were introduced by Gabszewicz and Michel (1997) and pursued by Bloch and Ghosal (1997), Bloch and Ferrer (2001), Dickson and Hartley (2008), Julien and Tricou (2012). In these models, both sides of the market are linked by a price mechanism. This mechanism was developed by Shapley and Shubik (1977) and refined by Sahi and Yao (1989) and Amir et al. (1990). Our model is closely related to Julien and Tricou (2012)'s who assumes that all traders have the same preferences which can be represented by the same log linear utility function. We relax this assumption by considering that agents located on both sides of the market exhibit different preferences which are adequately captured by Cobb-Douglas utility functions. Morever, our economy enables agents to have different market power. The simple model we develop here allows for the investigation of the role of preferences on production strategies and pollution emissions in two scenarios that differ in terms of symmetric market power (Cournot equilibrium with pollution, namely CEP) and asymmetric market power (Stackelberg-Cournot equilibrium with pollution, namely SCEP). [1] To the best of our current knowledge, this paper is the first to examine the role of asymmetric preferences on emissions level, in bilateral oligopoly model when firms have asymmetric market power.

This market structure has several applications in ecological economics.[2]

We show that: 1) at the SCEP and the CEP, the strategic supply of each polluter only depends on non-polluters preferences while the emissions level depends on the preferences of all agents; 2) at the CEP and the SCEP, the elasticity of polluters emissions is more responsive to non-polluters preferences for the non-polluting good compared to their own preferences for this good; 3) at the CEP, the marginal variation of the polluters emissions is more elevated when non-polluters preferences for the non-polluting good rises compared to an increase in their own preferences for this good; 4) at the SCEP, when polluters have a higher preference for the non-polluting good than non-polluters', the marginal variation of the leader's emissions (the follower's) is more sensitive to non-polluters (polluters) preferences for the non-polluting good compared to those of the non-polluters'. 5) when the marginal cost of the firm is low (high) compared to its competitor, the polluters emissions flexibility when their preferences for the non-polluting good raise, decreases (increases) with the market power.

The article is structured as follows. The next section outlines the model. In sections 3 and 4, we present and analyze the SCEP and CEP respectively. Section 5 provides a comparison between the SCEP and the CEP. We conclude in Section 6.

[1] Environmental externalities are not studied in this paper. Also, issues related to the existence and uniqueness of oligopolistic equilibrium in general equilibrium models are beyond the scope of this paper (due to problems raised in Gabszewicz (2002)).

[2] *Example 1:* Consider a polluting company (for example McDonald's, a fish processing company) owned by several shareholders who have different market (decisional) powers. These shareholders consume another good (meat or fish) used as an input to produce the final good (burgers or canned fish). All agents of the economy consume the two goods. In such a context, we describe how the agents' preferences for the non-polluting good would affect strategies and, discuss climate change mitigation by asking how to act effectively on these preferences to alleviate pollution, and thus preserve the environment.

Example 2: Imagine two agri-food companies that produce a final good (good 1) with cereals, and several (n) other companies that own the cereals (good 2). It is assumed that producers are also consumers and the utility of each agent of the economy depends on the consumption of both goods. In addition, all the economic agent have Cobb-Douglas preferences that differ in the value of the preferences. The production of good 1 requires good 2 whose combustion generates pollution. We know that acting on preferences could reduce pollution. From the social planner point of view, the main questions are: in which cases should we act on producer-consumer preferences rather than pure-consumer preferences to better reduce pollution? Conversely, in which cases would an action on pure-consumers preferences reduce more pollution compared to an action on producer-consumers'preferences ?

2 The model

Let us consider an exchange economy with a productive sector. It consists in $n+2$ traders of two types (indexed respectively by $i = 1, 2$ and $j = 1, ...n$) and two divisible commodities (A and B). Good B which is not produced, is used as an input to produce good A. p_A denotes the price of good A in terms of good B so that good B is assumed to be a numeraire; i.e $p_B = 1$. Pollution results from the processing of good B. Firms i are consumer-producers and firms j are pure consumers. The preferences of each agent are captured by the following utility functions:

$$U(x_A^i, x_B^i) = \alpha \ln x_A^i + (1-\alpha) \ln x_B^i, \quad \alpha \in (0,1) \quad \forall i = 1, 2 \tag{1}$$

$$U(x_A^j, x_B^j) = \Omega \ln x_A^j + (1-\Omega) \ln x_B^j, \quad \Omega \in (0,1) \quad \forall j = 1, ..n \tag{2}$$

Following Gabszewicz and Michel (1997), Julien and Tricou (2012), the initial endowments in good A and B for both types of agents are respectively given by:

$$w^i = (0, 0), \qquad \forall i = 1, 2 \tag{3}$$

$$w^j = \left(0, \frac{1}{n}\right), \qquad \forall j = 1, ..., n \tag{4}$$

As in Gabszewicz and Michel (1997), an oligopolist must produce to consume. By using z^i quantity of good B, an oligopolist produces a quantity y^i of good A according to the linear technology:

$$y^i = \frac{1}{\beta^i} z^i, \quad \beta^i > 0, \quad \forall i = 1, 2 \tag{5}$$

Following Crettez et al. (2014), the use of an amount z^i of the polluting input generates a quantity of emissions:

$$e^i = \frac{1}{\gamma^i} z^i, \quad \gamma^i > 0, \quad \forall i = 1, 2 \tag{6}$$

where γ^i measures the magnitude of the pollution. From the last two equations, we express the production y^i of good 1 in terms of the emissions e^i and obtain:

$$y^i = \frac{\gamma^i}{\beta^i} e^i, \quad \forall i = 1, 2 \tag{7}$$

Traders try to manipulate the market price through their strategic supply. Let q^i denote the strategy of agent i; e^i their emissions level and b^j the strategy of agents j, the strategy sets for the supply of both oligopolists are:

$$s^i = \left\{ (q^i, e^i) \in \mathbb{R}_+^2 | 0 \leq q^i \leq \frac{\gamma^i}{\beta^i} e^i \right\}, \quad \forall i = 1, 2$$

$$s^j = \left\{ b^j \in \mathbb{R}_+ | 0 \leq b^j \leq \frac{1}{n} \right\}, \quad \forall j = 1, ..., n$$

Market price is then given by:

$$p_A = \frac{\sum_{j=1}^{n} b^j}{\sum_{i=1}^{2} q^i} = \frac{B}{Q} \tag{8}$$

Individual allocations are given by:

$$(x_A^i, x_B^i) = \left(y^i - q^i, \frac{B}{q^i + q^{-i}} q^i - \gamma^i e^i \right), \quad i = 1, 2 \tag{9}$$

$$(x_A^j, x_B^j) = \left(\frac{Q}{b^j + B^{-j}} b^j, \frac{1}{n} - b^j \right), \quad j = 1, ..., n \tag{10}$$

and yield the following indirect utility levels:

$$V^i(q^i, q^{-i}, B) = \alpha \ln \left(y^i - q^i \right) + (1 - \alpha) \ln \left(\frac{B}{q^i + q^{-i}} q^i - \gamma^i e^i \right), \quad i = 1, 2 \tag{11}$$

$$V^j(Q, b^j, B^{-j}) = \Omega \ln \left(\frac{Q}{b^j + B^{-j}} b^j \right) + (1 - \Omega) \ln \left(\frac{1}{n} - b^j \right), \quad j = 1, ..., n, \tag{12}$$

where $b = (b^1, b^2, ..., b^n)$ is the vector of equilibrium strategies of traders j and $q = (q^1, q^2)$ is the vector of equilibrium strategies of traders i.

3 The Stackelberg-Cournot equilibrium with pollution

In this game, polluting firms compete "*à la Stackelberg*"; agent 1 behaves as a Stackelberg leader with respect to the $(n + 1)$ remaining agents. The game consists in two stages. In the first, the leader solves the following program:

$$(q^1, e^1) \in \arg\max \quad \alpha \ln \left(\frac{\gamma^1 e^1}{\beta^1} - q^1 \right) + (1 - \alpha) \ln \left(\frac{\sum_{j=1}^{n} g^j(q^1)}{q^1 + f(q^1)} q^1 - \gamma^1 e^1 \right) \tag{13}$$

At the second stage, the $n + 1$ followers simultaneously solve the following problems:

$$(q^2, e^2) \in \arg\max \quad \alpha \ln \left(\frac{\gamma^2 e^2}{\beta^2} - q^2 \right) + (1 - \alpha) \ln \left(\frac{B}{q^1 + q^2} q^2 - \gamma^2 e^2 \right), \forall q^1 \tag{14}$$

$$b^j \in \arg\max \quad \Omega \ln \left(\frac{q^1 + q^2}{b^j + B^{-j}} b^j \right) + (1 - \Omega) \ln \left(\frac{1}{n} - b^j \right), \forall q^1 \quad j = 1, ..., n, . \tag{15}$$

Proposition 1 *: The solution of the SCEP is given by the following equilibrium strategy profiles $(\tilde{q}^1, \tilde{q}^2, b)$ and emissions level $(\tilde{e}^1, \tilde{e}^2)$:*

$$\tilde{q}^1 = \frac{\Omega\beta^2}{4(\beta^1)^2}\phi \tag{16}$$

$$\tilde{q}^2 = \frac{\Omega\phi}{4(\beta^1)^2}\left[2\beta^1 - \beta^2\right] \tag{17}$$

$$b^j = \frac{\Omega(n-1)}{n(n-\Omega)} \quad \forall j = 1, ..., n \tag{18}$$

$$\tilde{e}^1 = \frac{\Omega(1+\alpha)}{4\gamma^1}\frac{\beta^2}{\beta^1}\phi \tag{19}$$

$$\tilde{e}^2 = \frac{\Omega\alpha\phi}{2\gamma^2}\left(\left(2 - \frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}} + \frac{1-\alpha}{2\alpha}\frac{\beta^2}{\beta^1}\left(2 - \frac{\beta^2}{\beta^1}\right)\right), \tag{20}$$

where $\phi = \frac{n-1}{n-\Omega}$.

From these equilibrium strategies given in proposition 1, the market price is:

$$\tilde{p}_A = 2\beta^1 \tag{21}$$

The individual allocations are:

$$(\tilde{x}_A^1, \tilde{x}_B^1) = \left(\frac{\alpha\Omega\beta^2}{4(\beta^1)^2}\phi, \frac{\Omega(1-\alpha)\beta^2}{4\beta^1}\phi\right) \tag{22}$$

$$(\tilde{x}_A^2, \tilde{x}_B^2) = \left[\tilde{x}_A^2, \tilde{x}_B^2\right] \tag{23}$$

$$\tilde{x}_A^2 = \frac{\Omega\phi}{2}\left(\frac{\alpha}{\beta^2}\sqrt{2 - \frac{\beta^2}{\beta^1}} + \frac{1-\alpha}{2(\beta^1)^2}\left(2 - \alpha\frac{\beta^2}{\beta^1}\right)\right) \tag{24}$$

$$\tilde{x}_B^2 = \frac{\Omega(1-\alpha)}{2}\phi\left(\sqrt{2 - \frac{\beta^2}{\beta^1}} - \frac{\beta^2}{2\beta^1}\left(2 - \frac{\beta^2}{\beta^1}\right)\right) \tag{25}$$

$$(\tilde{x}_A^j, \tilde{x}_B^j) = \left(\frac{\Omega\phi}{2n\beta^1}, \frac{(1-\Omega)\phi}{n}\right) \quad \forall j = 1, ..., n. \tag{26}$$

Finally, equilibrium utility levels write as:

$$\tilde{U}^1 = \alpha \ln \frac{\alpha}{\beta^1(1-\alpha)} + \ln \frac{\Omega\beta^2\phi}{4\beta^1} + \ln(1-\alpha) \tag{27}$$

$$\tilde{U}^2 = \alpha \ln\left[\tilde{x}_A^2\right] + (1-\alpha)\ln\left[\tilde{x}_B^2\right] \tag{28}$$

$$\tilde{U}^j = \Omega \ln\left(\frac{\Omega\phi}{2n\beta^1}\right) + (1-\Omega)\ln\frac{1-\Omega\phi}{n}, \quad j = 1, ..., n. \tag{29}$$

We remark that: $\frac{\partial\tilde{e}^1}{\partial\Omega} = \frac{n}{n-\Omega}\frac{\alpha+1}{4\gamma^1}\frac{\beta^2}{\beta^1}\phi$; $\frac{\partial\tilde{e}^1}{\partial\alpha} = \frac{\Omega}{4\gamma^1}\frac{\beta^2}{\beta^1}\phi$; $\frac{\partial\tilde{e}^2}{\partial\alpha} = \frac{\Omega\phi}{2\gamma^2}\sqrt{2 - \frac{\beta^2}{\beta^1}}\left(1 - \frac{\beta^2}{2\beta^1}\sqrt{2 - \frac{\beta^2}{\beta^1}}\right)$; $\frac{\partial\tilde{e}^2}{\partial\Omega} = \frac{n}{n-\Omega}\frac{\alpha\phi}{2\gamma^2}\sqrt{2 - \frac{\beta^2}{\beta^1}}\left(1 + \frac{1-\alpha}{\alpha}\frac{\beta^2}{2\beta^1}\sqrt{2 - \frac{\beta^2}{\beta^1}}\right)$.

While partial equilibrium models with pollution usually assume an exogenous (often linear) market demand function (Hahn (1984), Montero (2009), Chen and Hobbs (2005), Sanin and Zanaj (2011), Sanin and Zanaj (2012)), the market demand in our model is endogenous and depends on agents' preferences.

Proposition 2 *: In the SCEP, marginal variations of the leader and the follower emission levels are greater when non-polluters preferences for the non-polluting good (Ω) increase, compared to an increase in their own preferences (α).*

The emissions level of polluters increase with their preferences for the product good, but this increase remains low compared with what we would have obtained if non-polluters preferences varied in the same proportions.

Proof: The differences of the marginal variations are $\frac{\partial \tilde{e}^1}{\partial \Omega} - \frac{\partial \tilde{e}^1}{\partial \alpha} = \frac{\tilde{e}^1}{\Omega(n-\Omega)(1+\alpha)}[n+n(\alpha-\Omega)+\Omega^2] \quad > 0$ and $\frac{\partial \tilde{e}^2}{\partial \alpha} - \frac{\partial \tilde{e}^2}{\partial \Omega} = \frac{\phi}{2\gamma^2}\sqrt{2-\frac{\beta^2}{\beta^1}}\left((\Omega - \alpha\frac{n}{n-\Omega}) - \frac{\beta^2\Omega}{2\beta^1}\sqrt{2-\frac{\beta^2}{\beta^1}}[\Omega + \frac{n}{n-\Omega}\frac{1-\alpha}{2}]\right) < 0, \quad$ if $\Omega < \alpha$. However, when α is high, the value obtained from $\frac{\partial \tilde{e}^2}{\partial \alpha} - \frac{\partial \tilde{e}^2}{\partial \Omega}$ is greater than that obtained with a small value of α, and the follower marginal variation of emissions become more and more sensitive to polluters preferences compared to those of the non-polluters. If $\alpha > \Omega$, $\alpha - \Omega > 0$, $(\alpha - \Omega) < 1$. This implies $n + n(\alpha - \Omega) + \Omega^2 > 0$, so that $\frac{\partial \tilde{e}^1}{\partial \Omega} - \frac{\partial \tilde{e}^1}{\partial \alpha} > 0$. Morever, the emission elasticities resulting from a variation of preferences yield the following expressions:

$$\varepsilon_{\tilde{e}^1/\alpha} = \frac{\alpha}{1+\alpha} \quad < \frac{1}{2}$$

$$\varepsilon_{\tilde{e}^2/\alpha} = \frac{1 - \frac{\beta^2}{2\beta^1}\left(2-\frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}}}{1 + \frac{1-\alpha}{\alpha}\frac{\beta^2}{2\beta^1}\left(2-\frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}}} \quad < 1$$

$$\varepsilon_{\tilde{e}^1/\Omega} = \varepsilon_{\tilde{e}^2/\Omega} = \frac{n}{n-\Omega} \quad > 1$$

The elasticity of emissions resulting from a variation of non-polluters' preferences is the same for the leader and the follower: this elasticity does not depend on market power and is greater than what recorded if non-polluters' preferences increase. Then, to reduce pollution when producers are also consumers, it is better to act on pure-consumers'preferences.

However, the emissions elasticity resulting from a variation of polluters' preferences is different for the leader and the follower. Indeed, $\varepsilon_{\tilde{e}^2/\alpha} < (>)\frac{1}{2} \Rightarrow \frac{1-X}{1+\frac{1-\alpha}{\alpha}X} < (>)\frac{1}{2}$ with $X = \frac{\beta^2}{2\beta^1}\left(2-\frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}}$. By solving this inequality, we get $\frac{\alpha}{1+\alpha} < (>)X$, so $\varepsilon_{\tilde{e}^2/\alpha} < (>)\frac{1}{2}$ if $\varepsilon_{\tilde{e}^1/\alpha} < (>)X$.

This result show that to reduce pollution, acting on the follower give a better result than acting on leader, if the elasticity of the leader's emission is $<$ to a certain threshold (X).

4 The Cournot equilibrium with pollution

Here, polluters compete "*à la Cournot*". The Cournot equilibrium is obtained as the solution of the following system of simultaneous optimization programs:

$$(q^1, e^1) \in \arg\max \quad \alpha \ln \left(\frac{\gamma^1 e^1}{\beta^1} - q^1 \right) + (1-\alpha) \ln \left(\frac{B}{q^1+q^2} q^1 - \gamma^1 e^1 \right), i = 1 \tag{30}$$

$$(q^2, e^2) \in \arg\max \quad \alpha \ln \left(\frac{\gamma^2 e^2}{\beta^2} - q^2 \right) + (1-\alpha) \ln \left(\frac{B}{q^1+q^2} q^2 - \gamma^2 e^2 \right), i = 2 \tag{31}$$

$$b^j \in \arg\max \quad \Omega \ln \left(\frac{q^1+q^2}{b^j + B^{-j}} b^j \right) + (1-\Omega) \ln \left(\frac{1}{n} - b^j \right), j = 1, ..., n. \tag{32}$$

Proposition 3 *: The solution of the CEP is given by the following strategy profiles* $(\widehat{q}^1, \widehat{q}^2, b)$ *and emissions level* $(\widehat{e}^1, \widehat{e}^2)$*:*

$$\widehat{q}^1 = \frac{\Omega \beta^2}{(\beta^1+\beta^2)^2} \phi \tag{33}$$

$$\widehat{q}^2 = \frac{\Omega \beta^1}{(\beta^1+\beta^2)^2} \phi \tag{34}$$

$$b^j = \frac{\Omega(n-1)}{n(n-\Omega)} = \frac{\Omega}{n} \phi \quad \forall j = 1, ..., n \tag{35}$$

$$\widehat{e}^1 = \frac{\Omega}{\gamma^1} \frac{\beta^2(\alpha\beta^2 + \beta^1)}{(\beta^1+\beta^2)^2} \phi \tag{36}$$

$$\widehat{e}^2 = \frac{\Omega}{\gamma^2} \frac{\beta^1(\alpha\beta^1 + \beta^2)}{(\beta^1+\beta^2)^2} \phi, \tag{37}$$

where $\phi = \frac{n-1}{n-\Omega}$.

From those strategies, we deduce the market price $p_A = \frac{\sum_{j=1}^n b^j}{\sum_{i=1}^2 q^i}$ which is:

$$\widehat{p}_A = \beta^1 + \beta^2 \tag{38}$$

Individual allocations are

$$\widehat{x}^1 = (\widehat{x}^1_A, \widehat{x}^1_B) = \left(\frac{\alpha\Omega}{\beta^1} \left(\frac{\beta^2}{\beta^1+\beta^2} \right)^2 \phi, \Omega(1-\alpha) \left(\frac{\beta^2}{\beta^1+\beta^2} \right)^2 \phi \right) \tag{39}$$

$$\widehat{x}^2 = (\widehat{x}^2_A, \widehat{x}^2_B) = \left(\frac{\alpha\Omega}{\beta^2} \left(\frac{\beta^1}{\beta^1+\beta^2} \right)^2 \phi, \Omega(1-\alpha) \left(\frac{\beta^1}{\beta^1+\beta^2} \right)^2 \phi \right) \tag{40}$$

$$\widehat{x}^j = (\widehat{x}^j_A, \widehat{x}^j_B) = \left(\frac{\Omega\phi}{n(\beta^1+\beta^2)}, \frac{(1-\Omega)\phi}{n} \right). \tag{41}$$

and yield the following utility levels

$$\widehat{U}^1 = 2\ln\left(\frac{\beta^2}{\beta^1+\beta^2}\right) + (1-\alpha)\ln(1-\alpha) + \ln\alpha\Omega\phi - \alpha\ln\beta^1 \tag{42}$$

$$\widehat{U}^2 = 2\ln\left(\frac{\beta^1}{\beta^1+\beta^2}\right) + (1-\alpha)\ln(1-\alpha) + \ln\alpha\Omega\phi - \alpha\ln\beta^2 \tag{43}$$

$$\widehat{U}^j = \Omega\ln\left(\frac{\Omega\phi}{(\beta^1+\beta^2)^n}\right) + (1-\Omega)\ln(1-\Omega\phi) + (\Omega-1)\ln n. \tag{44}$$

The elasticity are given by:

$$\varepsilon_{\widehat{e}^1/\alpha} = \frac{\alpha\beta^2}{\alpha\beta^2+\beta^1} \quad < 1$$

$$\varepsilon_{\widehat{e}^1/\Omega} = \varepsilon_{\widehat{e}^2/\Omega} = \frac{n}{n-\Omega} \quad > 1$$

Polluters react more to an increase in non-polluters' preferences than to an increase in their own preferences.

5 Comparison of equilibrium outcomes

We now proceed with the comparison of equilibrium outcomes in the SCEP and CEP. The strategic supplies of goods A and B exclusively depend on non-polluters preferences while emissions level depend on the preferences of all agents (polluters and non-polluters).

Proposition 4 *: In the SCEP and CEP, when preferences are asymmetric, agents' supplies only depend on the non-polluters preferences. But, when preferences are symmetric, equilibrium supplies depend on the preferences of all agents.*

Proof: If preferences are asymmetric, $\frac{\partial\tilde{q}^i}{\partial\Omega} = \frac{n}{n-\Omega}\frac{\beta^{-i}}{(\beta^i+\beta^{-i})^2} > 0$; $\frac{\partial\tilde{q}^1}{\partial\Omega} = \frac{n}{n-\Omega}\frac{\beta^2}{4(\beta^1)^2}\phi > 0$; $\frac{\partial\tilde{q}^2}{\partial\Omega} = \frac{n}{n-\Omega}\frac{\phi}{4(\beta^1)^2}[2\beta^1-\beta^2] > 0$; $\frac{\partial\tilde{q}^i}{\partial\alpha} = \frac{\partial\tilde{q}^1}{\partial\alpha} = \frac{\partial\tilde{q}^2}{\partial\alpha} = 0$; $\frac{\partial b^j}{\partial\alpha} = 0$. However, when all agents have the same utility function $U(x_1^h, x_2^h) = \alpha\ln x_1^h + (1-\alpha)\ln x_2^h, \quad \alpha \in (0,1) \quad \forall i = 1,2, \quad \forall j = 1,2,....n$, the strategic supplies are given by $\widehat{q}^1 = \frac{\alpha\beta^2}{(\beta^1+\beta^2)^2}\phi$; $\quad \widehat{q}^2 = \frac{\alpha\beta^1}{(\beta^1+\beta^2)^2}\phi$; $b^j = \frac{\alpha(n-1)}{n(n-\alpha)} \quad j = 1,...,n$.

We now focus on the effect of a change in agent preferences on the level of emissions. Indeed we have: $\frac{\partial\widehat{e}^i}{\partial\Omega} = \frac{n}{n-\Omega}\frac{\beta^{-i}(\alpha\beta^{-i}+\beta^i)}{\gamma^i(\beta^{-i}+\beta^i)^2}\phi \quad > 0$ and $\frac{\partial\widehat{e}^i}{\partial\alpha} = \frac{1}{\gamma^i}\frac{\Omega(\beta^{-i})^2}{(\beta^{-i}+\beta^i)^2}\phi \quad > 0$. The emissions level of a relatively inefficient firm (with a high value for β) is less sensitive to a variation in preferences compared to that of a relatively efficient firm: indeed, if $\beta^{-i} < \beta^i$, $\frac{1}{\gamma^i}\frac{\Omega(\beta^{-i})^2}{(\beta^{-i}+\beta^i)^2}\phi < \frac{1}{\gamma^{-i}}\frac{\Omega(\beta^i)^2}{(\beta^{-i}+\beta^i)^2}\phi$ and $\frac{1}{\gamma^{-i}}\frac{\beta^{-i}(\alpha\beta^{-i}+\beta^i)}{(\beta^{-i}+\beta^i)^2}\phi < \frac{1}{\gamma^i}\frac{\beta^i(\alpha\beta^i+\beta^{-i})}{(\beta^{-i}+\beta^i)^2}\phi$. Then, the difference in the marginal variation of emissions level resulting from a variation of the preferences is represented by:

$$\frac{\partial\widehat{e}^i}{\partial\alpha} - \frac{\partial\widehat{e}^i}{\partial\Omega} = \frac{\beta^{-i}\phi}{\gamma^i(n-\Omega)(\beta^i+\beta^{-i})^2}\left[\beta^{-i}(n(\Omega-\alpha)-\Omega^2) - n\beta^i\right] \tag{45}$$

Proposition 5 *: At both equilibria, if $\alpha > \Omega$, polluters' marginal variations of emissions remain more elevated when non-polluters preferences increase compared with an increase in their own preferences. However, at the CEP, the polluter is more sensitive to their own preferences rather than non-polluters preferences if $\frac{\beta^{-i}}{\beta^i} > \frac{n}{n(\Omega-\alpha)-\Omega^2}$.*

At both equilibria, the polluters' emissions elasticity is greater when non-polluters' preferences increase, compared to an increase recorded when polluters' preferences increase.

Proof: $\forall \beta^i$, if $\alpha > \Omega$, $\frac{\partial \widehat{e}^i}{\partial \alpha} - \frac{\partial \widehat{e}^i}{\partial \Omega} < 0$. In addition, solving $\frac{\partial \widehat{e}^i}{\partial \alpha} - \frac{\partial \widehat{e}^i}{\partial \Omega} > 0$, needs $\beta^{-i}(n(\Omega-\alpha)-\Omega^2) - n\beta^i > 0$ ie $\frac{\beta^{-i}}{\beta^i} > \frac{n}{n(\Omega-\alpha)-\Omega^2}$; the ratio of marginal costs remaining positive and $\frac{1}{n(\Omega-\alpha)-\Omega^2}$ being negative ($\alpha > \Omega$).

Remark 1: When $\alpha > \Omega$, this condition is sufficient to yield $\frac{\partial \widehat{e}^i}{\partial \alpha} - \frac{\partial \widehat{e}^i}{\partial \Omega} < 0$. This indicates that a higher preference of polluters for their own good, their level of emissions increases more with non-polluters preferences compared to the increase observed if their own preferences varied.

Proposition 6 *: An increase in non-polluters preferences for the non-polluting good affects more the polluter's marginal variation of emissions if their market power is sufficiently high. However, when their market power decreases, polluters preferences have no impact.*

Proof: At the SCEP, if $\alpha > \Omega$, we have $\frac{\partial \tilde{e}^1}{\partial \Omega} - \frac{\partial \tilde{e}^1}{\partial \alpha} = \frac{\tilde{e}^1}{\Omega(n-\Omega)(1+\alpha)}[n+n(\alpha-\Omega)+\Omega^2] \quad > 0$ and $\frac{\partial \tilde{e}^2}{\partial \alpha} - \frac{\partial \tilde{e}^2}{\partial \Omega} = \frac{\phi}{2\gamma^2}\sqrt{2-\frac{\beta^2}{\beta^1}}\left[(\Omega - \alpha\frac{n}{n-\Omega}) - \frac{\beta^2\Omega}{2\beta^1}\sqrt{2-\frac{\beta^2}{\beta^1}}[\Omega + \frac{n}{n-\Omega}\frac{1-\alpha}{2}]\right] < 0$. However with $\alpha > \Omega$, at the CEP we get $\frac{\partial \widehat{e}^2}{\partial \alpha} - \frac{\partial \widehat{e}^2}{\partial \Omega} = \frac{\beta^1\phi}{\gamma^2(n-\Omega)(\beta^2+\beta^1)^2}\left[\beta^1(n(\Omega-\alpha)-\Omega^2) - n\beta^2\right] > 0$ because $\frac{\beta^1}{\beta^2}$ is always positive.

Remark 2: If polluters market power is below a certain threshold, their emissions' marginal variation is higher when their own preferences vary compared to a variation in that of non-polluters. From this threshold, their emissions becomes more responsive to non-polluters preferences. Moreover, at the CEP, the impact on polluters' emissions level resulting from a variation of their preferences may be identical to that which would be recorded if the preferences of non-polluters varied in the same proportion. However, if the same firm competed at the SCEP as a leader, this coincidence could not be observed. Indeed, $\frac{\partial \widehat{e}^i}{\partial \alpha} - \frac{\partial \widehat{e}^i}{\partial \Omega} = 0$ if $\frac{\beta^i}{\beta^{-i}} = \frac{n(\Omega-\alpha)-\Omega^2}{n}$ while $\frac{\partial \tilde{e}^1}{\partial \Omega} - \frac{\partial \tilde{e}^1}{\partial \alpha} \neq 0$ because the value recommended for α is greater than 1, i.e $\alpha = 1 + \Omega - \frac{\Omega^2}{n} < 0$. The ratio of marginal costs remaining positive, $\frac{\partial \widehat{e}^i}{\partial \alpha} - \frac{\partial \widehat{e}^i}{\partial \Omega} = 0$ if and only if $\Omega > \alpha$.

Remark 3: The emission elasticity does not depend on firms' marginal costs (β^i and β^{-i}) when Ω increases while it depends on it when α increases. At the SCEP as at the CEP, when firms have the same marginal cost, they have also the same vulnerability in their emission. At the SCEP as at the CEP, polluting firms react in the same way to a change in the preferences of non-polluting agents ($\varepsilon_{\widehat{e}^i/\Omega} = \varepsilon_{\tilde{e}^i/\Omega}$), and this value is greater than what recorded with a variation in their own preferences.[3] However, at the

[3]This result is not obvious when producers consume the good they produce.

CEP, $\varepsilon_{\widehat{e}^1/\alpha} > \frac{1}{2}$ if and only if $\frac{\beta^i}{\beta^{-i}} > 1$ while at the SCEP, $\varepsilon_{\tilde{e}^1/\alpha}$ is always $< \frac{1}{2}, \forall \beta^i, \beta^{-i}$.

Proposition 7 *: If $\beta^i < \beta^{-i}$, firm's emission elasticity decreases when their market power increase. However, if $\beta^i > \beta^{-i}$, firm's emission elasticity increases when their market power increases.*

Proof : Indeed, $\varepsilon_{\widehat{e}^1/\alpha} = \frac{\alpha}{\frac{\beta^1}{\beta^2}+\alpha}$ and $\varepsilon_{\tilde{e}^1/\alpha} = \frac{\alpha}{1+\alpha} \Rightarrow \varepsilon_{\widehat{e}^1/\alpha} > \varepsilon_{\tilde{e}^1/\alpha}$ if $\beta^1 < \beta^2$. Also, $\varepsilon_{\tilde{e}^2/\alpha} < \varepsilon_{\widehat{e}^2/\alpha}$ if and only if $\frac{\beta^1}{\beta^1+\beta^2} < \frac{\beta^2}{2\beta^1}\left(2 - \frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}}$. This is equivalent to solve $\frac{\beta^2}{2\beta^1}\frac{\beta^1+\beta^2}{(\beta^1)^2}\left(2 - \frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}} > 1$.

- If $\beta^1 > \beta^2$, ie $\frac{\beta^2}{\beta^1} < 1$, we get $\frac{\beta^2}{2\beta^1} < \frac{1}{2}, \left(2 - \frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}} < \sqrt{2}$: this yields $\frac{\beta^2}{2\beta^1}\frac{\beta^1+\beta^2}{(\beta^1)^2}\left(2 - \frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}} < 1 \quad \Rightarrow \varepsilon_{\widehat{e}^2/\alpha} < \varepsilon_{\tilde{e}^2/\alpha}$.
- If $1 < \frac{\beta^i}{\beta^{-i}} < 2$, ie $\beta^i > \beta^{-i}$, then we get $\begin{cases} \varepsilon_{\widehat{e}^2/\alpha} > \varepsilon_{\tilde{e}^2/\alpha} & i = 2 \\ \varepsilon_{\tilde{e}^1/\alpha} > \varepsilon_{\widehat{e}^1/\alpha} & i = 1 \end{cases}$
- When $\beta^1 = \beta^2$, we get $\varepsilon_{\tilde{e}^1/\alpha} = \varepsilon_{\tilde{e}^2/\alpha} = \varepsilon_{\widehat{e}^i/\alpha} = \frac{\alpha}{1+\alpha}$.

Finally, we determine the conditions under which the two equilibria coincide.

Proposition 8 *: When $\beta^1 = \beta^2 = \beta$ and $\gamma^i = \gamma$, the SCEP coincides with the CEP.*

Remark 4: Morever, $\frac{\partial \tilde{e}^1}{\partial \Omega} - \frac{\partial \tilde{e}^1}{\partial \alpha} = \frac{\partial \tilde{e}^2}{\partial \Omega} - \frac{\partial \tilde{e}^2}{\partial \alpha} = \frac{\partial \widehat{e}^i}{\partial \Omega} - \frac{\partial \widehat{e}^i}{\partial \alpha} = \frac{\phi}{4\gamma(n-\Omega)}[n + n(\alpha - \Omega) + \Omega^2] > 0$.

Proof: If $\beta^i = \beta$ and $\gamma^i = \gamma$, we get $\tilde{q}^1 = \tilde{q}^2 = \widehat{q}^i = \frac{\Omega}{4\beta}\phi$ and $\tilde{e}^1 = \tilde{e}^2 = \widehat{e}^i = \frac{\Omega(\alpha+1)}{4\gamma}\phi$.

6 Conclusion

In this note, we pioneered the analysis of asymmetric preferences on pollution in bilateral oligopoly market by extending the model of Julien and Tricou (2012). We set out to examine the consequences of asymmetric preferences on emissions when polluters have asymmetric and symmetric market power. Three results can be emphasized. First, the supply of each polluter depends on the non-polluters' preferences. Second, when the firm's market power increases, their emission increases or decreases according to the relationship between its marginal cost and that of its competitor. Third, when producers are also consumers, an action on pure-consumers'preferences reduce more the pollution, and this regardless of the producers' market power. From the point of view of the social planner, to reduce pollution it is more benefit to act on firms with big market power rather than small firms. This study can be extended to a dynamic context and in particular to the dynamics of government health expenditures.

References

Amir, R., Sahi, S., Shubik, M., and Yao, S. (1990). A strategic market game with complete markets. *Journal of Economic Theory*, 51(1):126–143.

Bloch, F. and Ferrer, H. (2001). Strategic complements and substitutes in bilateral oligopolies ‘. *Economics Letters*, 70:83–87.

Bloch, F. and Ghosal, S. (1997). Stable Trading Structures in Bilateral Oligopolies. *Journal of Economic Theory*, 74:368–384.

Chen, Y. and Hobbs, B. F. (2005). An Oligopolistic Power Market Model with Tradable NOx Permits. *IEEE Transactions on Power System*, 20(1):119–129.

Crettez, B., Jouvet, P.-A., and Julien, L. A. (2014). Tax Policy in a Simple General Oligopoly Equilibrium Model with Pollution Permits. *Les Cahiers de la chaire Economie du Climat, Working Paper Series*, (13).

Dickson, A. and Hartley, R. (2008). The strategic Marshallian cross. *Games and Economic Behavior*, 64(2):514–532.

Dickson, A. and Mackenzie, I. A. (2018). Strategic trade in pollution permits. *Journal of Environmental Economics and Management*, 87:94–113.

Gabszewicz, J. J. (2002). Strategic Multilateral Exchange: General Equilibrium with Imperfect Competition. (Cheltenham: Edward-Elgar).

Gabszewicz, J. J. and Michel, P. (1997). Oligopoly Equilibria in Exchange Economies,. *in Eaton, B. C. and Harris, R.G. (eds). Trade, Technology and Economics. Essays in Honour of R.G. Lipsey*, (Cheltenham: Edward-Elgar):217–40.

Hahn, R. (1984). Market Power and Transferable Property Rights. *Quarterly Journal of Economics*, 99(4):753–765.

Julien, L. A. and Tricou, F. (2012). Market price mechanisms and stackelberg general equilibria: An example. *Bulletin of Economic Research*, 64(2):239–252.

Kolstad, J. T. and Wolak, F. A. (2008). Using Environmental Emissions Permit Prices to Raise Electricity Prices : Evidence from the California Electricity Market by. *Working paper, Harvard University*, (March 2003).

Misiolek, W. S. and Elder, H. W. (1989). Exclusionary Manipulation of Markets for Pollution Rights. pages 156–166.

Montero, J. P. (2009). Market power in pollution permit markets. *Energy Journal*, 30(Special issue 2):115–142.

Sahi, S. and Yao, S. (1989). The non-cooperative equilibria of a trading economy with complete markets and consistent prices. *Journal of Mathematical Economics*, 18(4):325–346.

Salop, S. C. and Scheffman, D. T. (1983). Raising Rivals’ Costs. *American Economic Review Papers and Proceedings*, 73(2):267–271.

Sanin, M.-E. and Zanaj, S. (2011). A Note on Clean Technology Adoption and its Influence on Tradeable Emission Permits Prices. *Environmental and Resource Economics*, 48(4):561–567.

Sanin, M.-E. and Zanaj, S. (2012). Clean Technology Adoption Under Cournot Competition. *Strategic Behavior and the Environment*, 2(2):159–172.

Shapley, L. and Shubik, M. (1977). Trade Using One Commodity as a Means of Payment. *Journal of Political Economy*, 85:937–67.

Westskog, H. (1996). Market Power in a System of Tradeable C02 Quotas. *The Energy Journal*, 17(3):85–103.