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Market collusion with joint harm and liability sharing

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Abstract

When it is impossible to identify ex post the producer of a product causing harm, or the damage caused is indivisible although caused by multiple injurers, courts must apportion the total damage among tortfeasors. In this model we examine how such liability sharing rules affect the likelihood of tacit collusion. For this we use a standard Cournot oligopoly model where firms are collectively held liable for joint harm inflicted on third parties. With repeated market interaction and grim strategies, we investigate the sustainability of collusion to derive some policy implications.

Keywords: Cournot oligopoly, liability sharing rules, market collusion

JEL codes: L41, L13, K13

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1 Introduction

Collusive behavior, be it tacit or explicit, is under constant scrutiny by competition authorities throughout the world. Substantial losses for consumers/buyers and total welfare as well pushed competition agencies to toughen up the sanctions against horizontal market coordination. In Europe, for instance, the fines imposed for cartel infringements by the European Commission alone amount to 28 464 092 824 € over the 1990-2018 period, from which 8 522 679 000 € were imposed during the 2014-2018 period alone.[1] Detection of collusive behavior is crucial for the effectiveness of antitrust enforcement, and a thorough understanding of circumstances that may facilitate collusive agreements is indisputably needed. One feature that is so far missing from the economic analysis of collusion is the impact of apportioning rules for civil liability. This is the object of our paper.

We focus on situations where courts hold industry firms collectively liable for an injury caused to third parties. For instance, courts and even affected parties might be eventually unable to identify the actual producer of a defective good (Marino, 1991). This producer identification problem occurred for the famous DES litigation[2], as well as for airplane beverage carts[3], and even rowing exercise machines.[4] Alternatively, the aggregate industry output may cause an indivisible injury, which is current in environmental liability cases, such as leakage of gasoline

[1]These are fines not adjusted for Court judgments. See the EC's cartel statistics, available at http://ec.europa.eu/competition/cartels/statistics/statistics.pdf, retrieved on December 22, 2018.

[2]The diethylstilbestrol (DES) was a generic drug sold by various pharmaceutical firms to pregnant women to prevent miscariage. Years later, children born from these women developed medical conditions caused by the *in utero* exposition to the drug. Due to the long time lapse the exposition and the injury occurence, victims were unable to determine which pharmaceutical firm sold the defective product. See Sindell versus Abbott Laboratories, 26 Cal. 3d 588; 163 Cal. Rptr. 132, 607 P.2d 924, March 1980.

[3]See Russo v Material Handling Specialties Co, 1995 WL 1146853: a flight attendant for USAir was injured by an unsecured run-a-way beverage cart while working on a flight. It was unknown who supplied that specific cart to USAir.

[4]See Mahar v Hanover House Indus, Inc, 1995 WL 1146188: at the time there was no inventory control in place to determine which defendant supplied the defective rowing-type exerciser that injured the plaintiff.

products[5] or airplane noise pollution.[6] Environmental liability cases are a prime example for the involvement of third-party victims in contrast to cases where consumers are harmed by defective products.

One important feature of all these examples is that they may relate to oligopoly markets, and hence the firms' strategic market behavior might be affected by the courts' choice in terms of apportioning collective liability among competitors. In this paper we examine to what extent the possibility of cooperative behavior on oligopoly markets, i.e. the likelihood of horizontal collusive agreements, hinges on the choice of liability sharing rules.

There are various ways to apportion liability among market firms. Starting with the DES Sindell v. Abbott litigation case, US courts often used market shares as a criterion to share damages among liable firms.[7] According to this rule, the apportioning of liability is fully proportional to a firm's output level. This is in stark contrast to the per capita sharing rule, which is often enforced as a default solution,[8] and according to which the share of the damage caused and ensuing liability incurred by a single firm are independent from output, but only depends on the number of (comparable) firms active on the market. Instead of focusing alternatively on one or another of these extreme cases (i.e. full or zero proportionality of individual liability

[5]The methyl tertiary butyl ehter (MTBE) is a gasoline additive used by oil companies and constitutes one of the major sources of groundwater pollution in the United States. Both the storage and the distribution of gasoline require to mix the products of different refiners. This makes it possible that "certain gaseous or liquid products (e.g., gasoline, liquid propane, alcohol) of many suppliers were present in a completely commingled or blended state at the time and place that the risk of harm occurred, and the commingled product caused a single indivisible injury" - quote from In re Methyl Tertiary Butyl Ether ("MTBE") Prods. Liab. Litig. (MTBE 1), 379 F. Supp. 2d 348, 377-78 (S.D.N.Y. 2005).

[6]See Cour de Cassation, Paris (2e Civ), July 20, 1988, n° 86-12.543: ninety seven inhabitants living close to Orly Paris airport filed a suit against Pakistan Airlines, Air India, Alitalia, El Al Ly, Lufthansa and Swissair for excessive noise due to air traffic.

[7]This was also recently enforced in France, for the appeal in a DES litigation case - see Versailles Appeal Court, June 30, 2016, n° 14/04397.

[8]For instance, even though the French law does not impose one or another method to apportion strict liability among tortfeasors, the French case law actually establishes the per capita rule as the default sharing rule and provides several examples where Courts used the per capita sharing rule, including one case of DES litigation (specifically, Paris Appeal Court, October 26, 2012, n°10/18297). The presumption that the per capita sharing rule is the default rule is actually made in the US (see Section 2 of the 1995 version of the Uniform Contribution Among Tortfeasors Act), as well as in several European countries, such as Austria, Greece, Italy, or Poland - see Ranouil (2014).

w.r.t. output), we opt to model courts' choice of apportioning liability among market firms by means of a composite sharing rule. This is a linear combination of the equal sharing (i.e. the per capita rule) and the market share rule, and allows us to examine to what extent the importance of the output proportionality in the apportioning of liability is likely to impact the cooperative market behavior.

To model market interaction, we use the workhorse model of symmetric Cournot oligopoly with linear demand. We assume that firms are held collectively and strictly liable for the harm generated by their aggregate output, and consider alternative cost structures for expected harm: first linear, then cumulative/quadratic in aggregate output. In order to meaningfully examine the emergence of cooperative behavior, we assume repeated market interaction and grim strategies. Note that although this framework is designed to study the sustainability of collusion that is reached in a "tacit", i.e. non-cooperative way, to the extent that firms may not resort to explicit agreements to establish quotas or share markets and subsequently discipline deviations, the analysis is equally relevant for explicit agreements such as cartels. Indeed, sustainability is a necessary condition for cartels formed through explicit coordination as long as the punishment of a deviation cannot be legally binding. Hence for our analysis we use indistinctly the terms "(tacit) collusion" and "cartel".

Based on this framework we derive several results. We find that when harm increases proportionally to aggregate output, the apportioning rule is neutral w.r.t. the likelihood of market collusion. In contrast, with cumulative harm (i.e. the damage increases more-than-proportionally w.r.t. aggregate output), we find that the sustainability of collusion does depend on the liability system used: the more weight the apportioning rule puts on market shares, the less likely the collusive behavior. By comparing the two scenarii, we also find that market collusion is relatively easier to sustain on markets where the harm jointly inflicted by industry firms is cumulative w.r.t. aggregate output.

The intuition for our results hinges on how a change in liability allocation impacts the relative gain from deviation from a cartel agreement to the ensuing relative loss in profits in the punishment phase. Both are affected by the double externality that firms exert among themselves: the one due to imperfect competition, and the one derived from the sharing of liability. A shift towards the market share rule makes deviation from the cartel agreement less attractive, since the increase in output by the deviating firm becomes relatively costlier in terms of liability. But then the punishment becomes less severe too, since the shift towards the market share rule also reduces competitors' output in Cournot oligopoly, thereby increasing the equilibrium price and reducing the extent of the negative externality via the liability system. Cournot profits in the punishment phase become higher. Given the full linearity (both demand- and cost-wise) of the proportional harm scenario, the two effects are of equal weight, leaving the conditions for cartel stability unaffected.

With convex cost from harm, this neutrality result no longer holds. With a higher weight on market share liability, deviation from the cartel agreement by increasing own output still becomes less attractive and due to higher Cournot profits punishment is less severe. However, given that with convex harm the extent of the harm externality between firms is more important, firms gain more from the reduction in output achieved by a cartel than in the case of linear harm, especially if liability is not aligned to market shares. As a result, a move towards market share liability allocation reduces the loss in profits due to punishment relatively more than it reduces deviation profits, thus making collusion less likely.

Note that the non-neutrality result we obtain in the cumulative harm scenario is essentially a reminder that judicial decisions may actually impact the market outcomes/equilibrium, and possibly in a wrong way. By doing so, we add one more argument to the literature questioning the separability between antitrust and liability performance (see Daughety and Reinganum, 2014). However, policy conclusions in our setting that combines imperfect competition and harm externalities may be even more complicated as the two market imperfections may theoretically result in socially too high or low output levels with Cournot competition. A shift towards market share liability *per se* reduces competitive output levels and this repercussions must also be taken into account for a welfare analysis. We conduct such a welfare analysis and find that the linear harm scenario argues for a move towards the per capital liability allocation, especially since the likelihood of collusion is not affected by such a shift. In contrast, for the scenario with cumulative harm a trade-off might exist between increasing output by a shift towards per capita liability and the higher likelihood of collusion in this case.

The paper proceeds as follows: first we briefly review the literature closely related to our topic (Section 2), before presenting the framework and notations we use (Section 3). We first solve the proportional harm case in Section 4, then go on to examine the cumulative harm one (Section 5). We discuss the welfare and policy implications of our results in Section 6, and check robustness of our results to the application of some alternative assumptions in Section 7. We consider price instead of quantity competition, partial instead of full market collusion, optimal punishments instead of grim trigger strategies, and also non-linear demand specifications. Section 8 concludes. Mathematical derivations are provided in a Technical Appendix available on request.

2 Related literature

Our analysis contributes to an increasing bulk of academic research dedicated to the consequences of assigning strict liability in case of joint harm and producer identification problem by means of a sharing rule based on the injurers' market shares (Sheiner, 1977). Starting with Marino (1991), several contributions have thus examined the efficiency of the strict market share liability, in particular in terms of associated levels of precautionary effort to be undertaken by firms, in a context of non-cooperative market interaction. The common assumption of these analyses is that strict liability is enforced based on a market share apportioning, and that firms make market behavior decisions given this rule. For instance, Hamilton and Sunding (2000) determine, in an asymmetric oligopoly with conjunctural variations, the conditions under which an increase in the liability exposure may trigger entry, whereas Daughety and Reinganum (2014) use instead a symmetric Cournot oligopoly to determine a modified market-share liability sharing rule that may induce firms to choose the socially optimal level of care for any given level of output.

We, in contrast, discuss the case of cooperative market behavior, and also, do not exclusively consider the market shares sharing rule. As such, our analysis is related to papers allowing for alternative ways to share liability among the multiple injurers.[9] Golbe and White (2000) for instance compare the market share liability with no liability (i.e. victim liability) and also multiple joint and several with no contribution (i.e. placing fully the industry's entire liability on each defendant), to argue that efficiency often recommends increasing liability beyond the market-share standard. Fees and Hege (1998) relate the problem of allocation liability to problems of team production and show that an efficient liability sharing rule might require deviations from a proportional splitting rule. They recommend an apportioning rule that takes into account the possibility for injurers to have an asymmetric impact on the occurrence of a specific accident: in that case, the entity more likely to be at fault should assume a disproportionately larger share of the losses. More recently, Gluttel and Leshem (2013, 2014) consider in a tort model the equal sharing rule in addition to the one based on market shares, in order to identify the impact on the precautionary effort. Leshem (2017) uses a composite liability sharing rule to determine the optimal mix that provides firms with the necessary incentives to undertake the target level of precaution. Finally, Charreire and Langlais (2017) consider alternatively an equal (per capita) sharing rule and one based on market shares to examine, in a Cournot oligopoly with precautionary effort, the efficiency properties of the resulting market equilibrium.[10] We equally examine these very same sharing rules, although not alternatively, but within a linear-combination com-

[9]Early contributions to the literature on multiple causation of harm and problems of unobservability are also found in the books of Shavell (1987) and Landes and Posner (1987) or in Kornhauser and Revesz (1989) who do not explicitly focus on strict liability.

[10]Charreire and Langlais (2017) consider the situation of symmetric and asymmetric firms, whereas we concentrate on the case of symmetric firms.

posite rule, following Leshem (2017). Our purpose is to better grasp the implications of a possible gradual shift towards the enforcement of an output-proportional liability sharing rule. Also, our model abstracts from precautionary effort but instead concentrates on output, i.e. activity, choices.

Finally, and in contrast to the rest of the literature on liability with the exception of only one paper, we focus on cooperative market behavior. To study the impact of liability sharing rules on the likelihood of market collusion we use the workhorse setting of an infinitely repeated market interaction between firms, in which collusion may emerge in equilibrium provided that firms value sufficiently the future losses in case of deviation. This framework allows to discuss the likelihood of collusion by means of a comparative statics exercise on the critical discount factor. Thus, our paper equally belongs to the vast industrial organization literature addressing this topic. Feuerstein (2005) and Kaplow and Shapiro (2007) provide quite comprehensive surveys on the various factors that may hinder or facilitate collusion. Interestingly, institutional features and more generally speaking, legal or liability systems have been so far mostly overlooked.[11] To our knowledge, the only other formal analysis of how the enforcement of liability may impact the sustainability of collusion is provided by Friehe (2014). In a symmetric oligopoly with either strict liability or negligence applying, Friehe (2014) finds that when harm increases proportionally w.r.t. output, the liability rules are neutral for the stability of collusion. Instead, when harm increases at a more-than-proportional rate in the output level, collusion is more likely under strict liability than under negligence. However, Friehe (2014) restricts the analysis to cases in which the harm is generated by individual output, and therefore does not consider different liability sharing rules. We, in contrast, focus on the role played by the liability allocation in case of joint harm.

[11] For rare exceptions, see Haufler and Schjelderup (2004) and Schindler and Schjelderup (2009), which examine the impact of different tax systems on the critical discount factor.

3 Framework and notations

We consider a homogeneous goods market with a given number of $n > 2$ identical firms competing in quantities.[12] Firms face linear demand. The inverse demand function is given by $P(Q) = a - Q$, $a > 0$, where Q denotes aggregate output. We denote by q_i the individual output level of firm i, $i = 1, ..., n$. We assume constant unit production cost, which – w.l.o.g. – we normalize to zero. Aggregate industry output generates expected harm $H(Q)$ to third parties. Harm incurred by individual third parties cannot be ascribed to single firms, and firms are collectively subject to strict liability. As a result, every firm incurs a liability cost in addition to its production cost, where the expected damage payments for a given firm depends on the liability sharing rule. Total expected damages payments by firms equal total expected harm.

We follow Leshem (2017) for the sharing of liability among the multiple injurers on the market, and consider a "composite" sharing rule: this is a linear combination of per capita sharing of liability (PC henceforth) and proportional sharing according to firms' market shares (MS henceforth). With γ denoting the weight put on partitioning of damages according to market share, firm i's share in liability, s_i, amounts to[13]

$$s_i = \gamma \frac{q_i}{Q} + (1 - \gamma)\frac{1}{n}. \tag{1}$$

An increase in the parameter γ, where $\gamma \in [0, 1]$, represents a shift from the equal sharing rule to the market-shares rule. Whereas this "composite" sharing rule also allows us to discuss the two polar cases of courts using a pure PC or a pure MS rule, our analysis is mostly concerned with a (marginal) increase in the weight γ put on market-shares. An increase in γ may be interpreted as

[12] Recall that the assumption of Cournot competition also covers the two-stage capacity-then-price setting game (see Kreps and Scheinkman, 1983).

[13] This interpretation of γ is in line with the one used in Leshem (2017). However, note that γ may lend itself to another interpretation: *ex ante* there might be some degree of uncertainty on the liability sharing rule used by courts. Thus, γ may represent the *ex ante* probability that a court uses the market shares based allocation of damages instead of a simple per capita allocation.

courts expending more effort to obtain precise measures of individual firms' output levels instead of relying on a default rule of apportioning damages equally among the formally identical firms.

With a firm's liability share specified by (1), the remaining determinant of expected liability cost is given by the amount of total expected harm inflicted on third parties, $H(Q)$. The distinguishing feature that is important for our analysis is how expected harm impacts the firm's objective function. In a first scenario, expected harm is assumed to be linear in the aggregate output level, $H(Q) = h^{Lin} \times Q$, $h^{Lin} < a/n$. Each unit produced by any firm contributes equally to expected harm, but it is not possible to link harm to any individual output unit. In a second scenario, the expected harm increases at a more-than-proportional rate in aggregate output.[14] For this cumulative harm framework we assume a quadratic damage function: $H(Q) = \frac{h^{Cum}}{2} \times Q^2$, with $h^{Cum} < 2/(n-2)$.

With respect to timing, we consider repeated market interaction during an infinite number of periods, making room for sustainable collusion to emerge among firms. In the main analysis, we consider the often used grim trigger strategies (Friedman, 1971): industry firms jointly maximize profits and continue to do so unless a deviation occurred in the previous period. If this is the case, then the punishment involves firms returning to the Cournot/non cooperative equilibrium forever. We establish the critical discount factor that has to be surpassed for collusion to be stable.

We use our set-up to conduct a comparative statics exercise, in which we determine the impact of the weight placed on the MS rule on the critical discount factor, and therefore on the likelihood of collusion. In each of the two scenarii, linear and cumulative damage, we first examine the outcome of Cournot competition. Afterwards we determine the collusive and deviation profits, as well as the critical discount factor.

[14] This is particularly relevant for environmental pollution and exposure to toxins, for instance - see Daughety and Reinganum (2014).

4 Linear Harm Model

In this section we tackle the case of linear harm - we use the upperscript Lin to distinguish from the case of cumulative harm. We assume here that harm inflicted to third parties increases proportionally to aggregate industry output, and use the following linear damage cost function: $H(Q) = h^{Lin} \times Q$.

4.1 Non-cooperative behavior: market outcome under Cournot-Nash competition

We start by computing the output and profit levels for the Cournot-Nash equilibrium. Individually, a firm i solves the following:

$$\begin{aligned} \max_{q_i} \pi_i^{Lin} &= P(Q)q_i - s_i(\gamma, q_i, Q)H(Q) \\ &= (a-Q)q_i - \left(\gamma\frac{q_i}{Q} + (1-\gamma)\frac{1}{n}\right) \times h^{Lin} \times Q \ , \\ &= (a - q_i - Q_{-i})q_i - \left((1-\gamma)\frac{1}{n} + \gamma\right) \times h^{Lin} \times q_i - (1-\gamma)\frac{Q_{-i}}{n} \times h^{Lin} \end{aligned} \tag{2}$$

with $Q = \sum_{j=1}^{n} q_j$ as overall and $Q_{-i} = \sum_{j \neq i} q_j$ as competitors' output. The above illustrates that, besides the price-related externality, joint harm implies a second kind of externality among firms, which results from liability sharing and is reflected in the last term in (2).

The first order condition satisfies

$$a - (q_i + Q) = \left((1-\gamma)\frac{1}{n} + \gamma\right) \times h^{Lin}. \tag{3}$$

Condition (3) has a standard interpretation: each firm determines its output level such that marginal revenue, the LHS in (3), equals its full marginal cost, i.e. the marginal cost of production (here equal to zero, given our normalization) plus the marginal cost associated with expected liability - the RHS in (3).

Two remarks seem relevant here. First, with linear harm, the marginal cost of liability is independent of both individual and aggregate output, regardless of the sharing rule used. Second, a firm's marginal liability costs increase in the weight put on the MS rule γ. Consider a pure PC liability sharing rule, i.e. $\gamma = 0$: then each firm internalizes only the share $1/n$ of marginal expected harm coming with an increase in output. Instead, with a liability system purely based on market shares, i.e. $\gamma = 1$, marginal liability coincides with marginal harm.

Using symmetry among firms, we obtain the equilibrium individual output and profit levels:[15]

$$\begin{aligned} q_{CN}^{Lin} &= \frac{an - h^{Lin}(1 + \gamma(n-1))}{n(1+n)} \\ \pi_{CN}^{Lin} &= (a - h^{Lin} - nq_{CN}^{Lin})q_{CN}^{Lin}. \end{aligned}$$

It is straightforward that a shift from the PC to the MS sharing rule lowers individual equilibrium output. This is due to the aforementioned increase in marginal liability costs associated with a higher weight on the MS rule for liability allocation. In turn, individual Cournot profit increases in γ:

$$\frac{d\pi_{CN}^{Lin}}{d\gamma} = -\frac{(n-1)\left(a + h^{Lin}(1-2\gamma)\right)}{n+1}\frac{dq_{CN}^{Lin}}{d\gamma} > 0.$$

In other words, the closer to the MS rule the liability allocation gets, the higher are profits for the Cournot firms. The intuition behind this result is straightforward, and is associated to the two types of externalities among firms described above. First, aggregate output in the symmetric Cournot equilibrium surpasses monopoly output resulting in lower than maximal profits for firms. Any decrease in equilibrium output leads to higher individual profits for each single firm due to lower output by its competitors resulting in a higher equilibrium price. This effect is standard in oligopoly models. In addition, a second effect reinforces the increase in

[15] Note that the assumption of $a > nh^{Lin}$ introduced in Section 3 guarantees positive output and profit levels for $\gamma \in [0, 1]$.

profits. For $\gamma < 1$, i.e. part of harm is allocated independently of market shares, the second negative externality among firms becomes relevant as well. Any increase in output by one single firm not only reduces the market price (the first effect mentioned above) but directly increases competitors costs as additional liability is shared between firms (see the last term in (2)). A higher weight on the MS rule reduces the extent of the harm externality and therefore increases profit levels. To sum up, Cournot firms are better off under the MS sharing rule, which will be important when we discuss punishment in the event of deviation from the collusive behavior.

4.2 Cooperative behavior and sustainability of collusion

We now turn to firms' collusive behavior, and first determine the cartel's profit. The firms' joint maximization problem writes

$$\max_Q P(Q)Q - \sum_{i=1}^{n} s_i(\gamma, q_i, Q) \times H(Q) = (a - Q)Q - h^{Lin} \times Q,$$

where $\sum_{i=1}^{n} s_i(\gamma, q_i, Q) = 1$. The corresponding FOC is

$$a - 2Q = h^{Lin},$$

indicating that in equilibrium the cartel's marginal revenue equals the full marginal cost associated with the cartel's overall production. Note that as compared with the non-cooperative game, the marginal cost is still constant, but now independent of the liability sharing rule. When all industry firms collude, the cartel's total marginal cost (i.e. including liability) is always equal to the social cost, irrespective of the weight γ.

Solving the FOC for the equilibrium cartel output, we obtain the symmetric equilibrium outcomes:

$$
\begin{aligned}
q_c^{Lin} &= \frac{a-h^{Lin}}{2n}, \\
\pi_c^{Lin} &= n(q_c^{Lin})^2.
\end{aligned}
$$

Of course, the fact that collusion profit is independent from the liability sharing rule is easily explained: with all-encompassing collusion, every firm fully internalizes the externality due to liability sharing among them, hence the precise liability sharing rule does not impact the individual gain from collusion.

Next we determine the profit obtained in case of individual deviation in a given period. A firm in the cartel would deviate by choosing a level of output which maximizes its own profit given that all other insiders keep on colluding and producing q_c^{Lin}. Thus, the deviant firm's program is given by:

$$
\begin{aligned}
\max_{q_d} \pi_d(q_d) &= P(q_d+Q_{-d})q_d - s_d(\gamma, q_d, q_d+Q_{-d})H(q_d+Q_{-d}) \\
&= (a-(q_d+Q_{-d}))q_d - \left(\gamma\frac{q_d}{q_d+Q_{-d}} + (1-\gamma)\frac{1}{n}\right) h^{Lin} \times (q_d+Q_{-d})
\end{aligned}
$$

where $Q_{-d} = (n-1)q_c^{Lin}$. As with Cournot competition, marginal costs of the deviant firm are independent w.r.t. both individual and aggregate output, but increasing in the weight γ put on the MS rule in apportioning liability. The resulting deviant firm's output and profit level, denoted by q_d^{Lin} and π_d^{Lin}, respectively, are given by:

$$
\begin{aligned}
q_d^{Lin} &= \frac{a(n+1)+h^{Lin}(n-3)-2\gamma h^{Lin}(n-1)}{4n}, \\
\pi_d^{Lin} &= \left(a-q_d^{Lin}-(n-1)q_c^{Lin}\right) q_d^{Lin} - h^{Lin}\left(\gamma q_d^{Lin} + (1-\gamma)\frac{q_d^{Lin}+(n-1)q_c^{Lin}}{n}\right).
\end{aligned}
$$

Output of the deviant firm is decreasing in the weight on the MS rule γ. This directly follows from the observations of cartel output being independent of the liability sharing rule, whereas marginal costs of the deviant firm increase in the weight put on the MS rule.

Note that the competitors' output levels do not depend on the liability sharing rule and the adjustment in output by the deviant firm in response to an increase in the weight γ has no first order effect on its profit level. Accordingly, the only remaining effect is the direct effect of a redistribution of liability between firms, which is given by

$$\frac{d\pi_d^{Lin}}{d\gamma} = \frac{\partial\pi_d^{Lin}}{\partial\gamma} = -h^{Lin} \times \frac{n-1}{n} \underbrace{\left(q_d^{Lin} - q_c^{Lin}\right)}_{>0} < 0.$$

With the deviant firm producing a higher output level than its competitors, a shift toward a higher weight of the MS rule in liability sharing necessarily increases the cost of the deviant firm, thereby reducing its profit level. This is the second crucial effect to take into account when determining the conditions under which collusion is sustainable.

As a final step, we establish the critical discount factor which has to be surpassed for collusion to be an equilibrium behavior. With grim trigger strategies and δ denoting the discount factor, full collusion is sustainable as long as

$$\frac{1}{1-\delta}\pi_c^{Lin} \geqslant \pi_d^{Lin} + \frac{\delta}{1-\delta}\pi_{CN}^{Lin} \ ,$$

where the LHS is the present value of profits from colluding while the RHS gives the present value of profits from deviation. The critical discount factor, $\bar{\delta}^{Lin}$, for which an insider is indifferent between staying in the cartel and deviating, results as:

$$\bar{\delta}^{Lin} = \frac{\pi_d^{Lin} - \pi_c^{Lin}}{\pi_d^{Lin} - \pi_{CN}^{Lin}} = \frac{(\pi_d^{Lin} - \pi_c^{Lin})}{(\pi_d^{Lin} - \pi_c^{Lin}) + (\pi_c^{Lin} - \pi_{CN}^{Lin})} = \frac{(n+1)^2}{6n + n^2 + 1}.$$

Clearly, the critical discount factor does not depend on γ, and we establish the following:

Proposition 1 *In a model of linear demand, if symmetric firms compete in quantities and expected harm is proportional to output, the liability sharing rule has no impact on the sustainability of collusion.*

Proposition 1 delivers a benchmark result for the model with linear costs and demand. To better seize the intuition for the result, recall that the individual profit of a firm within the cartel, π_c^{Lin}, is independent of the weight γ. In contrast, the liability system impacts both the gain from deviation $(\pi_d^{Lin} - \pi_c^{Lin})$ and the punishment incurred in future periods after deviation $(\pi_c^{Lin} - \pi_{CN}^{Lin})$. On the one hand, a shift towards market share-liability decreases the gain from deviation as the deviating firm has to bear a higher share of the additional harm due to its output expansion. On the other hand, a higher weight on market-share liability increases Cournot profits, which implies lower punishment in case of defection. The latter effect is due to the harm externality between firms becoming less severe and making coordination of output in a cartel less gainful. Generally, depending on which effect dominates, a move towards liability sharing according to the MS-rule could lead to an increase as well as a decrease in the critical discount factor. With linear cost and demand, however, we find that the sustainability of collusion is independent of the liability sharing system as the relative lower gain from deviation is mirrored by an equal relative reduction in punishment. This is to be put down to the full linearity of the setting we consider, and may no longer hold when we consider nonlinearities within the model which we do in the next sections.

The result described in Proposition 1 is comparable to the finding in Friehe (2014). Recall that for linear but firm-specific harm, Friehe (2014) established a neutrality result regarding the application of negligence or strict liability. Thus, Proposition 1 qualitatively extends this result to the analysis of liability sharing rules in the case of industry-specific harm.

5 Cumulative Harm Model

In this section we assume that the harm inflicted upon third parties increases more-than-proportionally with aggregate output. A simple way to obtain this effect is to use the quadratic damage function - the upperscript *Cum* applies throughout this section to better distinguish the analysis from the previous case of linear harm. Explicitly, we now use the following damage function: $H(Q) = \frac{h^{Cum}}{2} \times Q^2$. As before, we start by determining the non-cooperative outcome before going on to analyze cooperative behavior and deviation incentives.

5.1 Non cooperative behavior: market outcome under Cournot-Nash competition

Analogously to the last section, the profit of firm i now writes:

$$\begin{aligned}
\max_{q_i} \pi_i(q_i) &= P(Q)q_i - s_i(\gamma, q_i, Q)H(Q) \\
&= (a - Q)q_i - \left(\gamma \frac{q_i}{Q} + (1-\gamma)\frac{1}{n}\right) \times \frac{h^{Cum}}{2} \times Q^2 \\
&= (a - q_i - Q_{-i})q_i - \left(\gamma + \frac{1-\gamma}{n}\right) \times \frac{h^{Cum}}{2} \times q_c^2 \\
&\quad - \left(\gamma + 2\frac{1-\gamma}{n}\right) \times \frac{h^{Cum}}{2} \times q_i \times Q_{-i} - \frac{1-\gamma}{n} \times \frac{h^{Cum}}{2} \times Q_{-i}^2. \qquad (4)
\end{aligned}$$

The last two lines in (4) again illustrate the two externalities, price- and liability-related, among firms. In comparison to the linear harm case, see (2), the harm liability is more complicated and can no longer be completely separated from the firm's liability cost associated with own output. The competitors' output increases firm i's costs through two channels: first, higher competitor output is like an increase in firm i's fixed costs as long as no pure market-share liability system applies (last term in (4)); in addition higher competitor output increases firm i's liability costs proportional to firm i's output level (second to last term in (4)).

The first order condition for a profit-maximum yields:

$$a - (q_i + Q) = h^{Cum} \left(\frac{2 + \gamma(n-2)}{2n} Q_{-i} + \frac{1 + \gamma(n-1)}{n} q_i \right). \tag{5}$$

The interpretation of this condition mirrors the corresponding condition (3) in the linear harm model. Note however that marginal liability costs are no longer constant, but increasing in both individual and competitors' output. Similar to the case of linear harm, given competitor output, marginal costs increase with a higher weight on the MS rule in apportioning liability.

In the symmetric equilibrium, individual output and profit levels, denoted by q_{CN}^{Cum} and π_{CN}^{Cum} amount to:[16]

$$\begin{aligned} q_{CN}^{Cum} &= \frac{2a}{2(n+1) + h^{Cum}\left(2 + (n-1)\gamma\right)}, \\ \pi_{CN}^{Cum} &= \left(a - n\left(1 + \frac{h^{Cum}}{2}\right) q_{CN}^{Cum}\right) q_{CN}^{Cum}. \end{aligned} \tag{6}$$

Note that positive profits are ensured by the assumption of $h^{Cum} < 2/(n-2)$, introduced in Section 3.

As in the linear harm scenario, individual output decreases in the weight γ due to the accompanying increase in marginal liability costs, which cannot be offset by the drop in competitor output in the symmetric equilibrium. In turn, equilibrium profits increase with a higher weight on the MS rule in apportioning liability:

$$\frac{d\pi_{CN}^{Cum}}{\gamma} = \frac{a\,(n-1)\left(\gamma h^{Cum} - 2h^{Cum} - 2\right)}{2(n+1) + h^{Cum}\left(2 + (n-1)\gamma\right)} \frac{dq_{CN}^{Cum}}{d\gamma} > 0.$$

The intuition is the same as before: as in standard models of competition, the decrease in competitor output due to the shift towards a MS sharing rule results in a higher price in equilibrium.

[16] An interior equilibrium with firms producing different levels of output is not possible since (5) cannot be simultaneously fulfilled for several different levels of individual output.

Again, in our setting with indivisible harm, this effect is reinforced, since the extent of the negative externality due to liability sharing is reduced. This second effect is even more important than in the linear harm model, given the higher elasticity of harm with respect to output as well as the additional effect competitor output has on marginal costs for each single firm. Firms only partially internalize the full marginal cost of harm, and an increase in the weight γ brings marginal costs closer to full marginal costs. This effect is similar to the scenario with a linear harm function, but note that with quadratic harm, the full internalization of marginal harm costs would require $\gamma = n(n-1)$, which is clearly higher than one.[17]

5.2 Cooperative behavior and sustainability of collusion

Repeating the steps from the linear harm scenario, we formulate the cartel's maximization problem as

$$\max_{Q} P(Q)Q - \sum_{i=1}^{n} s_i(\gamma, q_i, Q) \times H(Q) = (a-Q)Q - \frac{h^{Cum}}{2}Q^2,$$

for which the corresponding FOC is

$$a - 2Q = h^{Cum}Q.$$

Note that naturally there is still no influence of the liability sharing rule on the full marginal liability cost, which now increases in the overall output level and again coincides with full marginal expected harm. From the FOC we directly obtain symmetric individual output and profit levels as:

$$\begin{aligned} q_c^{Cum} &= \frac{a}{n\left(h^{Cum}+2\right)}, \qquad (7)\\ \pi_c^{Cum} &= \left(a - n\left(1+\frac{h^{Cum}}{2}\right)q_c^{Cum}\right)q_c^{Cum}. \end{aligned}$$

[17] From (5), in the symmetric equilibrium marginal liability costs amount to $h^{Cum}q_{CN}^{Cum}(1+\gamma/n)$. Marginal harm is given by $h^{Cum}Q_{CN}^{Cum}$.

Again, the all-inclusive cartel assumption leads to the same remark as with linear harm: every firm on the market fully internalizes the price and the liability externality, hence collusion output and collusion profits are independent from the liability sharing rule.

Next we consider individual profit in the event of deviation: a firm in the cartel will deviate by choosing a level of output which maximizes its own profit, considering that all other firms keep on colluding. Thus, the deviant firm's maximization program writes:

$$\begin{aligned}
\max_{q_d} \pi_d(q_d) &= P(q_d + Q_{-d})q_d - s_d(\gamma, s_d, q_d + Q_{-d})H(q_d + Q_{-d}) \\
&= (a - (q_d + Q_{-d}))q_d - \left(\gamma \frac{q_d}{q_d + Q_{-d}} + (1-\gamma)\frac{1}{n}\right)\frac{h^{Cum}}{2}(q_d + Q_{-d})^2,
\end{aligned}$$

where $Q_{-d} = (n-1)q_c^{Cum}$, and in combination with (7) enables us to eventually derive both the deviant firm's output and profit level, q_d^{Cum} and π_d^{Cum}:

$$\begin{aligned}
q_d^{Cum} &= \frac{a\left(2n^2\left(h^{Cum}+2\right) - (n-1)\left(h^{Cum}(\gamma(n-2)+2)+2n\right)\right)}{2n\left(h^{Cum}+2\right)\left(h^{Cum}(\gamma(n-1)+1)+2n\right)}, \\
\pi_d^{Cum} &= \left(a - q_d^{Cum} - (n-1)q_c^{Cum}\right)q_d^{Cum} \\
&\quad - \frac{h^{Cum}}{2}(q_d^{Cum} + (n-1)q_c^{Cum})\left(\gamma q_d^{Cum} + (1-\gamma)\frac{q_d^{Cum} + (n-1)q_c^{Cum}}{n}\right).
\end{aligned}$$

Due to the fact that its marginal costs increase with the weight put on the MS rule for liability sharing, the deviant firm's output decreases in the weight γ.[18] Again, because the cartel output is independent from liability sharing, the only effect of the liability system on the deviant firm's profits is the direct effect due to the change in cost sharing:

$$\frac{d\pi_d^{Cum}}{d\gamma} = \frac{\partial \pi_d^{Cum}}{\partial \gamma} = -\frac{h^{Cum}Q_d^{Cum}}{2}\frac{n-1}{n}\underbrace{(q_d^{Cum} - q_c^{Cum})}_{>0} < 0.$$

Finally, we establish the critical discount factor based on grim trigger strategies in the event

[18]It holds that $\partial^2 \pi_d^{Cum}/\partial q_d^{Cum}\partial\gamma < 0$ and $\partial^2 \pi_d^{Cum}/\partial\left(q_d^{Cum}\right)^2 < 0$ resulting in $\partial q_{d}^{Cum}/\partial\gamma < 0$.

of a deviation:

$$
\begin{aligned}
\bar{\delta}^{Cum} &= \frac{\pi_d^{Cum} - \pi_c^{Cum}}{\pi_d^{Cum} - \pi_{CN}^{Cum}} \\
&= \frac{\left(h^{Cum}(\gamma(n-1)+2)+2(n+1)\right)^2}{(h^{Cum})^2 \begin{pmatrix} (\gamma(n-1)(\gamma(n-1)+8)+8) \\ +4h^{Cum}(\gamma(n-1)(n+3)+4(n+1))+4(n(n+6)+1) \end{pmatrix}}.
\end{aligned}
$$

It holds that $\frac{\partial \bar{\delta}^{Cum}}{\partial \gamma} > 0$, and we have:

Proposition 2 *In a model of linear demand, if symmetric firms compete in quantities and expected harm is quadratic w.r.t. aggregate output, then a higher weight on the MS rule in apportioning liability leads to an increase in the critical discount factor* $\bar{\delta}^{Cum}$*. In this sense, full collusion becomes less stable.*

Proposition 2 states that the sharing rule used by courts to apportion liability does impact cartel sustainability for our scenario with non-linear harm. The scenarii of linear and quadratic harm are similar to the extent that both the relative short-term gain from deviation, $\pi_d^{Cum} - \pi_c^{Cum}$, as well as the long-run loss from punishment, $\pi_c^{Cum} - \pi_{CN}^{Cum}$, are always decreasing in the weight put on the MS-rule. Indeed, just as in the linear scenario, the cartel profit is independent of γ, the deviation profit is decreasing in it, and the stand-alone Cournot profit is increasing with it. However, and in contrast to the linear damage case, the relative changes in the short-term gain and the long-term loss from deviation no longer cancel out one another: the relative decrease in punishment is more pronounced than the relative decrease in the short-term gain from deviation, making the cartel more difficult to sustain.

The explanation lies with the difference in the harm function which, as can be seen by a comparison of (2) and (4), results in differences in a firm's costs. With quadratic harm, each firm's marginal cost is increasing w.r.t. own output. Absent the harm externality among firms (for example if harm were divisible), this would rather argue for the cartel becoming more stable: a convex cost structure makes deviation by increasing own output less attractive, whereas the decrease in punishment is less pronounced given firms' limited incentives for expanding production levels in Cournot competition. The difference between the linear and the quadratic harm scenario therefore cannot be explained by this effect.

Instead, the difference between the linear and the quadratic harm scenario is explained by the different extent of the harm externality between firms. The higher elasticity of harm with respect to output in the quadratic harm scenario implies that the coordination on lower output levels is especially beneficial for the firms, and the more so the lower the weight on the MS-rule. Furthermore, the coordination in the cartel is doubly beneficial, since the implied reduction in output reduces each firm's marginal costs. In contrast, given that the deviation is the best reply to competitors' output which is independent of the liability system, the larger extent of the harm externality is less consequential for the gain from deviation. Consequently, an increase in the weight on the MS-rule for apportioning of liability has a larger bearing on the punishment profits than on the deviation profits. Moving towards the MS-rule, the decrease in the loss in profits during punishment is more pronounced than the relative decrease in the (short-term) gain from deviation. The cartel becomes more difficult to sustain.

Finally, we can compare the critical discount rates between the two scenarii considered, the linear and the cumulative one. For this, it is enough to note that $\bar{\delta}^{Cum}(\gamma = 1) = \bar{\delta}^{Lin}$. Given Proposition 2, this remark implies that $\bar{\delta}^{Cum} \leq \bar{\delta}^{Lin}$ for any $\gamma \in [0, 1]$. In other words, the following holds:

Corollary 1 *In a model of linear demand with symmetric firms competing in quantities, full collusion is easier to sustain in industries where the joint harm inflicted is not proportional but cumulative w.r.t. the total market output.*

This may be relevant for the detection strategy of competition agencies, which need to allocate scarce resources to monitor different markets. Accordingly, competition authorities should focus their detection efforts on markets characterized by cumulative damages, since they are relatively more conducive of collusion as long as $\gamma < 1$. The result points again to the fact that the coordination achieved by firms in a cartel is more important for them in the scenario with cumulative harm which makes the punishment in the event of deviation more severe.

6 Welfare implications

Our model is characterized by both imperfect competition and a harm externality between firms. In this section we discuss the welfare implications of liability allocation and some resulting policy implications.

6.1 Linear harm

Starting with the linear harm model, recall first that the critical discount factor enabling sustainable collusion as well as the market equilibrium with a cartel are independent from the liability sharing rule. Nonetheless, to the extent that the competitive outcome may not coincide with the socially optimal output, the choice of the liability sharing rule is relevant for welfare. In the linear harm model social welfare is given by

$$SW^{Lin} = \int_0^{Q^{Lin}} (a - Q)dQ - h^{Lin}Q^{Lin} = aQ^{Lin} - h^{Lin}Q^{Lin} - \frac{\left(Q^{Lin}\right)^2}{2},$$

and the first-best output is therefore

$$Q^{Lin,*} = a - h^{Lin}.$$

Under the assumption that $a > nh^{Lin}$, introduced in Section 3 and ensuring positive Cournot profit levels even for pure PC-liability sharing, we have that

$$Q^{Lin,*} > Q_{CN}^{Lin} = \frac{n}{n+1}\left(a - h^{Lin}\right) + \frac{n-1}{n+1}h^{Lin}(1-\gamma) > Q_{c}^{Lin} = \frac{a - h^{Lin}}{2}. \tag{8}$$

Cournot output lies in between the cartel and socially optimal output level: firms' market power leads to lower output levels than socially optimal, but the harm externality among firms increases output levels. As described in (8), the effect via the harm externality is proportional to the factor $1-\gamma$, but, with firms earning positive profits in the market equilibrium, it can never dominate the effect from firm's market power. Any increase in the weight of MS-liability sharing reduces the extent of the harm externality between firms, and leads to a shift in Cournot output away from the socially optimal one. With output being always lower than socially optimal in any market setting, the ranking in output levels directly translates into a social welfare ranking.

From a policy perspective, we have that the contraction in output by a cartel is still detrimental to welfare, despite the possible harm externality between firms at the non-cooperative equilibrium. Authorities should fight cartels, but adjustments in the liability sharing rule are no instrument to be used to influence collusion incentives. In the end, with market power preventing a first-best solution, relying on a greater weight for PC-liability sharing can indeed be welfare improving. This can be interpreted as an application of the theory of second-best (see, Lipsey and Lancaster 1956). Given that the distortion in market outcomes due to market power cannot be avoided, allowing for a second imperfection in the form of the harm externality among firms actually leads to a welfare gain.

6.2 Cumulative harm

Turning to the cumulative harm framework, recall that the liability sharing rule does impact the sustainability of collusion: the critical discount factor increases with the weight γ put on MS-liability sharing. This potentially makes room for a trade-off (by means of a particular choice of γ) between reducing the opportunities for collusion and, conversely, reducing the welfare loss resulting from the externality accompanying the firms' output.

With cumulative harm, the social welfare function is given by

$$SW^{Cum} = \int_0^{Q^{Cum}} (a - Q)dQ - \frac{h^{Cum}}{2}(Q^{cum})^2 = aQ^{Cum} - (h^{Cum} + 1)\frac{\left(Q^{Cum}\right)^2}{2},$$

where Q^{Cum} is total industry output and first-best output necessarily surpasses cartel output:

$$Q^{Cum,*} = \frac{a}{h^{Cum} + 1} > Q_c^{Cum} = \frac{a}{h^{Cum} + 2}.$$

In contrast to the linear harm scenario, the harm externality among firms with cumulative harm is more pronounced, and the full marginal cost internalization would require a weight on the MS-rule of more than one, $\gamma > 1$. As a result, and in contrast to the case of linear harm, the Cournot equilibrium output, which is higher than the cartel output, may even exceed the first-best output level $Q^{Cum,*}$. This is more likely for higher levels of harm and may even be true for a liability system relying exclusively on MS-liability, i.e. $\gamma = 1$. More precisely, from (6) we have that $Q_{CN}^{Cum} = nq_{CN}^{Cum} > Q^{Cum,*}$ if

$$(2 - \gamma)h^{Cum} > \frac{2}{n - 1}.$$

Our requirement of $h^{Cum} < 2/(n - 2)$ to guarantee positive profit levels does not exclude the above condition to be fulfilled.

When Cournot output does surpass the first-best value, theoretically a cartel could result in a higher welfare level. However, in our setting, and requiring positive profit levels for every liability system, this is not possible for more than two firms in the market.[19] Consequently, we

[19] With only two firms in the market, harm levels could be very high, resulting in severe welfare consequences from too high output levels. With more than two firms, the requirement of positive profit levels instills an upper

can finally distinguish three possible situations:

(i) For low levels of harm ($h^{Cum} < 1/(n-1)$), Cournot firms underproduce for every value of $\gamma \in [0,1]$: therefore reducing the weight on the MS-rule in liability sharing brings the equilibrium output level closer to the first best - in other words, reducing γ also reduces the welfare loss. This would go against the policy recommendation of choosing a higher weight γ in order to reduce the opportunities for collusion though.

(ii) For high levels of harm ($h^{Cum} > 2/(n-1)$), Cournot firms overproduce: as a result, a shift towards MS-liability, i.e. an increase in γ, reduces the welfare loss with Cournot competition. Hence this case strengthens the argument of enforcing a liability sharing rule based on the MS apportioning, as this would also reduce opportunities for collusion.

(iii) For intermediate levels of harm ($1/(n-1) \leq h^{Cum} \leq 2/n-1$), there exists an intermediate value of the weight put on the MS-rule $\gamma^* \in [0,1]$ that aligns Cournot output with the first-best output. In this case, the relevant range for the optimal choice of the weight γ is $\gamma \geq \gamma^*$. Fighting collusion incentives by choosing a weight for the MS-rule that is actually higher than the level γ^* has to be traded-off against increasing the welfare loss in the non-cooperative equilibrium.

7 Extensions and robustness checks

In what follows we undertake several robustness checks of our results, by varying some central assumptions. The detailed derivation for all mathematical results in this section is provided in the Technical Appendix.

7.1 Bertrand competition

In our main analysis, we consider Cournot competition with firms choosing quantities. In this section, we investigate Bertrand competition as the main alternative model of competition and

bound on the harm level, which negates the possibility of the cartel being welfare superior to Cournot competition.

let firms set prices. With perfectly homogeneous products and total demand served by the firms that charge the lowest price, we show that the neutrality result survives for linear harm and even extends to the case of cumulative harm.

We consider standard Bertrand competition. Only firms charging the lowest price are active and share total demand $Q = a - P$ in equal proportions. Denoting by P_{-i} the lowest price charged by competitors and by k the number of competitors charging this price, we obtain the profits for firm i charging price p_i as

$$\pi_i = \begin{cases} 0 & p_i > P_{-i} \\ p_i \frac{a-p_i}{k+1} - \frac{H(a-p_i)}{k+1} \quad \text{if} & p_i = P_{-i} \\ p_i(a-p_i) - H(a-p_i) & p_i < P_{-i} \end{cases}$$

for either the linear or the cumulative harm case. Note that symmetric allocation of demand among firms charging the lowest price implies that profits are independent of the liability sharing rule. Price competition will result in at least two firms charging a price equal to average costs, i.e.,

$$P^{Lin} = h^{Lin};\ P^{Cum} = \frac{ah^{Cum}}{2 + h^{Cum}}$$

and we obtain zero profits in the competitive equilibrium.

The profit levels in a full cartel are the same as with Cournot competition as it doesn't matter which instrument, price or quantity, is used to determine the joined monopolistic profit maximum. The optimal deviation strategy with price competition is to set a price marginally below the monopoly price resulting in total demand being served by the deviating firm. The deviating firm obtains approximately the full cartel profit, i.e., $\pi_d = n\pi_c - \epsilon$. Since cartel profits do not depend on the liability sharing rule, deviation profits do neither. Accordingly, the critical discount factor is given by $\delta = (n-1)/n$ and independent of the liability sharing rule for both linear and cumulative harm. The result is due to the fact that with Bertrand competition and homogeneous goods neither deviation profits nor profits in the subsequent punishment phase depend on the liability sharing rule.[20]

7.2 Partial collusion with grim trigger strategy

Many functional cartels are not all-inclusive.[21] By considering the case of partial collusion we can examine the likely impact of liability sharing rules in a more realistic context.[22] In particular, a shift towards the MS-rule may now also impact cartel profits as the asymmetry between cartel and fringe firms means that the effective sharing of liability depends on the specific sharing rule applied. In order to check whether this may alter our results in terms of cartel sustainability, we perform the analysis in both scenarii.

We consider two groups of firms: k out of n firms, the insiders, collude, whereas the remaining firms, the outsiders, compete as the competitive fringe and play the Cournot-Nash game against the partial cartel. We assume that both k and n are exogenous but restrict attention to values of k for which a cartel firm's profits are at least as high as profits in simple Cournot competition; otherwise collusion is irrational.[23] To determine output levels, insiders now maximize the sum of insiders' profits:

$$\max_{q_i} \sum_{i=1}^{k} \pi_i = (a-Q)Q_k - \left(\gamma \frac{Q_k}{Q} + (1-\gamma)\frac{k}{n}\right) \times H(Q)$$

[20]The analysis regarding the critical discount factor with homogeneous goods yields standard results as they have been described for example in Motta (2004). The model could be extended to firms offering heterogeneous goods. In this case, with linear demand, we again obtain that the critical discount factor is independent of the liability rule for the linear harm scenario whereas the critical discount factor increases in the weight of the MS-rule in the event of cumulative harm. The findings are therefore similar to the one for Cournot competition in the main analysis.

[21]See Harrington (2006), and also Griffin (1989), who studies 54 well-known international cartels, 53 of which were incomplete.

[22]See Schwalbe (2010) for a brief survey of the theoretical literature on partial cartels.

[23]The profit comparison provides in each scenario the lower bound for the partial cartel size, $\underline{k}^j$, $j = Lin, Cum$. Note that $\underline{k}^{Lin}$ is the same as the minimum size for the endogenous partial cartel model with linear cost of Escriuhela–Villa (2008), and also the same as the minimum threshold for industry concentration that guarantees the profitability of a horizontal merger within a linear Cournot model as identified by Salant et al. (1983). Alternatively, it would be possible to endogenize the size of the partial cartel - see for instance Escrihuela-Villar (2008, 2009), or Chapter 14 in Belleflamme and Peitz, (2010).

with $Q_k = \sum_{i=1}^{k} q_i$. Fringe members still maximize own individual Cournot profit.

Solving for equilibrium quantities, we can calculate the resulting profit levels and find that the profit of a partial cartel member is increasing in the weight on market-share liability γ :

$$\frac{\partial \pi_k^{Lin}}{\partial \gamma} > 0;\ \frac{\partial \pi_k^{Cum}}{\partial \gamma} > 0$$

This is different from the case of full collusion, and due to the presence of fringe firms and the ensuing asymmetry among firms on the market. A shift towards market liability makes fringe firms reduce output levels which benefits cartel firms as the negative externality exerted by fringe firms on cartel members liability is reduced and the equilibrium price increases.

In order to tackle the sustainability of partial cartels based on infinitely repeated interaction, we further compute the profit obtained in case of individual deviation. An insider will deviate by choosing a level of output which maximizes its own profit, considering that all other insiders keep on colluding, and also that the competitive fringe outsider firms still produce non-cooperative outputs. As with a full cartel, a shift towards market share liability makes deviation less attractive due to the necessary expansion in output being associated with higher liability costs. However, contrary to the analysis above, there is also an effect favoring deviation, since with a higher weight on the MS-rule output levels of the other members of the partial cartel and the fringe firms is lower, which results in higher deviation profits.

In sum, the fact that the output levels in the event of partial collusion is still a function of the liability system implies additional effects on gains from deviation as well as the loss from later punishment. The fact that a higher weight on the MS-rule increases cartel profits *per se* implies that punishment becomes harsher with a system closer to market share liability. But at the same time, the reduced output levels in the presence of a cartel *ceteris paribus* make deviation more attractive. Assuming grim trigger strategies, we calculate the critical discount factor and obtain

$$\bar{\delta}_k^{Lin} = \frac{(k-1)(1+n)^2}{4k^2(3+2n) - 4k^3 - (1+n)^2 - 3k(1+n)^2}$$

for the case of linear harm whereas the critical factor $\bar{\delta}_k^{Cum}$ for the cumulative case is a quite complex polynomial function of k, n, γ and h^{Cum}.[24]

Clearly, the neutrality result identified in the case of linear harm and full market collusion still holds in the case of a partial cartel, since the critical discount rate is independent from γ. The fully linear setting (both on the demand and cost sides) still applies, and ensures perfect proportional changes in both the short-term gain $(\pi_d^{Lin} - \pi_k^{Lin})$ and the long-term loss $(\pi_k^{Lin} - \pi_{CN}^{Lin})$ from deviation whenever there is a shift towards the MS-rule. Hence the critical discount factor is independent from γ. Note however that this occurs despite the fact that, in contrast to the full cartel case, the liability system does now affect the profit of an insider.

By the same token, we find that the liability sharing rule does impact the sustainability of partial collusion when the damage is cumulative in aggregate output. More precisely, for a high enough number of firms on the market, a shift towards the MS sharing rule will make partial collusion less stable. When there are enough firms on the market, i.e. $n > 6$ actually, we find that, as with the full cartel, the closer the liability sharing rule is to the MS-rule, the lower the relative gain from deviation $(\pi_d^{Cum} - \pi_k^{Cum})$, but also much lower the long-run loss from the ensuing punishment $(\pi_k^{Cum} - \pi_{CN}^{Cum})$. Thus, the higher γ, the larger the critical discount factor.

However, for $n = 6$ which allows a partial cartel with $k = 5$ to potentially exist, the critical discount factor is indeed decreasing with the weight γ which implies that collusion becomes easier to sustain. With such a limited number of firms in the market the effect of the one fringe firm reducing its output in response to a shift towards the MS-rule is especially pronounced such that the new effects pointed out can eventually override our original finding. Table 1 below illustrates this case with an example. Note that given the opposing overall effects, the absolute

[24]It can be shown that the larger the partial cartel, the easier it is to sustain (i.e. $\frac{\partial \bar{\delta}_k^j}{\partial k} < 0, j = Lin, Cum$). This is actually a confirmation of the result put forward in Escrihuela-Villar and Guillén (2011).

change in the critical discount factor is rather limited.

Numerical values: $a = 15$, $h^{Cum} = 0.2$, $n = 6$, $k = 5$					
γ	0	0.25	0.5	0.75	1
π_{CN}^{Cum}	2.60417	3.04022	3.44579	3.82315	4.1744
π_{k}^{Cum}	3.10547	3.50618	3.87828	4.22402	4.54545
π_{d}^{Cum}	8.08498	8.11225	8.13729	8.16039	8.18182
$\pi_{d}^{Cum} - \pi_{k}^{Cum}$	4.97951	4.60607	4.25901	3.93637	3.63636
$\pi_{d}^{Cum} - \pi_{CN}^{Cum}$	5.48081	5.07203	4.69150	4.33724	4.00742
$\overline{\delta}_{k}^{Cum}$	0.90854	0.90813	0.90782	0.90758	0.90741
$\partial\overline{\delta}_{k}^{Cum}/\partial\gamma$	-.00180	-.00143	-.00111	-.00081	-.00055

Table 1: Partial cartel with decreasing discount factor for $n = 6$, $k = 5$ (limit case)

7.3 Full collusion with optimal punishment strategy

In this section we consider an alternative way to sustain cooperation among industry firms as a subgame-perfect Nash-equilibrium (SPNE) of the infinitely repeated game: namely, optimal punishment strategies (Abreu, 1986). These are more severe punishments, whose credibility (i.e. firms optimally enforce them) is ensured by shortening the punishment phase and rewarding the firms that punish the deviator by eventually returning to the collusive outcome.

The optimal punishment strategies that we consider below have the punishment for deviation occur for only one period.[25] The collusion game begins with every firm producing the collusive output, and they continue to do so as long as collusion is observed in all previous periods. In case of an individual deviation at some period, all firms produce the punishment output (denoted $\widehat{q}$) at the next period, and return to the collusive output afterwards. If a firm plays a different quantity than the punishment output during the punishment stage, punishment output is also applied in the following period. This two-phase output path displays a stick-and-carrot pattern, since the profit loss during a period following a deviation is used as a stick, whereas the possibility to return to the higher collusive profit after just one period represents the carrot.

[25]Here we closely follow the formal analysis of Belleflamme and Peitz 2010 (chapter 14).

In this framework, collusion will be sustainable for a high enough discount factor, and is supported through an optimal punishment output. The latter maximizes the scope for collusion by minimizing the deviating firm's profit under the constraint that firms do not deviate from enforcing the punishment. The critical discount factor ensuring that full collusion is a SPNE of the infinitely repeated game will be identified by checking that both the punishment/stick stage and the collusion/carrot phase are deviation-proof.

The no-deviation condition at the punishment stage is actually a credibility condition: the cost of deviating from enforcing the punishment (i.e. the cost inflicted by the one-period delay of the return to collusion) must be larger than the immediate gain from deviating from the enforcement of punishment. In our symmetric quantity competition framework, this can be written as follows:

$$\delta\left(\pi_c^j - \pi^j(\widehat{q})\right) \geq \pi_d^j(\widehat{q}) - \pi^j(\widehat{q}), \tag{Credibility cond.}$$

where $j = Lin, Cum$, π_c^j is the individual collusive profit, $\pi^j(\widehat{q})$ is the symmetric individual profit when every firm plays the punishment output, and $\pi_d^j(\widehat{q})$ is the individual deviation profit obtained by reacting optimally to all other firms playing the punishment output. The optimal punishment output with full collusion is determined by solving the credibility condition as equality.

The second condition is the no-deviation condition at the collusion stage, and is given by the following inequality:

$$\delta\left(\pi_c^j - \pi^j(\widehat{q})\right) \geq \pi_d^j(q_c^j) - \pi^j(\widehat{q}), \tag{Sustainability cond.}$$

where q_c^j is the individual collusive output, $\pi_d^j(q_c^j)$ is the individual deviation profit obtained by reacting optimally to all other firms playing the collusive output, and $j = Lin, Cum$. This inequality provides the sustainability of the cooperation among firms, which requires that the cost of a deviation from the collusive output, to be incurred at the next period, must be larger than the immediate gain from deviation at the current period. Plugging into this inequality the optimal punishment output determined above yields the critical discount factor with optimal punishment strategies.

Note that given symmetric collusion and punishment output levels, the liability sharing rule affects neither collusion profits π_c^j nor punishment profits $\pi^j(\widehat{q})$ directly. However, there is an indirect effect. Punishment implies high output levels. In consequence, optimal deviation from punishment is actually associated with a lower output level than competitor output. This implies that a shift in liability towards market share allocation makes deviating at the punishment stage more attractive. This requires lower output levels by all firms at the punishment stage to make punishment credible again. Since output levels at the punishment stage exceed the profit-maximizing ones, the decrease in output implies a higher profit level at the punishment stage; consequently, as with grim trigger strategies, a shift towards market share liability leads to punishment becoming less severe. Likewise, since deviation from the cartel agreement implies choosing a higher level of output, deviation profits $\pi_d^j(q_c^j)$ decrease in the weight on the MS-rule for liability allocation, just as in the model with grim-trigger strategies.

For the linear harm scenario, we obtain for the critical discount factor

$$\overline{\delta}_{opt-punish}^{Lin} = \frac{n^2 + 2n + 1}{16n},$$

which clearly does not depend on the choice of a particular liability sharing rule. As a result, the neutrality result still holds. In contrast, for the cumulative harm model we obtain that

$$\frac{\partial \overline{\delta}_{opt-punish}^{Cum}}{\partial \gamma} = \frac{h^{Cum}(n-1)}{16(2 + h^{Cum})} \times \left(1 - \frac{(2 + h^{Cum})^2}{(h^{Cum} + 2n + h^{Cum}(n-1)\gamma)^2}\right) > 0,$$

indicating that the likelihood of collusion is still decreasing with the weight put on the MS-rule.

The qualitative results are similar to the grim trigger strategies case. The novel element of the analysis of optimal punishment is that instead of the Cournot profit level, the profit levels in the punishment stage have to be compared to deviation and cartel profits. As shown above, like Cournot profits, the punishment profits are increasing in the weight on the MS-rule for liability sharing, because the higher weight γ makes it more attractive to deviate from the agreed-on punishment strategy. In this respect, changes in the liability system are still more important with cumulative harm than with linear harm: as it was the case for the change in Cournot profits, the shift in liability bears more heavily on the relative reduction in the loss from punishment for the cumulative harm model than for the linear harm model.

7.4 Non-linear demand

Alternative demand specifications may be considered instead of the widely used linear case. We do so in this section, so as to better assess to what extent our results hinge on the introduction of nonlinearities into the modelling framework.

We test two alternative inverse demand functions: $P(Q) = a - Q^2$, i.e. a concave case, and $P(Q) = a - Q^{\frac{1}{2}}$, i.e. a convex one. For both we use the linear damage function and perform the full cartel analysis with grim trigger strategies We find that a shift towards the MS-rule lowers the likelihood of collusion when demand is convex, but increases it when demand is concave. These results confirm that the structural features, i.e. the cost and demand specifications, are actually crucial for the impact of liability sharing rules on the likelihood of collusive market behavior. In particular, the neutrality of the apportioning rule w.r.t. the likelihood of market collusion does not survive the introduction of non-linearity into the framework, be it on the cost or on the demand side.

The non-neutrality results stem again from the different-sized impact of an increase in the weight placed on the MS criterion (γ) for the relative gain from deviation and for the relative loss from the ensuing punishment respectively, since both the deviation profit and the Cournot/punishment profit qualitatively behave as in the main analysis.[26] Given the propor-

[26] With a higher weight γ on market share liability, deviation from the cartel agreement by increasing own

tional damage/ linear cost assumption used in this section, the harm externality that firms exert on each another does not have a crucial bearing on the relative profitability of deviation as opposed to that of sustaining collusion. This then leaves the price-related externality as the alternative explanation for the non-neutrality results. In order to explain the relative larger bearing of the price-related externality on the non-cooperative profits as compared to that on the deviation profit, we examine the industry-wide cost pass-through, i.e. the degree to which a given change in cost causes a change in the equilibrium price. The cost pass-through generally depends on the curvature of demand and also of cost functions,[27] and for Cournot competition with homogenous products it will be relatively large if inverse demand is convex (the curve becoming steeper as output decreases), and relatively small if inverse demand is concave (the curve becoming flatter in this case as output decreases), all else being equal.[28]

As compared with the linear demand case, the convex demand implies a larger pass-through:[29] consequently, the increase in marginal cost due to a higher weight γ will lead to a more pronounced increase in the market price, which requires a more substantial drop in aggregate output. This will induce a relatively stronger increase in Cournot profits. Hence, a move towards the MS-rule reduces the loss in profits due to punishment relatively more than it reduces the gain from deviation, thus making collusion less likely.

With concave demand, the pass-through is lower than with linear demand.[30] This lower price increase will require a lower aggregate output contraction. As a consequence, this smaller output reduction results in a relatively lower increase in Cournot profits compared with the fully

output still becomes less attractive. Due to Cournot profits still being increasing with γ, the punishment is also less severe.

[27]Seade (1985) provides important early insight into pass-through in the case of Cournot competition. See Weyl and Fabinger (2013) for a general expression for industry-wide cost pass-through with symmetric firms, which nests perfect competition, monopoly, homogeneous goods Cournot competition, and differentiated product Bertrand competition.

[28]In our context with constant marginal costs, the industry-wide pass-through is $\frac{n}{(n+1)+Q\frac{P''(Q)}{P'(Q)}}$.

[29]With constant marginal cost and our convex demand, the industry-wide cost pass-through is $\frac{n}{n+\frac{1}{2}}$. Note that this exceeds the pass-through in case both demand and cost are linear, which equals $\frac{n}{n+1}$.

[30]The industry-wide pass-through equals $\frac{n}{n+2}$ in our constant-marginal cost setting for the concave demand considered.

linear case. Thus, a move towards the MS-rule will induce a decrease in the loss in profits due to punishment which is less pronounced than the relative decrease in the (short-term) gain from deviation: the cartel becomes easier to sustain.[31]

8 Conclusion

In many circumstances, liability sharing rules are applied to apportion liability among several jointly liable tortfeasors. Distinguishing per capita and market share apportionment, this paper studies how the design of the sharing rule impacts on the sustainability of collusion in the market. With linear demand and quantity competition, we show that the type of harm stemming from the industry output is crucial for the impact of different liability sharing rules on incentives for collusion. Explicitly, the apportioning rule is neutral w.r.t. the likelihood of market collusion when harm increases proportionally to aggregate output. In contrast, with cumulative harm (i.e. the damage increases more-than-proportionally w.r.t. aggregate output), the sustainability of collusion does depend on the liability system used: the more biased the apportioning rule in favor of an allocation based on market shares, the less likely collusive behavior. Considering several extensions we *inter alia* found that the results transfer to the analysis of optimal punishment instead of grim-trigger strategies but that the introduction of further non-linearieties like non-linear demand function challenges the results.

[31] Note that the assumption of concave demand can also result in a reversal of the conclusion obtained with cumulative harm, i.e. the fact that the critical discount factor was found to be increasing in γ. With concave demand and cumulative harm, the opposite holds if the number of firms is low enough. The intuition relies on the impact of the intensity of competition on the pass-through rate: the latter is increasing in the number of firms. Hence for a low number of firms the demand-externality effect is more important and can therefore overturn that from the harm externality. For a larger number of firms the impact of the harm externality becomes dominant, and in this case the result of the main analysis (i.e. linear demand and cumulative harm) holds again: the critical discount factor increases with γ.

References

[1] Abreu, D. (1986) "Extremal Equilibria of Oligopolistic Supergames." Journal of Economic Theory 39 (1), 191-225.

[2] Belleflamme, P. and M. Peitz (2010) *Industrial Organization: Markets and Strategies*, Cambridge University Press

[3] Charreire, M., Langlais, E. (2017) "Imperfect competition, joint harm and market share liability", mimeo.

[4] Daughety, A.F., Reinganum, J.F. (2014) "Cumulative Harm and Resilient Liability Rules for Product Markets". Journal of Law, Economics and Organization 30(2), 371-400.

[5] Escriuhela–Villar, M. (2008) "On Endogenous Cartel Size under Tacit Collusion". Investigaciones Economicas, 32, 325-338.

[6] Escriuhela–Villar, M. (2009) "A note on cartel stability and endogenous sequencing with tacit collusion". Journal of Economics, 96, 137-147.

[7] Escrihuela-Villar, M. and J. Guillén (2011) "On Collusion and Industry Size". Annals of Economics and Finance, 12-1, 31-40.

[8] Feess, E. and U. Hege (1998) "Efficient Liability Rules for Multi-Party Accidents With Moral Hazard". Journal of Institutional and Theoretical Economics 154(2), 422-450.

[9] Feuerstein, S. (2005) "Collusion in industrial economics - a survey". Journal of Industry, Competition and Trade, 5, 163-198.

[10] Friedman, J. W. (1971) "A Non-cooperative Equilibrium for Supergames". Review of Economic Studies 28, 1-12.

[11] Friehe, T. (2014) "Tacit collusion and liability rules". European Journal of Law and Economics 38(3), 453-469.

[12] Golbe, L.D. and White, J. L. (2000) "Market Share Liability and Its Alternatives". SSRN Electronic Journal. 10.2139/ssrn.209809.

[13] Gluttel, E., Leshem, S. (2013) "Optimal allocation of joint liability". WP available at https://editorialexpress.com/cgi-bin/conference/download.cgi?db_name=ALEA2013&paper_id=209.

[14] Gluttel, E., Leshem, S. (2014) "The Uneasy Case of Injurers' Liability" Theorical Inquiries in Law 15 (2), 261 - 292.

[15] Griffin, J.M. (1989) "Previous Cartel Experience: Any Lessons for OPEC?" In L.Theory and Practice: An Eclectic Approach. Dordrecht: Kluwer Academic Publishers.

[16] Harrington, J.E. Jr.(2006) "How Do Cartels Operate?" Foundations and Trends in Microeconomics, Vol. 2, Issue 1.

[17] Hamilton, S.F., Sunding, D.L. (2000) "Product Liability, Entry Incentives and Market Structure". International Review of Law and Economics 20 (2), 269-283.

[18] Haufler, A., Schjelderup, G. (2004) "Tacit collusion and international commodity taxation". Journal of Public Economics, 88, 591–617.

[19] Kaplow, L., Shapiro, C. (2007). Antitrust. In: A. M. Polinsky, & S. Shavell (Eds.), Handbook of law and economics. Amsterdam: North-Holland.

[20] Kornhauser, L.A., Revesz, R.L. (1989) "Sharing liability among multiple tortfeasors". Yale Law Journal, 98, 831–884.

[21] Kreps and Scheinkman (1983) "Quantity Precommitment and Bertrand Competition Yields Cournot Outcome". The Bell Journal of Economics 14, 326-337.

[22] Landes, W.S., Posner R.A. (1987). The economic structure of tort law. Cambridge MA: Harvard University Press.

[23] Leshem, S. (2017) "Allocation of Liability : On the Efficiency of Composite Sharing Rules". Journal of Institutional and Theorical Economics , 25-43.

[24] Li, Y. (2018) "Apportioning indivisible damage and strategic diffusion of pollution abatement technology". Journal of Economics https://doi.org/10.1007/s00712-018-0610-8, forthcoming.

[25] Lipsey, R. G., Lancaster, K. (1956) "The general theory of second best". Review of Economic Studies 24, 11-32.

[26] Marino, A. M. (1991) "Market Share Liability and Economic Efficiency." Southern Economic Journal 58, 667-675.

[27] Motta, M. (2004) *Competition policy - theory and practice*, Cambridge University Press.

[28] Ranouil, M. (2014) *Les recours entre coobligés.* IRJS Editions, Collection: Bibliothèque de l'Institut de Recherche Juridique de la Sorbonne - André Tunc, ISBN : 978-2-919211-34-0.

[29] Salant, S., Switzer, S., Reynolds, R. (1983) "The effects of an exogenous change in industry structure on Cournot-Nash equilibrium". Quarterly Journal of Economics, 98, pp. 185–99.

[30] Schindler, D., Schjelderup, G. (2009) "Harmonization of corporate tax systems and its effect on collusive behavior". Journal of Public Economic Theory 11 (4), 599-621.

[31] Schinkel, M. P., Spiegel, Y. (2017) "Can collusion promote sustainable consumption and production?". International Journal of Industrial Organization 53, 371-398.

[32] Schwalbe, U. (2010) "Welfare Effects of Partial CartelsSome results from the theoretical literature". Paper prepared for the Economic Ex-

pert Forum on Quantification of Damages, Brussels, retrieved at http://ec.europa.eu/competition/antitrust/actionsdamages/schwalbe.pdf

[33] Seade, J. (1985) ”Profitable Cost Increases and the Shifting of Taxation : Equilibrium Response of Markets in Oligopoly”. The Warwick Economics Research Paper Series (TWERPS) 260, University of Warwick, Department of Economics.

[34] Shavell, S. (1987). Economic analysis of accident law. Cambridge MA: Harvard University Press.

[35] Sheiner, N. (1978) ”DES and a Proposed Theory of Enterprise Liability.” Fordham Law Review 46, 963-1007.

[36] Weyl, G. and M. Fabinger (2013) ”Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition”. Journal of Political Economy 121 (3), 528-583.