Learning, house prices and macro-financial linkages

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Document de Travail
Working Paper
2020-10

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May 2020

Abstract: In the US, the linkages between the housing market, the credit market and the real sector have been striking in the past decades. To explain these linkages, I develop a small-scale DSGE model in which agents update non-rational beliefs about future house price growth, in accord with recent survey data evidence. Conditional on subjective house price beliefs, expectations are model-consistent. In the model with non-rational expectations, both standard productivity shocks and shocks in the credit sector can generate endogenously persistent booms in house prices. Long-lasting excess volatility in house prices, in turn, affects the financial sector (because housing assets serve as collateral for household and entrepreneurial debt), and propagates to the real sector. This amplification and propagation mechanism improves the ability of the model to explain empirical puzzles in the US housing market and to explain the macro-financial linkages during 1985-2019. The learning model can also replicate the predictability of forecast errors evidenced in survey data.

JEL classification: D83, D84, E32, E44, G12.

Keywords: Housing booms, Financial Accelerator, Business Cycles, Non-rational Expectations, Learning.

*pgandre@parisnanterre.fr, Université Paris Nanterre and EconomiX, CNRS UMR 7235, France. I am very grateful to Fabian Winkler for detailed explanation on the implementation of the approximation method for the learning equilibrium and for feedback. I also thank Zouhair Ait Benhamou, Sophie Béreau, Camille Cornand, Frédéric Dufourt, Valérie Mignon, Fabien Tripier, Mirko Wiederholt and participants at the 24th CEF Conference at Università Cattolica del Sacro Cuore, at the 32nd EEA Congress at the University of Lisbon, at the 34th Symposium on Money, Banking and Finance at the Université Paris Nanterre, at the ADRES Doctoral Conference at the Toulouse School of Economics, at the 21st T2M Conference at the Lisbon Catolica University and at the EconomiX lunch seminar for helpful comments.

1 Introduction

The subprime financial crisis, which started in the US mortgage credit market in 2007 following a sudden decrease in house prices and finally propagated to the real sector, has revealed the strong linkages between the housing sector, the credit market and the real sector. Such macro-financial linkages have been accentuated since the early 2000s because of the fast development of mortgage debt contracts, in which house prices determined how much agents could borrow. Therefore, it seems crucial to explain the dynamics of house prices to understand the credit and business cycles over the last decades. However, as for other assets, such as stocks, patterns of excess volatility in house prices relative to fundamentals have been apparent.

As stated by Piazzesi and Schneider (2016), "a major outstanding puzzle is the volatility of house prices – including but not only over the recent boom-bust episode. Rational expectations models to date cannot account for house price volatility – they inevitably run into "volatility puzzles" for housing much like for other assets. Postulating latent "housing preference shocks" helps understand how models work when prices move a lot, but is ultimately not a satisfactory foundation for policy analysis. Moreover, from model calculations as well as survey evidence, we now know that details of expectation formation by households – and possibly lenders and developers – play a key role" (p. 5).

To simultaneously explain several puzzling features of the dynamics of house prices and expectations, along with the dynamics of credit and standard macroeconomic variables in the US since the mid-1980s, this paper presents a stylized small-scale DSGE model in the spirit of Iacoviello (2005). In this setting, impatient households and entrepreneurs can borrow from patient households against a fraction of the expected value of their real-estate assets. The model features standard real frictions (capital adjustment costs) and financial frictions (credit market frictions) related to the asymmetry of information between lenders and borrowers.[1] The specificity of the model mechanism is that it relies on non-rational expectations about future housing returns. This assumption is motivated by two distinct recent pieces of evidence. First, survey data about expected house price growth have recently developed (see the Michigan Survey of Consumers), thereby

[1]In addition, the housing supply is fixed, which can be interpreted as an additional rigidity in the housing market.

filling a gap in the understanding of the formation of house price expectations. Similar to what has been long established for macroeconomic variables and other assets, recent survey data reveal that US households make systematic forecast errors when predicting future house price growth and that these forecast errors are predictable. This means that they are correlated with variables that are observable at the time of the forecast. These features obviously conflict with the implications of the rational expectations assumption. Second, the recent theoretical literature shows that modeling non-rational expectations about future house prices enables a better explanation of the empirical behavior of house prices (e.g. Granziera and Kozicki (2015) and Glaeser and Nathanson (2017)).

In this paper, non-rational expectations about future house price growth relate to the assumption that agents do not understand how house prices form endogenously through market clearing. Agents, instead, believe that house price growth is exogenous and equals the sum of two components: a persistent time-varying component and a transitory component. Because agents cannot observe the two components separately, the agents learn over time the unobservable persistent component of house price growth, by using past data. To introduce the smallest degree of freedom into the model and the smallest deviation from the rational expectations assumption, I follow Winkler (2019) in assuming that, conditional on house price expectations, expectations of all variables are rational. This assumption does not imply that expectations of other variables are fully rational (such an implication would not accord with survey data), as errors in the estimation of house prices propagate to other variables. However, expectations of other variables are model consistent. Under this assumption, the solving method is close to the standard perturbation method used for models with rational expectations.

In contrast to current literature on learning about future house prices, the learning mechanism is embedded into a production economy. Thus, both the asset price and the business cycles implications of the learning mechanism can be investigated. In addition to standard total factor productivity shocks, the model features credit sector shocks, namely, lenders' intertemporal shocks that mimic sudden variations in the willingness to lend independently of borrowers' ability to repay the debt. Indeed, as emphasized by Iacoviello (2015), productivity shocks, which are the traditional drivers of business cycles in most DSGE models, are unlikely to fully explain the Great

Moderation dynamics, the 2007-2008 financial crisis and its aftermath, when business cycles have been mainly financial.

Our approach yields the following main results. First, the learning model explains several puzzling features of housing market dynamics. The model is able to generate endogenously persistent booms in house prices in response to small macroeconomic and credit sector shocks. In addition, in contrast with the rational expectations version of the model, the learning model replicates the strong autocorrelation in house prices and the positive sign of the autocorrelation in housing returns observed in US data during 1985-2019. These results arise because learning about future house prices generates a strong feedback mechanism between house prices and beliefs. When housing returns are expected to be higher, housing demand and, thus, house prices increase. This increase, in turn, makes housing returns expectations more optimistic as long as realized returns are higher than expected returns. More optimistic expectations then further fuel the increase in house prices.

Second, the learning model generates an amplified response of credit and macroeconomic variables to shocks. This amplification is made apparent by the fact that the shocks variance that is required to simultaneously replicate the volatility in house prices and in additional variables observed during 1985-2019 in the US is significantly smaller under learning. This result arises because housing assets serve as collateral in the borrowing sector and are a factor of production and because investment in housing assets enables the transference of money over time. Therefore, the dynamics of forecast errors and beliefs concerning housing returns, which are slow-moving state variables in the learning model, amplify the effects of shocks on the other model variables. Third, the learning model replicates the predictability of forecast errors in both housing returns and macroeconomic variables, in accord with survey data.

The remainder of the paper is organized as follows. Section 2 presents the related literature. Section 3 describes the baseline model with collateralized borrowing constraints, credit frictions and capital adjustment costs. Section 4 explains the formation of beliefs about future house prices and describes the equilibrium under learning. Section 5 displays the simulated results obtained in the learning model, compares them to those of the rational expectations model and discusses how they can help explain features of the joint dynamics of house prices, credit, real variables and expectations since the mid-1980s in the US. Finally, Section 6 concludes.

2 Related literature

This paper is related to two strands of the literature that have for the most part remained separate. The first strand relates to the relaxation of the rational expectations assumption in standard asset pricing models, whereas the second strand relates to the role of the housing market in the business cycle. The first strand aims at modelling expectations that are more consistent with the results of survey data and at better replicating house price dynamics. Thus, Gelain and Lansing (2014) and Granziera and Kozicki (2015) explain house price volatility by introducing intuitive, but not microfounded, extrapolative models of house price expectations.

By contrast, I directly follow Adam et al. (2012), Adam et al. (2016), Caines (2016) and Adam et al. (2017) in specifying the perceived law of motion of asset prices. Expectations of future house price growth are then microfounded, relying on Bayesian updating. These papers model exchange economies in which consumption and output streams are exogenous. Therefore, they cannot account for the impact of asset prices on the business cycle. An exception is the recent paper by Winkler (2019). In contrast to this paper, the model developed in the present paper focuses on house prices rather than on stock prices and includes credit sector shocks and household debt to account for the specificites of the recent period in the US.

The literature on the role of the housing market in the business cycle is the second strand of literature this paper relates to. Several papers investigate the linkages between asset markets, the credit market and the real sector in production economies with financial frictions, where consumption and production are endogenous, featuring a well-known financial accelerator mechanism (Kiyotaki and Moore (1997), Bernanke et al. (1999)). Most papers featuring a financial accelerator mechanism have focused on firms' borrowing but more recent papers have also modeled financial accelerator dynamics related to household borrowing (Aoki et al. (2004), Iacoviello (2005), Iacoviello (2015)). This mechanism seems consistent with the role of household debt in the subprime financial crisis.

However, in most of the papers that feature housing assets as collateral, at least part of the dynamics of house prices is driven by exogenous changes directly related to the housing sector. The most common approach consists in introducing housing price shocks, housing demand shocks, or housing technology shocks (Iacoviello (2005), Darracq-Paries and Notarpietro (2008), Iacoviello and Neri (2010)). Such ingredients are not very helpful in understanding house price dynamics, as the latter thus remain largely exogenous. Other elements of explanations resort to monetary policy shocks and financial conditions shocks (Aoki et al., 2004), or to non-time separable preferences (Jaccard, 2012). In all cases, this set of explanations, based on standard rational expectations specifications, is difficult to reconcile with survey data about expected future house price growth. Even in papers modelling non-rational expectations and learning about financial variables such as leverage, house price shocks are introduced to replicate the boom-bust pattern observed in the 2000s (Pintus and Suda (2019)).

By contrast, I only introduce discount factor shocks in the lending sector (in addition to standard productivity shocks), such that the dynamics of the housing market are initially driven by shocks related to the credit market and not directly by shocks related to the housing market. The response of house prices to exogenous shocks is thus more endogenous, less close to the shock, and more consistent with patterns observed during the last boom-and-bust episode in the US housing market. Indeed, the steep increase in house prices that started in 2001 in the US arised as a consequence of relaxed financial conditions and the fast development of mortgage credit (e.g. Mian and Sufi (2009), Demyanyk and Van Hemert (2011), Dell'Arricia et al. (2012)).[2] Therefore, by explaining house price dynamics more endogenously, it is possible to investigate the feedback transmission channels between the credit sector, the housing market and the real sector. In the next section, I turn to the description of the baseline model.

3 The baseline model

The baseline model is close to the extended model presented in Iacoviello (2005), except that it focuses on real and financial frictions. The model features a discrete-time, infinite horizon econ-

[2] The 2011 U.S. Financial Crisis Inquiry Commission Final Report on the Causes of the Financial and Economic Crisis in the United States presents a similar view on the chain of events that triggered the crisis: increased willingness to lend fueled credit and housing demand, and thus fueled a housing boom, which in turn fueled credit. When house prices fell, the mortgage-credit sector then collapsed.

omy with three sectors: lenders in the form of patient households and borrowers in the form of both entrepreneurs and impatient households. The housing stock in the economy is exogenous and normalized to 1. All variables are expressed in units of a single consumption good, which also serves as an investment good.

3.1 Lenders

Following Iacoviello (2005), it is assumed that a set of households displays a high discount factor relative to other households. Those households put more weight on future periods in their intertemporal utility function; they are more patient and thus more willing to postpone consumption and save money through lending. Their preferences take the standard following form:

$$\max E_0 \sum_{t=0}^{\infty} \beta_P^t d_t [\ln(C_{t,P}) + j \ln(H_{t,P}) + \psi \ln(1 - N_{t,P})]. \tag{1}$$

Patient households thus value consumption $C_{t,P}$, housing services provided by real-estate assets $H_{t,P}$ and leisure hours equal to $1 - N_{t,P}$, where $N_{t,P}$ are working hours. Patient households discount future periods with the discount factor β_P, j is the weight allocated to housing services in the utility function and ψ is the weight allocated to leisure. d_t is the discount factor shock, i.e., a time preference shock which follows an autoregressive process in the form of:

$$\ln(d_t) = \rho_d \ln(d_{t-1}) + \varepsilon_{d,t}, \tag{2}$$

where $\rho_d < 1$ and $\varepsilon_{d,t}$ follows a normal distribution with mean zero and variance σ_d. The interpretation of this shock is that time preferences are time varying: patient households can suddenly display more or less preference for current consumption, housing services and leisure. This shock is introduced to mimic a context of higher willingness to lend, independently of borrowers' net worth. An exogenous decrease in the current period discount factor of lenders thus increases the credit supply, independently of borrowers' ability to pay back the debt.

The intertemporal flow of funds constraint of patient households writes as follows:

$$C_{t,P} + q_t H_{t,P} + B_t = w_t N_{t,P} + R_{t-1} B_{t-1} + q_t H_{t-1,P}, \tag{3}$$

where q_t is the price of houses, B_t is the debt held by patient households, R_t is the (gross) interest rate on debt and w_t is the wage. Housing assets are traded in each period.

The inter-temporal first-order conditions with respect to housing, debt and hours worked are standard, except that the preference shock is included:

$$d_t \frac{1}{C_{t,P}} q_t = \beta_P E_t \left[\frac{1}{C_{t+1,P}} q_{t+1} d_{t+1} \right] + j d_t \frac{1}{H_{t,P}}. \tag{4}$$

$$\frac{d_t}{C_{t,P}} = \beta_P E_t \left[\frac{d_{t+1}}{C_{t+1,P}} R_t \right]. \tag{5}$$

$$\frac{w_t}{C_{t,P}} = \frac{\psi}{1 - N_{t,P}}. \tag{6}$$

3.2 Entrepreneurs

Entrepreneurs own the capital stock and the firm and maximize the intertemporal utility of consumption streams:

$$\max E_0 \sum_{t=0}^{\infty} \beta_F^t [\ln(C_{t,F})], \tag{7}$$

subject to the following flow of funds constraint:

$$C_{t,F} + q_t H_{t,F} + R_{t-1} B_{t-1,F} + w_t N_t + I_t = Y_t + B_{t,F} + q_t H_{t-1,F}, \tag{8}$$

where β_F is the entrepreneurs' discount factor, $C_{t,F}$ is consumption, $H_{t,F}$ represents real-estate holdings, $B_{t,F}$ is debt, N_t is labor demand, I_t is investment and Y_t is output.[3] The production function is a typical Cobb-Douglas production function, with three factors of production: labor,

[3]Entrepreneurs' consumption being residual income after investment, labor costs, housing and interest payments have been made, the decision problem of entrepreneurs is equivalent to maximizing a concave function of discounted dividends.

capital and housing. Both capital and housing become productive only after one period:

$$Y_t = A_t K_{t-1}^{\alpha} H_{t-1}^{v} N_t^{1-\alpha-v}. \tag{9}$$

Total factor productivity A_t follows a standard $AR(1)$ process in the log:

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_{a,t}, \tag{10}$$

where $\varepsilon_{a,t}$ follows a normal distribution with mean zero and variance σ_a. Adjusting capital too fast is assumed to be costly (notably because installing new machines implies temporarily not running some of the existing machines). Therefore, the capital accumulation equation takes the standard following form under capital adjustment costs (Hayashi, 1982):

$$K_t = I_t + (1-\delta)K_{t-1} - \frac{\phi}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1}, \tag{11}$$

where K_t is the capital stock, δ is the capital depreciation rate and ϕ is a parameter governing the size of the capital adjustment cost.

Entrepreneurs can borrow a limited amount of debt and face a collateralized borrowing constraint in which housing assets play the role of pledgeable assets:

$$B_{t,F} \leq m E_t \left[q_{t+1} \frac{H_{t,F}}{R_t}\right], \tag{12}$$

where the term $E_t[q_{t+1}\frac{H_{t,F}}{R_t}]$ represents expected future asset value and m is the loan-to-value ratio. The lender can recover only some fraction of the pledgeable assets in case of default, due to asymmetry of information between lenders and borrowers, implying that $m < 1$. Even though the entrepreneurs' borrowing constraint is not directly microfounded in the present model, it is both standard (e.g. Kiyotaki and Moore (1997), Iacoviello (2005) and Iacoviello (2015) to mention just a few) and intuitive. Indeed, the borrowing constraint (12) implies that the borrowing capacity of agents depends on the expected future value of their assets, because assets can be seized and sold by the lender in case of default. In addition, the borrowing constraint implies that transaction costs arising when the lender seizes borrowers' assets reduce the final recovery value.

The first-order conditions for firms with respect to labor, debt and real-estate assets write:

$$w_t = \frac{(1-\alpha-v)Y_t}{N_t}, \tag{13}$$

$$\frac{1}{C_{t,F}} = \beta_F E_t \left[\frac{1}{C_{t+1,F}}\right] R_t + \mu_{F,t}, \tag{14}$$

and

$$\frac{q_t}{C_{t,F}} = \beta_F E_t \left[\frac{1}{C_{t+1,F}} \left(q_{t+1} + \frac{vY_{t+1}}{H_{t,F}}\right)\right] + \mu_{F,t} m E_t \left[\frac{q_{t+1}}{R_t}\right], \tag{15}$$

where $\mu_{F,t} \geq 0$ is the Lagrange multiplier associated with the borrowing constraint. The complementary slackness condition writes:

$$\mu_{F,t}\left[B_{t,F} - mE_t\left[q_{t+1}\frac{H_{t,F}}{R_t}\right]\right] = 0. \tag{16}$$

The first-order condition with respect to labor is standard, except that the share of labor in the production function depends not only on the share of capital but also on the share of real-estate assets in the production function.

The Lagrange multiplier $\mu_{F,t}$ associated with the borrowing constraint appears in the previous two equations, which shows that financial frictions act as an inter-temporal wedge in the first-order conditions by comparison to standard first-order conditions. The Lagrange multiplier of the borrowing constraint enters the first-order condition with respect to housing, because buying more of this asset today relaxes the borrowing constraint. House price expectations enter the borrowing constraint, they thus matter for determining the entrepreneurs' demand for credit. As a consequence, learning proves able to generate dynamics that are distinct from rational expectations ones even for similar states of the economy.

Note also that in the non-stochastic steady state, the Lagrange multiplier associated with the borrowing constraint of firms μ_F is equal to $(\frac{\beta_P - \beta_F}{\beta_F})\frac{1}{C_F}$. Therefore, the discount factor of lenders must be strictly higher than the discount factor of borrowers to ensure that the Lagrange multiplier associated with the borrowing constraint is strictly positive, and thus that the borrowing constraint is binding. Therefore, $\beta_F < \beta_P$ is a necessary condition for ensuring that the borrowing constraint is binding in a neighborhood of the steady state.

The combination of the first-order conditions with respect to capital and to investment yields:

$$\frac{1}{C_{t,F}\left(1-\phi(\frac{I_t}{K_{t-1}}-\delta)\right)} = E_t\left[\beta_F \frac{1}{C_{t+1,F}}\left(\frac{\alpha Y_{t+1}}{K_t} + \frac{1}{1-\phi(\frac{I_{t+1}}{K_t}-\delta)}(1-\delta-\frac{\phi}{2}\left(\frac{I_{t+1}}{K_t}-\delta\right)^2 + \phi(\frac{I_{t+1}}{K_t}-\delta)\frac{I_{t+1}}{K_t})\right)\right]. \tag{17}$$

When the capital adjustment cost parameter ϕ is null, this equation reduces to the standard first-order condition with respect to capital.

3.3 Impatient households

The preferences of impatient households are similar to those of patient households except that their time preference rate β_I differs ($\beta_I < \beta_P$). This assumption, which is standard in a borrower-saver model, makes impatient households willing to borrow rather than lend. The maximization program of impatient households is thus the following:

$$\max E_0 \sum_{t=0}^{\infty} \beta_I^t [\ln(C_{t,I}) + j\ln(H_{t,I}) + \psi \ln(1-N_{t,I})] \tag{18}$$

s.t.

$$C_{t,I} + R_{t-1}B_{t-1,I} + q_t H_{t,I} = w_t N_{t,I} + q_t H_{t-1,I} + B_{t,I} \tag{19}$$

$$B_{t,I} \leq m E_t\left[q_{t+1}\frac{H_{t,I}}{R_t}\right]. \tag{20}$$

All variables indexed by I for impatient households are equivalent to similar variables indexed by P for patient households. Impatient households face a borrowing constraint similar to that of entrepreneurs.

The first-order conditions with respect to housing, labor supply and debt write:

$$\frac{q_t}{C_{t,I}} = \beta_I E_t\left[\frac{q_{t+1}}{C_{t+1,I}}\right] + j\frac{1}{H_{t,I}} + \mu_{I,t} m E_t\left[\frac{q_{t+1}}{R_t}\right], \tag{21}$$

$$\frac{w_t}{C_{t,I}} = \frac{\psi}{1 - N_{t,I}}, \tag{22}$$

$$\frac{1}{C_{t,I}} = \beta_I E_t \left[\frac{1}{C_{t+1,I}} R_t\right] + \mu_{I,t}, \tag{23}$$

where $\mu_{I,t} \geq 0$ is the Lagrange multiplier associated with the borrowing constraint of the impatient households. The complementary slackness condition writes:

$$\mu_{I,t}\left[B_{t,I} - mE_t\left[q_{t+1}\frac{H_{t,I}}{R_t}\right]\right] = 0. \tag{24}$$

In the first-order condition with respect to housing, the last term is what differentiates the impatient households' first-order condition from that of patient households. Indeed, for borrowers, buying real-estate assets also presents the advantage of relaxing the borrowing constraint. Similar to the entrepreneurs case, the Lagrange multiplier associated with the borrowing constraint of impatient households μ_I is equal to $(\frac{\beta_P - \beta_I}{\beta_I})\frac{1}{C_I}$ in the deterministic steady state, requiring $\beta_I < \beta_P$ for the borrowing constraint to be binding in the steady state.

3.4 Market clearing

Finally, the model is closed by adding market clearing conditions, and standard transversality conditions are imposed. The model features four markets: a goods market, credit market, labor market and housing market.The market clearing condition on the goods market is:

$$Y_t = I_t + C_{t,P} + C_{t,F} + C_{t,I}. \tag{25}$$

Bonds are assumed to be in zero-net supply:

$$B_t = B_{t,F} + B_{t,I}. \tag{26}$$

The equilibrium condition on the labor market is:

$$N_t = N_{t,P} + N_{t,I}. \tag{27}$$

Finally, the market clearing condition on the housing market is:

$$H_{P,t} + H_{F,t} + H_{I,t} = 1. \tag{28}$$

In the rational expectations case, the model is solved relying on standard perturbation methods. Appendix A provides a summary of the model's equilibrium equations in level under the assumption that the borrowing constraint of both entrepreneurs and impatient households is binding (and thus that the associated Lagrange multipliers $\mu_{F,t}$ and $\mu_{I,t}$ are strictly positive). When numerically solving the model both under rational expectations and under learning, I verify that this assumption holds true in all simulations, as in Iacoviello (2005) and Iacoviello (2015).

4 The learning model

We now describe the model under subjective expectations, when agents no longer form rational expectations about the law of motion of house prices, while still holding model-consistent beliefs for all other variables.

4.1 Optimal Bayesian learning

Following a recent trend in the literature on learning regarding asset prices (Adam et al. (2012), Adam et al. (2016), Adam et al. (2017), Winkler (2019)), I now assume that agents in the economy do not understand the endogenous process through which house prices form. The actual equilibrium price results from the equalization of the demand for housing of the three sectors in the model to the exogenous supply of housing. However, in the learning model, market participants have imperfect knowledge of the market process and do not properly understand how house prices form. Instead, agents observe house prices realizations and try to determine whether the actual evolution is permanent or temporary. They thus try to evaluate the persistence of the current variation in house prices based on their past experience. Price determination is indeed a difficult task for atomistic agents. It implies perfect knowledge of the mapping between state variables and prices, and thus perfect knowledge about other agents' knowledge. Therefore, instead of taking into account the housing market clearing condition, atomistic market participants believe that logged house prices follow an exogenous process which takes the following form:

$$\ln(q_t) - \ln(q_{t-1}) = \ln(\mu_t) + \ln(\eta_t), \tag{29}$$

where η_t is a temporary disturbance, and where the time-varying persistent component μ_t follows the process:

$$\ln(\mu_t) = \ln(\mu_{t-1}) + \ln(\nu_t), \tag{30}$$

where ν_t is an additional disturbance. This assumption is grounded on several empirical elements. First, the learning mechanism is intuitive: when house prices rise, it is hard to disentangle whether the increase is persistent or wether it is only temporary. Observers thus try to evaluate the persistence of the increase based on their past experience. Second, the perceived law of motion for house prices is consistent with the short-term empirical behavior of house prices. Indeed, US house prices present episodes of persistent increase followed by episodes of persistent decrease. Third, Adam et al. (2012) and Caines (2016) show that this specification for perceived house price growth helps better understand the recent dynamics of house prices in G7-countries prior to the subprime financial crisis. Fourth, recent papers (Adam et al. (2016), Adam et al. (2017) and Winkler (2019)) show that this specification is successful in explaining several features of survey data about future stock returns, which are similar to patterns observed in survey data about future housing returns.

Agents perceive the innovations η_t and ν_t to be normally distributed according to the following joint distribution:

$$\begin{pmatrix} \ln(\eta_t) \\ \ln(\nu_t) \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\nu^2 \end{pmatrix} \right). \tag{31}$$

Note that all sectors in the economy share common beliefs on the house price process. Agents observe house price realizations q_t without noise, but they are not able to separately observe the persistent component and the transitory component of what they believe to be the exogenous process driving house price dynamics. Therefore, they face an optimal filtering problem and come up with the best statistical estimate $\ln(\widehat{\mu}_t)$ of the persistent component $\ln(\mu_t)$ in each period t. Due to normality of residuals and the linearity of the process, Bayesian filtering amounts to standard Kalman filtering in the set-up. Again following the related literature, the prior distribution of beliefs is assumed to be a normal distribution with mean parameter $\ln(\widehat{\mu_0})$ and dispersion parameter σ_0. Because the deterministic steady state is the starting point in the simulations below, as is usual in DSGE models analysis, I set the prior mean and dispersion parameters at their steady state values.[4] The prior mean belief about house price growth is thus set at $\ln(\widehat{\mu_0}) = 0$, and prior uncertainty σ_0^2 is set at its Kalman filter steady state value σ^2:[5]

$$\sigma^2 = \frac{-\sigma_\nu^2 + \sqrt{(\sigma_\nu^2)^2 + 4\sigma_\nu^2\sigma_\eta^2}}{2}. \tag{32}$$

Agents' subjective probability measure P is specified jointly by equations 29, 30 and 31, by prior beliefs and by knowledge of the productivity and lenders' discount factor random processes. The posterior distribution of beliefs in time t following some history up to period t, ω_t, is $\ln(\mu_t)|\omega_t \sim N(\ln(\widehat{\mu_t}), \sigma^2)$, where $\ln(\widehat{\mu_t})$ is given by the following optimal updating rule:

$$\ln(\widehat{\mu_t}) = \ln(\widehat{\mu_{t-1}}) + g\left[\ln(q_t) - \ln(q_{t-1}) - \ln(\widehat{\mu_{t-1}})\right]. \tag{33}$$

This unique recursive equation – in which g is the Kalman filter gain, which optimal expression is $\frac{\sigma^2}{\sigma^2+\sigma_\nu^2}$ – fully characterizes agents' beliefs about house price growth, which are summarized in each period t by the state variable $\widehat{\mu_t}$. The Kalman filter gain governs the size of the updating in the direction of the last forecast error. Logically, the Kalman filter gain increases in the signal-to-noise ratio $\frac{\sigma_\nu^2}{\sigma_\eta^2}$. A higher signal-to-noise ratio means that changes in house prices are driven to a higher extent by changes in the persistent component μ_t relative to changes in the transitory noise η_t. Thus, the last forecast error is more informative for predicting future house prices.

Given the perceived law of motion for house prices, agents believe that house prices in period t are such that:

$$\ln(q_t) = \ln(q_{t-1}) + \ln(\widehat{\mu_{t-1}}) + z_{1t}, \tag{34}$$

[4] Note that in the steady state, agents assume both $\bar{\eta}$ and $\bar{\nu}$ to be equal to zero.

[5] This implies that posterior uncertainty will remain at its steady state value following new house price realizations, because it is already starting at its minimal value.

where z_{1t} is seen by agents as an exogenous forecast error, normally distributed with mean 0 and variance σ_z, whereas it is actually endogenous: it is equal to the difference between the expected growth rate of house prices and the actual growth rate formed endogenously on the housing market.

4.2 Internally rational expectations equilibrium

Given the perceived law of motion for the growth rate of house prices, agents form optimal beliefs and make optimal decisions. Therefore, agents are 'internally rational' despite not holding rational expectations on the future dynamics of house prices.[6]

Definition 1: Internal rationality for each sector in each period t

- Patient households are internally rational if they choose $(C_{t,P}, H_{t,P}, N_{t,P}, B_t)$ to maximize the expected utility (1) subject to the budget constraint (3), for the given subjective probability measure P.

- Entrepreneurs are internally rational if they choose $(C_{t,F}, H_{t,F}, B_{t,F}, N_t, K_t, I_t)$ to maximize the expected utility (7) subject to the budget constraint (8), the production function (9), the capital accumulation equation (11) and the complementary slackness condition (16), for the given subjective probability measure P.

- Impatient households are internally rational if they choose $(C_{t,I}, H_{t,I}, N_{t,I}, B_{t,I})$ to maximize the expected utility (18) subject to the budget constraint (19) and to the complementary slackness condition (24), for the given subjective probability measure P.

The internally rational expectations equilibrium of the model is thus defined as follows.

[6]The concept of internal rationality was defined by Adam and Marcet (2011): "[i]nternal rationality requires that agents make fully optimal decisions given a well-defined system of subjective probability beliefs about payoff relevant variables that are beyond their control or "external", including prices." By contrast, "[e]xternal rationality postulates that agents' subjective probability belief equals the objective probability density of external variables as they emerge in equilibrium" (p. 1225).

Definition 2: The internally rational expectations equilibrium

The internally rational expectations equilibrium is defined by:

- The subjective probability measure P over the space Ω of all possible realizations of variables which are external to agents' decisions.[7]

- A sequence of contingent choices $\{C_{t,P}, C_{t,F}, C_{t,I}, H_{t,P}, H_{t,F}, H_{t,I}, B_t, B_{t,F}, B_{t,I}, N_t, N_{t,P}, N_{t,I}, K_t, I_t\} : \Omega^t \to \mathbb{R}_+^{14}$ such that the internal rationality of each agent defined above is satisfied.

- A sequence of equilibrium prices $\{q_t, R_t, w_t\}_{t=0}^{\infty}$ where $(q_t, R_t, w_t) : \Omega^t \to \mathbb{R}_+^3$ such that markets clear in each period t and all realizations in Ω are almost surely in P.

4.3 Solving the model under imperfect market knowledge

To solve the model under subjective expectations, I resort to lagged beliefs updating to avoid the simultaneous determination of beliefs and house prices. Indeed, according to equation (33), the mean belief about house price growth $\ln(\widehat{\mu}_t)$ in period t depends on current house prices q_t. At the same time, house prices in period t depend on the expectations of future house price growth and, thus, on the current mean belief $\ln(\widehat{\mu}_t)$. To avoid this issue, which is inherent in self-referential learning, lagged beliefs updating is assumed, that is, agents rely on lagged information when updating their beliefs. This assumption is common to all papers that model the same specification of asset prices and is also standard in the general self-referential learning literature. Adam et al. (2017) provide microfoundations for this updating rule with delayed information. Lagged beliefs updating consists in rewriting the beliefs updating equation rule (33) as:

$$\ln(\widehat{\mu}_t) = \ln(\widehat{\mu_{t-1}}) + g\left[\ln(q_{t-1}) - \ln(q_{t-2}) - \ln(\widehat{\mu_{t-2}})\right]. \tag{35}$$

[7] Note that under learning the space of realizations of variables external to agents' decisions includes the realizations of house prices. Under rational expectations, house price realizations provide information redundant with that provided by exogenous fundamental realizations because agents understand the mapping of fundamentals into house prices.

The slightly modified updating rule means that in period t, agents update their mean belief in the direction of the forecast error of the previous period rather than of the current period. Consequently, the mean belief $\ln(\widehat{\mu_t})$ is now predetermined at time t, and equilibrium house prices are determined by the housing market clearing condition. Lagged beliefs updating thus ensures that the equilibrium is unique. Following Winkler (2019), I treat the lagged forecast error as an exogenous disturbance z_{2t} in the belief system of agents in period t while ensuring that z_{2t} is equal to the lagged forecast error (i.e., to $\ln(q_{t-1}) - \ln(q_{t-2}) - \ln(\widehat{\mu_{t-2}})$).

Under imperfect market knowledge, all expected future realizations are conditional on the subjective probability measure P. The system of equations characterizing the policy function under P includes the first-order conditions (4-6), (13-15), (17) and (21-23), the flow of fund constraints (3), (8), and (19), the production function (9), the capital accumulation equation (11), the complementary slackness conditions (16-24), market clearing conditions (26) and (27), random processes (2) and (10) and the beliefs' updating equation (35). The market clearing condition on the housing market is not included in the system under the subjective probability measure P because agents do not understand how house prices form.

Solving this subjective system of equations yields the subjective policy functions, obtained under the probability measure P. However, subjective policy functions do not characterize the actual equilibrium house prices, which, despite being seen as exogenous, arise endogenously in the model through the market clearing condition.

Therefore, to solve the model under imperfect market knowledge, I rely on two steps, following the method proposed in Winkler (2019). First, I numerically solve for the coefficients of the approximate subjective policy function h of the above system of equations in the neighborhood of the deterministic steady state, relying on standard second-order perturbation methods. Second, I solve for the approximate actual policy function, i.e., the objective policy function g, by deriving actual endogenous house prices from the subjective policy function h, relying on chain rules derivation. Therefore, I obtain the derivatives of the Taylor expansion of the actual policy function g in the neighborhood of the deterministic steady state. This yields a numerical approximation for g, which fully characterizes the numerical solution to the learning model. The method is explained with more details in Appendix B and closely follows that presented in Winkler (2019).

5 Results: house price dynamics and macro-financial linkages

The model is solved under both the assumptions of rational expectations and imperfect market knowledge, to assess the relevance of the model in explaining the US asset price, credit and business cycles during 1985-2019. Appendix C details the quarterly data used in the calibration and for assessing the results.

5.1 Calibration strategy

The first set of parameters consists of static parameters $(\beta_P, \beta_I, \beta_F, \alpha, \psi, m_1, m_2, \delta, j)$, which affect only the steady state. The discount factor of patient households β_P is set at 0.9934 so that the steady-state mortgage rate $\bar{R}$ equals the mean of the average 1-year adjustable mortgage rate in the US over the period of interest. The discount factors of impatient households and firms (β_I, β_F) are set at 0.94, following Iacoviello (2015).

The weight on leisure in the household utility function is set at $\psi = 2$. This value implies that households allocate approximately one-third of their active time to work and that the Frisch elasticity of labor supply is around 2, which is consistent with values used in the macroeconomic literature (see Peterman (2016)) and with the calibration in Iacoviello (2015). The weight on housing in the household utility function is set at $j = 0.075$, whereas the share of housing in the consumption goods production function is set at $v = 0.05$. These parameter values imply a steady-state entrepreneurial share of housing of 24%. Given the US average labor share to output during 1985-2019, we obtain $\alpha = 0.34$ for the share of capital in the production function. The capital depreciation rate δ is set at the standard value of 0.025, corresponding to a 10% annual depreciation.

The loan-to-value ratio m is set at 0.5 to match the steady state debt-to-GDP ratio in the model with the average debt-to-GDP ratio in the data. This value is consistent with the values estimated in Iacoviello (2005) and Kuang (2014) for the household sector. It implies a significant degree of financial frictions and is such that the borrowing constraints are always binding throughout the simulations.

In what regards the dynamic parameters of the model, the persistence parameter of the productivity shock ρ_a is estimated from the US data during 1985-2019 by using the perpetual inventory method. The linearly detrended Solow residual displays relatively strong persistence, with $\rho_a = 0.93$. For the persistence parameter of the lenders' preference shock, I rely on the literature on intertemporal disturbances and set $\rho_d = 0.83$ (Primiceri et al., 2006).

Finally, the four remaining dynamic parameters $(g, \phi, \sigma_a, \sigma_d)$ are estimated by using the generalized method of moments (GMM), in both the rational expectations and learning models. Parameters are chosen to minimize the distance function between four targeted second-order moments in the data (namely, the variances of production, investment, consumption and house prices) and corresponding theoretical moments. In the minimization procedure, higher weight is allocated to the empirical moments that are estimated more precisely. Therefore, the estimated parameters $\hat{\theta}$ are those that minimize the following distance function:

$$\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmin}} (m(\theta) - \hat{m})' \hat{W}_T (m(\theta) - \hat{m}),$$

where $m(\theta)$ denotes the vector of theoretical moments, $\hat{m}$ denotes the vector of moments estimated in the data during 1985-2019, and $\hat{W}$ denotes the weighting matrix estimated in the data.[8] An upper bound on the value of the Kalman filter gain g is imposed to exclude cases in which oscillations in the model variables are too high; high oscillations are not consistent with the empirical behavior of these variables. The estimated parameter g reaches the upper bound of 0.05.[9] Table 1 gathers the values of all parameters used in the model simulations.

[8] $\hat{W}_T$ is equal to $\text{diag}(\hat{\Sigma})^{-1}$, with $\hat{\Sigma}$ the variance-covariance matrix of the data moments estimated with the standard Newey-West kernel.

[9] The value of the upper bound for g is chosen by relying on the learning literature (an upper bound of 0.05 is consistent with the value estimated in Winkler (2019) in what regards stock price fluctuations) and on the analysis of the impulse response functions.

Parameter	Calibrated value (Learning)	Calibrated value (Rational expectations)
β_P	0.9934	0.9934
β_I	0.94	0.94
β_F	0.94	0.94
ψ	2	2
j	0.075	0.075
v	0.05	0.05
α	0.34	0.34
m	0.5	0.5
δ	0.025	0.025
ϕ	9.7111	13.6399
ρ_a	0.93	0.93
ρ_d	0.83	0.83
σ_a	0.0060	0.0062
σ_d	0.0070	0.0259
g	0.005	NA

Table 1: Calibration

The estimated variances of the productivity shock are roughly similar in both the rational expectations and learning model, even though the variance is slightly lower under learning. In both models, the estimated variance of the lenders' preference shock is higher than that of the productivity shock. This result is consistent with the idea that financial shocks recently became stronger and play a larger part in business cycles, notably in the run-up to the Great Recession. However, the estimated variance is more than three times larger in the rational expectations model than in the learning model. This result reveals the strong amplification in the responses to shocks generated by the learning mechanism.[10] Due to the high variance of the financial shock, the capital adjustment cost parameter is higher in the rational expectations model than in the learning model.

5.2 Business cycle, credit and house price statistics

Table 2 compares the standard business cycle, credit and house price moments in the data with those of the learning model. The table also reports the moments obtained in the rational expecta-

[10] As a comparison, in his model, Iacoviello (2015) estimates the volatility of the housing demand shock, which is introduced to replicate the volatility of house prices, at 0.0346. This value is higher than the sum of the volatility of the two shocks in our model. Our model can, however, fully replicate the dynamics of house prices while providing a more endogenous explanation of these dynamics, i.e., without resorting to housing sector shocks.

tions model for values of dynamic parameters that are identical to those estimated in the learning model ("RE Calibration 1") and for values of dynamic parameters specifically estimated for this version of the model ("RE Calibration 2"). The use of the first calibration helps identify differences in the size of the propagation mechanisms between the rational expectations and the learning model, whereas the use of the second calibration enables the study of the version of the rational expectations model that best fits the targeted data.

Table 2 presents both the moments that were directly targeted in the estimation strategy (output, house prices, investment and consumption volatility) and a large set of standard moments that were not targeted. Both the empirical quarterly data and the model-generated data are logged (except for the house price growth rate) and hp-filtered with a parameter of 1600. Moments targeted in the estimation strategy are displayed with an asterisk. The model-generated consumption is the sum of the consumptions of patient households, impatient households and entrepreneurs.

	US Data Q1 1985-Q4 2019	Learning	RE Calibration 1	RE Calibration 2
$\sigma_{Y_t}*$	0.0102	0.0099	0.0086	0.0095
$\frac{\sigma_{I_t}}{\sigma_{Y_t}}*$	3.36	3.48	2.84	3.69
$\frac{\sigma_{C_t}}{\sigma_{Y_t}}*$	0.71	0.77	0.83	0.78
$\frac{\sigma_{N_t}}{\sigma_{Y_t}}$	1.50	0.38	0.21	0.59
$\frac{\sigma_{B_t}}{\sigma_{Y_t}}$	1.95	2.73	1.45	2.71
$\frac{\sigma_{q_t}}{\sigma_{Y_t}}*$	1.91	1.92	0.94	1.93
$\rho(I_t, Y_t)$	0.89	0.95	0.95	0.83
$\rho(C_t, Y_t)$	0.81	0.99	0.99	0.96
$\rho(N_t, Y_t)$	0.87	0.77	0.57	0.52
$\rho(B_t, Y_t)$	0.41	0.90	0.91	0.73
$\rho(q_t, Y_t)$	0.54	0.74	0.87	0.69
$\rho(Y_{t-1}, Y_t)$	0.88	0.76	0.72	0.72
$\rho(I_{t-1}, I_t)$	0.91	0.77	0.70	0.69
$\rho(C_{t-1}, C_t)$	0.84	0.73	0.73	0.76
$\rho(N_{t-1}, N_t)$	0.94	0.79	0.66	0.65
$\rho(B_{t-1}, B_t)$	0.96	0.80	0.67	0.62
$\rho(q_{t-1}, q_t)$	0.93	0.86	0.74	0.73
$\rho(\ln(\frac{q_t}{q_{t-1}}), \ln(\frac{q_{t-1}}{q_{t-2}}))$	0.68	0.20	-0.03	-0.03

Table 2: Business cycles, credit and house price moments

Despite the parsimony of the baseline model, both the learning and the rational expectations model with calibration 2 replicate relatively well the volatility of production and the relative volatility of most other variables. However, both models tend to overpredict the volatility of debt, and they unsurprisingly have difficulties in replicating the volatility of hours. These difficulties are a well-known issue in standard basic real business cycle models due to the simplicity of labor market decisions.[11] When the rational expectations model is solved with the same parameter values as the learning model (calibration 1), the model cannot match the volatility of the distinct variables, thus revealing that amplification mechanisms arise under learning. In addition, the learning model replicates the high autocorrelations observed in the dat, better than both versions of the rational expectations model. The learning mechanism indeed acts as an endogenous source of persistence without needing to resort to habit or other exogenous sources of persistence. The learning model also replicates the strong procyclicality in the model variables, even though the model tends to overpredict the correlation of debt and house prices with output. In addition, interestingly, the learning model predicts a positive autocorrelation in house price growth $\rho(\ln \frac{q_t}{q_{t-1}}, \ln \frac{q_{t-1}}{q_{t-2}})$, as observed in the data. The rational expectations model is unable to replicate this feature of the data. Indeed, unlike the learning model which generates extrapolation in house price beliefs and thus autocorrelation in housing returns, the rational expectations model generates a mean-reverting behavior in housing returns.

Our quantitative results suggest that learning about future house prices offers a promising and intuitive mechanism for explaining the joint dynamics of macroeconomic variables, credit and house prices in a simple and standard production economy, while considering that the empirical validity of the rational expectations assumption is called into question.

5.3 Impulse response functions analysis

To better understand the amplification mechanism in operation in the learning model, the impulse response functions to the two shocks under learning and under rational expectations are displayed below. The impulse response functions represent log-deviations from the steady state in response to a one-standard-deviation positive productivity shock and a one-standard-deviation negative lenders' preference shock.[12]

[11] Several assumptions have been made in the literature to overcome this limitation inherent in standard real business cycle theory, such as indivisible labor and the search model of the labor market. An avenue for future research is to incorporate the learning mechanism into larger DSGE models that feature more realistic labor market decisions.

[12] To compare impulse response functions under subjective and rational expectations for similar shocks, the standard deviations of shocks that I retain are those estimated for the learning model.

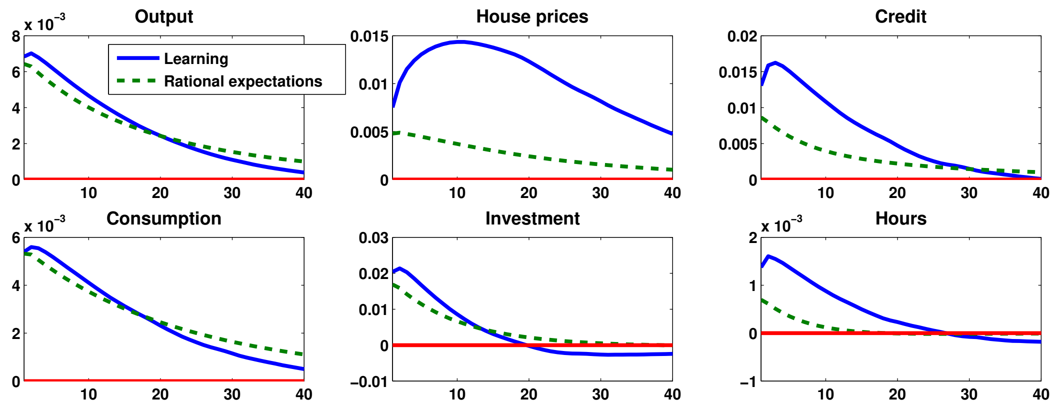

Figure 1: Impulse response functions following a positive productivity shock

Following a productivity shock (Figure 1), entrepreneurs increase labor demand and investment, and output grows. The labor supply of patient households rises as well, while the labor supply of impatient households decreases. Demand for housing grows, and this growth in demand is reflected in higher house prices. The shock generates a reallocation of the fixed housing stock: the share of the housing stock held by the productive sector and by impatient households increases at the expense of patient households. Credit increases in equilibrium, notably because borrowers hold more housing assets, thereby relaxing the borrowing constraint. These redistributive effects thus affect the financial accelerator mechanism of the model, both in the household sector and in the productive sector. Consumption of the three sectors increases due to a rise in wealth.

Despite this common mechanism, there are significant differences between the impulse response functions in the learning and rational expectations model. In the learning model, house prices are booming following the shock. Indeed, the initial effect of the shock is amplified over time; house prices display a persistent hump-shaped response. The response of credit to the shock is initially stronger and is amplified over time relative to the response in the rational expectations model. Hours worked also react more strongly in the learning model. The responses of output, consumption and investment are also stronger and slightly hump-shaped compared to the same responses in the rational expectations model.

A negative lenders' preference shock (Figure 2) implies that patient households value the current period less and are, thus, more willing to transfer money into the future through lending. This shock thus mimics a context of higher willingness to lend and easier access to credit independent of borrowers' net worth.

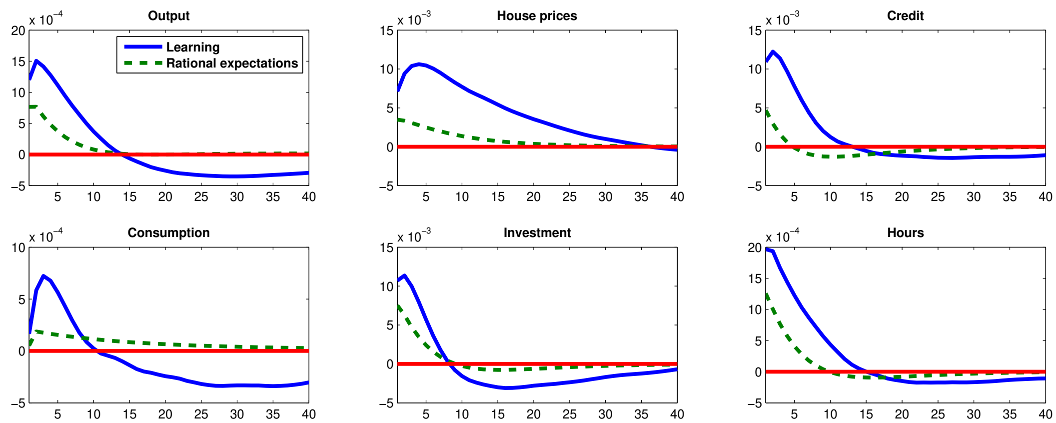

Figure 2: Impulse response functions following a negative lenders' preference shock

Following the shock, patient households increase the credit and labor supply and decrease consumption. In equilibrium, credit is, thus, higher, and impatient households increase investment in housing assets, while firms invest more in both housing assets and capital. House prices rise in response to the increase in housing demand. Output grows because hours worked rise. Consumption of both impatient households and firms increases. Once again, in the learning model, the shock generates an endogenous boom in house prices, whereas the response of house prices is much smaller and not persistent in the rational expectations model. The responses of the other variables are amplified, and the initial effect of the shock on these variables is propagated over time. The impulse response functions are thus hump-shaped.

Differences in the impulse response functions in the learning and rational expectations model relate to differences in the evolution of expected housing returns and prices, which affects current housing demand in the three sectors of the economy. Expected housing returns and prices, thus, affect intertemporal trade-offs and affect the financial accelerator mechanism. To understand the mechanism at play in the learning model, Figure 3 presents the joint dynamics of expected and realized housing returns and their impact on prices in response to a one-standard-deviation negative lenders' preference shock. The mechanism is similar for the productivity shock.

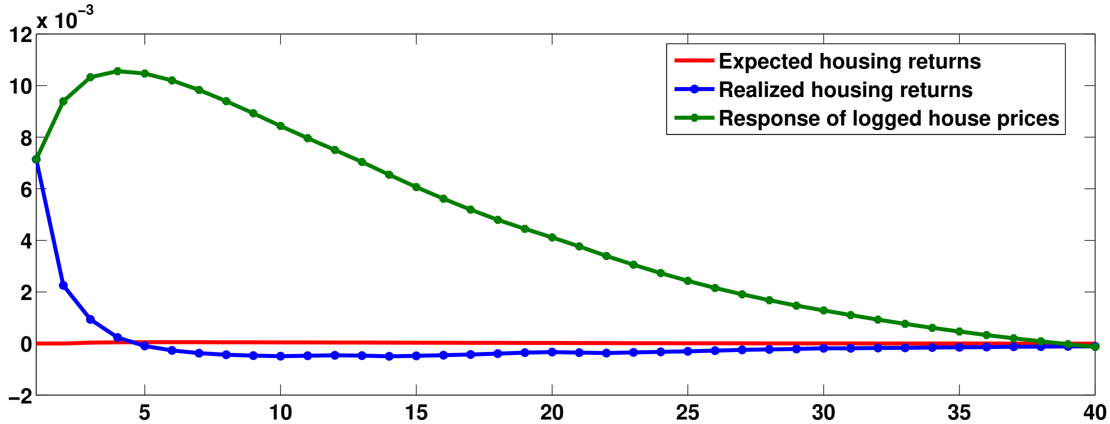

Figure 3: House prices and the dynamics of beliefs

House prices grow in response to the shock (green curve). Therefore, the realized growth rate at the time of the shock $\ln(q_1) - \ln(q_0)$ (blue curve) is higher than the expected growth rate before the shock $\ln(\widehat{\mu_0})$ (red curve)[13], thus implying that forecast errors – which are measured as the difference between the two curves – are positive. According to equation (35), when lagged forecast errors are positive, beliefs are updated upward. Therefore, expected housing returns increase. Housing demand rises, thereby further propagating the initial increase in house prices.

As long as the realized housing returns are higher than the expected housing returns, the forecast error is positive, and expected future housing returns continue to increase, fueling the demand for housing. The forecast error then gradually decreases. Eventually, the forecast error becomes null, i.e., the expected growth rate is equal to the actual growth rate. The demand for housing no longer increases. The realized growth rate of house prices thus decreases below the expected value, and forecast errors become negative. Expected housing returns thus start to decrease, thereby generating an endogenous reversal in house prices.

Therefore, in the learning model, surprise effects in housing returns endogenously propagate the initial effect of the shock. In contrast, under rational expectations, surprise effects arise only in the first period, when the shock hits. This result explains that the learning model better replicates

[13]Due to the scale of the figure, expected housing returns look constant but they actually fluctuate around zero.

the autocorrelations observed in the data. Even though the learning mechanism generates very small variations in expected housing returns, these variations strongly affect the dynamics of house prices. Housing returns expectations and forecast errors are state variables; consequently, their variations also affect the other model variables' realized and expected values.

5.4 Explaining non-rational patterns in expectations: forecast errors predictability

I now investigate the ability of the learning model to explain some features of macroeconomic and house price expectations, as measured by survey data. In particular, survey data reveal that forecast errors are predictable. Therefore, forecast errors are correlated with variables that were observable at the time of the forecast. By contrast, the rational expectations hypothesis implies that forecast errors are unpredictable because all information available at the time of the forecast is already incorporated into the forecast. Forecast errors, thus, only result from unpredictable shock realizations. Consequently, by nature, models that assume rational expectations fail to explain the data in what regards the formation of expectations.

Table 3 presents evidence of forecast errors predictability in survey data about expected future macroeconomic variables and housing returns and compares this predictability to that obtained by the learning model. Forecast errors for house price growth ε_R are defined as the difference between realized annual house price growth and one-year-ahead household forecasts from the US Michigan Survey of Consumers available for 2007-2019. Forecast errors for output, investment and consumption (ε_Y, ε_I and ε_C) are retrieved from the Survey of Professional Forecasters and are computed for annualized quarter-on-quarter growth rates of the variables for 1985-2019.

	US Survey data	Learning	RE
$\rho(\varepsilon_{R_{t+4}}, q_t)$	-0.55	-0.22	0
$\rho(\varepsilon_{R_{t+4}}, \ln(\frac{q_t}{q_{t-1}}))$	0.56	0.30	0
$\rho(\varepsilon_{Y_{t+1}}, \ln(\frac{q_t}{q_{t-1}}))$	0.13	0.13	0
$\rho(\varepsilon_{I_{t+1}}, \ln(\frac{q_t}{q_{t-1}}))$	0.03	0.06	0
$\rho(\varepsilon_{C_{t+1}}, \ln(\frac{q_t}{q_{t-1}}))$	0.16	0.32	0

Table 3: Forecast errors predictability

The results reveal that the learning model is able to replicate the sign, and, to some extent, the size, of the predictability of forecast errors, even though predictability was not targeted in the calibration method. Thus, the learning model predicts a negative correlation between housing returns forecast errors and house prices observed at the time of the forecast, whereas the model predicts, in accord with the data, a positive correlation between these same forecast errors and house price growth at the time of the forecast. These features emerge because agents tend to underpredict housing returns when house prices start to rise (i.e., when house price growth is high) and to overpredict housing returns when house prices are high (i.e., close to the peak). For the forecast errors of macroeconomic variables, the learning model succeeds in replicating the positive correlation of these errors with the observed house price growth at a short-term horizon. This positive correlation suggests that agents tend to underpredict future macroeconomic variables at the beginning of a housing boom.

Embedding a learning mechanism about housing prices into a stylized production economy framework enables an explanation, in accord with survey data, of non-rational patterns in expectations of both housing returns and macroeconomic variables. Even though expectations of macroeconomic variables are model consistent, these expectations are conditional on house price growth expectations and are, thus, not rational either. Therefore, a small deviation from rational expectations (regarding only the perceived process of the housing returns) enables improving the explanation of subjective features of macroeconomic expectations.

6 Conclusion

The present paper proposes an interpretation of the recent US macro-financial linkages based on imperfect knowledge regarding the formation of house prices. This assumption, which is consistent with the rejection of the rational expectations hypothesis evidenced by survey data, is introduced into an otherwise standard borrower-saver model with a production sector, collateralized borrowing constraints and financial frictions.

The learning mechanism about house price growth generates amplification and propagation over time of small macroeconomic and credit sector aggregate disturbances involving feedback transmission channels between the housing market, the credit market and the real sector.

Compared with the rational expectations model, the subjective expectations model, by introducing learning about house prices, yields three main empirical results. First, the learning mechanism enables the replication of persistent booms in house prices in response to shocks, followed by endogenous reversals, in accord with what was observed in the US in recent decades, particularly in the run-up to the subprime crisis. Second, the setup helps simultaneously replicate several aspects of standard business cycle moments in the last 30 years and additional features of the dynamics of house prices. Estimated shock variances are smaller in the learning model than in the rational expectations model because the learning mechanism acts as an endogenous source of persistence. Third, the learning model can explain non-rational patterns in expectations of both future housing returns and macroeconomic variables, namely, the predictability of forecast errors.

The parsimonious and stylized model offers an explanation with intuitive appeal of the excess volatility in house prices and the macroeconomic consequences of such volatility. The promising results that are obtained despite the small scale of the model pave the way for several extensions in larger DSGE models, notably those that model more complex labor market decisions. An important avenue for future research is to derive optimal policies when asset prices display excess volatility. In particular, the setup could be extended by introducing nominal frictions and investigating optimal monetary policy under non-rational expectations.

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A Equilibrium conditions in the rational expectations model

A.1 The first-order conditions

$$d_t \frac{1}{C_{t,P}} q_t = \beta_P E_t \left[\frac{1}{C_{t+1,P}} q_{t+1} d_{t+1} \right] + j d_t \frac{1}{H_{t,P}}.$$

$$\frac{d_t}{C_{t,P}} = \beta_P E_t \left[\frac{d_{t+1}}{C_{t+1,P}} R_t \right].$$

$$\frac{w_t}{C_{t,P}} = \frac{\psi}{1 - N_{t,P}}.$$

$$w_t = \frac{(1 - \alpha - v) Y_t}{N_t}.$$

$$\frac{1}{C_{t,F}} = \beta_F E_t \left[\frac{1}{C_{t+1,F}} \right] R_t + \mu_{F,t}.$$

$$\frac{q_t}{C_{t,F}} = \beta_F E_t \left[\frac{1}{C_{t+1,F}} \left(q_{t+1} + \frac{v Y_{t+1}}{H_{t,F}} \right) \right] + \mu_{F,t} m E_t \left[\frac{q_{t+1}}{R_t} \right].$$

$$\frac{1}{C_{t,F} \left(1 - \phi(\frac{I_t}{K_{t-1}} - \delta) \right)} =$$

$$\beta_F \frac{1}{C_{t+1,F}} \left(\frac{\alpha Y_{t+1}}{K_t} + \frac{1}{1 - \phi(\frac{I_{t+1}}{K_t} - \delta)} (1 - \delta - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_t} - \delta \right)^2 + \phi(\frac{I_{t+1}}{K_t} - \delta) \frac{I_{t+1}}{K_t}) \right).$$

$$B_{t,F} = m E_t \left[\frac{q_{t+1}}{R_t} \right] H_{t,F}.$$

$$\frac{q_t}{C_{t,I}} = \beta_I E_t \left[\frac{q_{t+1}}{C_{t+1,I}} \right] + j \frac{1}{H_{t,I}} + \mu_{I,t} m E_t \left[\frac{q_{t+1}}{R_t} \right].$$

$$\frac{w_t}{C_{t,I}} = \frac{\psi}{1 - N_{t,I}}.$$

$$\frac{1}{C_{t,I}} = \beta_I E_t \left[\frac{1}{C_{t+1,I}} R_t \right] + \mu_{I,t}.$$

$$B_{t,I} = m E_t \left[\frac{q_{t+1}}{R_t} \right] H_{t,I}.$$

A.2 The flow of fund constraints

$$C_{t,P} + q_t H_{t,P} + B_t = w_t N_{t,P} + R_{t-1} B_{t-1} + q_t H_{t-1,P}.$$

$$C_{t,F} + q_t H_{t,F} + R_{t-1} B_{t-1,F} + w_t N_t + I_t = Y_t + B_{t,F} + q_t H_{t-1,F}.$$

$$C_{t,I} + R_{t-1} B_{t-1,I} + q_t H_{t,I} = w_t N_{t,I} + q_t H_{t-1,I} + B_{t,I}.$$

A.3 The production function and the capital accumulation equation

$$Y_t = A_t K_{t-1}^{\alpha} H_{t-1}^{v} N_t^{1-\alpha-v}.$$

$$K_t = I_t + (1-\delta) K_{t-1} - \frac{\phi}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}.$$

A.4 The shock processes

$$\ln(d_t) = \rho_d \ln(d_{t-1}) + \varepsilon_{d,t}.$$

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_{a,t}.$$

A.5 The market clearing equations

$$Y_t = I_t + C_{t,P} + C_{t,F} + C_{t,I}.$$

$$B_t = B_{t,F} + B_{t,I}.$$

$$N_t = N_{t,P} + N_{t,I}.$$

$$H_{P,t} + H_{F,t} + H_{I,t} = 1.$$

B The approximation method

Appendix B presents a summary of the original method presented in Winkler (2019) applied to the specific case of our model.

A general way to describe the learning model is to write it as the combination of the two following systems of equations under the subjective probability measure P:

$$E_t^P[f(y_{t+1}, y_t, x_t, u_t, z_t)] = 0, \tag{36}$$

$$E_t^P[\phi(y_{t+1}, y_t, x_t, u_t, z_t)] = 0, \tag{37}$$

where y_t is a vector of endogenous variables in period t, y_{t+1} is a vector of these variables in the next period, x_t is a vector of state variables such that $x_t = Cy_{t-1}$, where C is a matrix including 0 and 1 values, and y_{t-1} is a vector of the endogenous variables in the previous period. u_t is a vector of stochastic disturbances with variances σ. z_t is a vector of the variables that are perceived by agents as iid exogenous disturbances. They affect the house price process and the belief updating process, and have zero mean and variances σ_z. They are assumed to be uncorrelated with u_t. They can be interpreted as house price growth forecast errors and are assumed to be null in the steady state. E_t^P is the expectations operator under the subjective probability measure P. As discussed in Section 3, the elements of z_t are actually not exogenous. They are determined endogenously by market clearing on the housing market, and thus by a second set of equilibrium conditions, given by the system of equations (37), which is unknown to agents.

In the specific case of our model, the second set of equilibrium conditions takes the form of two equations:

$$H_{I,t} + H_{F,t} + H_{P,t} - 1 = 0, \tag{38}$$

and

$$z_{2t} - z_{1,t-1} = 0. \tag{39}$$

Solving the model under the subjective probability measure P first amounts to finding the function h, which is the subjective policy function, such that all endogenous variables are expressed as functions of the state variables x_t and of the two types of exogenous disturbances u_t and z_t (and of the shock variances): $h(x_t, u_t, z_t, \sigma)$. We obtain a numerical approximation for the subjective policy function by implementing standard perturbation methods for solving the system of equations under P that includes the first-order conditions (4-6), (13-15), (17) and (21-23), the flow of fund constraints (3), (8), and (19), the production function (9), the capital accumulation equation (11), the complementary slackness conditions (16-24), market clearing conditions (26) and (27), random processes (2) and (10), and the beliefs' updating equation (35). This step yields the coefficients of the Taylor expansion of h around the deterministic steady state. This is the first step of the approximation method.

Secondly, the aim is to find the values of z_t such that the market clearing condition on the housing market is actually satisfied, that is, such that $H_{I,t} + H_{F,t} + H_{P,t} = 1$. The elements of z_t can thus be written as a function r of state variables x_t, stochastic disturbances u_t and shock variances σ: $z_t = r(x_t, u_t, \sigma)$. We thus need to approximate the function r (at this point, it is assumed to exist and to be unique, which is then verified ex-post in the case of our model) in order to approximate the objective policy function g. The latter is such that, in equilibrium, $y_t = g(x_t, u_t, \sigma)$. By applying chain rules for derivation, g can be approximated in the neighborhood of the non-stochastic steady state at the first order as follows:

$$y_t = g(x_t, u_t, \sigma) = h(x_t, u_t, r(x_t, u_t, \sigma), \sigma)$$
$$\simeq g(\bar{x}, 0, 0) + (h_x + h_z r_x)(x_t - \bar{x}) + (h_u + h_z r_u)u_t + (h_\sigma + h_z r_\sigma)\sigma,$$

where $\bar{x}$ is the vector of steady state values of the state variables. The first order derivatives of h and r are written without time subscripts (e.g. $h_x = \frac{\partial h}{\partial x_t}$).

To derive the objective policy function g, we still need to find the approximate values of r_x, r_u and r_σ in the steady state. These derivatives can be obtained by total differentiation of the second set of equilibrium conditions (37) at the deterministic steady state. Indeed, the second set of equilibrium conditions (37) can be rewritten as:

$$0 = \Phi(x_t, u_t, \sigma) = E_t^P\left[\phi(y_{t+1}, y_t, x_t, u_t, z_t)\right] =$$

$$E_t^P\left[\phi\left[\underbrace{h\left(Ch(x_t, u_t, r(x_t, u_t, \sigma), \sigma), u_{t+1}, z_{t+1}, \sigma\right)}_{y_{t+1}}, \underbrace{h\left(x_t, u_t, r(x_t, u_t, \sigma), \sigma\right)}_{y_t}, x_t, u_t, \underbrace{r\left(x_t, u_t, \sigma\right)}_{z_t}\right]\right].$$

Total differentiation at the steady state makes it possible to obtain the derivatives of r, which are, in our case, as follows (given that y_{t+1} and u_t do not appear in the second set of equilibrium conditions (38-39), we have: $\phi_{y_{t+1}} = 0$ and $\phi_{u_t} = 0$):

$$\frac{d\Phi}{dx}(\bar{x}, 0, 0) = \phi_y h_x + \phi_x + (\phi_y h_z + \phi_z) r_x = 0$$
$$\Leftrightarrow r_x = -(\phi_y h_z + \phi_z)^{-1}(\phi_y h_x + \phi_x).$$

First-order derivatives are written without time subscripts in the above equation because they all involve variables in t only.

$$\frac{d\Phi}{du}(\bar{x}, 0, 0) = \phi_y h_u + (\phi_y h_z + \phi_z) r_u = 0$$
$$\Leftrightarrow r_u = -(\phi_y h_z + \phi_z)^{-1}\phi_y h_u.$$

$$\frac{d\Phi}{d\sigma}(\bar{x}, 0, 0) = \phi_y h_\sigma + (\phi_y h_z + \phi_z) r_\sigma = 0$$
$$\Leftrightarrow r_\sigma = -(\phi_y h_z + \phi_z)^{-1}\phi_y h_\sigma = 0,$$

because $h_\sigma = 0$.

Because the matrix $\phi_y h_z + \phi_z$ is invertible in our model, the function r exists and is unique.

The method is similar for higher order approximations (see Winkler (2019) for more details).

C Data series

All data series are extracted from the Federal Reserve Bank of St Louis database (FRED), mainly from the US Flow of Funds Statistics, and are expressed in real terms using the GDP Implicit Price Deflator.

Variable	**Series**
Output	Gross Domestic Product (GDP)
House prices	All-transactions House Prices Index (USSTHPI)
Investment	Private Non-Residential Fixed Investment (PNFI) + Durable Consumption (PCDG)
Consumption	Personal Consumption Expenditures, Nondurable Goods and Services (PCND + PCESV)
Debt	Entrepreneur Loans (MLBSNNCB+NNBTML+BLNECLBSNNCB+OLALBSNNCB+OLALBSNNB +BLNECLBSNNB) and Household Loans (HHMSDODNS+HCCSDODNS)
Hours	Hours of All Persons, Nonfarm Business Sector (HOANBS)
Labor share	Share of Labour Compensation in GDP at Current National Prices (LABSHPUSA156NRUG)
Mortgage rate	1-year adjustable rate (average in the US) (MORTGAGE1US)
Price Deflator	GDP Implicit Price Deflator (GDPDEF)