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ABSTRACT

We investigate the ability of the Fama equation to compute proper conditional densities and currency risks. Based on quantile regressions, we fit a Skewed t-distribution to estimate the conditional densities on the monetary policy of eight currency pairs. We demonstrate that the conditional densities are highly sensitive to the monetary policy stance. Then, we use the estimated conditional densities to measure the currency risks. Our results highlight that the depreciation/appreciation risks are extremely heterogeneous and that the currencies are more exposed to depreciation risks, especially during turmoils. Our findings can be used as a supplementary tool to assess whether a currency behaves as a safe-haven currency. We also investigate the relative and absolute performance of our model in forecasting densities. We find that the predictive densities are perfectly well-calibrated. Moreover, our results also demonstrate that our methodology can outperform the random walk in forecasting densities.

Keywords: Quantile regressions, Predictive densities, Currency risks, Safe-haven currency.
JEL codes: C22, C53, F31.

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1 Introduction

According to the Bank for International Settlements in the last Triennial Survey, the trading volumes in the foreign exchange (FX) markets averaged 6.6 trillion per day in April 2019 against 5.1 trillion per day three years earlier. This important volume of trading makes this market the most important but also the most liquid and volatile. These characteristics are obviously due to the great importance of exchange rates in the daily decisions of agents. Hence, there is a real need to develop models not only to analyze the current evolution but also to predict the future paths of exchange rates. Large numbers of theories emerged in the literature to propose explanations and drivers for exchange rates future movements. However, in practice, forecasting exchange rates turns out to be one of the most complicated puzzles in international finance. Since the works of Meese and Rogoff (1983), large numbers of researchers evaluated the out-of-sample performances of different empirical and theoretical models but failed to outperform the random walk (see Rossi (2013) for a literature review). Although the prediction of the exchange rate level has been widely explored in the literature, forecasting the density of exchange rates remains somewhat unexplored. Nowadays, there is a growing interest in predicting the full distribution of exchange rates insofar as forecasting densities depicts all the uncertainty associated with a prediction. Economic agents, practitioners, and policymakers do no longer focus only on a point forecast but also and particularly on the full predictive density of a variable. Moreover, the expanding field of financial risk management is also a reason to focus more on predictive densities. As some risk measures largely depend on distributions, the better the predictive densities are calibrated, the better the risks are measured. In this paper, we investigate this issue for eight currency pairs, all against the US dollar.

This paper has three main objectives. First, we try to estimate the conditional densities of exchange rates. We exploit the ability of the quantile regressions developed by Koenker and Bassett (1978) to convert conditional quantile functions into conditional densities. We begin by estimating the traditional Fama equation regarding four different horizons : one month, three months, six months and twelve months. Then, we employ the quantile interpolation methodology to smooth the estimated conditional quantiles using the skewed t-distribution developed by Azzalini and Capitanio (2003) and obtain the estimated conditional densities of the currency pairs. The choice of a skewed t-distribution is justified by its flexibility and its ability to encompass several class of distributions according to the value of its parameters. Second, we evaluate the depreciation and appreciation risks. We rely on the Expected Shortfall, which has become the most widely used measure in risk management. The Basel Committee On Banking Supervision (2016) imposed the Expected Shortfall as the new benchmark for managing risk. It measures the total probability mass that the conditional density assigns to the left tail. As we are also interested in the upside risk, we consider the upper tail counterpart of Expected Shortfall that we term Expected Longrise as in Adrian et al. (2019). To backtest our Expected Shortfall/Longrise, we use the test proposed by Bayer and Dimitriadis (2019). We make two robustness analyses. We begin by fitting a Normal distribution and explore its ability to compute currency risk measures. After that, we augment the Fama equation with the economic policy uncertainty (EPU) of Baker et al. (2016) to evaluate the currency risks related to both monetary policy and uncertainty events. Third, we compute out-of-sample density forecasts and evaluate them. Two kinds of evaluations are performed. On the one hand, we use the test of Rossi and Sekhposyan (2019) to carry out an absolute evaluation. On the other hand, the relative evaluation our of model is performed by comparing it to the traditional benchmark when forecasting exchange rates, namely the random walk. We rely on the notion of predictive score and the framework proposed by Amisano and Giacomini (2007).

We obtain the following findings. First, evidence from the quantile regressions suggests that, for most currency pairs, a common pattern appears for most of the currency pairs. For lower quantiles of exchange rates, the Fama coefficient is positive. But for the upper quantiles, the Fama coefficient decreases and tends to be negative. This result is an evidence of the nonlinear features of the Fama equation and suggests that the Fama coefficient depends on the distribution of exchange rates. We also find that the estimated conditional densities are sensitive to the monetary policy stance. The entire distribution of exchange rates exhibits a time-varying pattern and depicts significant and robust asymmetries. The Expected Shortfall/Longrise highlight an important feature : for most currency pairs, the depreciation risks are more volatile than the appreciation risks, especially during crises. Furthermore, the risk of depreciation drops significantly during turmoils. In other words, the currency pairs are more exposed to a downside risk, except for the Swiss Franc and the Japanese Yen. Our methodology can also be used to assess whether a currency behaves like a safe-haven currency. If the volatility of the appreciation risk is higher than the volatility of the depreciation risk, then the currency behaves as a safe-haven currency. We find that the US Dollar but also the Japanese Yen and the Swiss Franc can be considered as safe-haven. The backtesting procedure of our depreciation/appreciation risks demonstrates that the risk measures are correctly specified. These measures can be beneficial for policymakers or practitioners insofar as they will be able to evaluate the risk of depreciation/appreciation and use them to monitor their different strategies or policies. The robustness analysis made with the EPU provides the same conclusions. Moreover, based on the backtesting procedure, we find that the Normal distribution is not suitable to fit the currency risks. Finally, the results of the test of Rossi and Sekhposyan (2019) suggest that our predictive conditional densities are well specified. In other words, our model is suitable to estimate the predictive densities of the exchange rates. Concerning the relative performance, our model overperforms the random walk for the one-month-ahead framework. For the three-month-ahead, six-month-ahead and twelve-month-ahead, the random walk provides more accurate predictive densities.

Our paper is closely linked to several strands of the literature. First, from a methodological point of view, our research is part of a field that uses quantile regressions to investigate exchange rates issues. Several studies used quantile regressions to analyze exchange rate dynamics. For example, Nikolaou (2008), with semi-parametric and non-parametric frameworks based on quantile autoregressions, investigates the question of mean reversion in real exchange rates. The same, using quantile regressions for six Asian countries, Tsai (2012) demonstrates that exchange rate and stock price index are negatively correlated for extreme quantiles of the exchange rate. More recently, Kuck and Maderitsch (2019), using quantile autoregression, find a negative autocorrelation for central quantiles of exchange rates and positive autocorrelation for extreme quantiles. Our paper goes beyond quantile regressions and uses the estimated quantile functions to fit a distribution. Koenker (2005) provides a formula to transform the estimated conditional quantile functions into conditional densities. After him, Gaglianone and Lima (2012) build on this framework and fit an Epanechnikov Kernel to estimate the conditional densities of the US unemployment rate. The same, Korobilis (2017) uses the same methodology in a panel framework. More closely to our paper, Adrian et al. (2019) fit a Skewed t-distribution and estimate the conditional distribution of the US growth.

Second, our paper is also related to the literature that investigates the field of currency risk. This strand of the literature has not been widely explored. A first way to evaluate the currency risk is to use the value-at-risk (VaR). As an example, based on the value-at-risk methodology and three different models, Sarno and Valente (2005) evaluate the risk associated with eight currencies. The same, Bredin and Hyde (2004) compute different VaR based on various methodologies for a portfolio of exchange rates. In the same vein, Akhtekhane and Mohammadi (2012) evaluate the daily VaR for the Rial-Euro exchange rate. More recently, Chulia et al. (2018) evaluate the downside risk spillover for twenty advanced and emerging foreign exchange markets using a Conditional Autoregressive value-at-risk (CAViAR) model.

Finally, we are also close to a strand that has focused on forecasting exchange rate densities and evaluating them. One of the first articles to consider the issue of the density forecast of exchange rate is Diebold et al. (1999). With high-frequency data, they provide a framework to compute half-hour-ahead density forecasts for the Deutschmark and the Yen. After this study, a few researchers investigate this subject. Christoffersen and Mazzotta (2005), with a dataset of daily data on over-the-counter currency options prices, provide one-month-ahead and three-month-ahead density and interval forecasts. Sarno and Valente (2005) also investigate this field and evaluate the performance of a vector error correction model to forecast exchange rate density. In the same vein, Rapach and Wohar (2006) evaluate the performance of nonlinear models to forecast densities of the US dollar. More recently, Gaglianone and Marins (2017) use a broad range of statistical and economic-based models and construct multi step-ahead density forecast of the Brazilian Real. Most of the previous studies use the methodology based on the probability integral transform (PIT) proposed by Diebold et al. (1998) to evaluate the accuracy of the predictive densities.

Our paper differs from the existing literature in various dimensions. First, we use a better approach to measure the risk. Indeed, we rely on the Expected Shortfall and its upper tail counterpart, the Expected Longrise. The literature broadly demonstrates that the Expected Shortfall has several advantages over the VaR. The main advantage of the Expected Shortfall is that it permits us to take into account the tails of the distribution. As the exchange rates are fat-tailed, considering a measure which does not account for extreme tails is not appropriate. Moreover, we also consider the Expected Longrise. In some cases, the appreciation of a currency turns out to be a problem. For example, the Brazilian Real in 2012 largely appreciated after rate cuts in the United States, Europe, and Japan and Brazilian authorities talked about "currency war". Hence, the need to account for appreciation risk. Concerning the densities forecasts, we rely on a more recent test namely the test of Rossi and Sekhposyan (2019). Contrariwise to existing procedures that assess the predictive densities at their pseudo-true values, the test of Rossi and Sekhposyan (2019) permits to evaluate the predictive densities given the specific model and estimation techniques of the researcher and thus evaluate the predictive densities at the estimated parameter values of the model. Our paper is a complementary tool in this flourishing field and tries to apply more parsimonious methodologies to compute density forecasts and to evaluate them with very recent evaluation test.

The remainder of the paper is organized as follows. Section 2 presents the methodological framework and the data. Section 3 depicts the results. In section 4, we provide the out-of-sample densities and their absolute and relative evaluation. And section 5 concludes.

2 Methodological framework

We follow a two-step approach to compute the conditional densities. We begin by estimating the conditional quantiles on the differential of interest rates. Then we fit a skewed t-distribution based on the quantile functions obtained in the first step. The empirical framework is centered around the traditional Fama equation. We rely on this equation because it permits us to evaluate the risk associated with the monetary policy stance. Moreover, it is simple, founded and represents a benchmark when modeling exchange rates. We investigate four different horizons : one month, three months, six months and twelve months. In this way, we will compare the evolution of the results according to the horizon forecast. We consider the following Fama equation :

$$s_{t+h} - s_t = \alpha + \beta(i^{us}_{t,h} - i^{*}_{t,h}) + \epsilon_{t+h}, \qquad h = 1, 3, 6, 12 \tag{1}$$

with s_t the logarithm of exchange rate measured as the number of US dollars per foreign currency unit, i^{us} and i^* are respectively the US interest rate and the foreign interest rate.

2.1 Conditional densities and risk measures

We rely on quantile regressions to estimate equation 1. Quantile regressions permit to estimate conditional quantile functions. This method is based on the asymmetric minimization of the weighted absolute errors, in opposition to the ordinary least squares (OLS), which minimizes the sum of squared errors. The most important motivation for preferring the quantile regressions to the OLS is that the former method depicts in a better way and with more accuracy the relationship between random variables. Moreover, as argued by Koenker and Bassett (1978), in a non-Gaussian settings, quantile regressions frameworks provide more robust and efficient estimates compared to conventional least squared estimators. Another justification for using quantile regression in our paper is that the impact of the conditioning variables (the interest rates differential and the intercept) may depend on the distribution of exchange rates.

To simplify the equations that will follow, let us denote y_{t+h} the exchange rate growth $s_{t+h} - s_t$ and x_t the vector of the conditioning variables. By minimizing the quantile weighted absolute value of errors, we obtain the following slope estimator :

$$\hat{\beta}_\tau = \underset{\beta_\tau}{\operatorname{argmin}} \sum_{t=1}^{T-h} (\tau.\mathbb{1}_{(y_{t+h} \geq x_t\beta)} \mid y_{t+h} - x_t\beta_\tau \mid + (1-\tau).\mathbb{1}_{(y_{t+h} < x_t\beta)} \mid y_{t+h} - x_t\beta_\tau \mid), \quad (2)$$

where $\mathbb{1}_{(.)}$ is the indicator function. $\hat{Q}_{y_{t+h}|x_t}(\tau|x_t) = x_t\hat{\beta}_\tau$ represents the predicted values and are a consistent linear estimator of the conditional quantile function of y_{t+h}, as argued by Koenker and Bassett (1978). For low (high) level of τ, the τth conditional quantile of the exchange rate growth $(x_t\hat{\beta}_\tau)$ describes the behavior of the dependent variable y_{t+h} at the left (right) tail of the distribution.

The quantile functions from the quantile regressions can be used to obtain the probability density function. The literature investigates various ways to use the inverse CFD (ICDF) obtained from quantile regressions and transform them into estimated conditional densities. For example, Gaglianone and Lima (2012) and Korobilis (2017) use the Epanechnikov Kernel to smooth their ICDF and to estimate the conditional densities. In the same vein, Adrian et al. (2019) use the quantile interpolation framework to fit a skewed t-distribution. We employ the same methodology as Adrian et al. (2019). Previous researchers consider different distributions to model the exchange rate. The possibility that the Normal distribution can fit exchange rate has been discussed in the literature. For instance, Coppes (1995) argues that the exchange rates monthly returns are normally distributed. However, theoretical and empirical studies demonstrate that exchange rates follow distributions with fatter and more asymmetric tails compared with those that prevail under Normal distribution (see De Vries and Leuven (1994) for some properties of exchange rates). The skewed t-distribution, developed by Azzalini and Capitanio (2003), presents the great advantage of encompassing a large set of distributions. According to the value of its four parameters, the skewed t-distribution can become a particular distribution. That way, we are almost sure to get closer to the true distribution of the exchange rate growth. The probability density function of the skewed t-distribution is written as follow :

$$f(y;\mu,\sigma,\alpha,\nu) = \frac{2}{\sigma} t\left(\frac{y-\mu}{\sigma};\nu\right) T\left(\alpha\frac{y-\mu}{\sigma}\sqrt{\frac{\nu+1}{\nu+(\frac{y-\mu}{\sigma})^2}};\nu+1\right), \tag{3}$$

with the location μ, the scale σ, the fatness ν and the shape α. $t(.)$ and $T(.)$ are respectively the PDF and the CDF of the student t-distribution. Three particular cases can be mentioned. First, if the shape parameter α is null, the skewed t-distribution reduces to a t-distribution. Second, the distribution is a traditional Gaussian distribution if $\alpha = 0$ and $\nu = \infty$. Lastly, when $\alpha \neq 0$ and $\nu = \infty$, the distribution is a Normal skewed.

The quantile interpolation methodology consists in estimating the four parameters of the skewed t-distribution by minimizing the squared distance between the estimated conditional quantiles and the ICDF of the skewed t-distribution. As the skewed t-distribution has four parameters, we run the quantile interpolation for four different percent quantiles targets : 5%, 25%, 75% and 95%. In other words, for each month, we choose the four parameters of the skewed t-distribution to match the four different percentiles mentioned above. The optimization equation can be written as follow :

$$\{\hat{\mu}_{t+h}, \hat{\sigma}_{t+h}, \hat{\alpha}_{t+h}, \hat{\nu}_{t+h}\} = \underset{\mu,\sigma,\alpha,\nu}{\operatorname{argmin}} \sum_{\tau} \left(\hat{Q}_{y_{t+h}|x_t}(\tau|x_t) - F^{-1}(\tau;\mu,\sigma,\alpha,\nu)\right)^2, \tag{4}$$

with $F^{-1}(.)$ the inverse cumulative skewed t-distribution. The result of this optimization is a set of the four estimated parameters for each period and thus the evolution of the fitted conditional distributions.

The tails of the estimated conditional distributions can be used to estimate the risk of depreciation/appreciation. One of the most widely used risk measures is the value-at-risk (VaR). The VaR is defined as the maximum loss for an investment given a confidence level and a specific investment period. However, the drawbacks of the VaR lead researchers to compute new risk measures. A well-known drawback of the VaR is its incapacity to account for severe losses. Moreover, the VaR does not respect some mathematical properties. More precisely, it does not satisfy the subadditivity property [1]. In opposition, the Expected Shortfall is a coherent measure and is capable to account for extreme losses. The Expected Shortfall is defined as the average of all losses which are greater (or equal) than the VaR. For all these reasons, the Expected Shortfall is preferred to the VaR. As we are also interested in the appreciation risk, we consider the upper tail counterpart of the Expected Shortfall named the Expected Longrise.

The Expected Shortfall/Longrise can be expressed as follow :

$$ES_{t+h} = \frac{1}{\pi}\int_0^{\pi} \hat{F}^{-1}_{y_{t+h}|x_t}(\tau|x_t)\,\mathrm{d}\tau \qquad EL_{t+h} = \frac{1}{\pi}\int_{1-\pi}^{1} \hat{F}^{-1}_{y_{t+h}|x_t}(\tau|x_t)\,\mathrm{d}\tau, \tag{5}$$

with π the risk level and $\hat{F}^{-1}_{y_{t+h}|x_t}(\tau|x_t)$, the estimated conditional cumulative skewed t-distribution. In this paper, we consider a risk level of 5% for the Expected Shortfall/Longrise. The Expected Longrise and the Expected Shortfall are respectively interpreted as the appreciation risk and the depreciation risk of the foreign currency.

The computation of risk measure (VaR or Expected Shortfall/Longrise) is generally followed by the backtesting procedure, which is " *a set of statistical procedures designed to check if the real losses are in line with the value-at-risk forecasts*", as defined by Jorion (2007). The literature documents different methodologies to backtest the Expected Shortfall (see Engle and Manganelli (2004), Christoffersen (2011), McNeil and Frey (2000) or Nolde and Ziegel (2017)). In this paper, we rely on the test of Bayer and Dimitriadis (2019). This test is based on a joint regression that models both the VaR and the Expected Shortfall. If the Expected Shortfall is correctly specified, then the slope parameter associated with the Expected Shortfall and the intercept should be respectively equal to 1 and 0. A Wald statistic permits to test for the values of the parameters. More precisely, their procedure is based on the following joint equation :

1. According to the subadditivity property, the VaR of a portfolio must not be greater than the sum of the VaRs of the individual assets in the portfolio.

$$y_{t+h} = \gamma + \theta \hat{v}_{t+h} + \epsilon^v_{t+h} \qquad and \qquad y_{t+h} = \alpha + \beta \hat{es}_{t+h} + \epsilon^e_{t+h} \tag{6}$$

where y_{t+h} represents the exchange rate growth, $\hat{es}_{t+h}$ and $\hat{v}_{t+h}$ are respectively the estimated Expected Shortfall and VaR. ϵ^v and ϵ^e are error terms. The null hypothesis of correct specification of the Expected Shortfall is given by :

$$H_0 : (\alpha, \beta) = (0, 1) \quad against \quad H_1 : (\alpha, \beta) \neq (0, 1), \tag{7}$$

The test statistic is given by :

$$T_{ESR} = T \left((\hat{\alpha}, \hat{\beta}) - (0, 1) \right) \hat{\Sigma}^{-1}_{ES} \left((\hat{\alpha}, \hat{\beta}) - (0, 1) \right)^{'}, \tag{8}$$

with $\hat{\Sigma}_{ES}$ the estimator of the covariance matrix of the parameters (α, β) and T, the size of the sample. We also backtest the expected longrise. The procedure is quite the same except that we use the negative returns instead of the returns themselves.

2.2 Data

The data set for this paper includes monthly data spanning from 2000M1 to 2017M12. We collect data on end-of-month exchange rates and interest rates for nine advanced economies, namely : Australia, Canada, Euro Zone, Japan, Norway, Sweden, Switzerland, United Kingdom, and the USA as the reference country. The exchange rates are extracted from the Federal Reserve database, H.10 release. The interest rates are from Datastream, and they are measured as the midpoint of bid and offer rates for h-month Eurocurrency rates. For the robustness analysis, we use the economic policy uncertainty of Baker et al. (2016). [2]

2. The EPU is available for only six countries of our sample : Australia, Canada, Euro Zone, Japan, Sweden and the United Kingdom.

3 Results

3.1 Conditional densities

The results of the quantile regression estimations are presented in Figure 2. It presents the evolution of the estimated Fama parameter through quantiles and suggest that the impact of interest rates differential is heterogeneous across exchange rate quantiles. A common pattern emerges from these results. While the slope parameter is positive for the left tail of the distribution, it decreases over quantiles and switches to negative for the right tail. This result is strengthened for the twelve-month-ahead framework. According to Figure 2, a positive shock of the US interest rate generates different effects according to the distribution. In a period of high appreciation of the US dollar (left tail of the distribution), a rise of the US interest rate tends to induce a depreciation of the US dollar. Alternatively, in a period of high depreciation of the US dollar (right tail of the distribution), a rise of the US interest rate induces an appreciation of the US dollar. This result is another evidence of the presence of nonlinearity in the relationship between exchange rates and interest rates differential. The literature widely investigates this issue and finds many explanations for this nonlinear dynamic among them transaction costs (Baldwin (1990), Michael et al. (1997)), central bank interventions (McCallum (1994), Mark and Moh (2007)) or limits to speculation (Lyons (2001), Baillie and Kilic (2006), Sarno et al. (2006)). The quantile regression permits to deepen the question and reveals that the impact of the interest rates differential depends on the distribution of exchange rates. For the lower quantiles, the Fama coefficient is found to be positive. On the contrary, we find the traditional Fama puzzle for the upper quantiles.

The quantile interpolation provides us with the conditional densities on interest rates differential. The results are presented in Figure 3. We plot the fitted conditional densities for two periods : a normal period and a stress period [3]. The conditional densities are very different from a currency pair to another. However, this graph permits us to highlight the time-varying feature of all the conditional densities. Indeed, they clearly evolve, especially the tails of the distribution. In other words, the tails of the distribution, even the central tendency, are very sensitive to particular events such as crises.

3. The normal period referred to 2006 January and the stress period referred to the height of the financial crisis, around 2008 October

For several currency pairs, the changes appear principally in the left tail (depreciation of the foreign currency). More explicitly, the left tail is fatter during the 2007-2008 crisis, meaning by the way that, during the 2008 crisis, the foreign currency is subject to an increased chance of depreciation. Moreover, the conditional densities shifts to the left during the great financial crisis. These results reflect the increasing probability of depreciation of the currencies against the US Dollar during crises and are not surprising insofar as the US dollar is a safe-haven. As a safe-haven currency, the US dollar remains one of the most reliable currencies during a crisis. Kaul and Sapp (2006) highlight that during the Y2K (Year 2000) fears [4], investors believed in the USD, and evidence demonstrated that the presence of safe-haven flows towards the US dollar. The same, during the 2007 financial crisis, even if the USA was the epicenter of the crisis, Habib and Stracca (2012) argue that the USD acts as a safe-haven currency.

Switzerland turns out to be a particular case as it acts as a go-to choice during turmoils. Conditional to the differential of interest rates, the conditional distributions shift to the right during the 2008 crisis.

3.2 Expected Shortfall/Longrise

Figure 4 depicts the evolution of the currency risks and highlights a real asymmetry between the upside and the downside risks. We notice that, globally, the depreciation risks are more volatile than the appreciation risk. The impact of the crises is clearly identifiable by significant drops in the depreciation risks. Thus, the currency pairs are exposed to more considerable depreciation risks, and appreciation risks are more stable and less significant. In other words, the monetary polisy stance has a significant impact on the depreciation risk while it has much less impact on the appreciation risk. As a result, against the USD and conditional to the monetary policy stance, most of the global currencies face an important downside risk, while the upside risk is generally less important.

However, the risk measures are heterogeneous across the currency pairs. Some of them are more resilient to shocks and crises. The Swiss Franc and the Japanese Yen present more moderate falls during the crisis and less volatilities in their depreciation risks. Yet,

4. The fear of Y2K refers to the phenomenon at the beginning of the 21st century where many computer users feared huge errors in the calculation of dates because many programs represented four-digit years with only the last two digits, which could lead to not distinguishing the years 1900 to 2000.

their upside risks are extremely volatile. [5]

The panel A of table 1 presents the variance for the Expected Shortfall/Longrise. This table clearly confirms the previous results. The variance of the Expected Shortfall is generally larger than the variance of the Expected Longrise, reflecting a more significant importance of the downside risk over the upside risk. The particular cases of the Swiss Franc and the Japanese Yen, especially for the one-month-ahead, are highlighted by this table.

The risk measures perfectly highlight some particular events. For example, all the currencies depict a sharp drop in the depreciation risk around the global financial crisis in 2007-2008. This period is associated with global uncertainty and strong expansionary monetary policies. This leads to a large flight-to-quality towards the US dollar. For example, the one-month-ahead depreciation risk of the euro goes from -0.05 at the beginning of 2007 to -0.08 in the height of the crisis (October 2008).

The panel A of table 2 presents the results of the test of Bayer and Dimitriadis (2019). The p-values of the test are presented. For most of the currency pairs and horizons, the null hypothesis is not rejected at 5 % : the Expected Shortfall and Longrise are well specified. This result means that our depreciation risk and appreciation risk can be reliable and be used to monitor exchange rate risk measurement.

3.3 Robustness analysis

In this part, we try to evaluate the out-of-sample performance of our methodology. We repeat the quantile matching estimations and reestimate the measures of risks by considering a Normal distribution instead of a skewed t-distribution. We try to compare the ability of these two distributions to compute the best risk measure. We use the test of Bayer and Dimitriadis (2019) to compare the ability of these two distributions to evaluate the risk measures. The more a risk measure reject the misspecification, the more this measure is better specified. The panel B of table 2 presents the results of the test of Bayer and Dimitriadis (2019) under the Normal distribution. This table confirms the previous results of the literature and shows clearly that the Normal distribution is not able to model currency risks. Most of the p-values are inferior to the confidence level 5% rejecting

5. To a less extend for the Japanese Yen.

the null hypothesis of correct specification of the Expected Shortfall/Longrise.

The main reason that justifies these results is that the Normal distribution systematically underestimates the risk as it does not account for fat tails. As the exchange rate series exhibit extreme values and fat tails, these results illustrate perfectly that the Normal distribution is not well designed to model exchange rates.

Concerning the augmented Fama equation, figure 5 confirms the previous findings. All the currency pairs depict strong asymmetries in the evolution of their risk measures and suggest that the appreciation risk is less volatile than the depreciation risk. As a result, the monetary policy stance and the uncertainty largely drive the downside risk of the currency pairs. Introducing the uncertainty in the Fama equation permit us to highlight some particular events. For example, if we consider the the British case, we denote a large drop, especially for the one-month-ahead and the three-month-ahead, in the depreciation risk around 2016 reflecting the risk associated with the Brexit referendum. The panel B of table 1 presents the variance of the risk measures under the augmented Fama. As previously mentioned for the Fama equation, the extended Fama equation also permits to highlight a more volatile downside risk compared to the upside risk.

3.4 Implications of our results

Even if the documentation on the safe-haven currencies is large, several aspects remain unexplored. Moreover, there is no real consensus about the safe-haven status of the currencies. The role of the US Dollar as safe-haven currency has been largely discussed in the literature and several researchers demonstrate that the international status of reserve currency of the US Dollar could make this currency a safe-haven. However, conclusions for the Swiss Franc, the Japanese Yen or even the Euro are more mitigated. This is surely due to the various definitions and characteristics of a safe-haven currency among studies. Hossfeld and MacDonald (2015) define a safe-haven currency as a currency whose effective returns are negatively correlated to global stock market returns during turmoils. They find that only the US Dollar and the Swiss Franc can be qualified as safe-haven currencies. In opposition, according to Coudert et al. (2014), safe-haven currencies depict positive excess returns during crises. They argue that the US Dollar and the Japanese Yen are the only candidates for the role of safe-haven currency. Other studies do not focus on which currency acts as a safe-haven but on the fundamental drivers of the safe-haven currencies.

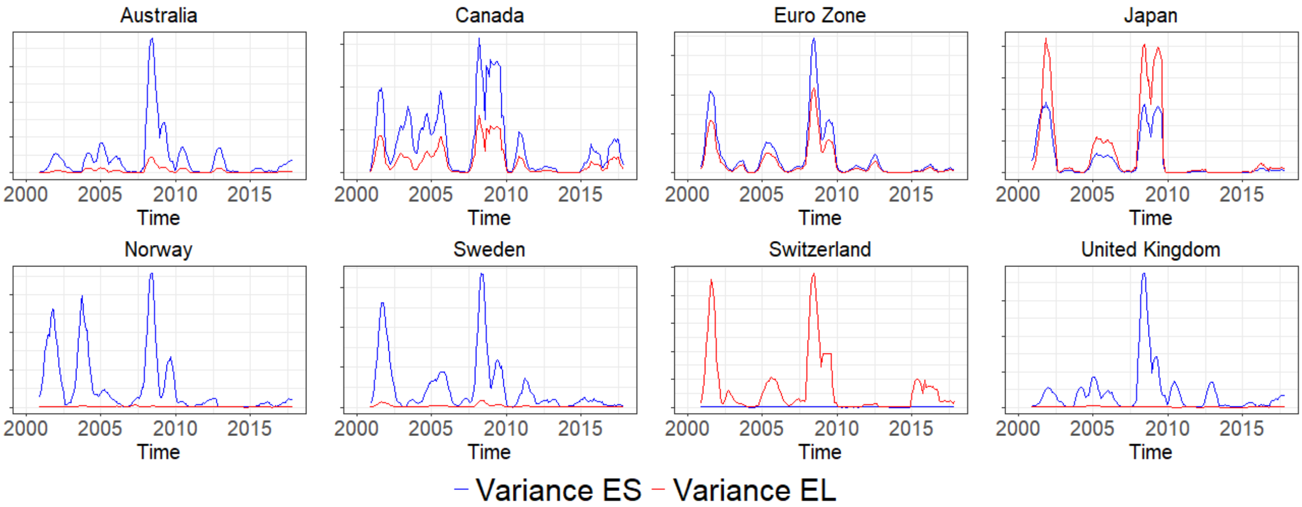
Habib and Stracca (2012) argue that factors such the net foreign asset position, the size of the stock market and the interest rate spread against the US drive the safe-haven behavior of a currency. However, among this diversity of results, a common result emerges. Indeed, the safe-haven behavior of a currency follows a nonlinear fashion insofar as, the safe-haven behavior is more pronounced during period of stress.

Our paper is a complementary tool to investigate this question and our methodology can be used to assess whether a currency behaves as a safe-haven. We define safe-haven currencies as those able to depict an appreciation risk more volatile than a depreciation risk, especially during crises. Indeed, the more the downside/upside risk is stable over time, the less the currency is vulnerable to it. In opposition, when the downside/upside risk largely comoves with the interest rates spread and depicts strong volatilities, then, conditional to the monetary policy stance, the currency is more vulnerable to the risk. Consequently, if the volatility of the upside risk is higher than that of the downside risk, particularly during turbulences, the currency is then more exposed to a greater appreciation risk, thereby reflecting a flight-to-quality/safe-haven phenomenon.

We compute the rolling variances of the one-month-ahead [6] Expected Shortfall/Longrise, with a window of twelve months. Figure 1 presents the evolution of the rolling variances over time. Our results can be separated into two groups. The first group of countries contains Australia, Canada, the Euro Zone, Norway, Sweden and the United Kingdom. All these currencies present a more volatile depreciation risk. Moreover, the volatility of the depreciation risks depict a surge during crises, especially during the 2007-2008 crisis. In opposition, the volatility of the appreciation risk of these currencies remains flat. The second group refers to Japan, Switzerland. In opposition to the first group, the appreciation risk is more volatile than the depreciation risk. Furthermore, the volatility of the appreciation risk largely increases during the last financial crisis. As a result, three currencies emerge as safe-haven namely the US Dollar, the Swiss Franc and the Japanese Yen.

6. As the flight-to-quality refers to short-term phenomena, we focus only on the one-month-ahead framework.

Figure 1: Evolution of the variance of the Expected Shortall and the Expected Longrise.



4 Out-of-sample densities

4.1 The out-of-sample estimation schemes

In this section, based on a rolling window estimation scheme, we compute a sequence of out-of-sample conditional densities. More in detail, we estimate the equation 1 using quantile regression with data from 2000M1 to 2004M12. Then, we predict the h-step ahead conditional quantile (1 month, three months, six months and twelve months). Once the out-of-sample conditional quantile is computed, we smooth it by fitting a skewed t-distribution and estimate the h-step ahead out-of-sample densities. We repeat the operation for data ranging from 2000M2 to 2005M1. And so on, until the end of the sample. At the end of the process, we obtain a sequence of P out-of-sample predictive densities denoted $\{\hat{f}_{t+h}(y_{t+h}|\zeta^t_{t-R+1})\}^T_{t=R}$ where $\hat{f}_{t+h}$ denotes the probabilitity density function of a skewed t-distribution, ζ^t_{t-R+1}, the set of information available at each iteration, and R the size of the in-sample portion of the procedure. The rolling window procedure is preferred to the recursive process insofar as a rolling window permit to get better predictive densities in presence of breaks in the conditional moments of the predictive densities. As we work with financial data, the presence of breaks is obvious (see Clark (2011) and Jore et al. (2010)).

In a key paper in the field, Diebold et al. (1998) evaluate the conditional densities by testing the properties of the probability integral transforms (PITs), namely the uniformity and the independence. After them, many researchers tried to find other ways to evaluate the predictive densities (see Tay and Wallis (2000) or Corradi and Swanson (2006) for an overview of the existing tests). In this paper, we rely on the test of Rossi and Sekhposyan (2019). For each probability density function $\hat{f}_{t+h}$, the PIT is defined as the cumulative density function evaluated at the realized value :

$$w_{t+h} = \int_{-\infty}^{y_{t+h}} \hat{f}_{t+h}(y|\zeta_{t-R+1}^{t}) \,\mathrm{d}y \equiv \hat{F}_{t+h}(y_{t+h}|\zeta_{t-R+1}^{t}), \tag{9}$$

Diebold et al. (1998) demonstrate that if the predictive densities are well calibrated, then the sequence of $\{w_{t+h}\}$ is an independent and identically distributed Uniform $U(0,1)$ and its cumulative distribution function is the 45-degree line. Rossi and Sekhposyan (2019) develop a test to assess the correct specification of the predictive densities. They test the null hypothesis that the predictive densities are correctly specified :

$$H_0 : \hat{F}_{t+h}(y|\zeta_{t-R+1}^{t}) = \hat{F}_0(y|I_t) \qquad for \;\; all \;\; t = R, ..., T, \tag{10}$$

where $\hat{F}_0(y|I_t) \equiv Pr(y_{t+h} \leq y|I_t)$ is the distribution specified under the null and I_t is the true set of information.

The test is based on the probability of the out-of-sample forecasts :

$$\Psi_P(r) \equiv P^{-1/2} \sum_{t=R}^{T} \xi_{t+h}(r), \tag{11}$$

where :

$$\xi_{t+h}(r) \equiv \left(\mathbb{1}\{\hat{\Phi}_{t+h}(y_{t+h}|\zeta_{t-R+1}^{t}) \leq r\} - r \right), \tag{12}$$

$\mathbb{1}\{.\}$ is the indicator function and $r \in [0,1]$ the different quantiles.

The test statistic of Rossi and Sekhposyan (2019)' test is as follow :

$$\kappa_P = \sup_{r \in [0,1]} |\Psi_P(r)| \tag{13}$$

At α significance level, if $\kappa_P < \kappa_\alpha$, the test does not reject the null hypothesis of correct specification ; with κ_α the critical value tabulated by Rossi and Sekhposyan (2019).

Instead of applying the test on the whole distribution, an interesting way to apply the test of Rossi and Sekhposyan (2019) is to implement a graphical analysis which focuses on specific parts of the distribution such as the left tail, the right rail or the central part. The idea is to plot the CDF of the PITs, the 45 degree line and the critical values. The critical values for each quantile can be tabulated as follow : $r \pm \kappa_\alpha / sqrt(P)$. The graphical interpretation permits to go deeper in the analysis and evaluate the predictive densities at particular tails of the distribution.

We conclude the out-of-sample exercises by analyzing the relative performance of our model. Using the predictive scores defined as the predictive densities evaluated at the outturn, we compare our model to the well-known random walk. Higher predictive scores mean more accurate predictive densities. A way to compare two models is to compute their predictive scores and compare their evolutions over time or simply their average : a higher average score indicates a better out-of-sample performance (see Adrian et al. (2019) for example). However, we rely on a more formal framework and apply the test of Amisano and Giacomini (2007) to compare the scores of our models. Amisano and Giacomini (2007) derive their test from the difference between the loss functions (here, the log predictive scores) associated with the two competing models. The test statistic follows a standard Normal distribution. Given two alternative density forecasts $\hat{f}$ and $\hat{f}^{rw}$, let us define the following Log Predictive Scores Difference(LPSD) :

$$LPSD_{t+1} = \log \hat{f}_t^{rw}(y_{t+1}) - \log \hat{f}_t(y_{t+1}), \tag{14}$$

where, $\hat{f}$ and $\hat{f}^{rw}$ correspond to the predictive densities respectively associated with the Fama equation and the random walk. More formally, we test the following null hypothesis of equal predictive accuracy :

$$H_0 : E[LPSD_{t+1}] = 0, \tag{15}$$

The test statistic is written as follow :

$$t = \frac{\overline{LPSD}}{\hat{\sigma}_P/\sqrt{(}P)}, \tag{16}$$

with $\overline{LPSD} = \frac{1}{P}\sum_{t=R}^{T-1} LPSD_{t+1}$, $\hat{\sigma}_P$ is a consistent estimator of the standard error of $LPSD_{t+1}$ and P corresponds to the size of the out-of-sample portion. The test statistic follows a standard Normal distribution. If the null hypothesis of equal predictive accuracy is rejected, the choice between the two competing models depends on the sign of $\overline{LSPD}$. If $\overline{LSPD}$ is negative, one will choose the Fama equation ; otherwise, one will choose the random walk.

4.2 The out-of-sample results

Figure 6 depicts the results of the Rossi and Sekhposyan (2019)'test. The blue line depicts the CDF of the PITs. The 45 degree red line represents the CDF of the PITs under the null hypothesis. The confidence interval based on the κ_P test are also reported in dotted green line. The results of the Rossi and Sekhposyan (2019) demonstrate that our predictive densities are well calibrated. Indeed, the CDF of the PITs evolves into the confidence interval for all currencies and for all horizons. Globally, figure 6 suggests proper calibration for the predictive densities.

This result is interesting from a political point of view. Indeed, our model and specification can be used to forecast the downside and upside risks of exchange rates since they can calibrate properly the predictive densities.

Table 3 presents the results of the test of Amisano and Giacomini (2007). For the medium-run and long-run, the random walk is statistically better than the Fama equation. However, this is no longer true in the one-month-ahead framework where our the Fama equation outperforms the random walk.

The literature already investigates the ability of some models to outperform the random walk. For example, Sarno and Valente (2005) find that a Markov-switching VECM produces more accurate predictive densities than the random walk. Contrariwise to them, we employ a simple linear methodology which is more easily to estimate and replicate. The same, Gaglianone and Marins (2017) find that, with a daily frequency, the random walk can be beaten in many cases. However, they did not work out to outperform the random walk in the monthly frequency but only in a daily frequency. In that sense, our results are a complementary analyses to previous ones insofar as they are based on simple model and methodologies, and second they propose a reliable framework to estimate proper predictive densities of exchange rates, and, moreover, they produce more accurate predictive densities than the random walk in the short-run.

5 Conclusion

We investigate in this paper two critical issues concerning exchange rates. On the one hand, the measurement of currency risks. And on the other hand, the question of the predictive densities. The currency risks are measured based on the Expected Shortfall and its upper tails counterpart, the expected longrise. To do so, we begin by computing the conditional quantiles that we use to fit a skewed t-distribution. Based on this estimated distribution, we compute our risk measures. The predictive densities are evaluated using two criteria : An absolute criterion which referred to the test of Rossi and Sekhposyan (2019) and a relative criterion based on the predictive scores.

We find that the conditional densities are extremely time-varying and are affected by crises. Another interesting result is the measurement of the currency risks. We find that the depreciation risks are more volatile than the appreciation risk. In other words, most of the global currencies usually face depreciation episodes against the US dollar. These results are robust to the extension of our Fama with the economic policy uncertainty. We dive into the literature of safe-haven currencies and find that the US Dollar, the Swiss Franc and the Japanese Yen behave as safe-haven. We also investigate the ability of the Normal distribution to compute risk measures. Our results confirm the previous findings of the literature and evidence that the Normal distribution is not adequate to model exchange rates. Finally, we find that the predictive densities are well calibrated. The PIT-based test suggests, for all the currency pairs and all horizons, a proper calibration of the out-of-sample densities. More importantly, our simple linear model has better predictive accuracy in forecasting densities, especially in the short-run.

The findings of this paper are quite interesting for practitioners and also for policymakers. Predictive densities can be an excellent means to evaluate future uncertainty around exchange rates. The same, the out-of-sample depreciation/appreciation risks are proper tools for risk management of exchange rates.

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6 Appendix

Table 1: Descriptive statistics for the risk measures.

Panel A : Variance of the risk measures for the Fama equation								
Country	H = 1		H = 3		H = 6		H = 12	
	ES	EL	ES	EL	ES	EL	ES	EL
Australia	6.48e-04	7.40e-05	6.55e-06	1.24e-04	3.00e-03	9.61e-04	5.33e-03	3.87e-04
Canada	7.65e-05	3.10e-05	7.93e-04	3.38e-05	1.44e-03	3.85e-04	5.45e-03	1.49e-04
Euro Zone	1.36e-04	8.65e-05	1.46e-04	1.07e-05	1.57e-03	2.07e-04	6.07e-04	2.49e-03
Japan	2.80e-05	4.86e-05	4.06e-04	7.73e-05	4.39e-04	3.41e-06	1.19e-03	4.63e-04
Norway	4.39e-04	4.59e-06	1.00e-03	7.31e-06	1.98e-02	1.21e-03	2.09e-02	7.54e-04
Sweden	1.96e-04	9.75e-06	1.95e-04	8.54e-06	8.08e-03	3.44e-05	4.75e-03	4.04e-04
Switzerland	4.76e-07	2.36e-04	2.24e-05	9.82e-06	6.12e-04	7.76e-04	1.09e-04	1.15e-03
United Kingdom	1.17e-04	2.76e-06	1.19e-03	2.51e-04	1.25e-02	2.30e-04	1.51e-02	5.33e-05
Panel B : Variance of the risk measures for the augmented Fama equation								
Country	H = 1		H = 3		H = 6		H = 12	
	ES	EL	ES	EL	ES	EL	ES	EL
Australia	1.16e-03	3.65e-04	2.97e-05	1.67e-04	2.85e-03	1.28e-03	5.52e-03	1.24e-02
Canada	9.55e-05	3.77e-05	1.13e-03	5.82e-05	1.76e-03	5.88e-04	7.03e-03	5.82e-04
Euro Zone	2.33e-04	1.38e-04	1.02e-03	3.63e-05	1.60e-03	4.34e-04	3.01e-03	2.49e-03
Japan	2.91e-05	3.36e-04	5.42e-04	9.26e-05	8.46e-04	2.89e-04	1.80e-03	3.65e-04
Sweden	2.56e-04	3.94e-06	4.91e-04	1.27e-04	1.25e-02	2.06e-04	3.33e-03	6.79e-04
United Kingdom	9.73e-04	8.56e-05	1.60e-03	2.90e-04	9.53e-03	4.05e-04	1.46e-02	4.58e-05

Table 2: Results of the regression-based backtesting. The p-values of the test are presented. When the p-value is superior to the confidence level 5% (in bold in the table), the null hypothesis of correct specification is not rejected.

Panel A : Results of the test for the Fama equation under the Skewed t-distribution.

Country	Expected Shortfall				Expected Longrise			
	H = 1	H = 3	H = 6	H = 12	H = 1	H = 3	H = 6	H = 12
Australia	**0.74**	**0.66**	**0.95**	**0.15**	**0.36**	**0.46**	**0.37**	**0.63**
Canada	**0.11**	**0.90**	**0.37**	**0.82**	**0.32**	**0.81**	**0.34**	**0.62**
Euro Zone	**0.93**	**0.89**	**0.18**	**0.51**	**0.52**	**0.74**	**0.97**	0.00
Japan	**0.99**	**0.40**	**0.96**	**0.05**	**0.84**	**0.59**	**0.19**	**0.60**
Norway	**0.91**	**0.23**	0.00	0.00	**0.28**	**0.62**	**0.60**	**0.88**
Sweden	**0.99**	**0.30**	**0.02**	**0.13**	**0.73**	**0.51**	**0.38**	**0.94**
Switzerland	**0.41**	0.00	**0.96**	**1.00**	**0.48**	**0.22**	**0.76**	**0.75**
United Kingdom	**0.48**	**0.99**	0.00	0.00	**0.31**	**0.40**	**0.78**	**0.70**

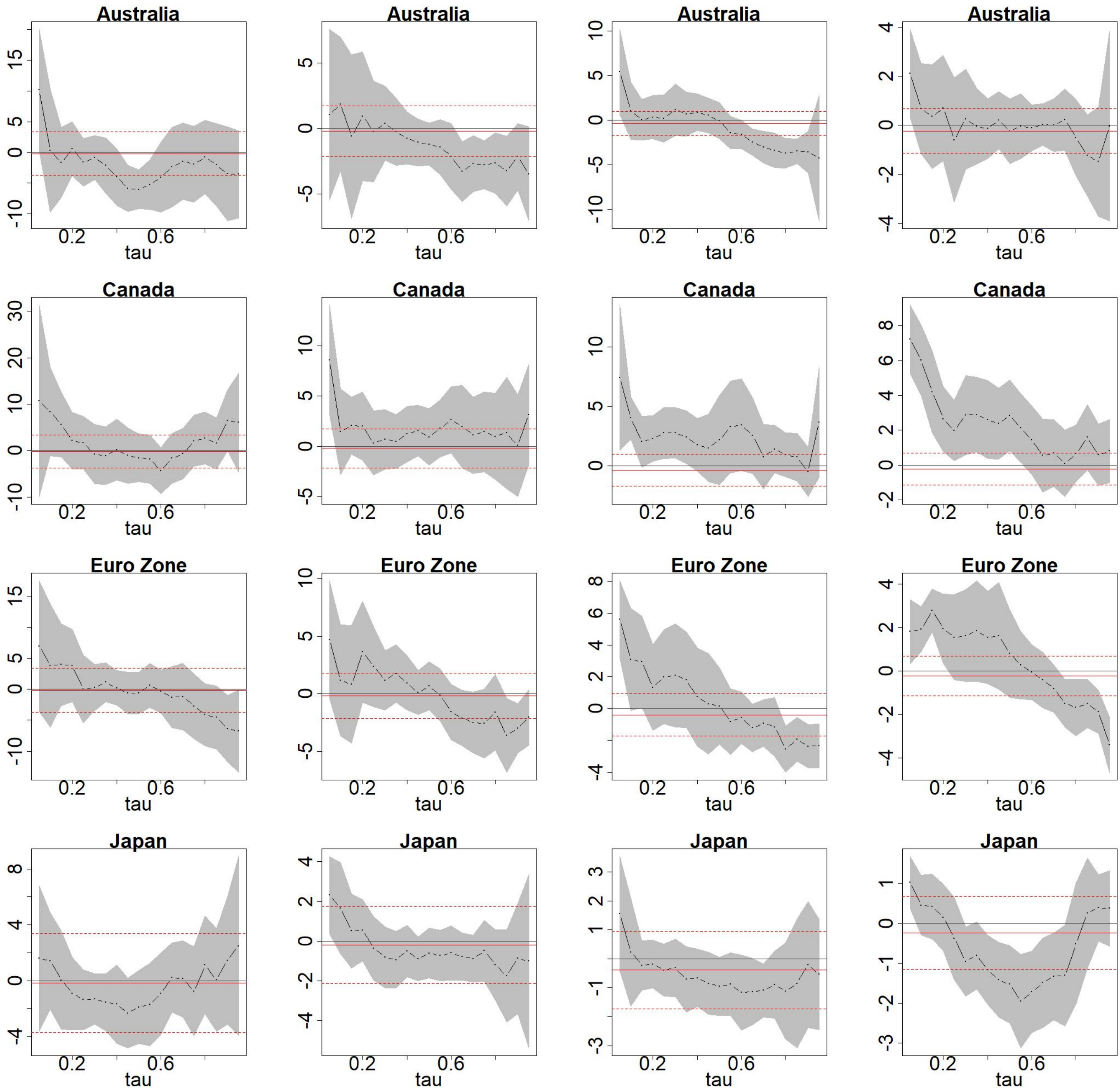
Panel B : Results of the test for the Fama equation under the Normal distribution.

Country	Expected Shortfall				Expected Longrise			
	H = 1	H = 3	H = 6	H = 12	H = 1	H = 3	H = 6	H = 12
Australia	0.03	**0.11**	0.00	0.04	0.03	**0.44**	**0.14**	0.04
Canada	0.02	0.04	0.04	0.02	**0.08**	0.01	0.00	**0.25**
Euro Zone	0.00	0.02	0.00	**0.10**	0.02	**0.46**	**0.73**	**0.72**
Japan	0.00	0.02	0.00	**0.06**	0.03	**0.24**	**0.74**	**0.72**
Norway	0.03	0.03	0.00	0.00	0.04	**0.29**	0.04	**0.80**
Sweden	**0.11**	**0.11**	0.00	0.04	0.03	**0.38**	**0.41**	**0.28**
Switzerland	**0.15**	0.00	0.00	0.03	0.01	**0.24**	0.04	0.00
United Kingdom	**0.19**	0.03	0.00	0.00	**0.06**	**0.10**	**0.10**	**0.71**

Table 3: Fama model VS Random Walk : Results of the test of Amisano and Giacomini (2007). The test statistic and the p-values of the test are presented. When the p-value is superior to the confidence level 5% and the test statistic negative (in bold in the table), then Fama outperformed the random walk.

Country	H = 1	H = 3	H = 6	H = 12
Australia	-2.19	6.95	8.33	8.08
	0.03	0.00	0.00	0.00
Canada	-1.54	4.30	5.81	7.87
	0.13	0.00	0.00	0.00
Euro Zone	-1.84	9.58	9.20	9.23
	0.07	0.00	0.00	0.00
Japan	-2.08	10.03	11.66	12.24
	0.04	0.00	0.00	0.00
Norway	-2.74	9.78	5.08	8.05
	0.01	0.00	0.00	0.00
Sweden	-1.89	10.94	9.60	10.67
	0.06	0.00	0.00	0.00
Switzerland	-1.67	7.12	11.09	9.65
	0.10	0.00	0.00	0.00
United Kingdom	-1.36	8.49	10.78	11.57
	0.17	0.00	0.00	0.00

Figure 2: Results of quantile regressions. The first, the second, the third and the last column represent, respectively, h=1, h=3, h=6 and h=12. The dotted black line represents the estimated coefficients of the quantile regression. The solid red line represents the OLS coefficient.



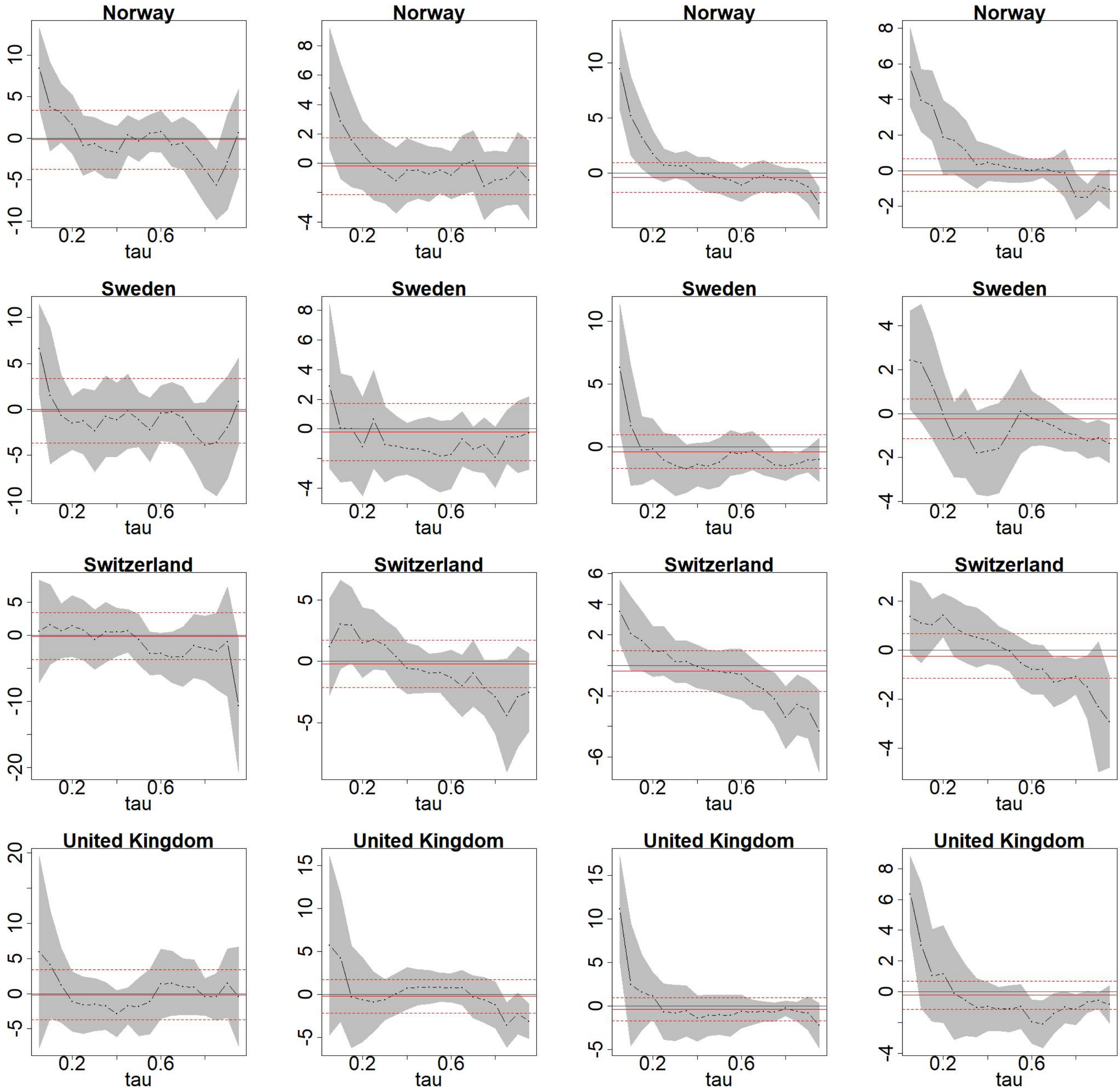

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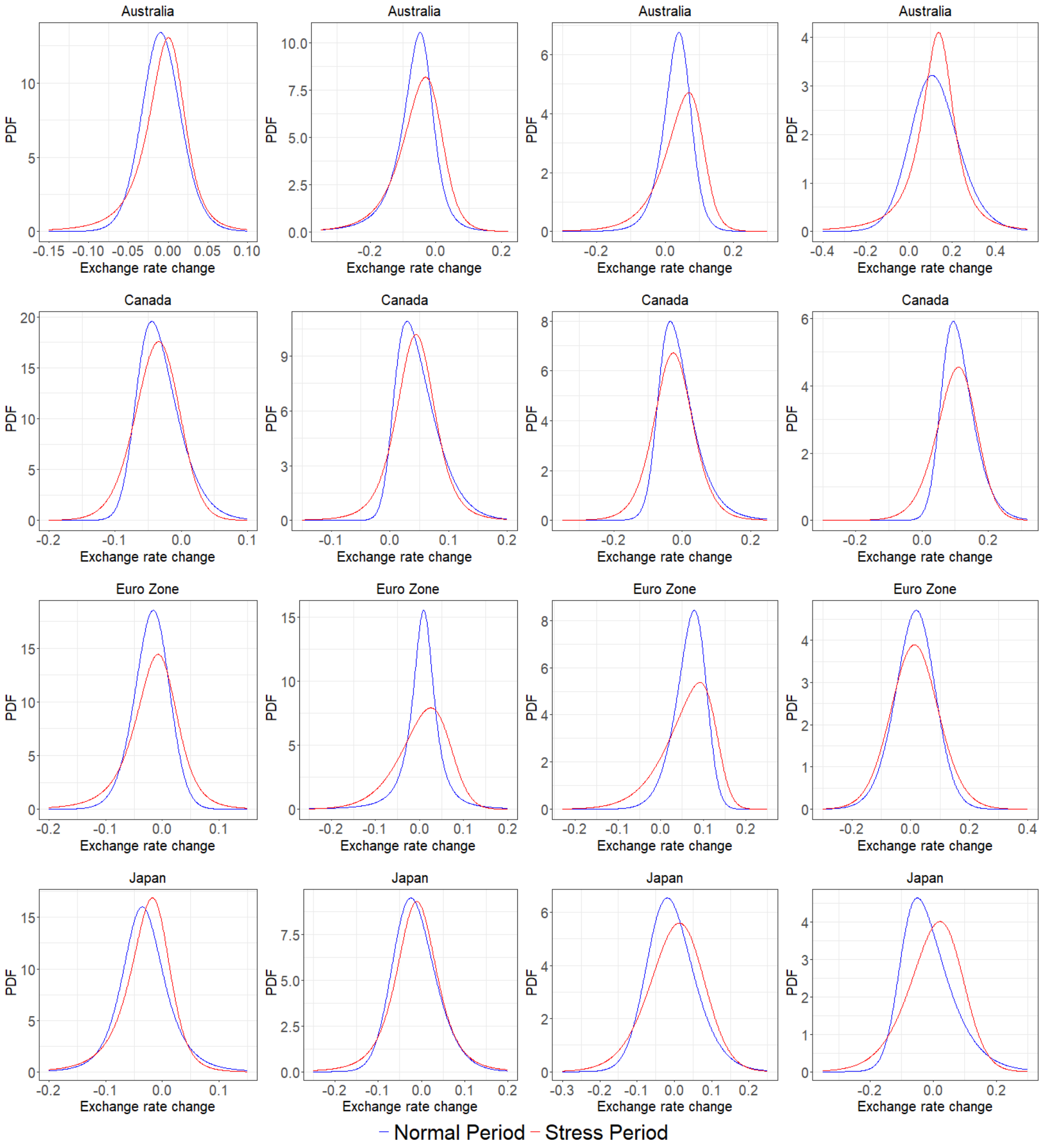
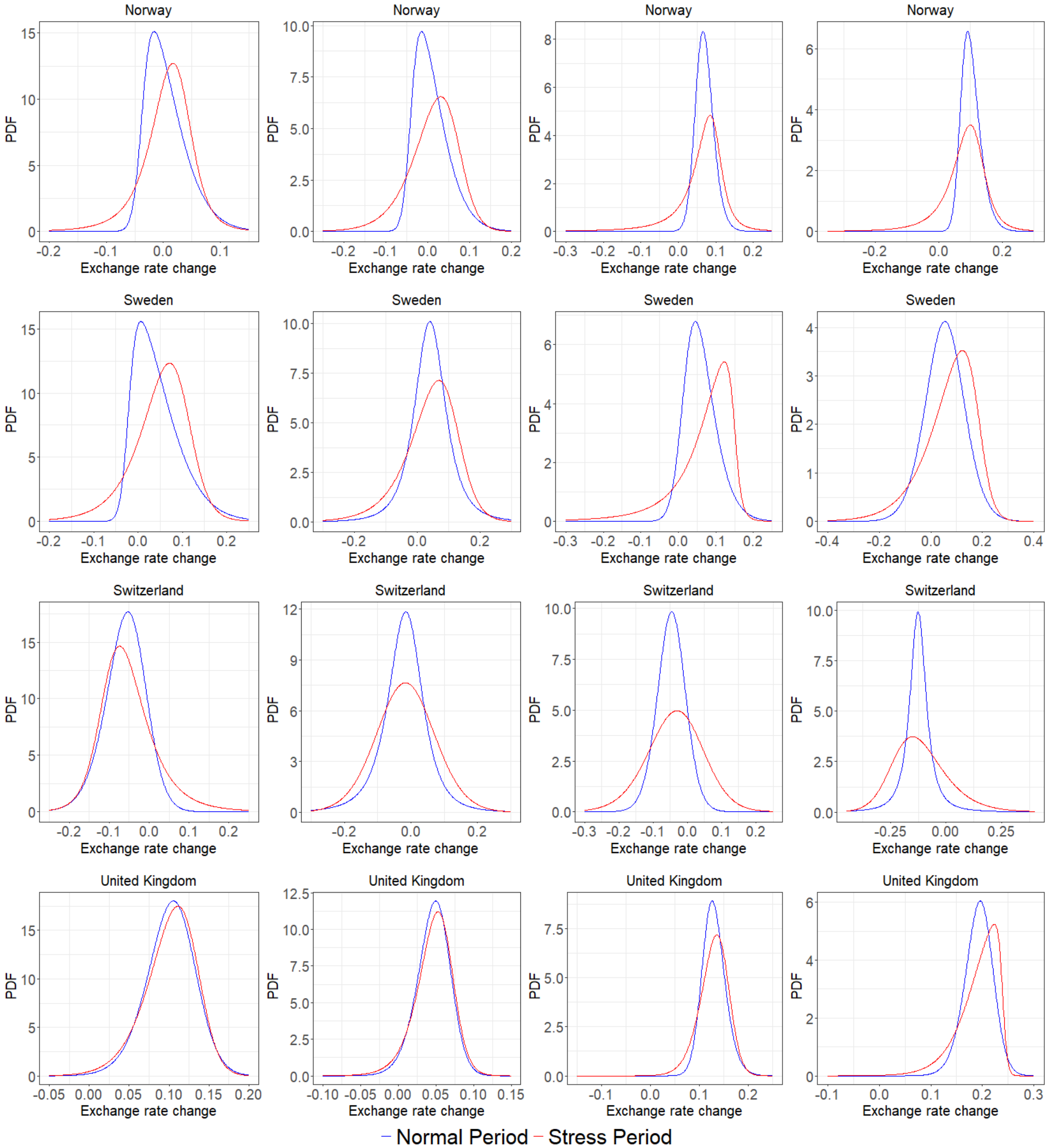
Figure 3: Estimated conditional densities. The first, the second, the third and the last column represent, respectively, h=1, h=3, h=6 and h=12. The blue and the red lines corresponds respectively to a normal period and a period of stress




Norway
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Switzerland
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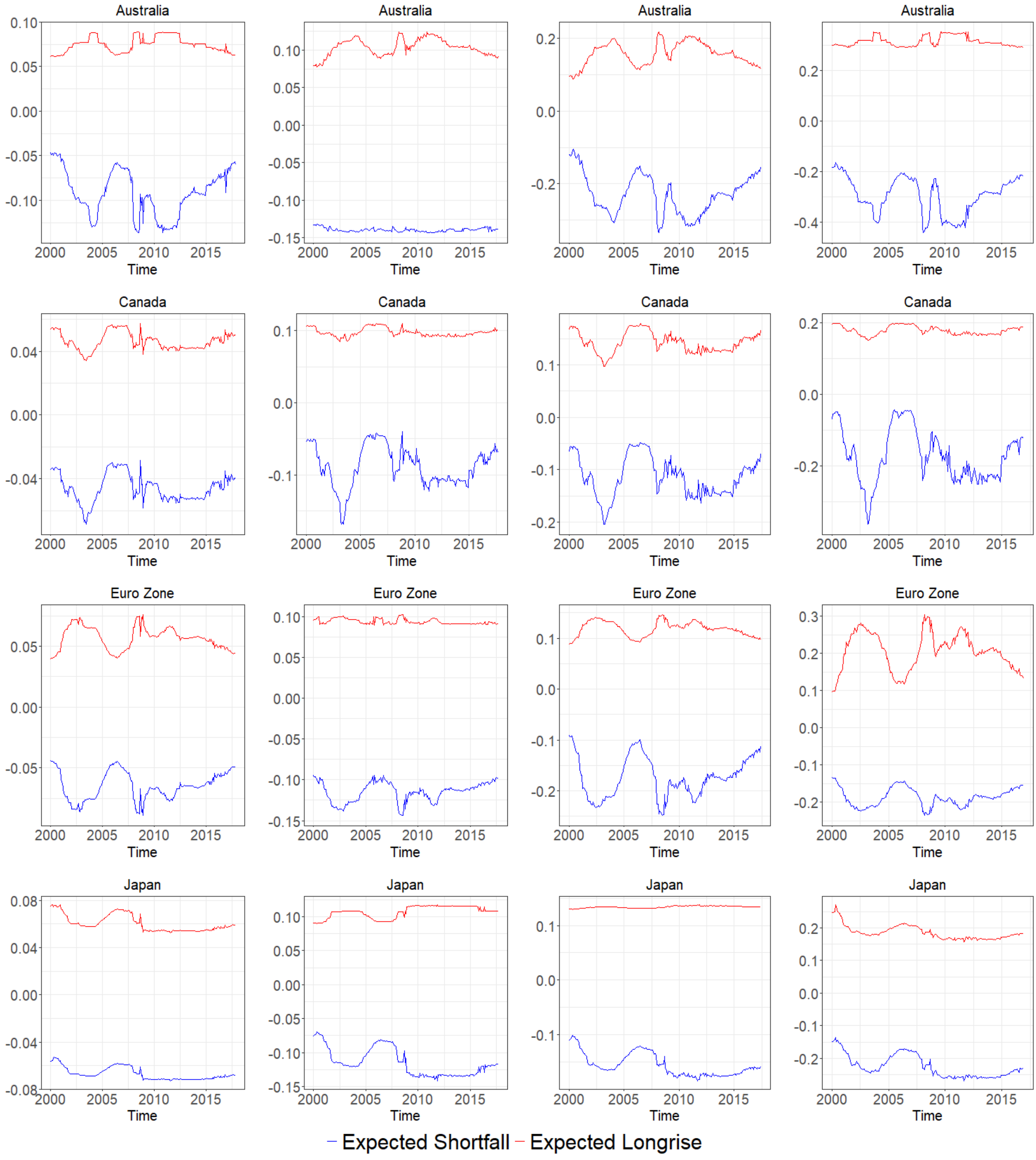
Switzerland
United Kingdom
United Kingdom
United Kingdom
United Kingdom
PDF
Exchange rate change
Normal Period
Stress Period

Figure 4: Expected Shortfall/longrise. Expected Shortfall is in blue and the red line is the Expected Longrise. The first, the second, the third and the last column represent, respectively, h=1, h=3, h=6 and h=12.



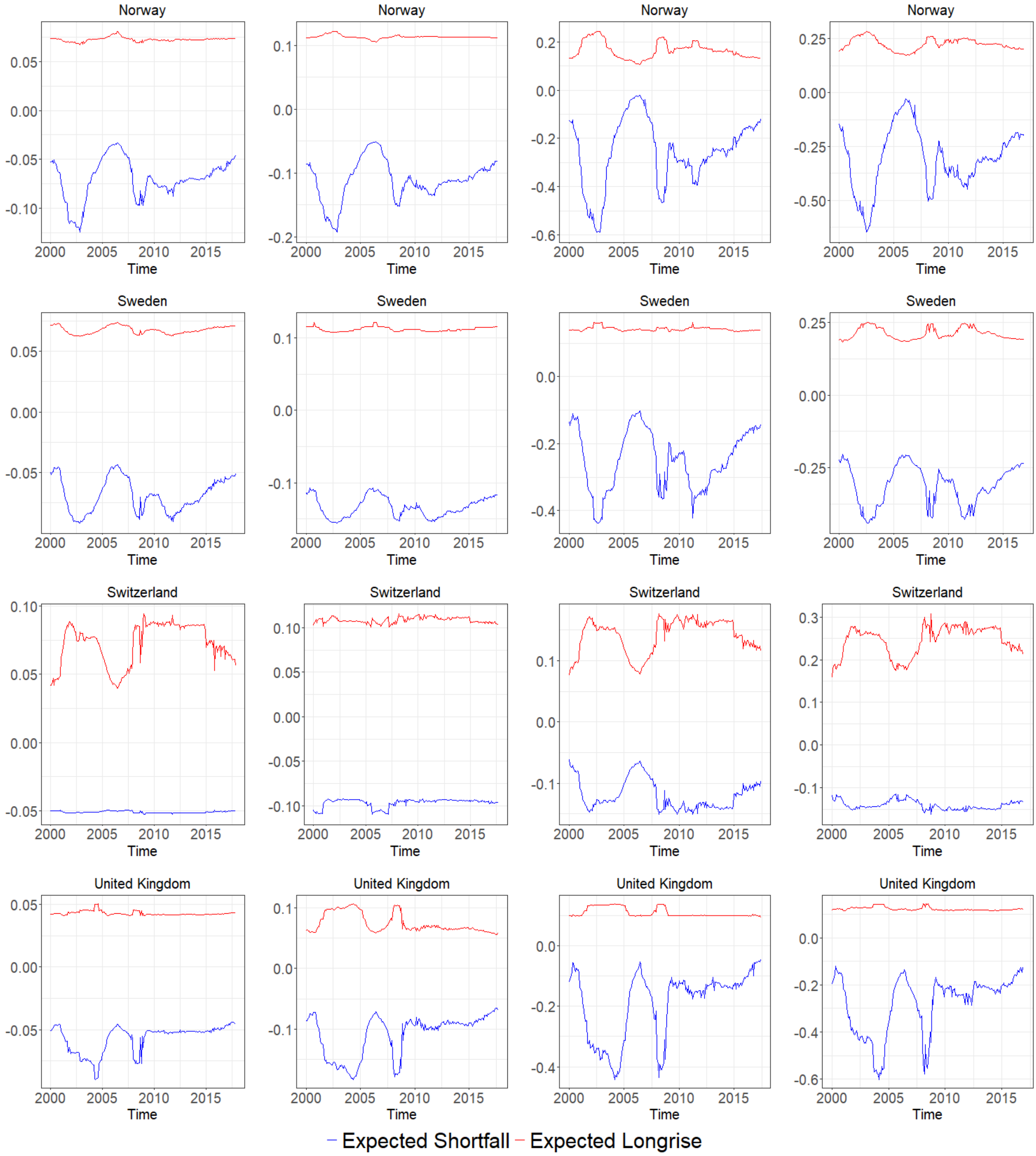

Norway
Sweden
Switzerland
United Kingdom
Time
Expected Shortfall
Expected Longrise

Figure 5: Expected Shortfall/Longrise based on the augmented Fama.

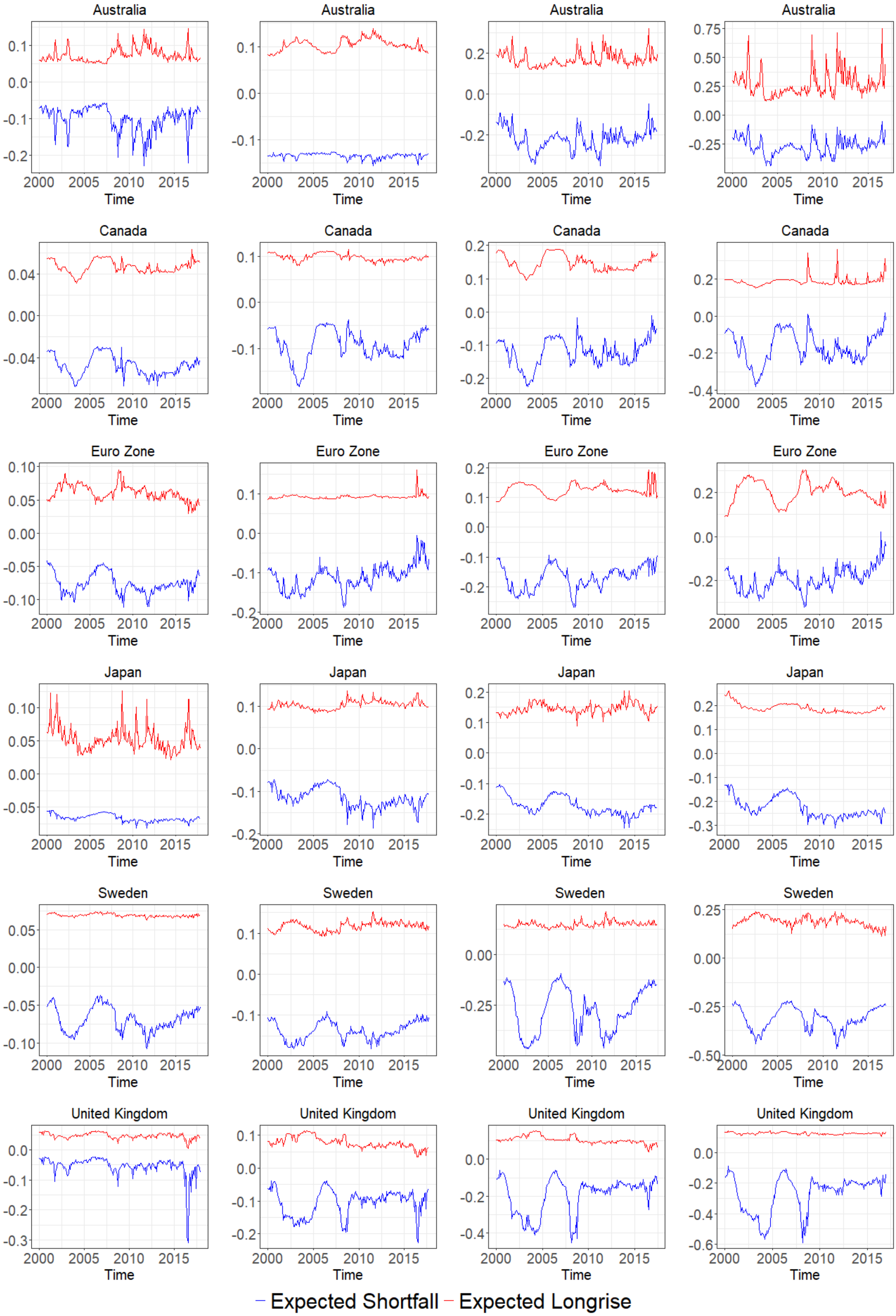
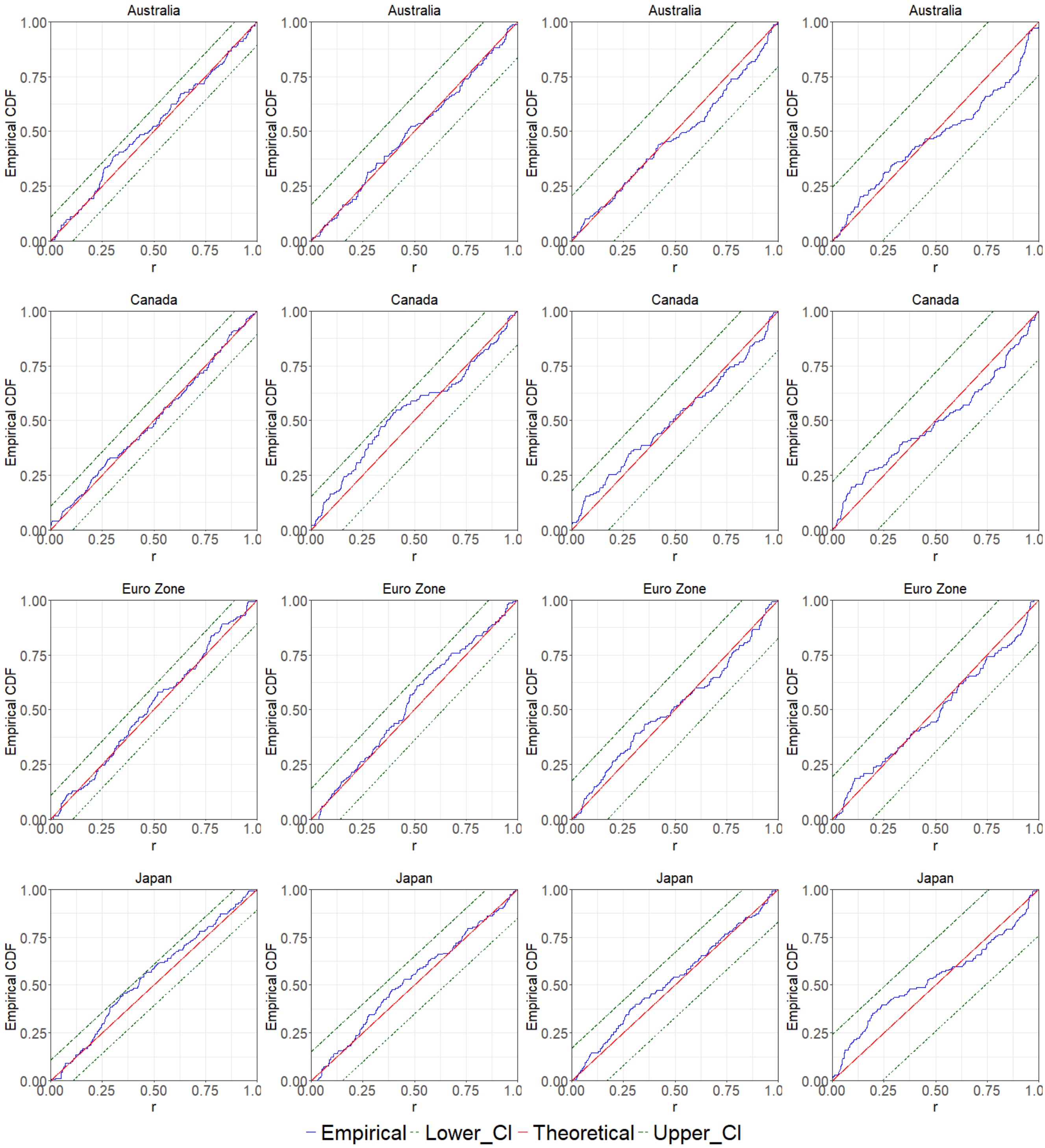
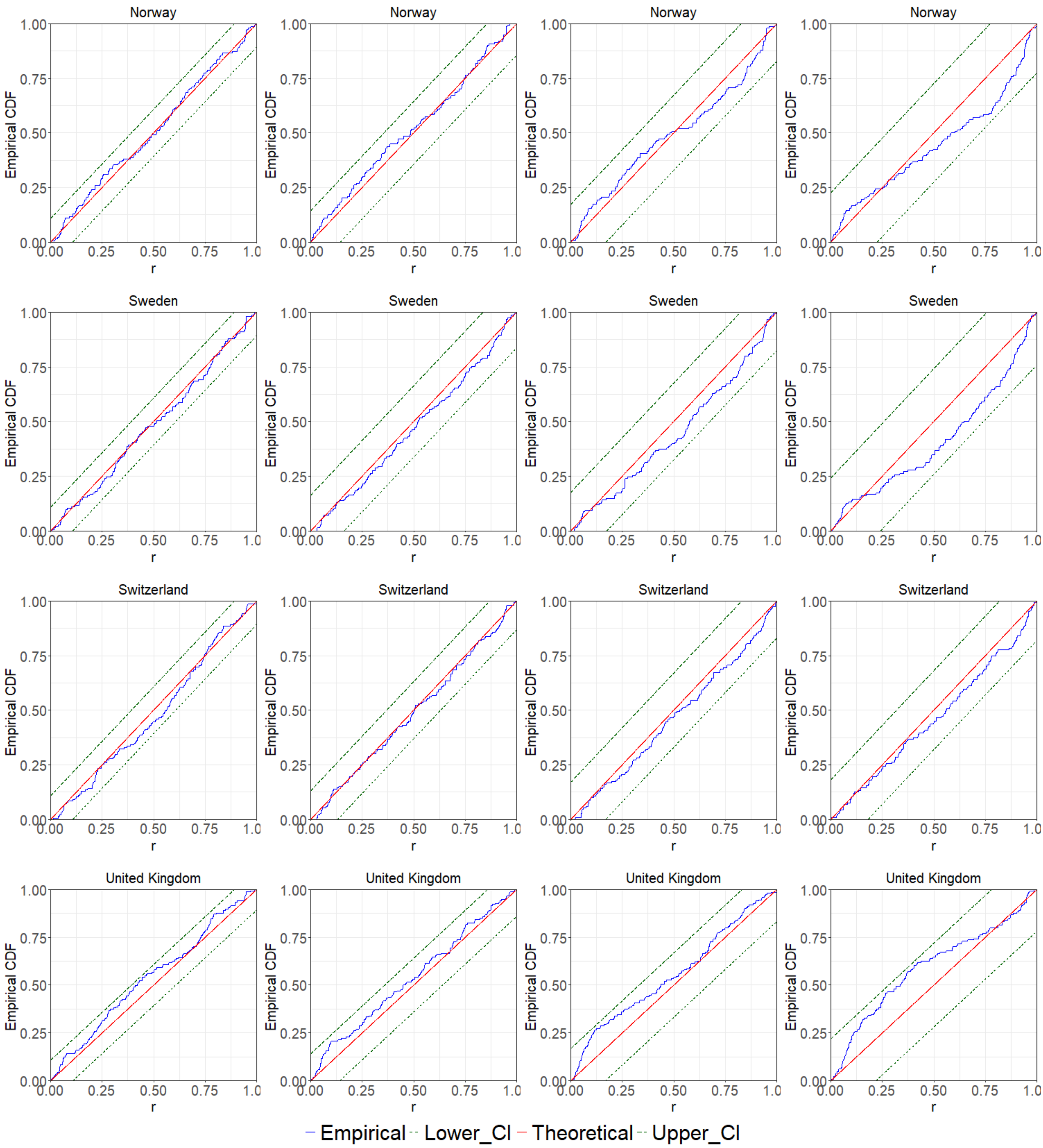


Figure 6: Results of the Rossi and Sekhposyan (2019)'s tests. The first, the second, the third and the last column represent, respectively, h=1, h=3, h=6 and h=12.

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