The Hicksian Traverse

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Abstract

We consider the disturbances due to the introduction of a new machine in an economy moving on a regular path. Hicks names traverse the intertemporal path leading from the present to another regular path. The dynamics depend on the characteristics of the old and the modern techniques. We simplify and extend Hicks' analysis, which relies on a neo-Austrian model, and show that the conditions for the existence of a full employment traverse are restrictive.

Keywords. Austrian model, technical change, employment, traverse.

JEL classification. B25, E32, O33.

1 Introduction

Consider an economy moving on a regular growth path sustained by a given technique, with a permanent stock of capital per head, and imagine that a cheaper production method is invented at some date. The new method is operated from that date onwards and the new technique is associated with another long-run stock of capital per head. What about the transition from a regular path to the other? The problem was studied by Hicks (1970, 1973), who named traverse the corresponding intertemporal path (Section 2). We critically analyse and extend Hicks' construction, which relies on a neo-Austrian model. In such a model, a flow of labour inputs generates a flow of a final good, a formalization which allows Hicks to take fixed capital, here

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referred to as a machine, into account. The question is therefore to study the disturbances induced by the introduction of a new type of machine, when an old type becomes obsolete but the installed old machines continue to be utilized up to the end of their lifetime. To simplify the analysis, Hicks considers a particular category of machines with a 'simple profile', i.e. which obey specific rules concerning their construction and their utilization. In the first phase of the transition, the two machines coexist, whereas the modern machines are only operated in a second phase but a new regular path is not yet reached. We reexamine the role of the simple profile hypothesis and set the question in a general framework when the dynamics are governed by a full employment condition. The study identifies the conditions for the existence of a traverse from the initial path to the long-run regime associated with the new method (Section 3). We compare these results with Hicks' and examine the difficulties met by the transition (Section 4). Section 5 concludes. Details are discussed in the Appendices.

Technical change is a permanent feature of evolving societies. The question of the impact of technological progress on employment is back in the spotlight with the massive introduction of artificial intelligence in the coming years. The analysis of the traverse is the simplest exercise on dynamics one can imagine and which avoids setting arbitrary assumptions on the economic agents' behaviours.

2 The notion of traverse

2.1 The standard case

The Austrian theory represents a production method as an intertemporal flow of labour inputs which generates a final good at a later date (flow input-point output process). The temporal lag allows it to shed light on the notions of roundaboutness and of average period of production which were at the heart of the Austrian project (Böhm-Bawerk, 1889). The neo-Austrian theory takes intertemporal joint production into account by extending the formalization to processes for which a flow of labour inputs generates a flow of dated outputs

$$(l_0, l_1, ..., l_{T-1}) \rightarrow (b_1, b_2, ..., b_T) \tag{1}$$

On the left-hand side, the indices refer to the dates of application of labour and, on the right-hand side, to those at which the final good is obtained, the total duration of the process being T periods. Against that tradition, Hicks assumes that the final good is obtained with no delay: "[I]nput produces output instantaneously!" (Chapter II, section 6).[1] The simultaneity convention simplifies some proofs, as illustrated in Appendix A, but the results are unchanged and we stick to the standard neo-Austrian formalization. More generally, the changes we introduce in Hicks' study aim at extending its scope.

A significant advantage of the neo-Austrian formalization is to cover the case of production with fixed capital: if $b_1 = ... = b_m = 0$, the first m periods are devoted to the construction of a 'machine', which does not appear explicitly, and the last n periods to the production of successive amounts $b_{m+1}, ..., b_T$ of final good ($m + n = T$) by means of that machine. Its ageing explains the possible variations in the labour and output coefficients.

The point Hicks examines concerns the effects of a technical innovation leading to the replacement of some type of machine ('old' or 'stale' machine) by another ('new' or 'fresh' machine). Because of the complexity of the phenomenon, Hicks restricts his study to a model which "retains the vital distinction between construction and utilization" (Chapter VII, section 2) and deals with machines with a 'Simple Profile' which obey the following conditions:

(i) The amount of labour l_c per period during the phase of construction is constant;

(ii) The amount of labour l_u per period during the phase of utilization is constant;

(iii) The machine has constant efficiency over its lifetime.

The analysis is moreover restricted to the 'Standard Case':

(iv) The durations m of construction and n of utilization are the same for both machines, and n is a multiple of m.

By choosing the common duration of construction as the new time unit (we call it a year), the construction of both machines takes one year ($m = 1$) and the general neo-Austrian model (1) is then reduced to a process

$$(l_c; l_u, ..., l_u) \to (0; 1, ..., 1) \tag{2}$$

which lasts $T = 1 + n$ years. In the Standard Case, two Simple Profiles are fully characterized by their respective coefficients (l_c^*, l_u^*) for the old process and (l_c, l_u) for the new one. These "drastic simplifications" (Chapter VII,

[1] All quotations refer to *Capital and Time*. We have changed some notations slightly.

section 2) imply the absence of truncation and re-switching (see Appendix A) but call for a reflection on the robustness of the conclusions.

2.2 The traverse

For the present problem, there is no inconvenience to replace the reference to regular paths with a constant growth rate by that to steady states. Let the amount $L^*$of labour available at any date be constant and given. Consider a steady state associated with a simple profile (l_c^*, l_u^*) and assume that a new and cheaper process (l_c, l_u) with the same duration is invented at date 0. From that date, the investments are made in the new machine exclusively (see Appendix B). Though the old machines are no longer produced, those already installed continue to be used, so that the two machines coexist during a lapse of time at the end of which the stale machine disappears. That step of progressive substitution defines the 'Early Phase' of the transition, which lasts n years. When the old machine is no longer present, its effects continue to appear indirectly: since all generations of the new machine are not equally numerous, the economy is not in a steady state. Economic fluctuations persist during subsequent years, and the question is whether they are dampened during the 'Late Phase' and lead to a new steady state. The whole dynamics are named 'Traverse'. They depend on the assumption retained to define the successive activity levels. Hicks himself examines two alternative hypotheses, that of Full Employment and that of Fixwage. The analyses have many common points and we limit ourselves to the study of the first, even if the second has specific features.

3 An extension

3.1 A more general model

In the process of extension of Hicks' analysis, we proceed by steps and do not jump directly to the most general framework. It is assumed here that the initial state is associated with the use of a process of total duration $1 + n$

$$(l_0^*; l_1^*, ..., l_n^*) \rightarrow (0; 1, ..., 1) \tag{3}$$

The left-hand side is that of a general neo-Austrian method, whereas the right-hand side is that of a simple profile. Note that the economic interpretation of a neo-Austrian process as production with fixed capital relies on the structure of the right-hand side only. In the present case, the construction of the machine takes one year and its production is constant, but the labour inputs during the period of utilization vary with age.

At date 0, more precisely a few days before that date, a new machine of the same type is invented. We do not retain the Standard Case hypothesis and allow its lifetime v to differ from that of the old machine:

$$(l_0; l_1, ..., l_v) \to (0; 1, ..., 1) \tag{4}$$

The new machine is substituted for the old one from date 0 onwards, the present stock of old machines being exhausted at date n. However, even if the new machines are only operated from that date, they are not yet fully installed if they have a longer lifespan ($v > n$): then, the end of the early phase occurs when all generations of the new machines are installed. The late phase begins at date $\max(n, v)$ and their dynamics are entirely determined by the characteristics of the modern machine.

3.2 The equations of the dynamics

In the initial steady state, the number x^* of stale machines built at each date is also that of machines of any age and is determined by the full employment condition. The investments from date 0 are made in the new machine. Let x_t be the number of new processes initiated at date t ($t = 0, 1, ...$). At date 0, the change in employment with regard to the steady state comes from the elimination of $l_0^* x^*$ jobs in the construction of obsolete machines and the creation of $l_0 x_0$ jobs in that of modern machines. As the total number $L^*$of workers is constant, the full employment condition is written

$$l_0 x_0 = l_0^* x^* \tag{5}$$

and determines x_0. At date 1, there is neither construction nor use of new-born machines of the old type, but workers are employed to produce with the help of the existing x_0 modern machines and in the construction of x_1 new machines. The full employment condition is now written

$$l_1 x_0 + l_0 x_1 = (l_0^* + l_1^*) x^* \tag{6}$$

and determines x_1. More generally, the successive activity levels $x_0, ..., x_n$ during the first n years are defined by equalities

$$\forall t = 0, 1, ..., n \qquad l_t x_0 + l_{t-1} x_1 + ... + l_1 x_{t-1} + l_0 x_t = (\sum_{i=0}^{t} l_i^*) x^* \tag{7}$$

These equalities hold if the two types of machines have the same lifetime ($v = n$). If the lifetime v of the modern machine is shorter ($v < n$), the coefficients l_t on the left-hand side are not defined for $t > v$ but the equalities (7) still hold by setting $l_t = 0$. The convention does not mean that the new machines remain active beyond their lifetime and may well be set when the point is to count the number of jobs.

If the lifetime of the new machine exceeds that of the old machine ($v > n$), all generations of new machines are not yet installed at date n. The full employment dynamics between dates n and v are governed by equalities

$$\forall t = n, .., v \quad l_t x_0 + l_{t-1} x_1 + ... + l_1 x_{t-1} + l_0 x_t = L^* \tag{8}$$

Eventually, the late phase starts at date $\max(n, v)$. There is no reason why a steady state would be reached at that date and the dynamics during that phase obey the rule

$$\forall t \geq \max(n, v) \quad l_v x_{t-v} + l_{v-1} x_{t-v+1} + ... + l_1 x_{t-1} + l_0 x_t = L^* \tag{9}$$

3.3 Study of the dynamics

The study of the dynamics sets two questions. The first is to check if the activity levels defined by the above relationships are positive, the second to examine if the dynamics converge towards a new steady state. Positive answers are ensured only under restrictions relative to the distribution of the labour coefficients in the modern technique: it is assumed here that the labour coefficients are decreasing through time (Hicks' All Downs condition, Chapter XII) and more precisely that

$$l_0 > l_1 \geq ... \geq l_v \tag{10}$$

Condition $l_0 > l_1$ appears at the very beginning of the early phase. Indeed, equations (5) and (6) imply

$$x_1 = (1 - \frac{l_1}{l_0}) l_0^* x^* + l_1^* x^*$$

so that the dynamics might fail very early if $l_1 > l_0$.[2] The following inequalities are justified in a similar way. Conversely, let the All Downs condition

[2] Though the weak inequality $l_0 \geq l_1$ suffices to guarantee the positivity of x_1, we anticipate the study of the late phase and retain the strict inequality. Hicks himself writes down the All Downs condition (10) as a set of strict inequalities (see Appendix C).

(10) be met. By comparing equalities (7) for dates t and $t+1$, it turns out that

$$l_0 x_{t+1} = l^*_{t+1} x^* + \sum_{i=0}^{t} (l_i - l_{i+1}) x_{t-i}$$

therefore x_{t+1} is positive if its previous values are. By induction, all activity levels are positive during the early phase (note that, if $v > n$, the introduction of the convention $l_t = 0$ for $t > n$ is compatible with the All Downs hypothesis and the argument applies). The same for the late phase.

Let us now examine the long-run convergence. The dynamics of the late phase are defined by the unique formula (9). By taking as unit of measure of labour the amount necessary for the construction of a new machine ($l_0 = 1$), the comparison of the full employment condition (9) at dates $t-1$ and t allows us to express the activity level at date t as a weighted average of its $v+1$ previous values

$$x_t = (1 - l_1) x_{t-1} + (l_1 - l_2) x_{t-2} + ... + (l_{v-1} - l_v) x_{t-v} + l_v x_{t-v-1}$$

with nonnegative weights by the All Downs condition. The average property shows that the sequence remains between the minimum and the maximum of its initial values and admits an upper limit M. By ignoring the first terms of the sequence, it may be assumed that inequality $x_t \leq M + \varepsilon$ always holds. Let x_t be close to M (this is indeed the case for some t, by definition of the upper limit). As x_t is an average of values smaller than $M + \varepsilon$, all these values must themselves be close to M. This is the case in particular for x_{t-1} (inequality $1 - l_1 = l_0 - l_1 > 0$ is used here), and also by induction for $x_{t-2}, ..., x_{t-v}$. Again by the average property, the whole sequence after t stays close to M, so that M is the limit of the sequence. In short, the economy tends towards a steady state. The final state does not keep track of the initial state, as there exists a unique steady state making use of the new machines and ensuring full employment.

The introduction of the All Downs condition is explained in economic terms as follows: in period t, total employment amounts to $L^* = x_t l_0 + ... + x_{t-n-1} l_{n-1} + x_{t-n} l_n$. The very nature of an Austrian process introduces intertemporal rigidities as, in period $t+1$, employment $L_p = x_t l_1 + ... + x_{t-n-1} l_n$ is pre-empted in order to continue the processes in progress. If inequality $L_p > L^*$ held, no new machine could be launched at date $t+1$ and some of the existing processes should even be abandoned. The role of the All Downs assumption is to ensure the opposite inequality

$$x_t l_1 + ... + x_{t-n-1} l_n < x_t l_0 + ... + x_{t-n-1} l_{n-1} + x_{t-n} l_n \tag{11}$$

in the absence of any further information on activity levels during the transition. The interpretation shows that, in the presence of an incremental technical change, the traverse works without the All Downs hypothesis: all activity levels then remain close to their initial values x^* and inequality (11) holds automatically. But a change in the duration of construction or utilization of a machine does not fall into that category.

The interpretation in terms of complementarities is also confirmed by the following thought experiment: if it were possible to associate any amount of labour with a machine of age t (the amount of the product varying with that of labour), a full employment traverse could be found.

4 Discussion

4.1 Hicks' comments

On the Simple Profile hypothesis $l_1 = ... = l_n$, the All Downs condition is reduced to $l_0 > l_1$ (the inequality is necessary and sufficient for the existence of a full employment traverse). As Hicks considers that "a typical economic process would have a rising and then (probably) a falling phase" (Chapter XII), "the Simple Profile [...] is a rather poor approximation to this economic shape" (*ibid.*). He therefore examines some generalizations.

The assumption according to which the old and the new machines have the same duration of construction ($b_1^* = b_1 = 0$) is contrary to the very spirit of the Austrian theory: Böhm-Bawerk stresses the variability of the waiting period. On that topic, Hicks writes: "Since time is irreversible, the effects of shortening and of lengthening are not quite symmetrical" (Chapter XI, section 1). Moreover, "[i]t would be better to make an analysis of changes in the [...] life of the machine. Unfortunately, [...] it is hard to do much about these" (*ibid.*). The above analysis takes into account that case but does not confirm Hicks' opinion.

The weakening of the constant labour hypothesis $l_1 = ... = l_n$ leads Hicks to consider a more general case and to introduce the All Downs condition (details in Appendix C). Hicks does not discuss the constant production hypothesis $b_2 = ... = b_T$.

4.2 A general framework

The intertemporal production processes (3) and (4) we have studied are more general than Hicks' Simple Profiles on the input side (variable productivity of labour and different lifetimes) but retain the same intertemporal structure for the products: construction takes one year and the production of a machine is constant over its lifetime. Can these hypotheses be alleviated? What is the impact of the right-hand side vectors $(b_1^*, ..., b_{n+1}^*)$ and $(b_1, ..., b_{\nu+1})$ on the dynamics? The question admits a neat answer: the activity levels are determined independently of them. Indeed, since the age structure of the stale and fresh processes at any date t determines the pre-empted employment at date $t+1$, the full employment condition defines the number of new fresh processes at date $t+1$ and the whole age structure at that date: the successive activity levels only depend on the labour input coefficients, whereas the output coefficients matter for the gross product fluctuations. All that has been said above on the positivity and convergence of activity levels is unchanged if the construction of the machine requires several years, if its production varies with age, or even if no reference is made to the notion of fixed capital.

The remark reduces the above analysis to a single statement: for the most general neo-Austrian models, the constancy of employment determines the number of new processes launched at any date. The decreasingness of the labour coefficients for the new process guarantees the existence of a full employment path leading from the initial stationary state to a new one.

4.3 A fragile traverse

The extension of the analysis to general processes modifies some economic interpretations on which Hicks' discussion relies. It turns out that the notions of Simple Profile and Standard Case have no specific economic significance and may even be misleading: for instance, the scalar l_0 cannot be interpreted as the total amount of labour required for the construction of a machine.

The hypothesis of decreasing labour coefficients for the new process (and, in particular, for a modern machine if fixed capital is taken into account) is crucial. It has been introduced as a sufficient condition, but it is also necessary to ensure the existence of a traverse starting from *any* previous stationary state and leading to the one associated with the modern technique. The All Downs condition is difficult to justify economically and, when it is not met, the existence of a full employment traverse is dubious. The worry is reinforced by the fact that the study ignores the value side, which will also be affected by the innovation and will induce further disturbances in the economy. It must be stressed that, even if the effects of a technical change on employment are clear in the case of a labour-saving innovation, the above analysis shows that the difficulties to define a full employment traverse are much more general.

The impact of technological shocks on employment manifested itself in social reality before being examined by economists. Ricardo (1817, 1821) is one of the very first theoreticians to have studied it, even if, for the Classicals, the question is not that of full employment but at most that of constant employment, and the effects on the real wage must also be taken into account. A common point between the classical approach and Hicks' study is to take only real magnitudes into account, leaving aside psychological considerations linked to expectations as well as monetary phenomena. This is the meaning of the remark Hicks addresses to Hayek (1931), to whom he attributes the discovery of "the relevance, to economic fluctuations, of the time-structure of production" (Chapter XI, section 6), but who claims that "the disturbances have a monetary origin" (*ibid.*): in the present study, the causes of fluctuations, or even crises, are entirely real (the point has clearly an echo in contemporary debates).

Eventually, the neo-Austrian model is a model with a unique factor and a unique final good. Since it may be considered as a specific case of a multisector model (Burmeister, 1974), it is also rich in general lessons and it cannot be expected that the pessimistic conclusions here reached may be reversed for more complex models.

5 Conclusion

According to Solow, "traverse is the easiest part of skiing, but the hardest part of economics". In spite of the success met by *Capital and Time*, the analytical side of the transitional dynamics has not been explored further than the study Hicks himself made fifty years ago. The notion of simple profile, which is at the heart of Hicks' analysis, is unnecessarily restrictive and, we dare say, clouds the debate: the crux of the question lies in a condition on the decreasingness of the labour coefficients which, being independent of any specific profile and even of the notion of fixed capital, has a universal value. The extension leads us to modify the judgement on the existence of a full employment traverse: Hicks' opinion relies on the use of a very simplified model, but the more general All Downs condition has fragile empirical foundations. The fact remains that Hicks has provided tools for the study of a significant and permanent economic phenomenon.

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APPENDICES

Appendix A. Re-switching

Even if Hicks (1973) considers that the attention paid to the paradoxes of capital theory is misplaced, he takes care to show that, in the case of equal durations m of construction and n of utilization for both types of machines, re-switching is excluded. Re-switching occurs when one method is cheaper than the other at two interest rates but more expensive at some intermediate rate. Then the $w-r$ curves associated with one or the other process have two intersection points, w being the wage in terms of final good and r the rate of interest.

The $w-r$ curve (or efficiency curve, in Hicks' words) associated with the Simple Profile $(l_c, ..l_c; l_u, ..., l_u) \rightarrow (0, ..., 0; 1, ..., 1)$ expresses that the present value at rate r of net incomes is zero. Let us first assume with Hicks that the product is obtained instantaneously. The equation is written

$$wl_c \sum_{t=0}^{m-1} (1+r)^{-t} + wl_u \sum_{t=m}^{T-1} (1+r)^{-t} = \sum_{t=m}^{T-1} (1+r)^{-t} \tag{12}$$

hence

$$\frac{wl_c}{1-wl_u} = \frac{\sum_{t=m}^{T-1} (1+r)^{-t}}{\sum_{t=0}^{m-1} (1+r)^{-t}}$$

In the Standard Case (same m and same T for the two machines), equality

$$\frac{wl_c}{1-wl_u} = \frac{wl_c^*}{1-wl_u^*} \tag{13}$$

holds at a switch point and determines one positive value of w at most: a switch point, if any, is unique (Chapter IV).

Suppose alternatively a one-period lag between labour and the product. The $w-r$ curve is now defined by equation

$$wl_c \sum_{t=0}^{m-1} (1+r)^{-t} + wl_u \sum_{t=m}^{T-1} (1+r)^{-t} = \sum_{t=m+1}^{T} (1+r)^{-t} \tag{14}$$

Proving the absence of re-switching between two functions $w = w(r)$ of that type requires additional calculations. These are avoided by means of the following trick: by setting $w = \omega(1+r)^{-1}$, equation (14) is rewritten as

$$\omega l_c \sum_{t=1}^{m} (1+r)^{-t} + \omega l_u \sum_{t=m+1}^{T} (1+r)^{-t} = \sum_{t=m+1}^{T} (1+r)^{-t} \tag{15}$$

which is close to (12) in that the coefficient of ωl_u coincides with the scalar on the right-hand side. At a switch point, equality (13) holds when w is replaced by ω, hence the uniqueness property. The result does not hold if the machines have different periods of construction or utilization, but the question of re-switching has little to do with the traverse.

Appendix B. Stale machines

Following Hicks, it has been admitted that the construction of stale machines is abandoned as soon as a modern machine is invented. This is indeed the case if the duration of construction is one year and the year is an unbreakable time unit. Hicks, however, subdivides the year into 'weeks', new production processes being launched every Monday. The economic magnitudes may then be considered as continuous variables. But let us assume with Hicks that the construction of an old machine takes 52 weeks and suppose that a new machine is invented at the end of December. The next Monday, stopping the production of all obsolete machines will cause an irremediable loss. It may be less costly to finish the construction of the old machines that have been launched at the beginning of the previous year rather than to count them as dead losses. The study ignores that phenomenon.

Appendix C. Hicks on decreasing labour coefficients

Hicks first shows the existence of a full employment traverse in the Standard Case upon assumption $l_0 > l_1 = ... = l_n$. In section 20 of the mathematical Appendix, he considers more general profiles for machines with identical lifetimes and shows that hypothesis $l_0 > l_1 > ... > l_n$ ensures the existence of a traverse. However, since Hicks states the condition as a set of strict inequalities (his mathematical argument to prove convergence is more complex but weaker than the one used above), it does not apply to the Standard Case: hence the necessity of two distinct proofs, as Hicks pointed at in Chapter XII. The significant gap with the formulation we retain does not lie in strict or weak inequalities, but in the universality of the All Downs condition: when Hicks examines the case of two machines with different durations of construction (Appendix, section 5), he maintains the hypothesis of constant labour coefficients during construction and utilization.