The Macroeconomics of Free Digital Services

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Abstract

Digital technology has enabled the rise of free digital services financed by advertising. These services are increasingly popular and enable a few digital firms to generate significant revenue, although GDP does not directly consider them. This paper presents a growth model with digital services providers collecting household data in exchange for their services, which are used to sell targeted advertising to traditional firms. It enables us to study the impacts of this sector on key macroeconomic aggregates and welfare within the American context. Our results highlight that enhanced activity among large providers (new entry, greater efficiency in producing service quality or advertising) positively impacts the economy. Data collection enables small providers to compete with large ones, which benefit from greater user attention. Household preferences, such as sensitivity to privacy, also play a role, potentially hindering the free digital services sector's economic impact.

JEL Classification– D21; D43; O40

Keywords– Economic growth; Digital economy; Data economy; Privacy; Advertising

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1 Introduction

In recent years, many free digital services have emerged and are expanded (see Figures 1 and 2). Google and Facebook are two examples, along with many other websites and applications. These services are not only growing in number but also in use. For instance, the number of monthly active Facebook users has grown from 100 million in 2008 to 3,030 million in 2023, and the average time spent on social media by internet users has increased from 90 minutes per day in 2012 to 153 minutes in 2023.[1] These services have no monetary cost to the consumer but require time and attention. To be financed, free digital service providers collect users' personal data in exchange for usage and analyze the collected information to sell targeted advertising services (Zuboff, 2019). This business model enables them to generate significant revenue, particularly for the most popular platforms, as digital has considerably reduced the cost of targeting (Goldfarb, 2014).

Figure 1: Number of available apps in Google Play and the Apple App Store from 2008 to 2022

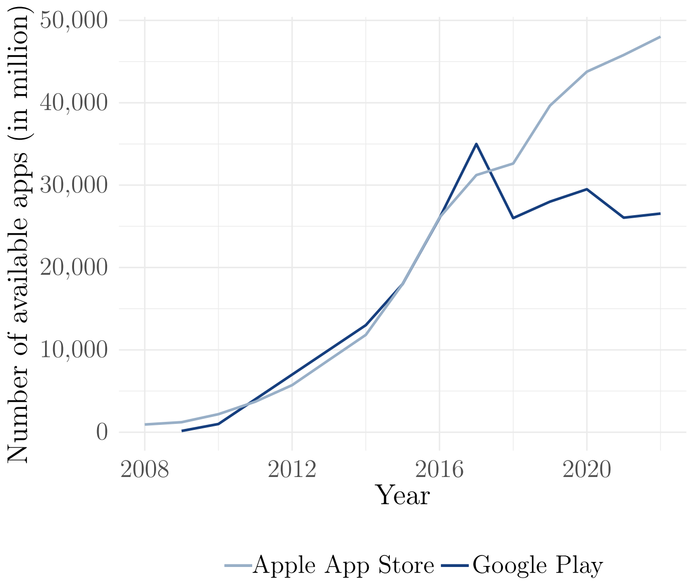
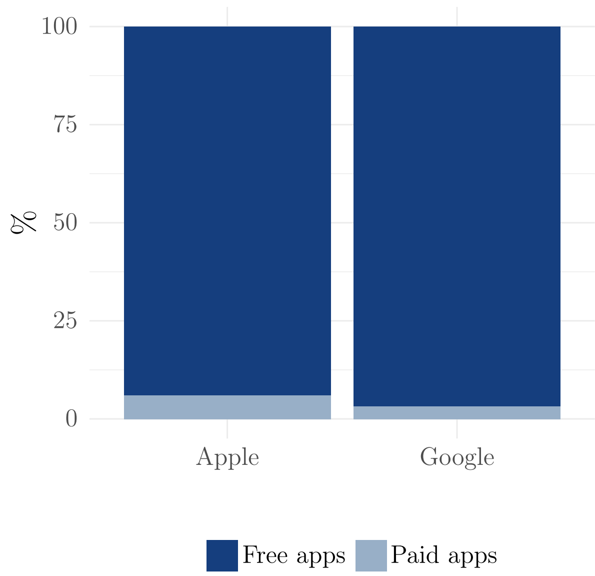


Figure 2: Share of free and paid apps in Google Play and Apple App Store in 2022



Source: Statista

The impact of the free digital services sector on macroeconomic aggregates is ambiguous. The

[1]Data are from Statista and were downloaded on 20 August 2023 on the following webpages: https://www.statista.com/statistics/264810/number-of-monthly-active-facebook-users-worldwide/ and https://www.statista.com/statistics/433871/daily-social-media-usage-worldwide/.

consumption of free digital services is not considered as final consumption of households in the national accounting system, as their price is zero. However, they can be a substitute for traditional services and harm the Gross Domestic Product (GDP). Advertising revenues are considered intermediate consumption and, therefore, deducted from a country's value-added calculation. Their impact on real GDP will depend on how they are incorporated into the traditional goods price. Moreover, despite the significant increase in digital advertising spending, advertising represents a constant share of GDP (Greenwood *et al.*, 2021) because digital advertising substitutes for traditional advertising (Goldfarb & Tucker, 2011; Seamans & Zhu, 2014). Nevertheless, Brynjolfsson *et al.* (2018) show evidence that even if free digital services do not contribute substantially to GDP, consumers benefit from them.[2] As a consequence, they can increase welfare without impacting economic growth (Hulten & Nakamura, 2019).

Building on these facts, this paper aims to study the impact of free digital services financed by advertising on the macroeconomic aggregates and welfare. To this aim, we develop an endogenous growth model without scale effects *à la* Young (1998). It includes the free digital services sector and its interactions with the traditional sector and households. The traditional sector comprises firms in monopolistic competition selling differentiated varieties of goods to households as in Dixit & Stiglitz (1977). Following Grossmann (2008), traditional firms must invest both in research & development (R&D) and advertising to improve the perceived quality of their goods. We stand out by considering that advertising is outsourced and sold by the digital sector. The latter comprises a fixed number of large digital service providers and a continuum of small providers (Shimomura & Thisse, 2012). They propose differentiated digital services to households in exchange for personal data. Each digital service provider uses the collected information and the attention of its users (measured by the time spent on the digital service) to supply targeted impressions to the traditional sector. Finally, households derive utility from consuming different varieties of traditional goods and spending time on digital services. They also face positive and

[2] Literature attempts to estimate the surplus brought by free digital services. Ahmad *et al.* (2017) and Nakamura *et al.* (2018) have measured it through the cost of production (e.g., advertising, data). However, as pointed out by Spence & Owen (1977), advertising revenues are not a good measure of the value of free goods ad-supported. Therefore, Goolsbee & Klenow (2006) and Brynjolfsson *et al.* (2023) used the time spent on free digital services to estimate their value. Brynjolfsson *et al.* (2018) and Coyle & Nguyen (2023) measured the willingness to accept giving up access to digital and non-digital goods.

negative impacts from the data collection.[3] As data are non-rivals (Jones & Tonetti, 2020), digital services providers use them to personalize their service for each household. On the other hand, data collection raises privacy issues that harm households' utility. As a consequence, users have to choose the quantity of data they accept to disclose to the digital service providers.

The purpose of the model is twofold. First, it contributes to understanding the link between the free digital services sector, macroeconomic aggregates, and welfare. Second, it enables the analysis of the economic impacts of free digital services. To this aim, we derive comparative-static to highlight the economic implications of the free digital services sector characteristics and households preferences. It is worth noting that the model is set in the context of the American economy, which owns the majority of the large providers of free digital services. It is consistent with some empirical trends experienced in the United States of America (USA), such as the increase in digital leisure time (Wallsten, 2015; Aguiar *et al.*, 2020), the decrease in worked hours (Vandenbroucke, 2009; Kopytov *et al.*, 2023), and the productivity slowdown (Cette *et al.*, 2016). We also capture that welfare growth has been higher than income growth since the 1980s, as highlighted by Jones & Klenow (2016).

Our results highlight that the free digital services sector impacts households' welfare and has several macroeconomic consequences, without involving economic growth. The market structure of the free digital services sector, characterized by a few large providers and many small ones, plays an important role. Large providers have a quality advantage over small ones, leading to strong household preferences for these services. These preferences, influenced by factors like true quality of the service as well as lock-in, popularity, or habits effects, imply that large providers positively impact the economy and welfare through a redistribution mechanism. We find that enhancements in the production efficiency (quality or impression) of large providers or the entry of new large providers increases individual wealth, stimulating the demand for traditional goods. However, only the number of varieties of goods increases, not the effort of R&D and, therefore, economic growth. This finding aligns with the empirical evidence presented by Baslandze *et al.* (2023), who demonstrate a proliferation of varieties resulting from digital advertising. We also highlight that the weight of users' data and attention in advertising production has economic implications,

[3] Users can also experience security problems, misinformation, addiction, and others from using digital services (OECD, 2019) that we are not considering in this paper.

particularly for small providers. Indeed, if attention is an important production factor in impression production, small providers are harmed by the high competition with large providers. Data collection provides an opportunity for small providers to compensate for households' preferences for spending time on the services of large providers. Finally, household preferences such as sensitivity to privacy must be considered as they can hinder the impact of the free digital service sector on the economy.

The remainder of this paper is structured as follows. Section 2 provides a literature review, and Section 3 describes the free digital services market. Section 4 presents the model. Equilibrium is presented in section 5. In section 6, we derive comparative-static to study the impacts of the free digital services sector on the main macroeconomic variables. Section 7 concludes and discusses the main results.

2 Related literature

This paper is related to the theoretical literature on the macroeconomic impacts of digital technology. More specifically, it is part of the recent literature on data economy. While the use of data in production is not a novel concept, the emergence of digital technology has significantly reduced the cost of collecting and storing data, increasing their use (Goldfarb, 2014). In a growth model, Farboodi & Veldkamp (2022) highlight that data and new prediction algorithms improve the quality of goods and increase economic growth. However, in the long run, data accumulation is similar to capital accumulation in the model of Solow (1956) and cannot sustain economic growth. Jones & Tonetti (2020) emphasize the non-rival nature of the data. In their growth model *à la* Romer (1990), data, which are a by-product of consumption, improve the quality of ideas in the production function and can be sold and used by other firms. They study three cases: (i) firms own data, (ii) consumers own their data and can sell them, and (iii) the government outlaws the selling of data. They highlight that data regulation can have significant economic implications and that enabling consumers to sell their data leads to the optimal situation regarding privacy, output, and consumer welfare. In the same line, Arrieta-Ibarra *et al.* (2018) propose considering data as labor, not capital. They argue that digital firms should pay consumers for their data. Cong *et al.* (2021) enable consumers to sell their data to firms depending on their privacy concerns. As in Jones & Tonetti (2020), the non-rivalry of data is horizontal (they are only used in the R&D sector, not in the final goods sector). They find that data use results in an over-accumulation of data and under-employment, leading to sustainable economic growth with lower welfare. In addition to horizontal non-rivalry, Cong *et al.* (2022) enable data to be used across sectors. They find different results: data are under-used because of privacy concerns. They also highlight that data is more efficient in innovation than in the production sector.

Most articles on the data economy consider privacy sensitivity, which is modeled as a disutility resulting from disclosing personal data. In the present paper, data is a by-product of free digital services consumption. Only free digital service providers collect and use data. We consider the horizontal non-rivalry nature of data: digital services and advertising are personalized according to individual data. Households also have a sensitivity for privacy. They must choose how much data they disclose to free digital services providers by making a trade-off between data collection's positive and negative impacts.

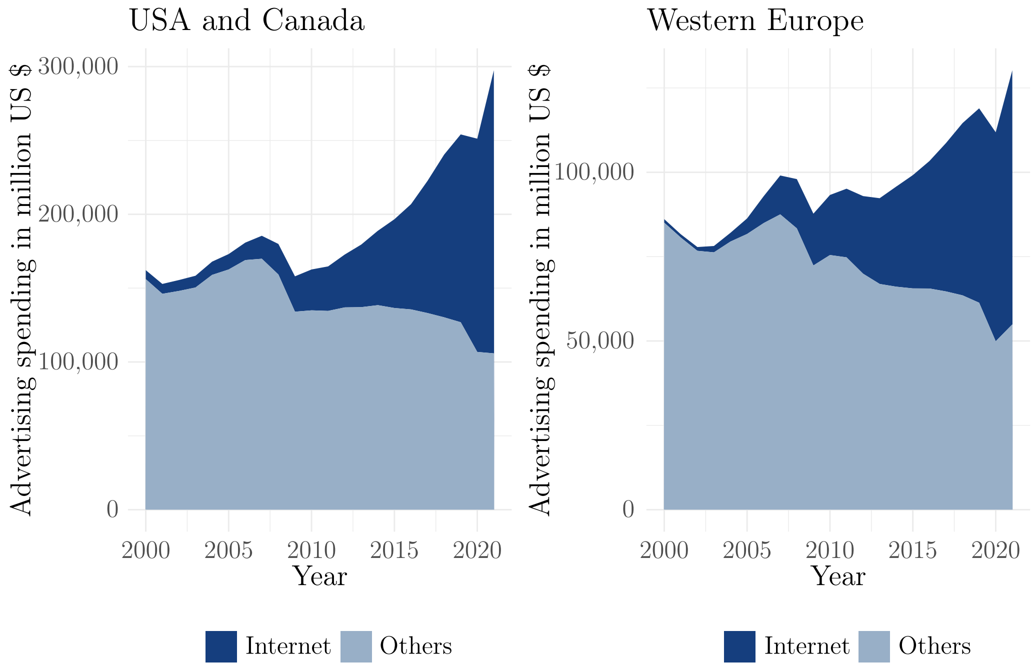
This paper is also related to the theoretical literature on the link between advertising, innovation, and economic growth. The paper of Grossmann (2008), on which we build our approach, is the first to consider the interaction between advertising and R&D in a growth model. He follows Young (1998)'s theoretical framework, which enables qualitative growth and the absence of scale effects in the growth process. There is a debate about the effect of advertising on consumers. In a literature review on the economics of advertising, Bagwell (2007) identifies three types of advertising views: the persuasive (advertising modifies consumers' tastes and preferences) (Robinson, 1933; Braithwaite, 1928), informative (advertising is supplementary information on the product) (Ozga, 1960; Stigler, 1961), and complementary (advertising is complementary to the advertised good) (Stigler & Becker, 1977). In the model of Grossmann (2008), advertising modifies the perceived quality of the selling good and is combative in the sense that *"an increase in marketing expenditure of a single firm creates a negative externality on demand faced by other firms."* In contrast, R&D investment improves the true quality of the goods and provides a positive externality in the research activity. However, the advertising sector is not included. Firms in the final goods sector realize advertising activity. We stand out by modeling digital service providers and studying their impacts on welfare and their role in advertising activity. Further research extends the literature by focusing on the role of firm size (Cavenaile & Roldan-Blanco, 2021), market concentration and markups (Cavenaile *et al.*, 2022), and informative advertising (Klein & Şener, 2022). These papers find that advertising and innovation are substitutes, which can depress economic growth.

Few papers model digital advertising in a macroeconomic framework. Greenwood *et al.* (2021) model the interaction between traditional and digital advertising, highlighting the targeting advantage of digital advertising. As in our model, consumers are exposed to advertising by spending time on free media-leisure goods. The authors focus on the impact on welfare and price competition, arguing that GDP is not a good measure of free media goods. In the same line, Cavenaile *et al.* (2023) highlight the adverse impact of the increase in targeted advertising on welfare. They show that it can both increase welfare by improving consumer-product matching and decrease it by increasing the average level of markups and reducing the creation of new product categories. Rachel (2021) models free digital leisure in a growth model. He is interested in the impacts on productivity and seeks to explain the decline in hours worked due to the emergence of the attention economy. In these papers, consumers do not reveal personal information through free digital services usage. Therefore, there are no privacy and data aspects. We contribute to the literature by considering digital advertising, which depends on users' data and attention, and privacy issues in an endogenous growth model.

3 The free digital services market

Firms in the free digital services market, also called the attention market, act like two-sided platforms (Rochet & Tirole, 2003; Armstrong, 2006). On the first side, they offer free digital services to consumers, enabling them to collect various data on their behavior. Users can directly provide this information by filling in a subscription form on the service. Data can also be collected indirectly by creating cookies that follow the user's online path inside and outside the website. Digital service providers use this information to build a profile of their users based on their demographic characteristics and online behavior. The non-rivalry nature of the data enables the platforms to improve and personalize the digital service and to produce and sell targeted advertising to the second side: traditional firms. These digital firms benefit from important economies of scale due to their characteristics. Firstly, the marginal cost of digital services is close to zero as the production cost is mainly fixed (Shapiro & Varian, 1998). Moreover, they benefit from significant network effects on the same side (user's utility increases with the number of users) and between the two-side of the market. The more users and usages a digital platform has, the more personal data it obtains and the more it can improve its advertising service. Increasing usage also increases users' exposure to advertising. Therefore, digital firms are incentivized to attract as many users as possible for as long as possible by investing in the quality of their service. Online advertising is also attractive to traditional firms because the cost of targeting is reduced compared to offline advertising, as digital facilitates the collection and storage of data (Goldfarb, 2014). This is illustrated by the increase in internet advertising spending in the United States, Canada, and Western Europe, to the detriment of traditional advertising (see Figure 3).

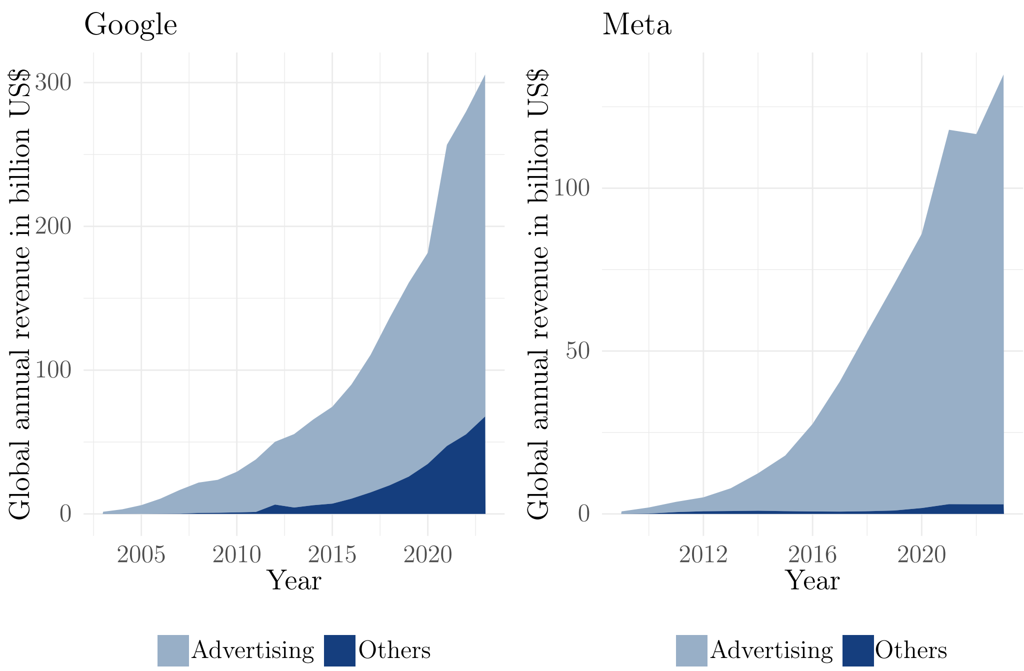
Figure 3: Advertising spending in the United States, Canada, and Western Europe between 2000 and 2021



Source: Statista.

Large returns to scale, which benefited digital firms, foster dominant positions. Consequently, few firms mainly dominated the market. Among the large providers of free digital services, the most well-known and meaningful ones are Google and Meta. The two firms do not offer the same free digital service. Google had a 91% share of the search engine market and Meta 70% of the social media market in 2020 (Bourreau & Perrot, 2020). However, they shared half of the online advertising market share (30% for Google and 22% for Meta). Their advertising revenues, which represent their primary revenue source, are growing exponentially, reaching $237.86 billion for Google and $131,948 billion for Meta in 2023 (see Figure 4).[4]

Figure 4: Revenue of Google between 2003 and 2023 and Meta (formerly Facebook Inc.) between 2009 and 2023



Source: Statista.

A continuum of websites and applications shares the rest of the market. Note that Amazon is the third largest online advertising supplier (it represents 9% of the market share) but is not included in the free digital services market as its main activity is e-commerce.[5] In addition to being the most visited platforms, the two firms have acquired several platforms, some very

[4]To illustrate the magnitude, in 2022, Google's advertising revenue was higher than Greece's GDP, according to World Bank data.

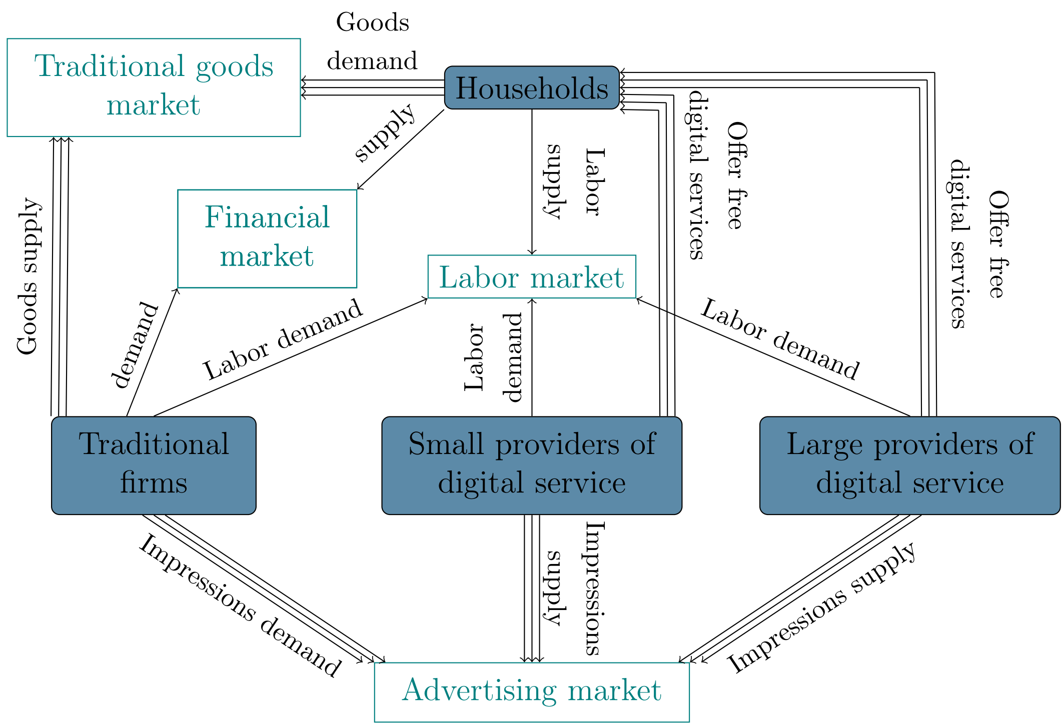
[5]This point can be discussed as Amazon is sometimes used as a search engine for goods. However, this paper focuses on pure-player firms, i.e., firms that only sell advertising.

popular today, such as YouTube for Google and Instagram for Meta. It has enabled them to consolidate their position in various digital sectors and accumulate a significant amount of data and attention, creating barriers to entry. Nevertheless, despite the important revenues earned by some free digital service providers, there has been little research (theoretical and empirical) on the macroeconomic impacts of the free digital services sector.

4 Model

This section presents of the model structure composed of households and two sectors (traditional and free digital services). The main interactions between the economic agents are summarized in Figure 5. Model variables are summarized in Table A.1 in Appendix A to facilitate reading.

Figure 5: Main interactions between the model's economic agents



→ means that goods/services are homogeneous.
⇉ means that goods/services are differentiated.

4.1 Households

The economy comprises H identical households, and the population is constant over time. Each household seeks to maximize its utility by consuming differentiated varieties of goods, spending time on free digital services, and choosing the quantity of data she discloses to the digital service providers. The intertemporal utility of an individual $h \in \mathcal{H} = \{1, ..., H\}$ is given by:

$$U_{h,0} = \sum_{t=0}^{\infty} \rho^t u(C_{h,t}, D_{h,t})$$

with $0 < \rho < 1$ the discount factor, $C_{h,t}$ a goods consumption index, and $D_{h,t}$ a free digital services consumption index.

Following Grossmann (2008), we model the utility derived from consumption as in Dixit & Stiglitz (1977) adding a variable for the perceived quality of the different varieties of good. The goods consumption index for the household h is given by:

$$C_{h,t} = \left(\int_0^{I_t} (\tilde{z}_{h,t}(i) c_{h,t}(i))^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $\sigma > 1$ is the elasticity of substitution between the different varieties of good, I_t the number of differentiated varieties of good in period t, $c_{h,t}(i)$ is the quantity of variety of good $i \in \mathcal{I} = [0, I_t]$ consumed by household h in period t, and $\tilde{z}_{h,t}(i)$ the perceived quality of the variety of good i by household h in period t (optimally chosen by traditional firms).

Households divide their online time between large and small digital services. Following Shimomura & Thisse (2012), the free digital service sector is modeled as a "mixed market" with a fixed number J of large providers, indexed $j \in \mathcal{J} = \{1, ..., J\}$, and a mass of K_t small providers, indexed $k \in \mathcal{K} = [0, K_t]$, offering differentiated free digital services. The number of small providers is not fixed, as there is free entry into this sub-category of digital services. The utility derived from digital services depends on the time spent on each digital service, the perceived quality of each digital service, and the quantity of data collected by the digital service providers. Perceived quality of the digital service by a household h depends on the quality of the service (q), which is uniformly determined by the service provider for all users, and on the data collected by the digital service providers on the household (d_h). Indeed, data are used to personalize the digital service tailored to the user's profile.[6] The more a household discloses its data, the more the perceived quality of the digital services is high for her (perceived quality is individual). However, data collection also harms utility as it raises privacy concerns. Following Wein (2022), the (des)utility of spending time on free digital services j and k takes the form of a Cobb-Douglas such as:

$$(s_{h,j,t}q_{j,t}d_{h,j,t})^{\alpha}(\overline{d}_h - d_{h,j,t})^{1-\alpha}$$

$$(s_{h,t}(k)q_t(k)d_{h,t}(k))^{\alpha}(\overline{d}_h - d_{h,t}(k))^{1-\alpha}$$

where $s_{h,j,t}$ and $s_{h,t}(k)$ are, respectively, the time spent on the digital service j and k by household h, $q_{j,t}$ and $q_t(k)$ the quality of the digital service j and k, $d_{h,j,t}$ and $d_{h,t}(k)$ the quantity of data the household h chooses to disclose to the digital service providers j and k, $\overline{d}_h$ the maximum quantity of data that digital service providers can collect and $\alpha \in]0,1[$ a preference parameter for privacy. As α increases, households are more inclined to share their data. We assume that consumers has the choice of how much data they want to give.[7] Therefore, households choose the optimal quantity of data they will disclose by making a trade-off between data collection's positive and negative impact on utility. As households are homogeneous in the model, this is also a convenient approach to modeling average data collection among all users, rather than modeling heterogeneous households where some give everything and others nothing.

To consider the substitution between the different free digital services, the utility derived from online time takes the form of a nested Constant Elasticity of Substitution. Households can substitute digital services belonging to the same category and between the digital services of the small and large providers. Overall, it is given by:

$$D_{h,t} = \Big[\sum_{j=1}^{J}\Big((s_{h,j,t}q_{j,t}d_{h,j,t})^{\alpha}(\overline{d}_h - d_{h,j,t})^{1-\alpha}\Big)^{\epsilon} + \int_0^{K_t}\Big((s_{h,t}(k)q_t(k)d_{h,t}(k))^{\alpha}(\overline{d}_h - d_{h,t}(k))^{1-\alpha}\Big)^{\epsilon}dk\Big]^{\frac{1}{\alpha\epsilon}} \tag{2}$$

[6] Note that data are non-rivals. Therefore, they will also be used in the production of impressions.

[7] A growing number of data regulations have been implemented such as the General Data Protection Regulation (GDPR) in Europe in 2018 and the California Consumer Privacy Act (CCPA) in California in the US in 2020. Users no longer have the choice between giving away all their data or only the essential ones, but they can choose which type of data they are willing to provide.

where $\epsilon \in]0, 1[$ is a substitution parameter between the digital services.[8] Note that the mixed market as in Shimomura & Thisse (2012) implies that each digital service of large providers has a positive and significant impact on household utility. In contrast, each digital service of small providers has a negligible impact. Digital services of large providers have a higher quality than those of small providers. Therefore, users spend significantly more time. Nevertheless, the aggregate impact on utility from the continuum of digital services offered by small providers can be comparable to that of digital services provided by large providers.

To simplify, we assume that the instantaneous utility function of the household h takes the form of an additive function given by:

$$u(C_{h,t}, D_{h,t}) = \ln C_{h,t} + \ln D_{h,t}$$

The total time available for each household is equal to ℓ. Consequently, if a household spends a time $S_{h,t} = \sum_{j=1}^{J} s_{h,j,t} + \int_0^{K_t} s_{h,t}(k)dk$ online, its work time is $(\ell - S_{h,t})$. It allows us to capture the long-term decline in working hours associated with the rise in leisure activities, mainly digital leisure (Rachel, 2021).[9]

As digital services are free, only the consumption of differentiated commodities has a monetary cost. The intertemporal budget constraint of the representative household h is given by:

$$b_{h,t+1} = (1 + r_t)b_{h,t} + \left(\ell - \sum_{j=1}^{J} s_{h,j,t} - \int_0^{K_t} s_{h,t}(k)dk\right)w_t + \frac{1}{H}\sum_{j=1}^{J} \pi_{j,t} - \int_0^{I_t} p_t(i)c_{h,t}(i)di \tag{3}$$

with $b_{h,t}$ the individual wealth, r_t the interest rate, w_t the wage rate, $\pi_{j,t}$ the profit of the large providers of digital services equally redistributed to households (large providers are the only type

[8] The elasticity of substitution is equal to $\frac{1}{1-\alpha\epsilon} \in]1, \infty[$ and the taste for variety is equal to $\frac{1}{\alpha\epsilon} - 1$. When α or ϵ increases, the elasticity of substitution rises while the taste for variety decreases.

[9] Empirical research has shown a trend toward declining hours worked and increasing leisure time over the past few decades. Aguiar & Hurst (2007) estimate that leisure time has increased by roughly six to nine hours per week for men and four to eight hours for women between 1965 and 2003 in the US. Vandenbroucke (2009) and Kopytov *et al.* (2023) show evidence that decreased work hours in several countries since 1900 are due to decreased leisure prices. Some studies highlight the importance of digital leisure in the reallocation of time. Wallsten (2015) shows that digital leisure is a substitute for other types of leisure and work. Aguiar *et al.* (2020) show that innovation in digital, more precisely the quality improvement of recreational computing and gaming, is responsible for the decline in worked hours by young men.

of firms to make profits in the model), and $p_t(i)$ is the price of the variety of good i.

The representative household maximizes the intertemporal utility function under the law of wealth evolution given by (3):

$$
\begin{aligned}
&\max_{c_{h,t}(i),s_{h,j,t},s_{h,t}(k),d_{h,j,t},d_{h,t}(k)} U_{h,0} = \sum_{t=0}^{\infty} \rho^t \Big[\ln C_{h,t} + \ln D_{h,t}\Big] \\
&s.t \quad b_{h,t+1} = (1+r_t)b_{h,t} + \left(\ell - \sum_{j=1}^{J} s_{h,j,t} - \int_0^{K_t} s_{h,t}(k)dk\right)w_t + \frac{1}{H}\sum_{j=1}^{J}\pi_{j,t} - \int_0^{I_t} p_t(i)c_{h,t}(i)di
\end{aligned}
\tag{4}
$$

with $b_{h,0}$ is given.

We solve this program in several steps. In the first step, the household decides the optimal quantity of data she discloses to each digital service provider. To this aim, at each period, she chooses the quantity of data $d_{h,j}$ and $d_h(k)$ which maximize the utility function of spending time online given by equation (2).[10] The optimal quantity of data disclosed by the household h to each digital service provider j and k is obtained by solving the following First Order Conditions (FOCs):

$$\frac{\partial D_h}{\partial d_{h,j}} = 0$$

and

$$\frac{\partial D_h}{\partial d_h(k)} = 0$$

It gives us:

$$d_{h,j} = d_h(k) = \alpha \bar{d}_h \tag{5}$$

Households disclose the same quantity of personal information to each digital service provider. Indeed, the use of tracking cookies enables digital providers to follow the online activity of each household. A single provider can, therefore, record users' online behavior on all the digital services if the user has accepted cookies. The sensitivity to privacy is identical for all digital services. Data collected by digital services is higher when the sensitivity to privacy is low (high α). Note that if the privacy parameter α equals 1, the household consents to provide all personal data (within

[10] We omit the time index t for all static programs to simplify notation.

the limit of what the digital firm can collect). On the contrary, if α equals 0, households do not provide any data. This situation means that households do not consume digital services, which is irrelevant to this paper. It could also correspond to the case where a regulation prohibits data collection. In this case, digital service providers should change their business model, for example, by pricing access to the service.[11]

By substitution and using the optimal quantity of data disclosed to each digital services provider given by equation (5), we can rewrite the equation (2) as:

$$D_h = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \overline{d}_h^{-\frac{1}{\alpha}} \Big[\sum_{j=1}^{J} (s_{h,j} q_j)^{\alpha\epsilon} + \int_0^K (s_h(k) q(k))^{\alpha\epsilon} dk \Big]^{\frac{1}{\alpha\epsilon}} \tag{6}$$

We have:

$$s_{h,0} q_0 = \Big(\int_0^K (s_h(k) q(k))^{\alpha\epsilon} dk \Big)^{\frac{1}{\alpha\epsilon}}$$

We can, therefore, rewrite the equation (6) as:

$$D_h = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \overline{d}_h^{-\frac{1}{\alpha}} \Big(\sum_{j=0}^{J} (q_j s_{h,j})^{\alpha\epsilon} \Big)^{\frac{1}{\alpha\epsilon}} \tag{7}$$

The household now has to determine (i) the optimal time spent on each digital service of small providers and (ii) the optimal time spent on each digital service (large and total small) by solving the two following programs:

$$\begin{aligned} &\min_{s_h(k)} \int_0^K s_h(k) dk \\ &s.t. \quad s_{h,0} q_0 = \Big(\int_0^K (s_h(k) q(k))^{\alpha\epsilon} dk \Big)^{\frac{1}{\alpha\epsilon}} \end{aligned} \tag{8}$$

[11]Some platforms have expressed concerns about potential threats to their existence arising from restrictions on data collection. As a response, some are now offering consumers the option of giving their data or paying for the service. This was recently the case with Meta in Europe.

and

$$\begin{aligned} &\min_{s_{h,j}} \sum_{j=0}^{J} s_{h,j} \\ &s.t. \quad QS_h = \Big(\sum_{j=0}^{J} (s_{h,j} q_j)^{\alpha\epsilon}\Big)^{\frac{1}{\alpha\epsilon}} \end{aligned} \tag{9}$$

The optimal demand for the digital service of small provider k (or optimal time spent on the digital service k) is given by:[12]

$$s_h(k) = s_{h,0}\Big(\frac{q(k)}{q_0}\Big)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}} \tag{10}$$

with $s_{h,0} = \int_0^K s_h(k)dk$ the total time spent on all digital services of small providers and $q_0 = (\int_0^K q(k)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}} dk)^{\frac{1-\alpha\epsilon}{\alpha\epsilon}}$ an index of the quality of all digital services of small providers.

The optimal time spent on the digital service j is given by:

$$s_{h,j} = S_h\Big(\frac{q_j}{Q}\Big)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}} \tag{11}$$

with $S_h = \sum_{j=0}^{J} s_{h,j}$ the total time spent on all digital services and $Q = (\sum_{j=0}^{J} q_j^{\frac{\alpha\epsilon}{1-\alpha\epsilon}})^{\frac{1-\alpha\epsilon}{\alpha\epsilon}}$ an index of the quality of all digital services.

According to equation (11), we have $s_{h,0} = S_h\Big(\frac{q_0}{Q}\Big)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}$. We can easily deduce that equation (10) can be rewritten as:

$$s_h(k) = S_h\Big(\frac{q(k)}{Q}\Big)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}$$

Time spent on each digital service depends on the total online time, the relative quality of the service, but also on the sensitivity to privacy (equations (10) and (11)). Consequently, an increase in the quality of a digital service negatively impacts the demand for other digital services. Moreover, the lower the privacy sensitivity (high α), the higher the household spends time on the digital service. Indeed, the sensitivity to privacy decreases the utility of spending time online through disutility from data collection.

[12] See proof 1 in appendix B.1.

At the last stage, the household determines its optimal consumption $c_h(i)$ of good i at each period by solving the following program:

$$\max_{c_h(i)} C_h = \left(\int_0^I (\tilde{z}_h(i) c_h(i))^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$
$$s.t. \quad E_h = \int_0^I p(i) c_h(i) di$$

With this traditional Dixit & Stiglitz (1977) formulation and for a given expenditure level E_h, the optimal demand of household h for the good i is:

$$c_h(i) = \tilde{z}_h(i)^{\sigma-1} \frac{E_h}{P} \left(\frac{p(i)}{P} \right)^{-\sigma} \tag{12}$$

with P is an index of all variety's prices given by:

$$P = \left(\int_0^I \left(\frac{p(i)}{\tilde{z}_h(i)} \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$$

and $E_h = PC_h = \int_0^I p(i) c_h(i) di$.

Finally, the household has to choose the optimal level of the consumption index $C_{h,t}$ and the optimal time spent on digital services $S_{h,t}$ by solving the intertemporal program (4). Note that we can rewrite the budget constraint given by equation (3) as:

$$b_{h,t+1} = (1 + r_t) b_{h,t} + (\ell - S_{h,t}) w_t + \frac{1}{H} \sum_{j=1}^{J} \pi_{j,t} - P_{h,t} C_{h,t}$$

and the online time utility function given by (7) as:

$$D_{h,t} = \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} \bar{d}_h^{-\frac{1}{\alpha}} S_{h,t} Q_t \tag{13}$$

Therefore, the intertemporal utility program can be rewritten as:

$$\max_{C_{h,t},S_{h,t}} U_{h,0} = \sum_{t=0}^{\infty} \rho^t \left[\ln C_{h,t} + \ln S_{h,t} Q_t + \ln \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \overline{d}_h^{\frac{1}{\alpha}} \right]$$

$$s.t \quad b_{h,t+1} = (1+r_t)b_{h,t} + (\ell - S_{h,t})w_t + \frac{1}{H}\sum_{j=1}^{J} \pi_{j,t} - P_t C_{h,t}$$

By solving the following intertemporal Lagrangian with respect to $C_{h,t}$, $S_{h,t}$, and the state variable $b_{h,t+1}$, we obtain the optimal online time which depends on the optimal consumption expenditure level of the representative household and the wage rate:

$$S_{h,t} = \frac{P_t C_{h,t}}{w_t} = \frac{E_{h,t}}{w_t}$$

and the optimal dynamics of total consumption expenditure (Keynes-Ramsey equation):

$$E_{h,t+1} = \rho(1 + r_{t+1})E_{h,t} \tag{14}$$

The transversality condition is given by:

$$\lim_{t\to\infty} \rho^t \frac{b_{h,t}}{E_t} = 0$$

4.2 Traditional firms

The traditional sector is composed of a continuum of I_t firms indexed by i in monopolistic competition, each selling a (horizontally) differentiated variety *à la* Dixit & Stiglitz (1977). The number of firms is endogenously determined for each period as there is free entry of firms into the traditional (or final goods) market. The production function of traditional firms is given by:

$$x_t(i) = L_t^x(i)$$

where $x_t(i)$ is the production of the variety of good i in period t and $L_t^x(i)$ is the quantity of labor used in the production of the variety of good i in period t.

Following Grossmann (2008), each firm can enhance the perceived quality of its good to stimulate its demand. For this purpose, firms can improve the true quality of the good and buy advertising in the digital sector. In Grossmann (2008), firms incur advertising expenditures by employing a quantity of marketing labor. In this paper, we model the specificity of digital advertising. Firms buy a quantity of personalized impressions from digital service providers to display their advertising to users on digital services.[13] Impressions are personalized because digital service providers use personal information collected to target advertising. The more impressions of a good i a household h views, the more she will value the quality of the good i.

To improve the true quality $z_t(i)$ of the good i in period t, the firm i have to invest in R&D by employing in period $t-1$ a quantity of labor $L^R_{t-1}(i)$, such as:

$$z_t(i) = \overline{z}_{t-1} L^R_{t-1}(i)^{\beta^r}$$

with $\beta^r > 0$ a parameter of the effectiveness of R&D and $\overline{z}_t = \frac{1}{I_t} \int_0^{I_t} z_t(i)di$ the state of technology in $t-1$ representing the average knowledge accumulation acquired by previous R&D activities. Knowledge acquired by each firm is private information for one period. Therefore, $\overline{z}_{t-1}$ provides a positive externality in the research activity of firms in the following period (intertemporal spillover). Improving $\overline{z}_{t-1}$ enables less labor to obtain the same quality of goods. Research activity is, therefore, the growth driver of our model.[14]

As in Grossmann (2008), advertising is combative in the sense that *"an increase in marketing expenditure of a single firm creates a negative externality on demand faced by other firms"*. As a consequence, the perceived quality of the variety of good i by household h is not influenced by the number of impressions she saw but by the quantity of impressions on good i view by household

[13]There is a wide range of payment methods in the digital advertising market. In this paper, we model the Cost Per Impression method, where firms pay for the number of advertising displayed and viewed by a user on the website. Other popular methods include the Cost Per Click, where the firms pay only when a user clicks on the advertising. In general, impressions price is determined by an auction system (Goldfarb, 2014). For the sake of simplicity, we do not model auction systems.

[14]This approach to model endogenous growth was introduced by Young (1998) and extended by Dinopoulos & Thompson (1998), Peretto (1998), and Howitt (1999). It enables to have endogenous growth without scale effects on the growth rate. In these papers, a rise in population proportionally increases the number of varieties in the economy. Therefore, the size of each sector and the effort of R&D in each sector are unaffected by the population increase. Population growth does not impact the economic growth rate but only has scale effects on income per capita. The reader may refer to Jones (1999) for more details.

h relative to the average quantity of impressions view by household h:

$$m_{h,t}(i) = \frac{a_{h,t}(i)}{\overline{a}_{h,t}} \tag{15}$$

where $a_{h,t}(i)$ is the quantity of impressions purchased by the firm i in period t displayed to household h and $\overline{a}_{h,t} = \frac{1}{I_t}\int_0^{I_t} a_{h,t}(i)di$ the average quantity of impressions by firms displayed to household h.

Finally, the perceived quality $\tilde{z}_{h,t}(i)$ of good i by household h in period t is given by:

$$\tilde{z}_{h,t}(i) = z_t(i)m_{h,t-1}^{\beta^a}(i) \tag{16}$$

where $\beta^a \geqslant 0$ measures the effectiveness of advertising on the perceived quality by household h of good i.

The profit $\pi_t^x(i)$ of the traditional firm i in period t is given by:

$$\pi_t^x(i) = p_t(i)x_t(i) - w_t L_t^x(i)$$

By substitution and using the optimal demand of household h for the variety of good i given by equation (12) and the perceived quality function (16), we can rewrite the profit function of the firm i as:

$$\pi_t^x(i) = (p_t(i) - w_t)\sum_{h=1}^{H}\left(\left(\overline{z}_{t-1}{L_{t-1}^R}^{\beta^r}(i)\left(\frac{a_{h,t-1}(i)}{\overline{a}_{h,t-1}}\right)^{\beta^a}\right)^{\sigma-1}\frac{E_{h,t}}{P_t}\left(\frac{p_t(i)}{P_t}\right)^{-\sigma}\right)$$

Each firm also incurs a fixed labor cost $f^x > 0$ at period $t-1$, which can be interpreted as organization costs. Therefore, in period $t-1$, firms face three costs to improve the perceived quality of their goods in t: fixed, R&D and advertising costs. Firms borrow in the perfect financial market in $t-1$ to be financed and repay their debt with interest in the next period. Traditional firms are in monopolistic competition, and there is free entry into the market. Therefore, they do not have any assets in $t-1$ since they will only make profits in t, which will cover the repayment of the loan generated in the previous period.

At period $t-1$, the firm i chooses the quantity of labor $L^R_{t-1}(i)$, the number of impressions $a_{h,t-1}(i)$ destined to each household h, and the price $p_t(i)$ that maximize the firm's value:[15]

$$\begin{aligned} V_{t-1}(i) =& \frac{\pi^x_t(i)}{1+r_t} - w_{t-1}(L^R_{t-1}(i) + f^x) - p^a_{t-1}\sum_{h=1}^{H} a_{h,t-1}(i) \\ =& \frac{p_t(i) - w_t}{1+r_t}\sum_{h=1}^{H}\left(\left(\overline{z}_{t-1}{L^R_{t-1}}^{\beta^r}(i)(\frac{a_{h,t-1}(i)}{\overline{a}_{h,t-1}})^{\beta^a}\right)^{\sigma-1}\frac{E_{h,t}}{P_t}\left(\frac{p_t(i)}{P_t}\right)^{-\sigma}\right) \\ & - w_{t-1}(L^R_{t-1}(i) + f^x) - p^a_{t-1}\sum_{h=1}^{H} a_{h,t-1}(i) \end{aligned} \tag{17}$$

where p^a_{t-1} is the price of one impression in period $t-1$ and $p^a_{t-1}\sum_{h=1}^{H} a_{h,t-1}(i)$ the total value of impressions bought by firm i in period $t-1$.

FOCs with respect to $p_t(i)$, $L^R_{t-1}(i)$, and $a_{h,t-1}(i)$ gives us the optimal price of variety of good i, the optimal quantity of labor employed in the R&D of the variety of good i, and the optimal quantity of impressions bought by the firm i destined for the household h:

$$p_t(i) = w_t\frac{\sigma}{\sigma-1}$$

$$L^R_{t-1}(i) = (\sigma-1)\beta^r\left(\frac{p_t(i)-w_t}{1+r_t}\right)\frac{\sum_{h=1}^{H} c_{h,t}(i)}{w_{t-1}}$$

$$a_{h,t-1}(i) = (\sigma-1)\beta^a\left(\frac{p_t(i)-w_t}{1+r_t}\right)\frac{c_{h,t}(i)}{p^a_{t-1}}$$

The demand for labor for R&D activity of good i and impressions of good i displayed to household h positively depends on the price of the variety of good i, the efficiency parameter associated, and the demand for the good i and negatively on the wage rate. The demand for impressions also negatively depends on prices.

We deduce that the ratio between the labor allocated to research activity and the quantity of impressions purchased destined to the household h depends on the ratio of their respective

[15]To ensure the concavity of the objective function (17), we impose that $(\beta^a + \beta^r)(\sigma - 1) < 1$.

effectiveness and the price of the advertising:

$$\frac{L_t^R(i)}{a_{h,t}(i)} = \frac{\beta^r}{\beta^a} \frac{p_{t-1}^a}{w_{t-1}} \frac{\sum_{h=1}^H c_{h,t}(i)}{c_{h,t}(i)}$$

4.3 Free digital service providers

The free digital services sector is composed of a constant number of large digital service providers, indexed $j \in \mathcal{J} = \{1, ..., J\}$ and a competitive fringe with a continuum of small providers, indexed $k \in \mathcal{K} = [1, K_t]$ as in Shimomura & Thisse (2012). Each digital service provider is identical within its category. The difference between the two categories is that large providers have an advantage in digital service quality over small providers. The quality production functions of the digital service for large and small providers are given by the two following equations:

$$q_{j,t} = \beta^j L_{j,t}^q \tag{18}$$

$$q_t(k) = \beta^k L_t^q(k) \tag{19}$$

with $\beta^j > \beta^k$ and $L_{j,t}^q$ and $L_t^q(k)$ are, respectively, the quantity of labor employed by the digital service provider j and k which determine the quality of the digital service. Households spend more time on the digital services of large providers as their quality is much higher than small ones. Indeed, these services benefit from significant network effects, making using services used by other users more attractive.

Digital service providers generate revenue by selling advertising to traditional firms. They need households' attention (households must spend time on their digital service) and household data to produce impressions. Time and data are substitutable in the production function. The more time the household spends on the digital service, the more impressions the providers can display to her. Moreover, digital service providers use data to produce personalized impressions for each household h. Data enables digital service providers to match advertising with users interested in the advertised good. Therefore, they supply impressions tailored to each household to traditional firms. The more information the firm has about the household, the more impressions it can display. The production function of impressions destined for the household h for large and small providers is given by:

$$a^s_{h,j,t} = s^{\gamma}_{h,j,t} d^{1-\gamma}_{h,j,t}$$

$$a^s_{h,t}(k) = s_{h,t}(k)^{\gamma} d_{h,t}(k)^{1-\gamma}$$

where $0 < \gamma < 1$ is an indicator of the importance of data collection in the production of impressions. To simplify, we now assume that $\overline{d}_h$, the maximum quantity of data that a provider can collect on the household h, is equal to $S_{h,t}$, the total online time of the household h. Indeed, by using cookies, digital service providers can track their users' online activity. Therefore, we assume that the more a household spends time online, the more data she can disclose. However, we assume that households do not consider that $\overline{d}_h$ depends on the time spent on digital services and, therefore, they take as given this variable.[16] By substitution and using the optimal quantity of data given by households (equation (5)) and the optimal time spent on each digital service (equations (10) and (11)), the production function of impressions destined for the household h can be rewritten as:

$$a^s_{h,j,t} = \alpha^{1-\gamma} S_{h,t} \left(\frac{q_{j,t}}{Q_t} \right)^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon}} \tag{20}$$

$$a^s_{h,t}(k) = \alpha^{1-\gamma} S_{h,t} \left(\frac{q_t(k)}{Q_t} \right)^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon}} \tag{21}$$

There is free entry into the competitive fringe but not into the digital service of large providers. Large digital service providers have a higher fixed cost than small providers. This creates an entry barrier to joining large digital service providers (the incumbents). The profit functions for large and small digital service providers are given by:

$$\pi_{j,t} = p^a_t \Big(\sum_{h=1}^{H} a^s_{h,j,t} \Big) - w_t \Big(L^q_{j,t} + f^j \Big)$$

$$\pi_t(k) = p^a_t \Big(\sum_{h=1}^{H} a^s_{h,t}(k) \Big) - w_t \Big(L^q_t(k) + f^k \Big)$$

[16] If $\overline{d}_h$ were a household control variable, with the functional form retained in our model, households could increase their welfare by increasing this maximum amount while keeping the volume of data collected unchanged. This does not seem realistic.

with f^j and f^k the fixed labor cost incurred by the large and small providers and $f^j > f^k$. By substitution and using the production functions of quality (equations (18) and (19)) and impressions (equations (20) and (21)), the profit functions are:[17]

$$\pi_{j,t} = p_t^a \alpha^{1-\gamma} \Big(\frac{q_{j,t}}{Q_t}\Big)^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon}} \sum_{h=1}^{H} S_{h,t} - w_t \Big(\frac{1}{\beta^j} q_{j,t} + f^j\Big)$$

$$\pi_t(k) = p_t^a \alpha^{1-\gamma} \Big(\frac{q_t(k)}{Q_t}\Big)^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon}} \sum_{h=1}^{H} S_{h,t} - w_t \Big(\frac{1}{\beta^k} q_t(k) + f^k\Big) \tag{22}$$

All digital service providers are price and global quality takers, as large providers have no strategic behaviors.[18] Therefore, each provider seeks to maximize its profit by choosing the optimal digital service quality. FOC's give us the optimal quality of each type of digital service:

$$q_{j,t} = \Big(\frac{\beta^j}{w_t} p_t^a \frac{\alpha^{2-\gamma}\gamma\epsilon}{1-\alpha\epsilon} \sum_{h=1}^{H} S_{h,t}\Big)^{\frac{\alpha\epsilon-1}{\alpha\epsilon(1+\gamma)-1}} Q^{\frac{\gamma\alpha\epsilon}{\alpha\epsilon(1+\gamma)-1}} \tag{23}$$

$$q_t(k) = \Big(\frac{\beta^k}{w_t} p^a \frac{\alpha^{2-\gamma}\gamma\epsilon}{1-\alpha\epsilon} \sum_{h=1}^{H} S_{h,t}\Big)^{\frac{\alpha\epsilon-1}{\alpha\epsilon(1+\gamma)-1}} Q^{\frac{\gamma\alpha\epsilon}{\alpha\epsilon(1+\gamma)-1}} \tag{24}$$

Using equations (23) and (24), we obtain the ratio between the quality of the large digital service j and the small one k:

$$\frac{q_{j,t}}{q_t(k)} = \beta^{\frac{\alpha\epsilon-1}{\alpha\epsilon(1+\gamma)-1}} \tag{25}$$

which mainly depends on $\beta = \frac{\beta^j}{\beta^k}$ the gap between the quality efficiency of large and small digital service providers. As $\beta^j > \beta^k$, the quality level of large providers is higher than that of small providers. Households, therefore, consume a higher quantity of digital services from large providers than small ones.

[17] To ensure the concavity of the profit functions, we have to impose that $\alpha\epsilon(1+\gamma) < 1$.

[18] Modeling the strategic behavior of large digital service providers does not provide a general equilibrium analytical solution. There is a literature focusing on the strategic behavior of firms in a mixed market. However, even with simple functional forms (linear demand functions), Huppmann (2013) must perform numerical simulations. Okuguchi (1985) find equilibrium but with specific functional forms and strict conditions on the functions. To incorporate strategic behavior, we must adopt a partial equilibrium approach. Moreover, we have chosen to obtain an analytical solution without numerical simulation.

5 Equilibrium

We now characterize the macroeconomic equilibrium. We choose labor as the numeraire ($w_t = 1,\ \forall t$). We are at the symmetric equilibrium. All households, traditional firms, large digital service providers, and small ones are identical. Therefore, households consume the same quantity of each variety of good, such as:

$$c_{h,t}(i) = c_t = \frac{E_t}{pI_t} \tag{26}$$

and the time spent on digital services of large and small providers is given by:

$$s_{h,j,t} = s_t^j = \frac{S_t}{J + \beta^{\frac{\alpha\epsilon}{\alpha\epsilon(1+\gamma)-1}} K_t}$$

$$s_{h,t}(k) = s_t^k = \frac{S_t}{K_t + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J}$$

The total time spent online by each household is given by:

$$S_{h,t} = S_t = E_t \tag{27}$$

The price index of the traditional variety of goods at the symmetrical equilibrium is given by:

$$P_t = \frac{p}{\tilde{z}_t} I_t^{\frac{1}{1-\sigma}}$$

with the equilibrium price of each traditional good:

$$p = \frac{\sigma}{\sigma - 1} \tag{28}$$

At the symmetric equilibrium, all traditional firms employ the same quantity of labor in the R&D activity and buy the same quantity of impressions destined for each household:

$$L_{t-1}^R(i) = L_{t-1}^R = (\sigma - 1)\beta^r \left(\frac{p-1}{1+r_t}\right) x_t \tag{29}$$

$$a_{h,t-1}(i) = a_{t-1} = (\sigma - 1)\beta^a \left(\frac{p-1}{1+r_t}\right) \frac{x_t}{H p_{t-1}^a} \tag{30}$$

with $x_t = Hc_t$ the total supply of goods of a traditional firm.

Advertising is combative in this model. Therefore, at the symmetric equilibrium, the perceived quality given by equation (16) is equal to the true quality as the quantity of impressions bought by a firm relative to the average impressions is equal to 1 (equation (15)).

There is a free entry of the firms into the traditional sector. Consequently, the firm value is equal to zero. Using this condition and the optimal demand of good given by the equation (26), we obtain:

$$\frac{p-1}{1+r_t}\frac{HE_t}{pI_t} = L^R_{t-1} + p^a_{t-1}Ha_{t-1} + f^x \tag{31}$$

At the symmetric equilibrium, the quality index of all digital services is given by :

$$Q_t = \left(\int_0^{K_t} q_t(k)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}} dk + \sum_{j=1}^{J} q_{j,t}^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}\right)^{\frac{1-\alpha\epsilon}{\alpha\epsilon}} = \left(K_t q_t(k)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}} + J q_{j,t}^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}\right)^{\frac{1-\alpha\epsilon}{\alpha\epsilon}}$$

Using equation (25), we obtain:

$$Q_t = q_t^j \left(\beta^{\frac{\alpha\epsilon}{\alpha\epsilon(1+\gamma)-1}} K_t + J\right)^{\frac{1-\alpha\epsilon}{\alpha\epsilon}} = q_t^k \left(K_t + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\right)^{\frac{1-\alpha\epsilon}{\alpha\epsilon}} \tag{32}$$

Using the quality index of digital services given by equation (32), the total impressions equations of digital service providers j and k (equations (20) and (21)) can be rewritten as:

$$a_t^j = H\alpha^{1-\gamma} S_t \left(\beta^{\frac{\alpha\epsilon}{\alpha\epsilon(1+\gamma)-1}} K_t + J\right)^{-\gamma} \tag{33}$$

$$a_t^k = H\alpha^{1-\gamma} S_t \left(K_t + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\right)^{-\gamma} \tag{34}$$

and the quality of each digital service (equations (23) and (24)) as:

$$q_t^j = (\beta^j p^a \frac{\alpha^{2-\gamma}\gamma\epsilon}{1-\alpha\epsilon} HS_t)\left(\beta^{\frac{\alpha\epsilon}{\alpha\epsilon(1+\gamma)-1}} K_t + J\right)^{-\gamma} \tag{35}$$

$$q_t^k = (\beta^k p^a \frac{\alpha^{2-\gamma}\gamma\epsilon}{1-\alpha\epsilon} HS_t)\left(K_t + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\right)^{-\gamma} \tag{36}$$

Following an increase in impression price, digital service providers improve the quality of their services to incentivize households to spend more time on their services. It enables them to collect more data and attention and increase the quantity of impressions supplied.

Using the profit function of small digital service providers (equation (22)), the production function of quality (equation (19)), and the supply of impressions by small digital service providers (equation (21)), the free entry among small digital service providers imposes:

$$p_t^a \alpha^{1-\gamma} \left(\frac{q_t^k}{Q_t} \right)^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon}} HS_t = \frac{1}{\beta^k} q_t^k - f^k \tag{37}$$

Finally, the macroeconomic equilibrium must satisfy the different markets' equilibrium conditions. Advertising, traditional goods, and labor market clearing conditions are given by:[19]

$$I_{t+1} H a_t = K_t a_t^k + J a_t^j \tag{38}$$

$$H c_t = x_t = L_t^x \tag{39}$$

$$H(\ell - S_t) = I_{t+1}(L_t^R + f^x) + I_t L_t^x + K_t(L_t^k + f^k) + J(L_t^j + f^j) \tag{40}$$

At the symmetric equilibrium, we find a unique stationary value of E (see proof 2 in appendix B.2):

$$E^* = \left(\frac{\sigma}{\rho - 1 + 2\sigma} \right) \left[\ell + \frac{J}{H} \left(\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} f^k - f^j \right) \right] \tag{41}$$

The equilibrium expenditure for each household positively depends on the worked hours and the profit of the large digital service providers[20] and negatively on the discount rate and the substitution parameter σ (an increase in substitution between traditional goods will decrease the price of traditional goods p). Note that to ensure that their profits are positive, we have $\beta^{\frac{\gamma\alpha\epsilon}{\alpha\epsilon(1+\gamma)-1}} f^k > f^j > f^k$. The gap between the fixed cost of large and small digital service providers cannot be too high. We also impose that $\ell > \frac{\rho+2\sigma-1}{\rho+\sigma-1} \frac{J}{H} \left[\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} f^k - f^j \right]$ to ensure that the individual time of a household is strictly superior to the time spent online. We assume also that the labor force is significantly higher than the labor fixed costs ($H(\ell - S^*) >> f^x + f^k + f^j$) to

[19] According to Walras' law, the financial market is in equilibrium if the labor, advertising, and traditional goods markets are in equilibrium.

[20] At symmetric equilibrium, $\pi_t^j = p_t^a a_t^j - \frac{1}{\beta^j} q_{t-1}^j - f^j = f^k \beta^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} - fj$ (see proof 2 in appendix B.2).

ensure labor resources necessary to produce traditional goods and digital services.

The stationarity of E at symmetric equilibrium implies that the interest rate also immediately jumps to its stationary value (equation (14)), with $r^* = \frac{1-\rho}{\rho}$. All other variables, with the exception of the true quality of traditional goods (z_t), the consumption index (C_t), and well-being, are therefore also in a stationary state from the initial moment.

The stationary equilibrium values of the number of firms in the traditional sector, the demand for labor in the R&D activity by each firm, and the quantity of impressions bought by each firm are given by (see the proof 2 in appendix B.2):

$$I^* = \frac{\rho}{\sigma}\frac{E^* H}{f^x}(1-(\sigma-1)(\beta^r+\beta^a))$$

$$L^{R*} = \frac{(\sigma-1)\beta^r f^x}{1-(\sigma-1)(\beta^r+\beta^a)} \tag{42}$$

$$a^* = \beta^a \frac{\sigma-1}{\sigma}\rho \frac{E^*}{I^* p^{a^*}}$$

At equilibrium, most of the traditional sector's endogenous variables are unaffected by the free digital services sector. This disconnect between the traditional goods and free digital services sectors comes from the linearity of household preferences. In our model, the main connections between the two sectors come from labor and advertising.

In the free digital services sector, the equilibrium values of the number of small providers, the quality of digital services provided by small and large providers, and the price of advertising are given by (see proof 2 in appendix B.2):

$$K^* = \beta^a \rho \frac{\sigma-1}{\sigma}\frac{1-\alpha\epsilon(1+\gamma)}{1-\alpha\epsilon}\frac{H}{f^k}E^* - \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J \tag{43}$$

$$q^{k*} = \frac{\alpha\gamma\epsilon}{1-\alpha\epsilon(1+\gamma)}\beta^k f^k \tag{44}$$

$$q^{j*} = \frac{\alpha\gamma\epsilon}{1-\alpha\epsilon(1+\gamma)}\beta^{k\frac{\alpha\epsilon\gamma}{\alpha\epsilon(1+\gamma)-1}}\beta^{j\frac{\alpha\epsilon-1}{\alpha\epsilon(1+\gamma)-1}} f^k$$

$$p^{a*} = \beta^a \frac{\sigma-1}{\sigma}\rho\alpha^{\gamma-1}\Big[K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\Big]^{\gamma}\Big[K^* + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J\Big]^{-1}$$

GDP is given by the sum of final consumption of traditional goods. At the symmetric equilibrium, it is equal to $H_t I_t c_t$. These three variables are stationary from the initial moment. Therefore, in our model, there is no GDP growth.[21] GDP does not grow at the equilibrium, but the instantaneous individual well-being can increase through the increase in the quality of the goods included in the index of consumption C_h.[22] We find that the consumption index is given by (see proof 3 in appendix B.2):

$$C_{h,t} = \frac{\sigma - 1}{\sigma} I^{*\frac{1}{\sigma-1}} \overline{z}_0 E^* (L^{R*})^{\beta^r \times t} \tag{45}$$

and the growth rate of the consumption index by:

$$g_C = \beta^r \ln(L^{R^*}) \tag{46}$$

To ensure that g_C is strictly positive, we have to impose that $\beta^r(\sigma - 1)f^x > 1 - (\sigma - 1)(\beta^r + \beta^a)$. The consumption index given by equation (45) depends on the number of goods varieties, the state of technology (and thus the quality of the goods), and the optimal level of expenditure. As a consequence, it can be impacted by the free digital services sector through a change in optimal expenditure as the latter is related to the online time (equation (27)). Nevertheless, at the symmetric equilibrium, the optimal expenditure is constant. Therefore, the only driver of the consumption index growth given by equation (46) is R&D. The latter depends solely on parameters specific to the traditional goods sector and is therefore not impacted by the free digital services sector (equation (42)). The higher the investment in R&D (L^R) or the higher the efficiency of R&D (β^r), the higher the perceived quality of the goods will be and, therefore, the consumption index growth. Indeed, due to the spillover effects of R&D, the true quality of the differentiated goods is constantly improving. In our model, in contrast to traditional goods, free digital services do not benefit from a permanent increase in quality. Indeed, technical progress is significantly lower in services activities than in manufacturing (Herrendorf *et al.*, 2015).

[21] GDP growth is approximately equal to $\ln\left(\frac{H_t I_t c_t}{H_{t-1} I_{t-1} c_{t-1}}\right)$.

[22] Jones & Klenow (2016) empirically highlighted that income and welfare growth are different. They find that the average welfare growth in the US was around 3.1% between the 1980s and mid-2000s while income growth was around 2.1%.

The intertemporal utility is given by (see proof 3 in appendix B.2):

$$U_0 = X + \frac{1}{1-\rho}\Big[2\ln E^* + \frac{\rho}{\sigma-1}\ln I^* + \ln\left(\frac{\alpha\gamma\epsilon}{1-\alpha\epsilon(1+\gamma)}\beta^k f^k\right) + \frac{1-\alpha\epsilon}{\alpha\epsilon}\ln\left(K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\right)\Big]$$

with $X = \frac{1}{\sigma-1}\ln I_0 + \frac{1}{1-\rho}\ln(\frac{\sigma-1}{\sigma}\overline{z}_0\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\overline{d}_h^{\frac{1}{\alpha}}) + \frac{\rho}{(\rho-1)^2}\beta^r\ln(L^{R^*})$. Several components impact the intertemporal utility. First, it depends on the consumption of differentiated goods and the true quality of these goods. Moreover, households benefit from an increase in the number of traditional firms due to the taste for variety (Dixit & Stiglitz, 1977). The intertemporal utility also depends on the free digital services sector; it positively depends on online time and free digital service quality.

6 The macroeconomic impacts of free digital services

The main aim of this paper is to determine the interactions between the free digital services sector and the rest of the economy, focusing on the macroeconomic impacts of this sector and its potential developments. Our model dynamics, inspired by Young's approach, are relatively straightforward. With the exception of the consumption index of traditional goods and instantaneous well-being, all other endogenous variables are in a steady state from the outset. Therefore, we can perform a conventional comparative static analysis to assess how changes in the key characteristics of the free digital services sector affect macroeconomic aggregates. Firstly, we explain the reason behind the independence between free digital services sector and consumption growth. Then, the analysis focuses on three parameters and exogenous variables that characterize the free digital services sector: the number of large free digital services providers (J), the gap between the quality efficiency of large and small providers (β) and the weight given to attention in impression production (γ). The study then shifts toward the parameters defining household preferences regarding the free digital services sector: the preference parameter for privacy (α) and the substitution parameter between digital services (ϵ). In these two stages of the analysis, comparative statics are conducted by differentiating between two categories of endogenous variables: those concerning the free digital services sector and those related to the traditional sector and household welfare. The most important result's proofs of the comparative-static are presented in appendix B.3.

6.1 Free digital services and consumption growth

The only growth driver in the model is the effort in R&D given by L^r (see equation (46)). However, at the symmetric equilibrium, the optimal level of firms' R&D is independent of the free digital services sector characteristics due to the free entry in the traditional sector. Indeed, by integrating the equations for optimal demand of R&D effort (equation (29)) and impressions demand (equation (30)) with the equation for free entry into the traditional goods market (equation (31)), we directly obtain the optimal level of production of traditional goods:

$$x^* = \frac{\sigma - 1}{\rho} \frac{1}{1 - (\beta^R + \beta^a)(\sigma - 1)} f^x$$

This value of x^* determines the level of R&D effort and total advertising expenditures, which are also independent of the free digital services sector characteristics.[23] This does not imply that this sector does not impact the traditional sector. The traditional sector responds by adjusting the number of varieties available in the market (I), rather than altering the quality of goods and, consequently, the investment in R&D. This is due to the free entry into the traditional market. Consequently, a shock in the free digital services sector can instantaneously modify the level of consumption index but not its growth rate.

6.2 Key characteristics of free digital services sector

Entry of a new large digital service provider

We investigate the effects of market structure in the free digital services sector by examining the economic dynamics resulting from the entry of a new large provider. This entry increases the number of large digital service providers (J). Our findings indicate that an elevation in J exerts

[23]By integrating the value of x^* in equations (29) and (30), we obtain that $L^{r^*} = \frac{(\sigma-1)\beta^r f^x}{1-(\sigma-1)(\beta^r+\beta^a)}$ as in the equation (42) and $p^a Ha = \frac{\sigma-1}{1-(\beta^R+\beta^a)(\sigma-1)}\beta^a f^x$.

a positive influence on the optimal expenditure level E^*, time spent online S^*, the number of traditional firms I^*, and the advertising price p^{a^*}.[24] Conversely, it negatively affects the quantity of small providers K^* and does not alter the quality of digital services.

Consistent with Shimomura & Thisse (2012), the entry of new large providers does not similarly affect large and small providers in a mixed market. Households demonstrate a strong preference for the services of large providers due to their superior quality compared to that of small ones. At the symmetric equilibrium, all large providers maintain equivalent quality levels; therefore, households reallocate a portion of their time from the services of small providers to those of the new large one. This reallocation diminishes the mass of small providers by reducing their impression supply through a decrease in household attention and consequently, potential profits (see proof 4 in appendix B.3). Conversely, it benefits large providers by enhancing their profits through an increase in the supply of impressions. If the number of small providers is relatively high compared to that of large ones, the profits of large providers can further increase due to a rise in the price of impressions (see proof 5 in appendix B.3). In this case, the augmented impression supply from large providers is inadequately compensated by the diminished supply from small providers, resulting in a net decrease in total impression supply and a consequent rise in prices. Finally, the increase in profits for large providers boosts individual wealth as these profits are redistributed to households, thereby elevating households' optimal level of expenditure. This surge in expenditure enhances the value of traditional firms, enabling new firms to enter the traditional goods market due to free entry. The shock instantaneously increases the consumption index level through the introduction in new variety, but as previously emphasized, it does not affect the consumption growth rate. The production levels of each firm remain unchanged. Finally, the increase in household expenditure also results in extended online time, benefiting all providers through heightened household attention and increased data collection from the augmented online activity. However, these benefits are insufficient to preclude small providers from exiting the market but merely slow their reduction.

Finally, the entry of new large providers leads to an increase in household intertemporal utility. As households exhibit strong preferences for large providers services, they benefit from

[24] The positive effect on p^{a^*} arises when the number of small providers is significantly higher in comparison to that of large providers (a sufficient condition), see proof 5 in appendix B.3.

the enhanced offerings despite the reduced variety of small providers services. The increased consumption, the augmented number of goods varieties, and the extended time spent online collectively positively impact intertemporal utility. Households gain from the presence of new large firms.

The quality advantage of large providers over small ones

Let us now turn our attention to another important characteristic of this sector: the advantage of large providers over small ones in the quality of services provided. We are interested in the consequences of an increase in this advantage, i.e. an increase in β, which can be due to an increase in the service quality efficiency parameter of large providers β^j or a decrease in that of small providers β^k. We find that an increase in the quality advantage β has a positive impact on the total expenditures E^*, the time spent online S^*, the number of traditional firms I^*, the impression price p^{a*} and a negative one on the number of small providers K^*.

The first and direct impact of an increase in β is the enhancement of the quality of large providers' services relative to those of small ones ($\frac{q^{j*}}{q^{k*}}$). Large provider's services are now more attractive, leading households to prefer spending more time on them compared to those of small providers. This increase in attention enables large providers to boost their profits by expending the supply of impressions. As profits are redistributed to households, their optimal expenditure level increases, which benefits traditional firms. These effects collectively contribute to increased production of traditional goods, firm value, and, consequently, the number of traditional firms due to the assumption of free entry. However, the reallocation of households' online time toward large provider's services harms the activity of small providers. It diminishes their potential profits and shrinks the mass of small providers (see proof 4 in appendix B.3). Moreover, the decrease in small providers' supply of impressions drives up impressions price (see proof 5 in appendix B.3). It benefits large providers whose impressions supply increases.

It is important to note that the impact of a shock on β differs depending on whether it arises from an increase in β^j or β^k. An increase in β^k lowers the digital service quality of large providers, while a change in β^j does not affect that of small providers. This discrepancy is due to the presence of free entry among small digital service providers, unlike large ones.[25] If the quality of small providers services improves, it adversely affects large providers. With the narrowing gap in quality between large and small providers, households reallocate some of their time to small providers. This reduction in attention to large providers' services leads to a decline in their profits. This drop in resources forces them to adapt to the shock by lowering the quality of their services. However, no symmetrical effect exists when β^j changes. When large providers services quality increases, the adjustment among small providers is not related to service quality but rather to the number of small providers.

An increase in the quality advantage of large providers among small ones positively impacts households' intertemporal utility when it results from an increase in the efficiency of producing quality of large provider's services β^j (β^k remains constant). In this scenario, households benefit from a higher quality of large provider's services and an increase in wealth and traditional goods varieties. It offsets the decrease in the number of small provider's services. However, in the case of a decrease in the quality of small provider's services, the impact is uncertain. While households benefit from increased wealth and a wider selection of traditional goods, they are negatively affected by the decline in the quality of small providers' services, which is not necessarily compensated by higher quality from large providers.

The importance of data and time in impressions production

Digital service providers need users' attention and data to produce impressions. Service quality directly influences attention, while the volume of data collected depends solely on total online time.[26] The weight given to attention in impression production (γ) compared to data has significant implications. A higher γ favors large providers, as they benefit from greater user attention

[25] By integrating the impressions supply of small providers (equation 21) and the optimal quality of small providers services (equation 24) in the free entry equation (equation 26), we obtain that $q^k = f^k \beta^k \frac{\alpha\gamma\epsilon}{1-\alpha\epsilon(1+\gamma)}$. At the symmetric equilibrium, the quality of small providers services does not depend on large providers characteristics.

[26] Large digital service providers, such as Google and Meta, have an advantage in the number of data collected. However, most small providers do not have the financial capacity to produce their advertising services. They, therefore, use Google AdSense to monetize their service. Using their user databases, Google displays advertisers' impressions of small providers' spaces. In this case, the users' attention (or clicks on the impression) makes the difference between digital services.

compared to small providers. We find that an increase in γ positively impacts the optimal level of expenditure E^*, the time spent online S^*, the number of traditional firms I^*, the quality of large digital services providers relative to small one $\frac{q^{j*}}{q^{k*}}$, the supply of impressions of large providers relative to small one $\frac{a^{j*}}{a^{k*}}$, and decreases the number of small digital service providers K^*.

With an increased γ, attention becomes a more important production factor of impressions. Providers are incentivized to invest in the quality of their services to attract users for a longer time. This quality improvement increases online time, leading to more data collection as households engage in lengthier online activities, divulging more information. There is no free entry among the large digital service providers; they have, therefore, non-zero profits. Consequently, their profits rise through the increase in the two production factors of impressions. The optimal expenditure level increases through this increase in wealth through profit redistribution. This rise in household expenditure increases the value of traditional firms. New firms enter the traditional goods market as there is free entry.

Two opposite effects occur on small providers. Firstly, quality improvement leads to an increase in the time spent on their services. Combined with the rise of data collected, small providers are expected to produce more impressions, thereby increasing their potential profits. Due to free entry, new small providers should enter the market. Nevertheless, impression production is significantly more efficient among large providers than small ones because they capture users' attention more effectively. An additional increase in γ widens the gap in the advantage between large and small providers. The positive effect of the increase in production factors is offset by the negative one; small providers exit the market due to decreased potential profits (see proof 4 in appendix B.3). Using data allows small providers to compete with large ones. This is because all providers collect the same amount of data, as it only depends on the total online time. Therefore, the relative importance of data compared to attention in impression production significantly impacts the competition between small and large providers.

The increase in the weight of attention compared to data in the production of impressions improves the intertemporal utility. Households benefit from the increase in optimal expenditure level, time spent online, and quality of digital services, as well as from the broader variety of available goods due to their taste for diversity.

6.3 Households preferences

We now focus on the impact of household preferences for free digital services on the free digital services sector, macroeconomic aggregates, and well-being.

Sensitivity for privacy

Firstly, we investigate the impact of privacy sensitivity, measured by the parameter α. The higher α, the lower the household is sensitive to privacy issues. We find that an increase in α positively impacts the optimal level of expenditure E^*, the number of traditional firms I^*, the quality of large digital services relative to small ones $\frac{q^{j*}}{q^{k*}}$, and the supply of impressions of large providers relative to small ones $\frac{a^{j*}}{a^{k*}}$ and decreases the number of small digital service providers K^*.

It is important to keep in mind that α is a parameter for privacy but also influences the elasticity of substitution between digital services (see equation (2)). Therefore, a variation in α simultaneously affects these two characteristics of household preferences. Nevertheless, a shock to ϵ only measures the effects of a change in the elasticity of substitution (or taste for variety). Thus, the differences in impacts between α and ϵ reveal the specific effects of changes in household preferences regarding privacy protection. We shall see, further on, that we encounter the same results when we focus on the impacts of the variation in the taste for diversity alone (ϵ), except for the effects on the price of impressions. The sign of the variation in p^a is not determined with certainty when the value of α is altered, whereas it is consistently positive whenever ϵ increases. We will revisit this difference later, to concentrate here on the mechanisms inherent to privacy sensitivity that contribute to the same outcomes.

When households are less sensitive to privacy, they place a greater value on the benefits derived from spending time on services due to their quality than on the negative consequences of data collection. Therefore, they spend more time online, leading to increased collected data. Since large providers' offer higher quality services than small ones, the growth in time spent on large providers' services surpasses that on small providers services, further widening the advantage of large providers over small ones. This amplified advantage results in increased production of impressions by large providers relative to small ones, leading to higher profits for the large providers. Through the redistribution mechanism, households' optimal expenditure level and the number of traditional firms increase. However, it hurts small providers, resulting in lower potential profit and the exit of some small providers (see proof 4 in appendix B.3). Despite the increased data collection, it is insufficient to counterbalance the growing advantage between large and small providers, as all providers collect the same data in our model.

A decreasing sensitivity to privacy has a positive impact on intertemporal utility through the increase in optimal expenditure level, time spent online, and the wider variety of traditional goods. It also increases because the disutility of data collection is lower.

Preference for digital services diversity

A second parameter that can influence household consumption of digital services is the preference for diversity, measured by ϵ a substitution parameter between digital services. The higher ϵ, the lower the preference for digital services diversity. We find that an increase in ϵ leads to a rise in the optimal level of expenditure E^*, the number of traditional firms I^*, the quality of large digital services relative to small ones $\frac{q^{j*}}{q^{k*}}$, the supply of impressions of large providers relative to small ones $\frac{a^{j*}}{a^{k*}}$, and the advertising price p^{a*}, and decreases the number of small digital service providers K^*.

Households allocate their online time to different services according to the quality of each service and their preference for diversity. If the preference for diversity of digital services decreases, the importance of service quality in decision-making becomes more pronounced. As large providers' services have a higher quality than small ones, households will spend more time on the large than on the small ones. Through the mechanisms highlighted above, the rise in attention for large providers increases their impressions' production and profit, the optimal expenditure level, and the number of traditional firms. In contrast, small providers are harmed. They previously benefited from the preference for diversity, which compensated for the lower quality of their service. The reduced attention to these services diminishes their potential profits, leading to the exit of some small providers from the market (see proof 4 in appendix B.3).

As explained before, the differences in impacts between α and ϵ reveal the specific effects of changes in household preferences regarding privacy protection. The key difference is that an increase in ϵ will also increase the impressions price. A lower preference for digital services diversity only amplifies the preference for large providers' services, while a lower preference for privacy benefits to both types of providers. In the first case, the total advertising supplied decreases because of the exit of some small providers, thereby increasing the price of impressions. In the second case, the exit of small providers is limited by the increase in data collected. The total amount of advertising remains unchanged; the decrease in impressions from small providers is counterbalanced by the increase in impressions from large providers. In both cases, intertemporal utility experiences an enhancement.

7 Conclusion

The emergence of free digital services has changed household consumption as well as the allocation of their time. It has also resulted in the creation of large digital service providers, often called Big Tech, which generate revenues similar to those of the largest traditional firms. However, their business model differs from that of traditional firms, as the service is offered free of charge to users. The novelty lies in the fact that digital services are utilized to collect data about users, which is then employed in the production of targeted advertising. Nevertheless, advertising is not directly considered in the GDP measure. Consequently, firms with significant revenues can have no direct impact on economic growth, while it can impact the other sectors of the economy and the households.

This paper proposes an endogenous growth model, including the free digital services sector, to understand the link between the primary macroeconomic aggregates and welfare. It models the interactions between free digital services and traditional sectors and households. The traditional sector comprises a continuum of monopolistic firms that sell differentiated goods. To improve their good's perceived quality, they can increase the effort in the R&D activity and buy targeting impressions to the free digital services sector. The latter comprises a fixed number of large digital service providers and a continuum of small providers. To produce targeted impressions, they need users' attention and data. To this aim, they improve the quality of their service to attract users for as long as possible. Finally, households optimally choose their level of consumption of traditional goods and the time spent online. They suffer from data collection disutility according to their sensitivity to privacy and benefit from it as it is used to personalize the service. Households must choose how much data they disclose to maximize their utility.

The model highlights that the free digital services sector can have macroeconomic implications without affecting economic growth. We stress the significance of the market structure within the free digital services sector, which comprises large and small providers, as well as users' data and attention that have economic implications. Firstly, we find that using data in impression production enables small providers of digital services to compete with large providers. It enables them to counter the advantage of large providers' quality and, therefore, the households' preferences in favor of large providers' digital services. Our findings highlight that factors fostering the activities of large providers, such as the entry of a major player and improvements in production efficiency (e.g., service quality and advertising), positively impact the economy and household welfare. The increase in their profits positively impacts the economy and households' welfare by increasing the optimal expenditure and the number of traditional firms. Therefore, the economy and households benefit more from large providers than small ones. The positive impact is amplified when multiple large providers exist.

It is important to remember that this model is developed within the American framework. One of the main mechanisms leading to an increase in wealth and household well-being comes from the assumption of profit redistribution. This redistribution hypothesis aligns more closely with the American market, which hosts most of the large providers of free, ad-supported digital services (e.g., Google and Meta). Nevertheless, this result contributes to the ongoing debate regarding the relevance of regulating US digital giants in the European Union. Policies to support the development of digital players in the European market could also be implemented to benefit from the positive impact of the free digital economy. Implementing regulations to prevent large firms from imposing barriers to entry, such as the European Digital Market Act, also appears relevant. Moreover, taxation can be a way of benefiting from the redistribution of large providers' profits. Acemoglu & Johnson (2024) also recommend to tax digital advertising revenues. In addition to negatively impacting mental health and facilitating the spread of false information, political radicalization and all forms of extremism, they stress that this type of activities distorts innovation.

Our results also underscore the importance of household preferences, such as sensibility for privacy and the taste for diversity among free digital services, in developing the free digital services market. We highlight that privacy sensitivity harms the economy through the difficulty of collecting data. Firms are incentivized to collect as much information as possible without data regulation to improve their profits. Data regulations such as GDPR and CCPA have been implemented to protect users' privacy. Besides protecting privacy, the aim is also to increase the responsibility of firms to avoid data sharing, which can result in user manipulation, as illustrated by the Facebook-Cambridge Analytica data scandal in the 2010s. The sharing of personal information can also lead to security or addiction issues. These negative impacts and scandals increase household concerns about digital risks (OECD, 2020). Data protection regulation can be a way to reduce privacy fear and, therefore, counterbalance its adverse impact on the economy.

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A Variables description

Table A.1: Variables description

Variable	Description
Households	
- Endogenous variables	
$s_{h,j}$	Time spent on the digital service of the large provider j by the household h
$s_h(k)$	Time spent on the digital service of the small provider k by the household h
$s_{h,0}$	Total time spent on all digital services of small providers by the household h
S_h	Total time spent online by the household h
$D_{h,t}$	Index of online time spent on digital services of the household h
$c_h(i)$	Consumption of the variety of good i by the household h
C_h	Consumption index of differentiated goods of the household h
b_h	Individual wealth of the household h
E_h	Consumption expenditure level of the household h
$d_{h,j}$	Data displayed by household h to the large digital service provider j
$d_h(k)$	Data displayed by household h to the small digital service provider k
$\overline{d_h}$	Maximum quantity of data on household h that can be collected by digital service providers
- Parameters	
ρ	Time preference rate
σ	Substitution parameter between varieties of consumption goods
ϵ	Substitution parameter between digital services
α	Preference parameter for privacy
- Exogenous variables	
H	Population's size
ℓ	Total time available for each household
Traditional sector	
- Endogenous variables	
I	Number of traditional firms
$\tilde{z}(i)$	Perceived quality of the variety of good i by the household h
$z(i)$	True quality of the variety of good i
$\overline{z}$	State of technology (average investment in R&D)
$x(i)$	Production of the variety of good i by firm i
$a_h(i)$	Quantity of impressions bought by the firm i to be displayed to household h

$\overline{a}$	Average quantity of impressions displayed to household h
$m_h(i)$	Quantity of impressions bought by firm i to be displayed to household h relative to average impressions displayed to this household
$L^x(i)$	Labor used in the production of the variety of good i
$L^R(i)$	Labor used in the activity of R&D of firm i
$\pi^x(i)$	Profit of the firm i
$V(i)$	Value of the firm i

- Parameters

β^a	Effectiveness parameter of advertising
β^r	Effectiveness parameter of R&D

- Exogenous variables

f^x	Fixed labor cost incurred by traditional firm

Free digital services sector

- Endogenous variables

q_j	Quality of the digital service of the large provider j
$q(k)$	Quality of the digital service of the small provider k
q_0	Index of the quality of digital services of small providers
Q	Index of the quality of all digital services
K	Number of small digital service providers
L_j^q	Labor used to improve the quality of the digital service of the large provider j
$L^q(k)$	Labor used to improve the quality of the digital service of the small provider k
π_j	Profit of the large digital service provider j
π_k	Profit of the small digital service provider k
$a^s_{h,j}$	Production of impressions by the large digital service provider j for display to household h
$a^s_h(k)$	Production of impressions by the small digital service provider k for display to household h

- Parameters

β^j	Effectiveness parameter of the digital service quality of large providers
β^k	Effectiveness parameter of the digital service quality of small providers
β	The gap between the quality efficiency of large and small digital service providers
γ	Indicator of the importance of attention in the production of impressions

- Exogenous variables

f^j	Fixed labor cost incurred by large digital service provider

f^k	Fixed labor cost incurred by small digital service provider
J	Number of large digital service providers
Prices	
$p(i)$	Price of the variety of good i
P	Price index of differentiated goods
r	Interest rate
w	Wage rate
p^a	Price of one unit of impression

B Proofs

B.1 Optimum results

Proof 1: the optimum values of $s_h(k)$ and $s_{h,j}$

The Lagrangian associated with the minimization program given by (8) is:

$$\mathcal{L} = \int_0^K s_h(k)dk + \lambda\Big(s_{h,0}q_0 - \Big(\int_0^K (s_h(k)q(k))^{\alpha\epsilon}dk\Big)^{\frac{1}{\alpha\epsilon}}\Big)$$

FOC's with respect to $s_h(k)$ gives us:

$$1 = \lambda\Big(\int_0^K (s_h(k)q(k))^{\alpha\epsilon}dk\Big)^{\frac{1}{\alpha\epsilon}-1} q(k)^{\alpha\epsilon}s_h(k)^{\alpha\epsilon-1}$$

We can write the ratio between the FOCs with respect to $s_h(k)$ and $s_h(k')$ as:

$$s_h(k) = s_h(k')\Big(\frac{q(k)}{q(k')}\Big)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}$$

Multiplying by $q(k)$ and integrating with respect to k, we obtain:

$$\Big(\int_0^K (s_h(k)q(k))^{\alpha\epsilon}dk\Big)^{\frac{1}{\alpha\epsilon}} = s_h(k')q(k')^{\frac{\alpha\epsilon}{\alpha\epsilon-1}}\Big(\int_0^K q(k)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}dk\Big)^{\frac{1}{\alpha\epsilon}}$$

By setting $s_{h,0} = \int_0^K s_h(k)dk$ the total time spent on digital services of small providers and $q_0 = (\int_0^K q(k)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}dk)^{\frac{1-\alpha\epsilon}{\alpha\epsilon}}$ a quality index of the digital services of small providers, the optimal time spent on the digital service k is given by:

$$s_h(k) = s_{h,0}\Big(\frac{q(k)}{q_0}\Big)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}$$

The same methodology is used to solve the minimization program given by (9). The Lagrangian associated is:

$$\mathcal{L} = \sum_{j=0}^{J} s_{h,j} + \lambda\Big(QS_h - \Big(\sum_{j=0}^{J}(q_j s_{h,j})^{\alpha\epsilon}\Big)^{\frac{1}{\alpha\epsilon}}\Big)$$

By setting $S_h = \sum_{j=0}^{J} s_{h,j}$ the total time spent on all digital services and $Q = (\sum_{j=0}^{J} q_j^{\frac{\alpha\epsilon}{1-\alpha\epsilon}})^{\frac{1-\alpha\epsilon}{\alpha\epsilon}}$ an index of the quality of all digital services, we obtain the optimal time spent on each digital services of large provider:

$$s_{h,j} = S_h\Big(\frac{q_j}{Q}\Big)^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}$$

B.2 Equilibrium results

PROOF 2: macroeconomic equilibrium values of E^*, K^*, q^{j*}, q^{k*}, I^*, L^{R*}, and p^{a*}

By substitution and using the Keynes-Ramsey equation (equation (14)) and the equilibrium price of traditional goods (equation (28)) in the equation of free entry in the traditional market (equation (31)), we obtain:

$$\frac{\rho}{\sigma}\frac{H}{I_t}E_{t-1} = L^R_{t-1} + p^a_{t-1}Ha_{t-1} + f^x \tag{B.2.1}$$

By adding the total supply of goods given by equations (26) and (39) in the previous equation and combining with the labor market equilibrium (equation (40)), we have:

$$\frac{\rho+\sigma-1}{\sigma}HE_{t-1} = H(\ell - S_{t-1}) - K_{t-1}(L^k_{t-1} + f^k) - J(L^j_{t-1} + f^j) + I_t p^a_{t-1}Ha_{t-1}$$

Finally, using the free entry condition on the small digital service market (equation (37)) and the equilibrium in the advertising market (equation (38)), the previous equation can be rewritten as:

$$\frac{\rho+\sigma-1}{\sigma}HE_{t-1} = H(\ell - S_{t-1}) - J(L^j_{t-1} + f^j) + Jp^a_{t-1}a^j_{t-1} \tag{B.2.2}$$

Using the advertising market equilibrium (equation (38)) and the two previous equations, we obtain:

$$K_t a_t^k + J a_t^j = H\alpha^{1-\gamma} S_t \Big(K_t + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\Big)^{-\gamma} \Big(K_t + \beta^{\frac{\gamma\alpha\epsilon}{\alpha\epsilon(1+\gamma)-1}} J\Big) \tag{B.2.3}$$

Introducing the Keynes-Ramsey rule (equation (14)) in demand for impressions by household (equation (30)), we obtain:

$$I_t p_{t-1}^a a_{t-1} = \beta^a \rho \frac{\sigma - 1}{\sigma} E_{t-1} \tag{B.2.4}$$

By substitution and using equations (B.2.3) and (B.2.4), we can rewrite the advertising market equilibrium as:

$$\beta^a \rho \Big(\frac{\sigma - 1}{\sigma}\Big) E_{t-1} = p_{t-1}^a \alpha^{1-\gamma} S_{t-1} \Big(K_{t-1} + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\Big)^{-\gamma} \Big(K_{t-1} + \beta^{\frac{\gamma\alpha\epsilon}{\alpha\epsilon(1+\gamma)-1}} J\Big) \tag{B.2.5}$$

By substitution and using equation (34) in the free entry in the small digital service market condition, we obtain:

$$\Big(K_{t-1} + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\Big)^{-\gamma} = f^k (p_{t-1}^a H S_{t-1})^{-1} \Big(\frac{\alpha^{1-\gamma}(1 - \alpha\epsilon(1-\gamma))}{1 - \alpha\epsilon}\Big)^{-1} \tag{B.2.6}$$

By combining equations (B.2.5) and (B.2.6), we obtain an increasing linear relationship between the value of the number of small digital service providers and total household spending:

$$K_t = \beta^a \rho \frac{\sigma - 1}{\sigma} \frac{1 - \alpha\epsilon(1+\gamma)}{1 - \alpha\epsilon} \frac{H}{f^k} E_t - \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J$$

Using equation (33), we obtain that:

$$p_{t-1}^a a_{t-1}^j - \frac{1}{\beta^j} q_{t-1}^j = p_{t-1}^a H S_{t-1} \frac{\alpha^{1-\gamma}(1 - \alpha\epsilon(1-\gamma))}{1 - \alpha\epsilon} \Big(\beta^{\frac{\alpha\epsilon}{\alpha\epsilon(1+\gamma)-1}} K_{t-1} + J\Big)^{-\gamma}$$

By substituting equation (B.2.6), we have:

$$p_{t-1}^a a_{t-1}^j - \frac{1}{\beta^j} q_{t-1}^j = f^k \beta^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}$$

By including this previous equation in equation (B.2.2) and using the optimal online time (equation (27)), we find that the value of total household spending at the symmetric equilibrium is unique and stationary, equal to:

$$E_{t-1} = E_t = E^* = \Big(\frac{\sigma}{\rho + 2\sigma - 1}\Big)\Big[\ell + \frac{J}{H}\Big(\beta^{\frac{\alpha\epsilon\gamma}{\alpha\epsilon(1+\gamma)-1}} f^k - f^j\Big)\Big]$$

As in Young's model, economic equilibrium implies that the economy is immediately at a stationary level of consumption expenditures and therefore the interest rate is also stationary from the initial moment. The absence of transitional dynamics comes from the linearity of the spillover effect in the evolution of perceived quality.

By substitution and using the free entry condition into the small digital service providers and the equilibrium value of a^{k*} (equation (34)) in equation (36), we obtain the equilibrium value of q^k:

$$q^{k^*} = \beta^k \frac{\alpha\epsilon\gamma}{1 - \alpha\epsilon(1+\gamma)} f^k$$

We deduce the equilibrium value of q^{j*} by including the previous equation in the equation (35):

$$q^{j^*} = \frac{\alpha\epsilon\gamma}{1 - \alpha\epsilon(1+\gamma)} \beta^{j\frac{\alpha\epsilon-1}{\alpha\epsilon(1+\gamma)-1}} \beta^{k\frac{\alpha\epsilon\gamma}{\alpha\epsilon(1+\gamma)-1}} f^k$$

Combining equations (B.2.1) and (B.2.4), we obtain:

$$\frac{\rho}{\sigma}\frac{HE^*}{I_t} = L^R_{t-1} + \beta^a \rho \frac{\sigma - 1}{\sigma}\frac{HE^*}{I_t} + f^x \tag{B.2.7}$$

Moreover, by substitution and using the demand of variety of good (equation (26)), and the equilibrium price of traditional good (equation 28)), we can rewrite the labor demand in the R&D activity (equation (29)) as:

$$L^R_{t-1} = \frac{\sigma - 1}{\sigma}\beta^r \rho \frac{HE^*}{I_t}$$

we deduce the number of traditional firms at equilibrium by introducing the previous equation in equation (B.2.7):

$$I^* = \frac{\rho}{\sigma}(1 - (\beta^r + \beta^a)(\sigma - 1))\frac{HE^*}{f^x}$$

and the equilibrium value of labor in R&D such as:

$$L^{R^*} = \frac{\beta^r(\sigma - 1)f^x}{1 - (\beta^r + \beta^a)(\sigma - 1)}$$

Finally, by combining the equations (B.2.4) and (38), we obtain that:

$$\beta^a \frac{\sigma - 1}{\sigma}\rho\frac{E^*}{I^*} = p^a_{t-1}(Ka^k_{t-1} + Ja^J_{t-1})$$

By substitution and using the equilibrium values of a^k and a^j given by equations (34) and (33), we obtain the equilibrium value of advertising price:

$$p^{a*} = \beta^a \frac{\sigma - 1}{\sigma}\rho\alpha^{\gamma-1}\Big[K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}J\Big]^{\gamma}\Big[K^* + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}}J\Big]^{-1}$$

Proof 3: equilibrium values of $C_{h,t}$ and U_0

At the symmetric equilibrium, the consumption index given by equation (1) can be rewritten as:

$$C_t = I_t^{\frac{\sigma}{\sigma-1}}\tilde{z}_t c_t$$

All traditional firms buy the same number of impressions at the symmetric equilibrium. Therefore, the perceived quality of traditional goods equals the true quality. By substitution and using the optimal demand of each variety of good by household (equation (26)), we have:

$$C_t = \overline{z}_{t-1}(L^R_{t-1})^{\beta^r}\frac{E^*}{p}I^{*\frac{1}{\sigma-1}}$$

Moreover, the state of technology at the symmetric equilibrium is equal to $\overline{z}_{t-1} = z_{t-1} = \overline{z}_{t-2}(L^{R^*})^{\beta^r} = \overline{z}_0(L^{R^*})^{\beta^r \times t-1}$. Therefore, using the equilibrium price of good (equation (28)), we have:

$$C_t = \frac{\sigma - 1}{\sigma}I^{*\frac{1}{\sigma-1}}\overline{z}_0 E^*(L^{R*})^{\beta^r \times t}$$

Note that at period 0, consumption index is simply equal to $C_0 = \frac{\sigma-1}{\sigma} I_0^{*\frac{1}{\sigma-1}} \overline{z}_0 E^*$ as the investment in R&D occurs in t-1.[27] The intertemporal utility can be rewritten as:

$$\begin{aligned}
U_0 &= \ln C_{h,0} + \sum_{t=1}^{\infty} \rho^t \ln C_{h,t} + \sum_{t=0}^{\infty} \rho^t \ln D_{h,t} \\
&= \frac{1}{\sigma-1} \ln I_0 + \ln(\frac{\sigma-1}{\sigma} \overline{z}_0 E^*)(1 + \sum_{t=1}^{\infty} \rho^t) + \sum_{t=1}^{\infty} \rho^t \ln(I^{*\frac{1}{\sigma-1}} (L^{R^*})^{\beta^r \times t}) + \sum_{t=0}^{\infty} \rho^t \ln D_{h,t}
\end{aligned}$$

We have $\sum_{t=0}^{\infty} \rho^t = \frac{1}{1-\rho}$, $\sum_{t=1}^{\infty} \rho^t = \frac{\rho}{1-\rho}$ and $\sum_{t=1}^{\infty} \rho^t t = \frac{\rho}{(\rho-1)^2}$. Therefore:

$$U_0 = \frac{1}{\sigma-1} \ln I_0 + \frac{1}{1-\rho} \ln(\frac{\sigma-1}{\sigma} \overline{z}_0 E^*) + \frac{1}{\sigma-1} \frac{\rho}{1-\rho} \ln I^* + \frac{\rho}{(\rho-1)^2} \beta^r \ln(L^{R^*}) + \frac{1}{1-\rho} \ln D_{h,t}^*$$

At the symmetric equilibrium and using equation (32), the digital consumption index given by equation (13) can be rewrite as:

$$D^* = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \overline{d}_h^{\frac{1}{\alpha}} E^* q^{j^*} \left(\beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} K^* + J\right)^{\frac{1-\alpha\epsilon}{\alpha\epsilon}}$$

By substitution and using the previous equation, the intertemporal utility can be rewrite as:

$$\begin{aligned}
U_0 =& \frac{1}{\sigma-1} \ln I_0 + \frac{1}{1-\rho} \ln(\frac{\sigma-1}{\sigma} \overline{z}_0) + \beta^r \frac{\rho}{(\rho-1)^2} \ln(L^{R^*}) + \frac{1}{1-\rho} \ln(\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \overline{d}_h^{\frac{1}{\alpha}}) \\
&+ \frac{2}{1-\rho} \ln E^* + \frac{1}{\sigma-1} \frac{\rho}{1-\rho} \ln I^* + \frac{1}{1-\rho} \ln(q^{j^*} \left(\beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} K^* + J\right)^{\frac{1-\alpha\epsilon}{\alpha\epsilon}})
\end{aligned}$$

Finally, using the equilibrium value of the quality of small provider's services given by equation (44), we find:

$$U_0 = X + \frac{1}{1-\rho} \Big[2 \ln E^* + \frac{\rho}{\sigma-1} \ln I^* + \ln\left(\frac{\alpha\gamma\epsilon}{1-\alpha\epsilon(1+\gamma)} \beta^k f^k\right) + \frac{1-\alpha\epsilon}{\alpha\epsilon} \ln\left(K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\right)^{\frac{1-\alpha\epsilon}{\alpha\epsilon}} \Big]$$

with $X = \frac{1}{\sigma-1} \ln I_0 + \frac{1}{1-\rho} \ln(\frac{\sigma-1}{\sigma} \overline{z}_0 \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \overline{d}_h^{\frac{1}{\alpha}}) + \beta^r \frac{\rho}{(\rho-1)^2} \ln(L^{R^*})$

[27] $\overline{z}_0$ and I_0 are given, they come from previous decisions made by firms that have engaged in R&D and advertising and therefore have production activity at $t = 0$.

B.3 Static-comparative results

In this appendix, we only present the calculations of comparative statics for which the sign of the derivatives is not trivial.

PROOF 4: the sign of $\frac{\partial K^*}{\partial J}$, $\frac{\partial K^*}{\partial \beta}$, $\frac{\partial K^*}{\partial \gamma}$, $\frac{\partial K^*}{\partial \alpha}$, and $\frac{\partial K^*}{\partial \epsilon}$.

According to equations (41) and (43), the number of small providers at the equilibrium is given by:

$$K^* = \frac{\beta^a \rho(\sigma-1)}{\rho+2\sigma-1}\frac{1-\alpha\epsilon(1+\gamma)}{1-\alpha\epsilon}\frac{H}{f^k}\Big(\ell+\frac{J}{H}\Big(\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}}f^k - f^j\Big)\Big) - \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}}J$$

The derivative of K^* with respect to J is equal to:

$$\frac{\partial K^*}{\partial J} = \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}}\Big[\frac{\rho\beta^a(\sigma-1)}{\rho+2\sigma-1}\frac{1-\alpha\epsilon(1+\gamma)}{1-\alpha\epsilon} - 1\Big] - \beta^a\rho\frac{\sigma-1}{\rho+2\sigma-1}\frac{1-\alpha\epsilon(1+\gamma)}{1-\alpha\epsilon}f^j$$

The second term of the previous equation is positive. Therefore, this derivative is negative if the first term is negative, which implies the following sufficient condition:

$$\gamma > \frac{1-\alpha\epsilon}{\alpha\epsilon}\Big(\frac{\rho(\beta^a(\sigma-1)-1)+(1-2\sigma)}{\beta^a\rho(\sigma-1)}\Big) = \underline{\gamma}$$

The threshold $\underline{\gamma}$ is strictly negative as $(\beta^a+\beta^r)(\sigma-1) < 1$ (concavity condition on the firm value function of traditional firms, which implies $\beta^a(\sigma-1) < 1$). As $\gamma > 0$, γ is superior to $\underline{\gamma}$ and, therefore, $\frac{\partial K^*}{\partial J}$ is negative without any additional condition.

The derivative of K^* with respect to β is equal to:

$$\frac{\partial K^*}{\partial \beta} = \frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}-1}J\Big[\frac{\rho\beta^a(\sigma-1)}{\rho+2\sigma-1}\frac{1-\alpha\epsilon(1+\gamma)}{1-\alpha\epsilon} - 1\Big]$$

The sign of the previous equation depends on the sign of term in the brackets which is negative if: $\gamma > \underline{\gamma}$, which has already been demonstrated previously, we conclude that $\frac{\partial K^*}{\partial \beta} < 0$.

The derivative of K^* with respect to γ is equal to:

$$\begin{aligned}\frac{\partial K^*}{\partial \gamma} =& -\frac{\beta^a\rho(\sigma-1)}{\rho+2\sigma-1}\frac{\alpha\epsilon}{1-\alpha\epsilon}\frac{H}{f^k}\Big(\ell+\frac{J}{H}\Big(\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}}f^k-f^j\Big)\Big)\\ &+J\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}}\frac{\alpha\epsilon(1-\alpha\epsilon)}{(1-\alpha\epsilon(1+\gamma))^2}\ln(\beta)\Big[\frac{\beta^a\rho(\sigma-1)}{\rho+2\sigma-1}\frac{1-\alpha\epsilon(1+\gamma)}{1-\alpha\epsilon}-1\Big]\end{aligned}$$

The first term of the equation is always negative. The sign of the second term depends on the sign of the term in the brackets. Therefore, we can conclude that $\frac{\partial K^*}{\partial \gamma}<0$ if the term in the brackets is negative, i.e., if $\gamma > \underline{\gamma}$. By the same reasoning as above, we conclude that $\frac{\partial K^*}{\partial \gamma}<0$.

The derivative of K^* with respect to α is equal to:

$$\frac{\partial K^*}{\partial \alpha} = \beta^a\rho\frac{\sigma-1}{\sigma}\frac{H}{f^k}\frac{(-\epsilon\gamma)}{(1-\alpha\epsilon)^2}E^*+J\beta^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}\frac{\gamma\epsilon}{(1-\alpha\epsilon(1+\gamma))^2}\ln(\beta)\Big[\frac{\beta^a\rho(\sigma-1)}{\rho+2\sigma-1}\frac{1-\alpha\epsilon(1+\gamma)}{1-\alpha\epsilon}-1\Big]$$

and the derivative of K^* with respect to ϵ is equal to:

$$\frac{\partial K^*}{\partial \epsilon} = \beta^a\rho\frac{\sigma-1}{\sigma}\frac{H}{f^k}\frac{-\epsilon\gamma}{(1-\alpha\epsilon)^2}E^*+J\beta^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}\frac{\gamma\epsilon(1-\alpha\epsilon)}{(1-\alpha\epsilon(1+\gamma))^2}\ln(\beta)\Big[\frac{\beta^a\rho(\sigma-1)}{\rho+2\sigma-1}\frac{1-\alpha\epsilon(1+\gamma)}{1-\alpha\epsilon}-1\Big]$$

Once again, the sign of the two derivatives above depends on the sign of the bracketed term, which is negative. Therefore $\frac{\partial K^*}{\partial \alpha}$ and $\frac{\partial K^*}{\partial \epsilon}$ are negative.

PROOF 5 of the sign of $\frac{\partial {p^a}^*}{\partial J}$ and $\frac{\partial {p^a}^*}{\partial \beta}$.

The equilibrium price of advertising is given by:

$${p^a}^* = \beta^a\frac{\sigma-1}{\sigma}\rho\alpha^{\gamma-1}\Big[K^*+\beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}J\Big]^{\gamma}\Big[K^*+\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}}J\Big]^{-1}$$

The derivative of ${p^a}^*$ with respect to J is equal to:

$$\begin{aligned}\frac{\partial {p^a}^*}{\partial J} =&\beta^a\frac{\sigma-1}{\sigma}\rho\alpha^{\gamma-1}\Big(K^*+\beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}J\Big)^{\gamma-1}\Big(K^*+\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}}J\Big)^{-2}\\ &\Big[\frac{\partial K^*}{\partial J}(\gamma(K^*+\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}}J)-(K^*+\beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}J))\\ &+\beta^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}(\gamma K^*+\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}}J\beta^{\frac{\alpha\epsilon(1-\gamma)}{1-\alpha\epsilon(1+\gamma)}}-K^*+\beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}J\Big]\end{aligned}$$

The sign of $\frac{\partial p^{a^*}}{\partial J}$ depends on the bracketed term's sign. The first part of the bracketed term is positive as $\frac{\partial K^*}{\partial J}$ and $(\gamma(K^* + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J) - (K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J))$ are negative (because $\beta > 1$ and $0 < \gamma < 1$). Therefore, the derivative is positive if the third line of the above equation is positive. We are not able to conclude otherwise. We find that $\frac{\partial p^{a^*}}{\partial J} > 0$ if and only if $\gamma\beta^{\frac{\alpha\epsilon(1-\gamma)}{1-\alpha\epsilon(1+\gamma)}} > 1$ and $K^* > J\Big(\frac{\beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}(1-\gamma)}{\beta^{\frac{\alpha\epsilon(1+\gamma)}{1-\alpha\epsilon(1+\gamma)}}\gamma}\Big)$.

The derivative of p^{a^*} with respect to β is equal to:

$$\begin{aligned}\frac{\partial p^{a^*}}{\partial \beta} =&\beta^a \frac{\sigma-1}{\sigma}\rho\alpha^{\gamma-1}\Big(K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\Big)^{\gamma-1}\Big(K + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J\Big)^{-2} \\ &\Big[\gamma\Big(K^* + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J\Big)\Big(\frac{\partial K^*}{\partial \beta} + \frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}\beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}-1} J\Big) \\ &- \Big(K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\Big)\Big(\frac{\partial K^*}{\partial \beta} + \frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}-1} J\Big)\Big]\end{aligned}$$

The sign of the above derivative depends on the bracketed term, that we can rewrite as:

$$\begin{aligned}A =&\frac{\partial K^*}{\partial \beta}\Big[\gamma\Big(K^* + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J\Big) - \Big(K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\Big)\Big] \\ &+ \frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}\beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}-1} J\Big[(K^* + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J)\beta^{\frac{\alpha\epsilon(1-\gamma)}{1-\alpha\epsilon(1+\gamma)}} - (K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J)\Big]\end{aligned}$$

The first part of the above equation is positive as $\frac{\partial K^*}{\partial \beta}$ and $\gamma\Big(K^*+\beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J\Big)-\Big(K^*+\beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\Big)$ are negative. The second term is positive if $(K^* + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J)\beta^{\frac{\alpha\epsilon(1-\gamma)}{1-\alpha\epsilon(1+\gamma)}} > K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J$ which is always the case. Therefore, $\frac{\partial p^{a^*}}{\partial \beta}$ is positive.

The derivative of p^{a^*} with respect to ϵ is equal to:

$$\begin{aligned}\frac{\partial p^{a^*}}{\partial \epsilon} &= \beta^a \frac{\sigma-1}{\sigma}\rho\alpha^{\gamma-1}\Big[K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J\Big]^{\gamma-1}\Big[K^* + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J\Big]^{-2} \\ &\Big[\frac{\partial K^*}{\partial \epsilon}[\gamma(K^* + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J) - (K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J)] \\ &+ \frac{\gamma\alpha}{(1-\alpha\epsilon(1+\gamma))^2} J\beta^{\frac{\gamma\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}}\ln(\beta)[(K^* + \beta^{\frac{\alpha\epsilon\gamma}{1-\alpha\epsilon(1+\gamma)}} J)\beta^{\frac{\alpha\epsilon(1-\gamma)}{1-\alpha\epsilon(1+\gamma)}} - (K^* + \beta^{\frac{\alpha\epsilon}{1-\alpha\epsilon(1+\gamma)}} J)]\Big]\end{aligned}$$

which is always positive