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Multimarket Contact, Cross-Market Externalities and Platform Competition

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Abstract

Antitrust authorities are concerned with the dominant market position of Tech Giants such as Google, Meta, or Amazon. These digital conglomerates are characterized by platform-based business models and multimarket contact (MMC). In traditional one-sided markets, theory and empirical evidence show that MMC tends to relax competition. In this paper, we revisit this result in the context of platform competition with competitive bottleneck and cross-market externalities, and provide new insights into the impact of MMC on platform competition. In this context, when platforms charge the two groups of users (bilateral pricing), we find that MMC always decreases the profitability of platforms regardless of the nature and magnitude of cross-market externalities. Then we consider the case in which platforms can only charge one group of users (unilateral pricing). When platforms charge the side on which they are not directly competing for users (i.e. the side that is not the competitive bottleneck), MMC may relax competition only if cross-group externalities and cross-market externalities are both sufficiently small. From a competition policy perspective, our paper provides insights into how antitrust authorities should review conglomerate mergers in digital markets and assesses the effects of the diversification strategies of digital platforms in the context of cross-market externalities and competitive bottleneck.

Keywords: two-sided markets, platform competition, digital markets, multimarket contact, cross-market externalities, competitive bottleneck, competition policy.

JEL codes: D43, L13, L41, L86.

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1 Introduction

The market dominance of Tech Giants such as Google, Apple, Meta, Amazon and Microsoft poses a serious challenge to antitrust authorities (Evans and Schmalensee, 2013; Peitz and Valletti, 2015; Cremer et al., 2019; Parker et al., 2020). These Tech Giants have monopolized many digital markets, by taking advantage of network effects and massive data collection. Their expansion relies on both internal and external (mergers & acquisitions) growth strategies that strengthen their position in their core markets. They have also achieved a high level of diversification, not only by entering markets that are complementary to their core market, but also by expanding into unrelated markets (Bourreau and De Streel, 2019). For instance, while Google's core market was originally centered on its search engine, it has diversified into operating systems, video/music streaming services, cloud services, voice assistance, etc. Similarly, Amazon also offers a wide variety of products and services, ranging from e-commerce to payment services, cloud computing, electronic devices, gaming and video services. Although these conglomerate strategies generate synergies (through e.g., economics of scope, network effects, data sharing), they may reinforce dominance (through e.g., self-preferencing, tying and bundling) and create insurmountable barriers to entry. Among competitive concerns and potential harm, these strategies can limit consumer choice and stifle innovation (Bourreau and De Streel, 2019; Parker et al., 2021).

In this context, the purchase of WhatsApp by Facebook in 2014 for $19 billion was highly debated. The European Commission (EC) eventually approved the deal based on the argument that the two platforms were not operating in the same market: at that time, Facebook operated mainly in the market for social networking services whereas WhatsApp was active in the market of communication services.[1] If the purchase had been evaluated a few years later, the arguments and the decision of the EC would probably have been different, given the increasing domination of Facebook in the markets for social networking and for communication services. Following this debate, the acquisition of Fitbit by Google in 2020 was much more cautiously investigated by the

[1] According to the EC, *"consumers would continue to have a wide choice of alternative communications apps after the transaction [and] the merged entity would continue to face sufficient competition after the merger. (...) Furthermore, even in the event of an integration between WhatsApp and Facebook such that Facebook's position in social networking services could be strengthened, the net gain in terms of new members of the social network would be limited, since the user base of WhatsApp already overlaps to a significant extent with that of Facebook."* European Commission Press Release no. IP/14/1088, released on 3 October 2014, available from `https://ec.europa.eu/commission/presscorner/detail/en/ip_14_1088`

European antitrust authority and was eventually approved subject to conditions on data sharing and interoperability with third-party applications.[2]

In recent years, the regulation of digital markets has been reviewed by competition authorities both in the United States (Scott Morton et al., 2019) and in the European Union (Cremer et al., 2019). Several antitrust scholars have argued that there is a need for more stringent merger controls and for structural remedies against the dominance of large digital platforms (Shapiro, 2019; Cabral et al., 2021; Parker et al., 2021; Spiegel, 2024). The European response came in the shape of the Digital Markets Act (DMA) that entered into force in November 2022 in the European Union. The DMA targets companies that provide "core platform services" and hold a strong economic position in terms of size and number of users. These gatekeeper platforms must comply with a list of "do's and don'ts". For instance, such platforms must enable third parties to interoperate with their own services or allow their business users to access the data that they generate through the use of the platform (Cabral et al., 2021). These ex-ante obligations apply to all the markets in which the platform is active regardless of whether or not it actually dominates a specific market (Franck and Peitz, 2021).

In their objection to stringent regulation, Tech Giants claim that they are struggling to attract and retain the same groups of users (advertisers, developers, third-party sellers, end users, etc.). Evans (2017) collected many examples of market overlaps between Google, Facebook, Apple, Microsoft, and Amazon, including cloud computing & storage, streaming services, voice & video calling, and payment wallets. According to Evans (2017), these factual findings of extensive multimarket contact (MMC) serve as evidence of intense competition in digital markets.[3] However, a large body of theoretical and empirical literature shows that MMC creates *mutual forbearance* and reduces rivalry among firms (Edwards, 1955; Bernheim and Whinston, 1990). Yet, such findings apply to the context of traditional (i.e., one-sided) industries while most digital services are provided by or through two-sided platforms (Rochet and Tirole, 2003). The impacts of MMC in two-sided mar-

[2]According to the EC, *"the commitments will determine how Google can use the data collected for ad purposes, how interoperability between competing wearables and Android will be safeguarded and how users can continue to share health and fitness data, if they choose to."* European Commission Press Release no. IP/20/2484, released on 17 December 2020, available from `https://ec.europa.eu/commission/presscorner/detail/en/ip_20_2484`

[3]For Evans (2017), none of these Tech Giants can *"act as if it has a moat around it, protected by data, network effects, or anything else. There don't seem to be sleepy monopolies littering the digital economy. Few who participate in this sector can sleep well at night."*

kets appear to be unexplored and the aim of our paper is to fill this gap and better understand how platforms compete in the presence of MMC. From a competition policy perspective, this paper investigates if increased multimarket contact between conglomerates such as Google, Facebook, or Apple (through internal growth or acquisitions) may strengthen or relax competition in digital markets.

To answer this question, we consider two symmetric markets and develop a model of platform-based competition in the specific context of a competitive bottleneck. In each market, two platforms need to attract two groups of users. The competitive bottleneck model supposes that one group of users singlehomes (i.e. joins at most one platform), while the other group makes its decision to join one platform independently from its decision to join the other platform, and so can possibly multihome (i.e. platforms do not directly compete to attract the second group of users). This situation is very common in many digital markets and has therefore received a lot of attention in the two-sided market literature (Armstrong, 2006; Belleflamme and Peitz, 2019). For instance, in the case of application stores, end users have either access to the Google Play Store or the Apple Store (i.e. they singlehome), whereas app developers are present on both stores (i.e. they multihome). Similarly, most consumers use only one food delivery platform whereas restaurants are available on multiple delivery platforms. Other examples include short term rental platforms (with singlehoming hosts facing multihoming travelers) or contactless payment systems (with consumers using only one payment system and merchants accepting all payment systems). In our framework, a multimarket platform refers to a firm which operates a platform in both markets. In reference to previous examples, this is the case of Apple which operates several plaform-based services such as the App Store and Apple pay as well as of Uber, which operates ride-hailing and food delivery platforms. This particular situation allows those firms to interconnect and/or integrate their platforms.

The key feature of our theoretical framework is that platform integration or interconnection generates positive externalities across markets that benefit the users of those platforms. In other terms, users may gain higher utility as they join a multimarket platform, because they can interact with more users or have access to a wider variety of goods and services offered by third-party providers. In digital markets, these cross-market externalities are mainly data-driven. A company that connects two platforms can collect and share data about the users of both platforms. Through data sharing, a multimarket platform can improve its recommendations, targeting, search or matching tools which increases the utility of its users in all markets. The more users (and user data), the more attractive the platform becomes in every market. As an example of this, consider the change in WhatsApp's terms of service and privacy policy in January 2021, a few years after being bought by Facebook.[4] This change clearly helped interconnect data from the two services and amplified the magnitude of the positive externalities that users can get from third party developers on the two platforms. In related markets, Li and Agarwal (2017) also provide empirical evidence of these benefits by showing that the acquisition of Instagram and its integration within the Facebook ecosystem added value not only for consumers, but also for Facebook application developers.

Our model develops a comprehensive framework that accounts for both intra- and inter-market externalities. In our setting, two kinds of effects arise. The first effects are caused by cross-group externalities (intra-market network effects). In each market, the utility of platform users positively depends on the number of users on the other side of the platform. In addition to these intra-market network effects, the users of a multimarket platform face cross-market externalities, such that the utility of a user in one market increases with the number of users of the platform in the second market. These demand-side cross-market externalities interplay with the cross-group externalities, and reinforce the attractiveness of the platform in both markets. When platforms charge the two sides (bilateral pricing), our main finding is that platform profitability is always lower under MMC regardless of the magnitude of the cross-market externalities. MMC actually increases competition and improves the surplus of the two sides as well as the total surplus. However, when the side on which platforms do not directly compete benefits more from cross-market externalities, MMC increases the access price charged to this side.

We also consider the case where only one side pays to join the platform and the other side gets free access (unilateral pricing). This case is of high practical relevance because it describes a very common pricing model of two-sided platforms. We find that under certain conditions, MMC relaxes price competition and increases platform profitability, but still improves users surplus. This occurs when the singlehoming side has free access and when cross-group and cross-market externalities are

[4]See https://faq.whatsapp.com/5623935707620435/, last consulted on January 2025, 15th.

both sufficiently low.

Our results suggest that users benefit from MMC in platform-based markets with competitive bottleneck. In this context, we also identify the conditions under which multimarket platform competition is more or less intense than in the absence of multimarket contact. From a competition policy perspective, our paper provides insight into how antitrust authorities should review conglomerate mergers between digital platforms with competitive bottleneck and assess their effects on competition across markets. Specifically, when authorities examine the integration or merger of two platforms that operate in different markets, they should consider the effect on multimarket contact (does it create or strengthen multimarket contact between competitors?), the nature of cross-market externalities (which side benefits more from these external effects?) and the pricing model used by these platforms (which side must pay an access fee?). When both sides are charged, there is little reason for competition concerns. However, when platforms only charge the side on which they have monopoly power, antitrust authorities should assess the magnitude of cross-group and cross-market externalities before challenging such mergers and acquisitions.

Our paper builds on the theoretical and empirical literature on platform competition and mergers. A large body of papers analyzes the pricing behavior under platform competition (Armstrong, 2006; Rochet and Tirole, 2006; Caillaud and Jullien, 2003; Belleflamme and Peitz, 2019; Cabral et al., 2019; Jullien et al., 2021; Etro, 2023; Xie et al., 2021; Amelio et al., 2020; Choi and Jeon, 2021; Atad and Yehezkel, 2024). For instance, Cabral et al. (2019) study how multihoming on one side of the platform can affect pricing and whether it benefits or harms each side of the platform. Xie et al. (2021) consider platform competition when users can benefit both from intra-group and cross-group externalities and compare several configurations (singlehoming, partial multihoming). Other papers focus on the effect of mergers between two-sided platforms (Correia-da Silva et al., 2019; Evans and Noel, 2008; Katz, 2021). Compared to traditional markets, horizontal mergers in two-sided markets may exhibit less anti-competitive effects, because consumers can be more price sensitive (Behringer and Filistrucchi, 2015). In that respect, some empirical studies have measured the effects of mergers in two-sided markets. For instance, Filistrucchi et al. (2012) study the effect of the merger of newspapers on their advertising revenues and the prices paid by readers. More recent papers have focused their attention on conglomerate mergers and their effect on innovation and barriers to entry (Gautier and Lamesch, 2021; Motta and Peitz, 2021). To the extent of our knowledge, there is no theoretical paper which considers platform competition in multiple markets with the presence of cross-market externalities.

The remainder of the paper is organized as follows. Section 2 presents the baseline model. Section 3 presents the equilibrium outcomes and analyzes the effect of MMC on platform competition with bilateral pricing. Section 4 extends the model to the case with unilateral pricing. Section 5 discusses policy implications and concludes.

2 The baseline model of platform competition

2.1 Platform competition without multimarket contact

We consider two markets i ($i = \{1, 2\}$), with two platforms competing in each market. In the absence of multimarket contact, markets 1 and 2 are independent. Each platform brings together two sides (or groups) of users that benefit from cross-group externalities.[5] For convenience, we label the two groups as buyers (b) and sellers (s). We denote Ji the platform J ($J = \{A, B\}$) in market i ($i = \{1, 2\}$) and we note $n_{b,Ji}$ and $n_{s,Ji}$ the number of active buyers and active sellers on this platform. In this paper, we focus on a competitive bottleneck model of platform competition that is a very common configuration in digital markets (Armstrong, 2006; Belleflamme and Peitz, 2019). In this setting, one group of users singlehomes (here the buyers) and the other group multihomes (here the sellers). An example of this is the competition between app store platforms where app developers are present on multiple platforms (e.g. Google Play Store and Apple App Store) while buyers tend to singlehome when purchasing their devices (e.g. smartphones, tablets, etc.). Our model also applies to e-payment systems or to food delivery platforms with merchants/restaurants on one side and consumers on the other side.[6] There are several reasons for a group of users (in this case, the buyers) to singlehome. First, it can be costly to access and manage distinct accounts in multiple platforms. Moreover, platforms often implement loyalty or exclusivity programs to strengthen the

[5]Contrary to Xie et al. (2020), our model does not account for intra-group externalities. Intra-group or within-group externalities would refer to externalities derived from the same side in the same market.

[6]Our model does not account for configurations in which both groups of users engage in multi-homing, as in social media platforms (with advertisers and end-users). However, our framework is quite general to encompass various types of digital platforms, particularly most transactional platforms.

incentives for singlehoming. It can also be time-consuming to learn how each platform works and to configure settings. Once buyers become accustomed to one platform, they have limited incentives to maintain a presence on a second platform. This is particularly true if sellers multihome. In this case, buyers can access the same sellers, regardless of the platform they choose to join.

Buyers' choice: our model assumes that on the buyer side, platforms are horizontally differentiated and located at the extreme points of a unit interval. Buyers are uniformly distributed on this unit interval and have a unit mass in each market. Buyers join at most one platform (singlehoming) and pay a price $p_{b,Ji}$ to access platform J in market i. They also incur an opportunity cost of joining this platform that linearly increases in distance at the rate $t_b > 0$. We assume that each platform provides the same intrinsic benefit v to buyers and that the markets 1 and 2 are fully-covered. Let α_s ($\alpha_s > 0$) denote the cross-group externality parameter that represents the value for a buyer when an additional seller is connected to the same platform. For instance, in the case of a food delivery platform, buyers' utility increases with the number of restaurants on the other side, as it offers a greater variety of meal options. Formally, the utility that a buyer located at x ($x \in [0, 1]$) derives from platforms Ai and Bi in market i ($i = \{1, 2\}$) is given by

$$\begin{cases} U_{Ai}(x) = v + \alpha_s n_{s,Ai} - t_b x - p_{b,Ai} \\ U_{Bi}(x) = v + \alpha_s n_{s,Bi} - t_b(1-x) - p_{b,Bi} \end{cases} \tag{1}$$

The type-x buyer chooses platform Ai if $U_{Ai}(x) \geq U_{Bi}(x)$ and platform Bi if $U_{Bi}(x) > U_{Ai}(x)$.

Sellers' choice: as buyers singlehome, sellers can only reach the buyers that are present on one platform by having direct access to that platform. Thus, there is no direct competition between platforms to attract sellers since the latter make their decision to join one platform independently of the decision to join the other platform. Our specification of sellers' decisions to join a platform follows that of Economides and Tag (2012) or Rasch and Wenzel (2013). Specifically, sellers are heterogeneous with respect to the fixed cost of setting up their business on a platform (e.g. costs to adapt their services to the platform requirements or to advertise their products). We assume that sellers are uniformly distributed on a unit interval and have a unit mass in each market. Formally, a type-y seller (with $y \in [0, 1]$) incurs a cost $t_s y$ to be present on a platform. Moreover, sellers pay access price $p_{s,Ji}$ to join platform Ji. Let α_b ($\alpha_b > 0$) denote the cross-group externality parameter that represents the value for a seller to have an additional buyer connected to the same platform.

For instance, on a food delivery platform, having an additional customer can lead to an increase in the expected online orders and sales for restaurants. Thus $\alpha_b n_{b,Ji}$ depicts the expected gain for a seller to join platform J in market i. The profit of a y-type seller on platform Ji is

$$\pi_{Ji}(y) = \alpha_b n_{b,Ji} - t_s y - p_{s,Ji} \tag{2}$$

Each seller such that $\pi_{Ji}(y) \geq 0$ joins platform Ji. The number of active sellers on Ji is given by the location of the marginal seller who is indifferent between joining the platform and staying out of it.

2.2 Platform competition with multimarket contact

In the previous setting, the two markets are independent and each platform is operated by a single-market firm. Now, we suppose that the platforms $A1$ and $A2$ are interconnected and operated by firm A . Similarly, platforms $B1$ and $B2$ are interconnected and operated by firm B. Thus, firms A and B have multimarket contact through their platforms which compete in both markets. This setting is illustrated in the right panel of Figure 1.

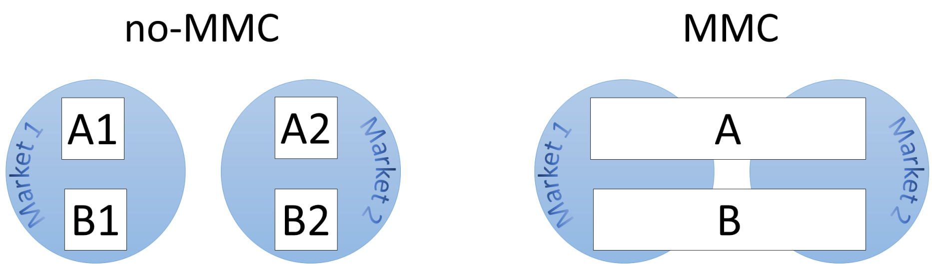

Figure 1: Comparison of market structures with and without MMC.

In addition to cross-group externalities within each market, a multimarket platform allows for cross-market externalities that benefit users. There are potentially two sources of cross-market externalities. First, sellers in a given market (say market 1) who join a multimarket platform may benefit from being directly or indirectly connected to the buyers enrolled by the platform in market 2. Indeed, the multimarket platform can collect data about buyers in market 2 that can be shared with sellers in market 1. These sellers can use these data to reach new consumers or better serve their current ones. For instance, consider a multimarket platform like Uber that operates two platforms, one for food delivery and the other for ride-hailing services. The ride-hailing platform typically collects data about the trip habits and locations of its customers. Once collected, these data provide relevant information for sellers (i.e. affiliated restaurants) on the food delivery platform. The latter may better estimate the potential demand in each city area and choose the best location to open a new branch. By accessing more relevant and fine-grained data, restaurants can also better advertise their menus and improve consumer targeting. Therefore, restaurants benefit from joining a food delivery platform with a large consumer base on the other side (cross-group externalities) and across its other platforms (cross-market externalities). Formally, when a seller joins a platform in a given market, its profit increases with both the number of buyers on the other side and the number of buyers on the second platform operated by the same firm. These buyer-driven cross-market externalities benefit sellers and are measured by the parameter γ_b ($\gamma_b > 0$) multiplied by the number of buyers in the second market.

Symmetrically, buyers who join a multimarket platform in a given market may benefit from being directly or indirectly connected to the sellers that are present on the platform of the second market. These benefits are typically derived from the increased interoperability between the services or products offered by sellers in both markets, through better APIs and platform integration.[7] Consider for instance the case of Google which operates Google Play Store (application store) and Google Home services (smart home and connected devices). Many third-party applications are available on both platforms. End users may join the Google ecosystem because they appreciate synchronizing their data and activities related to these third-party applications. These seller-driven cross-market externalities are measured by the parameter γ_s ($\gamma_s > 0$) multiplied by the number of sellers in the other market.

With cross-market externalities, the utility that a x-type buyer derives from platforms Ai and Bi in market i ($i = \{1, 2\}$ and $i' \neq i$) is given by

$$\begin{cases} U_{Ai}(x) = v + \alpha_s n_{s,Ai} + \gamma_s n_{s,Ai'} - t_b x - p_{b,Ai} \\ U_{Bi}(x) = v + \alpha_s n_{s,Bi} + \gamma_s n_{s,Bi'} - t_b(1-x) - p_{b,Bi} \end{cases} \tag{3}$$

and the profit of a y-type seller on platform Ji ($J = \{A, B\}$, $i = \{1, 2\}$ and $i' \neq i$) is given by

$$\pi_{Ji}(y) = \alpha_b n_{s,Ji} + \gamma_b n_{b,Ji'} - t_s y - p_{s,Ji} \tag{4}$$

We restrict our attention to the equilibrium outcomes for which the two rival platforms in each

[7]APIs (Application Programming Interface) allow secured information transfer from one service to another.

market are active. Formally, these equilibria must meet the following conditions: (i) the buyer side is fully-covered in both markets ($n_{b,Ai}+n_{b,Bi}=1$); (ii) the number of active sellers on each platform is strictly positive ($n_{s,Ji}>0$); and *(iii)* platforms earn positive profits ($\Pi_{Ji}>0$). In the no MMC case, these conditions are met if Assumption 1 holds.

Assumption 1 *In a platform competition setting without MMC, parameters must satisfy*
$8t_bt_s > \alpha_b^2+6\alpha_b\alpha_s+\alpha_s^2$.

Assumption 1 implies that transportation cost parameters t_b and t_s have to be sufficiently large with respect to cross-group externality parameters α_b and α_s.[8] Similarly, Assumption 2 guarantees the existence of equilibria with MMC that meet the three conditions above.

Assumption 2 *In a platform competition setting with MMC, parameters must satisfy*
$8t_bt_s > \alpha_b^2+\alpha_s^2+\gamma_b^2+\gamma_s^2+6\gamma_b\gamma_s+2\alpha_s(3\gamma_b+\gamma_s)+2\alpha_b(3\alpha_s+\gamma_b+3\gamma_s)$.

Under multimarket contact, Assumption 2 states that both platforms are active and the market is fully covered on the buyer side if transportation cost parameters t_b and t_s are sufficiently large with respect to the cross-group and cross-market externality parameters. Let us note that Assumption 1 can be derived from Assumption 2 by setting $\gamma_b=0$ and $\gamma_s=0$ and that Assumption 2 introduces stricter conditions on parameters than Assumption 2. Therefore, we will compare the equilibrium outcomes with and without MMC when Assumption 2 holds.

2.3 Timing of the game

The model is a two-stage game. In stage 1, platforms simultaneously choose the access prices they charge buyers and sellers. Without loss of generality, we assume that the platforms do not bear variable cost nor fixed cost. In stage 2, buyers and sellers decide which platforms to join and the platforms earn their equilibrium profits.

In stage 1, firm Ji that operates a single platform on the market i chooses the prices $p_{b,Ji}$ and $p_{s,Ji}$ to maximize its profit $\Pi_{Ji}\equiv n_{b,Ji}p_{b,Ji}+n_{s,Ji}p_{s,Ji}$. In the case of a multimarket firm J ($J=\{A,B\}$), it will maximize its profit on both markets $\Pi_J=\Pi_{J1}+\Pi_{J2}$ with respect to prices $p_{b,J1}$, $p_{s,J1}$, $p_{b,J2}$ and $p_{s,J2}$.

[8] A similar condition is found in Armstrong (2006) and Belleflamme and Peitz (2019).

As it is usual in most of the literature on two-sided platforms, we assume that all agents have full information about all prices so that they can have perfect expectation about the effect of price changes on demands (i.e., number of buyers and sellers) on each platform (fulfilled expectations equilibrium). The computation of equilibrium outcomes with and without MMC is detailed in Appendix A.1.

3 The impact of MMC on platform competition with bilateral pricing

This section examines the impact of cross-market externalities when platforms charge both buyers and sellers (bilateral pricing). Lemmas 1 and 2 depict the equilibrium outcomes (prices, number of sellers and buyers, and profits).[9]

Lemma 1 ***(Equilibrium prices and number of users with bilateral pricing)*** *Under platform competition with MMC, the price charged to sellers is* $p_s^* = \frac{1}{4}(\alpha_b - \alpha_s + \gamma_b - \gamma_s)$ *and the price charged to buyers is* $p_b^* = \frac{4t_b t_s - (\alpha_b + \gamma_b)(\alpha_b + 3\alpha_s + \gamma_b + 3\gamma_s)}{4t_s}$. *Each platform attracts* $n_b^* = \frac{1}{2}$ *buyers and* $n_s^* = \frac{\alpha_b + \alpha_s + \gamma_b + \gamma_s}{4t_s}$ *sellers. (proof: see Appendix A.1).*

Lemma 2 ***(Equilibrium profit with bilateral pricing)***

Under platform competition with MMC, the equilibrium platform profit (per market)[10] *is*

$\Pi^* = \frac{1}{16t_s}\left(8t_b t_s - \left[\alpha_b^2 + \alpha_s^2 + \gamma_b^2 + \gamma_s^2 + 6\gamma_b\gamma_s + 2\alpha_s(3\gamma_b + \gamma_s) + 2\alpha_b(3\alpha_s + \gamma_b + 3\gamma_s)\right]\right)$

(proof: see Appendix A.1).

Based on Lemmas 1 and 2, Table 1 summarizes the equilibrium outcomes (prices, number of sellers and buyers, platforms' profits, and surplus) in three cases: i) with both cross-group and cross-market externalities (Column 1); ii) with only cross-group externalities ($\gamma_b = 0$ and $\gamma_s = 0$, Column 2); iii) with only cross-market externalities ($\alpha_s = 0$ and $\alpha_b = 0$, Column 3).

[9] Since platforms are symmetric in both markets, buyers (resp. sellers) are charged the same access price $p_{b,Ji}^* = p_b^*$ (resp. $p_{s,Ji}^* = p_s^*$) in equilibrium. Hence, equilibrium profit and surplus are also similar with respect to platforms and markets. To ease notations, we further omit the i and J subscripts in lemmas and propositions.

[10] The total profit of a multimarket platform is thus twice the profit it earns on each market, i.e., $2 \times \Pi^*$.

	$\alpha_b > 0, \alpha_s > 0, \gamma_b > 0, \gamma_s > 0$	$\alpha_b > 0, \alpha_s > 0, \gamma_b = 0, \gamma_s = 0$	$\alpha_b = 0, \alpha_s = 0, \gamma_b > 0, \gamma_s > 0$
$p^*_{b,A,i} = p^*_{b,B,i}$	$\frac{4t_b t_s - (\alpha_b+\gamma_b)(\alpha_b+3\alpha_s+\gamma_b+3\gamma_s)}{4t_s}$	$\frac{4t_b t_s - \alpha_b(\alpha_b+3\alpha_s)}{4t_s}$	$\frac{4t_b t_s - \gamma_b(\gamma_b+3\gamma_s)}{4t_s}$
$p^*_{s,A,i} = p^*_{s,B,i}$	$\frac{1}{4}\left(\alpha_b - \alpha_s + \gamma_b - \gamma_s\right)$	$\frac{1}{4}\left(\alpha_b - \alpha_s\right)$	$\frac{1}{4}\left(\gamma_b - \gamma_s\right)$
$n^*_{b,A,i} = n^*_{b,B,i}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$n^*_{s,A,i} = n^*_{s,B,i}$	$\frac{\alpha_b+\alpha_s+\gamma_b+\gamma_s}{4t_s}$	$\frac{\alpha_b+\alpha_s}{4t_s}$	$\frac{\gamma_b+\gamma_s}{4t_s}$
$\Pi^*_{A,i} = \Pi^*_{B,i}$	$\frac{\left(\begin{array}{c} 8t_b t_s - \alpha_b^2 - \alpha_s^2 - 6\alpha_s\gamma_b - \gamma_b^2 - \gamma_s^2 \\ -2(\alpha_s+3\gamma_b)\gamma_s - 2\alpha_b(3\alpha_s+\gamma_b+3\gamma_s) \end{array}\right)}{16t_s}$	$\frac{8t_b t_s - \alpha_b^2 - \alpha_s^2 - 6\alpha_b\alpha_s}{16t_s}$	$\frac{8t_b t_s - \gamma_b^2 - \gamma_s^2 - 6\gamma_b\gamma_s}{16t_s}$
BS^*	$2v + \frac{\left(\begin{array}{c} -5t_b t_s + \alpha_b^2 + \alpha_s^2 + 4\alpha_s\gamma_b + \gamma_b^2 + \gamma_s^2 \\ +2(\alpha_s+2\gamma_b)\gamma_s + 2\alpha_b(2\alpha_s+\gamma_b+2\gamma_s) \end{array}\right)}{2_s}$	$2v + \left(\frac{-5t_b t_s + \alpha_b^2 + \alpha_s^2 + 4\alpha_b\alpha_s}{2t_s}\right)$	$2v + \left(\frac{-5t_b t_s + \gamma_b^2 + \gamma_s^2 + 4\gamma_b\gamma_s}{2t_s}\right)$
SS^*	$\frac{(\alpha_b+\alpha_s+\gamma_b+\gamma_s)^2}{8t_s}$	$\frac{(\alpha_b+\alpha_s)^2}{8t_s}$	$\frac{(\gamma_b+\gamma_s)^2}{8t_s}$
W^*	$2v + \frac{-4t_b t_s + 3(\alpha_b+\alpha_s+\gamma_b+\gamma_s)^2}{8t_s}$	$2v + \frac{-4t_b t_s + 3(\alpha_b+\alpha_s)^2}{8t_s}$	$2v + \frac{-4t_b t_s + 3(\gamma_b+\gamma_s)^2}{8t_s}$

Table 1: Equilibrium outcomes with bilateral pricing.

Comparison of these three settings highlights that cross-market externalities amplify cross-group externalities on both sides of the platforms. They work in the same direction whether we consider the seller-driven externalities (α_s and γ_s) or the buyer-driven externalities (α_b and γ_b).

Let us first examine the effect of cross-group externalities (α_s and α_b) on price competition on the buyer side. As already shown in the two-sided literature with horizontally-differentiated platforms, we find that those externalities intensify price competition on the buyer side. This is because an increase in α_b makes sellers more sensitive to the number of buyers. By lowering the access price for buyers, each platform seeks to increase its market share given that the total number of buyers is fixed. By attracting more buyers, platforms expect to have more sellers on the other side and thus gain more revenues. Similarly, an increase in α_s implies that buyers value more the presence of sellers on the other side. All things equal, an indirect way to attract more sellers and make the platform much more attractive on the buyer side is to lower the access price paid by buyers. The fact that buyers join at most one platform make them a critical asset for the success of each platform, driving platforms to aggressively compete to attract them.

Consider now the effects of cross-group externalities (α_s and α_b) on the seller side. In contrast, we find that the effects of cross-group externalities on price competition on the seller side depend on who directly benefits from those externalities. More precisely, an increase in the cross-group externalities that benefit buyers (α_s) reduces the price charged to sellers, as sellers become more valuable. Thus, platforms have more incentives to lower prices to expand the number of sellers, in order to indirectly attract more buyers. Conversely, an increase of the cross-group externalities that benefit sellers (α_b) rises the price charged to sellers. The intuition is that a larger α_b increases the willingness-to-pay of sellers. Then, each platform is able to exert market power by raising its price without the risk of losing any seller. This is due to the fact that on the seller side, sellers' demands for the two platforms are independent and an increase in the access price does not generate any direct substitution effect.

Multimarket contact amplifies these intra-market network effects by adding cross-market effects. Simple comparative statics show that the price charged to buyers decreases with seller-driven cross-market externalities (γ_s). Similarly, the price charged to sellers decreases when γ_s increases. Hence, since buyers benefit from being connected with sellers in the other market (when $\gamma_s > 0$), a multimarket firm has stronger incentives to lower prices on the seller side in both markets. As the rival firm also has the same incentives, this leads to fiercer price competition. Consequently, multimarket platforms attract more sellers, but earn lower profit per market when seller-driven cross-market externalities increase.

Looking at buyer-driven cross-market externalities (γ_b), an increase in γ_b relaxes price competition on the seller side, whereas it intensifies price competition on the buyer side. This is typically a seesaw effect already emphasized by Rochet and Tirole (2006). This seesaw effect states that a factor that is conducive to higher price on one side tends to call for a lower price on the other side. When buyer-driven cross-market externalities increase, sellers have a higher willingness-to-pay for the access to a multimarket platform, and both firms can take advantage of this to increase the price charged to sellers. However, the additional revenues obtained from sellers are transferred to buyers who enjoy lower prices. Both firms end up with lower profits as buyer-driven cross-market externalities increase.

Propositions $1-3$ are based on the comparison of the outcomes with MMC (Column 1 of Table 1) and without MMC (Column 2). They emphasize that when platforms charge both sides (bilateral pricing), MMC never relaxes competition. The first proposition presents the impact of MMC on the prices and the two other propositions focus on how multimarket contact (and cross-market externalities) affects the number of buyers and sellers joining each platform, as well as platform profits and users' surpluses.

Proposition 1 *With bilateral pricing, multimarket contact always decreases the access price paid by buyers, but increases the price paid by sellers if $\gamma_s < \gamma_b$. (proof: see Appendix B.1).*

Proposition 2 *With bilateral pricing, platforms attract more sellers, but earn lower profits under MMC competition. (proof: see Appendix B.2).*

Proposition 3 *With bilateral pricing, buyer surplus, seller surplus and total surplus are higher under MMC competition (proof: see Appendix B.3).*

The results on the buyer side are driven by the combined effects of the cross-group and cross-market externalities. To illustrate this, consider market 1. If a platform chooses to lower buyers' access price in that market, this price cut has a positive direct effect on the number of buyers and a positive indirect effect on the number of sellers on the other side, through cross-group externalities. These intra-market network effects generate a positive feedback loop, where more users on one side attract more users on the other side. Thus, each platform has strong incentives to cut its access prices to increase its market share on buyers, making its service much more valuable for sellers than the service provided by the rival platform. These effects explain why cross-group externalities intensify price competition between platforms: they have been widely analyzed in the two-sided literature since Armstrong (2006). However, our framework sheds light on a second type of effects, namely cross-market effects. Indeed, buyers' decision to join a multimarket platform depends on the number of sellers on the other side of the platform in both markets. All other things being equal, if a platform lowers the price charged to its buyers in market 1, then the number of buyers on the platform in market 1 increases, which also increases the revenues of sellers present on this platform in market 2. If the platform attracts more sellers in market 2, it will also attract more buyers in market 2 and these additional buyers will in return attract more sellers in market 1, *etc.* Hence a multimarket platform has more incentives to decrease its access price for buyers than a single-market platform, as each buyer exerts a larger positive externality on sellers. This results in fiercer platform competition and in lower access price for buyers under MMC.

The results on the seller side are more ambiguous and depend on the magnitude of buyer-driven and seller-driven cross-market externalities. When cross-market externalities benefit more buyers than sellers ($\gamma_s > \gamma_b$), MMC lowers the access price charged to sellers. The reason is that sellers in one market are also highly valuable for buyers in the other market, and the best strategy for a multimarket platform is to reduce the sellers' price to attract more sellers. However, when $\gamma_s < \gamma_b$, a multimarket platform charges a higher price to sellers. As sellers here benefit more from cross-market externalities, platforms are able to exert higher market power.

To sum up, our results establish that price competition under MMC is always fiercer on the buyer side regardless of the nature of cross-market externalities. The fact that buyers single-home in both markets explains why multimarket platforms are fighting more fiercely to attract them. By contrast, the impact of MMC on the access prices paid by sellers depends on which side benefits more from cross-market externalities. When cross-market externalities benefit more sellers than buyers, multimarket platforms charge higher prices to sellers than single-market platforms.

Proposition 2 highlights an interesting paradox. In our context, even if more sellers decide to join platforms under MMC, platforms are unable to capture the value generated by the cross-market externalities. Eventually, the surpluses of buyers and sellers increase along with total surplus. These propositions tend to confirm the claim of Evans (2017) that MMC between digital conglomerates strengthens competition and benefits platform users. We find that both platforms are better off in the no MMC case (i.e., in the absence of cross-market externalities). An interesting implication is that multimarket platforms have a shared interest in breaking down cross-market externalities (or at least in reducing their scope). Suppose for instance that each firm has the ability to choose the level of cross-market externalities through contractual, technological or design decisions, such as restricting data sharing or reducing integration and interoperability between its platform-based services. If both firms agree to reduce cross-market externalities, then Proposition 2 suggests that such coordination would be mutually profitable. However, this does not mean that this outcome would be stable, as a unilateral deviation (where a firm unilaterally increases its level of cross-market externalities) could be more profitable.[11]

Public authorities may also directly or indirectly influence cross-market externalities, for instance by enforcing stricter data regulation (e.g., increasing restrictions to Tech Giants like Google or Apple on the collection and the crossing of users' data across their platforms). According to our model, a stricter data regulation that would mitigate cross-market externalities could have ambiguous effects on platform users (i.e. protecting user privacy while softening price competition).

4 The impact of MMC with unilateral pricing

In this section, we consider the case in which platforms can charge only one side while the other side is granted free access (unilateral pricing). Granting free access to one of the two sides is a pricing model that is commonly observed in digital markets and that can be justified by several factors. First, technical or regulatory constraints may sometimes prevent a platform from charging

[11] This paradox and its implication are further discussed in Section 5.

one side. Secondly, depending on the type of platform, it may be possible for users to bypass the platform (see e.g., Gu and Zhu, 2021). For instance, in online marketplaces, buyers may directly contact sellers after a first deal so that these agents no longer use the platform if the access is not free. More generally, free access can be explained by platforms' non neutral price structure and be a profit-maximizing strategy for these platforms (Wright, 2004). Finally, as it has been emphasized by Tirole and Bisceglia (2023), this situation is frequently observed and thus deserves special attention to further investigate multimarket platform competition.

In this section, we focus on the case in which buyers are granted free access and only sellers can be charged.[12] The structure of the model is similar to that of the baseline model except that buyers' access prices (p_b) are removed from the utility function of buyers (with $i = 1, 2$ and $i' \neq i$) :

$$\begin{cases} U_{Ai}(x) = v + \alpha_s n_{s,Ai} + \gamma_s n_{s,Ai'} - t_b x \\ U_{Bi}(x) = v + \alpha_s n_{s,Bi} + \gamma_s n_{s,Bi'} - t_b(1-x) \end{cases} \tag{5}$$

In this case, platforms' profits solely rely on the revenues generated from sellers. Similarly to the baseline model, we focus on equilibrium outcomes in which the two rival platforms are active in both markets. This is the case if Assumption 3 (resp. 4) holds in the no MMC case (resp. in the MMC case).

Assumption 3 *With unilateral pricing and without MMC, parameters must satisfy* $t_b t_s > \alpha_b \alpha_s$.

Assumption 4 *With unilateral pricing and with MMC, parameters must satisfy* $t_b t_s > \alpha_b \alpha_s + \alpha_b \gamma_s + \alpha_s \alpha_b + \gamma_b \gamma_s$.

Full computation of the equilibrium outcome is detailed in Appendix A.2 and summarized by Lemmas 3 and 4.

Lemma 3 ***(Equilibrium prices and number of users when buyers have free access)*** *Under platform competition with MMC, the price charged to sellers is* $\overline{p}_s^* = \frac{(\alpha_b+\gamma_b)(t_b t_s-(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))}{4t_b t_s-3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s)}$. *Each platform attracts* $\overline{n}_b^* = \frac{1}{2}$ *buyers and* $\overline{n}_s^* = \frac{(\alpha_b+\gamma_b)(2t_b t_s-(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))}{2t_s(4t_b t_s-3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))}$ *sellers (proof: see Appendix A.2).*

[12] The case in which sellers are granted free access and platforms only charge buyers is presented in Appendix A.3. This case leads to propositions that are qualitatively similar to those derived from the baseline model (bilateral pricing) and it is therefore less interesting to present this case in detail compared to the case where only sellers are charged.

Lemma 3 states that the access price paid by sellers decreases with respect to both α_s (seller-driven cross-group externalities) and γ_s (seller-driven cross-market externalities) i.e., multimarket platforms have more incentives to reduce the price charged to sellers when buyers derive more value from additional sellers in both markets. Through a lower price, a multimarket platform can attract more sellers, which increases the number of buyers on the other side of the platform in both markets. This, in turn, reinforces the attractiveness of the platform for sellers. This creates a positive feedback loop between the two sides of the platform and across the two markets. The larger the magnitude of the cross-group and of the cross-market externalities, the higher the incentives to decrease sellers' access price. Consequently, the number of sellers increases as seller-driven cross-group and cross-market externalities (α_s and γ_s) increase, due to the lower price charged to sellers.

The impact of buyer-driven cross-group and cross-market externalities (α_b and γ_b respectively) on sellers' access price is less clear since these two parameters can have positive or negative effects depending on their values (non linear effects) and also depending of the value of other parameters as Propositions 4 and 5 will further show.

Lemma 4 ***(Equilibrium profits of platforms when buyers have free access)*** *Under platform competition with MMC, the equilibrium (per market) profit of a multimarket platform is* $\overline{\Pi}^* = \frac{(\alpha_b+\gamma_b)^2(2t_bt_s-(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))(t_bt_s-(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))}{t_s(4t_bt_s-3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))^2}$ *(proof: see Appendix A.2).*

Lemma 4 highlights that an increase in seller-driven cross market externalities (γ_s) is always detrimental to platforms' profit. Although a larger γ_s helps a multimarket platform to attract more sellers, it eventually intensifies price competition and reduces the revenues earned on the seller side. By contrast, the magnitude of buyer-driven cross-market externalities (γ_b) has ambiguous effects on platform profits, which can either increase or decrease.

Based on Lemmas 3 and 4, we are now able to compare the equilibrium outcomes with MMC and without MMC when only sellers are charged. Propositions 4 and 5 characterize the impact of MMC on platform competition in this setting.[13]

Proposition 4 *When buyers have free access, the access price charged to sellers is higher under MMC only if* $\alpha_s < \tilde{\alpha}_s^{(1)}$, $\gamma_b < \tilde{\gamma}_b^{(1)}$ *and* $\gamma_s < \tilde{\gamma}_s^{(1)}$ *(proof: see Appendix B.4).*

[13] Due to some lengthy expressions, the presentation of the cutoff values of parameters γ_s, γ_b and α_b that appear in these propositions is reported to the related appendix.

Two mechanisms are here at work. On the one hand, MMC intensifies price competition in order to attract sellers and buyers. On the other hand, MMC generates cross-market externalities that increase sellers' willingness to pay to access the platform which allows platforms to increase their prices. If the magnitude of cross-market externalities is sufficiently large ($\gamma_b > \tilde{\gamma}_b^{(1)}$ or $\gamma_s > \tilde{\gamma}_s^{(1)}$), the price effect dominates and the sellers' access price decreases under MMC. Conversely, a sufficient condition for MMC to relax price competition requires cross-market externalities to exhibit moderate values.

Proposition 5 *When buyers have free access, platforms always attract more sellers with MMC. Moreover, they earn higher profits with MMC only if $\alpha_s < \tilde{\alpha}_s^{(2)}$, $\gamma_b < \tilde{\gamma}_b^{(2)}$ and $\gamma_s < \tilde{\gamma}_s^{(2)}$. (proof: see Appendix B.5).*

Proposition 5 states that multimarket contact may here benefit platforms if the cross-market and cross-group externalities are both sufficiently low. This result can be explained by the fact that platforms do not compete directly for the side that is charged (i.e. sellers), allowing them to leverage their market power to capture higher profits under certain conditions. Indeed, sellers are willing to pay more to access a multimarket platform because they benefit from positive externalities generated by buyers present on the other side of the platform in both markets. The platform may take advantage of this situation by charging a higher price. This is the case if cross-market externalities (γ_b and γ_s) are both low enough. However, if either γ_b or α_b is relatively large, it implies that sellers place a high value on the number of buyers on the other side of the platform (in both markets). In this case, the best strategy of a multimarket platform for enrolling additional buyers is to increase the number of sellers, by charging a lower access price. As both multimarket platforms do the same, competition to attract both sides of users is fiercer and increasing prices for sellers is no longer a profitable strategy.

The conditions under which MMC increase platform profitability clearly highlight the role played by cross-market externalities. However, it should be pointed that the ability for platforms to take advantage of MMC also depends on intra-platform factors (i.e. cross-group externalities). To illustrate this, we can perform numerical simulations to exhibit the range of parameters for which MMC competition is more profitable.

Figure 2 displays the profit gap (i.e. the difference between the platform profit with MMC and that without MMC) for different values of cross-market externalities γ_s (x-axis, with $\gamma_s \in [0,2]$) and γ_b (y-axis, with $\gamma_b \in [0,2]$) all other parameters being constant (i.e., $t_b = 1, t_s = 1, \alpha_b = 0.4$ and $\alpha_s = 0.6$). In this figure, the colored area depicts all the (γ_s, γ_b) pairs whose profit gap is positive (i.e. platform profit higher with MMC than without MMC).[14] The grey area corresponds to the pairs whose profit gap is negative.[15] When both cross-market externalities are sufficiently low (lower left quadrant), MMC competition leads to higher platform profits. Now, consider successive increases in γ_b or γ_s. At some point, these increases will lead to a shift to the grey area where platform profits are lower under MMC. In this example, if at least one of the two cross-market externalities is too large, MMC is not profitable for platforms.

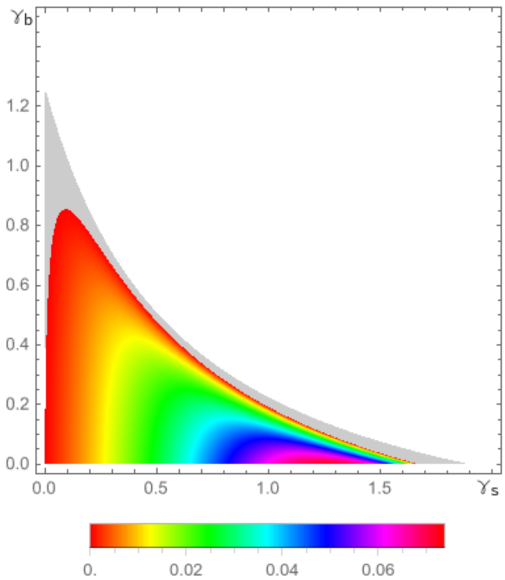

Figure 2: Profit gap (MMC - no MMC) with unilateral pricing as a function of γ_s (x-axis) and γ_b (y-axis), with $t_b = 1, t_s = 1, \alpha_b = 0.4$ and $\alpha_s = 0.6$. The grey area corresponds to a negative profit gap (MMC $<$ no MMC) and the colored area to a positive profit gap (MMC $>$ no MMC).

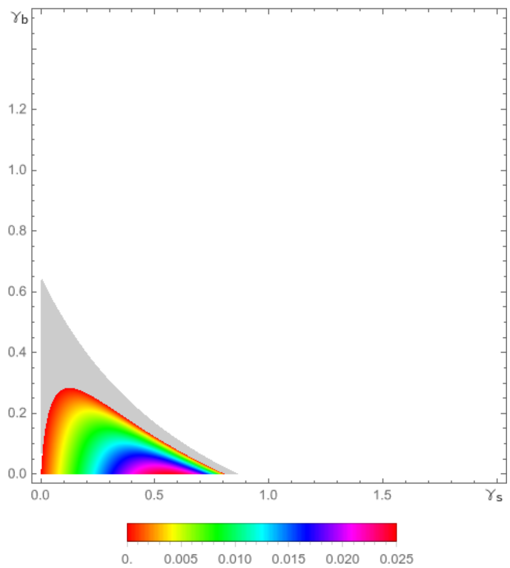

Figure 3: Profit gap (MMC - no MMC) with unilateral pricing as a function of γ_s (x-axis) and γ_b (y-axis), with $t_b = 1, t_s = 1, \alpha_b = 0.6$ and $\alpha_s = 0.8$. The grey area corresponds to a negative profit gap (MMC $<$ no MMC) and the colored area to a positive profit gap (MMC $>$ no MMC).

[14]The color used highlights the amount of the profit gap with the magenta color depicting the highest value of the profit gap and the red color depicting the lowest (i.e., close to 0) values.

[15]In this figure, areas in white correspond to parameter values that are not compatible with the equilibrium existence conditions. These cases correspond to relatively high values of cross-group and/or cross-market externalities. In these cases, externalities are so high that competition is so intense that one platforms will be deterred from the market on buyers' side. Since we restrict on competitive outcomes in this paper, these winner-takes-all outcomes are not investigated here.

Now, consider Figure 3 that exhibits the profit gap for the same range of cross-market externality values ($\gamma_s \in [0,2]$ and $\gamma_b \in [0,2]$), but assuming larger cross-group externalities (with $\alpha_b = 0.6$ and $\alpha_s = 0.8$ instead of 0.4 and 0.6 respectively in Fig.2.). Other parameters are held constant (i.e. $t_b = 1$ and $t_s = 1$). These higher cross-group externalities produce two effects. First, they narrow the range of parameters compatible with a competitive outcome (as evidenced by the expansion of the white zone), meaning that platform competition is more likely to end up with a winner-takes-all outcome. Secondly, they also limit the potential for MMC to enhance platform profits (shrinkage of the colored area). In Figure 3, there are fewer (γ_s, γ_b) pairs for which MMC competition is mutually profitable for platforms. These outcomes arise from intensified competition to attract buyers and sellers in the two markets, forcing multimarket platforms to lower access prices for sellers.

Proposition 6 *When buyers have free access, buyer surplus, seller surplus, and total surplus are higher with MMC (proof: see Appendix B.6).*

Proposition 6 completes the case with unilateral pricing with respect to surplus analysis. In this case, total surplus is always higher under MMC. In this setting, buyers obtain a higher surplus because they enjoy having more sellers on the other side of the platform in both markets and they are not charged for this. Similarly to the bilateral pricing model, sellers are also able to capture part of the value generated by cross-market externalities: more sellers join both platforms and they derive a higher surplus as a whole even though access price may be higher in some situations.[16] In this respect, these results are similar to those obtained with bilateral pricing. However, unlike the case with bilateral pricing, it is here possible for platforms to increase their profits and capture a part of the surplus, when cross-market and cross-group externalities are low.

5 Conclusion and policy implications

In this paper, we examine whether multimarket contact may relax competition among two-sided platforms. We focus on a competitive bottleneck setting and highlight two types of effects that drive platform competition under MMC: *(i)* intra-market network effects between the two sides that are connected through the platform within each market; and *(ii)* inter-market network effects that

[16]These situations are defined by Proposition 4, i.e. $\alpha_s < \tilde{\alpha}_s^{(1)}$, $\gamma_b < \tilde{\gamma}_b^{(1)}$ and $\gamma_s < \tilde{\gamma}_s^{(1)}$.

emerge when one side of the platform in a market is connected to the other side of the platform in another market. The first effect is well documented in models of platform competition (Armstrong, 2006). The second effect is original and refers to cross-market demand-side synergies that benefit users in a given market as the number of users in the other markets increases.

In this framework, we find that when platforms use bilateral pricing (i.e. they charge an access price to both sides), multimarket contact actually increases platform competition. In this setting, multimarket platforms are unable to capture any value from cross-market externalities. MMC is welfare-increasing but detrimental to platforms. We also investigate whether this conclusion extends to the case of unilateral pricing strategy (i.e. when platforms can only charge one side but grant free access to the other side). When the paying side is the group of users who are not the competitive bottleneck, we identify market conditions in which MMC relaxes competition and increases platform profits. Specifically, this is the case if cross-group and cross-market externalities are both sufficiently low. In all other cases, platforms are always worse off under MMC. To summarize, in a competitive bottleneck setting, multimarket contact between digital platforms is, in most cases, not effective in relaxing price competition and tends to reduce the profitability of platforms.

Our results may seem somewhat puzzling when we look at what is happening in digital markets. In these markets, all Tech Giants are using diversification strategies that lead to numerous multimarket contacts. This raises the question: Why would Tech Giants pursue such strategies if they are detrimental to their profits? Two arguments can be put forward.

Firstly, our analysis suggests that the motivations for multi-market or conglomerate strategies cannot be just market power. These strategies may be explained by other motives like innovation or cost saving from economies of scope, and more generally by synergies and efficiency effects. These potential effects are not considered in our theoretical framework. Therefore, it could be a possible extension of our model, taking into account that platform integration might lower platform costs, enhance the quality of service, or enable the creation of innovative services that buyers or sellers would be willing to pay for. Such effects would increase the likelihood of multimarket strategies, as they could help mitigate the negative impact of multimarket competition on platform profits.

Secondly, it is important to emphasize that our results are based on a comparison between a market configuration where neither firm has a multimarket presence and a market configuration where both firms operate across multiple markets. Therefore, it does not directly address how firms reach the multimarket outcome. Nevertheless, our theoretical framework can be extended to analyze the strategic decisions to operate multiple platforms across different markets, by introducing a pre-competition stage in which firms can decide whether to be a multimarket platform or a single-market platform. Three potential equilibrium outcomes can occur: both firms choose a multimarket strategy (*full multimarket* outcome), only one firm chooses a multimarket strategy and faces a single-market platform on both markets (*partial multimarket* outcome), and neither firm adopts a multimarket strategy (*no multimarket* outcome).[17] Therefore, a conglomerate (or multimarket) strategy may actually be a best-response in our specific framework of platform competition.

Our paper makes four contributions. First, we contribute to the theory of multimarket contact. The academic literature claims that MMC relaxes competition in traditional markets. We revisit this question in the context of two-sided platforms with competitive bottleneck and in a static setting, and find that, in most cases, MMC intensifies competition. The situations in which multimarket platform competition is relaxed are rather limited. For this result to hold, several conditions need to be met. Firstly, platforms must only charge the side on which they do no directly compete (i.e. the side on which they have market power). Secondly, the magnitude of cross-group and cross-market externalities has to both be relatively low.

The second contribution concerns competition policy. In recent years, there have been increasing calls from politicians and citizens for a greater regulation of Tech Giants. However the regulation of digital markets poses many challenges to antitrust authorities (Cremer et al., 2019) and there is still a need to better understand the multimarket strategies and the acquisition motives of these Tech Giants (Bourreau and De Streel, 2019; Gautier and Lamesch, 2021; Parker et al., 2021). In particular, the mergers & acquisitions of digital platforms that operate in distinct markets can lead to more

[17] The additional material related to the resolution of the model with the pre-competition stage is available upon request. Specifically, it provides the characterization of the equilibria (prices, profits, number of sellers) in an asymmetric setting in which one of the two platforms adopts a multimarket strategy whereas the other chooses a single-market strategy. The calculations when solving this extension are highly involved and require to run numerical simulations, whatever the pricing design considered (i.e. bilateral pricing, unilateral pricing where buyers have free access, and unilateral pricing where sellers have free access). Precisely, the outcome where only one firm adopts a multimarket strategy (*partial multimarket* outcome), as well as that where the two firms choose a multimarket strategy (*full multimarket* outcome), can be found as Nash equilibria, depending on the parameters of our model (i.e. cross-group and cross market externalities) and the pricing setting (i.e. bilateral pricing or unilateral pricing).

platform integration (e.g. through the crossing of data and the interoperability of applications) after merger has occurred, and this raises competitive concerns. Although our paper focuses on a competitive bottleneck setting, it provides some insights for competition authorities to review mergers & acquisitions dealing with platforms which operate in multiple markets. In particular, our results imply that antitrust authorities should first identify the cross-market externalities which are likely to arise from the merger, i.e. examine the nature (which side will most benefit from these externalities) and the magnitude of cross-market externalities after platform integration. If this acquisition strengthens multimarket contact and cross-market externalities, it is likely to be welfare-enhancing for the users of these digital platforms. However, authorities should be more cautious when the magnitude of cross-market externalities is relatively low since these situations may end up with higher platform profit. This is more likely to happen when the platforms' business models rely on unilateral pricing and platforms have some market power on the side that is charged. In this case, competition authorities also need to pay attention to the level of cross-group externalities.

The third contribution focuses on the interplay between privacy-protecting policies and competition policy. There are also growing concerns about how these two policies may interact (see e.g. Sokol and Zhu, 2021). Our paper highlights that stricter privacy and data regulation which would reduce the magnitude of cross-market externalities may eventually play against competition policy in the context of multimarket platform competition.

Finally, a last contribution of our paper is to provide some insights into the strategic dimension of platform design, such as interoperability or data sharing between platform-based services. Through these design decisions, multimarket platforms could be able to adjust the magnitude of cross-market externalities. We find that they have a shared interest in mitigating these externalities to relax price competition, and could mutually increase their profits if they are able to coordinate their design decisions. Investigating the stability of this kind of cooperation is outside the scope of this paper and opens up new paths for future research.

A limitation of our model is that it does not explicitly consider advertising as a source of revenues, despite being a pervasive feature of many digital markets. However, as highlighted by Belleflamme and Peitz (2021), a significant number of digital platforms, such as Netflix, Amazon, and AirBnB primarily generate revenue by directly charging users. Since the main objective of our paper is to examine the pricing strategies of platforms in the presence of both cross-group and cross-market externalities, we have chosen to rule out advertising from our framework. However, we believe that this question could be explored in future research, using other model setups, such as those proposed by de Corniere and Taylor (2014), de Corniere (2016), or Sato (2019). Another direction for future research is to consider other settings than competitive bottleneck. Due to its prevalence in many digital markets, we have focused on a configuration where buyers singlehome and sellers multihome. However, other configurations can arise in real-world markets. For instance, markets may vary in terms of multihoming and singlehoming behaviors, leading to heterogeneity across markets. Investigating such heterogeneity and its effect on mutimarket platform competition would offer valuable insights and further enrich the findings presented in this paper.

Appendix

The derivations of the equilibrium outcomes (lemmas) are presented in Appendix A where we successively consider the baseline model (bilateral pricing) and its two extensions (unilateral pricing when buyers have free access and unilateral pricing when sellers have free access). The proofs of the propositions are detailed in Appendix B.

Appendix A. Equilibria

A.1. Equilibria in the baseline model (bilateral pricing)

Remember that both buyers and sellers are charged access prices in the baseline model (bilateral pricing). We first derive the equilibrium outcomes with MMC (Appendix A.1.1), then without MMC (Appendix A.1.2).

A.1.1. MMC equilibrium

In the MMC case, firms A and B compete in markets 1 and 2. In this case, Ai and Ai' (with $i = 1, 2$, $i' = 1, 2$ and $i \neq i'$), as well as Bi and Bi', are multimarket platforms, which generates cross-market externalities (i.e., $\gamma_s > 0$ and $\gamma_b > 0$). On the buyers' side, platforms Ai and Bi are respectively located at 0 and 1 in market i. Assuming market i is fully-covered on the buyers' side, and denoting the expected number of buyers and sellers as $n^e_{s,Ai}$, $n^e_{s,Bi}$, $n^e_{b,Ai}$, and $n^e_{b,Bi}$, the marginal buyer who is indifferent between joining platform Ai and platform Bi in market i is located at x_i such that

$$v - t_b x_i + \alpha_s n^e_{s,Ai} + \gamma_s n^e_{s,Ai'} - p_{b,Ai} = v - t_b (1 - x_i) + \alpha_s n^e_{s,Bi} + \gamma_s n^e_{s,Bi'} - p_{b,Bi}. \tag{A1}$$

As buyers are uniformly distributed on the interval [0,1] and singlehome, x_i is the number of buyers joining platform Ai and $(1 - x_i)$ is the number of buyers joining platform Bi.

To find the number of sellers on each platform, we have to determine the location of the marginal seller on each platform in market i. The type y_{Ai} (resp. y_{Bi}) of the seller who is indifferent to join platform Ai (resp. Bi) or not is given by the zero profit condition so that

$$\alpha_b n^e_{b,Ai} + \gamma_b n^e_{b,Ai'} - t_s y_{Ai} - p_{s,Ai} = 0, \tag{A2}$$

and

$$\alpha_b n^e_{b,Bi} + \gamma_b n^e_{b,Bi'} - t_s y_{Bi} - p_{s,Bi} = 0. \tag{A3}$$

As sellers' platform specific costs are uniformly distributed over [0,1], the marginal seller y_{Ai} (respectively y_{Bi}) corresponds to the number of active sellers on platform Ai (respectively on platform Bi). From equations (A1-3), we can derive the number of buyers and sellers on platforms Ai and Bi:

$$n_{b,Ai} = \frac{t_b + \alpha_s \left(n^e_{s,Ai} - n^e_{s,Bi}\right) + \gamma_b \left(n^e_{s,Ai'} - n^e_{s,Bi'}\right) + \left(p_{b,Bi} - p_{b,Ai}\right)}{2t_b}, \tag{A4}$$

$$n_{b,Bi} = 1 - \frac{t_b + \alpha_s \left(n^e_{s,Ai} - n^e_{s,Bi}\right) + \gamma_b \left(n^e_{s,Ai'} - n^e_{s,Bi'}\right) + \left(p_{b,Bi} - p_{b,Ai}\right)}{2t_b}, \tag{A5}$$

$$n_{s,Ai} = \frac{\alpha_b n^e_{b,Ai} + \gamma_b n^e_{b,Ai'} - p_{s,Ai}}{t_s}, \tag{A6}$$

$$n_{s,Bi} = \frac{\alpha_b n^e_{b,Bi} + \gamma_b n^e_{b,Bi'} - p_{s,Bi}}{t_s}. \tag{A7}$$

We assume that users' expectations are fulfilled at equilibrium. Formally, we have $n^e_{s,Ai} = n_{s,Ai}$, $n^e_{s,Bi} = n_{s,Bi}$, $n^e_{b,Ai} = n_{b,Ai}$ and $n^e_{b,Bi} = n_{b,Bi}$. Solving the four-equation system given by (A4), (A5), (A6) and (A7), the number of buyers and sellers is

$$n_{b,Ai} = \frac{\begin{pmatrix} (\alpha_b\alpha_s - t_b t_s)^2 - (\alpha_s\gamma_b)^2 - (\alpha_b\gamma_s)^2 - \gamma_b\gamma_s\left(2t_b t_s - \gamma_b\gamma_s\right) \\ -\left(-\alpha_b\alpha_s^2 + t_b t_s\alpha_s + \alpha_b\gamma_s^2\right)\left(p_{s,Ai} - p_{s,Bi}\right) \\ -\left(\gamma_b\alpha_s^2 - \gamma_b\gamma_s^2 + t_b t_s\gamma_s\right)\left(p_{s,Ai'} - p_{s,Bi'}\right) \\ +t_s\left(\alpha_b\alpha_s + \gamma_b\gamma_s - t_b t_s\right)\left(p_{b,Ai} - p_{b,Bi}\right) \\ -t_s\left(\alpha_b\gamma_s + \gamma_b\alpha_s\right)\left(p_{b,Ai'} - p_{b,Bi'}\right) \end{pmatrix}}{2\left(t_b t_s - (\alpha_b - \gamma_b)(\alpha_s - \gamma_s)\right)\left(t_b t_s - (\alpha_b + \gamma_b)(\alpha_s + \gamma_s)\right)}, \tag{A8}$$

$$n_{b,Bi} = 1 - \frac{\begin{pmatrix} (\alpha_b\alpha_s - t_b t_s)^2 - (\alpha_s\gamma_b)^2 - (\alpha_b\gamma_s)^2 - \gamma_b\gamma_s\left(2t_b t_s - \gamma_b\gamma_s\right) \\ -\left(-\alpha_b\alpha_s^2 + t_b t_s\alpha_s + \alpha_b\gamma_s^2\right)\left(p_{s,Ai} - p_{s,Bi}\right) \\ -\left(\gamma_b\alpha_s^2 - \gamma_b\gamma_s^2 + t_b t_s\gamma_s\right)\left(p_{s,Ai'} - p_{s,Bi'}\right) \\ +t_s\left(\alpha_b\alpha_s + \gamma_b\gamma_s - t_b t_s\right)\left(p_{b,Ai} - p_{b,Bi}\right) \\ -t_s\left(\alpha_b\gamma_s + \gamma_b\alpha_s\right)\left(p_{b,Ai'} - p_{b,Bi'}\right) \end{pmatrix}}{2\left(t_b t_s - (\alpha_b - \gamma_b)(\alpha_s - \gamma_s)\right)\left(t_b t_s - (\alpha_b + \gamma_b)(\alpha_s + \gamma_s)\right)}, \tag{A9}$$

$$n_{s,Ai} = \frac{1}{4}\begin{pmatrix} \frac{2\left(-p_{s,Ai} - p_{s,Bi} + \alpha_b + \gamma_b\right)}{t_s} \\ +\frac{\left(-p_{s,Ai} + p_{s,Ai'} + p_{s,Bi} - p_{s,Bi'}\right)t_b - \left(p_{b,Ai} - p_{b,Ai'} - p_{b,Bi} + p_{b,Bi'}\right)(\alpha_b - \gamma_b)}{t_b t_s - (\alpha_b - \gamma_b)(\alpha_s - \gamma_s)} \\ +\frac{\left(-p_{s,Ai} - p_{s,Ai'} + p_{s,Bi} + p_{s,Bi'}\right)t_b + \left(p_{b,Ai} + p_{b,Ai'} - p_{b,Bi} - p_{b,Bi'}\right)(\alpha_b + \gamma_b)}{t_b t_s - (\alpha_b + \gamma_b)(\alpha_s + \gamma_s)} \end{pmatrix}, \tag{A10}$$

$$n_{s,Bi} = \frac{1}{4}\begin{pmatrix} \frac{2\left(-p_{s,Ai} - p_{s,Bi} + \alpha_b + \gamma_b\right)}{t_s} \\ +\frac{\left(p_{s,Ai} - p_{s,Ai'} - p_{s,Bi} + p_{s,Bi'}\right)t_b + \left(p_{b,Ai} - p_{b,Ai'} - p_{b,Bi} + p_{b,Bi'}\right)(\alpha_b - \gamma_b)}{t_b t_s - (\alpha_b - \gamma_b)(\alpha_s - \gamma_s)} \\ +\frac{\left(p_{s,Ai} + p_{s,Ai'} - p_{s,Bi} - p_{s,Bi'}\right)t_b + \left(p_{b,Ai} + p_{b,Ai'} - p_{b,Bi} - p_{b,Bi'}\right)(\alpha_b + \gamma_b)}{t_b t_s - (\alpha_b + \gamma_b)(\alpha_s + \gamma_s)} \end{pmatrix}. \tag{A11}$$

Solving backwards, we can replace $n_{s,Ai}$ (resp. $n_{s,Bi}$) according to (A8) and (A10) (resp. (A9) and (A11)) into the objective function of platform Ai (resp. Bi) in market i:

$$\Pi_{Ai} = n_{b,Ai}p_{b,Ai} + n_{s,Ai}p_{s,Ai}, \tag{A12}$$

and

$$\Pi_{Bi} = n_{b,Bi}p_{b,Bi} + n_{s,Bi}p_{s,Bi}. \tag{A13}$$

Multimarket firms A and B choose their access prices to maximize their total profits $\Pi_A = \Pi_{Ai} + \Pi_{Ai'}$ and $\Pi_B = \Pi_{Bi} + \Pi_{Bi'}$. The first-order conditions form a 4×4 system. We solve this system to define the Nash equilibrium of this game. Equilibrium prices are

$$p^*_{b,Ai} = p^*_{b,Bi} = \frac{4t_b t_s - (\alpha_b + \gamma_b)(\alpha_b + 3\alpha_s + \gamma_b + 3\gamma_s)}{4t_s}, \tag{A14}$$

and

$$p^*_{s,Ai} = p^*_{s,Bi} = \frac{1}{4}\left(\alpha_b - \alpha_s + \gamma_b - \gamma_s\right). \tag{A15}$$

Equilibrium number of buyers and equilibrium number of sellers are

$$n^*_{b,Ai} = n^*_{b,Bi} = \frac{1}{2}, \tag{A16}$$

and

$$n^*_{s,Ai} = n^*_{s,Bi} = \frac{\alpha_b + \alpha_s + \gamma_b + \gamma_s}{4t_s}. \tag{A17}$$

Equilibrium profits are

$$\Pi^*_A = \Pi^*_B = \frac{8t_b t_s - \alpha_b^2 - \alpha_s^2 - 6\alpha_s\gamma_b - \gamma_b^2 - 2(\alpha_s + 3\gamma_b)\gamma_s - \gamma_s^2 - 2\alpha_b(3\alpha_s + \gamma_b + 3\gamma_s)}{8t_s}. \tag{A18}$$

If Assumption 2 holds, both second-order conditions are satisfied and equilibrium prices and profits are positive. An interior profit-maximizing solution hence exists.

Equilibrium buyer (respectively seller) surplus in the two markets is denoted by BS^* (respectively SS^*) and is given by

$$BS^* = 2v + \left(\frac{-5t_b t_s + \alpha_b^2 + \alpha_s^2 + 4\alpha_s\gamma_b + \gamma_b^2 + 2(\alpha_s + 2\gamma_b)\gamma_s + \gamma_s^2 + 2\alpha_b(2\alpha_s + \gamma_b + 2\gamma_s)}{2t_s}\right), \tag{A19}$$

and

$$SS^* = \frac{(\alpha_b + \alpha_s + \gamma_b + \gamma_s)^2}{8t_s}. \tag{A20}$$

Equilibrium total surplus in the two markets is equal to

$$W^* \equiv BS^* + SS^* = 2v + \frac{-4t_b t_s + 3(\alpha_b + \alpha_s + \gamma_b + \gamma_s)^2}{8t_s}. \tag{A21}$$

A.1.2. No MMC equilibrium

The no MMC equilibrium is obtained from (A14), (A15), (A16), (A17), (A18), (A19), (A20), and (A21), by setting $\gamma_s = 0$ and $\gamma_b = 0$.

The equilibrium prices are thus

$$p_{b,Ai}^{**} = p_{b,Bi}^{**} = \frac{4t_b t_s - \alpha_b (\alpha_b + 3\alpha_s)}{4t_s}, \tag{A22}$$

and

$$p_{s,Ai}^{**} = p_{s,Bi}^{**} = \frac{1}{4}(\alpha_b - \alpha_s). \tag{A23}$$

The equilibrium number of buyers and equilibrium number of sellers are

$$n_{b,Ai}^{**} = n_{b,Bi}^{**} = \frac{1}{2}, \tag{A24}$$

and

$$n_{s,Ai}^{**} = n_{s,Bi}^{**} = \frac{\alpha_b + \alpha_s}{4t_s}. \tag{A25}$$

Equilibrium profits are

$$\Pi_{Ai}^{*} = \Pi_{Bi}^{*} = \frac{8t_b t_s - \alpha_b^2 - \alpha_s^2 - 6\alpha_b\alpha_s}{16t_s}. \tag{A26}$$

Equilibrium buyer surplus is

$$BS^{**} = 2v + \left(\frac{-5t_b t_s + \alpha_b^2 + \alpha_s^2 + 4\alpha_b\alpha_s}{2t_s}\right), \tag{A27}$$

and equilibrium seller surplus is

$$SS^{**} = \frac{(\alpha_b + \alpha_s)^2}{8t_s}. \tag{A28}$$

Equilibrium total surplus is

$$W^{**} \equiv BS^{**} + SS^{**} = 2v + \frac{-4t_b t_s + 3(\alpha_b + \alpha_s)^2}{8t_s}. \tag{A29}$$

A.2. Equilibria with unilateral pricing when buyers have free access

We here consider the setting with unilateral pricing where buyers have free access and only sellers pay an access price. We proceed as in A.1 yet we here impose the price charged to buyers to be zero ($p_{b,Ji}$, $i = 1, 2$, $J = A, B$). We derive the equilibrium outcomes with MMC (A.2.1), then without MMC (A.2.2).

A.2.1. MMC equilibrium

In the MMC case, firms A and B compete in markets 1 and 2. In this case, Ai and Ai' (with $i = 1, 2$, $i' = 1, 2$ and $i \neq i'$), as well as Bi and Bi', are multimarket platforms, which generates cross-market externalities (i.e., $\gamma_s > 0$ and $\gamma_b > 0$). On the buyers' side, platforms Ai and Bi are respectively located at 0 and 1 in market i. Assuming market i is fully-covered on the buyers' side, and denoting the expected number of buyers and sellers as $n_{s,Ai}^e$, $n_{s,Bi}^e$, $n_{b,Ai}^e$, and $n_{b,Bi}^e$, the marginal buyer who is indifferent between joining platform Ai and platform Bi in market i is located at x_i such that

$$v - t_b x_i + \alpha_s n_{s,Ai}^e + \gamma_s n_{s,Ai'}^e = v - t_b (1 - x_i) + \alpha_s n_{s,Bi}^e + \gamma_s n_{s,Bi'}^e. \tag{A30}$$

As buyers are uniformly distributed on the interval [0,1] and singlehome, x_i is the number of buyers joining platform Ai and $(1 - x_i)$ is the number of buyers joining platform Bi.

To find the number of sellers on each platform, we have to determine the marginal seller on each platform in market i. The type y_{Ai} (resp. y_{Bi}) of the seller who is indifferent to join platform Ai (resp. Bi) or not is given by the zero profit condition so that

$$\alpha_b n_{b,Ai}^e + \gamma_b n_{b,Ai'}^e - t_s y_{Ai} - p_{s,Ai} = 0, \tag{A31}$$

and

$$\alpha_b n_{b,Bi}^e + \gamma_b n_{b,Bi'}^e - t_s y_{Bi} - p_{s,Bi} = 0. \tag{A32}$$

As sellers' platform specific costs are uniformly distributed over [0,1], the marginal seller y_{Ai} (resp. y_{Bi}) corresponds to the number of active sellers on platform Ai (resp. the number of active sellers on platform Bi). From equations (A30-32), we can derive the number of buyers and sellers on platforms Ai and Bi:

$$n_{b,Ai} = \frac{t_b + \alpha_s \left(n^e_{s,Ai} - n^e_{s,Bi}\right) + \gamma_b \left(n^e_{s,Ai'} - n^e_{s,Bi'}\right)}{2t_b}, \tag{A33}$$

$$n_{b,Bi} = 1 - \frac{t_b + \alpha_s \left(n^e_{s,Ai} - n^e_{s,Bi}\right) + \gamma_b \left(n^e_{s,Ai'} - n^e_{s,Bi'}\right)}{2t_b}, \tag{A34}$$

$$n_{s,Ai} = \frac{\alpha_b n^e_{b,Ai} + \gamma_b n^e_{b,Ai'} - p_{s,Ai}}{t_s}, \tag{A35}$$

$$n_{s,Bi} = \frac{\alpha_b n^e_{b,Bi} + \gamma_b n^e_{b,Bi'} - p_{s,Bi}}{t_s}. \tag{A36}$$

We assume that users' expectations are fulfilled at equilibrium. Formally, we have $n^e_{s,Ai} = n_{s,Ai}$, $n^e_{s,Bi} = n_{s,Bi}$, $n^e_{b,Ai} = n_{b,Ai}$ and $n^e_{b,Bi} = n_{b,Bi}$. Solving the four-equation system given by (A33), (A34), (A35) and (A36), the number of buyers and sellers is

$$n_{b,Ai} = \frac{\begin{array}{c}(\alpha_b\alpha_s - t_b t_s)^2 - (\alpha_b\gamma_s)^2 - (\alpha_s\gamma_b)^2 - \gamma_b\gamma_s\left(2t_b t_s - \gamma_b\gamma_s\right) \\ + \left(\alpha_b\left(\alpha_s - \gamma_s\right)\left(\alpha_s + \gamma_s\right) - t_b t_s \alpha_s\right)\left(p_{s,Ai} - p_{s,Bi}\right) \\ - \left(\gamma_b\left(\alpha_s - \gamma_s\right)\left(\alpha_s + \gamma_s\right) + t_b t_s \gamma_s\right)\left(p_{s,Ai'} - p_{s,Bi'}\right)\end{array}}{2\left(t_b t_s - \left(\alpha_b - \gamma_b\right)\left(\alpha_s - \gamma_s\right)\right)\left(t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)}, \tag{A37}$$

$$n_{b,Bi} = 1 - \frac{\begin{array}{c}(\alpha_b\alpha_s - t_b t_s)^2 - (\alpha_b\gamma_s)^2 - (\alpha_s\gamma_b)^2 - \gamma_b\gamma_s\left(2t_b t_s - \gamma_b\gamma_s\right) \\ + \left(\alpha_b\left(\alpha_s - \gamma_s\right)\left(\alpha_s + \gamma_s\right) - t_b t_s \alpha_s\right)\left(p_{s,Ai} - p_{s,Bi}\right) \\ - \left(\gamma_b\left(\alpha_s - \gamma_s\right)\left(\alpha_s + \gamma_s\right) + t_b t_s \gamma_s\right)\left(p_{s,Ai'} - p_{s,Bi'}\right)\end{array}}{2\left(t_b t_s - \left(\alpha_b - \gamma_b\right)\left(\alpha_s - \gamma_s\right)\right)\left(t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)}, \tag{A38}$$

$$n_{s,Ai} = \frac{1}{4}\left(\begin{array}{c}\frac{2\left(-p_{s,Ai} - p_{s,Bi} + \alpha_b + \gamma_b\right)}{t_s} \\ + \frac{\left(-p_{s,Ai} + p_{s,Ai'} + p_{s,Bi} - p_{s,Bi'}\right)t_b}{t_b t_s - (\alpha_b - \gamma_b)(\alpha_s - \gamma_s)} \\ + \frac{\left(-p_{s,Ai} - p_{s,Ai'} + p_{s,Bi} + p_{s,Bi'}\right)t_b}{t_b t_s - (\alpha_b + \gamma_b)(\alpha_s + \gamma_s)}\end{array}\right), \tag{A39}$$

$$n_{s,Ai} = \frac{1}{4}\left(\begin{array}{c}\frac{2\left(-p_{s,Ai} - p_{s,Bi} + \alpha_b + \gamma_b\right)}{t_s} \\ + \frac{\left(p_{s,Ai} - p_{s,Ai'} - p_{s,Bi} + p_{s,Bi'}\right)t_b}{t_b t_s - (\alpha_b - \gamma_b)(\alpha_s - \gamma_s)} \\ + \frac{\left(p_{s,Ai} + p_{s,Ai'} - p_{s,Bi} - p_{s,Bi'}\right)t_b}{t_b t_s - (\alpha_b + \gamma_b)(\alpha_s + \gamma_s)}\end{array}\right). \tag{A40}$$

Solving backwards, we can replace $n_{s,Ai}$ (resp. $n_{s,Bi}$) according to (A8) and (A10) (resp. (A9) and (A11)) into the objective function of platform Ai (resp. Bi) in market i:

$$\Pi_{Ai} = n_{s,Ai} p_{s,Ai}, \tag{A41}$$

and

$$\Pi_{Bi} = n_{s,Bi} p_{s,Bi}. \tag{A42}$$

Multimarket firms A and B choose their access prices to maximize their total profits $\Pi_A = \Pi_{Ai} + \Pi_{Ai'}$ and $\Pi_B = \Pi_{Bi} + \Pi_{Bi'}$. The first-order conditions form a 2×2 system. We solve this system to define the Nash equilibrium of this game. Equilibrium prices are

$$\overline{p}^*_{s,Ai} = \overline{p}^*_{s,Bi} = \frac{\left(\alpha_b + \gamma_b\right)\left(t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)}{4t_b t_s - 3\left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)}. \tag{A43}$$

The equilibrium number of buyers and equilibrium number of sellers are

$$\overline{n}^*_{b,Ai} = \overline{n}^*_{b,Bi} = \frac{1}{2}, \tag{A44}$$

and

$$\overline{n}^*_{s,Ai} = \overline{n}^*_{s,Bi} = \frac{\left(\alpha_b + \gamma_b\right)\left(2t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)}{2t_s\left(4t_b t_s - 3\left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)}. \tag{A45}$$

Equilibrium profits are

$$\overline{\Pi}^*_A = \overline{\Pi}^*_B = \frac{\left(\alpha_b + \gamma_b\right)^2\left(2t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)\left(t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)}{t_s\left(4t_b t_s - 3\left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)^2}. \tag{A46}$$

If Assumption 4 holds, both second-order conditions are satisfied and equilibrium prices and profits are positive. An interior profit-maximizing solution hence exists.

Equilibrium buyer (respectively seller) surplus in markets 1 and 2 is denoted by $\overline{BS}^*$ (respectively $\overline{SS}^*$), and is given by

$$\overline{BS}^* = 2v + \left(\frac{-4\left(t_b t_s\right)^2 + 7 t_b t_s \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right) - 2\left(\alpha_b + \gamma_b\right)^2 \left(\alpha_s + \gamma_s\right)^2}{2 t_s \left(4 t_b t_s - 3\left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)}, \right) \tag{A47}$$

and

$$\overline{SS}^* = \frac{\left(\alpha_b + \gamma_b\right)^2 \left(2 t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)^2}{2 t_s \left(4 t_b t_s - 3\left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)^2}. \tag{A48}$$

Equilibrium total surplus in markets 1 and 2 is equal to

$$\overline{W}^* \equiv \overline{BS}^* + \overline{SS}^* = 2v + \frac{\begin{pmatrix} -16\left(t_b t_s\right)^3 + \left(\alpha_b + \gamma_b\right)^3 \left(\alpha_s + \gamma_s\right)^2 \left(5\left(\alpha_b + \gamma_b\right) + 6\left(\alpha_s + \gamma_s\right)\right) \\ +4\left(t_b t_s\right)^2 \left(\alpha_b + \gamma_b\right)\left(3\left(\alpha_b + \gamma_b\right) + 10\left(\alpha_s + \gamma_s\right)\right) \\ -\left(t_b t_s\right)\left(\alpha_b + \gamma_b\right)^2 \left(\alpha_s + \gamma_s\right)\left(16\left(\alpha_b + \gamma_b\right) + 29\left(\alpha_s + \gamma_s\right)\right) \end{pmatrix}}{2 t_s \left(4 t_b t_s - 3\left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)^2}. \tag{A49}$$

A.2.2. No MMC equilibrium

When platforms Ai and Bi only charge their sellers access prices, the no MMC equilibrium is obtained from (A43), (A44), (A45), (A46), (A47), (A48), and (A49), by setting $\gamma_s = 0$ and $\gamma_b = 0$. Specifically, the equilibrium prices here are

$$\overline{p}^{**}_{s,Ai} = \overline{p}^{**}_{s,Bi} = \frac{\alpha_b \left(t_b t_s - \alpha_b \alpha_s\right)}{4 t_b t_s - 3\alpha_b \alpha_s}. \tag{A50}$$

The equilibrium number of buyers and equilibrium number of sellers is

$$\overline{n}^{**}_{b,Ai} = \overline{n}^{**}_{b,Bi} = \frac{1}{2}, \tag{A51}$$

and

$$\overline{n}^{**}_{s,Ai} = \overline{n}^{**}_{s,Bi} = \frac{\alpha_b \left(2 t_b t_s - \alpha_b \alpha_s\right)}{2 t_s \left(4 t_b t_s - 3\alpha_b \alpha_s\right)}. \tag{A52}$$

Equilibrium profits are

$$\overline{\Pi}^{**}_{Ai} = \overline{\Pi}^{**}_{Bi} = \frac{\alpha_b^2 \left(2 t_b t_s - \alpha_b \alpha_s\right)\left(t_b t_s - \alpha_b \alpha_s\right)}{2 t_s \left(4 t_b t_s - 3\alpha_b \alpha_s\right)^2}. \tag{A53}$$

Equilibrium buyer surplus is

$$\overline{BS}^{**} = 2v + \left(\frac{-4\left(t_b t_s\right)^2 + 7 t_b t_s \alpha_b \alpha_s - 2\left(\alpha_b \alpha_s\right)^2}{2 t_s \left(4 t_b t_s - 3\alpha_b \alpha_s\right)} \right), \tag{A54}$$

and equilibrium seller surplus is

$$\overline{SS}^{**} = \frac{\alpha_b^2 \left(2 t_b t_s - \alpha_b \alpha_s\right)^2}{2 t_s \left(4 t_b t_s - 3\alpha_b \alpha_s\right)^2}. \tag{A55}$$

Equilibrium total surplus is

$$\overline{W}^{**} \equiv \overline{BS}^{**} + \overline{SS}^{**} = 2v + \frac{\begin{pmatrix} -16\left(t_b t_s\right)^3 + \alpha_b^3 \alpha_s^2 \left(5\alpha_b + 6\alpha_s\right) \\ +4\left(t_b t_s\right)^2 \alpha_b \left(3\alpha_b + 10\alpha_s\right) \\ -\left(t_b t_s\right)\left(\alpha_b\right)^2 \alpha_s \left(16\alpha_b + 29\alpha_s\right) \end{pmatrix}}{2 t_s \left(4 t_b t_s - 3\alpha_b \alpha_s\right)^2}. \tag{A56}$$

A.3. Equilibria with unilateral pricing when sellers have free access

We here consider the setting with unilateral pricing where sellers have free access and only buyers pay an access price. We proceed as in A.1 yet we here impose that the price charged to sellers is zero ($p_{s,Ji}$, $i = 1, 2$, $J = A, B$). We first derive the equilibrium outcome with MMC (A.3.1), then that without MMC (A.3.2). Precisely, as in A.2, we assume that $t_b t_s > \alpha_b \alpha_s + \alpha_b \gamma_s + \alpha_s \alpha_b + \gamma_b \gamma_s$ with MMC, and $t_b t_s > \alpha_b \alpha_s$ without MMC.

A.3.1. MMC equilibrium

In the MMC case, firms A and B compete in markets 1 and 2. In this case, Ai and Ai' (with $i = 1, 2$, $i' = 1, 2$ and $i \neq i'$), as well as Bi and Bi', are multimarket platforms, which generates cross-market externalities (i.e., $\gamma_s > 0$ and $\gamma_b > 0$). On the buyers' side, platforms Ai and Bi are respectively located at 0 and 1 in market $i = 1, 2$. Assuming market i is fully-covered on the buyers' side, and denoting the expected number of buyers and sellers as $n^e_{s,Ai}$, $n^e_{s,Bi}$, $n^e_{b,Ai}$, and $n^e_{b,Bi}$, the marginal buyer who is indifferent between joining platform Ai and platform Bi in market i is located at x_i such that

$$v - t_b x_i + \alpha_s n^e_{s,Ai} + \gamma_s n^e_{s,Ai'} - p_{b,Ai} = v - t_b \left(1 - x_i\right) + \alpha_s n^e_{s,Bi} + \gamma_s n^e_{s,Bi'} - p_{b,Bi}. \tag{A57}$$

As buyers are uniformly distributed on the interval [0,1] and singlehome, x_i is the number of buyers joining platform Ai and $(1 - x_i)$ is the number of buyers joining platform Bi.

To find the number of sellers on each platform, we have to determine the marginal seller on each platform in market $i = 1$. The type y_{Ai} (resp. y_{Bi}) of the seller who is indifferent to join platform Ai (resp. Bi) or not is given by the zero profit condition so that

$$\alpha_b n^e_{b,Ai} + \gamma_b n^e_{b,Ai'} - t_s y_{Ai} = 0, \tag{A58}$$

and

$$\alpha_b n^e_{b,Bi} + \gamma_b n^e_{b,Bi'} - t_s y_{Bi} = 0. \tag{A59}$$

As sellers' platform specific costs are uniformly distributed over [0,1], the marginal seller y_{Ai} (resp. y_{Bi}) corresponds to the number of active sellers on platform Ai (resp. platform Bi). From these equations, we can derive the number of buyers and sellers on platforms Ai and Bi:

$$n_{b,Ai} = \frac{t_b + \alpha_s \left(n^e_{s,Ai} - n^e_{s,Bi}\right) + \gamma_b \left(n^e_{s,Ai'} - n^e_{s,Bi'}\right) + \left(p_{b,Bi} - p_{b,Ai}\right)}{2t_b}, \tag{A60}$$

$$n_{b,Bi} = 1 - \frac{t_b + \alpha_s \left(n^e_{s,Ai} - n^e_{s,Bi}\right) + \gamma_b \left(n^e_{s,Ai'} - n^e_{s,Bi'}\right) + \left(p_{b,Bi} - p_{b,Ai}\right)}{2t_b}, \tag{A61}$$

$$n_{s,Ai} = \frac{\alpha_b n^e_{b,Ai} + \gamma_b n^e_{b,Ai'}}{t_s}, \tag{A62}$$

$$n_{s,Bi} = \frac{\alpha_b n^e_{b,Bi} + \gamma_b n^e_{b,Bi'}}{t_s}. \tag{A63}$$

We assume that users' expectations are fulfilled at equilibrium. Formally, we have $n^e_{s,Ai} = n_{s,Ai}$, $n^e_{s,Bi} = n_{s,Bi}$, $n^e_{b,Ai} = n_{b,Ai}$ and $n^e_{b,Bi} = n_{b,Bi}$. Solving the four-equation system given by (A60), (A61), (A62) and (A63), the number of buyers and sellers is

$$n_{b,Ai} = \frac{1}{2} + \frac{t_s}{4}\left(\frac{-p_{b,Ai} + p_{b,Ai'} + p_{b,Bi} - p_{b,Bi'}}{t_b t_s - \left(\alpha_b - \gamma_b\right)\left(\alpha_s - \gamma_s\right)} + \frac{-p_{b,Ai} - p_{b,Ai'} + p_{b,Bi} + p_{b,Bi'}}{t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)}\right), \tag{A64}$$

$$n_{b,Bi} = \frac{1}{2} - \frac{t_s}{4}\left(\frac{-p_{b,Ai} + p_{b,Ai'} + p_{b,Bi} - p_{b,Bi'}}{t_b t_s - \left(\alpha_b - \gamma_b\right)\left(\alpha_s - \gamma_s\right)} + \frac{-p_{b,Ai} - p_{b,Ai'} + p_{b,Bi} + p_{b,Bi'}}{t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)}\right), \tag{A65}$$

$$n_{s,Ai} = \frac{\alpha_b + \gamma_b}{2t_s} - \frac{1}{4}\left(\begin{array}{c}\frac{\left(p_{b,Ai} - p_{b,Ai'} - p_{b,Bi} + p_{b,Bi'}\right)\left(\alpha_b - \gamma_b\right)}{t_b t_s - \left(\alpha_b - \gamma_b\right)\left(\alpha_s - \gamma_s\right)} \\ + \frac{\left(p_{b,Ai} + p_{b,Ai'} - p_{b,Bi} - p_{b,Bi'}\right)\left(\alpha_b + \gamma_b\right)}{t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)},\end{array}\right) \tag{A66}$$

and

$$n_{s,Bi} = \frac{\alpha_b + \gamma_b}{2t_s} + \frac{1}{4}\left(\begin{array}{c}\frac{\left(p_{b,Ai} - p_{b,Ai'} - p_{b,Bi} + p_{b,Bi'}\right)\left(\alpha_b - \gamma_b\right)}{t_b t_s - \left(\alpha_b - \gamma_b\right)\left(\alpha_s - \gamma_s\right)} \\ + \frac{\left(p_{b,Ai} + p_{b,Ai'} - p_{b,Bi} - p_{b,Bi'}\right)\left(\alpha_b + \gamma_b\right)}{t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)}.\end{array}\right) \tag{A67}$$

Solving backwards, we can replace $n_{s,Ai}$ (resp. $n_{s,Bi}$) according to (A8) and (A10) (resp. (A9) and (A11)) into the objective function of platform Ai (resp. Bi) in market i:

$$\Pi_{Ai}\left(p_{b,Ai}, p_{b,Bi}, p_{b,Ai'}, p_{b,Bi'}\right) = n_{b,Ai} p_{b,Ai}, \tag{A68}$$

and

$$\Pi_{Bi}\left(p_{b,Ai}, p_{b,Bi}, p_{b,Ai'}, p_{b,Bi'}\right) = n_{b,Bi} p_{b,Bi}. \tag{A69}$$

Multimarket firms A and B choose their access prices to maximize their total profits $\Pi_A = \Pi_{Ai} + \Pi_{Ai'}$ and $\Pi_B = \Pi_{Bi} + \Pi_{Bi'}$. The first-order conditions form a 2×2 system. We solve this system to define the Nash equilibrium of this game. Equilibrium prices are

$$\widehat{p}^*_{b,Ai} = p^*_{b,Bi} = \frac{t_b t_s - (\alpha_b + \gamma_b)(\alpha_s + \gamma_s)}{t_s}. \tag{A70}$$

The equilibrium number of buyers and equilibrium number of sellers are

$$\widehat{n}^*_{b,Ai} = \widehat{n}^*_{b,Bi} = \frac{1}{2}, \tag{A71}$$

and

$$\widehat{n}^*_{s,Ai} = \widehat{n}^*_{s,Bi} = \frac{\alpha_b + \gamma_b}{2t_s}. \tag{A72}$$

Equilibrium profits are

$$\widehat{\Pi}^*_A = \widehat{\Pi}^*_B = \frac{t_b t_s - (\alpha_b + \gamma_b)(\alpha_s + \gamma_s)}{t_s}. \tag{A73}$$

Assumptions 3 and 4 guarantee the existence of the competitive equilibrium outcome, as both second-order conditions are satisfied and equilibrium prices and profits are positive. An interior profit-maximizing solution hence exists.

Equilibrium buyer (resp. seller) surplus in markets 1 and 2 is denoted by $\widehat{BS}^*$ (resp. $\widehat{SS}^*$), and is given by

$$\widehat{BS}^* = 2v + \left(\frac{-5t_b t_s + 6(\alpha_b + \gamma_b)(\alpha_s + \gamma_s)}{2t_s}\right), \tag{A74}$$

and

$$\widehat{SS}^* = \frac{(\alpha_b + \gamma_b)^2}{2t_s}. \tag{A75}$$

Equilibrium total surplus in markets 1 and 2 is

$$\widehat{W}^* \equiv \widehat{BS}^* + \widehat{SS}^* = 2v + \frac{-t_b t_s + (\alpha_b + \gamma_b)((\alpha_b + \gamma_b) + 2(\alpha_s + \gamma_s))}{2t_s}. \tag{A76}$$

A.3.2. No MMC equilibrium

When platforms Ai and Bi only charge their buyers access prices, the no MMC equilibrium is obtained from (A70), (A71), (A72), (A73), (A74), (A75), and (A76), by setting $\gamma_s = 0$ and $\gamma_b = 0$. Specifically, the equilibrium prices here are

$$\widehat{p}^{**}_{b,Ai} = \widehat{p}^{**}_{b,Bi} = \frac{t_b t_s - \alpha_b \alpha_s}{t_s}. \tag{A77}$$

The equilibrium number of buyers and equilibrium number of sellers are

$$\widehat{n}^{**}_{b,Ai} = \widehat{n}^{**}_{b,Bi} = \frac{1}{2}, \tag{A78}$$

and

$$\widehat{n}^{**}_{s,Ai} = \widehat{n}^{**}_{s,Bi} = \frac{\alpha_b}{2t_s}. \tag{A79}$$

Equilibrium profits are

$$\widehat{\Pi}^{**}_{Ai} = \widehat{\Pi}^{**}_{Bi} = \frac{t_b t_s - \alpha_b \alpha_s}{2t_s}. \tag{A80}$$

Equilibrium buyer surplus and equilibrium seller surplus are

$$\widehat{BS}^{**} = 2v + \left(\frac{-5t_b t_s + 6\alpha_b \alpha_s}{2t_s}\right), \tag{A81}$$

and

$$\widehat{SS}^{**} = \frac{\alpha_b^2}{2t_s}. \tag{A82}$$

Equilibrium total surplus is

$$\widehat{W}^{**} \equiv \widehat{BS}^{**} + \widehat{SS}^{**} = 2v + \frac{-t_b t_s + \alpha_b(\alpha_b + 2\alpha_s)}{2t_s}. \tag{A83}$$

Appendix B. Proofs of Propositions

B.1. Proof of Proposition 1

Consider the equilibrium outcomes with MMC and with no MMC when platforms Ji ($J = A, B$ and $i = 1, 2$) compete with bilateral pricing. These two equilibrium outcomes exist under Assumptions 1 and 2.

The difference between the equilibrium price paid by the buyers in the MMC case and that in the no MMC case is given by

$$\begin{aligned} \Delta p_{b,Ji} &\equiv p^{*}_{b,Ji} - p^{**}_{b,Ji} \\ &= \frac{4t_b t_s - (\alpha_b + \gamma_b)(\alpha_b + 3\alpha_s + \gamma_b + 3\gamma_s)}{4t_s} - \frac{4t_b t_s - \alpha_b(\alpha_b + 3\alpha_s)}{4t_s} \\ &= -\frac{\gamma_b(2\alpha_b + \gamma_b + 3\alpha_s) + 3\gamma_s(\alpha_b + \gamma_b)}{4t_s} < 0. \end{aligned} \tag{B1}$$

We thus have $\Delta p_{b,Ji} < 0$ and buyers pay lower equilibrium access price with MMC than without MMC when the platforms charge both buyers and sellers.

Similarly, the difference between the equilibrium price paid by the sellers in the MMC case and that in the no MMC case is

$$\begin{aligned} \Delta p_{s,Ji} &\equiv p^{*}_{s,Ji} - p^{**}_{s,Ji} \\ &= \frac{1}{4}(\alpha_b - \alpha_s + \gamma_b - \gamma_s) - \frac{1}{4}(\alpha_b - \alpha_s) \\ &= \frac{1}{4}(\gamma_b - \gamma_s). \end{aligned} \tag{B2}$$

Since $\Delta p_{s,Ji} > 0$ if $\gamma_b > \gamma_s$, sellers pay lower equilibrium access price only if $\gamma_b > \gamma_s$.

B.2. Proof of Proposition 2

Consider the equilibrium outcomes with MMC and with no MMC when platforms Ji ($J = A, B$ and $i = 1, 2$) compete with bilateral pricing. These two equilibrium outcomes exist under Assumptions 1 and 2.

The difference between the equilibrium number of sellers in the MMC case and that in the no MMC case is given by

$$\begin{aligned} \Delta n_{s,Ji} &\equiv n^{*}_{s,Ji} - n^{**}_{s,Ji} \\ &= \left(\frac{\alpha_b + \alpha_s + \gamma_b + \gamma_s}{4t_s}\right) - \left(\frac{\alpha_b + \alpha_s}{4t_s}\right) \\ &= \frac{\gamma_b + \gamma_s}{4t_s} > 0. \end{aligned} \tag{B3}$$

Thus, in market i, platform Ji attracts more sellers in equilibrium with MMC than without MMC when buyers and sellers are charged access prices.

Similarly, the difference between the equilibrium profit of platforms Ji in the MMC case and that in the no MMC case is

$$\begin{aligned} \Delta\Pi_{Ji} &\equiv \Pi^{*}_{Ji} - \Pi^{**}_{Ji} \\ &= \left(\frac{8t_b t_s - \alpha_b^2 - \alpha_s^2 - 6\alpha_s\gamma_b - \gamma_b^2 - 2(\alpha_s + 3\gamma_b)\gamma_s - \gamma_s^2 - 2\alpha_b(3\alpha_s + \gamma_b + 3\gamma_s)}{16t_s}\right) - \left(\frac{8t_b t_s - \alpha_b^2 - 6\alpha_b\alpha_s - \alpha_s^2}{16t_s}\right) \\ &= -\frac{2\gamma_s(3\alpha_b + 3\gamma_b + \alpha_s) + \gamma_b(2\alpha_b + \gamma_b + 6\alpha_s) + \gamma_s^2}{16t_s} < 0. \end{aligned} \tag{B4}$$

Thus, platform Ji achieves higher profit in equilibrium without MMC than with MMC when buyers and sellers are charged access prices.

B.3. Proof of Proposition 3

Consider the equilibrium outcomes with MMC and with no MMC when platforms Ji ($J = A, B$ and $i = 1, 2$) compete with bilateral pricing. These two equilibrium outcomes exist under Assumptions 1 and 2.

The difference between the equilibrium buyer surplus in the MMC case and the no MMC case is given by

$$\begin{aligned}
\Delta BS &\equiv BS^{*} - BS^{**} \\
&= \left(2v + \left(\frac{-5t_b t_s + \alpha_b^2 + \alpha_s^2 + 4\alpha_s\gamma_b + \gamma_b^2 + 2\left(\alpha_s + 2\gamma_b\right)\gamma_s + \gamma_s^2 + 2\alpha_b\left(2\alpha_s + \gamma_b + 2\gamma_s\right)}{2t_s}\right)\right) \\
&- \left(2v + \left(\frac{-5t_b t_s + \alpha_b^2 + \alpha_s^2 + 4\alpha_b\alpha_s}{2t_s}\right)\right) \\
&= \frac{\gamma_b^2 + 2\alpha_s\left(2\gamma_b + \gamma_s\right) + 2\alpha_b\left(\gamma_b + 2\gamma_s\right) + 4\gamma_b\gamma_s + \gamma_s^2}{2t_s} > 0.
\end{aligned} \tag{B5}$$

Similarly, the difference between the equilibrium seller surplus in the MMC case and that in the no MMC case is

$$\begin{aligned}
\Delta\overline{SS} &\equiv \overline{SS}^{*} - SS^{*} \\
&= \frac{\left(\alpha_b + \gamma_b + \alpha_s + \gamma_s\right)^2}{8t_s} - \frac{\left(\alpha_b + \alpha_s\right)^2}{8t_s} \\
&= \frac{\left(\gamma_b + \gamma_s\right)\left(2\alpha_b + \gamma_b + 2\alpha_s + \gamma_s\right)}{8t_s} > 0.
\end{aligned} \tag{B6}$$

Finally, the difference between total surplus in the MMC case and the no MMC case is

$$\begin{aligned}
\Delta W &\equiv W^{**} - W^{*} \\
&= \left(2v + \frac{-4t_b t_s + 3\left(\alpha_b + \alpha_s + \gamma_b + \gamma_s\right)^2}{8t_s}\right) - \left(2v + \frac{-4t_b t_s + 3\left(\alpha_b + \alpha_s\right)^2}{8t_s}\right) \\
&= \frac{3\left(\gamma_b + \gamma_s\right)\left(2\alpha_b + \gamma_b + 2\alpha_s + \gamma_s\right)}{8t_s} > 0.
\end{aligned} \tag{B7}$$

Hence, equilibrium buyer surplus, equilibrium seller surplus, and total surplus are larger with MMC than without MMC when buyers and sellers are charged access prices.

B.4. Proof of Proposition 4

Consider the equilibrium outcomes with MMC and with no MMC when platforms Ji ($J = A, B$ and $i = 1, 2$) compete with unilateral pricing by providing free access to buyers. These two equilibrium outcomes exist under Assumptions 3 and 4.

The difference between the equilibrium price paid by the sellers in the MMC case and that in the no MMC case is given by

$$\begin{aligned}
\Delta\overline{p}_{s,Ji} &\equiv \overline{p}^{*}_{s,Ji} - \overline{p}^{**}_{s,Ji} \\
&= \left(\frac{\left(\alpha_b + \gamma_b\right)\left(t_b t_s - \left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)\right)}{4t_b t_s - 3\left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right)}\right) - \left(\frac{\alpha_b\left(t_b t_s - \alpha_b\alpha_s\right)}{4t_b t_s - 3\alpha_b\alpha_s}\right) \\
&= \frac{3\alpha_b\gamma_b\alpha_s\left(\alpha_b + \gamma_b\right)\left(\alpha_s + \gamma_s\right) - t_b t_s\left(\alpha_b^2\gamma_s + \alpha_b\gamma_b\left(8\alpha_s + 5\gamma_s\right) + 4\gamma_b^2\left(\alpha_s + \gamma_s\right)\right) + 4\gamma_b t_b^2 t_s^2}{\left(3\alpha_b\alpha_s - 4t_b t_s\right)\left(3\gamma_b\alpha_s + 3\alpha_b\gamma_s + 3\alpha_b\alpha_s + 3\gamma_b\gamma_s - 4t_b t_s\right)}.
\end{aligned} \tag{B8}$$

Under Assumptions 3 and 4, the denominator of $\Delta\overline{p}_{s,Ji}$ is positive. Its numerator is positive only if

$$0 < \alpha_s < \frac{2t_b t_s}{3\alpha_b} \equiv \tilde{\alpha}_s^{(1)}, 0 < \gamma_b < \frac{8\alpha_b t_b\alpha_s t_s - 3\alpha_b^2\alpha_s^2 - 4t_b^2 t_s^2}{3\alpha_b\alpha_s^2 - 4t_b\alpha_s t_s} \equiv \tilde{\gamma}_b^{(1)}$$

and

$$0 < \gamma_s < \frac{4\gamma_b^2 t_b\alpha_s t_s - 3\alpha_b\gamma_b^2\alpha_s^2 - 3\alpha_b^2\gamma_b\alpha_s^2 + 8\alpha_b\gamma_b t_b\alpha_s t_s - 4\gamma_b t_b^2 t_s^2}{3\alpha_b^2\gamma_b\alpha_s + 3\alpha_b\gamma_b^2\alpha_s - 5\alpha_b\gamma_b t_b t_s + \alpha_b^2\left(-t_b\right)t_s - 4\gamma_b^2 t_b t_s} \equiv \tilde{\gamma}_s^{(1)}, \tag{B9}$$

which completes the proof of Proposition 4.

B.5. Proof of Proposition 5

Consider the equilibrium outcomes with MMC and with no MMC when platforms Ji ($J = A, B$ and $i = 1, 2$) compete with unilateral pricing by providing free access to buyers. These two equilibrium outcomes exist under Assumptions 3 and 4.

The difference between the equilibrium number of sellers in the MMC case and the no MMC case is given by

$$\begin{aligned}
\Delta\overline{n}_{s,Ji} &\equiv \overline{n}^{*}_{s,Ji} - \overline{n}^{**}_{s,Ji} \\
&= \left(\frac{(\alpha_b+\gamma_b)(2t_bt_s-(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))}{2t_s(4t_bt_s-3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))}\right) - \left(\frac{\alpha_b}{6}\left(\frac{1}{t_s}+\frac{2t_b}{4t_bt_s-3\alpha_b\alpha_s}\right)\right) \\
&= \frac{\left(\begin{array}{l} 3\alpha_b\gamma_b^2\alpha_s^2+3\alpha_b^2\gamma_b\alpha_s^2+3\alpha_b\gamma_b^2\alpha_s\gamma_s+3\alpha_b^2\gamma_b\alpha_s\gamma_s-4\gamma_b^2t_b\alpha_st_s \\ -8\alpha_b\gamma_bt_b\alpha_st_s+2\alpha_b^2t_b\gamma_st_s-2\alpha_b\gamma_bt_b\gamma_st_s+8\gamma_bt_b^2t_s^2-4\gamma_b^2t_b\gamma_st_s \end{array}\right)}{2t_s(3\alpha_b\alpha_s-4t_bt_s)(3\gamma_b\alpha_s+3\alpha_b\gamma_s+3\alpha_b\alpha_s+3\gamma_b\gamma_s-4t_bt_s)} \\
&= \frac{3\alpha_b\gamma_b\alpha_s(\alpha_b+\gamma_b)(\alpha_s+\gamma_s)+2t_bt_s\left(\alpha_b^2\gamma_s-\alpha_b\gamma_b(4\alpha_s+\gamma_s)-2\gamma_b^2(\alpha_s+\gamma_s)\right)+8\gamma_bt_b^2t_s^2}{2t_s(3\alpha_b\alpha_s-4t_bt_s)(3\gamma_b\alpha_s+3\alpha_b\gamma_s+3\alpha_b\alpha_s+3\gamma_b\gamma_s-4t_bt_s)}.
\end{aligned} \tag{B10}$$

Under Assumptions 3 and 4, the denominator of $\Delta\overline{n}_{s,Ji}$ is positive. The sign of $\Delta\overline{n}_{s,Ji}$ is thus given by the sign of its numerator. Let us introduce

$$f_1 = 3\alpha_b\gamma_b\alpha_s(\alpha_b+\gamma_b)(\alpha_s+\gamma_s)+2t_bt_s\left(\alpha_b^2\gamma_s-\alpha_b\gamma_b(4\alpha_s+\gamma_s)-2\gamma_b^2(\alpha_s+\gamma_s)\right)+8\gamma_bt_b^2t_s^2. \tag{B11}$$

The derivative of f_1 with respect to γ_b is

$$\frac{\partial f_1}{\partial\gamma_b} = 3\alpha_b^2\alpha_s\gamma_s+6\alpha_b\gamma_b\alpha_s^2+6\alpha_b\gamma_b\alpha_s\gamma_s+3\alpha_b^2\alpha_s^2+t_bt_s\left(6\alpha_b\gamma_s+(8t_bt_s-8\gamma_b\alpha_s-8\alpha_b\gamma_s-8\alpha_b\alpha_s-8\gamma_b\gamma_s)\right). \tag{B12}$$

From Assumption 4, we have $8(t_bt_s-\gamma_b\alpha_s-\alpha_b\gamma_s-\alpha_b\alpha_s-\gamma_b\gamma_s) > 0$. So $\frac{\partial f_1}{\partial\gamma_b} > 0$, and f_1 is increasing with γ_b. As a result and given the parameter set of definition ($\alpha_b > 0, \alpha_s > 0, \gamma_b > 0, \gamma_s > 0, t_b > 0$ and $t_s > 0$), we have

$$f_1 > f_1|_{\gamma_b=0} = 2\alpha_b^2t_bt_s\gamma_s > 0. \tag{B13}$$

Consequently, $\Delta\overline{n}_{s,Ji} > 0$, and the platform Ji attracts more sellers in equilibrium with MMC than without MMC when sellers have free access.

The difference between the equilibrium profit of each platform Ji in the MMC case and that in the no MMC case is given by

$$\begin{aligned}
\Delta\overline{\Pi}_{Ji} \equiv& \overline{\Pi}^{*}_{Ji} - \overline{\Pi}^{**}_{Ji} \\
=& \left(\frac{(\alpha_b+\gamma_b)^2(2t_bt_s-(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))(t_bt_s-(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))}{2t_s(4t_bt_s-3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))^2}\right) - \left(\frac{\alpha_b^2(2t_bt_s-\alpha_b\alpha_s)(t_bt_s-\alpha_b\alpha_s)}{2t_s(4t_bt_s-3\alpha_b\alpha_s)^2}\right) \\
=& \frac{\frac{(\alpha_b+\gamma_b)^2((\alpha_b+\gamma_b)(\alpha_s+\gamma_s)-2t_bt_s)((\alpha_b+\gamma_b)(\alpha_s+\gamma_s)-t_bt_s)}{(4t_bt_s-3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))^2} - \frac{\alpha_b^2(\alpha_b\alpha_s-2t_bt_s)(\alpha_b\alpha_s-t_bt_s)}{(4t_bt_s-3\alpha_b\alpha_s)^2}}{2t_s} \\
=& \frac{\left(\begin{array}{l} 9\alpha_b^2\gamma_b\alpha_s^2(\alpha_b+\gamma_b)^2(2\alpha_b+\gamma_b)(\alpha_s+\gamma_s)^2+32\gamma_bt_b^4t_s^4(2\alpha_b+\gamma_b) \\ -48\gamma_bt_b^3t_s^3(2\alpha_b+\gamma_b)(\gamma_b(\alpha_s+\gamma_s)+\alpha_b(2\alpha_s+\gamma_s)) \\ +2t_b^2t_s^2(\alpha_b^4(-\gamma_s)(2\alpha_s+\gamma_s)+2\alpha_b^3\gamma_b\left(50\alpha_s\gamma_s+52\alpha_s^2+7\gamma_s^2\right)+3\alpha_b^2\gamma_b^2\left(62\alpha_s\gamma_s+52\alpha_s^2+13\gamma_s^2\right) \\ +4\alpha_b\gamma_b^3\left(25\alpha_s\gamma_s+17\alpha_s^2+8\gamma_s^2\right)+8\gamma_b^4(\alpha_s+\gamma_s)^2) \\ +3\alpha_bt_b\alpha_st_s(\alpha_b+\gamma_b)(\alpha_b^3\gamma_s(\alpha_s+\gamma_s)-3\alpha_b^2\gamma_b\left(16\alpha_s\gamma_s+11\alpha_s^2+5\gamma_s^2\right) \\ -3\alpha_b\gamma_b^2\left(19\alpha_s\gamma_s+11\alpha_s^2+8\gamma_s^2\right)-8\gamma_b^3(\alpha_s+\gamma_s)^2) \end{array}\right)}{2t_s(4t_bt_s-3\alpha_b\alpha_s)^2(4t_bt_s-3\alpha_b\alpha_s-3\alpha_s\gamma_b-3\alpha_b\gamma_s-3\gamma_b\gamma_s)^2}.
\end{aligned} \tag{B14}$$

The denominator of $\Delta\overline{\Pi}_{Ji}$ is positive. Its sign is given by that of its numerator, which is positive if

$$0 < \alpha_s < \frac{2t_b t_s}{3\alpha_b} \equiv \tilde{\alpha}_s^{(2)},\ 0 < \gamma_b < 3\sqrt{\frac{\alpha_b^4\alpha_s^2 - 3\alpha_b^3 t_b\alpha_s t_s + 2\alpha_b^2 t_b^2 t_s^2}{(3\alpha_b\alpha_s - 4t_b t_s)^2}} - \alpha_b \equiv \tilde{\gamma}_b^{(2)},\ \text{and } 0 < \gamma_s < \tilde{\gamma}_s^{(2)}$$

with $\tilde{\gamma}_s^{(2)} \equiv$

$$\frac{1}{2}\sqrt{\frac{\begin{array}{l} 512t_b^6\alpha_b\gamma_b t_s^6 - 768t_b^5\alpha_b\alpha_s\gamma_b^2 t_s^5 - 1536t_b^5\alpha_b^2\alpha_s\gamma_b t_s^5 + 16t_b^4\alpha_b^4\alpha_s^2 t_s^4 + 864t_b^4\alpha_b^2\alpha_s^2\gamma_b^2 t_s^4 + 1728t_b^4\alpha_b^3\alpha_s^2\gamma_b t_s^4 \\ +256t_b^6\gamma_b^2 t_s^6 - 24t_b^3\alpha_b^5\alpha_s^3 t_s^3 - 432t_b^3\alpha_b^3\alpha_s^3\gamma_b^2 t_s^3 - 864t_b^3\alpha_b^4\alpha_s^3\gamma_b t_s^3 + 9t_b^2\alpha_b^6\alpha_s^4 t_s^2 + 81t_b^2\alpha_b^4\alpha_s^4\gamma_b^2 t_s^2 + 162t_b^2\alpha_b^5\alpha_s^4\gamma_b t_s^2 \end{array}}{(3t_b t_s\alpha_s\alpha_b^3 + 18\alpha_s^2\gamma_b\alpha_b^3 - 2t_b^2 t_s^2\alpha_b^2 + 9\alpha_s^2\gamma_b^2\alpha_b^2 - 48t_b t_s\alpha_s\gamma_b\alpha_b^2 - 24t_b t_s\alpha_s\gamma_b^2\alpha_b + 32t_b^2 t_s^2\gamma_b\alpha_b + 16t_b^2 t_s^2\gamma_b^2)^2}}$$
$$+ \frac{\left(\begin{array}{l} 171t_b t_s\alpha_s^2\gamma_b^2\alpha_b^2 - 3t_b t_s\alpha_s^2\alpha_b^4 - 36\alpha_s^3\gamma_b\alpha_b^4 - 54\alpha_s^3\gamma_b^2\alpha_b^3 + 4t_b^2 t_s^2\alpha_s\alpha_b^3 + 144t_b t_s\alpha_s^2\gamma_b\alpha_b^3 - 18\alpha_s^3\gamma_b^3\alpha_b^2 \\ -204t_b^2 t_s^2\alpha_s\gamma_b\alpha_b^2 + 48t_b t_s\alpha_s^2\gamma_b^3\alpha_b - 168t_b^2 t_s^2\alpha_s\gamma_b^2\alpha_b + 96t_b^3 t_s^3\gamma_b\alpha_b - 32t_b^2 t_s^2\alpha_s\gamma_b^3 + 48t_b^3 t_s^3\gamma_b^2 \end{array}\right)}{2(\alpha_b + \gamma_b)(3t_b t_s\alpha_s\alpha_b^3 + 18\alpha_s^2\gamma_b\alpha_b^3 - 2t_b^2 t_s^2\alpha_b^2 + 9\alpha_s^2\gamma_b^2\alpha_b^2 - 48t_b t_s\alpha_s\gamma_b\alpha_b^2 - 24t_b t_s\alpha_s\gamma_b^2\alpha_b + 32t_b^2 t_s^2\gamma_b\alpha_b + 16t_b^2 t_s^2\gamma_b^2)}, \tag{B15}$$

which demonstrates Proposition 5.

B.6. Proof of Proposition 6

Consider the equilibrium outcomes with MMC and with no MMC when platforms Ji ($J = A, B$ and $i = 1, 2$) compete with unilateral pricing by providing free access to buyers. These two equilibrium outcomes exist under Assumptions 3 and 4.

The difference between the equilibrium buyer surplus in the MMC case and the no MMC case is given by

$$\begin{aligned}\Delta\overline{BS} \equiv& \overline{BS}^* - \overline{BS}^{**} \\ =& 2v + \frac{-2(\alpha_b+\gamma_b)^2(\alpha_s+\gamma_s)^2 + 7t_b t_s(\alpha_b+\gamma_b)(\alpha_s+\gamma_s) - 4t_b^2 t_s^2}{2t_s(4t_b t_s - 3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))} - 2v - \left(\frac{-4(t_b t_s)^2 + 7t_b t_s\alpha_b\alpha_s - 2(\alpha_b\alpha_s)^2}{2t_s(4t_b t_s - 3\alpha_b\alpha_s)}\right) \\ =& \frac{(\gamma_b\alpha_s + \alpha_b\gamma_s + \gamma_b\gamma_s)\left(\begin{array}{l} 3\alpha_b\gamma_b\alpha_s^2 + 3\alpha_b^2\alpha_s\gamma_s + 3\alpha_b\gamma_b\alpha_s\gamma_s + 3\alpha_b^2\alpha_s^2 - 4\gamma_b t_b\alpha_s t_s \\ -4\alpha_b t_b\gamma_s t_s - 8\alpha_b t_b\alpha_s t_s - 4\gamma_b t_b\gamma_s t_s + 8t_b^2 t_s^2 \end{array}\right)}{t_s(4t_b t_s - 3\alpha_b\alpha_s)(-3\gamma_b\alpha_s - 3\alpha_b\gamma_s - 3\alpha_b\alpha_s - 3\gamma_b\gamma_s + 4t_b t_s)}.\end{aligned} \tag{B16}$$

The denominator of the expression above is positive under Assumptions 3 and 4. In addition, since $\alpha_b > 0, \alpha_s > 0, \gamma_b > 0, \gamma_s > 0, t_b > 0$ and $t_s > 0$), it is straightforward to see that the numerator is positive ($\gamma_b\alpha_s + \alpha_b\gamma_s + \gamma_b\gamma_s > 0$). Consequently, the sign of $\Delta\overline{BS}$ is given by that of the term in the second bracket of the numerator, f_2, with

$$\begin{aligned} f_2 =& 3\alpha_b\gamma_b\alpha_s^2 + 3\alpha_b^2\alpha_s\gamma_s + 3\alpha_b\gamma_b\alpha_s\gamma_s + 3\alpha_b^2\alpha_s^2 - 4\gamma_b t_b\alpha_s t_s - 4\alpha_b t_b\gamma_s t_s - 8\alpha_b t_b\alpha_s t_s - 4\gamma_b t_b\gamma_s t_s + 8t_b^2 t_s^2 \\ =& 3\alpha_b^2\alpha_s\gamma_s + 3\alpha_b\gamma_b\alpha_s^2 + 3\alpha_b\gamma_b\alpha_s\gamma_s + 3\alpha_b^2\alpha_s^2 + t_b t_s(-4\gamma_b\alpha_s - 8\alpha_b\alpha_s - 4\gamma_b\gamma_s + 8t_b t_s - 4\alpha_b\gamma_s) \\ =& 3\alpha_b^2\alpha_s\gamma_s + 3\alpha_b\gamma_b\alpha_s^2 + 3\alpha_b\gamma_b\alpha_s\gamma_s + 3\alpha_b^2\alpha_s^2 + t_b t_s(4\gamma_b\alpha_s + 4\gamma_b\gamma_s + (-8\gamma_b\alpha_s - 8\alpha_b\alpha_s - 8\gamma_b\gamma_s + 8t_b t_s - 8\alpha_b\gamma_s) + 4\alpha_b\gamma_s). \end{aligned} \tag{B17}$$

Under Assumption 4, $-8\gamma_b\alpha_s - 8\alpha_b\alpha_s - 8\gamma_b\gamma_s + 8t_b t_s - 8\alpha_b\gamma_s > 0$, so $f_2 > 0$. As a result, $\Delta\overline{BS} > 0$.

The difference between the equilibrium seller surplus in the MMC case and the no MMC case is given by

$$\begin{aligned}\Delta\overline{SS} \equiv& \overline{SS}^* - \overline{SS}^{**} \\ =& \frac{(\alpha_b+\gamma_b)^2((\alpha_b+\gamma_b)(\alpha_s+\gamma_s) - 2t_b t_s)^2}{2t_s(4t_b t_s - 3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))^2} - \frac{\alpha_b^2(\alpha_b\alpha_s - 2t_b t_s)^2}{2t_s(4t_b t_s - 3\alpha_b\alpha_s)^2} \\ =& \frac{\left(\begin{array}{l} 3\alpha_b\gamma_b^2\alpha_s^2 + 3\alpha_b^2\gamma_b\alpha_s^2 + 3\alpha_b\gamma_b^2\alpha_s\gamma_s \\ +3\alpha_b^2\gamma_b\alpha_s\gamma_s - 4\gamma_b^2 t_b\alpha_s t_s - 8\alpha_b\gamma_b t_b\alpha_s t_s \\ +2\alpha_b^2 t_b\gamma_s t_s - 2\alpha_b\gamma_b t_b\gamma_s t_s + 8\gamma_b t_b^2 t_s^2 \\ -4\gamma_b^2 t_b\gamma_s t_s \end{array}\right)\left(\begin{array}{l} 6\alpha_b^3\alpha_s\gamma_s + 9\alpha_b^2\gamma_b\alpha_s^2 + 9\alpha_b^2\gamma_b\alpha_s\gamma_s + 3\alpha_b\gamma_b^2\alpha_s^2 \\ +3\alpha_b\gamma_b^2\alpha_s\gamma_s + 6\alpha_b^3\alpha_s^2 - 10\alpha_b^2 t_b\gamma_s t_s - 20\alpha_b\gamma_b t_b\alpha_s t_s \\ -14\alpha_b\gamma_b t_b\gamma_s t_s - 4\gamma_b^2 t_b\alpha_s t_s - 20\alpha_b^2 t_b\alpha_s t_s + 16\alpha_b t_b^2 t_s^2 \\ +8\gamma_b t_b^2 t_s^2 - 4\gamma_b^2 t_b\gamma_s t_s \end{array}\right)}{2t_s(4t_b t_s - 3\alpha_b\alpha_s)^2(-3\gamma_b\alpha_s - 3\alpha_b\gamma_s - 3\alpha_b\alpha_s - 3\gamma_b\gamma_s + 4t_b t_s)^2} \\ =& \frac{\left(\begin{array}{l} 3\alpha_b\gamma_b\alpha_s(\alpha_b+\gamma_b)(\alpha_s+\gamma_s) + 8\gamma_b t_b^2 t_s^2 \\ 2t_b t_s\left(\alpha_b^2\gamma_s - \alpha_b\gamma_b(4\alpha_s+\gamma_s) - 2\gamma_b^2(\alpha_s+\gamma_s)\right) \end{array}\right)\left(\begin{array}{l} 3\alpha_b\alpha_s\left(3\alpha_b\gamma_b + 2\alpha_b^2 + \gamma_b^2\right)(\alpha_s+\gamma_s) + 8t_b^2 t_s^2(2\alpha_b+\gamma_b) - \\ 2t_b t_s\left(5\alpha_b^2(2\alpha_s+\gamma_s) + \alpha_b\gamma_b(10\alpha_s+7\gamma_s) + 2\gamma_b^2(\alpha_s+\gamma_s)\right) \end{array}\right)}{2t_s(4t_b t_s - 3\alpha_b\alpha_s)^2(-3\gamma_b\alpha_s - 3\alpha_b\gamma_s - 3\alpha_b\alpha_s - 3\gamma_b\gamma_s + 4t_b t_s)^2}.\end{aligned} \tag{B18}$$

Since the denominator of the expression above is positive, its sign depends on that of its numerator. Denote f_3 the term in the first bracket of the numerator, with

$$\begin{aligned} f_3 =& 3\alpha_b\gamma_b\alpha_s\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)+2t_bt_s\left(\alpha_b^2\gamma_s-\alpha_b\gamma_b\left(4\alpha_s+\gamma_s\right)-2\gamma_b^2\left(\alpha_s+\gamma_s\right)\right)+8\gamma_bt_b^2t_s^2 \\ =& 3\alpha_b\gamma_b^2\alpha_s^2+3\alpha_b^2\gamma_b\alpha_s^2+3\alpha_b\gamma_b^2\alpha_s\gamma_s+3\alpha_b^2\gamma_b\alpha_s\gamma_s-4\gamma_b^2t_b\alpha_st_s-8\alpha_b\gamma_bt_b\alpha_st_s+ \\ & 2\alpha_b^2t_b\gamma_st_s-2\alpha_b\gamma_bt_b\gamma_st_s+8\gamma_bt_b^2t_s^2-4\gamma_b^2t_b\gamma_st_s \\ =& 3\alpha_b^2\gamma_b\alpha_s^2+3\alpha_b^2\gamma_b\alpha_s\gamma_s+3\alpha_b\gamma_b^2\alpha_s^2+3\alpha_b\gamma_b^2\alpha_s\gamma_s+2\alpha_b^2t_b\gamma_st_s+\gamma_bt_bt_s\left(-4\gamma_b\alpha_s-2\alpha_b\gamma_s-8\alpha_b\alpha_s-4\gamma_b\gamma_s+8t_bt_s\right) \\ =& 3\alpha_b^2\gamma_b\alpha_s^2+3\alpha_b^2\gamma_b\alpha_s\gamma_s+3\alpha_b\gamma_b^2\alpha_s^2+3\alpha_b\gamma_b^2\alpha_s\gamma_s+2\alpha_b^2t_b\gamma_st_s+\gamma_bt_bt_s \\ & \left(4\gamma_b\alpha_s+6\alpha_b\gamma_s+4\gamma_b\gamma_s+\left(-8\gamma_b\alpha_s-8\alpha_b\gamma_s-8\alpha_b\alpha_s-8\gamma_b\gamma_s+8t_bt_s\right)\right). \end{aligned} \tag{B19}$$

Under Assumption 4, $-8\gamma_b\alpha_s-8\alpha_b\gamma_s-8\alpha_b\alpha_s-8\gamma_b\gamma_s+8t_bt_s>0$, so $f_3>0$. Then denote f_4 the second bracket of numerator, with

$$\begin{aligned} f_4 =& 3\alpha_b\alpha_s\left(3\alpha_b\gamma_b+2\alpha_b^2+\gamma_b^2\right)\left(\alpha_s+\gamma_s\right)+8t_b^2t_s^2\left(2\alpha_b+\gamma_b\right)-2t_bt_s\left(5\alpha_b^2\left(2\alpha_s+\gamma_s\right)+\alpha_b\gamma_b\left(10\alpha_s+7\gamma_s\right)+2\gamma_b^2\left(\alpha_s+\gamma_s\right)\right) \\ =& 6\alpha_b^3\alpha_s\gamma_s+9\alpha_b^2\gamma_b\alpha_s^2+9\alpha_b^2\gamma_b\alpha_s\gamma_s+3\alpha_b\gamma_b^2\alpha_s^2+3\alpha_b\gamma_b^2\alpha_s\gamma_s+6\alpha_b^3\alpha_s^2-10\alpha_b^2t_b\gamma_st_s \\ & -20\alpha_b\gamma_bt_b\alpha_st_s-14\alpha_b\gamma_bt_b\gamma_st_s-4\gamma_b^2t_b\alpha_st_s-20\alpha_b^2t_b\alpha_st_s+16\alpha_bt_b^2t_s^2+8\gamma_bt_b^2t_s^2-4\gamma_b^2t_b\gamma_st_s. \end{aligned} \tag{B20}$$

The derivative of f_4 with respect to t_b is given by

$$\begin{aligned} \frac{\partial f_4}{\partial t_b} =& -4\gamma_b^2\alpha_st_s-20\alpha_b\gamma_b\alpha_st_s-10\alpha_b^2\gamma_st_s-14\alpha_b\gamma_b\gamma_st_s+32\alpha_bt_bt_s^2-20\alpha_b^2\alpha_st_s+16\gamma_bt_bt_s^2-4\gamma_b^2\gamma_st_s \\ =& \gamma_bt_s\left(-4\gamma_b\alpha_s-4\gamma_b\gamma_s+16t_bt_s\right)+\alpha_bt_s\left(-20\gamma_b\alpha_s-10\alpha_b\gamma_s-20\alpha_b\alpha_s-14\gamma_b\gamma_s+32t_bt_s\right) \\ =& \gamma_bt_s\left(12\gamma_b\alpha_s+12\gamma_b\gamma_s+\left(-16\gamma_b\alpha_s-16\gamma_b\gamma_s+16t_bt_s\right)\right)+ \\ & \alpha_bt_s\left(12\gamma_b\alpha_s+22\alpha_b\gamma_s+12\alpha_b\alpha_s+18\gamma_b\gamma_s+\left(-32\gamma_b\alpha_s-32\alpha_b\gamma_s-32\alpha_b\alpha_s-32\gamma_b\gamma_s+32t_bt_s\right)\right). \end{aligned} \tag{B21}$$

Under Assumption 4, $-32\gamma_b\alpha_s-32\alpha_b\gamma_s-32\alpha_b\alpha_s-32\gamma_b\gamma_s+32t_bt_s>0$ and $-16\gamma_b\alpha_s-16\gamma_b\gamma_s+16t_bt_s>0$ (since $\alpha_b>0,\alpha_s>0,\gamma_b>0,\gamma_s>0,t_b>0$ and $t_s>0$)). Thus, $\frac{\partial f_4}{\partial t_b}>0$, which shows that f_4 is increasing with respect to t_b. As a result, we have

$$f_4>f_4|_{t_b=0}=3\alpha_b\left(\gamma_s+1\right)\left(4\alpha_b+3\gamma_b\right)+3\left(\gamma_s+1\right)\left(3\alpha_b\gamma_b+2\alpha_b^2+\gamma_b^2\right)>0. \tag{B22}$$

Since $f_3>0$ and $f_4>0$, we have $\Delta\overline{SS}>0$.

Finally, the difference between the equilibrium total surplus in the MMC case and the no MMC case is given by

$$\begin{aligned} \Delta\overline{W} \equiv& \overline{W}^*-\overline{W}^{**} \\ =& 2v+\frac{\begin{pmatrix} -16\left(t_bt_s\right)^3+\left(\alpha_b+\gamma_b\right)^3\left(\alpha_s+\gamma_s\right)^2\left(5\left(\alpha_b+\gamma_b\right)+6\left(\alpha_s+\gamma_s\right)\right) \\ +4\left(t_bt_s\right)^2\left(\alpha_b+\gamma_b\right)\left(3\left(\alpha_b+\gamma_b\right)+10\left(\alpha_s+\gamma_s\right)\right) \\ -\left(t_bt_s\right)\left(\alpha_b+\gamma_b\right)^2\left(\alpha_s+\gamma_s\right)\left(16\left(\alpha_b+\gamma_b\right)+29\left(\alpha_s+\gamma_s\right)\right) \end{pmatrix}}{2t_s\left(4t_bt_s-3\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)\right)^2} \\ & -2v-\frac{\begin{pmatrix} -16\left(t_bt_s\right)^3+\alpha_b^3\alpha_s^2\left(5\alpha_b+6\alpha_s\right)+4\left(t_bt_s\right)^2\alpha_b\left(3\alpha_b+10\alpha_s\right) \\ -\left(t_bt_s\right)\left(\alpha_b\right)^2\alpha_s\left(16\alpha_b+29\alpha_s\right) \end{pmatrix}}{2t_s\left(4t_bt_s-3\alpha_b\alpha_s\right)^2} \\ =& \frac{\begin{pmatrix} \left(\alpha_b+\gamma_b\right)^3\left(\alpha_s+\gamma_s\right)^2\left(5\alpha_b+5\gamma_b+6\alpha_s+6\gamma_s\right)+4t_b^2t_s^2\left(\alpha_b+\gamma_b\right)\left(3\alpha_b+3\gamma_b+10\alpha_s+10\gamma_s\right) \\ -t_bt_s\left(\alpha_b+\gamma_b\right)^2\left(\alpha_s+\gamma_s\right)\left(16\alpha_b+16\gamma_b+29\alpha_s+29\gamma_s\right)-16t_b^3t_s^3 \end{pmatrix}}{2t_s\left(4t_bt_s-3\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)\right)^2} \\ & -\frac{1}{18}\left(\frac{5\alpha_b^2}{t_s}-\frac{4\alpha_b^2t_b^2t_s}{\left(4t_bt_s-3\alpha_b\alpha_s\right)^2}+\frac{6\alpha_b\alpha_s}{t_s}+\frac{8t_b\left(\alpha_b^2+2t_bt_s\right)}{4t_bt_s-3\alpha_b\alpha_s}-13t_b\right). \end{aligned} \tag{B23}$$

Let us introduce f_5, with

$$f_5=\frac{\begin{matrix} \left(\alpha_b+\gamma_b\right)^3\left(\alpha_s+\gamma_s\right)^2\left(5\alpha_b+5\gamma_b+6\alpha_s+6\gamma_s\right)+4t_b^2t_s^2\left(\alpha_b+\gamma_b\right)\left(3\alpha_b+3\gamma_b+10\alpha_s+10\gamma_s\right) \\ -t_bt_s\left(\alpha_b+\gamma_b\right)^2\left(\alpha_s+\gamma_s\right)\left(16\alpha_b+16\gamma_b+29\alpha_s+29\gamma_s\right)-16t_b^3t_s^3 \end{matrix}}{2t_s\left(4t_bt_s-3\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)\right)^2}.$$

The derivative of f_5 with respect to γ_b is given by

$$\frac{\partial f_5}{\partial \gamma_b} =\frac{1}{9}\left(\begin{array}{l} -\frac{16t_b^3t_s^2(\alpha_b+\gamma_b)}{(4t_bt_s-3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))^3}+\frac{16t_b^2t_s\left((\alpha_b+\gamma_b)^2+2t_bt_s\right)}{(\alpha_b+\gamma_b)(4t_bt_s-3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s))^2} \\ +\frac{4t_b\left(2t_bt_s-(\alpha_b+\gamma_b)^2\right)}{(\alpha_b+\gamma_b)(3(\alpha_b+\gamma_b)(\alpha_s+\gamma_s)-4t_bt_s)}+\frac{5\alpha_b+5\gamma_b+3\alpha_s}{t_s}+\frac{3\gamma_s}{t_s}\end{array}\right) \tag{B24}$$

Since $\alpha_b > 0, \alpha_s > 0, \gamma_b > 0, \gamma_s > 0, t_b > 0$ and $t_s > 0$), we have $\frac{5\alpha_b+5\gamma_b+3\alpha_s}{t_s}+\frac{3\gamma_s}{t_s} > 0$ and we can focus on the remaining terms

$$\begin{aligned} f_6 =& -\frac{16t_b^3t_s^2\left(\alpha_b+\gamma_b\right)}{\left(4t_bt_s-3\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)\right)^3}+\frac{16t_b^2t_s\left(\left(\alpha_b+\gamma_b\right)^2+2t_bt_s\right)}{\left(\alpha_b+\gamma_b\right)\left(4t_bt_s-3\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)\right)^2} \\ &+\frac{4t_b\left(2t_bt_s-\left(\alpha_b+\gamma_b\right)^2\right)}{\left(\alpha_b+\gamma_b\right)\left(3\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)-4t_bt_s\right)} \\ =&\frac{4t_b\left(\begin{array}{l} 9\alpha_b^3\gamma_s^2+18\alpha_b^3\alpha_s\gamma_s+27\alpha_b^2\gamma_b\gamma_s^2+27\alpha_b^2\gamma_b\alpha_s^2+54\alpha_b^2\gamma_b\alpha_s\gamma_s+27\alpha_b\gamma_b^2\alpha_s^2+27\alpha_b\gamma_b^2\gamma_s^2 \\ +54\alpha_b\gamma_b^2\alpha_s\gamma_s+9\gamma_b^3\alpha_s^2+18\gamma_b^3\alpha_s\gamma_s+9\alpha_b^3\alpha_s^2+9\gamma_b^3\gamma_s^2-36\alpha_b^2t_b\gamma_st_s-18\alpha_bt_b\gamma_s^2t_s \\ -72\alpha_b\gamma_bt_b\alpha_st_s-36\alpha_bt_b\alpha_s\gamma_st_s-72\alpha_b\gamma_bt_b\gamma_st_s-36\gamma_b^2t_b\alpha_st_s-18\gamma_bt_b\alpha_s^2t_s-36\gamma_bt_b\alpha_s\gamma_st_s \\ -36\alpha_b^2t_b\alpha_st_s+28\alpha_bt_b^2t_s^2-18\alpha_bt_b\alpha_s^2t_s+24t_b^2\alpha_st_s^2-18\gamma_bt_b\gamma_s^2t_s+28\gamma_bt_b^2t_s^2+24t_b^2\gamma_st_s^2-36\gamma_b^2t_b\gamma_st_s. \end{array}\right)}{\left(-3\gamma_b\alpha_s-3\alpha_b\gamma_s-3\alpha_b\alpha_s-3\gamma_b\gamma_s+4t_bt_s\right)^3}. \end{aligned} \tag{B25}$$

Under Assumption 4, the denominator of this latter expression is positive, so the sign of f_6 depends on the sign of the term in the bracket of the numerator. Let us define f_7 so that

$$\begin{aligned} f_7 =& 9\alpha_b^3\gamma_s^2+18\alpha_b^3\alpha_s\gamma_s+27\alpha_b^2\gamma_b\gamma_s^2+27\alpha_b^2\gamma_b\alpha_s^2+54\alpha_b^2\gamma_b\alpha_s\gamma_s+27\alpha_b\gamma_b^2\alpha_s^2+27\alpha_b\gamma_b^2\gamma_s^2 \\ &+54\alpha_b\gamma_b^2\alpha_s\gamma_s+9\gamma_b^3\alpha_s^2+18\gamma_b^3\alpha_s\gamma_s+9\alpha_b^3\alpha_s^2+9\gamma_b^3\gamma_s^2-36\alpha_b^2t_b\gamma_st_s-18\alpha_bt_b\gamma_s^2t_s-72\alpha_b\gamma_bt_b\alpha_st_s \\ &-36\alpha_bt_b\alpha_s\gamma_st_s-72\alpha_b\gamma_bt_b\gamma_st_s-36\gamma_b^2t_b\alpha_st_s-18\gamma_bt_b\alpha_s^2t_s-36\gamma_bt_b\alpha_s\gamma_st_s-36\alpha_b^2t_b\alpha_st_s+28\alpha_bt_b^2t_s^2 \\ &-18\alpha_bt_b\alpha_s^2t_s+24t_b^2\alpha_st_s^2-18\gamma_bt_b\gamma_s^2t_s+28\gamma_bt_b^2t_s^2+24t_b^2\gamma_st_s^2-36\gamma_b^2t_b\gamma_st_s. \end{aligned} \tag{B26}$$

The derivative of f_7 with respect to t_b is

$$\begin{aligned} \frac{\partial f_7}{\partial t_b} =& 2t_s\left(\begin{array}{l} -18\alpha_b^2\gamma_s-9\alpha_b\gamma_s^2-36\alpha_b\gamma_b\alpha_s-18\alpha_b\alpha_s\gamma_s-36\alpha_b\gamma_b\gamma_s-18\gamma_b^2\alpha_s-9\gamma_b\alpha_s^2 \\ -18\gamma_b\alpha_s\gamma_s-18\alpha_b^2\alpha_s-9\alpha_b\alpha_s^2-9\gamma_b\gamma_s^2-18\gamma_b^2\gamma_s+28\alpha_bt_bt_s+24t_b\alpha_st_s+28\gamma_bt_bt_s+24t_b\gamma_st_s \end{array}\right) \\ =& 2t_s\left(\begin{array}{l} 8\alpha_b\alpha_s\gamma_s+\gamma_s\left(-9\alpha_b\gamma_s-18\alpha_b\alpha_s-9\gamma_b\gamma_s+24t_bt_s\right)+\gamma_b\left(-18\gamma_b\alpha_s-8\alpha_b\gamma_s-8\alpha_b\alpha_s-18\gamma_b\gamma_s+28t_bt_s\right) \\ +\alpha_s\left(-9\gamma_b\alpha_s-8\alpha_b\gamma_s-9\alpha_b\alpha_s-18\gamma_b\gamma_s+24t_bt_s\right)+\alpha_b\left(-28\gamma_b\alpha_s-18\alpha_b\gamma_s-18\alpha_b\alpha_s-28\gamma_b\gamma_s+28t_bt_s\right) \end{array}\right). \end{aligned} \tag{B27}$$

From Assumption 4, we have $-28\gamma_b\alpha_s-18\alpha_b\gamma_s-18\alpha_b\alpha_s-28\gamma_b\gamma_s+28t_bt_s > 0$, $-9\gamma_b\alpha_s-8\alpha_b\gamma_s-9\alpha_b\alpha_s-18\gamma_b\gamma_s+24t_bt_s > 0$, $-18\gamma_b\alpha_s-8\alpha_b\gamma_s-8\alpha_b\alpha_s-18\gamma_b\gamma_s+28t_bt_s > 0$, and $-9\alpha_b\gamma_s-18\alpha_b\alpha_s-9\gamma_b\gamma_s+24t_bt_s > 0$. As a result, we have $\frac{\partial f_7}{\partial t_b} > 0$, suggesting that f_7 is increasing with t_b. Therefore, we have

$$f_7 > f_7|_{t_b=0} = 9\left(\alpha_b+\gamma_b\right)^3\left(\alpha_s+\gamma_s\right)^2 > 0. \tag{B28}$$

With $f_7 > 0$, we have $f_6 > 0$, then $\frac{\partial f_5}{\partial \gamma_b} > 0$, suggesting that f_5 is increasing with γ_b. The derivative of f_5 with respect to γ_s is given by

$$\frac{\partial f_5}{\partial \gamma_s} =\frac{1}{3}\left(\alpha_b+\gamma_b\right)\left(-\frac{4t_b^2t_s\left(\alpha_b+\gamma_b\right)^2}{\left(4t_bt_s-3\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)\right)^3}+\frac{4t_b\left(\left(\alpha_b+\gamma_b\right)^2+2t_bt_s\right)}{\left(4t_bt_s-3\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)\right)^2}+\frac{1}{t_s}\right). \tag{B29}$$

Since $\frac{1}{3}\left(\alpha_b+\gamma_b\right) > 0$ and $\frac{1}{t_s} > 0$, we focus on the remaining terms:

$$\begin{aligned} f_8 =& \frac{4t_b\left(\left(\alpha_b+\gamma_b\right)^2+2t_bt_s\right)}{\left(4t_bt_s-3\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)\right)^2}-\frac{4t_b^2t_s\left(\alpha_b+\gamma_b\right)^2}{\left(4t_bt_s-3\left(\alpha_b+\gamma_b\right)\left(\alpha_s+\gamma_s\right)\right)^3} \\ =& \frac{4t_b\left(\begin{array}{l} -3\alpha_b^3\gamma_s-9\alpha_b^2\gamma_b\alpha_s-9\alpha_b^2\gamma_b\gamma_s-9\alpha_b\gamma_b^2\alpha_s-9\alpha_b\gamma_b^2\gamma_s-3\gamma_b^3\alpha_s-3\alpha_b^3\alpha_s-3\gamma_b^3\gamma_s \\ +6\alpha_b\gamma_bt_bt_s-6\alpha_bt_b\gamma_st_s-6\gamma_bt_b\alpha_st_s+3\alpha_b^2t_bt_s-6\alpha_bt_b\alpha_st_s+3\gamma_b^2t_bt_s-6\gamma_bt_b\gamma_st_s+8t_b^2t_s^2 \end{array}\right)}{\left(-3\gamma_b\alpha_s-3\alpha_b\gamma_s-3\alpha_b\alpha_s-3\gamma_b\gamma_s+4t_bt_s\right)^3}. \end{aligned} \tag{B30}$$

From Assumption 4, the denominator of the expression above is positive. Therefore, the sign of f_8 depends on that of its numerator. Let us introduce f_9, with

$$\begin{aligned}
f_9 =& -3\alpha_b^3\gamma_s - 9\alpha_b^2\gamma_b\alpha_s - 9\alpha_b^2\gamma_b\gamma_s - 9\alpha_b\gamma_b^2\alpha_s - 9\alpha_b\gamma_b^2\gamma_s - 3\gamma_b^3\alpha_s - 3\alpha_b^3\alpha_s - 3\gamma_b^3\gamma_s \\
&+ 6\alpha_b\gamma_b t_b t_s - 6\alpha_b t_b\gamma_s t_s - 6\gamma_b t_b\alpha_s t_s + 3\alpha_b^2 t_b t_s - 6\alpha_b t_b\alpha_s t_s + 3\gamma_b^2 t_b t_s - 6\gamma_b t_b\gamma_s t_s + 8t_b^2 t_s^2 \\
=&\alpha_b^2\left(-3\gamma_b\alpha_s - 3\alpha_b\gamma_s - 3\alpha_b\alpha_s - 3\gamma_b\gamma_s + 3t_b t_s\right) + \alpha_b\gamma_b\left(-6\gamma_b\alpha_s - 6\alpha_b\gamma_s - 6\alpha_b\alpha_s - 6\gamma_b\gamma_s + 6t_b t_s\right) \\
&+ \gamma_b^2\left(-3\gamma_b\alpha_s - 3\alpha_b\gamma_s - 3\alpha_b\alpha_s - 3\gamma_b\gamma_s + 3t_b t_s\right) + t_b t_s\left(-6\gamma_b\alpha_s - 6\alpha_b\gamma_s - 6\alpha_b\alpha_s - 6\gamma_b\gamma_s + 8t_b t_s\right).
\end{aligned} \tag{B31}$$

Under Assumption 4, we have $-6\gamma_b\alpha_s - 6\alpha_b\gamma_s - 6\alpha_b\alpha_s - 6\gamma_b\gamma_s + 8t_b t_s$, $-3\gamma_b\alpha_s - 3\alpha_b\gamma_s - 3\alpha_b\alpha_s - 3\gamma_b\gamma_s + 3t_b t_s$, and $-6\gamma_b\alpha_s - 6\alpha_b\gamma_s - 6\alpha_b\alpha_s - 6\gamma_b\gamma_s + 6t_b t_s > 0$. This shows that $f_9 > 0$, then $f_8 > 0$. As a result, $\frac{\partial f_5}{\partial \gamma_s} > 0$, which shows that f_5 is increasing with γ_s. To sum up, $\frac{\partial f_5}{\partial \gamma_b} > 0$ and $\frac{\partial f_5}{\partial \gamma_s} > 0$. It evidences that f_5 is increasing with both γ_b and γ_s. Consequently,

$$\Delta\overline{W} > \Delta\overline{W}|_{(\gamma_b=0,\gamma_s=0)} = 0. \tag{B32}$$

, which completes the proof of Proposition 6.

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