Towards a Demand for Money Measurement ? Application to the German hyperinflation of the early 1920s

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Application to the German hyperinflation of the early 1920s

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Abstract - An accounting measure of the demand for money is deduced from the Allais' "*Fundamental Equation of Monetary Dynamics".* Data from German hyperinflation in the early 1920s illustrate the method we propose. The spread between money supply and money demand is found to be rather moderate but is not white noise. Our approach can be applied to any country and over any period, provided that the aggregate expenditure can be approximated using available data. This new way can help improve the estimation of the money demand function while avoiding arbitrary assumptions about the dynamics of the spread between money supply and money demand.

***JEL Classification* :** C51, E41
***Keywords* :** demand for money, measure

1. Introduction

Because the demand for money is an unobservable macroeconomic variable, ad hoc assumptions about the dynamics of the spread between money supply and money demand are made in the economic literature to implement any econometric model of money demand. Authors assume that this spread is white noise or that the so-called "short-term" demand for money (i.e., the demand for money) equals the money supply and adjusts gradually to the so-called "long-term" demand for money represented by the money demand function to be estimated.[1] In this last approach, the supposed equality between supply and demand for money is all the more questionable since the spread between these two aggregates plays in principle a

[1] Prat (1990) discusses the various processes assumed in the literature, which take the form of an adaptive or an error correction model where money supply tends to adjusts to the target represented by the money demand function.

major role in the dynamics of aggregate expenditure. To avoid making such hypotheses, the present paper aims to propose a way for measuring (not explaining) the demand for money in the economy, allowing for a more direct and reliable estimation of the money demand function.

In several publications from 1953 to 1956, Allais introduced an equation establishing a relationship between change in aggregate expenditure and the spread between supply and demand for money.[2] In the textbook of his lecture at the *Ecole des Mines de Paris*, Allais (1968) established a much more complete version of his equation called the "*Fundamental Equation of Monetary Dynamics*" (FEMD), which remains largely unknown. In the present paper, we employ this version of the FEMD to suggest a method for measuring the money demand in the economy, what Allais did not undertake. As a first attempt, our approach is illustrated using data from the German hyperinflation during the early 1920s.

2. The Allais' FEMD

The FEMD is based on three hypotheses **:**

- H1 **:** at time *t*, economic agents receive the receipts from their activities of the last period $T1(t)$.

- H2 **:** at the same time *t*, economic agents decide the amount of their expenditures during the next period $T2(t)$.[3]

- H3 **:** $T1(t) = T2(t) = T(t)$. The equality between durations $T1(t)$ and $T2(t)$ allows to make comparable at t the amount of revenue received from activities of the period [t, t- $T1(t)$] with the amount of expenditures incurred over the period [t, t+ $T2(t)$].

[2] See especially Allais (1954).
[3] It is implicitly assumed that, during the time span [t, t+ $T2(t)$], nothing will disturb the aggregate expenditure incurred at *t*. This is all the more credible since the period $T2(t)$ generally short, in principle not exceeding one month. Moreover, at the aggregate level, disturbances in one direction for some agents are compensated by those in the opposite direction for other agents.

The period $T(t)$ "*represents a reaction time*" (Allais, 1986, note 11, p. 63), that is a holding period of money which generally varies over time since it depends notably on changes in the purchasing power of money and in liquidity constraints. At time *t*, let $D(t,\ t+T(t))$ be the aggregate expenditure incurred for the next period, $R(t,\ t-T(t))$ the aggregate receipts resulting from economic activities of the previous period, $[M(t)-M_d(t)]$ the spread between money supply $M(t)$ and money demand $M_d(t)$, and $[M(t+T(t)/2)-M(t-T(t)/2)]$ the change in money supply.[4] The variables $D(t,t+T(t))$ and $R(t,t-T(t))$ include all transactions giving rise to monetary payments, and only this type of transaction, so that non-bank debt is out of our scope of analysis.[5] Monetary payments are related to final production (distributed income, purchases of goods and services), but also to transactions issued directly from the production process (intermediate goods), from the second-hand market and from the financial market. The aggregate expenditure over the next period can be straightforwardly expressed by the following accounting relationship :

$$D(t,t+T(t)) = R(t,t-T(t)) + [M(t)-M_d(t)] + [M(t+T(t)/2) - M(t-T(t)/2)] \quad (1)$$

Eq(1) means that, at time *t*, the aggregate expenditure for the following period equals the revenue generated from economic activities at the previous period, plus the spread between money supply and money demand, plus the change in money supply. It seems indeed obvious that excess in cash balances ($M(t) > M_d(t)$) or shortage in cash balances ($M(t) < M_d(t)$) lead to increase or decrease the aggregate expenditure, respectively. Regarding the change in the money supply (notably generated by bank credit), the new money created (destroyed) also contributes to financing (restricting) the expenditure incurred at the time when this money is created (destroyed). It should be noted that, if the change in money supply were omitted in Eq(1), the FEMD would lead to inconsistency. Indeed, one can easily show that, during a dynamic equilibrium regime, the aggregate expenditure growth rate would be zero whatever the value of the money supply growth rate.

[4] Consistently with hypotheses made on receipts and expenditure, the change of $M(t)$ is centered at time *t* over the time span $T(t)$.

[5] The non-bank debt is the amount of credit that non-financial agents grant to each other (credits from producers to consumers and from suppliers to producers). As proposed by Allais, with a broader definition of aggregate expenditure, change in the non-bank debt should be added to the right hand side of Eq(1). However, according to Allais (1968, p.80), change in non-bank debt is negligible compared to the amount of debt from the creation of money by banks.

Since any receipts of an agent constitutes an expenditure for another agent, the aggregate expenditure equals at any time the aggregate receipts, thus we have $R(t, t - T(t)) = D(t, t - T(t))$.[6] Reporting this equality into Eq(1) and using the following approximations[7]

$$D(t, t + T(t)) = \int_{\tau=t}^{t+T(t)} D(\tau)\, d\tau \approx T(t). D(t + T(t)/2)$$

and

$$D(t, t - T(t)) = \int_{\tau=t-T(t)}^{t} D(\tau)\, d\tau \approx T(t). D(t - T(t)/2)$$

we get the following relation :

$$T(t)\ [D(t + T(t)/2)] - D(t - T(t)/2)] = [M(t) - M_d(t)] + [M(t + T(t)/2 - M(t - T(t)/2)] \quad (2)$$

Taylor series developments allow to write

$$D(t + T(t)/2) = D(t) + T(t)/2\ \ dD/dt(t) + T(t)^2/8\ \ d^2D/d\,t^2(t) + \cdots$$

$$D(t - T(t)/2) = D(t) - T(t)/2\ \ dD/dt(t) + T(t)^2/8\ \ d^2D/d\,t^2(t) + \cdots$$

and similar expressions hold for $M(t + T(t)/2)$ and $M(t - T(t)/2)$.

Supposing $T(t)$ to be small enough, the terms of order two or more can be neglected and we admit that

$$[D(t + T(t)/2)] - D(t - T(t)/2)] = T(t)\ \ dD/dt(t) \quad (3)$$

$$[M(t + T(t)/2) - M(t - T(t)/2)] = T(t)\ \ dM/dt(t) \quad (4)$$

Reporting Eq(3) and Eq(4) in Eq(2), we get the expression of the change in the aggregate expenditure :

$$dD/dt\,(t) = 1/T(t)^2\ [\,M(t) - M_d(t)] + 1/T(t)\ \ dM/dt(t) \quad (5)$$

Dividing all terms of Eq(5) by $D(t) = M(t)\,V(t)$ where $V(t)$ is the velocity of circulation of money, the Allais' FEMD allows writing the rate of change in the aggregate expenditure as

[6] This equality is directly intuitive when the money supply is constant (Allais 1968, p.77, note 319-3). However, it must be demonstrated when money supply is unstable, as has been done by Allais (1968, pp. 84-86).
[7] These approximations are of course all the more accurate than the spending period $T(t)$ is small.

$$\frac{1}{D}\frac{dD}{dt}(t) = \frac{1}{V(t)\,T(t)^2}\,\frac{M(t)-M_d(t)}{M(t)} + \frac{1}{V(t)\,T(t)}\,\frac{1}{M}\frac{dM}{dt}(t) \tag{6}$$

3. The measurement equation of money demand

For a given value of the money demand, the strength of the FEMD is to be an accounting relation.[8] Since $M_d(t)$ is the only unobservable variable, it is possible to assess its implicit value satisfying Eq(6). Setting for convenience $x(t) = \frac{1}{D}\frac{dD}{dt}(t)$, $m(t) = \frac{1}{M}\frac{dM}{dt}(t)$ and $log\, M(t)/M_d(t) \approx \frac{M(t)-M_d(t)}{M(t)}$, one can infer from Eq(6) the implicit value of the relative spread between supply and demand for money

$$log\, M(t)/M_d(t) = V(t)\,T(t)^2\,x(t) - T(t)\,m(t) \tag{7}$$

from which we can straightforwardly write the log-demand for money as

$$logM_d(t) = logM(t) - V(t)\,T(t)^2 x(t) + T(t)\,m(t) \tag{8}$$

Stability of the reaction time $T(t)$ would be a relevant assumption only during a period of stability of the velocity of money. In any case, such an assumption is implausible in times of turbulence. This is especially the case during a period of accelerated inflation when agents wish to quickly get rid of their depreciated cash balances (flight from money). Indeed, in such a situation, agents tend to shorten the duration of their money holdings - that is the length of the spending period $T(t)$ - which mechanically brings up the frequency with which money changes hands, this frequency representing the velocity of money. This is why Allais posits that the

[8] Of course, through the money demand, the FEMD captures expectations and uncertainty that underlie agents' decision-making regarding their future spending. For example, if at time *t*, uncertainty about their future receipts at $t+T(t)$ increases, agents would be prompted to increase their desired cash balances $M_d(t)$ for precautionary motive. In the same way, if at time *t* agents think that financial markets might offer them new opportunity gains over the time span $(t, t+T(t))$, they would increase their desired cash balances $M_d(t)$ in order to be able to seize investment opportunities between *t* and $t+T(t)$ (speculative motive).

duration $T(t)$ is proportional to the inverse of the velocity of money defined as $V(t) = D(t)/M(t)$.[9] Accordingly, we assume

$$T(t) = \gamma/V(t) \qquad \gamma > 0 \tag{9}$$

where γ is a structural parameter depending on payment habits in the country.[10]

Reporting Eq(9) in Eq(7), we obtain the following expression of the relative spread between money supply and money demand:[11]

$$log\, M(t)/M_d(t) = \frac{\gamma}{V(t)}(\gamma\, x(t) - m(t)) \tag{10}$$

Eq(10) leads directly to the value the money demand implicitly imbedded in the FEMD :

$$logM_d(t) = logM(t) - \frac{\gamma}{V(t)}(\gamma\, x(t) - m(t)) \tag{11}$$

By definition of undesired money held, the spread between money supply and money demand expressed by Eq(10) cannot last long, so the condition that its mean is zero is very insightful. One thus can choose γ under the condition

$$\gamma \mid \frac{1}{n}\sum_t^{t+n} \frac{\gamma\, x(t) - m(t)}{V(t)} = 0 \tag{12}$$

leading to the solution $\gamma = mean\frac{m(t)}{V(t)}/mean\frac{x(t)}{V(t)}$.

[9] See Allais (1982, pp.21 and following) and Allais (1986, footnote 11, p.63) who admits as a "*very simple and natural hypothesis*" that the theoretical value of $T(t)$ is $\hat{T}(t) = \gamma/\hat{V}(t)$ where $\hat{V}(t)$ is the theoretical velocity of circulation issued from his money demand function, with $\gamma = T_0/\emptyset_0$ where T_0 and $\emptyset_0$ are positive structural parameters depending on the country's payment habits that are supposed stable over the period considered. This directly implies the observed value of the spending period $T(t)$ to be given by Eq(9). Note that, as pointed out by Allais (1968, p.81), in a dynamic equilibrium regime where the aggregate expenditure and money supply grow at the same rate, the condition $\gamma = 1$ prevails (this can be shown by writing Eq(6) under the condition $M(t) = M_d(t)$).

[10] One can check up that time dimensions of variables in Eq(9) are consistent. Indeed, since the dimension of $V(t)$ is -1 while that of γ is 0, the dimension of $T(t) = \gamma/V(t)$ is 0-(-1) = +1, indicating a duration.

[11] Note that because the time dimension is -1 for $x(t)$, $m(t)$ and $V(t)$, the dimension of the right hand side of Eq(10) is -1-(-1)=0, which is consistent with the left hand side defining the monetary spread $log\, M(t)/M_d(t)$.

4. A case-study using data from the German hyperinflation of the early 1920s

This field of empirical application seems particularly interesting since agents' attitudes towards the money holding stand out more strikingly due to the exceptional rise in the price level.[12]

4.1 Data

Following Allais (1965), we assume that, during the German hyperinflation, $D(t) = k\,P(t)$ where $P(t)$ is the price level and k a positive constant. Indeed, the evolution of real quantities becomes truly negligible compared to the tremendous rise in the price level **:** in a context of stagnant production, the price index increased from 8 in December 1919 to 1093.10^6 in October 1923. Consequently, for measuring the spread between money supply and money demand using Eq(10), we only need the time series of price level and money supply (including fiat money and demand deposits). These series are given in Allais (1965, p.182) from December 1919 to October 1923 in the form of monthly indices in base 1913=1 and are dated in the middle of the month.[13] These two indices being denoted $P_i(t)$ and $M_i(t)$, the money velocity index expressed in base 1913=1 is thus given by the ratio $V_i(t) = P_i(t)/M_i(t)$, which is also dated at the middle of the month. Figure 1 exhibits indices $P_i(t)$, $M_i(t)$ and $V_i(t)$ over our sample period.

[12] Note that the book by Graham(1967) provides a detailed review of events that led to the German hyperinflation, including economic policies of the Weimar Republic, the impact of WWI and the role of international trade and finance.

[13] About the datations, see Cagan (1956, Appendix B, p.100) who used these data to estimate his money demand function based on inflation expectations. Data were provided directly to Allais by Cagan. Wholesale prices are considered in the index $P_i(t)$; retail prices index followed comparable trends in particular for the years 1922 and 1923 during which a parallelism can be observed in the two curves (Graham,1967, pp.97-99). Concerning the money supply $M_i(t)$, note that the index published by Cagan (1956, pp.102-103) do not include bank deposits whereas the one provided by Cagan to Allais include these deposits.

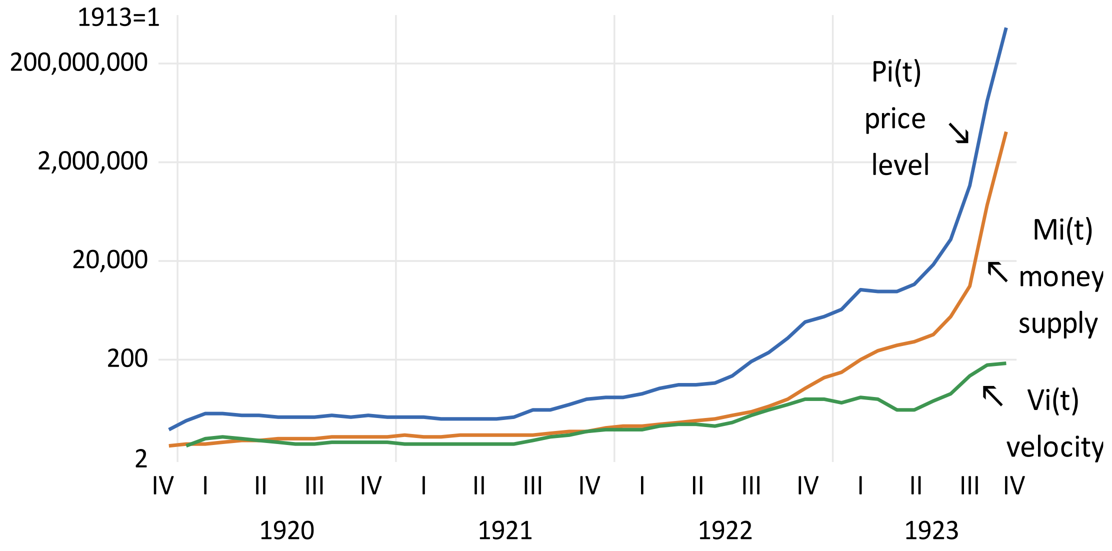

Figure 1 Price level, money supply and velocity of money
Germany, December 1919 - October 1923
(all series are indices base 1913=1)

Regarding Equation (10), and provided that $D(t) = k\,P(t)$, we have $x(t) = log\,P_i(t)/P_i(t-1)$ and $m(t) = \log M_i(t)/M_i(t-1)$, these two monthly rates of change being centered at the beginning of the month *t*. To also center the velocity index at the beginning of the month *t*, we consider the geometric average $\overline{V_i}(t) = [V_i(t).V_i(t-1)]^{1/2}$. However, in Eq10), the velocity $V(t)$ should not be expressed with index $\overline{V_i}(t)$ but as a number per month. Note that this scaling problem does not call into question the validity of determining the value of γ using Eq(11)).[14] Nevertheless, it can be seen that, according to Eq(10), although the *shape of the dynamics* of the spread between money supply and money demand is not affected by this problem of calibration, it is not the case for the *amplitude* of the spread. That is why we calibrate the velocity as $V(t) = \alpha\overline{V_i}(t)$ where α is a positive scalar.[15] Doing so, we obtain the value $\alpha = 1.38$ (see Appendix), so that the velocity in Eq(10) will be represented as $V(t) = 1.38\,\overline{V_i}(t)$.

[14] Indeed, with $V(t) = \alpha\overline{V_i}(t)$, the condition represented by Eq(11) implies that $\hat{\gamma}$ is independent of α since the solution is $\hat{\gamma} = mean\frac{m(t)}{\alpha\overline{V_i}(t)}/mean\frac{x(t)}{\alpha\overline{V_i}(t)} = mean\frac{m(t)}{\overline{V_i}(t)}/mean\frac{x(t)}{\overline{V_i}(t)}$.

[15] The velocity per month $V(t)$ is defined by the ratio between the global expenditure $D(t)$ and the money supply $M(t)$, both being expressed in monetary units. Since $D(t) = kP(t)$ while the price level and money supply are represented by indices $P_i(t)$ and $M_i(t)$, we have $\alpha = k\,b_P/b_M$ where the positive constants b_P and b_M depend on the arbitrary base 1913=1 of the two indices.

4.2 The spread between money supply and money demand

Eq(12) leads to the value $\hat{\gamma} = 0.62$. Figure 2 shows a downward trend in the estimated value of the spending period $T^*(t) = 0.62/V(t)$ during the course of hyperinflation, which clearly highlights the flight from money phenomenon. While $T^*(t) =$ 0.20 month or 6 days in January 1920, we get $T^*(t) = 0.00343$ month or 2.47 hours at the end of our period (October 16, 1923), which seems rather credible.

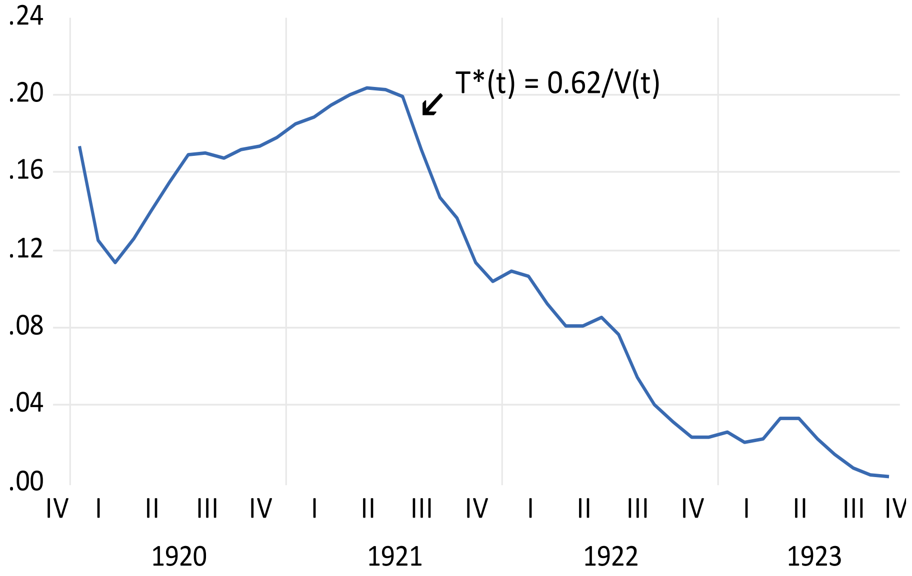

Figure 2 - Duration of the expenditure period expressed in month
Germany, January 1920 - October 1923

Replacing in Eq(10) γ by its estimated value, the empirical measure of the relative spread between $M(t)$ and $M_d(t)$ dated at the beginning of the month is

$$log\, M(t)/M_d(t) = \frac{0.62}{V(t)}(0.62\, x(t) - m(t)) \quad (13)$$

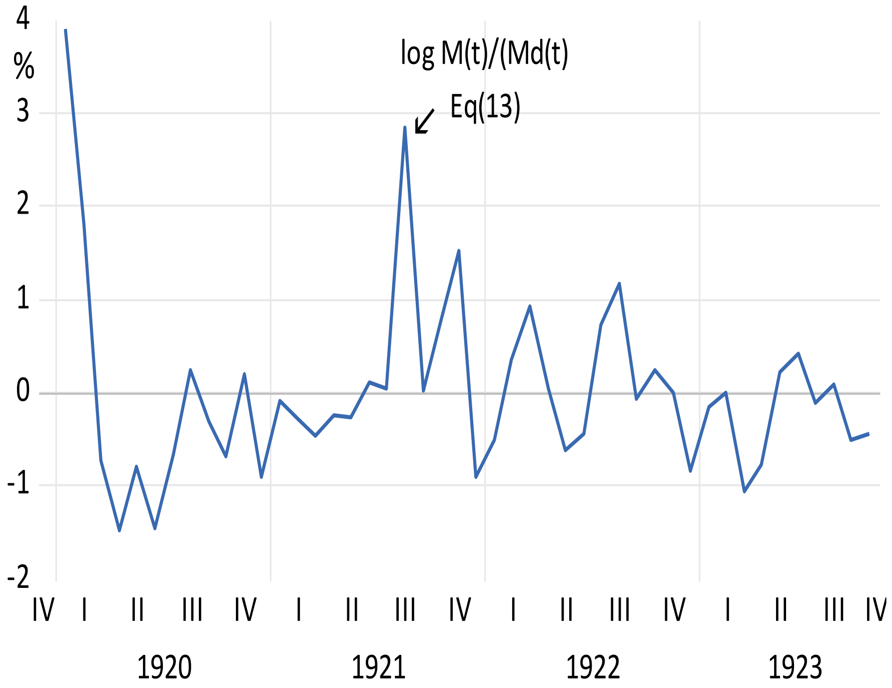

Figure 3 - Spread between money supply and money demand
Germany, january 1920 - October 1923

Figure 3 shows that the monetary spread $log\, M(t)/M_d(t)$ is characterized by a moderate amplitude with a standard deviation of 1.00%[16], a maximum of 3.90% and a minimum of -1.47%. Interestingly, using the same time series of price level and money supply over the same period, the standard deviation of residuals $log\, M(t)/M^*(t)$ of the Allais' model of money demand is 5.60% (Allais, 1965, p.113), which is significantly larger compared to the one of our monetary spread.[17] This suggests that replacing $M_d(t)$ by $M(t)$ to estimate the money demand function is indeed an acceptable approximation over this period. However, this ending is

[16] Due to approximations of the aggregate expenditure and thus of the velocity per month calibration, this standard error is of course estimated with a margin of error. But one can reasonably admit this margin to be moderate (see Appendix).

[17] $M^*(t)$ is the estimated value of $M(t)$ produced by the Allais' Hereditary, Relativist and Logistic (HRL) function of demand for money. This estimation clearly assumes the monetary spread to be *a priori* negligible since $M_d(t)$ is simply replaced by $M(t)$. In fact, $M^*(t)$ can as well be viewed as the estimated value of the money demand $M_d^*(t)$. In this view, the "true" residual issued from the money demand model can be written as $M_d(t) - M_d^*(t) = [M(t) - M_d^*(t)] - [M(t) - M_d(t)]$ where the first term of the right-hand side represents the residuals of the HRL money demand model while the second term is the monetary spread assessed by Eq(13). Residuals of the Allais' money demand model are found to be slightly negatively correlated with the monetary spread ($R = -0.30$). Consequently, the standard error of the residual $M(t) - M_d^*(t)$ (5.60 %) is found to somewhat underestimating the one of the "true" residual $M_d(t) - M_d^*(t)$ (6.15 %), meaning that the performance of the HRL demand for money model is slightly overvalued.

somewhat challenged by the fact that, although stationary, the monetary spread is not white noise since it is found to be autocorrelated, heteroskedastic, and non-normal.[18] Such statistical properties are not surprising since the spread depends on observed macroeconomic variables. These results suggest that, for estimating more accurately any money demand function, it would be relevant to consider as endogenous variable the money demand assessed by Eq(11) rather than the observed money supply. Of course, if money demand does not respond instantaneously to changes in its determinant factors, it would be justified to introduce an adjustment process of the money demand towards the money demand function.[19]

5. Conclusion

We propose an accounting measure of the demand for money in the economy using the Allais' "*Fundamental Equation of Monetary Dynamics*". As a first attempt, this implicit measure of money demand leads to credible outcomes during the German hyperinflation of the early 1920s. The spread between money supply and money demand turns out to be of rather moderate amplitude with a standard error of 1%. However, this spread is not white noise, which is likely to create a bias in the estimation of the money demand function if it is ignored or if it is assumed to be the result of a partial adjustment mechanism of money supply to the money demand function. The approach proposed in the present paper allows to avoid such a bias and can be applied to any country and period provided that an indicator of aggregate expenditure can be approximated. This is probably the main limitation for applying this method.

[18] These conclusions are at the 5% level of significance and are respectively from the Phillips-Perron Uroot stationary test (p-value=0.000, one lag and no constant), the Breusch-Godfrey serial correlation LM test (p-value=0.038 for one lag), the Arch heteroskedastic test (p-value=0.032 with one lag), and the Jarque-Bera normality test (p-value=0.000). Note that these statistical properties do not depend on our money velocity calibration.

[19] For example, $logM_d(t)$ being given by Eq(11) and assuming that the adjustment process is a simple error correction model, the equation to be estimated would be written as:

$$logM_d(t) - logM_d(t-1) = a\left[\hat{F}(.)(t-1) - logM_d(t-1)\right] + b\left[\hat{F}(.)(t) - \hat{F}(.)(t-1)\right] + \varepsilon(t)$$

$(a, b > 0)$ where $\hat{F}(.)(t)$ is the estimated log-money demand function including a set of observable variables.

Appendix. Calibration of the velocity of money

Let us write the level of velocity per month as $V(t) = \alpha V_i(t)$, where α is a positive scalar to be determined and $V_i(t)$ the velocity index (1913=1) whose values are centered at the middle of the month. Any month can be used to perform this calibration provided that $V(t)$ and $V_i(t)$ are both known. Accordingly, we assess $\alpha = V(dec\ 1919)/V_i(dec\ 1919)$ where $V(dec\ 1919)$ is the ratio between the aggregate expenditure (including all monetary transactions) and the money supply, these two variables being expressed in billions of Marks. Because there is no indicator of aggregate expenditure in Germany, α is determined in two steps. First, to the extent that, during the 1920s, Germany and US were countries with roughly comparable levels of economic development and monetary institutions within a floating exchange rate regime (from 1919 to 1926), we assume that the value of the ratio between aggregate expenditure and national income observed in US also applies to Germany. King (1930) gives statistics of bank clearings in US indicating that the amount of total monetary flows during the year 1919 was about 6 times the Gross National Product (GNP), this ratio remaining rather stable between 1920 and 1923. Second, the German GNP amounts to 296 billions of Marks in 1919 while the money supply is 50.1 billions of Marks at December 1919. Assuming the GNP for December 1919 to be roughly one-twelfth of the GNP for the whole year, one can approximate the level of velocity as $V(Dec\ 1919) = 6(296/12)/50.1 = 2.95$ per month, leading to the value $\alpha = V(Dec\ 1919)/V_i(Dec\ 1919) = 2.95/2.13 = 1.38$.

We can appreciate the sensitivity of the standard error of the spread $log\ M(t)/M_d(t)$ with respect to the value of the ratio between aggregate expenditure and GNP. For example, if this value was 7 in place of 6 (the retained value), the standard error of the spread over the period January 1920 – October 1923 would decrease from 1% to 0.9%, while if the value was 5 in place of 6, the standard error would rise from 1% to 1.2%. At any rate, it is worth noting that the value of this ratio does not affect the shape of the monetary spread dynamics.

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